A stochastic model for emission reduction by utilizing the Nordlink cable

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Abstract

With NordLink, a cable with a capacity of 1400 MW exists between Norway and Germany. Building a stochastic model based on existing data for Norwegian and German electricity production and demand [1], we found that the cable can be used as an effective way to reduce German emissions by exporting German renewable surplus production on windy days to Norway and importing that same amount later from Norway without affecting Norway's energy surplus probability distribution. Based on a mean reduction of CO_2 emissions of 710 g/kWh for electricity imported from Norway, we found that electrification of the oil/gas platforms is inferior to an increased export to Germany in terms of total emission reduction, giving only a reduction of 425 g/kWh based on numbers from the Norwegian oil directorate [2]. However, combining both approaches makes the best use of Norwegian surplus. Finally, we found that using German wind surplus could be used to cover Norwegian deficits if Norway did not overproduce, especially as German wind production increases in the years to come.

1 Introduction

In this article, we explore how the newly built Nordlink cable between Norway and Germany can be used to decrease both countries' combined carbon dioxide emissions. With Germany's electricity production heavily based on fossil fuels, importing green electricity from Norway can serve to reduce German emissions. However, there are some days where there is such a high wind and/or solar energy production that the production of renewable energy alone surpasses Germany's demand. Norway on the other hand, while only using renewable energies in its electricity production, requires extra power to electrify its oil platforms, which also serves to reduce CO_2 emissions. The cable can hence be used to either send German surplus to Norway, or to send Norwegian surplus to Germany, ignoring all connections Norway and Germany have to other countries. Based on time-series data for Norwegian and German electricity production and demand and emission numbers available online, as described in section 2.1, we built a stochastic model to consider different possible uses of this cable (called *cases*), which all aim at reducing the carbon dioxide emissions in Norway and Germany combined. The time series used in the stochastic model are modelled as vector autoregressive models, and described in detail in sections 2.2-2.4, where we explain the relevant theory and the models we used. In section 3, we describe the underlying assumptions of the model and the different cases, which try to model the following situations: 1. Germany imports from Norway, which does not electrify its platforms; 2. Germany uses Norway as battery for its own electricity, that is, German surplus is stored in Norwegian water reservoirs for later use; 3. Norway electrifies its platforms, but also imports from and exports to Germany, with export to Germany being prioritized over platform electrification; and finally 4. which is the same as 3., but Norway has an electricity deficit due to not producing wind power. In section 4, we describe how this was implemented computationally.

In section 5, we present our results and discuss them for 2020 and 2022 respectively, with the 2022 dynamics differing from the 2020 dynamics mainly stemming from an increased wind production in both Norway and Germany. Finally, we summarize our results and suggest different ways to build and improve upon our models in section 6. Different extra models and considerations, which serve as a robustness check, are considered in the Appendix.

2 Theory & Methods

2.1 Raw data

The time series for the electricity demand in Norway and Germany are acquired from [1], which contains time series for per-hour Norwegian and German electricity load, onshore wind production and, in the case of Germany, solar energy production, which is the majority of the data required. To estimate Norwegian water production, we use data from SSB's electricity balance.[1] Observe that the available data represents the electricity balance, not the total production of electricity from water, so these numbers are just an approximation to the real production.

In addition to solar energy and wind, Germany has three additional types of renewable energies: Water, bio mass & nuclear. The share of these energies can be found here.[2]

For water, biomass and nuclear in 2019 stood together for approx. 15 GW, with similar values in 2017, 2018 and 2020. I simply subtracted that constant number from the German load (which is the same as assuming a constant production) in the model. Observe that nuclear power stands for approximately half of the non-wind non-solar renewable energy sources. In this simulation, we ignore the fact that Germany plans to shut down its nuclear power plants in 2022.[3]

Based on our data, the Norwegian wind production would be 5.2 TWh in 2019, compared to the correct value of 5.5 TWh, which also takes into account offshore wind. To account for this, we multiplied the wind production with a factor of 5.5/5.2 as to take into account offshore wind which is not present in the data. Similarly, we added 2% of the mean value of the water production to each time step to take into account thermal energy used in Norway, which stands for 2% of the electricity production.

The time series for 2017-2020 are found here, as well as a trend and seasonal component in dotted lines, which is described in section 2.4. To get weekly data, we simply averaged the time data over 168 hours. There was a single gap in the data for the Norwegian load and the Norwegian wind production, which we simply interpolated. The original data and the corresponding trend can be seen in figure 1, with the log-transformed data in figure 2. The last day of each year (31st May) has been removed from the dataset, such that the number of points is divisible by 52 (the number of weeks in a year).

[1] https://www.ssb.no/statbank/table/12824
[2] https://energy-charts.info/charts/energy/chart.htm?l=en&c=DE&year=2020
[3] https://www.bundesregierung.de/breg-de/themen/energiewende/energie-erzeugen/ausstieg-aus-der-kernkraft-394280

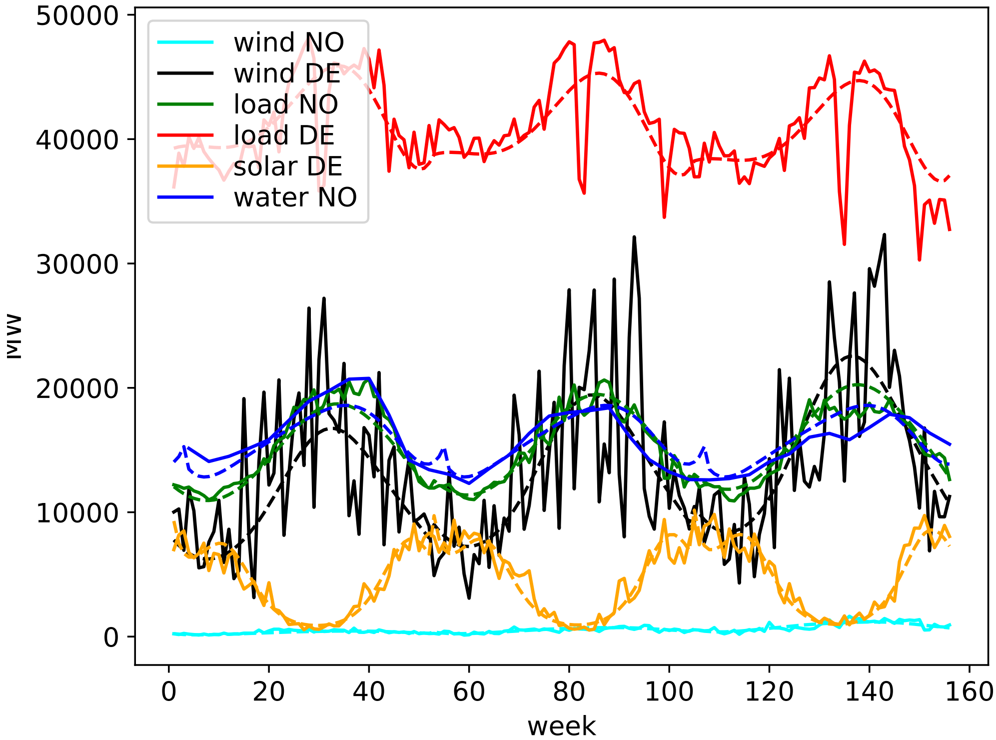

Figure 1: *Time series from june 2017 to june 2020 (weekly resolution) for the production and demand of electricity in Germany and Norway, "DE" standing for Germany and "NO" standing for Norway, with the trend + season as dotted lines.*

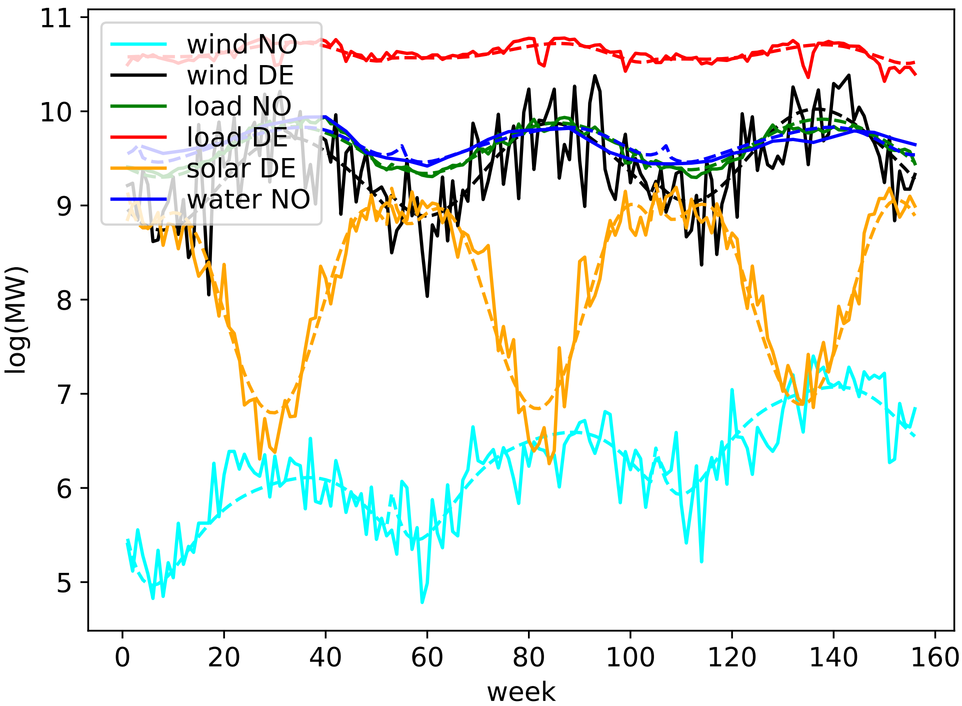

Figure 2: *Time series from june 2017 to june 2020 (weekly resolution) for the production and demand of electricity in Germany and Norway, with the trend + season as dotted lines.*

The trend is modeled as a linear function in log space, while the seasonal data is modelled as a simple-6th order polynomial. This function is noncontinuous at the boundary conditions, which is also visible in the graphs. However, we did not consider this as a too dangerous effect - the only drawback is that it possibly induces a sign change in the residuals at multiples of the period.

The water trend is slightly decreasing. As the water numbers represent the energy balance, not the water production, or even the "maximum possible water-production that is long-term possible", we will model the real trend as a constant function - we assume the water production not to be decreasing, and claim that the negative trend comes from warm summers, cold winters and the over-availability of cheap imported electricity.

The carbon dioxide emissions in Germany are estimated from data found in this report [3]. While there are some differences between hard coal and brown coal, these differences are not substantial and neglected. Table 1 shows the share of coal and gas of the total electricity production, and the corresponding emissions in Germany.[4]

Table 1: *Emissions and share of electricity production in Germany of coal and gas.*

Type	coal	gas
g CO_2/kWh (min/med/max)	740/820/910	410/490/650
share of electricity production in Germany in 2020	24.1%	11.6%

Averaging, we get that the mean carbon dioxide emissions per kWh fossil fuel are $\frac{24.1\% \cdot 820 + 11.6\% \cdot 490}{0.241 + 0.116} \approx$ 710 g CO_2/kWh. This number is very close to what the German Umweltbundesamt suggests.[5]

There are many different numbers for the effect of electrifying the platforms. We will here follow two different scenarios. In the "good" scenario, we follow the assumptions of this article,[6] about 70% of the CO_2 emissions of the platforms, which total at approx. 13.9 million tons CO_2-equivalent can be "electrified away".[7] While this is slightly outdated, they assumed that 6.3 million tons CO_2 could be saved by using 9 TWh, or 700g CO_2/kWh. In general an extra complication arises from the fact that the emissions per extracted unit oil and gas rise as it gets harder to extract from a field [4], but I will ignore these effects.

A more realistic "bad" scenario is based on recent calculations. To assess the value of platform electrification, we follow data from [2] and this website.[8] Assuming that the approx. 8 TWh per year already (or soon to be) sent from land to the platforms each year leads to a reduction of 3,2 million tons per year, as stated in the report, the planned projects will require an annual electricity of extra 4 TWh per year and further decrease the emissions by 1.7 million tons per year. This calculation means that the recent projects will lead to a decrease of $\frac{1.7\text{million tons CO}_2}{4\text{TWh}} = 425\text{g } CO_2/kWh$, which is a substantially lower number, which seems more reliable, as the data relies on a recent calculation of planned projects.

The NordLink cable between Norway and Germany has a capacity of 1400 MW, and we assume an effect loss of $\sim 5\%$.[9] The theoretical maximum energy per year sent through the cable is thus $1400 \cdot 24 \cdot 365 \approx 12.3$ TWh/year - which also serves as a benchmark later on.

On a site note, we would like to suggest an alternative to platform elecrification: It is forecast that Norway's oil production the next couple of years will be at around 100 million Sm^3 oil equivalents per

[4] https://energy-charts.info/charts/energy_pie/chart.htm?l=en&c=DE&interval=year&year=2020

[5] https://www.umweltbundesamt.de/themen/klima-energie/erneuerbare-energien/solarenergie

[6] https://www.norskoljeoggass.no/contentassets/7b83dbc5c17e47d682f1f5eea87e5f69/alternativ-kraft-til-norsk-sokkel.pdf

[7] https://miljostatus.miljodirektoratet.no/tema/klima/norske-utslipp-av-klimagasser/klimagassutslipp-fra-olje--og-gassutvinning/

[8] https://www.regjeringen.no/no/aktuelt/kraft-fra-land-til-norsk-sokkel-og-elektrifisering-av-stor-landbasert-industri/id2721239/

[9] https://www.statnett.no/vare-prosjekter/mellomlandsforbindelser/nordlink/

year 2020-2025, and 115 million Sm^3 gas equivalents 2020-2025, approximately.[10] It is rather hard to estimate the carbon dioxide emissions per liter extracted oil. A very simple estimate, according to this website (this is US data), [11] is to estimate that about 47% are transformed to gasoline and 28% are transformed to diesel, 10% jet fuel, with the rest being "lost" in production or used differently. Assuming that the rest has no emissions, we can approximate that one liter of these fuels produces $\sim$ 2.5 kg CO_2 per liter, or 2500 kg CO_2 per Sm^3.[12] With $(47 + 28 + 10)\%$ being burned, this calculation implies that each Sm^3 Norwegian oil burned will lead to emissions of $86\% \cdot 2500 = 2100$ kg CO_2/Sm^3. For gas, one Sm^3 oil equivalent corresponds to 1000 Sm^3 gas. According to data from this website,[13] 1000 Sm^3 natural gas creates $\sim$ 1880 kg CO_2 (tailpipe emissions). Combining this with the emissions from the production of oil, Norwegian oil stands for 440 million tons CO_2 equivalents per year. We assume that we can eradicate a realistic maximum of 70% of 13.9 million tons = 9.7 million tons CO_2 emissions by electrification of the platforms. How much less oil would we need to produce to reach the same number? Solving $x(440 + 13.9) = 9.7$ for x, we get that the same reduction in emissions could be reached by simply producing 2.1% less oil and gas (where we ignore the fact that electrifying the platforms might possibly lead to more gas being produced, as it is not used for electricity production). As no full electrification is planned, a reduction of oil production of less than 2% would have the same climate effect as the plans of the Norwegian goverment and Equinor.

To summarize the numbers used in this article:

- Emissions saved by electrified platforms: 425 g CO_2/kWh, assuming 4 TWh are used to eradicate 1.7 million tons (similar number for 12 TWh to eradicate 4.9 million tons). This will be called the *bad scenario*, which we consider to be the realistic scenario, as it is based on more recent calculations.
- Emissions saved by electrified platforms: 700 g CO_2/kWh. This will be called the *good scenario*.
- Emissions by using fossil energy in Germany: 710 g CO_2/kWh.
- Emissions of renewable energy (different ranges and different data in general is presented in [3] and [5], so these numbers should be interpreted with care):
  - Wind: 11 g CO_2/kWh [5]
  - Solar: 45 g CO_2/kWh [14]
  - Water: 18.5 g CO_2/kWh [5]
  - Nuclear: 12 g CO_2/kWh [3]
  - Biomass: 43 g CO_2/kWh [5]
- The same reduction of carbon dioxide emissions as a full electrification of the platforms would give, can also be reached by reducing the oil/gas production by 2.1%.
- The Nordlink cable can send up to 1400 MW, and 5% of effect is lost.
- Time series data for Norwegian and German load and wind production are available, as well as German solar energy and Norwegian water energy. From this data, a trend (linear in log space) and a seasonal component are extracted, which are assumed to be deterministic and also applicable in the future. The residuals are modelled as vector autoregressive models, which are described below.

[10] https://www.norskpetroleum.no/en/production-and-exports/production-forecasts/
[11] https://www.eia.gov/energyexplained/oil-and-petroleum-products/refining-crude-oil.php
[12] https://www.nrcan.gc.ca/sites/www.nrcan.gc.ca/files/oee/pdf/transportation/fuel-efficient-technologies/autosmart_factsheet_6_e.pdf
[13] https://www.eia.gov/environment/emissions/co2_vol_mass.php
[14] https://www.umweltbundesamt.de/themen/klima-energie/erneuerbare-energien/solarenergie

The total carbon dioxide emissions are the product of the production (in kWh) multiplied with the emissions per kWh from all energy sources, in addition to 13.9 million CO_2 from the platforms. (One might argue that a better model for wind and water would be to take the mean value of production as the emissions, but as this has no impact on the *reduction* in emissions through the model, this makes no difference anyways.)

2.2 Time series and autoregressive models

In this article, we model the data as time series.

In general, a time series is a function $f(t)$ of time t, where figures 1 and 2 are examples of such functions. We restrict the discussion to integer-values of t and hence use the two notations $X(t)$ and X_t interchangeably. We assume that the function $f(t)$ can be written as $f(t) = T(t) + S(t) + X(t)$, where $T(t)$ is a *deterministic* trend function, $S(t)$ is a deterministic seasonal function with period p, and $X(t)$ is a stochastic function. As $T(t)$ and $S(t)$ can be modelled as, for example, polynomial or harmonic functions, we will here focus on the stochastic part $X(t)$.

A desirable attribute of the stochastic function $X(t)$ is *stationarity*. A time series is stationary if its mean value and its covariance function are *independent of time*, that is

$$\begin{aligned} \mu_X &= \mathrm{E}(X_t) \quad \text{is independent of t.} \\ \gamma(h) = \gamma(t+h,t) &= \mathrm{Cov}(X_t+h, X_t) = \mathrm{E}[(X_{t+h}-\mu_X)(X_t-\mu_X)] \quad \text{is independent of t.} \end{aligned} \tag{2.1}$$

it should be mentioned that stationary is a weaker condition than *strict stationarity*, which apply to time series where $(X_t, X_{t+1}, \ldots, X_{t+n})$ needs to have the same joint probability distribution as $(X_{t+h}, X_{t+1+h}, \ldots, X_{t+n+h})$ for all values of h. Strict stationarity implies stationarity, but not vice versa. Stationarity is a reasonable assumption for time series, as it, in simple terms, indicates that "tomorrow is similar to today - if today is unrelated to the day before yesterday, then tomorrow is unrelated to today". Hence, the aim is to find functions $T(t)$ and $S(t)$ such that $X(t) = f(t) - T(t) - S(t)$ is stationary. This can sometimes be difficult/impossible, so it might be necessary to transform the original series $f(t)$ in such a way that the resulting series $X(t)$ becomes stationary, hence, we might instead have $T(t)+S(t)+X(t) = L(f(t))$, where $L(x)$ is a transform (for a strictly positive function $f(t)$, taking the logarithm $L(f) = \log(f)$ or the inverse $L(f) = \frac{1}{f}$ are examples of transforms) thus $f(t) = L^{-1}(T(t)+S(t)+X(t))$.

In this article, we will model the stochastic part $X(t)$ as an *autoregressive (AR)* model of order x (standard notation is p, but we use p for the period already). A time series $X(t)$ is an AR(x) process if it is stationary and has the form of a stochastic difference equation.

$$X_t = \phi_1 X_{t-1} + \cdots + \phi_x X_{t-x} + Z_t \tag{2.2}$$

where Z_t is white noise $\mathrm{WN}(0,\sigma^2)$, which often, such as in this article, is modelled as i.i.d. Gaussian noise $Z_t \sim N(0,\sigma^2)$. It has been shown that eq. (2.2) is stationary iff. $\phi(z) = 1 - \phi_1 z - \cdots - \phi_x z^x$ has no zeros for $|z| = 1$. Observe that detrending of the function $f(t)$ has the positive side effect that $X(t)$ has mean zero.

For given time series data, it is of interest to find parameters and the order that best explain the main trends in the data. To estimate the parameters and the number of parameters, there exist a plethora of methods. We will here use the BIC criterion and the maximum likelihood estimation, which is also what `statsmodels` uses on default. The details of what this entails can be read in [6].

Assume now that we have several time series that might or might not be independent of one another. After transforming and removing the trend and the seasonality, we will end up with a vector function $\mathbf{X}(t)$, which we will model as a *vector* AR-process (VAR)

$$\mathbf{X}_t = \Phi_1 \mathbf{X}_{t-1} + \cdots + \Phi_x \mathbf{X}_{t-x} + \mathbf{Z}_t + \mathbf{D}_t \tag{2.3}$$

where $\mathbf{Z}_t \sim \mathrm{WN}(\mathbf{0}, \mathbf{\Sigma})$ and Σ is the covariance matrix, and Φ_i are square matrices. $\mathbf{Z}_t$ will be modelled as multivariate Gaussian noise $\mathbf{Z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$. To test whether two time series are correlated, one fits an AR-model to each time series. The correlation function of the residuals (the "error" between the true data and the model's predictions, which should be white noise) of two time series can be used to discover correlations between time series. For values above/below $\pm 1.96/\sqrt{l}$ (where l is the number of data points in the time series), the correlations become statistically relevant, and a VAR-model might be more suitable than individual AR-models.

For a VAR-model, the parameters can be estimated using Ordinary Least Squares estimation, which is what we do in this article as this is the only estimator existing in `statsmodels`. To estimate a suitable order of autoregression, we use the "AIC" criterion to keep the order of autoregression low.

2.3 Stochastic modelling

Assume that we have one or several time series functions $\mathbf{X}_t = \Phi_1 \mathbf{X}_{t-1} + \cdots + \Phi_x \mathbf{X}_{t-x} + \mathbf{Z}_t + \mathbf{D}_t$ where $\mathbf{Z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ and $\mathbf{D}_t$ is a deterministic function. Assume that we consider the time series over a time range $\{t\}_{t=k}^{m}$ for $k \leq m$, and we are interested in the stochastic properties of a random variable which is a function of the time series at every step $Y = f(\{t\}_{t=k}^{m}) = f(\{\mathbf{X}_t\}_{t=k}^{m})$. As explicitly constructing the probability distribution P_Y of that random variable Y can be difficult or unfeasible, we can use the Monte Carlo method to approximate the true probability distribution. To do so, we

1. Initiate time step $j = 0$
2. For each time step considered, draw samples $\mathbf{z}_{t,j}$ following the distribution of the random variable $\mathbf{Z}_{t,j}$
3. For each time step considered, calculate $\mathbf{x}_{t,j}$ with $\mathbf{z}_{t,j}$
4. Calculate $y_j = f(\{\mathbf{x}_{t,j}\}_{t=k}^{m})$
5. Increase j by one, go back to step 2 until $j = n$, where n is the number of MC simulations.

Each sample y_j will follow the distribution of Y. For large n, a histogram over y_j will then resemble the true probability distribution of Y, and the law of large numbers guarantees convergence of all moments.

2.4 Model for the time series

As basis for the model building, we use the log-transformed model of figure 2. We assume that each of the log-transformed time series $f(t)$ is modelled as

$$f(t) = T(t) + S(t) + X(t) \tag{2.4}$$

where $T(t)$ is a deterministic trend function, $S(t)$ is a deterministic seasonal component with the period of one year, and $X(t)$ is a (stochastic) stationary time series.

2.4.1 Trend component

The trend component $T(t)$ is modelled as a linear function, and is simply the OLS fit. For water, the OLS fit is decreasing. As argued above, we will assume that it is, however, constant. With weekly resolution, the following parameters were found:

- wind NO: $T(t) = 0.009260t + 5.460278$
- wind DE: $T(t) = 0.002866t + 9.185670$

- load NO: $T(t) = 0.000761t + 9.557052$
- load DE: $T(t) = -0.000254t + 10.637291$
- water NO: $T(t) = 0.000000t + 9.656239$
- solar DE: $T(t) = 0.000868t + 8.113218$

where t is the number of weeks passed since june 1$^{\text{st}}$ 2017. It should be noted that the negative trend in the German electricity production is indeed predicted.[15] It should further be noted that this linear trend implies an exponential trend in the resulting series, which makes the time series a reasonable approximation for near-future predictions, but imprecise for far-future predictions.

2.4.2 Seasonal component

The seasonal components are modelled as a the best OLS fit of a 6th order polynomial of the function. This order was chosen as it leads to stationarity of the residual time series. Let n be the number of periods and p the period. Then we define the *seasonal mean*

$$h(t) = (g(t) + g(t+p) + g(t+2p) + \cdots + g(t+(n-1)p))/n \tag{2.5}$$

for $t \in [0, p)$, where $g(t) = f(t) - T(t)$, giving raise to a function $S'(t)$. $S(t)$ is then modelled to be $S'(t)$, repeating with period p. With weekly resolution $p = 52$ for all series but Norwegian water, which has $p = 13$ and t is number of four weeks, the following parameters were found

Table 2: *coefficients of the polynomials for the deterministic seasonal component of the time series*

series	a_6	a_5	a_4	a_3	a_2	a_1	a_0
wind NO	$5.27 \cdot 10^{-9}$	$-9.29 \cdot 10^{-7}$	$6.43 \cdot 10^{-5}$	$-2.22 \cdot 10^{-3}$	$3.82 \cdot 10^{-2}$	$-2.51 \cdot 10^{-1}$	$-9.39 \cdot 10^{-3}$
wind DE	$-2.13 \cdot 10^{-9}$	$3.41 \cdot 10^{-7}$	$-1.85 \cdot 10^{-5}$	$3.07 \cdot 10^{-4}$	$2.82 \cdot 10^{-3}$	$-6.33 \cdot 10^{-2}$	$-2.31 \cdot 10^{-1}$
load NO	$-8.43 \cdot 10^{-10}$	$1.41 \cdot 10^{-7}$	$-8.02 \cdot 10^{-6}$	$1.41 \cdot 10^{-4}$	$1.25 \cdot 10^{-3}$	$-2.78 \cdot 10^{-2}$	$-1.61 \cdot 10^{-1}$
load DE	$6.31 \cdot 10^{-10}$	$-5.84 \cdot 10^{-8}$	$9.60 \cdot 10^{-7}$	$3.09 \cdot 10^{-5}$	$-6.72 \cdot 10^{-4}$	$3.58 \cdot 10^{-3}$	$-5.88 \cdot 10^{-2}$
water NO	$1.14 \cdot 10^{-5}$	$-3.98 \cdot 10^{-4}$	$5.36 \cdot 10^{-3}$	$-3.62 \cdot 10^{-2}$	$1.26 \cdot 10^{-1}$	$-1.40 \cdot 10^{-1}$	$-1.49 \cdot 10^{-1}$
solar DE	$1.87 \cdot 10^{-8}$	$-3.26 \cdot 10^{-6}$	$2.06 \cdot 10^{-4}$	$-5.69 \cdot 10^{-3}$	$6.44 \cdot 10^{-2}$	$-2.73 \cdot 10^{-1}$	$10.23 \cdot 10^{-1}$

For Norwegian water, the data we collected has only monthly resolution. We extended the annual data in $p = 13$ intervals of same length by averaging such that the data is compatible with 52 weeks.

2.4.3 residuals

In the log-transformed case, the residuals $X(t) = f(t) - S(t) - T(t)$ are shown in figure 3.

[15] https://www.isi.fraunhofer.de/en/presse/2020/presseinfo-20-stromprognose.html

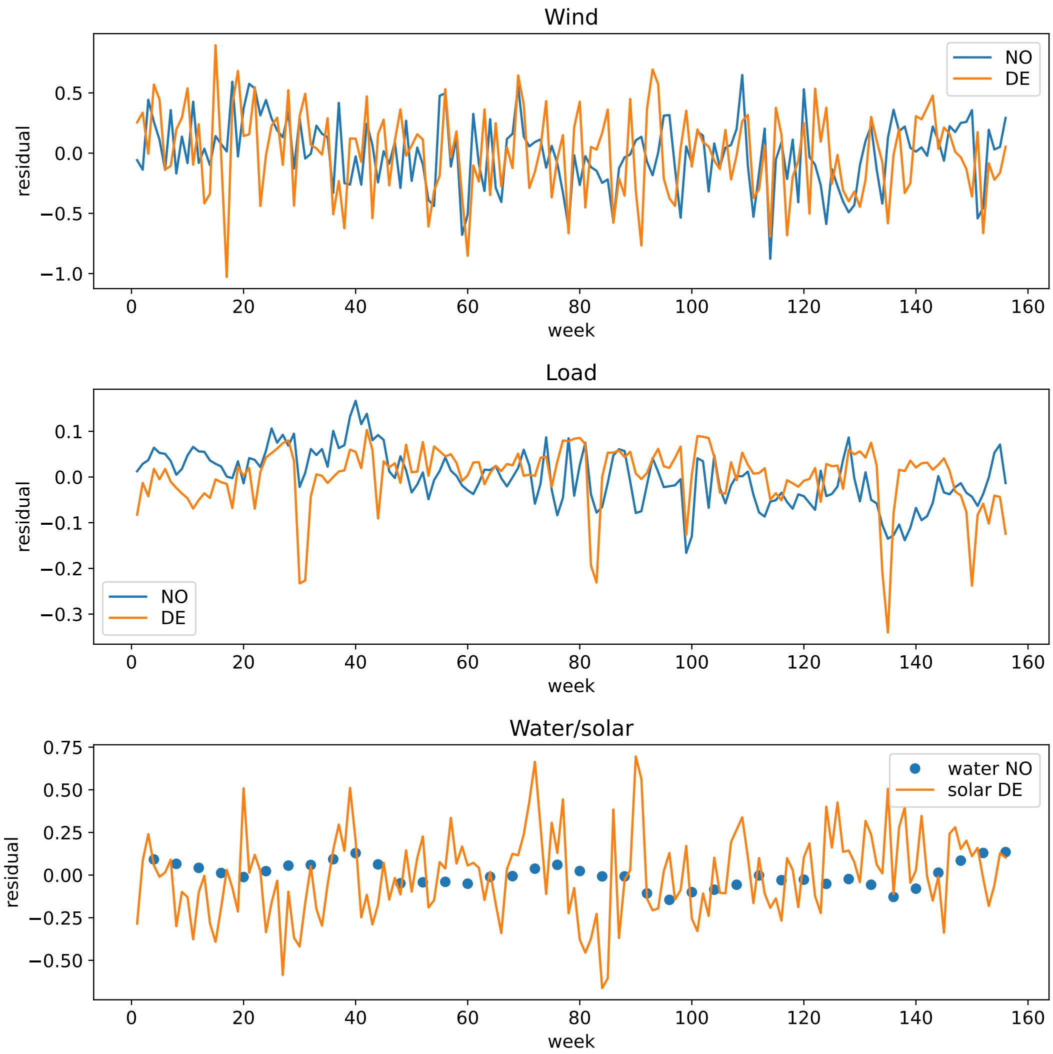

Figure 3: *residuals of the log-transformed loads and productions, separated after type*

Except for the Norwegian load, all series have "passed" the Augmented Dickey-Fuller ($p < 0.05$) and the KPSS-tests ($p > 0.05$) and we can hence assume they are stationary, but we will still model the Norwegian load as time series, despite the p-value of below 0.01 for the KPSS test - which we assume stems from the peculiarity that the Norwegian load, as a whole, is increasing, while the maximal values one gets in winter, are actually slightly decreasing, making it hard to fit a seasonal function to the data after the trend is removed - however, as we Norwegian load is only one single time series, we consider this to be an acceptable deficit.

We modelled each time series as an AR(x) series of degree x up to three, and found the following data shown in table 3 by maximum likelihood estimation.

Table 3: *coefficients of the AR(x) series for each of the residuals including standard error of the parameters, as gotten by statsmodels' fit function for x up to three. The degree was found by maximizing the BIC, which serves the purpose to not blow up the order of autoregression.*

Time series	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\sigma}^2$
wind NO	0.184604±0.077589	0	0	0.077322± 0.008715
wind DE	0	0	0	0.124229± 0.015349
load NO	0.669570±0.069938	-0.205016±0.074157	0.303462±0.074161	0.001737± 0.000173
load DE	0.626391±0.056887	-0.225152±0.085079	0	0.003344± 0.000305
water NO	1.189324±0.150155	-0.484997±0.143452	0	0.001406± 0.000382
solar DE	0.341299±0.067054	0	0	0.053133± 0.005824

With these parameters, we confirmed that the residuals' residuals (the error of the time series compared to the true data) are normally distributed for all time series by using the KPSS-test to compare to a sample of normally distributed variables with the same standard deviation, and that the sample ACF is close to zero (within bounds) for all lags≥ 1 to confirm the correctness of the estimated time series.

With this model, we created an example plot to see if it, in general, resembles the shape of the "correct" time series. This is shown in figure 4.

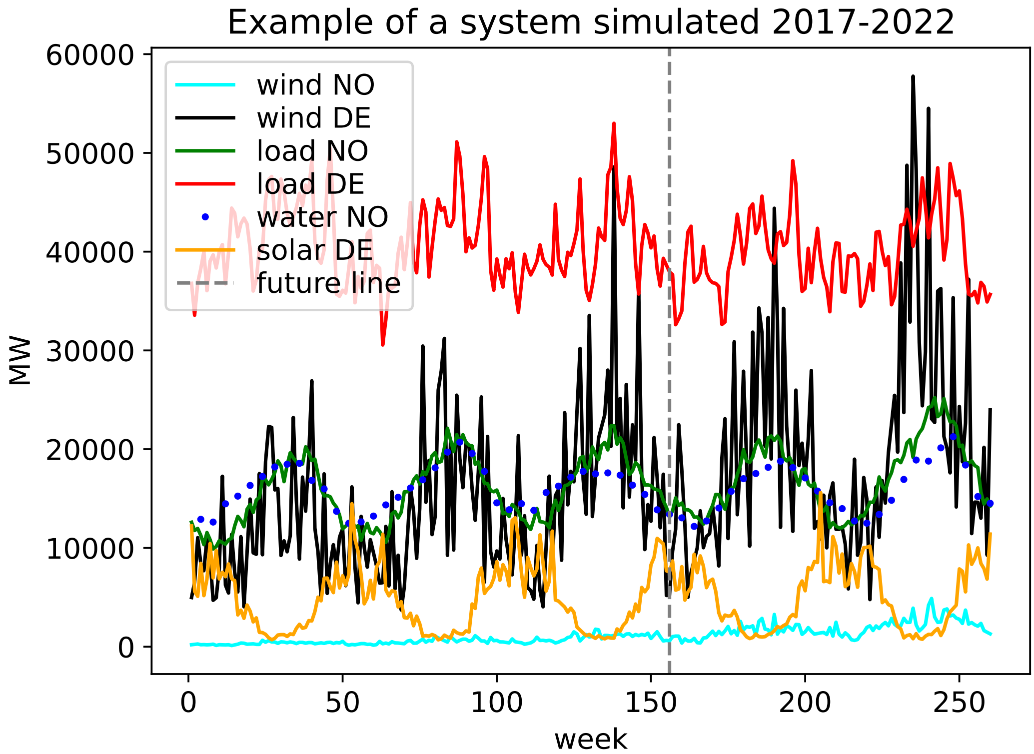

Figure 4: *Time series from june 2017 to june 2022 (weekly resolution) as simulated by the model*

We also checked for correlations between the time series by visually inspecting the cross-correlation of the deviations between real data and the models (that is, the residuals' residuals) between German wind and Norwegian wind, German wind and German sun, German load and Norwegian load as well as Norwegian load (in monthly resolution) and Norwegian water as described in [6]. This is shown in figure 5.

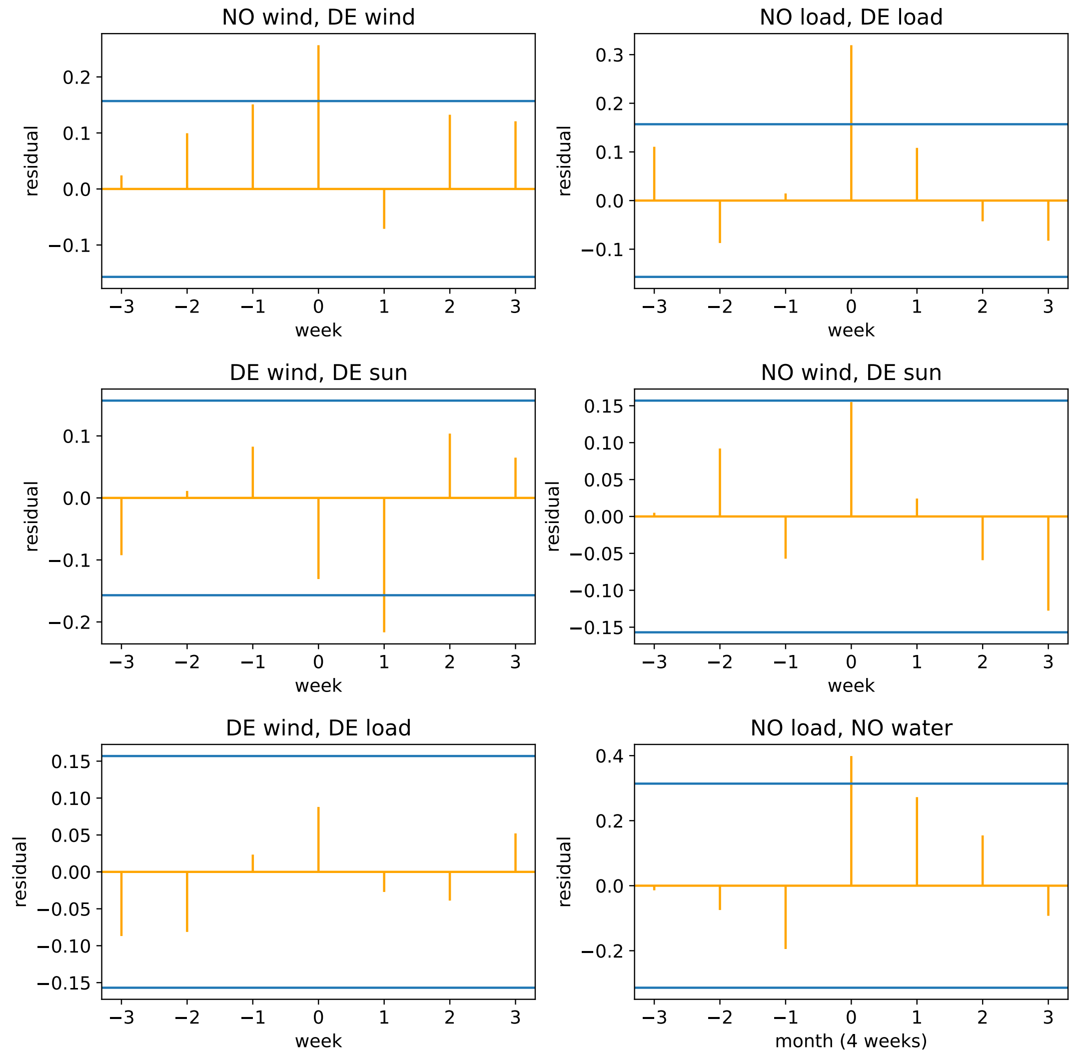

Figure 5: *Correlations between the residuals of the fitted AR-models. Weekly scale, except for the Norwegian load-water, which is the correlation between the mean value (over 4 weeks) of the load and the water. The drawn bounds represent $\pm 1.96/\sqrt{n}$. We see correlations between German wind and Norwegian wind and German sun, respectively, as well as between German/Norwegian load and Norwegian load/water.*

We see that Norwegian wind and German wind is correlated, as well as German wind and German solar. This is intuitively meaningful, as wind patterns across Europe are not independent, and lots of sun is negatively correlated with lots of wind. Similarly, we see that Norwegian and German loads are correlated at lag 0, which is also meaningful as both countries have the same seasons (to different extends) and similar holiday patterns. Finally, we see a correlation between Norwegian load and water - which is meaningful, as electricity production from water reservoirs can be matched to the demand.

We will hence model the wind and solar production as one dependent VAR(1) model, and the Norwegian and German loads as a VAR(3) model. The dependence between Norwegian load and water is a slight issue in terms of modeling due to the different time scales. The cheap solution would be to assume that water can be stored, such that the per-year production has to match the demand, but not the monthly production, but as water production is a function of weather, too, this is not a good solution and takes away a lot of the dynamics.

Alternatively, we can model water on a weekly scale and correlate it to the Norwegian (and German) load. As the dependence is at lag=0, yet another idea, which is what we end up doing, is to condition the Z_t value of the water production on the random value of the load, which we sample independently, and then use that the conditional distribution of a multivariate normal distribution (which we use for the residuals) is itself normal distributed,

$$X_1 \mid X_2 = x_2 \sim \mathcal{N}\left(\mu_1 + \frac{\sigma_1}{\sigma_2}\rho\left(x_2 - \mu_2\right), \left(1-\rho^2\right)\sigma_1^2\right). \tag{2.6}$$

In other words, with the white noise for the load produced by eq. (2.10), we then use eq. (2.6) to predict the white noise of the water, ignoring the cross-dependence (the white noise for the load is produced independently of water).

The final model will hence be the following:

Norwegian wind, German wind and German solar will be modelled as follows, where in all cases $\mathbf{Z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$.

$$\begin{bmatrix} X_{wind_{NO},t} \\ X_{wind_{DE},t} \\ X_{sol_{DE},t} \end{bmatrix} = \begin{pmatrix} 0.213521 & -0.066938 & -0.033662 \\ 0.291248 & -0.119213 & -0.321294 \\ -0.061305 & 0.071481 & 0.369008 \end{pmatrix} \begin{bmatrix} X_{wind_{NO},t-1} \\ X_{wind_{DE},t-1} \\ X_{sol_{DE},t-1} \end{bmatrix} + \mathbf{Z}_t \tag{2.7}$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.079352 & 0.024967 & 0.010698 \\ 0.024967 & 0.117833 & -0.008910 \\ 0.010698 & -0.008910 & 0.053757 \end{pmatrix} \tag{2.8}$$

Similarly, we get for the load

$$\begin{aligned} \begin{bmatrix} X_{load_{NO},t} \\ X_{load_{DE},t} \end{bmatrix} &= \begin{pmatrix} 0.612895 & 0.096800 \\ -0.031218 & 0.653266 \end{pmatrix} \begin{bmatrix} X_{load_{NO},t-1} \\ X_{load_{DE},t-1} \end{bmatrix} + \begin{pmatrix} -0.159204 & -0.063605 \\ -0.044460 & -0.260389 \end{pmatrix} \begin{bmatrix} X_{load_{NO},t-2} \\ X_{load_{DE},t-2} \end{bmatrix} \\ &+ \begin{pmatrix} 0.342012 & -0.078640 \\ 0.050070 & 0.056468 \end{pmatrix} \begin{bmatrix} X_{load_{NO},t-3} \\ X_{load_{DE},t-3} \end{bmatrix} + \mathbf{Z}_t \end{aligned} \tag{2.9}$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.001780 & 0.000843 \\ 0.000843 & 0.003491 \end{pmatrix} \tag{2.10}$$

For the water, we follow a different approach: We will model it as an AR(2) model with the same parameters as in table 3, but correlate the randomness with the load. For water and the Norwegian load, the main correlation happens at lag=0. We fitted a VAR(2) model, but decided to *only consider the correlation at lag zero*, ignoring the mutual effects of the lag past 0, as they are not relevant. This way, the white noise of the covariance matrix takes the shape on a monthly scale

$$\mathbf{\Sigma} = \begin{pmatrix} 0.001248 & 0.000523 \\ 0.000523 & 0.001514 \end{pmatrix} \qquad \mathbf{\Sigma}/4 = \begin{pmatrix} 0.000312 & 0.000131 \\ 0.000131 & 0.000378 \end{pmatrix} \tag{2.11}$$

The random variable for the Norwegian load will be modelled from eq. (2.10), and we will then use that the conditional distribution of $(Z_{water_{NO},t} | Z_{load_{NO},t} = z_{load_{NO},t})$ itself is normally distributed. We will however model the water as a weekly function, too. To do so, we use that two the sum of two independent multivariate distributions also is multivariate normal, thus $\mathbf{\Sigma}/4$ can be used on a weekly basis. Finally, the auto-regressive step is only performed every 4 weeks, and the average of the weekly steps is used such that

$$\begin{aligned} M_t &= \frac{1.189}{4}\left(X_{t-1} + X_{t-2} + X_{t-3} + X_{t-4}\right) - \frac{0.485}{4}\left(X_{t-5} + X_{t-6} + X_{t-7} + X_{t-8}\right) && \text{if } t \bmod 4 = 0 \\ M_t &= M_{t-(t \bmod 4)} && \text{otherwise} \\ X_t &= M_{t-(t \bmod 4)} + (Z_t | Z_{load_{NO},t} = z_{load_{NO},t}) \end{aligned} \tag{2.12}$$

An example of such a simulation is shown in figure 6.

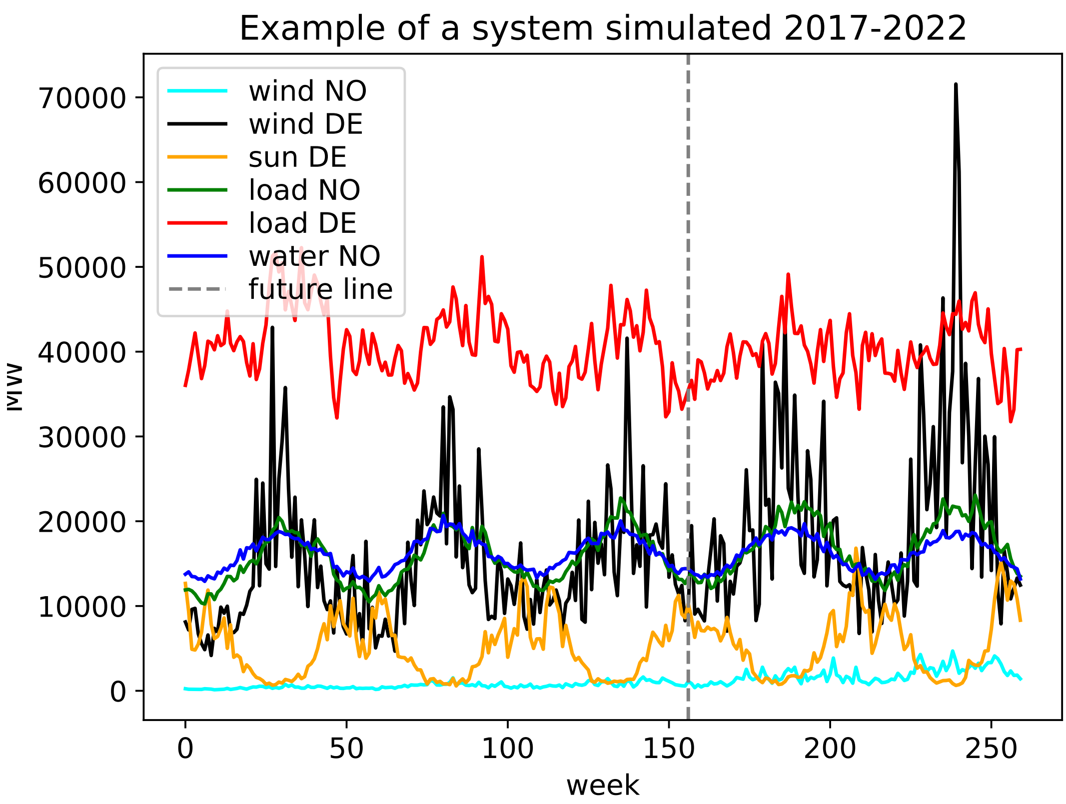

Figure 6: *Time series from june 2017 to june 2022 (weekly resolution) as simulated by the enhanced model*

3 Models

With the time series described, we will model a series of different cases. In all models, the basis assumptions are the following

- The energy production and consumption is described by the previously described time series.
- Germany uses green energy whenever available (wind/sun). The rest energy to cover the consumption is provided by coal and gas. *We do not model that coal and gas power plants cannot be turned off that easily in reality.*
- If Germany overproduces green energy (lots of sun/wind), the energy will either go to waste or be sent to Norway, depending on the case. If Germany overproduces energy, sending from Germany to Norway has priority over sending the other way around.
- Norway's consumption is first covered by its own wind, followed by what is sent from Germany, and finally water.
- Water not used can not be stored for later use, and all energy produced needs to be used the same time step. However, as it is in principle possible to store water energy (by producing less), we will report the "total energy surplus/deficit" in Norway, which will be composed of water not used (as wind and energy sent from Germany, in practice, never can cover the whole demand).
- The energy sent from Norway to Germany and vice versa is available at the same time step. The case of the energy available time step delayed is discussed in the appendix as a robustness test.

- Up to 18 TWh of energy can be sent to the platforms. As 8 TWh are sent there already (we assume this is included in the load already), and this is part of the load, we assume the existence of cables that can sent further 10 TWh to the platforms. This is based on numbers from this website.[16] In reality, not all of these cables are built yet, but we assume they already exist to consider the value of platform electrification.

The following cases will be considered:

Case 0: The cable between Norway and Germany is not used. That is, German overproduction will go to waste, while Norwegian overproduction is either stored or wasted (which of these, does not play a role in the model). The value of interest is the cumulative emissions of Norway and Germany combined in such a scenario, as well as the Norwegian "overproduction" and the German "overproduction" (how much energy goes to waste or remains unused). Case 0 will hence be the case which the other cases are weighed against.

Case 1: The cable between Norway and Germany is used. At each time step, Germany sends energy to Norway whenever there is a surplus of renewable energy in Germany and Norway is not fully covered by wind (the latter of which is practically always the case). This energy is used instead of using Norwegian water, which is "saved" (in the way described above). If the surplus surpasses the cable's capacity, the remaining surplus goes to waste. Norway sends energy to Germany whenever Germany is not fully covered by wind and solar energy and Norway has a surplus in energy production that week (but previously stored water is not used - only water produced at the same time step). There is an important design choice, namely when to send the energy (same or next time step). We will model it as same-step (what we call "delay 0"), unless otherwise stated (sending at the next time step is discussed in the appendix). No extra electricity is sent to the platforms, except for the 8 TWh already included in the load.

Case 2: This is very similar to Case 1, but under the restriction that Germany uses Norway as a "storage" and does not have net import from Norway. In practice, the following case will be modelled: Norway exports electricity to Germany, but not more than it itself expects to get from Germany in one year, see section 5.2. That means that there will still be net export from Norway to Germany (or the other way around) in most cases, but it is expected to peak at $\mu = 0$. Germany will still send wind to Norway whenever it can. No extra electricity is sent to the platforms, except for the 8 TWh already included in the load.

Case 3: Taking into account that Norway plans to electrify its platforms, we can use the Norwegian surplus to instead electrify the platforms (assuming this is easily possible and cables exist, which they don't at this time), instead of sending to Germany. At the same time, German surplus is still sent to Norway. We assume that the transfer to the platforms is loss-free, but the "bandwidth" to the platforms is restricted. Assuming the per-month oil and gas production to be constant, $\frac{10}{52}$TWh per week is the maximum number that can be send to the platforms. We assume "full saturation" when a total of 10 extra TWh has been send to the platforms, at which point we consider the platforms to not be able to run on more electricity, and no more can be sent to them. This is already taken care of by the bandwidth restriction. There are three possible versions of this scenario: Either one related to case 1, where Germany "exports" surplus energy to Norway to either cover negative electricity production, or to electrify the platforms. This will be called **Case 3-1**, where no electricity is sent from Norway to Germany. A related case is **Case 3-2**, where Norway has a deal with Germany to send the mean import back to Germany (just like in case 2), while the rest is used for platform electrification. We assume a simple scenario where sending to Germany has priority over sending to the platforms, up to the point where Germany has gotten its share. Finally, we define **Case 3-3**, which is maximizing the CO2-reduction by sending to Germany whenever there is a deficit there, and to the platforms otherwise - if Norway overproduces enough, then both Germany and the

[16] https://e24.no/det-groenne-skiftet/i/9vLA8d/elektrifisering-av-sokkelen-krever-mye-kraft-umulig-uten-vindkraft

platforms will get energy. Sending to Germany first is as good as sending to the platforms in the good scenario, but much better in the bad scenario (425 g CO_2/kWh compared to 710 g CO_2/kWh, despite the cable loss). We will play through both scenarios: Norway saving 700 g CO_2/kWh and 425 g CO_2/kWh on the platforms, and see if the qualitative result changes.

Case 4: Assuming Norway didn't have wind power, what would the carbon dioxide production look like? German surplus will first cover a lack of Norwegian load, and then go and electrify the platforms. Norwegian surplus will first cover German need, and then go and electrify the platforms. We ignore that Norway would get a rather strong negative surplus (which can be covered by import from neighbouring countries - or by increasing water production, which we do not consider). In a way, case 4 is really the same as case 3-3, just that Norway does not produce wind. In this model, we follow the same time series as before, but assume that wind energy somehow cannot be used.

For each of the cases, we will simulate the model n times for m years, starting at year Y under different random conditions and thus get probability distributions over the different values of interest, which usually is the (combined) carbon dioxide emissions of Norway and Germany.

3.1 Robustness tests

In order to evaluate the robustness of our results, we also tested the three following considerations:

1. What happens when the electricity sent through the cable is available at the next time step (a week later), not at the same time step? This question should be considered as it then becomes unknown whether Germany and Norway still require additional energy at the next time step, that is, whether the energy being sent based on last week's demand still is needed this week.

2. The model for the Norwegian water is based on several assumptions, which are up to debate. What happens when we model the Norwegian water as electricity-independent AR(2) model (the same model as before, but $Z_{NO,water}$ is not conditioned on the Norwegian load, but is normally distributed with standard deviation stated in table 3).

3. What happens if we instead of using a polynomial function, use trigonometric functions to model the seasonal trend? This serves to purposes: This way, the seasonal trend is continuous; in addition, we see how a different splitting of the time series data might impact our results.

These models will only be discussed briefly in the appendix. The purpose of this is partially to consider different scenarios that are worth to be considered on their own, and partially to assess the correctness of our qualitative results under different assumptions.

4 Computational Implementation

All results were created with Python. Libraries for Numerical Computing were used excessively, especially `statsmodels` which was used for parameter estimation for the (V)AR-models, as well as `NumPy`, `Matplotlib`,`scikit-learn`,`SciPy` and `pandas`. All code is available on GitHub.[17]

4.1 Test functions

Test functions were implemented to check the correctness of the data, such as the test that the sum of the deterministic functions and the residuals indeed is the real data. We tested that Cases 1-3 reduce to Case 0 when the cable capacity as well as the send-to-platforms limit is set to zero.

[17] https://github.com/schraderSimon/NorwayGermanyProject

We also tested that the time series models produce the expected data by comparing two different implementations of the same series and verify that they produce the same result.

4.2 Random Numbers

In order to compare runs of two different models (especially for a small number of simulation steps), we made sure that all models create the same random numbers by getting the same seed. Hence, the n^{th} simulation creates exactly the same electricity profile for each individual case.

5 Results & Discussion

Whenever we say "2020", we mean "june 2020 to june 2021", and similar for other years. We chose to simulate for 2020 and 2022, as the available time series stop at june 2020 and we wanted to see an immediate effect, as well as a postponed effect.

5.1 Production profiles

To see how annual production profiles for the loads and the different energy sources look like, we made a graph indicating the range of productions and loads for the different energy sources and loads. This graph does however not show how the different distributions are correlated. This is shown for 2020 and 2022 in figure 7.

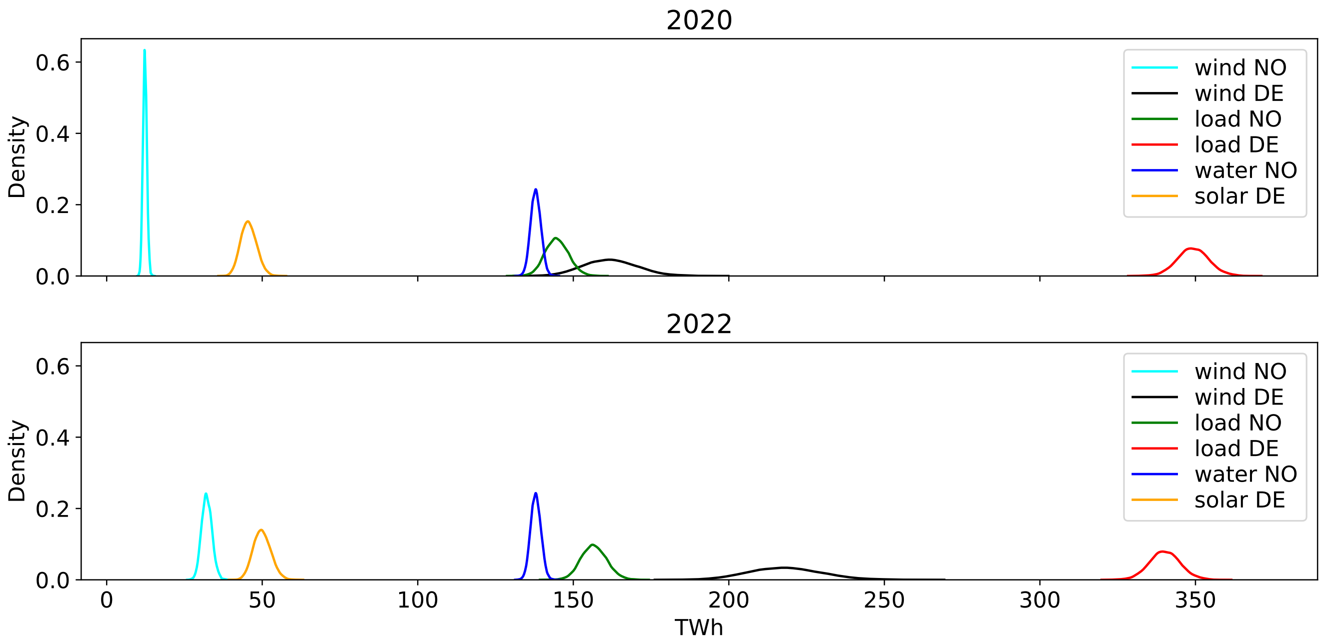

Figure 7: *Electricity production and use per year in 2020 and 2022.*

This shows well how the Norwegian water production is unchanged 2020 → 2022, how solar energy has a similar shape of distribution, just slightly moved 2020 → 2022, how German load is just so slightly reduced 2020 → 2022 while Norwegian load grows, and how wind is much more insecure than the other data. It also shows the need of Norwegian wind production (water does not suffice) and German dependency on non-renewable energy sources. We do think that the 2022 data is somewhat exaggerated - the Norwegian and German wind production will probably not raise as much, for example, and the Norwegian consumption will not increase that much. However, we do think that this is a good representation of what the two countries' energy profiles might look more at some further point in the future - if not 2022, then maybe 2025?

The German energy production depicted is much lower than Germany's real energy production. This is however not a mistake, as we have, as described previously, removed German nuclear energy, bio energy and water energy.

5.2 Wind overproduction

Figure 8 shows the probability distribution of the German wind overproduction (the energy that surpasses the German consumption and would go to "waste" if there was no cable), simulated for one year from June 2020 to June 2021, running with $n = 10,000$ simulations. It also shows the wind overproduction which can be sent to Norway - this is a different number, as the cable's capacity is finite.

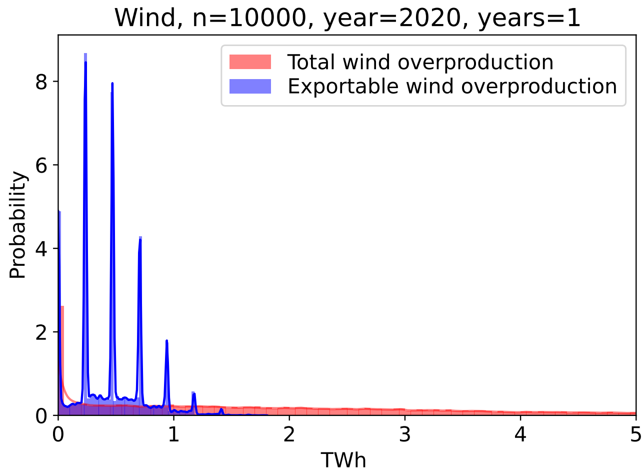

Figure 8: *Wind overproduction and wind that can be send to Norway for a simulation for one year from June 2020 to June 2021. Outlayers over 5 TWh are not included in the plot - which is approx. 10% of times. Graphs created using `seaborn`'s Kernel Density Estimator - which works a little bad for exponential-type distributions.*

We see that the wind overproduction is, in general, not so high. For the case of 2020-2021, this is expected - there will not be many days where Germany produces too much wind. In the model, the mean wind overproduction is 2.3 ± 2.3. The distribution of wind that can be send to Norway has quite a different shape, and it reflects the cable's maximum capacity (the peaks are at multiples of 1400 MW $\cdot 24 \cdot 7$ h=0.235 TWh). That way, we get that annually, 0.431 ± 0.303 TWh of wind are sent over to Norway.

Is this data reasonable? Yes. The jagged behaviour of the wind sent to Norway makes sense, as it simply reflects the number of weeks Germany produces too much wind. The cable's capacity is limited, so "very strong" wind cannot be sent completely. The shape of the total extra wind is reasonable, too. It is most likely that no wind will be overproduced, and the total reduction reminds of an exponential distribution, with higher overproduction values getting less and less likely.

For reference, the same plot for 2022-2023 is shown in figure 9. There, we see that the cable's capacity is simply not sufficient to take a hold of the majority of the German wind overproduction, and lots of overproduced wind goes to waste - which might imply the value of more export cables out of Germany like NordLink in the future.

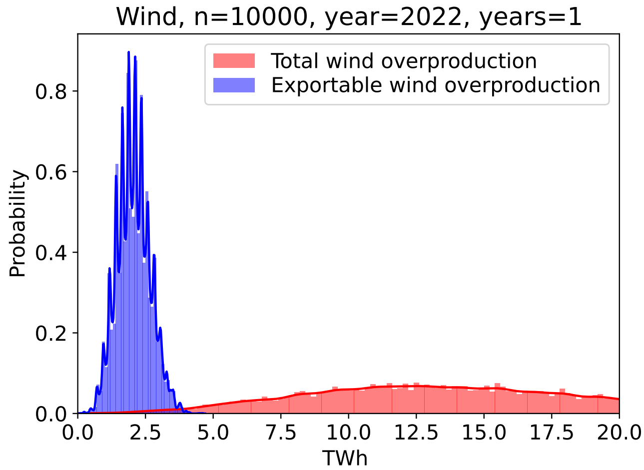

Figure 9: *Wind overproduction and wind that can be send to Norway for a simulation for one year from June 2022 to June 2023. Outlayers over 20 TWh are not included in the plot. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

5.3 Comparing Case 0 and Case 1

Running with $n = 10,0000$ simulations for a year from June 2020, the total carbon dioxide emissions of both Norway and Germany under case 0 and case 1 as well as the Norwegian energy surplus and the import-export balance for case 1are depicted in figure 10.

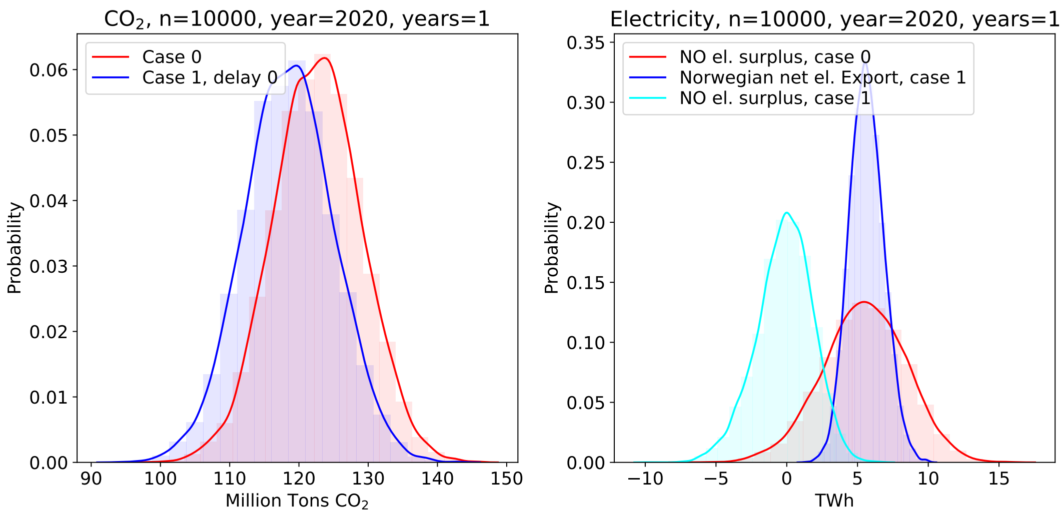

Figure 10: *Left: Combined CO_2 emissions for Case 0 and Case 1 in 2020. Right: Norwegian energy surplus and net electricity export (sent to Germany minus sent from Germany) for Case 0 and Case 1. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

We see that the total emissions are reduced by 4.1 ± 0.8 million tons, going from 123 ± 6 million tons CO_2 to 118 ± 6 by using Norwegian green energy in Germany instead of coal, and sending German weekly surpluses to Norway whenever possible. As no restrictions were placed on the Norwegian energy production other than that they follow the time series, the negative values are easily explained - even for case 0, when there is no export. We can however still see that the electricity surplus for case 1 lies quite close to 0 TWh (-0.2 ± 2.0TWh), with an average net export (that is, export minus import) of 5.6 ± 1.2 TWh. Qualitatively, we see that the energy export of case 1 in this model peaks around the same value as the energy surplus of case 0 - on average, all Norwegian surplus energy is hence sent Germany, sometimes more, sometimes less, so this type of model uses up Norwegian surplus and thus reduces carbon dioxide emissions with the conditions dictated by the time series in 2020.

It is an interesting observation that the surplus-curve has clearly less spread in case 1 than case 0 - this type of sending makes Norway's energy profile more predictable.

Is this data reasonable? We think so. Looking at the load, wind and water data of figure 1, the insecurity in this data make it seem reasonable that Norway may lie above or below the mean for consumption and the water production by several TWh, in addition to the innate insecurity of weather.

To confirm our intuition that the reduction of CO_2 is directly connected to the Norwegian overproduction, we checked that there is a (linear) relation between the carbon dioxide reduction and the Norwegian overproduction of case 0. This is indeed the case, as shown in figure 11. We see that there always is at least some reduction above 1 million ton Carbon dioxide in this type of model, as there is always something being exported from Norway. The slope of the linear fit is $m = 244$ g/kWh, which shows that Norway producing extra electricity is directly beneficial for the German CO_2 production. This should be compared to the value of $m = 710$ g/kWh saved of what arrives in Germany. We also see that German wind overproduction negatively impacts the carbon dioxide emissions - this is not related to the wind production itself, but follows from the fact that each day Germany sends to Norway is a day Norway does not send to Germany.

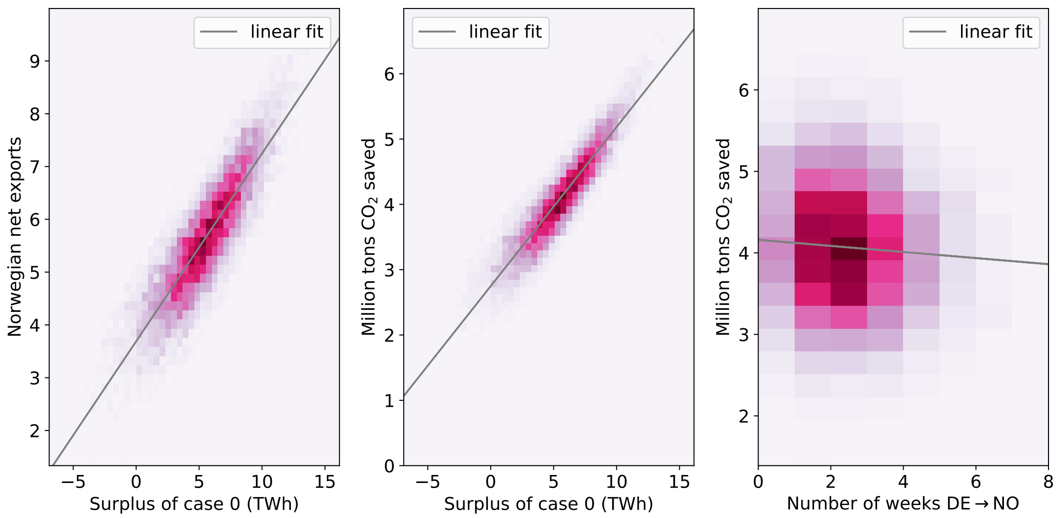

Figure 11: *2020: Left: 2D-histogram and linear fit of the case 0 surplus vs. the Norwegian net export. Middle: 2D-histogram and linear fit of the case 0 surplus vs. the saved Carbon dioxide emissions. Right: 2D-histogram and linear fit of number of days Germany exports to Norway vs. saved Carbon dioxide emissions.*

Figures 12 and 13 show the same plots for a simulation run in 2022, where Germany actually sends larger amounts of electricity to Norway. Similar results as in 2020 are found, and exactly the same trends apply. The emissions are reduced by 4.7 ± 0.7 million tons CO_2. This relatively small increase compared to 2020 can be explained by three factors - firstly, the capacity of the cable (the cable cannot send more than 0.23 TWh/week, so surplus remains unused), secondly, the fact that Germany sends on more weeks, reduces this number further, thirdly, as Norway cannot send on all days, so some of Norway's overproduction remains unused anyways. In addition, we observe more spread in the 2D-histograms, which we think might is caused by the insecurity and increased spread of wind and solar energy.

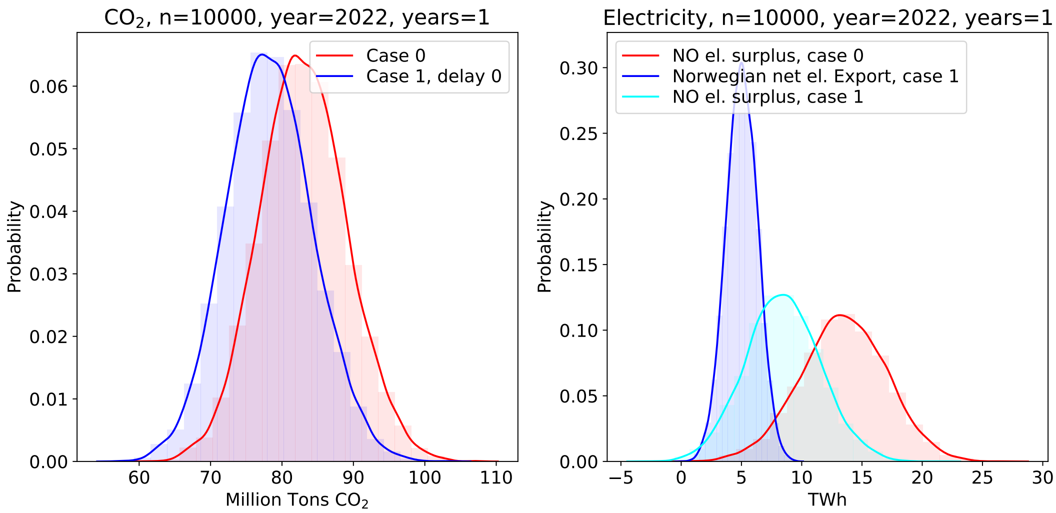

Figure 12: *Left: Combined CO_2 emissions for Case 0 and Case 1 in 2022. Right: Norwegian energy surplus and net electricity export (sent to Germany minus sent from Germany) for Case 0 and Case 1. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

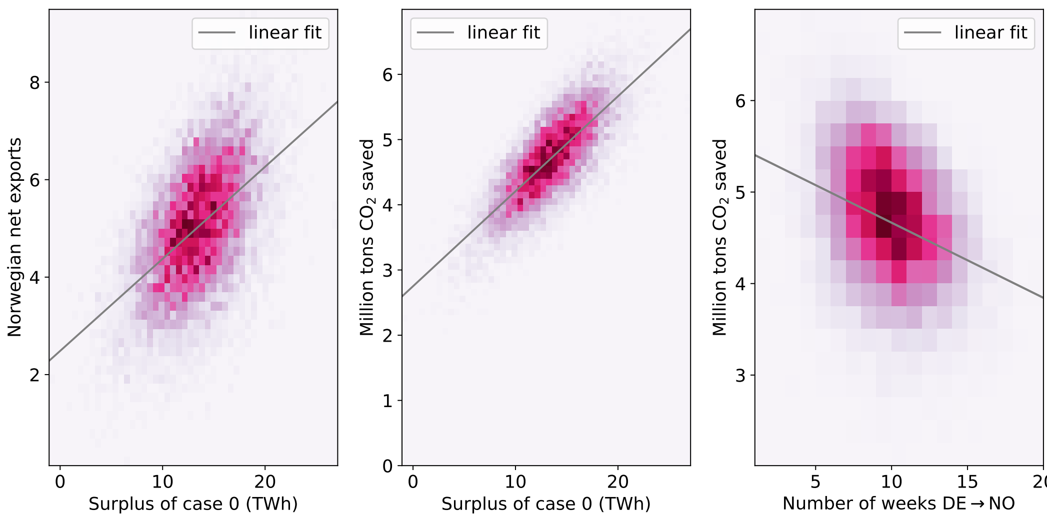

Figure 13: *2022: Left: 2D-histogram and linear fit of the case 0 surplus vs. the Norwegian net export. Middle: 2D-histogram and linear fit of the case 0 surplus vs. the saved Carbon dioxide emissions. Right: 2D-histogram and linear fit of number of days Germany exports to Norway vs. saved Carbon dioxide emissions.*

5.4 Comparing Case 0 and Case 2

First, we compared with $n = 10,000$ simulations for a year from June 2020 to June 2021. This is not a very interesting case, as Germany cannot import more than 0.44 TWh electricity from Norway, due to the very low number of overproduction of wind and the cable's capacity. We see a reduction of the German CO_2 emissions of 0.27 million tons, and the Norwegian export number peaks at 0 with a very small deviation, as expected, with jagged behaviour as in figure 8. The shape of the surplus curves for case 0 and case 2 is visually the same.

A much more interesting scenario arises when looking at 2022. The the combined carbon dioxide emissions and the Norwegian exports as well as the Norwegian energy surplus is shown in figure 14.

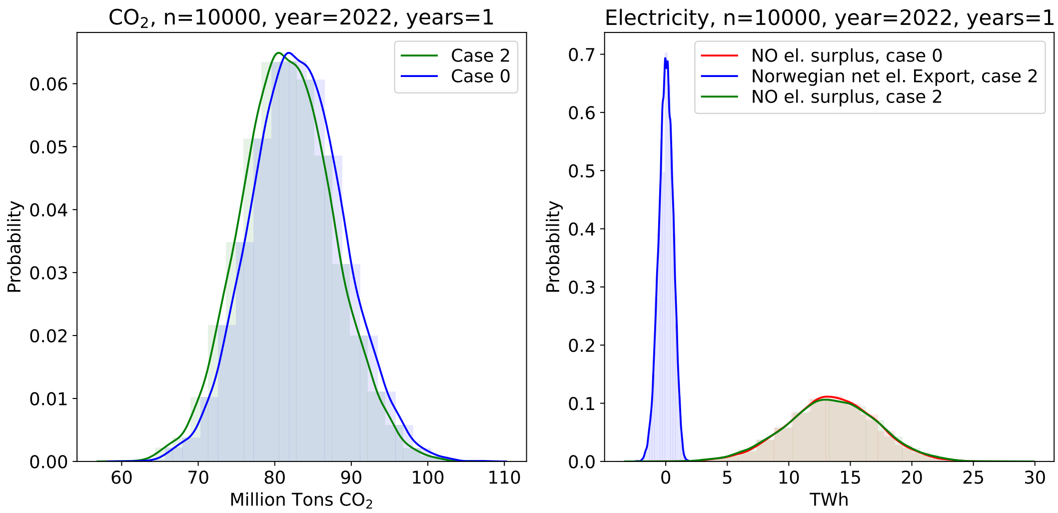

Figure 14: *Left: Combined CO_2 emissions for Case 0 and Case 2 in 2022. Right: Norwegian energy surplus and net electricity export for Case 0 and Case 2. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

In this scenario, Germany will send to Norway on average $\mu = 2.067 \pm 0.006$ TWh electricity, which leads to a CO_2 reduction of exactly 1.32 ± 0.00 Million tons CO_2 - there is no insecurity, as Norway always manages to export what it can get from Germany. Not surprisingly, the Norwegian surplus is almost the same in both cases, with 13.5 ± 3.6 TWh for case 0 and 13.5 ± 3.7 TWh for case 2. We see hence that Norway's electricity profile remains more or less the same, while Germany reduces its emissions by "borrowing" Norwegian water. Indeed, comparing the electricity surpluses of case 0 and case 2, the data "passes" the Kolmogorov-Smirnov test for equality of distributions with a p-value of 0.52 with $n = 10,000$. However, the net export balance of case 2 is not normally distributed, having a skewness of -0.2.

The same results apply to $2020 - 2021$ as well, but much less energy is imported or exported. As the probability distribution of the surplus is more or less unchanged, Norway has "nothing to lose" in terms of emissions and available electricity by participating in such a model. Even if the model should be extremely wrong in predicting the trends and the German or Norwegian wind do not raise as fast as predicted, or Norway's load should be indeed constant - in our opinion, this stands as a very strong result. Indeed, halving the slope of both the Norwegian load and the Norwegian wind (in log space) leaves this result unchanged, as does running the simulations in 2024.

5.5 Case 3-1, Case 3-2, Case 3-3 and Case 1

All simulations were run with $n = 10,000$.

5.5.1 Case 3-1

Comparing case 3-1 and case 1 means comparing electrification of platforms with the use of Norwegian surplus energy in Germany. The 2020 results are shown in figure 15, the 2022 data is shown in figure 16.

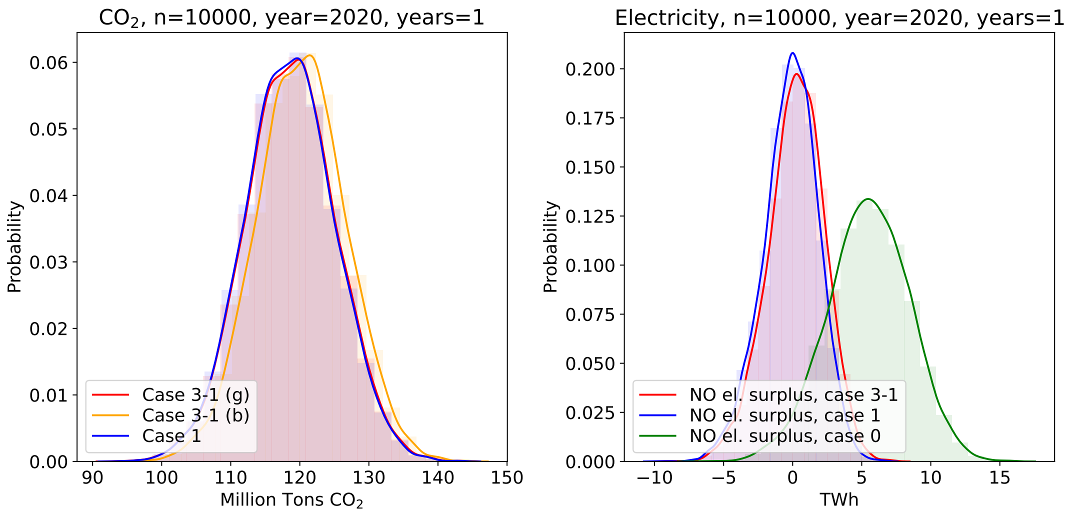

Figure 15: *Left: Combined CO_2 emissions for Case 1 and Case 3-1 in 2020. Right: Norwegian energy surplus and net electricity export for Case 1 and Case 3-1. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

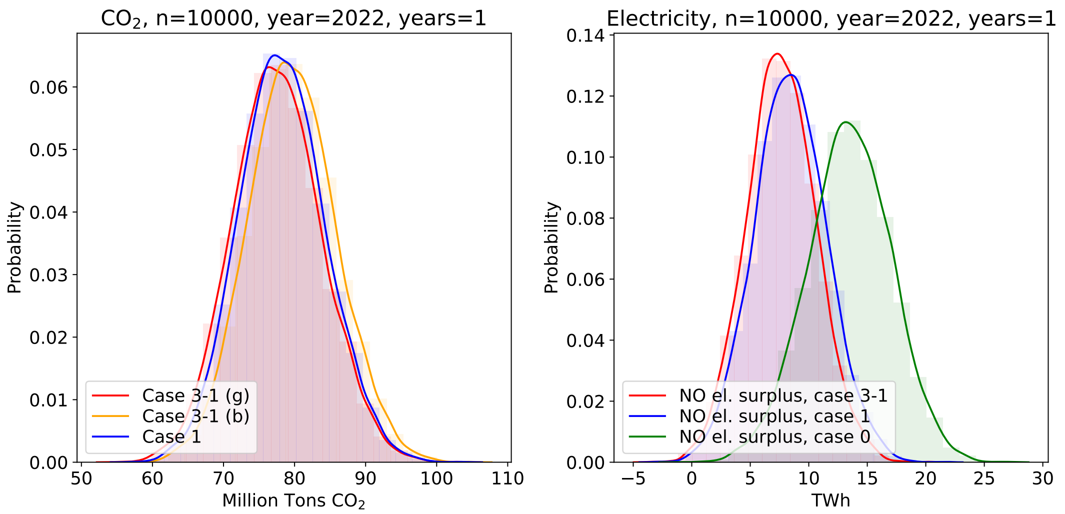

Figure 16: *Left: Combined CO_2 emissions for Case 1 and Case 3-1 in 2022. Right: Norwegian energy surplus and net electricity export for Case 1 and Case 3-1. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

We see that in 2020, sending extra electricity to Germany is as good as is sending to the platforms in the good scenario (Case 1 barely beats the good scenario with a CO_2-difference of 0.1 ± 0.2 million tons), but much better than the bad scenario (CO_2-difference: 1.7 ± 0.4 million tons). This is not the case in 2022: the good scenario of case 3-1 gives slightly lower emissions than case 1 (the good scenario beating case 1 with 0.7 ± 0.4 million tons, compared to the bad scenario, where case 1 beats the bad scenario with 1.4 ± 0.4).

At the same time, we see how the load surplus of case 3-1 is slightly higher in 2020, while it is slightly lower in 2022. The improvement of the carbon dioxide emissions of case 3-1 compared to case 1 is explained by the way we model: German surplus sent to Norway can go to the platforms in the same week - hence, German surplus electricity will likely end up on the platforms. Norwegian surplus is however only sent to Germany the days Germany is not overproducing. There are hence two factors: German surplus does not contribute to a reduction in Germany but it reduces platform emissions, which also explains why the electricity surplus of case 3-1 is lower than case 1 in 2022;

and German wind surplus reduces how much carbon dioxide emissions can be reduced in Germany (as no electricity is sent to Germany these days), as elaborated earlier. Wind in Germany hence reduces the possible emission reduction available through surplus from Norway, but increases the possible emission reduction in Norway (This effect would be maximised if Germany overproduced green electricity every week).

As we consider the German cable to be able to send up to $\sim$ 12 TWh/year, while the cables to the platform can send up to 10 TWh/year, with the proportional difference in what can be sent per week, it is not surprising that case 1 beats case 3-1 in 2020, where the cable mainly sends from Norway to Germany, while the good scenario of case 3-1 beats case 1 in 2022, where Germany sends to Norway more often.

Finally, as we consider the bad scenario to be the more realistic one, we see that sending Norwegian surplus energy to Germany with the aim of reducing German emissions, is a practicable alternative, given that it replaces gas and coal energy.

5.5.2 Case 3-2

In case 3-2, it should be worth to mention that we only send back to Germany a third of the cable's capacity when Norway is sending to Germany - this way, Norway is done sending to Germany roughly after week 30-40 (in 2022), instead of week 10-15. This has the advantage of increasing the chance that Norwegian surplus is sent to the oil platforms by spreading the surplus evenly between Germany and the platforms. The 2022 data comparing cases 3-1 and 3-2 are shown in figure 17.

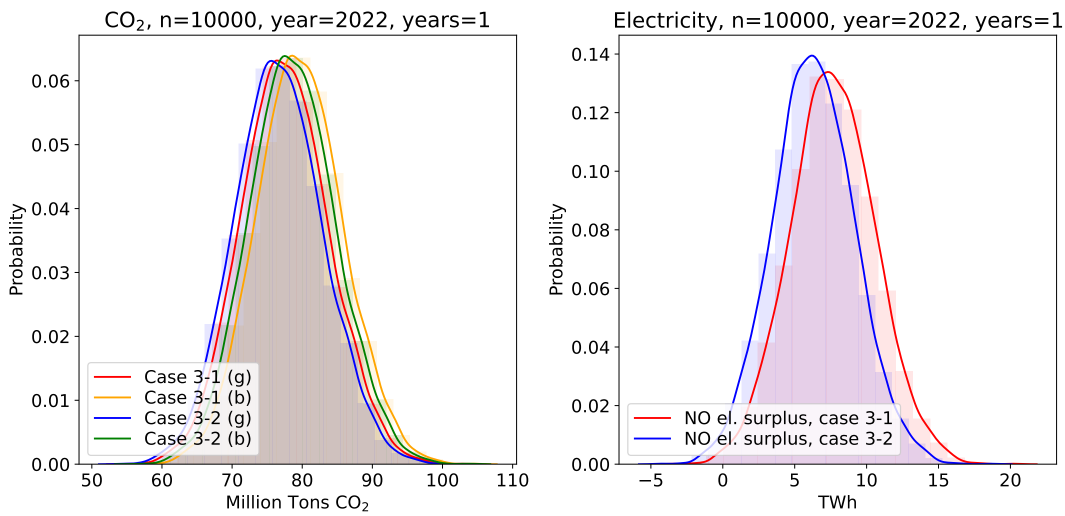

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Figure 17: *Left: Combined CO_2 emissions for Case 3-1 and Case 3-2 in 2022. Right: Norwegian energy surplus and net electricity export for Case 3-1 and Case 3-2. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

Again, we see that it is indeed worthwhile to use Norway as an "energy storage" for German electricity, with a reduction of 1.1 ± 0.1 in the bad scenario and 0.9 ± 0.2 in the good scenario a the cost of 1.3 ± 0.2 TWh compared to case 3-1. If German surplus is sent back to Germany, the emissions get further reduced, and Norway still keeps most of its surplus (which of course is reduced, as Norway sends back what Germany sends in, on average, instead of using it for itself on the platforms).

Compared to case 2, we see that the reduction is somewhat less in case 3-2. This is not surprising, as we here compare using the German surplus in Norway to sending it back to Germany. The fact that the reduction in surplus between case 3-2 and 3-1 (1.3 TWh) compared to what is sent to Germany (2.1 TWh) are different numbers, is easily explained by the fact that some less electricity is being sent to the platforms, which explains why the emission reduction is not the full 1.32 million tons, but less.

The study of case 3-2 shows that using Norway as German energy battery is still worthwhile, even when Norway instead could use German energy surplus for its own platform electrification.

5.5.3 Case 3-3

The 2020 and 2022 data comparing cases 3-1 and 3-3 are shown in figure 18 and 19.

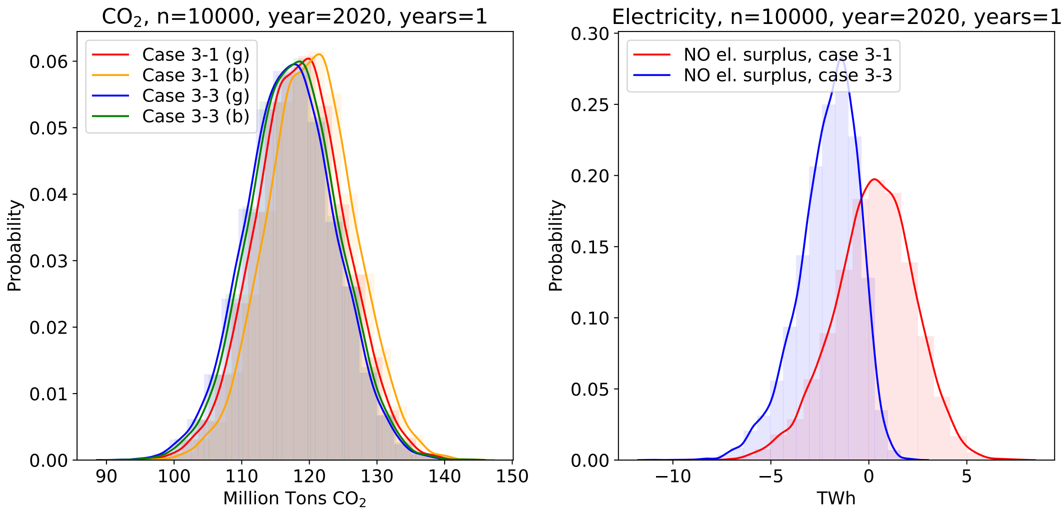

Figure 18: *Left: Combined CO_2 emissions for Case 3-1 and Case 3-3 in 2020. Right: Norwegian energy surplus and net electricity export for Case 3-1 and Case 3-3. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

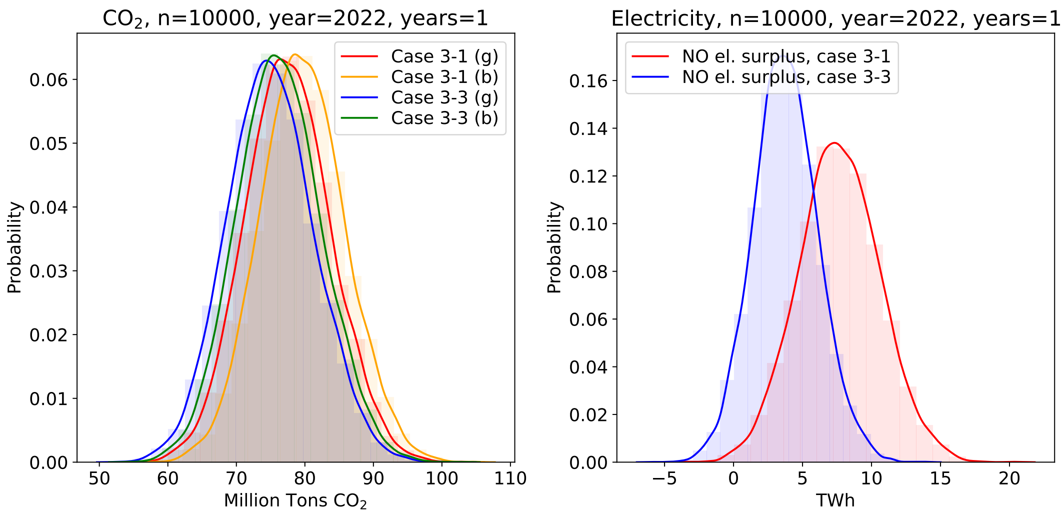

Figure 19: *Left: Combined CO_2 emissions for Case 3-1 and Case 3-3 in 2022. Right: Norwegian energy surplus and net electricity export for Case 3-1 and Case 3-3. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

We see that in 2020, by comparing the blue and the green line in the left graph of figure 16, that not so much extra electricity is sent to the platforms (the difference between the green and the blue line is the difference in the assumption how much emissions are saved at the platforms - if these two graphs are similar, not much is being saved at (and hence sent to) the platforms). There is however quite the reduction in CO_2 emissions in going from case 3-1 to case 3-3, with a reduction of 2.5 ± 0.6 million tons CO_2 in the bad scenario, compared to 1.5 ± 0.6 in the good scenario. At the same time, this scenario would not leave Norway with any relevant surplus, which is indeed negative (-2.1 ± 1.5 TWh).

In 2022, the trend is similar, but the numbers are higher with a reduction in CO_2 of 3.4 ± 0.7 (bad scenario) and 2.5 ± 0.7 (good scenario) million tons, respectively - a the same time, we see how more is sent to the platforms, again by comparing the good and the bad scenario of case 3-3. This is also an efficient use of Norwegian surplus, which now only lies around 3.8 ± 2.3 TWh (which should be compared to case 0 with no exports whatsoever, see figure 12).

We also did some direct comparison of what is sent to Germany and what is sent to the platforms in this scenario (what is sent to Norway, does not directly contribute to emission reductions). This is shown in figures 20 and 21 for 2020 and 2022, respectively. Going from 2020 to 2022, we see that the share of what is sent to Germany reduces from 75% to 60%. We see that more being sent to Germany also implies more being sent to the platforms in both 2020 and 2022, which is related to the fact that an overall higher energy production also implies a higher per-week overproduction, which is available in both Germany and at the platforms. In 2022, more energy is available, so more can be sent to the platforms compared to 2020, and the slope of the linear fit gets steeper ($0.45 \rightarrow 0.6$).

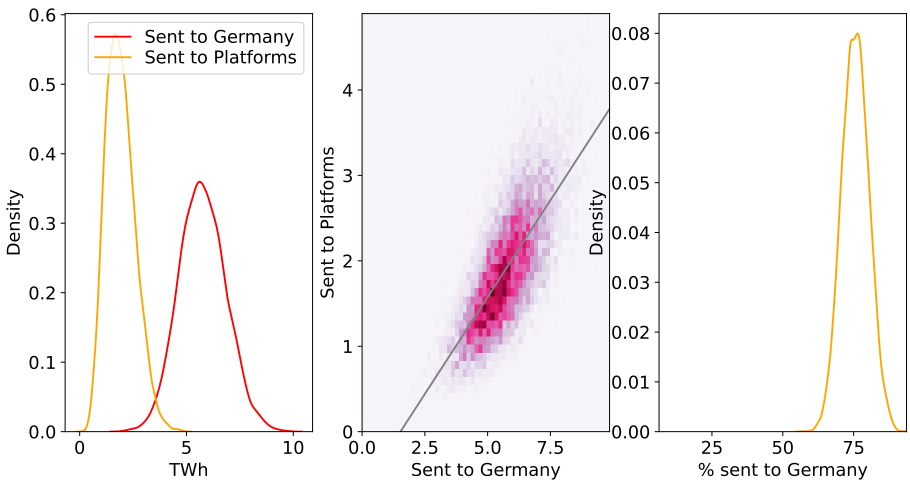

Figure 20: *Case 3-3 2020: Left: How much TWh is sent to Germany and the platforms, respectively. Middle: 2D-histogram showing a linear relation between what is sent to Germany and what is sent to the platforms. Right: The share of TWh sent to Germany (of what is being sent in total).*

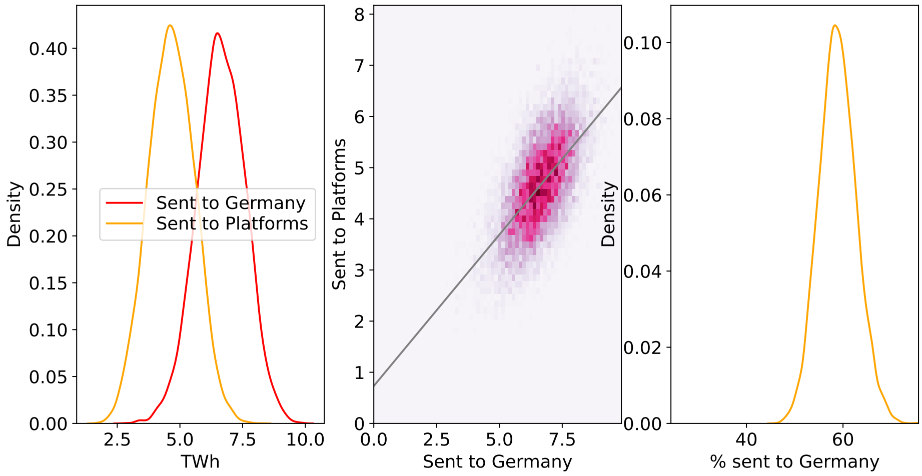

Figure 21: *Case 3-3 2022: Left: How much TWh is sent to Germany and the platforms, respectively. Middle: 2D-histogram showing a linear relation between what is sent to Germany and what is sent to the platforms. Right: The share of TWh sent to Germany (of what is being sent in total).*

Case 3-3 can be slightly modified and priority can be given to the platforms instead of Germany. As the virtual cable to the platforms has approximately the same bandwidth, we think that the distribution of the Norwegian surplus in case 3-3 would look very similar. The emission profile would be similar, too, in the good case, but somewhat higher in the bad case.

5.6 Case 4

We compare case 4 to a modified version of case 0, where Norway does not produce any wind. The 2020 results are shown in figure 22, the 2022 data is shown in figure 23.

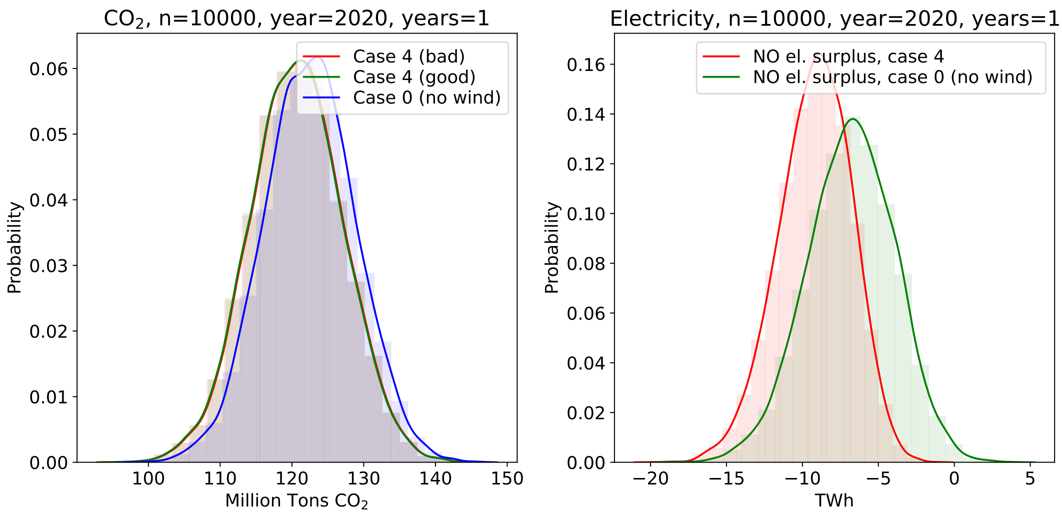

Figure 22: *Left: Combined CO_2 emissions for Case 4 in 2020. Right: Norwegian energy surplus and net electricity export for Case 4. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

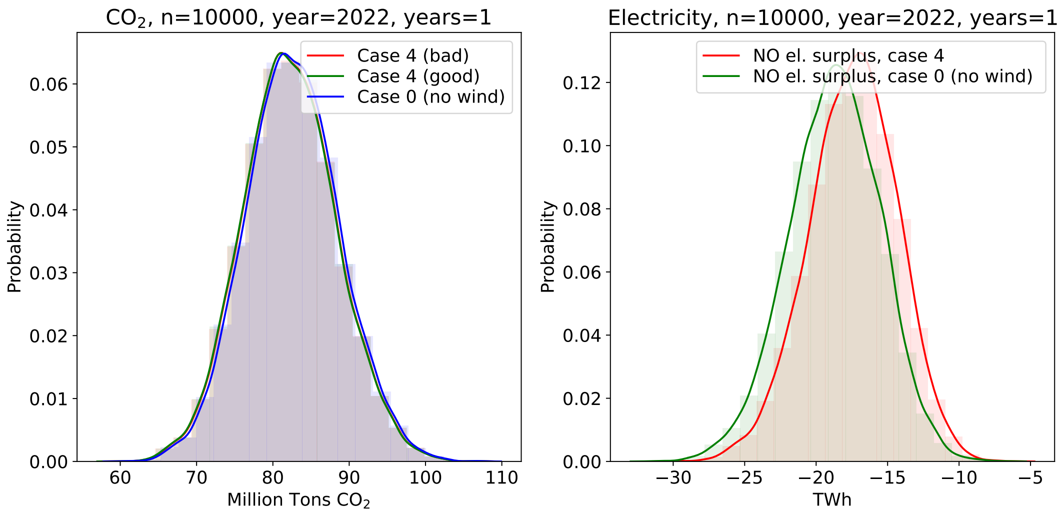

Figure 23: *Left: Combined CO_2 emissions for Case 4 in 2022. Right: Norwegian energy surplus and net electricity export for Case 4. Graphs created using* `seaborn`*'s Kernel Density Estimator.*

We see that the 2020 data resembles case 3-3 in behaviour: Norwegian weekly surplus is sent to Germany to reduce the emissions, almost nothing is sent to the platforms. The reduction is of course much less than case 3-3 or even case 1 would give, as Norway does not overproduce most of the days. The more interesting result is the 2022 data - here, the roles of who is the net receiver invert. As Norway's load increases, Norway ends up with large energy deficits. Norway compensates some of that net deficit by the overproduced wind from Germany. There is even still a very slight reduction of CO_2 occurring, with reductions of 0.5 ± 0.3 (good) and 0.4 ± 0.2 (bad) million tons. If Norway did not produce wind power, this would be a good deal: The overall emissions would still go down, and Norway would import emission-free electricity that would normally go to waste to partially cover its own demand. However, this requires that Norway is under-producing almost all the time, and that Germany is capable of sending "lots of wind" to Norway.

6 Conclusion

We have built a stochastic model for the energy production in Norway and Germany based on a deterministic trend and autoregressive models for the insecurities, which are based on true time series data. We have considered different models on how the NordLink cable can be used to reduce carbon dioxide emissions in Germany without making any changes in electricity production. First, we have shown that the cable can be effectively used to reduce German emissions by, on average, more than 4 million tons CO_2 equivalents by importing Norwegian energy surplus. In terms of emissions, sending surplus energy to Germany and importing surplus from Germany beats electrification of the oil platforms, as we deemed the platforms' electrification to be less effective (425 g CO_2/kWh) than sending to Germany (710 g CO_2/kWh). Even if no net energy is exported to Germany and Norway only wants to make up for what Germany overproduces, an emission reduction of over a million tons CO_2 is possible in the near future if Germany keeps building windmills - without affecting the Norwegian energy surplus profile, simply by using Norway as a type of battery. This results holds true no matter whether Norway electrifies its oil platforms or not. We have seen that if Norway keeps increasing its electricity production, cooperating with Germany and electrifying the oil platforms at the same time leads to the strongest emission reduction. A positive side effect of the cable is that the distribution function of surplus energy gets narrower - in that sense, the cable can also be used to stabilize fluctuations in energy production. Finally, we have seen that the cable can be used the other way around too - if Norway has an energy deficit, the cable can be used to import German surplus wind to Norway (given that Germany keeps building windmill) and reduce the energy deficit.

This holds true even if Norway sends surplus that rarely arises, to Germany.

We conclude that the cable can be used effectively to reduce carbon dioxide emissions, and that using it that way as long as the platforms are not fully electrified, can lead to a large overall reduction of carbon dioxide emissions. Even with the platforms being electrified, sending additional surplus energy to Germany reduces the overall emissions further. We have shown that using cables like NordLink with the aim of emission reduction is at least as good as platform electrification (per TWh). Even if no net electricity is planned to be exported, using the cable in such a way that Germany can use Norway as a battery, can reduce Germany's emissions.

Our models were very relatively simple, and there are more advanced models that can be considered in future research - including time series for import and export of energy, and integrate with other neighbouring countries that can import/export to Germany and Norway. The trend functions we used are simple linear fits; one could predict future models by the countries' predicted consumption and production. With NordNed and BritNed, similar cables exist between the Netherlands and Norway/the UK, and one could cleverly consider how energy can be transported along these cables. Finally, we did not address the elephant in the room - energy prices do not follow what is most ecologically effective. Making the import/export based on price models might lead to more realistic emission scenarios - we only described what is possible, and how the climate can profit double from Germany increasing its wind production. Let's hope Germany keeps doing that.

Figure 24: *A meme. The German text says "Armin Laschet when someone wants to build a wind mill". Armin Laschet is the conservative party's (CDU) chancellor candidate for the elections in September 2021 and the likely successor of Angela Merkel, if we can believe poll data.*

APPENDIX

A Robustness tests

A.1 Sending at the next time step

Assume that the cable is awfully slow, or that Germany and Norway do not manage to use the energy the same time step as it is sent, but the next one. How does this change the dynamics in our model? As an example, we looked at Case 1. This is shown in figure 25 and 26 for 2020 and 2020, respectively.

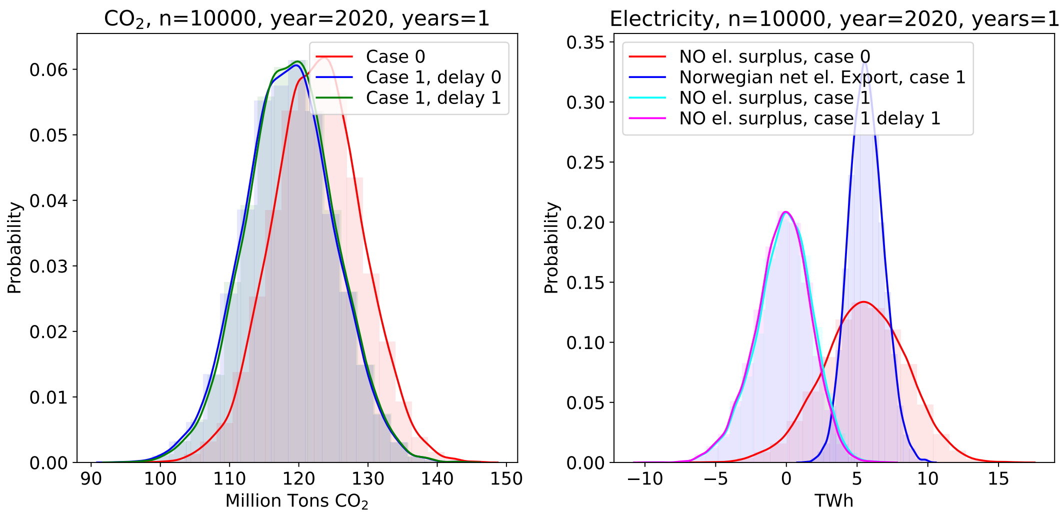

Figure 25: *Left: Combined CO_2 emissions for Case 0 and Case 1 in 2022, as well as when the energy is delayed by one. Right: Norwegian energy surplus and net electricity export (sent to Germany minus sent from Germany) for Case 0, Case 1 and Case 1 with a delay of 1. Graphs created using Seaborn's Kernel Density Estimator.*

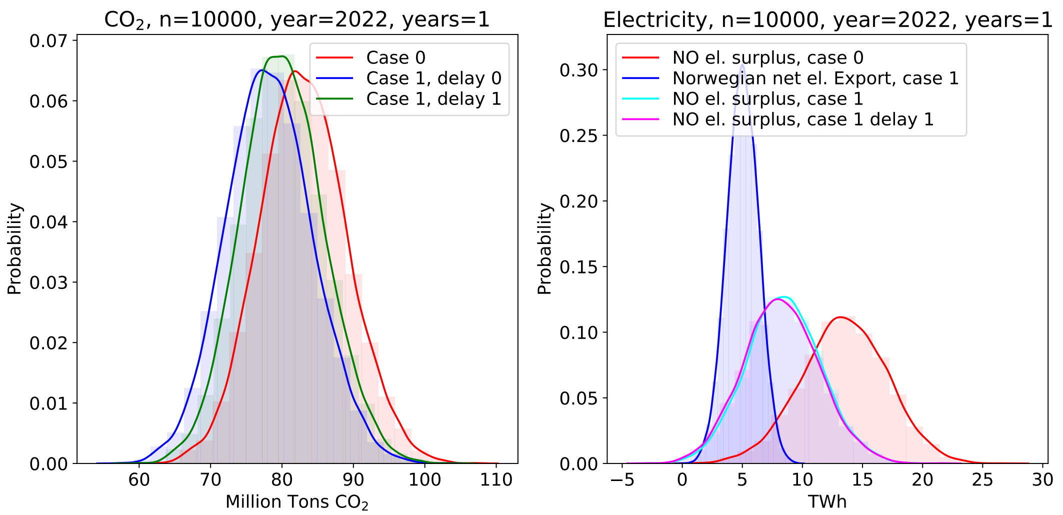

Figure 26: *Left: Combined CO_2 emissions for Case 0 and Case 1 in 2020, as well as when the energy is delayed by one. Right: Norwegian energy surplus and net electricity export (sent to Germany minus sent from Germany) for Case 0, Case 1 and Case 1 with a delay of 1. Graphs created using Seaborn's Kernel Density Estimator.*

We see that the 2020 scenario is almost identical to Case 1, while the 2022 is not. In 2020, even though the surplus profiles for the non-delayed and the delayed case 1 are almost identical, the delayed case 1, while still saving 2.9 ± 0.8 million tons CO_2 compared to case 0, has 1.81 ± 0.6 million tons less reduction than the nondelayed case 1. We explain this the following way: Sending same-day, we make sure that electricity is only sent when there's need for it - Norway sends energy to Germany when Germany needs energy, which is the same as it having a deficit. If that energy however arrives at the next time step, it is much less secure if Germany still has use for that energy. This plays more of a role in 2022, where there are many more weeks where Germany can send electricity to Norway. In addition, if energy sent from Norway at time t makes that Germany has an energy surplus at time step $t+1$. In that case, Germany will send to Norway. Thus, Norway cannot send to Germany, and as it is likely that Germany has an energy deficit at time step $t+2$, no energy can arrive that day. Indeed, Germany sends energy more often in this delayed scenario, see figure 27.

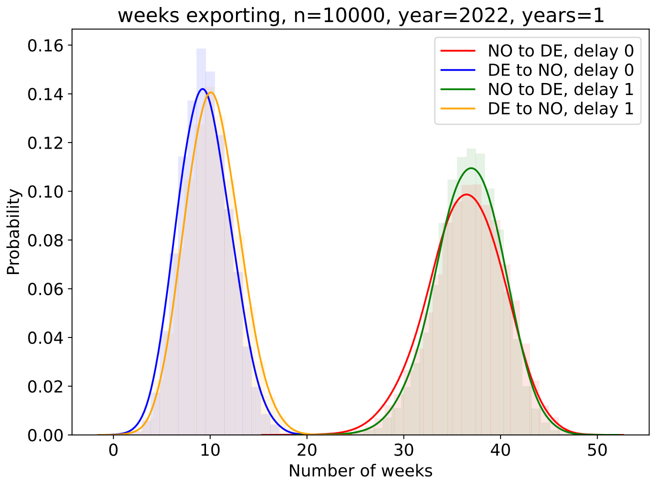

Figure 27: *Number of weeks the cable is used for Germany-to-Norway and number of weeks the cable is used for Norway-to-Germany, if sending same-week or one week delayed.*

A.2 Decoupling water and load

Here, we test whether coupling water and load the way we do has a strong effect on the results. We will still follow eq. (2.12), but instead of conditioning the water production on the load ($Z_t | Z_{load_{NO},t} = z_{load_{NO},t}$), we simply say that $Z_t \sim N(0, \sigma^2)$, where $\sigma^2 = 0.001406/4$ as seen in table 3. We verified for each case that the same qualitative results hold as for the original model, which is indeed the case. As an example, the production profiles for 2020 and 2022 are shown in figure 28, a simulation of cases 3-1 and 3-3 for 2022 is shown in figure 29.

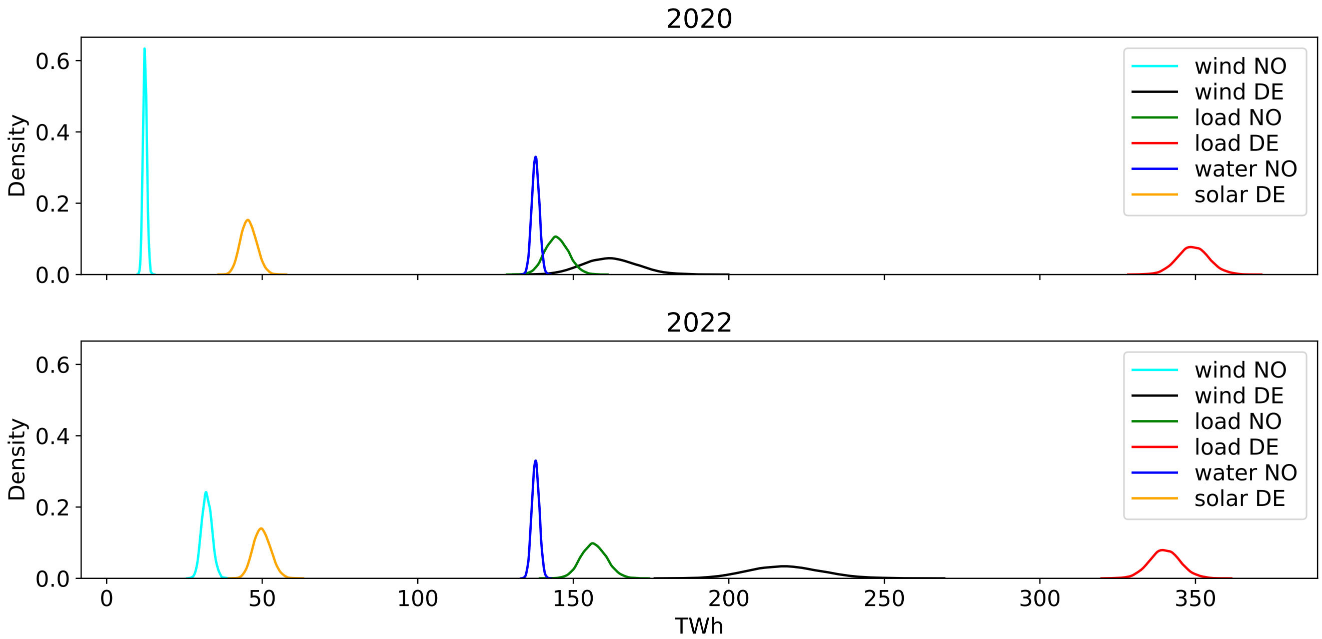

Figure 28: *Electricity production and use per year in 2020 and 2022 with water decoupled from load*

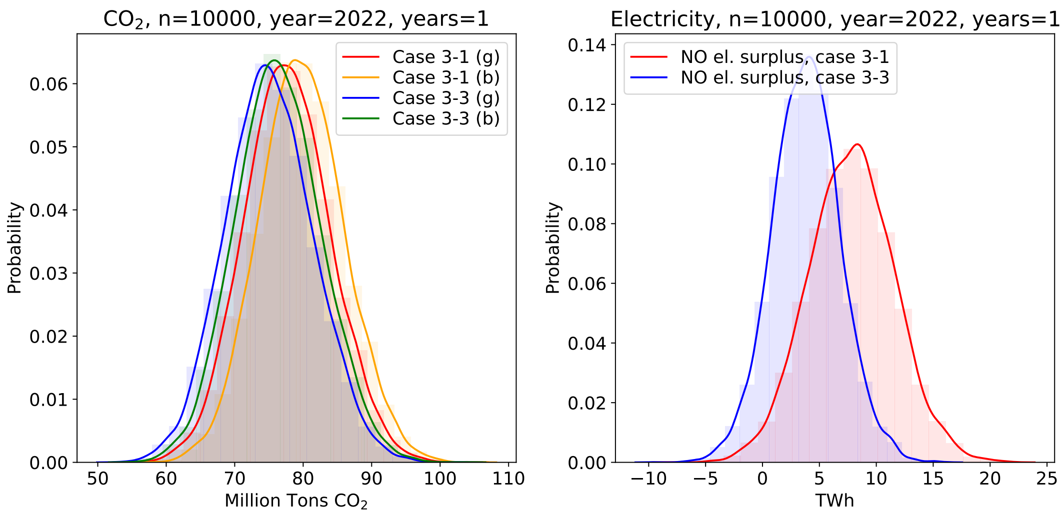

Figure 29: *Data produced with water decoupled from load.Left: Combined CO_2 emissions for Case 3-1 and Case 3-3 in 2022. Right: Norwegian energy surplus and net electricity export for Case 3-1 and Case 3-3. Graphs created using Seaborn's Kernel Density Estimator.*

We see that the water production profile gets narrower compared to the dependent scenario. We also see how the surplus profiles, on the other hand, get broader. However, the emission data is practically identical - as example, the good scenario of case 3-3 for decoupled water has 3.4 ± 0.8 million tons CO_2 less emissions than case 3-1 - hence, the mean value is the same as for when water production is dependent on load.

A.3 Fitting with trigonometric functions

Finally, we tested if the same qualitative results are achieved when modelling the seasonal coefficient of the log-transformed series as a trigonometric function. The same detrending was used for the trend functions. To each detrended time series, we fitted a function

$$S(t) = \sum_{k=1}^{6} b_k \sin\left(\frac{2\pi \cdot k}{p}(t - z)\right) \tag{A1}$$

were b_k and z are found using `SciPy`'s `optimize.curve_fit` function. The parameters (the number of sine and cosine functions (which is zero)) were chosen such that the residuals passed the KPSS and ADF-tests, which was the case for all time series but Norwegian load, for the reason explained in the main text. This is not a very elaborate model, and hence serves as a good benchmark to compare our model to. With eq. (A1), fitting AR-models to the time series, this gave the same relevant correlations between the time series, albeit slightly different degrees for the vector autoregression models where found, and the parameters of course were different. The correlations are shown in figure 30.

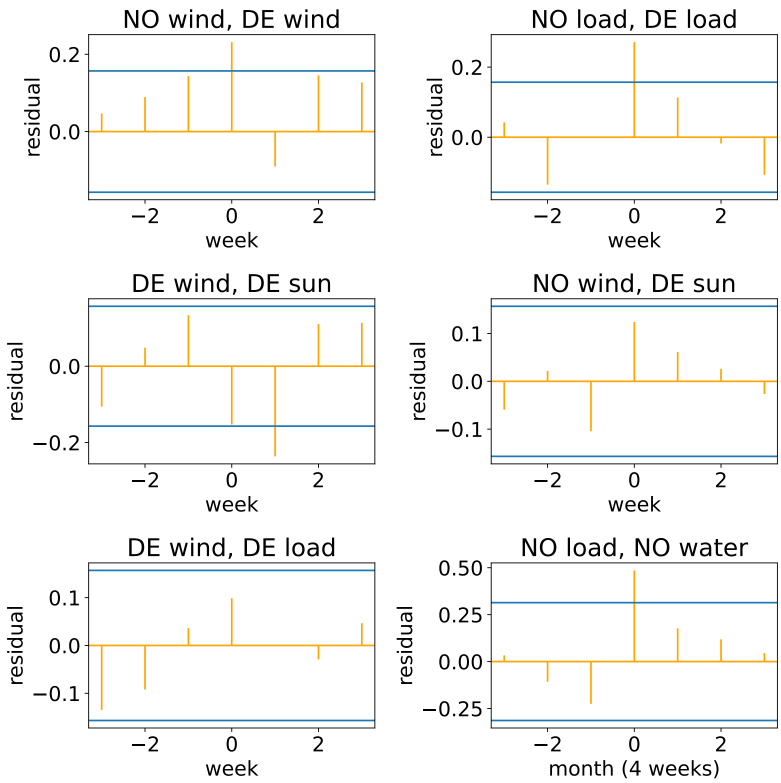

Figure 30: *Correlations between the residuals of the fitted AR-models. Weekly scale, except for the Norwegian load-water, which is the correlation between the mean value (over 4 weeks) of the load and the water. The drawn bounds represent $\pm 1.96/\sqrt{n}$. We see correlations between German wind and Norwegian wind and German sun, respectively, as well as between German/Norwegian load and Norwegian load/water.*

We verified for each case that the same qualitative results hold as for the original model, which is indeed the case. As an example, the production profiles for 2020 and 2022 are shown in figure 31, a simulation of cases 3-1 and 3-3 for 2022 is shown in figure 32.

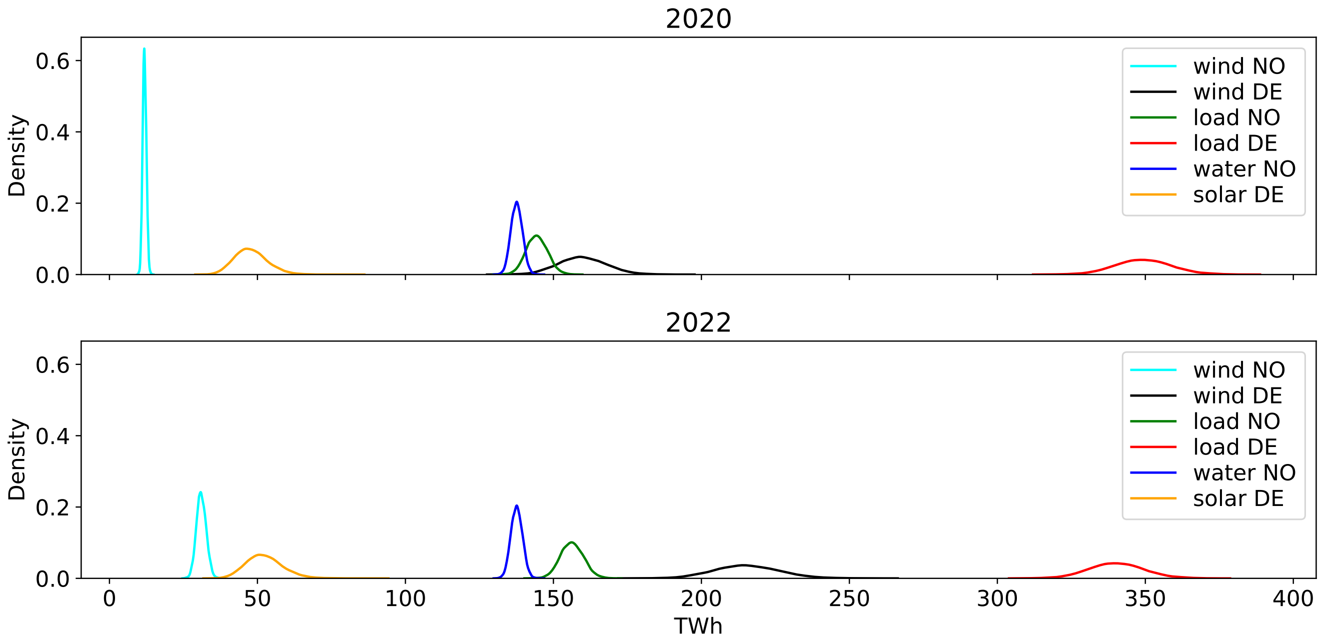

Figure 31: *Electricity production and use per year in 2020 and 2022 with the new seasonal decomposition* (A1).

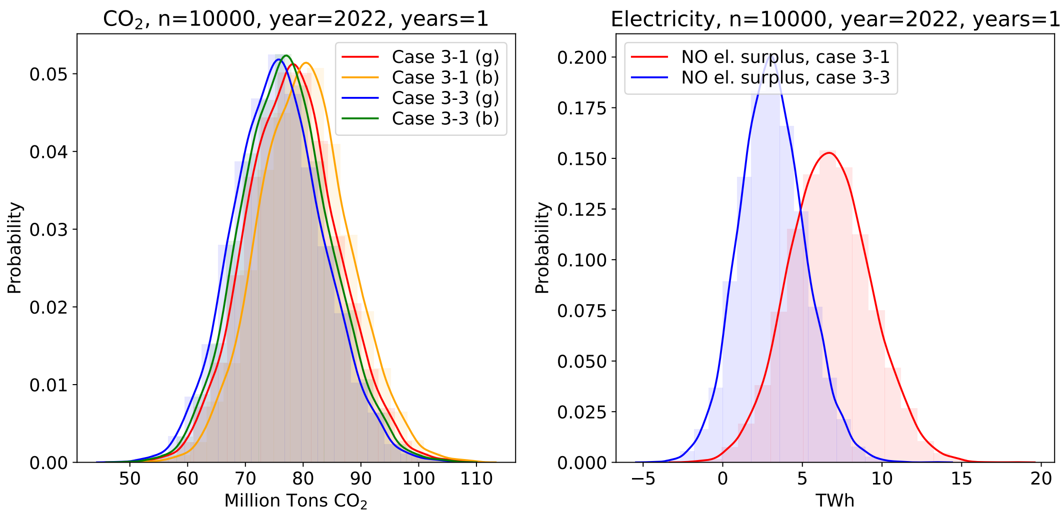

Figure 32: *Data produced with the new seasonal decomposition* (A1). *Left: Combined CO_2 emissions for Case 3-1 and Case 3-3 in 2022. Right: Norwegian energy surplus and net electricity export for Case 3-1 and Case 3-3. Graphs created using Seaborn's Kernel Density Estimator.*

We see that the per-year production/consumption profile get a little narrower or a little broader, which is expected, as the new model leaves larger variations for some time series. Comparing figure 32 with figure 19, while the individual curves change shape (the CO_2 profile gets broader, while the surplus profile gets more narrow), the overall trends remain the same. With the new way to remove the seasonality, case 3-3 has 3.2 ± 0.6 million tons CO_2 less emissions than case 3-1 in the good scenario, which differs less than 10% from the other model. This shows that the choice of how to de-season the data does not have a huge impact on the results and verifies their stability.

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