Signaling

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Signaling example: Jolly Roger

(story based on Leeson, "The invisible hook: the hidden economics of pirates", Princeton University Press, 2011)

Leeson's story I

- pirates
  - completely outside the law
  - hanged when caught
  - adopted policy: victims are left alive if they surrender and are killed if they (initially) fought/fled
- commissioned ships
  - commissioned to plunder other countries' ships
  - more treated like prisoners of war (not hanged!) when caught
  - lose this status if (i) kill surrendering prisoners or (ii) use a pirate flag ("jolly roger")
- pirates rarely had to fight as most merchants surrendered upon seeing jolly roger
- commissioned ships met more opposition

Leeson's story II

- important features to make this an "equilibrium"?
  - pirates and commissioned ships have same preferences over merchants behavior
  - merchants prefer different behavior against pirates and commissioned ships
  - using jolly roger is more costly for commissioned ships

Pirate model

timing:

1. nature chooses attacker type: Pirate (P) with prob p , commissioned (C) with $1 - p$
2. attacker chooses between Jolly Roger (J) and other flag (F)
3. defender decides whether to surrender (s) or fight (f)

Payoffs:

	Pirate (J or F)	commissioned (F)	commissioned (J)
surrender	-1,3	-1,3	-1,3-X
fight	-3,1	0,1	0,1-X

Pirate analysis

	Pirate (J or F)	commissioned (F)	commissioned (J)
surrender	-1,3	-1,3	-1,3-X
fight	-3,1	0,1	0,1-X

- if $X \geq 2$, then "separating" equilibrium:
  - P play J
  - C play F
  - defenders surrender iff J is played and fights otherwise
- "pooling" equilibrium
  - P and C play F
  - defender always surrenders (fights) if $p \geq 1/3$ ($p \leq 1/3$)

Job market signaling

- worker has type $\theta \in \{\theta_l, \theta_h\}$ with $\theta_h > \theta_l > 0$
  - interpret θ as productivity/ability
- worker chooses education level e at cost $c(e, \theta)$ with - education is costly: $c_e > 0$
  - marginal costs are increasing: $c_{ee} > 0$
  - no education has no costs: $c(0, \theta) = 0$
  - higher θ (productivity/ability) implies lower marginal costs: $c_{e\theta} < 0$
- competitive market offers wage equal to expected productivity θ
- information
  - θ is worker's private information
  - market views θ (ex ante) as being θ_h with probability $\lambda \in (0, 1)$
  - market might infer θ from education choice (and therefore update initial belief λ)
- payoff worker: $w - c(e, \theta)$

2 Model interpretations

either

- 1 worker with privately known θ
- mass 1 of workers with share λ of θ_h types
- competitive market: either many homogenous firms or at least two firms in Bertrand competition on the labor market
- note: education does not change productivity in this model, i.e. education is wasteful!
  - extreme assumption to focus entirely on signaling aspect!

Equilibrium in job market signaling

- market belief $\mu(e)$:
  - the probability that $\theta = \theta_h$ if education level e is observed
- worker's strategy optimal given belief μ
- market belief $\mu(e)$ derived by Bayes rule whenever e is in the support of one of the types' strategy
- off equilibrium path choices of e:
  - some arbitrary $\mu(e) \in [0, 1]$

- note: wage with education level e, i.e. $w(e)$, equals $\mu(e)\theta_h + (1 - \mu(e))\theta_l$

Preliminaries

- θ_h has flatter indifference curve than θ_l ("single crossing")
- wage $w(e) \in [\theta_l, \theta_h]$

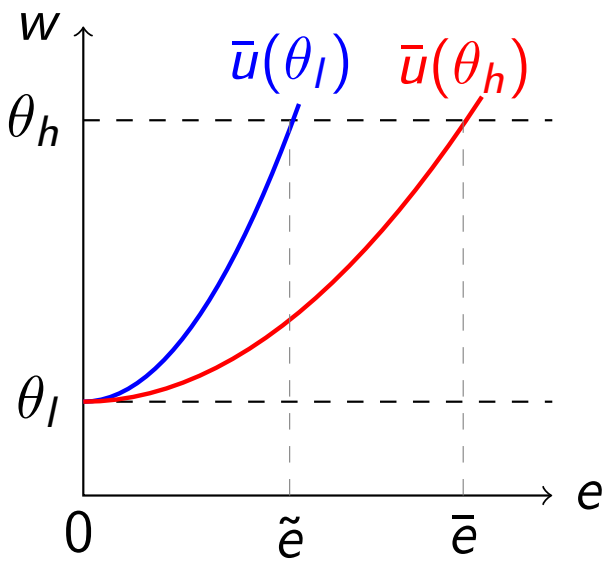
Separating equilibria I

- separating: two types have different education choices

Lemma

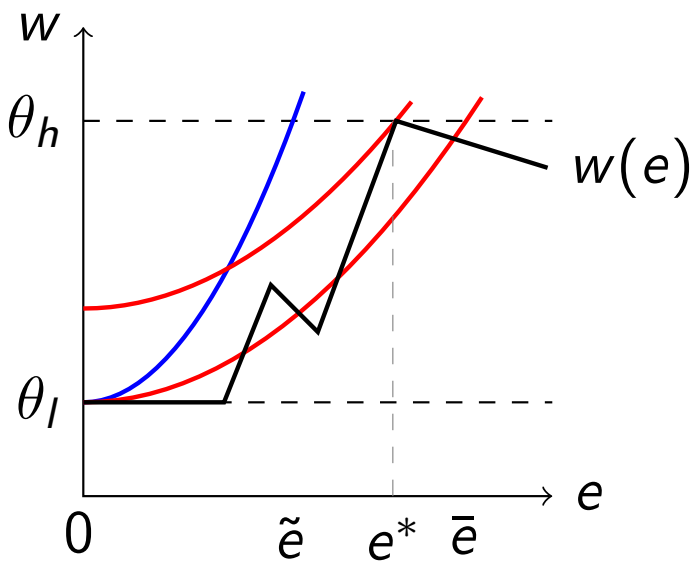
In a separating equilibrium, $w(e(\theta_l)) = \theta_l$ and $w(e(\theta_h)) = \theta_h$. Furthermore, $e(\theta_l) = 0$.

- what education levels are possible for θ_h in a separating equilibrium?



Separating equilibria II

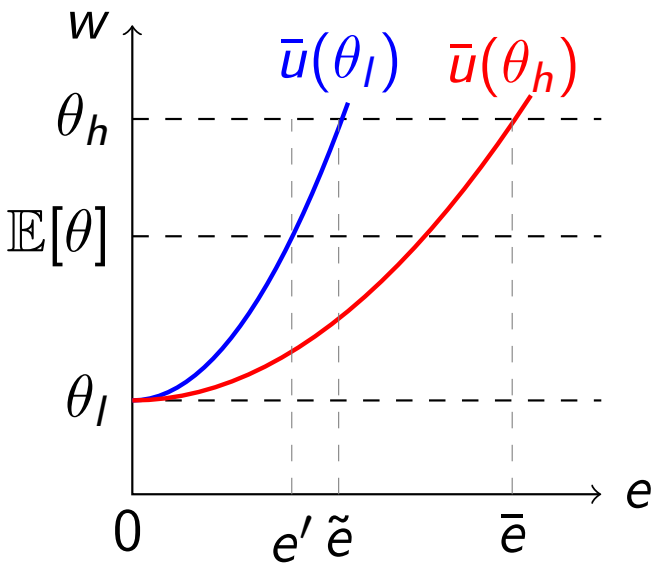
- what kind of beliefs (i.e. wage offers) sustain such an equilibrium?



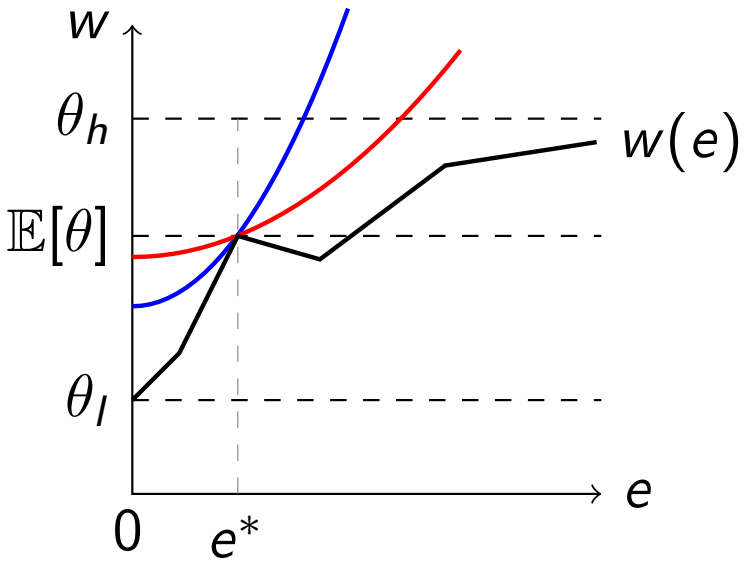
- $\mu(e) = (w(e) - \theta_l)/(\theta_h - \theta_l) \in [0, 1]$
- among separating equilibria, which are Pareto efficient?

Pooling equilibria I

- pooling: both types exert same education level e^*
- $\mu(e^*) = \lambda$ and $w(e^*) = \mathbb{E}[\theta] = \lambda\theta_h + (1-\lambda)\theta_l$
- which education levels can be sustained in a pooling equilibrium?



Pooling equilibria II

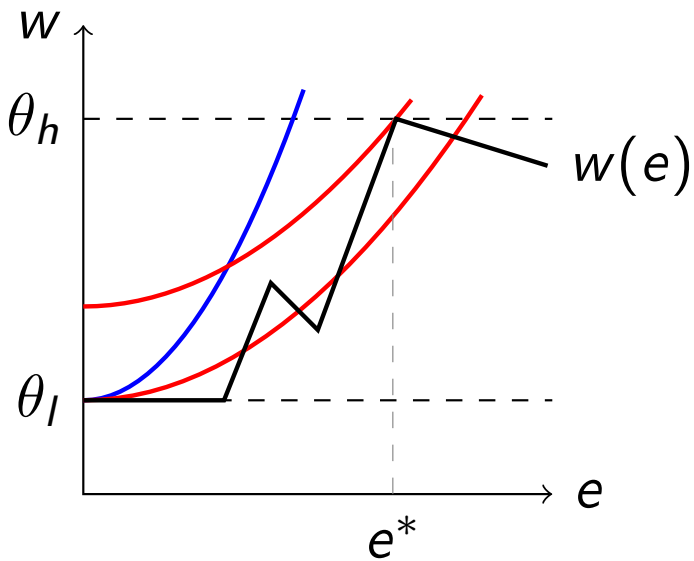


Welfare

- what is equilibrium without signaling possibility?
- who is better/worse off due to signaling possibility?

Refinements I

- multiplicity of equilibria supported by off path beliefs that can be freely chosen
- are these beliefs reasonable?



Refinements II

- let e_l be the equilibrium education of type θ_l
- let $\tilde{e}$ be the education level such that θ_l is indifferent between his equilibrium payoff and $(w, e) = (\theta_h, \tilde{e})$

$$w(e_l) - c(e_l, \theta_l) = \theta_h - c(\tilde{e}), \theta_l)$$

- equilibrium refinement: $\mu(e)$ equals 1 for all $e > \tilde{e}$

Lemma

The only equilibrium satisfying the equilibrium refinement is the "least cost separating equilibrium", i.e. the separating equilibrium in which $e(\theta_h) = \tilde{e}$.

- note: $\tilde{e}$ in separating equilibrium satisfies

$$\theta_l = \theta_h - c(\tilde{e}, \theta_l)$$

- note: $\tilde{e}$ in pooling equilibrium satisfies

$$\mathbb{E}[\theta] - c(e^*, \theta_l) = \theta_h - c(\tilde{e}, \theta_l)$$

Income tax

- assume least cost separating equilibrium
- suppose a revenue neutral income tax is introduced
  - wage of θ_l is subsidized by $s > 0$
  - wage of θ_h is taxed by amount $t = \frac{1-\lambda}{\lambda} s$

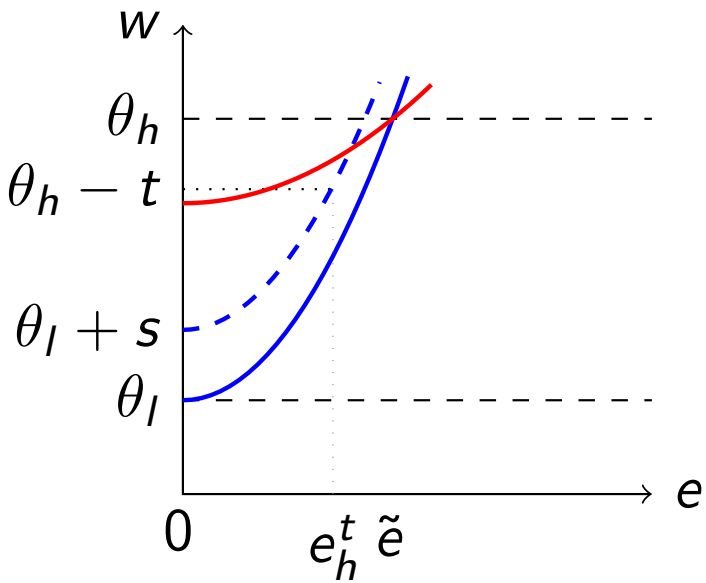


Figure: low λ

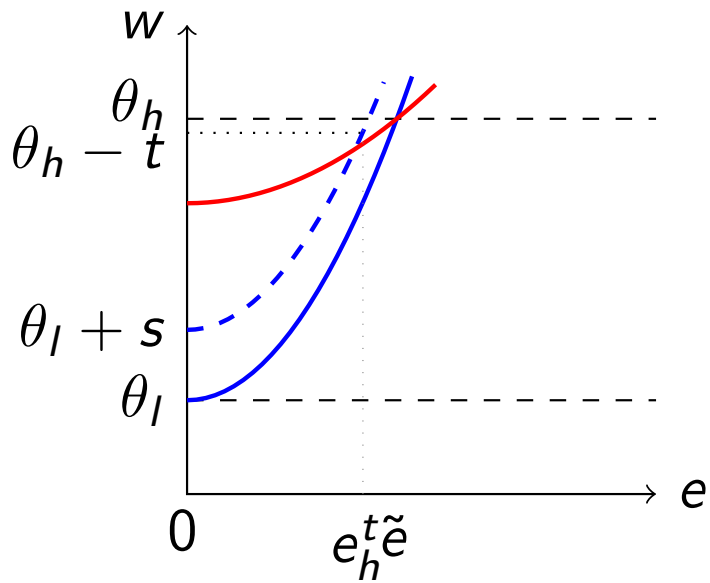


Figure: high λ

Mechanics of signaling summarized

- both types prefer higher wage (and less education)
- market is willing to pay higher wage to high types
- education is (marginally) more costly for lower type
  - wage increase $\theta_h - \theta_l$ is more than cost of education for high type
  - wage increase $\theta_h - \theta_l$ is less than cost of education for low type
- high types can signal their high type by obtaining education

Can you think of examples for signaling in practice?