Manufactured Ignorance: How Media Competition Lowers News Quality[*]

Ole Jann

CERGE-EI

Christoph Schottmüller

University of Cologne and TILEC

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Abstract

We argue that media outlets may deliberately lower the quality of their reporting to fragment society and thereby reduce competition. In our model, consumers have no biases for slanted content and value news as a source of information about the world and about what their neighbors believe. Broadcasters strategically reduce the informativeness of their reporting to make their products less interchangeable and soften price competition. This increases profits at the cost of consumer ignorance and social fragmentation. We show that increasing news quality may require limiting differentiation (e.g. through fairness regulations) or limiting some forms of competition.

1. Introduction

In an age of unprecedented access to news, trust in news media is at an all-time low.[1] Two related concerns stand out in public discourse: First, the role of news media in the fragmentation of society – for example, 74% of Americans say that it is "increasing political divisions" (AP-NORC, 2023). Second, the quality and reliability of news reporting – in the same poll, 58% blame news media for the spread of misinformation.

This makes it essential to understand whether and how competitive media markets can mitigate these developments. A classical argument is that if citizens are interested in being well-informed, "competition among media firms assures that voters and consumers obtain, on average, unbiased and accurate information" (Djankov et al., 2003, p. 342). Some studies have suggested, however, that media outlets may prefer catering to pre-existing biases or tastes rather than provide unbiased information (Gentzkow and Shapiro, 2008; Gentzkow et al., 2014; Cagé, 2020).

In this paper, we argue that competition itself can incentivize media outlets to provide low-quality reporting and thereby *create* societal fragmentation. This makes the outlets less interchangeable, which reduces price competition and increases profits. All consumers are worse off, though there can be multiple equilibria that correspond to different market outcomes. In symmetric equilibria, outlets report medium-quality content, leaving consumers equally half-informed. In asymmetric equilibria, some consumers get high-quality information while others receive low-quality content.

To make this argument, we construct a model with stylized assumptions that seemingly favor non-fragmented, high-quality reporting. In our model, two broadcasters compete for consumers whom they can charge a variable price. Broadcasters can freely and costlessly report with arbitrary precision (i.e. they can choose any information structure that maps the truth to any message space). Consumers perfectly observe each broadcaster's informativeness and freely choose which to follow. Consumers have two simple goals: First, to learn about the world (giving them an incentive to choose informative reporting). Second, to understand what other people in their neighborhood believe about the world – giving them an incentive to seek out reporting that is widely consumed. Consumers have no pre-existing political biases or demand for slant.

Our main result is that under many parameter configurations, no equilibrium exists in which both broadcasters provide accurate, high-quality reporting. To see why, consider broadcaster B when broadcaster A perfectly reports the state of the world. If B also reports perfectly, both broadcasts are interchangeable, consumers are highly price sensitive and profits low. If B instead reports less accurately, it provides a worse product and attracts fewer viewers. The remaining viewers, however, are less price-sensitive, since they

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watch B partly to learn what other B-viewers believe – a purpose for which A's reporting is now an imperfect substitute. This reduced price sensitivity lets both broadcasters charge higher prices in equilibrium.[2]

We show that symmetric equilibria (both broadcasters report with moderate noise) and asymmetric equilibria (one broadcaster reports perfectly, the other with large noise) can co-exist. We discuss these equilibria and their connection to public perceptions of media reporting and bias in section 3.2.

Our study makes three main contributions. First, we extend models of media competition by considering the social function of news – the idea that people consume news not only to stay informed, but also to communicate with others and to belong to a community. While this motivation is well-documented[3], its consequences on news choice and informativeness have been little explored.

Second, our main result identifies a mechanism by which broadcasters profit from deliberately fragmenting and misinforming the public.[4] We thus challenge conventional views that competition ensures high-quality reporting, or that polarization and misinformation stem from pre-existing demands for slanted or biased news.

Third, we use our framework to analyze policy responses. For example, fairness mandates or reporting standards may improve news quality and welfare not only by raising journalistic norms but also by increasing similarity in reporting. This intensifies competition and forces broadcasters to report with high precision, rather than create convenient niches of quasi-captive audiences. Seemingly anti-competitive policies – limiting competition, weakening the social function of news or making it harder to switch broadcasters – may also raise informativeness and welfare. While each approach has tradeoffs, a central insight is that increasing informativeness may require limiting competition and differentiation.

Connection to other research Our work is related to a large literature that studies outcomes of media competition, in particular whether competitive media markets provide citizens with accurate and unbiased information. It also connects to industrial organization studies of vertical and horizontal differentiation.

Our main departure from the literature is that we do not study "slant" or ideological bias. Many studies argue these can cause or enable horizontal differentiation, as media firms segment the market along ideological lines (cf. Anand et al., 2007, Gentzkow et al., 2014, Perego and Yuksel, 2022). Similarly, our media companies are not interested in pushing narratives or influencing opinions (cf. Anderson and McLaren, 2012, Chen and

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[2]"Higher prices" can also mean lower amenities, more advertising, or any other change that lowers consumer surplus while raising broadcaster revenue.

[3]"Talking about [the news] with family, friends and colleagues" is the most-cited reason for consuming news among US adults (Pew Research Center, 2010).

[4]It also implies that politically motivated media owners may find divisive reporting not only ideologically appealing, but also profitable.

Suen, 2023) or catering to biases Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2006. Our consumers prefer accurate reporting and have no pre-existing biases or demand for slant; broadcasters can only vary informativeness.

Empirical evidence on media competition is mixed. While Galvis et al. (2016) and Hong and Kacperczyk (2010) find that competition improves reporting quality, others (e.g. Cagé, 2020, Angelucci et al., 2024) find the opposite. Our model predicts that competition can reduce informativeness through a mechanism distinct from those in prior work.

Our key methodological innovation is modeling the social function of news – i.e., consumers' interest in what others believe. This motivation is empirically well-documented (cf. Pew Research Center, 2010). While studied in communications research (e.g. Palmgreen et al., 1980, Vincent and Basil, 1997), its implications for media competition have not been explored.

Our model connects to the IO literature on product differentiation, see Hotelling (1929); d'Aspremont et al. (1979) for horizontal and Gabszewicz and Thisse (1979); Shaked and Sutton (1982) for vertical differentiation. In our model, less accurate reporting creates both types of differentiation. As in d'Aspremont et al. (1979), firms reduce price competition through endogenous differentiation. In contrast to vertical differentiation models like Shaked and Sutton (1982), our consumers share quality preferences, so reduced quality strictly lowers demand. Important studies on joint differentiation (Neven and Thisse, 1989; Irmen and Thisse, 1998) or advertising-driven segmentation (Gal-Or and Dukes, 2003) do not consider the informational and social dynamics central to our mechanism.

2. Model

Overview and timing Two "broadcasters", A and B, disseminate information about the state of the world θ. A continuum of consumers uniformly distributed on $[0, 1]$ first decide which broadcaster to follow, then take actions based on the information they obtain from their chosen broadcaster.

We first outline the timing and then explain each part of the game in detail. We also discuss the main assumptions in section 4.1. The timing is:

1. Broadcasters A and B simultaneously choose information structures which determine how they inform consumers about the state of the world.

2. A and B observe each other's information structures and simultaneously set prices $p_i \in \mathbb{R}_+$ that consumers have to pay for following them.

3. Consumers observe information structures and prices. Each consumer j chooses to follow one of the two broadcasters or none at all.

4. The state of the world θ and broadcasters' signals realize; consumers observe the signal of their chosen broadcaster. (No one observes θ.)

5. Each consumer j gets randomly matched with another consumer (called $-j$), observes the broadcaster that $-j$ follows and takes actions whose payoff depends on the state of the world θ and the actions of $-j$.

The solution concept is perfect Bayesian equilibrium.[5]

State of the world The state of the world θ is distributed according to cumulative distribution function F on support $\Theta \subseteq \mathbb{R}$, where $|\Theta| \geq 2$.

Broadcaster actions and payoffs Each broadcaster i takes two choices:

1. In stage 1: An *information structure* $\mathcal{I}_i = (\mathcal{S}_i, \phi_i)$, which consists of a signal space $\mathcal{S}_i \subseteq \mathbb{R}$ and a mapping $\phi_i : \Theta \to \Delta \mathcal{S}_i$. ϕ_A and ϕ_B are independent conditional on the true state θ. This information structure maps each state of the world to a distribution over messages and determines how the broadcaster informs its followers about the state of the world.

2. In stage 2: A price $p_i \in \mathbb{R}_+$ that consumers pay to follow i.

Information structures are costless, i.e. broadcasters can report with any precision with no constraint. Broadcaster i's profit π_i is simply the revenue from all consumers that follow i, i.e.

$$\pi_i = \int_{\Psi_i} p_i \, dj$$

where $\Psi_i \subseteq [0, 1]$ is the set of consumers who follow i.

A consumer following broadcaster i receives signal $s_i \in \mathcal{S}_i$ in stage 4, which is the result of the mapping ϕ_i.

Consumer actions and payoff Each consumer j makes three choices: Which broadcaster i to follow (stage 3 of the timing) and two actions a_j and b_j (stage 5, after learning s_i). Consumer j's payoff (with $j \in [0, 1]$) from following A and taking actions a_j and b_j is given by

$$U_j(A, a_j, b_j) = \underbrace{v - p_A}_{\text{Benefit and cost}} - \underbrace{\tau j}_{\text{Transportation cost}} - \underbrace{(a_j - \theta)^2}_{\text{Information part}} - \underbrace{\alpha (b_j - a_{-j})^2}_{\text{Interaction part}} \tag{1}$$

where $\alpha > 0$ describes the relative importance of the interaction, and a_{-j} is the matched consumer's action. The payoff from consuming B is identical, except that transportation cost is $\tau(1 - j)$. We explain each part of this payoff in turn.

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[5]Strictly speaking, our analysis only needs sequential rationality. Consistency has no implications here since there is no relevant private information when actions are taken.

Benefit and cost   Consumers derive an exogenous payoff of v from following any broadcaster. This ensures a covered market equilibrium (every consumer follows one broadcaster); we choose v such that covered market equilibria exist. Each consumer following broadcaster i pays p_i (the price set by i).

Transportation cost   Consumers are also differentiated in their tastes, as in a classical Hotelling model. Consumer $j \in [0,1]$ incurs cost τj when following A (the "transportation costs" of Hotelling models) and $\tau(1-j)$ when following B. Methodologically, τ introduces a taste asymmetry among consumers so that broadcaster demand is continuous in price and our model has interior solutions in pure strategies. Our main result only requires $\tau > 0$.

Information part   The first informational benefit that consumers derive from broadcasts is learning about the state of the world. Formally, consumer j takes action $a_j \in \mathbb{R}$ which she tries to choose as close to the state of the world as possible.

Interaction part   The central theoretical innovation of our model is that consumers value knowing what others believe about the state of the world. This is the social function of news: The ability to predict others' beliefs, knowledge and interests.

Formally, each consumer j takes an action $b_j \in \mathbb{R}$ which she tries to take as close as possible to the action a_{-j} of randomly matched consumer $-j$. The optimal b_j depends on which broadcaster $-j$ is following and hence what information she has about the world. j is matched with $-j$ by the following matching algorithm:

Interaction matching   Each consumer j is randomly matched with another consumer in a neighborhood around j. This neighborhood is given by $[j - \tilde{\delta}_j, j + \tilde{\delta}_j]$, where $\tilde{\delta}_j = \min\{\delta, j, 1-j\}$ for some $\delta \in (0, 1/2)$. Thus the neighborhood is a δ-ball around j, truncated at the $[0,1]$ boundaries.[6] Matching is one-sided and independent: if j is matched to j', and j' to j'', then almost surely $j \neq j''$.

Committed consumers   A share $\lambda \in (0,1)$ of consumers are "committed": if $j \in [0, 0.5)$, they choose from $\{A, \emptyset\}$ in step 3; if $j \in (0.5, 1]$, from $\{B, \emptyset\}$. This may reflect limited access, awareness, or willingness to consume the distant broadcast. The remaining $1 - \lambda$ choose freely from $\{A, B, \emptyset\}$. Figure 1 illustrates this distribution.

Consumers' outside option   A consumer who follows no broadcaster pays no price or transportation cost but receives no signal.

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[6]This avoids discontinuities near endpoints. Our main results are unaffected by defining neighborhoods as $[j - \delta, j + \delta] \cap [0,1]$ and assuming that δ is small enough (below 1/4), though this would complicate the analysis with extra case distinctions.
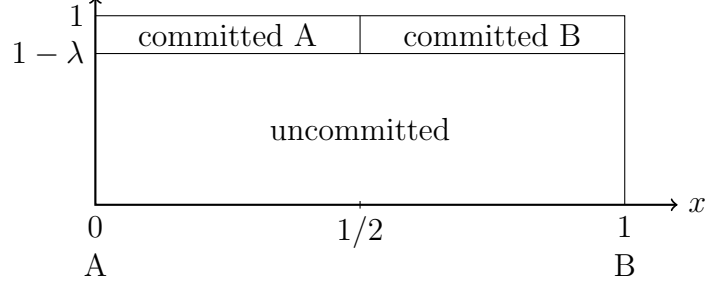
Figure 1: Distribution of consumers and "location" of broadcasters.

Reformulating the consumer's problem   Before our equilibrium analysis, we reformulate the uncommitted consumers' utility. The optimal a_j is $a_j^* = \mathbb{E}[\theta|s_i]$ (with i being the chosen broadcaster). If the matched consumer $-j$ follows the same broadcaster, $b_j^* = a_j^* = \mathbb{E}[\theta|s_i]$; otherwise $b_j^* = \mathbb{E}[\mathbb{E}[\theta|s_{-i}]|s_i]$ where $-i$ is the other broadcaster.

Thus, expected utility at the time of choosing broadcasters (and focusing on information terms) is:

$$
\begin{aligned}
E[U_j(i)] &= const - \mathbb{E}\left[(\theta - \mathbb{E}[\theta|s_i])^2\right] - \alpha q_{-i}\mathbb{E}\left[(\mathbb{E}[\mathbb{E}[\theta|s_{-i}]|s_i] - \mathbb{E}[\theta|s_{-i}])^2\right] \\
&= const - V_i - \alpha q_{-i} C_i
\end{aligned}
\tag{2}
$$

where

$$
V_i = \mathbb{E}\left[(\theta - \mathbb{E}[\theta|s_i])^2\right], \qquad C_i = \mathbb{E}\left[(\mathbb{E}[\mathbb{E}[\theta|s_{-i}]|s_i] - \mathbb{E}[\theta|s_{-i}])^2\right],
$$

q_{-i} is the consumer's belief that the consumer she will interact with follows the other broadcaster, and $const = v - p_A - \tau j$ if $i = A$ while $const = v - p_B - \tau(1-j)$ if $i = B$.

3. Analysis

Let $\mathcal{I}^f$ be the fully informative information structure in which $\mathcal{S}_i = \Theta$ and $\phi_i(\theta) = \theta$ with probability 1. Our main result shows that if α is above a threshold, full informativeness by both broadcasters cannot be an equilibrium: If one broadcaster chooses $\mathcal{I}^f$, the other's best response is an information structure with a small amount of noise.

In the spirit of backward induction, we first derive equilibrium consumer choice, then equilibrium prices, and finally characterize broadcasters' optimal precision.

Broadcaster choice by the consumer   We focus on covered markets (implying $q_{-i} = 1 - q_i$). There exists an indifference location $\hat{x}$: Consumers at $x < \hat{x}$ choose A, consumers located at $x > \hat{x}$ choose B, and those at $\hat{x}$ are indifferent, i.e.

$$
\begin{aligned}
& p_A + \tau\hat{x} + V_A + \alpha q_B(\hat{x})C_A = p_B + \tau(1-\hat{x}) + V_B + \alpha q_A(\hat{x})C_B \\
\Leftrightarrow\; & 2\tau\hat{x} - \alpha q_A(\hat{x})(C_A + C_B) = p_B - p_A + \tau + V_B - V_A - \alpha C_A.
\end{aligned}
\tag{3}
$$

Our matching protocol implies that

$$q_A(\hat{x}) = \begin{cases} (1-\lambda)\frac{1}{2} + \lambda * 1 & \text{if } \hat{x} < \frac{1}{2} - \delta \\ (1-\lambda)\frac{1}{2} + \lambda\frac{1/2-(\hat{x}-\delta)}{2\delta} & \text{if } \hat{x} \in [\frac{1}{2} - \delta, \frac{1}{2} + \delta] \\ (1-\lambda)\frac{1}{2} + \lambda * 0 & \text{if } \hat{x} > \frac{1}{2} + \delta. \end{cases} \tag{4}$$

In each of these expressions, the first summand represents the non-committed consumers – of whom, if $\hat{x}$ is indifferent, exactly half use each broadcaster. The second summand represents committed consumers, whose average choice in $\hat{x}$'s δ-neighborhood depends on the location of $\hat{x}$.

q_A is continuous, decreasing and piecewise linear, so for given prices and parameters, there is (at most) one $\hat{x}$ solving the indifference condition (3), and this $\hat{x}$ is continuous in prices and parameters.

Solving (3) yields

$$\hat{x} = \frac{p_B - p_A + Z}{Y} \tag{5}$$

where Z and Y depend on the relevant case of (4). As in Hotelling with quality differences, Z describes relative quality and Y the price sensitivity of demand. Without informational effects, we would have $Z = \tau$ and $Y = 2\tau$, but the information structure affects both. The exact expressions are as follows:

$$Z = \begin{cases} \tau + V_B - V_A + \alpha\frac{1+\lambda}{2}C_B - \alpha\frac{1-\lambda}{2}C_A & \text{if } \hat{x} < \frac{1}{2} - \delta \\ \tau - \alpha C_A + V_B - V_A + \alpha(C_A + C_B)\left(\frac{1}{2} + \frac{\lambda}{4\delta}\right) & \text{if } \hat{x} \in [\frac{1}{2} - \delta, \frac{1}{2} + \delta] \\ \tau + V_B - V_A + \alpha\frac{1-\lambda}{2}C_B - \alpha\frac{1+\lambda}{2}C_A & \text{if } \hat{x} > \frac{1}{2} + \delta \end{cases} \tag{6}$$

$$Y = \begin{cases} 2\tau & \text{if } \hat{x} < \frac{1}{2} - \delta \\ 2\tau + \alpha(C_A + C_B)\frac{\lambda}{2\delta} & \text{if } \hat{x} \in [\frac{1}{2} - \delta, \frac{1}{2} + \delta] \\ 2\tau & \text{if } \hat{x} > \frac{1}{2} + \delta. \end{cases} \tag{7}$$

Pricing At the pricing stage, broadcaster profits can be written as

$$\pi_A = p_A\left(\lambda\frac{1}{2} + (1-\lambda)\hat{x}\right) \quad \text{and} \quad \pi_B = p_B\left(\lambda\frac{1}{2} + (1-\lambda)(1-\hat{x})\right).$$

Equation (5) yields equilibrium prices and profits as functions of λ, Z and Y.[7] Equilibrium prices are:

$$p_A^* = \frac{1}{2}\frac{\lambda}{1-\lambda}Y + \frac{1}{3}Z + \frac{1}{3}Y$$
$$p_B^* = \frac{1}{2}\frac{\lambda}{1-\lambda}Y - \frac{1}{3}Z + \frac{2}{3}Y$$

implying that the indifferent consumer is located at

$$\hat{x}^* = \frac{1}{3}\frac{Y+Z}{Y} \tag{8}$$

and profits are

$$
\begin{aligned}
\pi_A &= \left(\frac{1}{2}\frac{\lambda}{1-\lambda}Y + \frac{1}{3}Z + \frac{1}{3}Y\right)\left(\lambda\frac{1}{6} + \frac{1}{3} + (1-\lambda)\frac{1}{3}\frac{Z}{Y}\right) \\
&= \left(\frac{1}{9}\lambda + \frac{2}{9}\right)Z + \left(\frac{1-\lambda}{9}\right)\frac{Z^2}{Y} + \left(\frac{1}{12}\frac{\lambda^2+2\lambda}{1-\lambda} + \frac{1}{18}\lambda + \frac{1}{9}\right)Y \\
\pi_B &= \left(\frac{1}{2}\frac{\lambda}{1-\lambda}Y - \frac{1}{3}Z + \frac{2}{3}Y\right)\left(-\lambda\frac{1}{6} + \frac{2}{3} - \frac{1-\lambda}{3}\frac{Z}{Y}\right) \\
&= \left(\frac{1}{9}\lambda - \frac{4}{9}\right)Z + \left(\frac{1-\lambda}{9}\right)\frac{Z^2}{Y} + \left(-\frac{1}{12}\frac{\lambda^2-4\lambda}{1-\lambda} - \frac{1}{9}\lambda + \frac{4}{9}\right)Y.
\end{aligned} \tag{9}
$$

Precision   We now turn to the precision choice in stage 1. Our main result states that if consumers care sufficiently about their interaction with others (i.e. α is sufficiently large), it is not an equilibrium for both broadcasters to be fully informative.[8]

Proposition 1. *Let $\alpha \geq 4\delta/(\lambda - 2\delta)$ and $\lambda > 2\delta$. Then no equilibrium exists in which both broadcasters choose the perfectly informative information structure $\mathcal{I}^f$.*

Proof: Suppose B chooses $\mathcal{I}^f$ which relays the true state with probability 1. We will show that A can profitably deviate by introducing noise.

Let A use a structure that transmits $s_A = \theta$ with probability $1 - \varepsilon$ and a random signal drawn from F (but independently of θ) with probability ε. When $\varepsilon = 0$, this is $\mathcal{I}^f$; we examine the payoff of increasing ε slightly.

The following lemma establishes two common and intuitive properties of Hotelling games: First, for arbitrarily small differences in noise (i.e. quality), the indifferent consumer is arbitrarily close to $1/2$. Second, the broadcaster offering a slightly worse product has slightly fewer customers.

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[7]These derivations rely on first-order conditions that may not hold at case boundaries. Lemma 1 below shows only one case is relevant and no equilibria involve boundary values, so the analysis is without loss of generality.

[8]Proposition 1 also applies to any structure that is equivalent to $\mathcal{I}^f$ apart from measure-zero deviations.

Lemma 1. *For $\varepsilon > 0$ sufficiently small, the indifference point $\hat{x}^*$ lies in $(1/2 - \delta, 1/2)$. (Proof in the appendix.)*

Using (9), we can write

$$\pi_A'(\varepsilon) = \left(\frac{1}{9}\lambda + \frac{2}{9}\right)\frac{dZ}{d\varepsilon} + \left(\frac{1-\lambda}{9}\right)\left(2\frac{Z}{Y}\frac{dZ}{d\varepsilon} - \frac{Z^2}{Y^2}\frac{dY}{d\varepsilon}\right) + \left(\frac{1}{12}\frac{\lambda^2 + 2\lambda}{1-\lambda} + \frac{1}{18}\lambda + \frac{1}{9}\right)\frac{dY}{d\varepsilon}.$$

As $Z_{\varepsilon=0,\mathcal{I}_B=\mathcal{I}^f} = \tau$ and $Y_{\varepsilon=0,\mathcal{I}_B=\mathcal{I}^f} = 2\tau$, this simplifies to

$$\pi_A'(0) = \frac{1}{3}\frac{dZ}{d\varepsilon} + \left(\frac{1}{12}\frac{\lambda^2 + 2\lambda}{1-\lambda} + \frac{1}{12}\lambda + \frac{1}{12}\right)\frac{dY}{d\varepsilon}.$$

By lemma 1, (6) and (7) apply for Z and Y and (given that B uses no noise) it is $V_B = 0$, $C_A = V_A \geq 0$ and $C_B \geq 0$. The latter inequalities are strict for $\varepsilon > 0$ and hold with equality if $\varepsilon = 0$. With the parameter restrictions of proposition 1, it is $Z \geq \tau$ and $Y \geq 2\tau$, where again the inequalities are strict for $\varepsilon > 0$ and hold with equality if $\varepsilon = 0$. Therefore, $dZ/d\varepsilon > 0$ and $dY/d\varepsilon > 0$ at $\varepsilon = 0$ which implies that $\pi_A'(0) > 0$. $\square$

Intuitively, noise dampens price sensitivity. If both broadcasters are fully informative, the information and interaction terms in consumers' payoffs vanish, making them highly price-sensitive: price is the only thing that matters for their broadcaster choice.

If A introduces some noise while B stays fully informative, it reduces all consumers' willingness to pay for A (since its broadcast becomes less informative about the state θ). In mixed neighborhoods, it further reduces consumers' willingness to pay for either broadcaster, since their reporting becomes less informative about the beliefs of some other consumers.

But noise also adds an additional consideration to broadcaster choice: Consumers care which broadcaster their neighbors follow. The more neighbors follow A, the higher a consumer's willingness to pay for A. For the indifferent consumer, half of all non-committed neighbors follow each broadcaster, but the share of *committed* neighbors following A increases as $\hat{x}$ moves left.

Thus indifferent consumers in two different locations differ in their willingness to pay for A due to both transport cost *and* neighborhood composition. This additional channel softens how price changes affect $\hat{x}$ and thereby demand, reducing price elasticity faced by each broadcaster.

Broadcasters therefore charge higher prices in equilibrium. A benefits from introducing noise if the gain from lower price sensitivity outweighs lost demand for its broadcast – which is true under the conditions in proposition 1.

Formally, equation (3) shows that without noise, a change of Δ in p_A or p_B shifts $\hat{x}$ by $\Delta/2\tau$. With noise, $C_A, C_B > 0$ and this shift becomes $\Delta/\left(2\tau + \alpha\left(C_A + C_B\right)\frac{\lambda}{2\delta}\right) < \Delta/2\tau$, showing that noise dampens price sensitivity of $\hat{x}$ and hence of demand.

As a complement to this result, we show that for small enough α, full information can be sustained in equilibrium:

Lemma 2. *There exists an $\underline{\alpha} > 0$ such that for all $\alpha \leq \underline{\alpha}$, an equilibrium with $\mathcal{I}_A = \mathcal{I}_B = \mathcal{I}^f$ exists.*

Proof of lemma 2: First, consider the extreme case $\alpha = 0$. We will show that full information is the essentially unique best response to $\mathcal{I}_B = \mathcal{I}^f$ in this case. Given $\alpha = 0$ and $\mathcal{I}_B = \mathcal{I}^f$, the indifferent consumer is given by

$$\hat{x} = \frac{p_B - p_A - V_A + \tau}{2\tau}.$$

Equilibrium prices are

$$p_A = \tau - \frac{V_A}{3} + \frac{\lambda}{1-\lambda}\tau \qquad p_B = \tau + \frac{V_A}{3} + \frac{\lambda}{1-\lambda}\tau$$

which implies $\hat{x} = 1/2 - V_A/(6\tau)$. As both equilibrium price and quantity for A are strictly decreasing in V_A, profits of A are strictly decreasing in V_A (with strictly negative derivative at $V_A = 0$). Hence, only information structures with $V_A = 0$ – i.e. full information – are best responses.

Profits of the pricing stage are smooth in all parameters around equilibria with interior $\hat{x}$. A's profit given $\mathcal{I}_B = \mathcal{I}^f$ hence remains strictly decreasing in V_A if α is positive but sufficiently small. By symmetry, the same applies for B. Consequently, information structures $\mathcal{I}_A = \mathcal{I}_B = \mathcal{I}^f$, where $V_A = V_B = 0$, are equilibrium choices for positive but sufficiently small α. $\qquad\square$

3.1. Normally distributed state and noise

In this section, we simplify our framework and assume that (i) the state θ is standard normally distributed and (ii) broadcasters use signals normally distributed around the state, i.e. $s_i = \theta + \varepsilon_i$ where $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2)$. The noise terms ε_A and ε_B are independent and each broadcaster chooses $\sigma_i^2 \in \mathbb{R}_+$ in stage 1.

Our earlier proofs apply to this setup. For small α, both broadcasters choose $\sigma_i^2 = 0$ in equilibrium, but no such equilibrium exists for sufficiently large α. In fact, the normal structure yields a tighter version of proposition 1:

Proposition 2. *Let θ be standard normally distributed. Full information, i.e. $\sigma_A^2 = \sigma_B^2 = 0$, is not an equilibrium if $\alpha > \frac{4\delta(1-\lambda)}{3\lambda}$.*

Proof: See appendix. $\qquad\square$

3.2. Symmetric and asymmetric equilibria

While general analytical statements about equilibrium noise levels are intractable, numerical simulations can illustrate what equilibria look like. Figure 2 illustrates proposition

2 by showing equilibrium noise levels in an example with normally distributed state and noise.

The left and right panels of figure 2, respectively, show that for sufficiently high α, there are symmetric equilibria (in which both broadcasters use noise) as well as asymmetric equilibria (in which only one does). In either case, noise levels are monotonic in α.
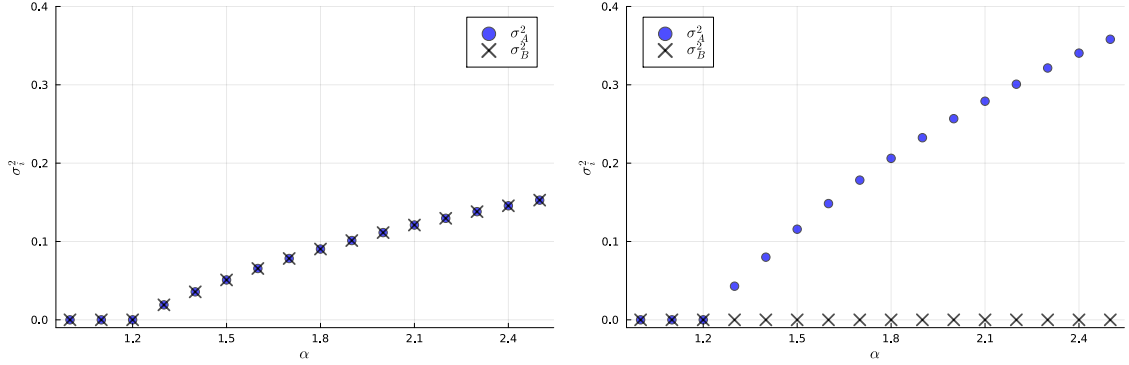
Figure 2: Equilibrium noise levels as a function of α (for $\delta = 0.1$, $\lambda = 0.1$, $v = 3$, $\tau = 1$).

Each broadcaster trades off lower demand from noisy reporting against the gain from differentiation (reduced price sensitivity). Each broadcaster benefits if its rival increases noise, as this differentiates the broadcasters, relaxes price competition and makes the rival's reporting less informative. In asymmetric equilibria, broadcaster A's noise level is so high that the best response of B is to report with zero noise (or vice versa). At the same time, A's noise level is a best response to zero noise, since choosing a lower noise level would reduce differentiation so much that it lowers A's profit.

Whether B best responds to noise with zero noise depends on A's noise level. If A's noise is low, the additional differentiation induced by noise in B's reporting outweighs B's loss in demand through reduced accuracy. Symmetric and asymmetric equilibria can hence coexist for the same model parameters.

These equilibria cannot easily be ordered in terms of informativeness (since they differ in terms of which consumers follow which broadcaster), but they yield substantially different levels of inequality, both in information and welfare. In symmetric equilibria, all consumers are moderately well-informed. In asymmetric ones, some are perfectly informed while others consume low-quality news. Even the well-informed suffer from interacting with badly informed neighbors.[9]

We do not take a stance on which equilibrium better reflects real-world outcomes, as this is beyond the scope of our analysis. Our aim is to highlight mechanisms, not make testable predictions. We can, however, note that U.S. survey data shows clear divides: Democrats and Republicans see the media as increasing political divisions, but differ

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[9]Average transportation costs are also higher in asymmetric equilibria as some consumers follow the more distant broadcast.

sharply in media trust and consumption habits.[10]

3.3. Public option

Many countries fund public broadcasters to provide high-quality reporting at zero cost. To see how this affects private incentives, suppose that B provides fully accurate news at zero price in the model with normally distributed noise, i.e. $\sigma_B^2 = p_B = 0$.

Proposition 3. *Let $\sigma_B^2 = p_B = 0$. Then $\sigma_A^2 > 0$ in a full-coverage equilibrium if $\alpha > \frac{4\delta(1-\lambda)}{\lambda}$.*

Proof: See appendix. □

Inaccurate reporting can still be a best response for A, but the threshold for α is three times higher than in proposition 2. The difference is that reducing accuracy does not increase the competitor's price (since p_B is fixed), but it still reduces own-demand elasticity.

4. Discussion

4.1. Model assumptions

We discuss some key assumptions and whether models with slightly different assumptions could produce similar results.

Social function of news Our main theoretical innovation is modeling the social function of news via the "interaction part" in consumer utility. This social function is empirically well-established[11], though our precise functional form is a stylized way to model it.

Our "interaction term" $-\alpha(b_j - a_{-j})^2$ in consumer utility (equation (1)) gives each consumer an additional action b_j to match the action a_{-j} of another consumer. This rewards j for being able to anticipate how $-j$ sees the world (which is represented by a_{-j}). We see this as an intuitive way of describing that j benefits from knowing what $-j$ believes, as j can then adjust to $-j$'s views when discussing a topic, or choose topics accordingly.

Our model does not just describe product differentiation or network effects; j also reasons about what she knows and what $-j$ knows about the world. While each consumer cares about which broadcaster others in his neighborhood (i.e. his "network") follow, the strength of this effect depends on the broadcaster's precision choices, j's location and his inference problem.

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[10]72% (Democrats) and 81% (Republicans) believe that news media is increasing political divisions in the U.S.; 26% (Democrats) and 60% (Republicans) report "little or no" trust that media reports the news "fully, accurately, and fairly" (AP-NORC, 2023). See Pew Research Center (2024b) for data on partisan news consumption.

[11]72% of US adults state that "one reason they consume news is because they enjoyed talking about it with family, friends and colleagues", more than "find[ing] information in the news that helps them improve their lives" (61%) (Pew Research Center, 2010).

Transportation costs Introducing τ to capture consumer heterogeneity mainly makes our model tractable. If $\tau = 0$, demand is discontinuous, there are no pure-strategy equilibria and we would have to resort to a mixed-strategy analysis similar to Varian (1980). Our results do not require a minimum τ – if λ and δ are small (see also next paragraph), our main results hold for τ arbitrarily small. In practice, such minor taste differences could reflect local or cultural affinity, preferences for a broadcaster's style or the ease of tuning in (Martin and Yurukoglu, 2017). The results also hold for large τ, provided the market remains covered.

Committed consumers A share λ of consumers is committed to a broadcaster – i.e. choosing either that broadcaster or none – and located near it in transportation-cost terms. This creates heterogeneity in neighborhood composition: The indifferent consumer's likelihood of meeting followers of either broadcaster depends on how the market is split among broadcasters.

Even a small share of committed consumers guarantees that changes in precision or price also affect the neighborhood composition around the indifferent consumer. For example, a small increase of $\hat{x}$ (e.g. from noisier reporting by A) increases the chance of interaction with B-followers, which reduces price elasticity.

The existence of committed consumers is empirically well-established.[12] Our results hold for small λ, provided neighborhood size (δ) is also small thereby ensuring sufficient heterogeneity in neighborhood composition. More complex setups – e.g. λ varying by location – could replicate our main mechanism, but we use the simplest functional form that captures the key dynamics.

Independent signals We assume that broadcasters' signals must be independent conditional on the state. This simplifies our analysis, but broadcasters would not benefit from correlating their signals even if they could. The point of introducing noise is making one's signal hard to predict, so that consumers following the other broadcaster cannot forecast one's signal well – correlation would undermine that.

No costs of precision We assume that broadcasters pay no direct costs for increasing signal precision. This creates a best-case environment for accurate reporting: if broadcasters avoid full precision under these conditions, they would do so even more with costly precision.

One-dimensional state of the world Since we model a one-dimensional state, broadcasters cannot differentiate themselves by topic. Allowing broadcasters to selectively report

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[12]See e.g. Pew Research Center (2016), section 3, on loyalty and constancy of US news consumers. Consumers can be committed for different reasons, e.g. to their local news media (Pew Research Center, 2024a) or to a specific ideological outlook (e.g. Iyengar and Hahn, 2009).

on different dimensions of a multi-dimensional state would likely yield topic-based differentiation instead of noise. Consumers would still lack information, as each broadcaster covers only part of the state. The core inefficiency – and its strategic origin – would remain.

4.2. Possible policy responses

Equilibria with noise may be undesirable for three reasons: First, consumers are badly informed. Second, transportation costs may not be minimized, which is inefficient. Finally, broadcaster profit is high and consumer surplus relatively low.[13] A policy intervention could target any of these dimensions – either because the dimension itself is of interest, or because it has implications that go beyond our model (see conclusion).

Our model suggests various ways to improve outcomes along these dimensions. We discuss several approaches.

Monopoly   A monopolist (e.g. after we simply remove one broadcaster from the model) always chooses full information: Noise lowers consumers' willingness to pay and increases the value of not following the broadcaster, since it makes the belief of followers easier to predict for non-followers. (More formally, noise increases the interaction utility of non-buyers without affecting the interaction utility of buyers.) Even if a monopolist prices some consumers out of the market, it would have no incentive to report with any noise.

Monopoly hence maximizes informativeness, albeit with low consumer surplus and with a possible inefficiency (due to transportation cost, unless the monopolist operates both broadcasters, and the possibility that consumers are priced out of the market). It may also create other problems outside our model – for example, a monopolist may become inefficient or be subject to government capture in ways that competing broadcasters are not.

Enforced similarity   Broadcasters in our model reduce informativeness to differentiate themselves and soften competition. Policies limiting this differentiation may counter this. One approach is to mandate similarity in broadcasting, for example through journalistic standards or requirements for balance. A classic example is the U.S. Federal Communications Commission's "fairness doctrine" (1949-1987), which required broadcasters to cover controversial public issues and present contrasting viewpoints.

Such regulation is complex and contentious: Defining "controversial issues" and ensuring balance is subjective and risks government overreach. The U.S. fairness doctrine was

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[13]Informativeness and efficiency are both maximized if broadcasters report perfectly and consumers choose broadcasters freely. But they are not the same thing: For example, an outcome with full information in which not all consumers follow the broadcaster that minimizes their transportation cost is most-informative but not efficient.

controversial throughout its lifetime and was ultimately repealed.[14] Yet many democracies maintain similar principles. France's ARCOM mandates how much airtime must be given to different groups. The U.K.'s Ofcom enforces broadcast impartiality, especially before elections. German public broadcasters are legally required to offer balanced reporting while being removed from direct government control; private German media are part of a self-regulatory consortium with a similar mandate.[15]

Such rules are usually intended to improve informativeness directly – but our model shows that there is another mechanism: By limiting differentiation, fairness regulation can increase correlation between reports. Broadcasters can no longer reduce correlation by strategically omitting topics or reporting with large slant or bias. This intensifies competition and incentivizes higher-quality, lower-price reporting – and, if effective, boosts informativeness, welfare and consumer surplus simultaneously.

Changes in the social function of news   In our model, differentiation reduces price sensitivity due to the social function of news, captured by a large enough α in proposition 1. Our numerical simulations (figure 2) show that equilibrium noise can be monotonic in α.

Thus, greater social importance of news – e.g. increased sharing on social media – may reduce reporting quality. Social media may worsen the information available to citizens through an indirect effect on the incentives of news media, rather than direct effects such as the sharing of misinformation.

This suggests a provocative question: Could reducing the social importance of news (i.e. lowering α) improve outcomes? Consumers would then be more willing to switch to a more informative broadcaster even if others in their neighborhood do not follow it (yet), and a fully informative equilibrium may exist.

We do not exactly know how this could be achieved, since underlying preferences matter as much as societal norms and customs. Effects on consumer surplus and efficiency, moreover, would be ambiguous, since people clearly value sharing and discussing news with their social environment.[16]

Changing the difficulty of switching broadcasters   In our model, broadcasters reduce quality to discourage switching. Policy measures facilitating switching, i.e. reducing τ, would increase the price elasticity and reduce the price level. Our main result – that firms introduce noise to relax price competition – would, however, be unaffected.

A counterintuitive intervention is to make switching *harder*. This could "crowd out" broadcasters' incentive to differentiate. For example, raising τ (without a corresponding

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[14]See US General Accounting Office (1979) and Simmons (2022).

[15]ERGA (2018) has an overview for EU countries. See Cage et al. (2024), sections 5 and 6 of the U.K. Broadcasting Code and §26 of the German *Medienstaatsvertrag* for details on national frameworks.

[16]These benefits are not reflected in the consumer utility (equation (1)), which is reduced by larger α. Our consumer utility is of course a reduced-form expression that includes the losses from disagreement but not the gains from socializing over news.

increase in v) could create an uncovered market, in which a group of consumers in the middle of the market does not follow any broadcaster. The two broadcasters would become local monopolists and no longer compete with each other but with the option to not follow any broadcaster. Consumers' willingness to pay (and hence broadcaster profit) is then maximized by fully informative reporting.

Such a change would have ambiguous effects on efficiency (since higher τ means direct welfare loss) and consumer surplus (since all consumers are now captive), but could increase equilibrium informativeness.

5. Conclusion

Conventional wisdom suggests that competition between profit-driven media companies should improve their news coverage: Accurate reporting should boost willingness to pay and broaden audience appeal, both of which should increase profits. We argue that under general, intuitive and empirically supported assumptions, profit-maximizing media may provide low-quality reporting which increases societal fragmentation – since this eases competitive pressures.

Policy can aim to improve efficiency, consumer surplus or news informativeness. Each of these is important in its own right but has implications beyond our model. A poorly informed electorate struggles to select effective leaders and hold government accountable. Media fragmentation has real political consequences (DellaVigna and Kaplan, 2007, Caprini, 2023). It can polarize citizens (cf. Levendusky, 2013) and increase affective polarization (cf. Iyengar et al., 2019) in ways that our simple one-shot model does not capture. Eventually, even well-meaning citizens may be unable to agree on basic facts.[17]

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[17]See e.g. Pew Research Center (2020) for a survey showing that Democrats and Republicans disagree on simple facts about the US election system and other topics and Bullock and Lenz (2019) for an overview on the topic.

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Appendix

Proof of lemma 1: Recall that $\pi_A = p_A (\lambda/2 + (1-\lambda)\hat{x})$. $\hat{x}$ is differentiable at all combinations of prices except those resulting in $\hat{x} = 1/2 - \delta$ and $\hat{x} = 1/2 + \delta$. At all but these two price vectors it is

$$\frac{d\,\pi_A}{d\,p_A} = \frac{1}{2}\lambda + (1-\lambda)\hat{x} + p_A(1-\lambda)\frac{d\,\hat{x}}{dp_A}.$$

Note that it follows from (5) that $d\hat{x}/dp_A = -1/Y$, which has a discontinuous upward jump at the p_A that leads to $\hat{x} = 1/2 + \delta$ (for a given p_B). Therefore, a p_A such that $\hat{x} = 1/2 + \delta$ cannot be a best response by A (as the upward jump of $\partial\pi_A/\partial p_A$ implies that π_A is "locally convex", i.e. π_A is higher either at prices slightly below or slightly above this prices). A similar argument shows that (for given p_A) the p_B that leads to $\hat{x} = 1/2 - \delta$ cannot be a best response by B. Consequently, prices (p_A, p_B) that lead to either $\hat{x} = 1/2 + \delta$ or to $\hat{x} = 1/2 - \delta$ cannot be equilibrium prices and $\hat{x} \notin \{1/2 - \delta, 1/2 + \delta\}$ in equilibrium.

We can therefore concentrate on prices at which the first order conditions are satisfied with equality. Note that $lim_{\varepsilon \to 0}Y = 2\tau$ and $lim_{\varepsilon \to 0}Z = \tau$ (regardless of where $\hat{x}$ lies and which of the expression determines Y and Z). This (with equation (8)) implies that $lim_{\varepsilon \to 0} \hat{x}^* = 1/2$ and therefore $\hat{x} \in (1/2 - \delta, 1/2 + \delta)$ for sufficiently small $\varepsilon > 0$.

Finally, $\hat{x}^* < 1/2$ is by (8) equivalent to $2Z < Y$. For $\hat{x}^* \in (1/2 - \delta, 1/2 + \delta)$, expressions (6) and (7) can be plugged into this inequality, which yields $V_B - V_A - \alpha C_A + \alpha C_B < 0$. This inequality is true if B uses $\mathcal{I}^f$, since then $V_B = 0$ and $C_B \leq C_A$ while $V_A > 0$ if $\varepsilon > 0$. $\qquad\square$

Proof of proposition 2: Let B choose $\mathcal{I}^f$ by choosing $\sigma_B^2 = 0$. We will show that $\sigma_A^2 = 0$ is not a best response for A if $\alpha > \frac{4\delta(1-\lambda)}{3\lambda}$ by showing that the derivative of A's profits with respect to σ_A^2 is positive at $\sigma_A^2 = 0$ (given that B chooses $\mathcal{I}^f$).

For $\varepsilon > 0$ sufficiently small, the arguments in the proof of lemma 1 imply that $\hat{x}$ will be in $(1/2 - \delta, 1/2)$. We can therefore work with the middle cases of expressions (6) and (7) for Z and Y when considering a marginal increase of σ_A^2 from 0.

As $\mathcal{I}_B = \mathcal{I}^f$,

$$C_A = \frac{\sigma_A^2}{1 + \sigma_A^2} = V_A \qquad \text{and} \qquad C_B = \frac{\sigma_A^2}{(1 + \sigma_A^2)^2}.$$

Therefore,

$$Z = \tau - (1 + \alpha)\frac{\sigma_A^2}{1 + \sigma_A^2} + \alpha\frac{2\sigma_A^2 + \sigma_A^4}{(1 + \sigma_A^2)^2}\left(\frac{1}{2} + \frac{\lambda}{4\delta}\right)$$

$$Y = 2\tau + \alpha\frac{\lambda}{2\delta}\frac{2\sigma_A^2 + \sigma_A^4}{(1 + \sigma_A^2)^2}$$

with derivatives

$$\frac{dZ}{d\sigma_A^2} = -(1+\alpha)\frac{1}{1+\sigma_A^2} + \alpha\frac{2}{(1+\sigma_A^2)^3}\left(\frac{1}{2}+\frac{\lambda}{4\delta}\right)$$

$$\frac{dY}{d\sigma_A^2} = \alpha\frac{\lambda}{2\delta}\frac{2}{(1+\sigma_A^2)^3}.$$

We are only interested in the derivatives evaluated at $\sigma_A^2 = 0$, where

$$\frac{dZ}{d\sigma_A^2}(0) = -1 + \frac{\alpha(\lambda)}{2\delta}$$

$$\frac{dY}{d\sigma_A^2}(0) = \frac{\alpha(\lambda)}{\delta}.$$

Using (9), we can write

$$\pi_A'(\sigma_A^2) = \left(\frac{1}{9}\lambda + \frac{2}{9}\right)\frac{dZ}{d\sigma_A^2} + \left(\frac{1-\lambda}{9}\right)\left(2\frac{Z}{Y}\frac{dZ}{d\sigma_A^2} - \frac{Z^2}{Y^2}\frac{dY}{d\sigma_A^2}\right) + \left(\frac{1}{12}\frac{\lambda^2+2\lambda}{1-\lambda} + \frac{1}{18}\lambda + \frac{1}{9}\right)\frac{dY}{d\sigma_A^2}.$$

As $Z_{\sigma_A^2=0, \mathcal{I}_B=\mathcal{I}^f} = \tau$ and $Y_{\sigma_A^2=0, \mathcal{I}_B=\mathcal{I}^f} = 2\tau$, this expression simplifies to

$$\pi_A'(0) = \frac{1}{3}\frac{dZ}{d\sigma_A^2} + \left(\frac{1}{12}\frac{\lambda^2+2\lambda}{1-\lambda} + \frac{1}{12}\lambda + \frac{1}{12}\right)\frac{dY}{d\sigma_A^2}.$$

Plugging in the expressions of the derivatives evaluated at 0 above, we obtain

$$\pi_A'(0) = \frac{1}{3}\left(-1 + \frac{\alpha\lambda}{2\delta}\right) + \left(\frac{1}{12}\frac{\lambda^2+2\lambda}{1-\lambda} + \frac{1}{12}\lambda + \frac{1}{12}\right)\left(\frac{\alpha\lambda}{\delta}\right)$$

$$= \frac{1}{3}\left[-1 + \alpha\frac{\lambda}{4\delta}\left(2 + \frac{1+2\lambda}{1-\lambda}\right)\right]$$

$$= \frac{1}{3}\left[-1 + \alpha\frac{3\lambda}{4\delta(1-\lambda)}\right].$$

For $\alpha > (4\delta(1-\lambda))/(3\lambda)$, we get $\pi_A'(0) > 0$ and therefore $\sigma_A^2 = 0$ is not a best response to $\mathcal{I}_B = \mathcal{I}^f$. $\square$

Proof of proposition 3: Profits for A in a full coverage equilibrium equal $p_A * (\lambda/2 + (1-\lambda)(-p_A + Z)/Y)$ and A's first order condition for the optimal price is therefore

$$\lambda/2 + (1-\lambda)\frac{-2p_A + Z}{Y} = 0 \qquad \Leftrightarrow \qquad p_A = \frac{Y}{4}\frac{\lambda}{1-\lambda} + \frac{Z}{2}.$$

Plugging the optimal price back into profits yields

$$\pi_A = \left(\frac{Y}{4}\frac{\lambda}{1-\lambda} + \frac{Z}{2}\right)\left(\frac{\lambda}{4} + (1-\lambda)\frac{Z}{2Y}\right) = \frac{\lambda^2}{1-\lambda}\frac{Y}{16} + \frac{\lambda}{4}Z + \frac{1-\lambda}{4}\frac{Z^2}{Y}.$$

The derivative of π_A with respect to σ_A^2 is

$$
\frac{d\,\pi_A}{d\sigma_A^2}(\sigma_A^2) = \frac{\lambda^2}{1-\lambda}\frac{1}{16}\frac{d\,Y}{d\sigma_A^2} + \frac{\lambda}{4}\frac{d\,Z}{d\sigma_A^2} + \frac{1-\lambda}{4}\frac{2ZY\frac{d\,Z}{d\sigma_A^2} - Z^2\frac{d\,Y}{d\sigma_A^2}}{Y^2}
$$

$$
= \frac{d\,Y}{d\sigma_A^2}\left(\frac{\lambda^2}{16(1-\lambda)} - \frac{1-\lambda}{4}\frac{Z^2}{Y^2}\right) + \frac{d\,Z}{d\sigma_A^2}\left(\frac{\lambda}{4} + \frac{1-\lambda}{2}\frac{Z}{Y}\right).
$$

At $\sigma_A^2 = 0$, $Z = \tau$ and $Y = 2\tau$ which then yields

$$
\frac{d\,\pi_A}{d\sigma_A^2}(0) = \frac{d\,Y}{d\sigma_A^2}\left(\frac{2\lambda - 1}{16(1-\lambda)}\right) + \frac{d\,Z}{d\sigma_A^2}\frac{1}{4}.
$$

Plugging in the expressions derived in the proof of proposition 2 for the derivatives Y and Z with respect to σ_A^2 evaluated at $\sigma_A^2 = 0$ yields

$$
\frac{d\,\pi_A}{d\sigma_A^2}(0) = \frac{\alpha\lambda}{\delta}\left(\frac{2\lambda - 1}{16(1-\lambda)}\right) + \left(-1 + \frac{\alpha\lambda}{2\delta}\right)\frac{1}{4} = \frac{\alpha\lambda}{16\delta(1-\lambda)} - \frac{1}{4}
$$

which is greater than 0 if $\alpha > 4\delta(1-\lambda)/\lambda$. $\qquad\square$