Notes on metric terms

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The average of some quantity $q(\lambda, \theta)$ on a regular longitude-latitude grid should be computed. We work in an angular spherical coordinate system with λ the longitude and θ the latitude. Let λ_i and θ_j be the points related to the lat-lon discrete cells (i, j). Then, we refer to the quantity stored in this cell as $q(\lambda_i, \theta_i) = q_{i,j}$

In order to compute the sum of all quantities on the sphere on a lat-lon grid, we consider the following issues:

- For each point (i, j) on the lon-lat grid which is related to the cell given at (λ_i, θ_j), there's
  - a cell of width $d\lambda_i = 2\pi/N_\lambda$ with N_λ the number of grid points along the latitude and
  - a cell of height $d\theta_i$ related to the latitude.
- In order to approximate average, it is sufficient to scale the quantity in each cell by
  $$d\lambda_i \cos(\theta_i) 2\pi$$
  to include the shrinking size of the cell torwards the poles. Finally, we can approximate the quantity on the unit sphere by
  $$Q^* = \sum_{i,j} q_{i,j} d\lambda_i d\theta_i \cos(\theta_i) 2\pi.$$
  Note, that the metric term for the latitude is already included since the coordinate system $[-\pi/2; \pi/2]$ represents the half circumference of a circle.
- The average is then given by
  $$Q_{avg} = \frac{Q^*}{4\pi}$$
  with 4π related to the surface of a unit sphere.