Straka test case

March 30, 2021

1 Initial condition for background state

The initial condition in the Straka benchmark of the background state is given for the temperature by

$$\overline{T} = T_s - zgC_p^{-1}$$

for the pressure by

$$\begin{aligned}\overline{p} &= p_0 \left(\overline{T}T_s^{-1}\right)^{R/C_p} \\ &= p_0 \left(\overline{T}T_s^{-1}\right)^{\kappa}\end{aligned}$$

with $\kappa = R/C_p$ and we get the density by the ideal gas equation

$$\overline{\rho} = \overline{p}\left(R\overline{T}\right)^{-1}.$$

2 Hydrostatic balance

Next, we show that the equations are in hydrostatic balance

$$\frac{d\overline{p}}{dz} = -\overline{\rho}g$$

holds for the equations above and we get

$$\begin{aligned}
\frac{d}{dz}\left(p_0\left(\overline{T}T_s^{-1}\right)^{\kappa}\right) &= -\overline{\rho}g && (1)\\
p_0 T_s^{-\kappa}\frac{d}{dz}\left(\overline{T}^{\kappa}\right) &= -\overline{\rho}g \text{ (OK)} && (2)\\
p_0 T_s^{-\kappa}\kappa\left(\overline{T}^{\kappa-1}\right)\frac{d}{dz}\overline{T} &= -\frac{\overline{p}}{R\overline{T}}g \text{ (OK)} && (3)\\
\overline{T}Rp_0 T_s^{-\kappa}\kappa\left(\overline{T}^{\kappa-1}\right)gC_p^{-1} &= \overline{p}g && (4)\\
Rp_0 T_s^{-\kappa}\kappa\left(\overline{T}^{\kappa}\right)C_p^{-1} &= \overline{p} && (5)\\
Rp_0 T_s^{-\kappa}\kappa\left(\overline{T}^{\kappa}\right)C_p^{-1} &= p_0\left(\overline{T}T_s^{-1}\right)^{\kappa} && (6)\\
\left(R/C_p\right)\kappa &= 1. && (7)
\end{aligned}$$

Here, we can observe that there's an error in the equations above, requiring a fix of κ

$$\Rightarrow \kappa = C_p/R$$