A stochastic model for emission reduction by utilizing the Nordlink cable

Simon Elias Schrader
University of Oslo

July 1, 2021

Abstract

Test

1 Introduction

2 Theory & Methods

2.1 Raw data

The time series for the electricity demand in Norway and Germany are acquired from [1], which contains time series for per-hour Norwegian and German electricity load, wind production and, in the case of Germany, solar energy production, which is the majority of the data required. One can safely assume that the rest of Norwegian electricity production stems from water, matching the total demand - alternatively, one can use data from SSB's electricity balance, which is what we did.[1] Observe that the available data represents the electricity balance, not the total water produced, so the accuracy of these numbers can be doubted.

In addition to solar energy and wind, Germany has three additional types of "emission-free" energy: Water, biomass & nuclear.The share of these energies can be found here.[2] For water, biomass and nuclear in 2019 (visual inspection) stood for approx. 15 GW, with similar values in 2017, 2018 and 2020. I simply subtracted that constant number from the German load (which is the same as assuming a constant production). Observe that uranium stands for approximately half of the non-wind non-solar renewables. As that will go to zero in the future, a given model could be done setting this value to zero, as Germany does not plan to produce Uranium in the future. Similar things were done to the Norwegian wind production, which is multiplied by 5.5/5.2 as to take into account offshore wind which is not present in my data. A factor of 2% of the mean water production is added to the water production to take into account thermal energy.

The time series for 2017-2020 are found here, as well as a trend and seasonal component in dotted lines, which is described below. The original data and the corresponding trend can be seen in figure 1, with the log-transformed data in figure 2. The last day of each year (31st May) has been removed from the dataset, such that the number of points is divisible by 52 (the number of weeks in a year).

[1] https://www.ssb.no/statbank/table/12824
[2] https://energy-charts.info/charts/energy/chart.htm?l=en&c=DE&year=2020

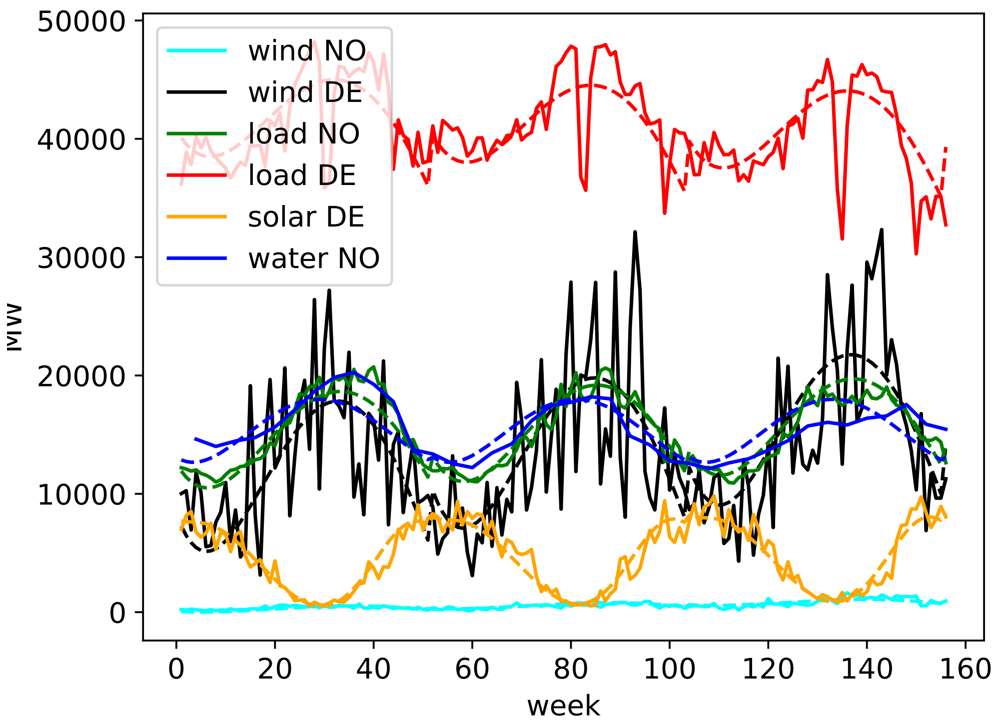

Figure 1: *Time series from june 2017 to june 2020 (weekly resolution) for the production and demand of electricity in Germany and Norway, "DE" standing for Germany and "NO" standing for Norway, with the trend + season as dotted lines.*

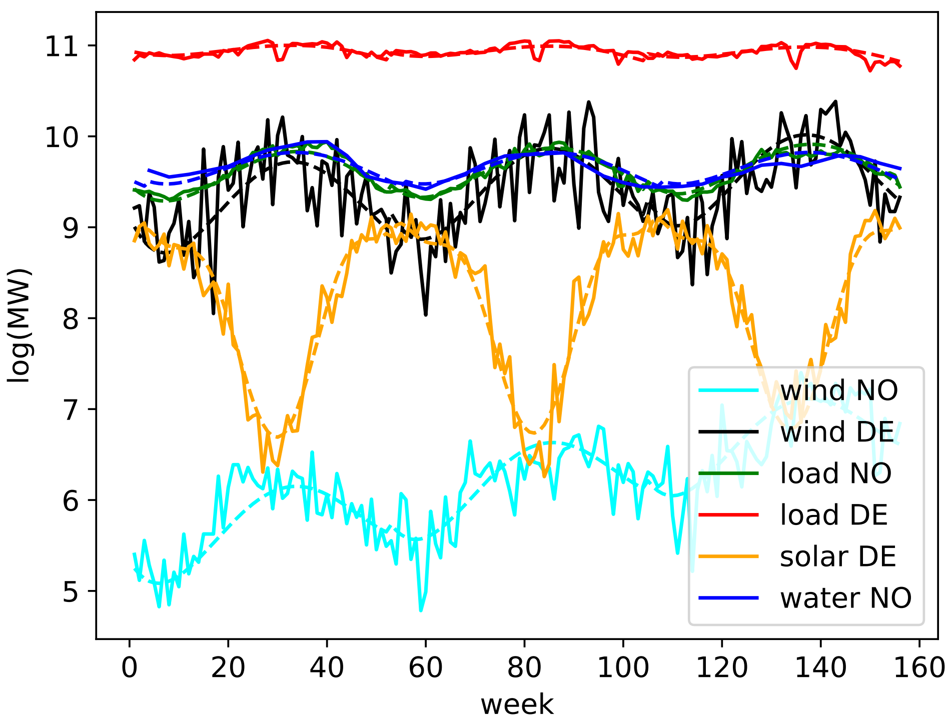

Figure 2: *Time series from june 2017 to june 2020 (weekly resolution) for the production and demand of electricity in Germany and Norway, with the trend + season as dotted lines.*

The trend is modeled as a linear function, while the seasonal data is modelled as a simple-6th order polynomial (noncontinuous at this moment) though I might try to do something more "correct" later on.

The water trend is slightly decreasing. As the water numbers represent the energy balance, not the water production, or even the "maximum possible water-production that is long-term possible", we will model the real trend as a constant function - we assume the water production not to be decreasing, and claim that the linear trend comes from warm summers, cold winters and the over-availabilty of cheap imported electricity. Another idea is to keep the seasonal data and the time series as predicted by the data; but model the trend not as a linear function (as was done here), but use data that is predicted (or aimed for) by the Norwegian and German authorities.

The carbon dioxide emissions in Germany are estimated from data found in this report.[3] While there are some differences between hard coal and brown coal, these differences are not substantial and neglected.

Type	coal	gas
g CO_2/kWh (min/med/max)	740/820/910	410/490/650
% in Germany in 2020[4]	24.1%	11.6%

Averaging, we get that the mean carbon dioxide emissions per kWh fossil fuel are $\sim$ 710 g CO_2/kWh (using the med-data).

Electrification of the oil platforms will cost around 15 TWh/year extra[5] to the 8 TWh/year we have right now. The same article also states the numbers 10 and 18 TWh/year - in addition, 8 TWh of electricity are already used by the platforms. The platforms' annual emissions equal 13 million tons CO_2 equivalents (in 2020), and power generation on the platforms as well as flaring account for 84.6% of total CO_2 emissions from the Norwegian continental shelf.[6] Believing this data to be approximately right, we approximate that each kWh electricity send to the platforms from the land saves 480g CO_2/kWh of emissions. These are rough estimates, and an extra complication arises from the fact that the emissions per extracted unit oil and gas rise as it gets harder to extract from a field [2], but I will mainly ignore these effects.

A completely different approach is to use data from this rapport[7] and this website.[8] Assuming that the approx. 8 TWh per year already send from land to the platforms each year leads to a reduction of 3,2 million tons per year, as stated in the report, the planned projects will require an annual electricity of extra 5.1 TWh per year and further decrease the emissions by 1.7 million tons per year. This calculation means that the recent projects will lead to a decrease of 333.3g CO_2/kWh, which is a substantially lower number, which seems more reliable.

It is forecasted that Norway's oil production the next couple of years will be at around 100 Million Sm^3 oil equivalents per year 2020-2025, and 115 Million Sm^3 gas equivalents 2020-2025, approximately.[9] It is rather hard to estimate the carbon dioxide emissions per liter extracted oil. A very simple estimate, according to this website (this is US data) [10], is to estimate that about 47% are transformed to gasoline and 28% are transformed to diesel, 10% jet fuel, with the rest being "lost" in production or used differently. Assuming that the rest has no emissions, we can approximate that one liter of

[3] https://www.ipcc.ch/site/assets/uploads/2018/02/ipcc_wg3_ar5_annex-iii.pdf#page=7
[5] https://e24.no/det-groenne-skiftet/i/9vLA8d/elektrifisering-av-sokkelen-krever-mye-kraft-umulig-uten-vindkraft
[6] https://www.equinor.com/en/what-we-do/electrification.html
[7] https://www.npd.no/globalassets/1-npd/publikasjoner/rapporter/2020/kraft-fra-land-til-norsk-sokkel/kraft-fra-land-til-norsk-sokkel-rapport-2020.pdf
[8] https://www.regjeringen.no/no/aktuelt/kraft-fra-land-til-norsk-sokkel-og-elektrifisering-av-stor-landbasert-industri/id2721239/
[9] https://www.norskpetroleum.no/en/production-and-exports/production-forecasts/
[10] https://www.eia.gov/energyexplained/oil-and-petroleum-products/refining-crude-oil.php

these fuels produces ~ 2.5 kg CO_2 per liter.[11] This calculation implies that each Sm^3 Norwegian oil burned will lead to emissions of ~ 2100 kg CO_2/Sm^3. For gas, one Sm^3 oil equivalent corresponds to 1000 Sm^3 gas. According to data from this website,[12] 1000 Sm^3 natural gas creates ~ 1880 kg CO_2 (tailpipe emissions). Combining this with the emissions from the production of oil, Norwegian oil stands for 440 million tons CO_2 equivalents per year.

Two different scenarios: Assuming electrification can eradicate all emissions with 15 TWh extra and get rid of 84.6% of the emissions of 13 million tons, Norway would only have to reduce its oil production by $\sim 2.5\%$ to reduce its emissions by the same number that electrifying of the platforms would - assuming that the electried platforms do not create extra oil and gas, which does not need to be burned.
The alternative scenario is: Using the extra 5.1 TWh to decrease the emissions by 1.7 million tons, the oil production would only need to be decreased by 0.4%, or the 13 TWh to decrease the emissions by 4.9 million ton, which gives a number of 1.1%.

The NordLink cable between Norway and Germany has a capacity of 1400 MW, and an effect loss of $\sim 5\%$.

To summarize:

- Emissions saved by electrified platforms: 333 g $\mathrm{CO_2}$/kWh, assuming 5.1 TWh are used to eradicate 1.7 million tons (similar number for 13.1 TWh to eradicate 4.9 million tons).
- Emissions saved by electrified platforms: 480g $\mathrm{CO_2}$/kWh, assuming 23 TWh are used to eradicate $85\% \cdot 13$ million tons.
- Emissions by using fossil energy in Germany: 710 g $\mathrm{CO_2}$/kWh.
- Emissions of renewable energy:
  - Wind: 11 g $\mathrm{CO_2}$/kWh
  - Solar: 11 g $\mathrm{CO_2}$/kWh
  - Water: 18.5 g $\mathrm{CO_2}$/kWh
  - Nuclear: 12 g $\mathrm{CO_2}$/kWh
  - Bio: 43 g $\mathrm{CO_2}$/kWh
- How much less $\mathrm{CO_2}$ to produce to decrease the $\mathrm{CO_2}$ production by 4.9 million tons per year, which is what recent planned and fulfilled models do (assuming linearity): 2.5% for the "ideal case" of electrifying all the platforms, 1.1% with the plans we have right now.
- The Nordlink cable can send up to 1400 MW, and 5% of effect is lost.
- Time series data for Norwegian and German load and wind production are available, as well as German solar energy and Norwegian water energy. From this data, a trend and a seasonal component (linear) are extracted, which are assumed to be deterministic and also applicable in the future. The residuals are modelled as (possibly depended) time series, as described below.

[11] https://www.nrcan.gc.ca/sites/www.nrcan.gc.ca/files/oee/pdf/transportation/fuel-efficient-technologies/autosmart_factsheet_6_e.pdf
[12] https://www.eia.gov/environment/emissions/co2_vol_mass.php

2.2 Time series - Theoretical aspects

2.3 Stochastic modelling

2.4 Model for the time series

As basis for the model building, we use the log-transformed model of figure 2. We assume that each (log-transformed) time series $f(t)$ can be modelled as

$$f(t) = T(t) + S(t) + X(t) \tag{2.1}$$

where $T(t)$ is a deterministic trend function, $S(t)$ is a deterministic seasonal component with the period of one year, and $X(t)$ is a (stochastic) stationary time series.

2.5 Trend component

The trend component $T(t)$ is modelled as a linear function, and is simply the OLS fit. For water, the OLS fit is decreasing. As argued above, we will assume that it is, however, constant. With weekly resolution, the following parameters were found:

- wind NO: $T(t) = 0.009260t + 5.460278$
- wind DE: $T(t) = 0.002866t + 9.185670$
- load NO: $T(t) = 0.000761t + 9.557052$
- load DE: $T(t) = -0.000254t + 10.637291$
- water NO: $T(t) = 0.000000t + 9.656239$
- solar DE: $T(t) = 0.000868t + 8.113218$

where t is the number of weeks passed since june 1^{st} 2017.

2.6 Seasonal component

The seasonal components are modelled as a the best OLS fit of a 4th or 8th order polynomial of the function. Let n be the number of periods and p the period. Then we define the *seasonal mean*

$$h(t) = (g(t) + g(t+p) + g(t+2p) + \cdots + g(t+(n-1)p))/n \tag{2.2}$$

for $t \in [0, p)$, where $g(t) = f(t) - T(t)$, giving raise to a function $S'(t)$. $S(t)$ is then modelled to be $S'(t)$, repeating with period p. With weekly resolution $p = 52$ for all series but Norwegian water, which has $p = 13$, the following parameters were found

Table 1: *coefficients of the polynomials for the deterministic seasonal component of the time series*

series	a_6	a_5	a_4	a_3	a_2	a_1	a_0
wind NO	$5.27 \cdot 10^{-9}$	$-9.29 \cdot 10^{-7}$	$6.43 \cdot 10^{-5}$	$-2.22 \cdot 10^{-3}$	$3.82 \cdot 10^{-2}$	$-2.51 \cdot 10^{-1}$	$-9.39 \cdot 10^{-3}$
wind DE	$-2.13 \cdot 10^{-9}$	$3.41 \cdot 10^{-7}$	$-1.85 \cdot 10^{-5}$	$3.07 \cdot 10^{-4}$	$2.82 \cdot 10^{-3}$	$-6.33 \cdot 10^{-2}$	$-2.31 \cdot 10^{-1}$
load NO	$-8.43 \cdot 10^{-10}$	$1.41 \cdot 10^{-7}$	$-8.02 \cdot 10^{-6}$	$1.41 \cdot 10^{-4}$	$1.25 \cdot 10^{-3}$	$-2.78 \cdot 10^{-2}$	$-1.61 \cdot 10^{-1}$
load DE	$6.31 \cdot 10^{-10}$	$-5.84 \cdot 10^{-8}$	$9.60 \cdot 10^{-7}$	$3.09 \cdot 10^{-5}$	$-6.72 \cdot 10^{-4}$	$3.58 \cdot 10^{-3}$	$-5.88 \cdot 10^{-2}$
water NO	$1.14 \cdot 10^{-5}$	$-3.98 \cdot 10^{-4}$	$5.36 \cdot 10^{-3}$	$-3.62 \cdot 10^{-2}$	$1.26 \cdot 10^{-1}$	$-1.40 \cdot 10^{-1}$	$-1.49 \cdot 10^{-1}$
solar DE	$1.87 \cdot 10^{-8}$	$-3.26 \cdot 10^{-6}$	$2.06 \cdot 10^{-4}$	$-5.69 \cdot 10^{-3}$	$6.44 \cdot 10^{-2}$	$-2.73 \cdot 10^{-1}$	$10.23 \cdot 10^{-1}$

For Norwegian water, the data we collected has only monthly resolution. We extended the annual data in $p = 13$ intervals of same length by averaging such that the data is compatible with 52 weeks.

2.7 residuals

In the log-transformed case, the residuals $X(t) = f(t) - S(t) - T(t)$ are shown in figure 3.

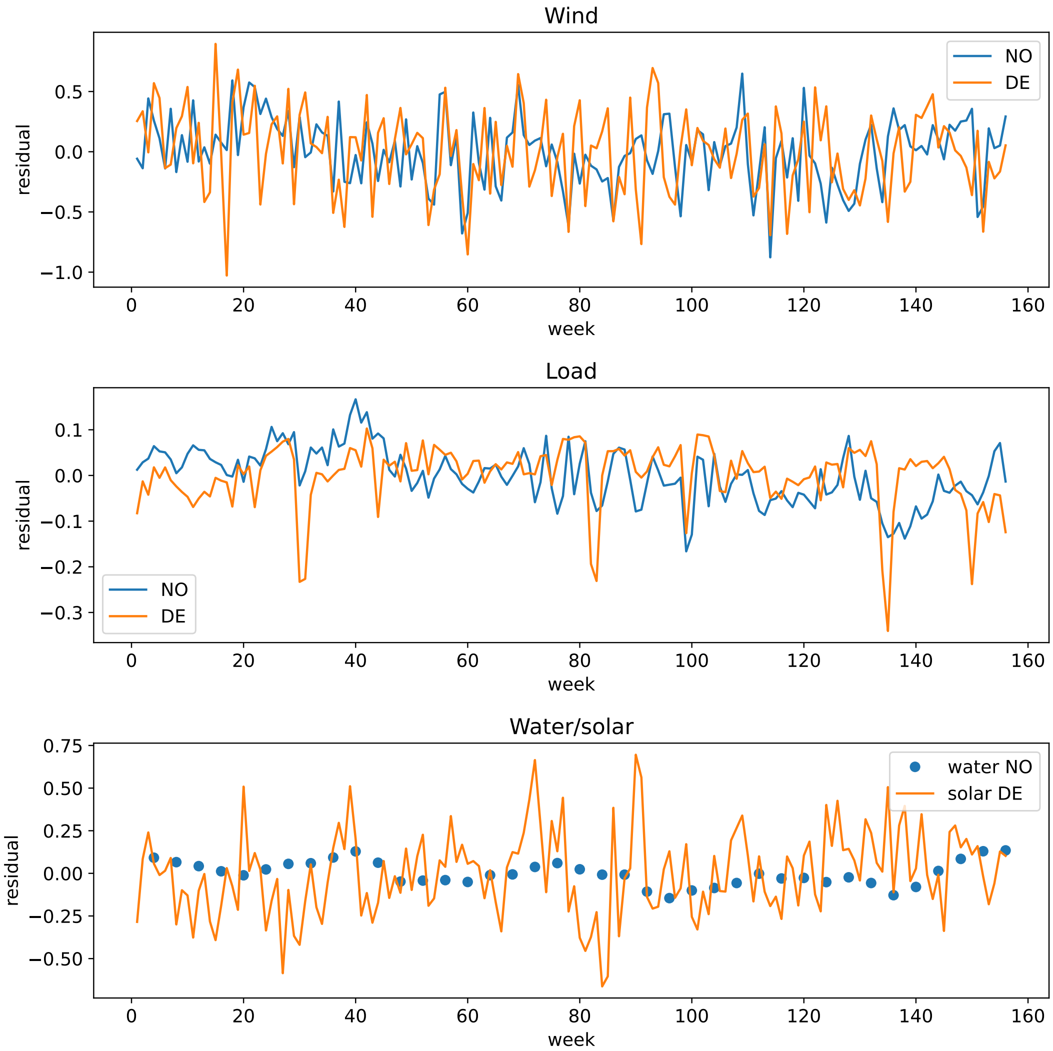

Figure 3: *residuals of the log-transformed loads, seperated after criterium*

Except for the Norwegian load, all series have "passed" the Augmented Dickey-Fuller and the KPSS-tests and we can hence assume they are stationary, but we will still model the Norwegian load as time series.

We modelled each time series as an AR(x) series of degree x up to three, and found the following data by maximum likelihood estimation

Table 2: *coefficients of the AR(x) series for each of the residuals including standard error of the parameters, as gotten by statsmodels' fit function for x up to three.*

Time series	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\sigma}^2$
wind NO	0.184604±0.077589	0	0	0.077322± 0.008715
wind DE	0	0	0	0.124229± 0.015349
load NO	0.669570±0.069938	-0.205016±0.074157	0.303462±0.074161	0.001737± 0.000173
load DE	0.626391±0.056887	-0.225152±0.085079	0	0.003344± 0.000305
water NO	1.189324±0.150155	-0.484997±0.143452	0	0.001406± 0.000382
solar DE	0.341299±0.067054	0	0	0.053133± 0.005824

With this model, we created an example plot to see if it, in general, resembles the shape of the

”correct” time series. This is shown in figure 4.

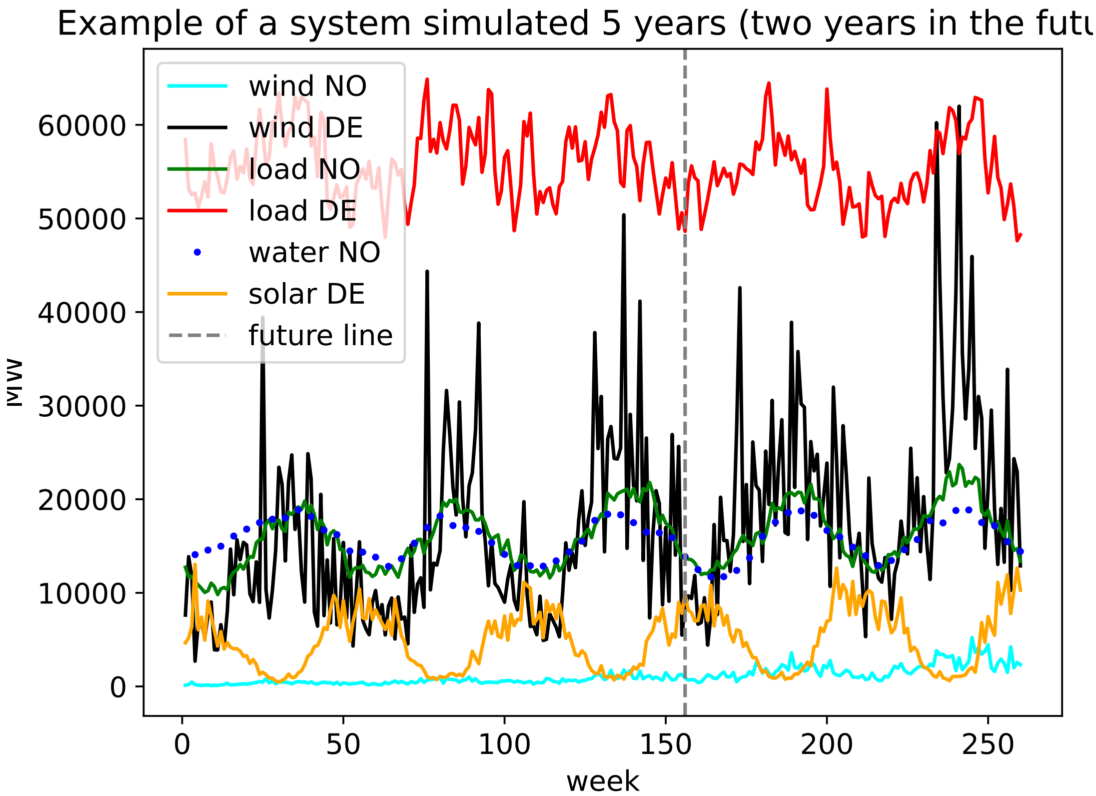

Figure 4: *Time series from june 2017 to june 2022 (weekly resolution) as simulated by the model*

We also checked for correlations between the time series by visually inspecting the cross-correlation of the deviations between real data and the models (that is, the residuals' residuals) between German wind and Norwegian wind, German wind and German sun, German load and Norwegian load as well as Norwegian load (in monthly resolution) (in monthly resolution) and Norwegian water as described in [3]. This is shown in figure 5.

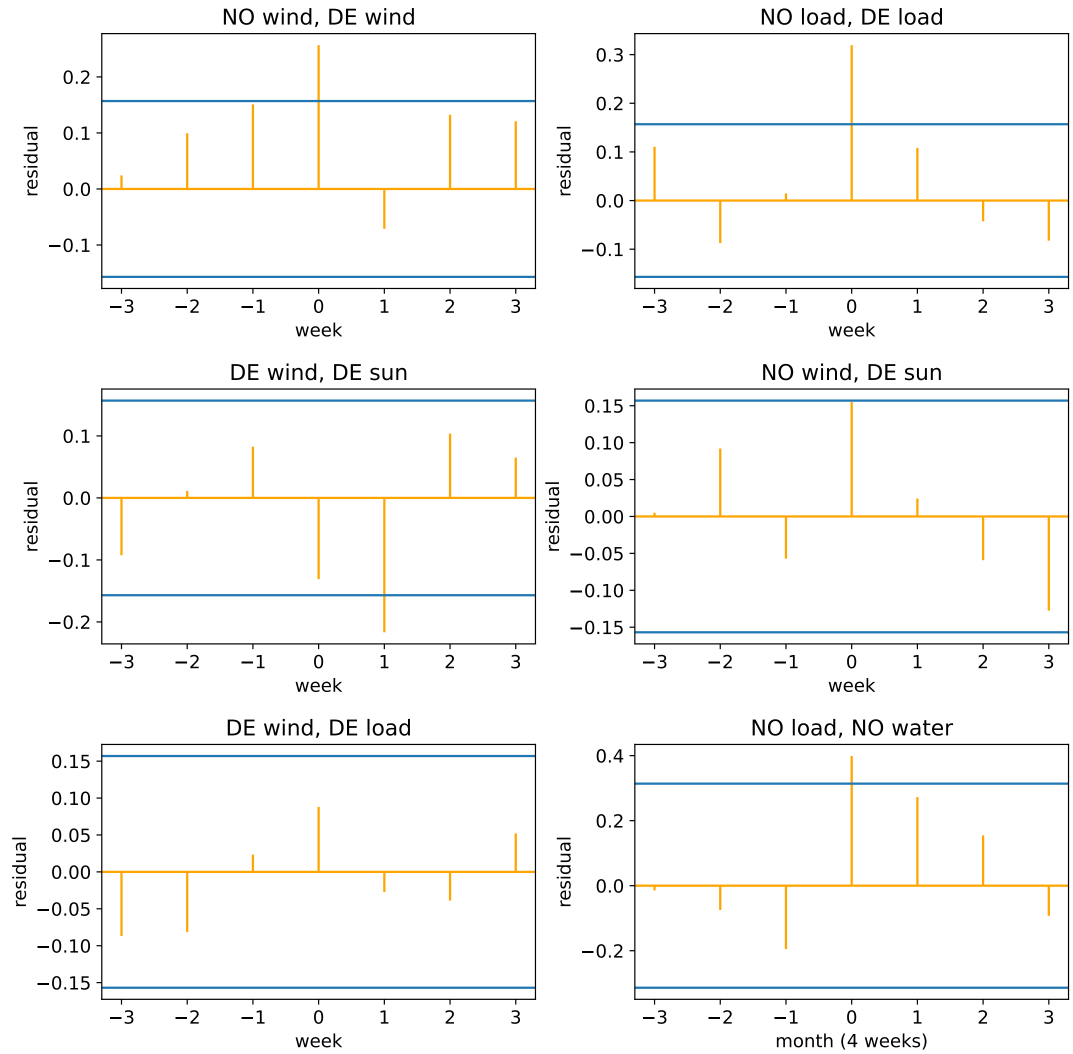

Figure 5: *Correlations between the residuals of the fitted AR-models. Weekly scale, except for the Norwegian load-water, which is the correlation between the mean value (over 4 weeks) of the load and the water. The drawn bounds represent* $\pm 1.96/\sqrt{n}$*. We see correlations between German wind and Norwegian wind and German sun, respectively, as well as between German/Norwegian load and Norwegian load/water.*

We see that Norwegian wind and German wind is correlated, as well as German wind and German solar. This is intuitively meaningful, as wind patterns across Europe are not independent, and lots of sun is negatively correlated with lots of wind. Similarly, we see that Norwegian and German loads are correlated at lag 0, which is also meaningful as both countries have the same seasons (to different extends) and similar holiday patterns. Finally, we see a correlation between Norwegian load and water - which is meaningful, as electricity production from water reservoirs can be matched to the demand.
We will hence model the wind and solar production as one dependent VAR(1) model, and the Norwegian and German loads as a VAR(3) modelThe dependence between Norwegian load and water is a slight issue in terms of modeling due to the different time scales. The cheap solution is to assume that water can be stored, such that the per-year production has to match the demand, but not the monthly production, but as water production is a function of weather, too, this is not a good solution..alternatively, we can model water on a weekly scale and correlate it to the Norwegian (and German) load. As the dependence is at lag=0, yet another idea, which is what we end up doing, is to condition the Z_t value of the water production on the random value of the load, which we sample from independently, and then use that the conditional distribution of a multivariate normal distribution (which we use for the residuals) is itself normal distributed,

$X_1 \mid X_2 = x_2 \sim \mathcal{N}\left(\mu_1 + \frac{\sigma_1}{\sigma_2}\rho\left(x_2-\mu_2\right), \left(1-\rho^2\right)\sigma_1^2\right)$. This is a little bit hacky and as a non-statistician, I feel like this is a little bit cheating (it is absolutely cheating!), but oh well...

The final model will hence be the following:

Norwegian wind, German wind and German solar will be modelled as follows, where in all cases $\mathbf{Z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$.

$$\begin{bmatrix} X_{wind_{NO},t} \\ X_{wind_{DE},t} \\ X_{sol_{DE},t} \end{bmatrix} = \begin{pmatrix} 0.213521 & -0.066938 & -0.033662 \\ 0.291248 & -0.119213 & -0.321294 \\ -0.061305 & 0.071481 & 0.369008 \end{pmatrix} \begin{bmatrix} X_{wind_{NO},t-1} \\ X_{wind_{DE},t-1} \\ X_{sol_{DE},t-1} \end{bmatrix} + \mathbf{Z}_t \tag{2.3}$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.079352 & 0.024967 & 0.010698 \\ 0.024967 & 0.117833 & -0.008910 \\ 0.010698 & -0.008910 & 0.053757 \end{pmatrix} \tag{2.4}$$

Similarly, we get for the load

$$\begin{aligned} \begin{bmatrix} X_{load_{NO},t} \\ X_{load_{DE},t} \end{bmatrix} &= \begin{pmatrix} 0.612895 & 0.096800 \\ -0.031218 & 0.653266 \end{pmatrix} \begin{bmatrix} X_{load_{NO},t-1} \\ X_{load_{DE},t-1} \end{bmatrix} + \begin{pmatrix} -0.159204 & -0.063605 \\ -0.044460 & -0.260389 \end{pmatrix} \begin{bmatrix} X_{load_{NO},t-2} \\ X_{load_{DE},t-2} \end{bmatrix} \\ &+ \begin{pmatrix} 0.342012 & -0.078640 \\ 0.050070 & 0.056468 \end{pmatrix} \begin{bmatrix} X_{load_{NO},t-3} \\ X_{load_{DE},t-3} \end{bmatrix} + \mathbf{Z}_t \end{aligned} \tag{2.5}$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.001780 & 0.000843 \\ 0.000843 & 0.003491 \end{pmatrix} \tag{2.6}$$

For the water, we follow a different approach: We will model it as an AR(1) model with the same parameters as in table 2, but correlate the randomness with the load. For water and the Norwegian load, As the main correlation happens at lag=0, we fitted a VAR(0) model and got on a monthly scale

$$\begin{bmatrix} X_{load_{NO},t} \\ X_{water_{NO},t} \end{bmatrix} = \mathbf{Z}_t \tag{2.7}$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.002465 & 0.002007 \\ 0.002007 & 0.005041 \end{pmatrix} \qquad \mathbf{\Sigma}/4 = \begin{pmatrix} 0.000616 & 0.000502 \\ 0.000502 & 0.001260 \end{pmatrix} \tag{2.8}$$

The random variable for the Norwegian load will be modelled from eq. (2.6), and we will then use that the conditional distribution of $(Z_{water_{NO},t} | Z_{load_{NO},t} = z_{load_{NO},t})$ itself is normally distributed. We will however model the water as a weekly function, too. To do so, we use that two the sum of two independent multivariate distributions also is multivariate normal, thus $\mathbf{\Sigma}/4$ can be used on a weekly basis. Finally, the auto-regressive step is only performed every 4 weeks, and the average of the weekly steps is used such that

$$\begin{aligned} M_t &= \frac{1}{4}\left(X_{t-1} + X_{t-2} + X_{t-3} + X_{t-4}\right) \qquad && \text{if } t \bmod 4 = 0 \\ M_t &= M_{t-(t \bmod 4)} && \text{otherwise} \\ X_t &= M_{t-(t \bmod 4)} + (Z_t | Z_{load_{NO},t} = z_{load_{NO},t}) \end{aligned} \tag{2.9}$$

An example of such a simulation is shown in figure 6.

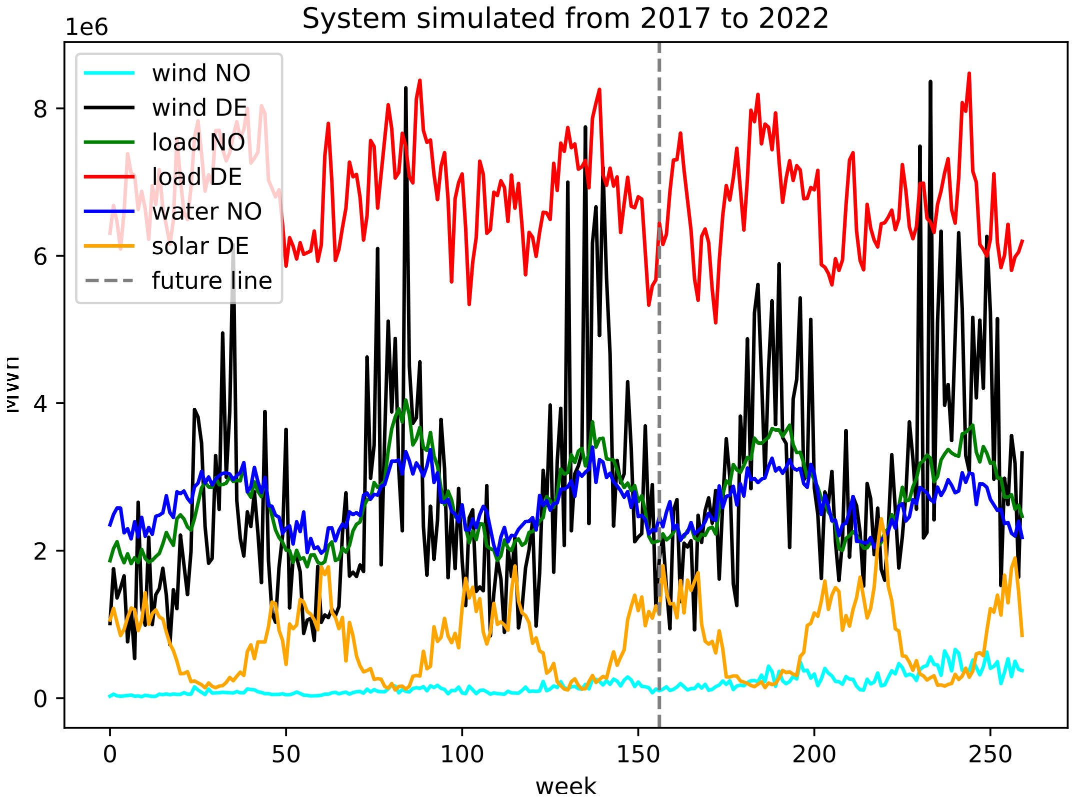

Figure 6: *Time series from june 2017 to june 2022 (weekly resolution) as simulated by the enhanced model*

3 Models

With the time series described, we will model a series of different cases. In all models, the basis assumptions are the following

- The energy production and consumption is described by the previously described time series.
- Germany uses green energy whenever available (wind/sun). The rest energy to cover the consumption is provided by coal and gas. *We do not model that coal power plants cannot be turned off that easily.*
- If Germany overproduces green energy (lots of sun/wind), the energy will either go to waste or be send to Norway, depending on the case.
- Norway's consumption is first covered by its own wind, followed by what is send from Germany, and finally water.
- Water not used can be stored for later use. This way, German wind can be used to save Norwegian water, which can either be send back to Germany or used in Norway in later time steps.

The following cases will be considered:

Case 0: The cable between Norway and Germany is not used. That is, Norwegian and German overproduction will go to waste. The value of interest is the cumulative in such a scenario, as well as the Norwegian "overproduction" and the German "overproduction" (how much energy goes to waste). This serves as a "ground" assumption.

Case 1: The cable between Norway and Germany is used. Germany sends energy to Norway whenever there is a surplus of wind/sun in Germany and Norway is not fully covered by wind (the latter of which is practically always the case). This energy is used instead of using Norwegian water, which is "saved". If the surplus surpasses the cable's capacity, it goes to waste. Norway sends energy to Germany whenever Germany is not fully covered by wind and solar energy and Norway has a surplus in energy production that week. If Germany is overproducing in the same week, the energy goes to waste. There are a couple of design choices, such as:

- When to send the energy (same or next time step). We will model it as same-step (what we call "delay 0"), unless otherwise stated.
- How long the "saved" water can be stored and how it can be tapped. Here,

Case 2: This is very similar to Case 1, but under the restriction that Germany uses Norway as a "storage" and does not have net import from Norway. In practice, the following case will be modelled: Norway exports electricity to Germany, but not more than it itself expects to get from Germany in one year, see section 5.1. That means that there will still be net export from Norway to Germany (or the other way around), but it is expected to peak at $\mu = 0$. Germany will still send wind to Norway whenever it can.

Case 3: Taking into account that Norway plans to electrify its platforms, we can use the Norwegian surplus to instead electrify the platforms (assuming this is easily possible and cables exist, which they don't at this time). This is then compared to Case 1.

For each of the five cases, we will simulate the model n times for m years, starting at year Y under different random conditions and thus get probability distributions over the different values of interest, which usually is the (combined) carbon dioxide emissions of Norway and Germany.

4 Computational Implementation

All data was created with Python. Libraries for Numerical Computing were used excessively, especially `statsmodels` which was used for parameter estimation for the (V)AR-models, as well as `NumPy`, `Matplotlib`,`scikit-learn`,`SciPy` and `pandas`. All code is available on GitHub.[13]

4.1 Test functions

Test functions were implemented to check the correctness of the data, such as the test that the sum of the deterministic functions and the residuals indeed is the real data. We tested that Case 1 reduces to Case 0 when the cable capacity is set to zero.

4.2 Random Numbers

In order to compare runs of two different models (especially for a small number of simulation steps), we made sure that both models create the same random numbers by getting the same seed.

5 Results & Discussion

5.1 Wind overproduction

Figure 7 shows the probability distribution of the German wind overproduction (the energy that surpasses the German consumption and would go to "waste" if there was no cable), simulated for one year from June 2020 to June 2021, running with $n = 100,000$ simulations. It also shows the

[13] https://github.com/schraderSimon/NorwayGermanyProject

wind overproduction which can be sent to Norway - this is a different number, as the cable's capacity is finite.

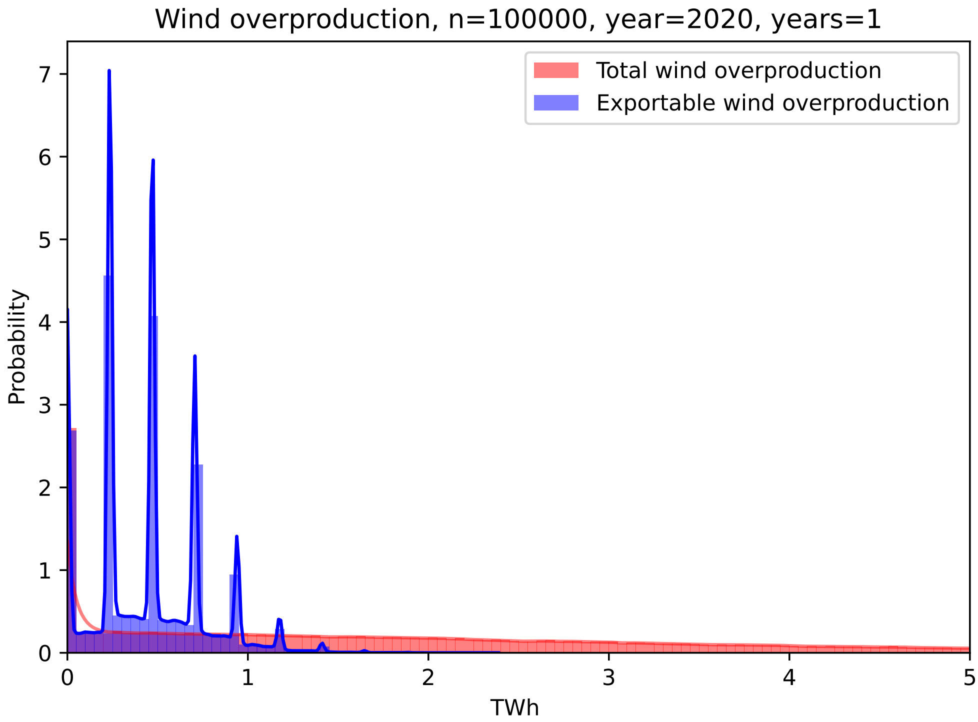

Figure 7: *Wind overproduction and wind that can be send to Norway for a simulation for one year from June 2020 to June 2021. Outlayers over 5 TWh are not included in the plot - which is approx. 10% of times. Graphs created using Seaborn's Kernel Density Estimator - which works badly for exponential-type distributions.*

We see that the wind overproduction is, in general, not so high. For the case of 2020-2021, this is expected - there will not be many days where Germany produces too much wind. In the model, the mean wind overproduction is 2.3 ± 2.3. The distribution of wind that can be send to Norway has quite a different shape, and it reflects the cable's maximum capacity. That way, we get that annually, 0.431 ± 0.303 TWh of wind are sent over to Norway.

Is this data reasonable? Mainly. The jagged behaviour of the wind send to Norway makes sense, as it simply reflects the number of weeks Germany produces too much wind. The cable's capacity is limited, so "very strong" wind is discarded. The shape of the total extra wind is reasonable, too. It is most likely that no wind will be overproduced, and the total reduction reminds of an exponential distribution, with higher overproduction values getting less and less likely. It might also be of interest to look at the average number of days Germany overproduces wind in this model, possibly in different years.

5.2 Comparing Case 0 and Case 1

Running with $n = 100,000$ simulations for a year from June 2020, the total carbon dioxide emissions of both Norway and Germany under case 0 and case 1 are depicted in figure 8. The Norwegian energy surplus is shown in figure 9, as well as the import-export balance (for case 1).

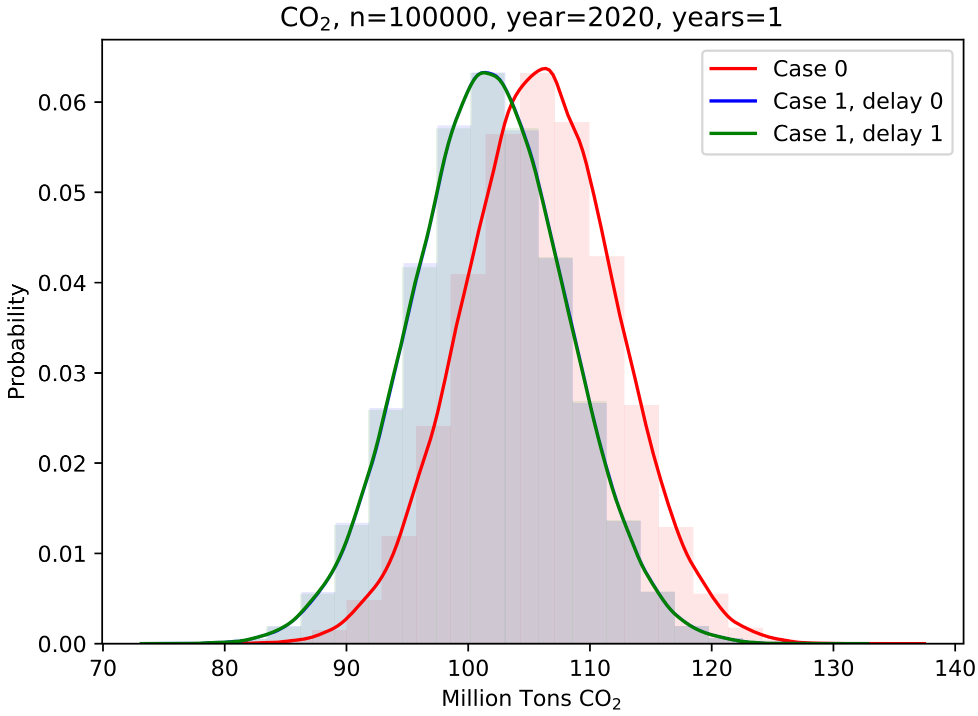

Figure 8: *Combined CO_2 emissions for Case 0 and Case 1, where electricity is send over either at the same time step as the surplus occurred, or at the next time step "delay". Graphs created using Seaborn's Kernel Density Estimator. Delay 0 is hidden behind delay 1.*

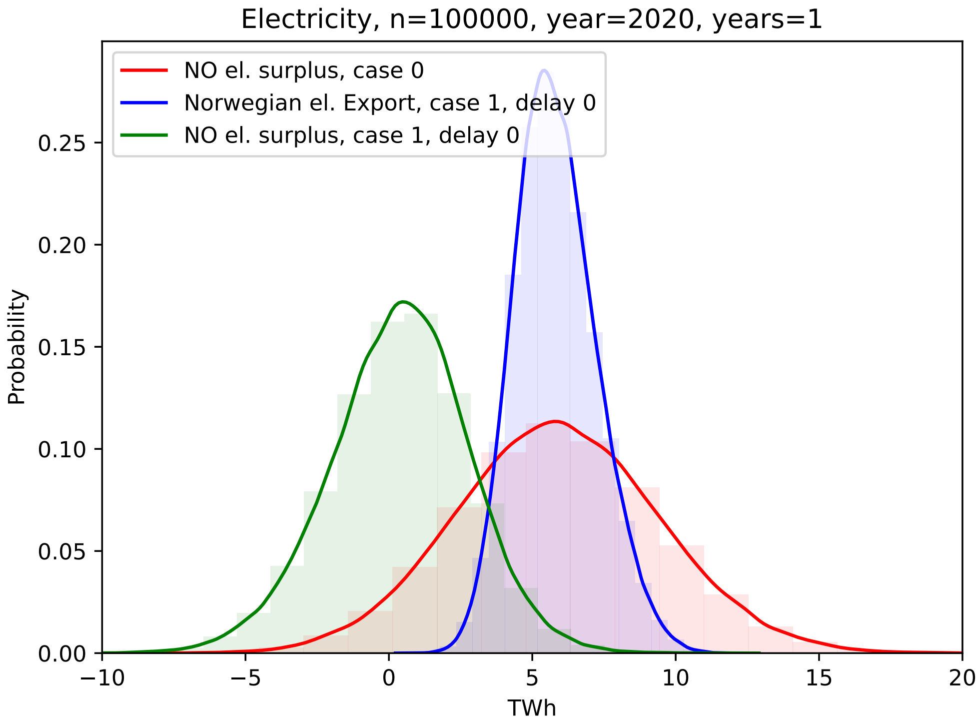

Figure 9: *Norwegian energy surplus and total electricity export for Case 0 and Case 1, where electricity is send over either at the same time step as the surplus occurred, or at the next time step "delay". Graphs created using Seaborn's Kernel Density Estimator.*

We see that the total emissions are reduced by 4.2 ± 0.9 million tons, going from 106 ± 6 Tons CO_2 to 102 ± 6. by using Norwegian "green" energy (which is already being produced in this model) in

Germany instead of coal, and sending German weekly surpluses to Norway whenever possible, as was done in model 1. As no restrictions were placed on the Norwegian energy production, the negative values are easily explained - even for Case 0, when there is no export! We can however still see that the mean value for case 1 lies above 0 (0.3 ± 2.4), with an average export of 5.8 ± 1.4 TWh. We see that the energy export of Case 1 in this model peaks around the same value as the energy surplus of Case 0 - on average, all Norwegian "surplus" is hence used by Germany, sometimes more, sometimes less.

It is an interesting observation that the surplus-curve has clearly less spread in Case 1 than Case 0 - combined with the fact that sending energy on surplus days from Norway to Germany still keeps an (on average) negative energy surplus.

Is this data reasonable? We think so. Looking at the load and water data of figure 1, the insecurity in the water production and the load make it seem reasonable that Norway may lie above or below the mean for consumption and the water production by several TWh, in addition to the innate insecurity of weather.

5.3 Comparing Case 0 and Case 2

First, we compared with $n = 10,0000$ simulations for a year from June 2020 to June 2021. This is not a very interesting case, as Germany cannot import more than 0.41 TWh electricity from Norway, due to the very low number of overproduction of wind. We see a reduction of the German CO_2 emissions of around $0.289 \pm 0.006\pm$ Million tons, and the Norwegian export number peaks at 0 with a very small deviation, as expected. The shape of the surplus is visually the same, with jagged behaviour as in figure 7.

A much more interesting scenario arises when assuming that the time series keep behaving the way they do also several years in the future. We simulated for a year from June 2022 to June 2023, where the wind production in both Norway and Germany keep rising, with the German load just so slightly decreasing, the Norwegian wind increasing and the Norwegian water production constant.

The change in the combined carbon dioxide emissions is shown in figure 10. The Norwegian exports as well as the Norwegian energy surplus is shown in figure 11.

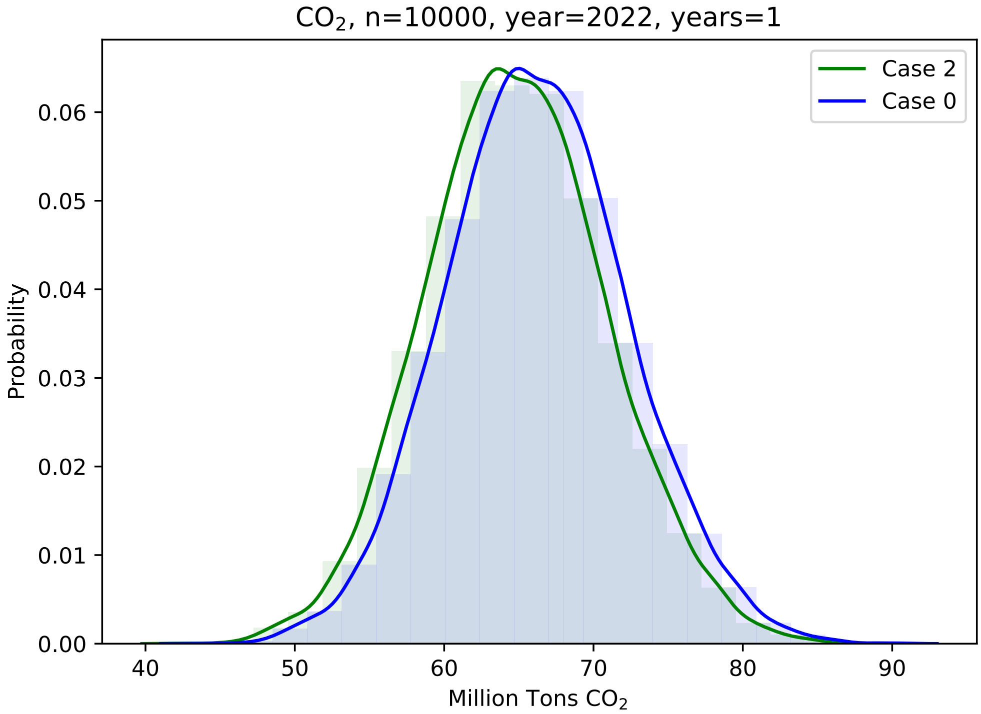

Figure 10: *Combined CO_2 emissions for Case 0 and Case 2. Graphs created using Seaborn's Kernel Density Estimator.*

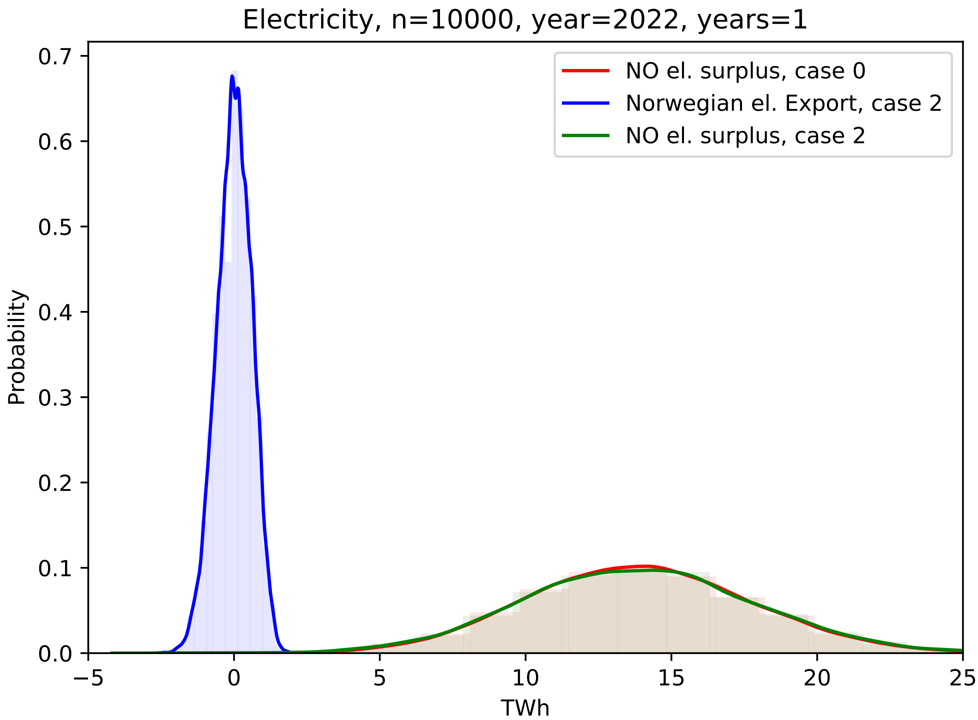

Figure 11: *Norwegian energy surplus and total electricity export for Case 0 and Case 2, where electricity is send over either at the same time step as the surplus occurred, or at the next time step "delay". Graphs created using Seaborn's Kernel Density Estimator.*

In this scenario, Germany will send to Norway on average 2.04 TWh electricity, which leads to a CO_2 reduction of 1.37 ± 0.03 Million tons CO_2. Not surprisingly, the Norwegian surplus like 13.9 ± 3.9 TWh for case 0 and 13.9 ± 4.0 TWh for case 2. We see hence that Norway's electricity profile remains more or less the same, but Germany reduces its emissions by "borrowing" Norwegian water. The same results do indeed hold for $2020 - 2021$, but much less energy is imported or exported. As the probability distribution of the surplus is more or less unchanged, Norway has "nothing to lose" [ignoring everything that has with cost and price optimization to do, of course] by participating in such a model. Even if the model is extremely wrong in predicting the trends and the German or Norwegian wind do not raise as fast as predicted, or Norways load should be indeed constant - in our opinion, this stands as a very strong result that I should double check by playing around with the trend parameters.

6 Conclusion

APPENDIX

References

[1] Open Power system data. “Data Package Time series. 2020”. In: (). DOI: 10.25832/time_series/2020-10-06 (cit. on p. 1).

[2] Rosendahl, K., Gavenas, E., and Skjerpen, T. “CO2-emissions from Norwegian oil and gas extraction”. In: *Working paper* 90 (July 2015). DOI: 10.1016/j.energy.2015.07.025 (cit. on p. 3).

[3] Brockwell, P. J. and Davis, R. A. *Introduction to time series and forecasting*. Springer, 2016 (cit. on p. 7).