Stats Library

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Definition 1.1 Mean

This finds the mean of a sample:

$$\bar{y} = \sum_{i=1}^{n} y_i$$

Definition 1.2 Variance

This finds the variance of a sample:

$$s^2 = \frac{1}{n-1} \sum_{i=0}^{n} (y_1 - \bar{y})^2$$

Definition 1.3: Standard Deviation

This finds the standard deviation of the sample:

$$s = \sqrt{\frac{\sum_{i=0}^{n} (x - x)^2}{n-1}}$$

This sample finds the standard deviation of the population

$$s = \sqrt{\frac{\sum_{i=0}^{n} (x - m)^2}{n}}$$

Theorem 2.1: MxN Rule

With m elements $a1, a2, ..., am$ and n elements $b1, b2, ..., bn$, it is possible to form $mn = mn$ pairs containing one element from each group.

Definition 2.2: Permutations

This finds the permutations of n distinct objects taken r times:

$$P_r^n = \frac{n!}{(n-r)!}$$

Theorem 2.3 Partition

The number of ways partitioning n distinct objects into k distinct groups where each object appears only once:

$$N = \frac{n!}{n_1!n_2!...n_k!}$$

Definition 2.8 Combinations

This finds the number of combinations of n objects taken of r amount:

$$P_r^n = \frac{n!}{r!\,(n-r)!}$$

Definition 2.9: Conditional Probability

This finds the conditional probability of event A, given that event B has occurred:

$$P\left(A|B\right) = \frac{P\left(A \wedge B\right)}{P\left(B\right)}$$

Definition 2.10: Independent Probability

Events A and B are independent if any of the following holds:

$$P\left(A|B\right) = P\left(A\right)$$

$$P\left(B|A\right) = P\left(B\right)$$

$$P\left(A \wedge B\right) = P\left(A\right)P\left(B\right)$$

Otherwise, the events are dependent.

Theorem 2.8: Theorem of Total Probability

Theorem of Total Probability:

$$P\left(A\right) = \sum_{i=1}^{n} P\left(A|B_i\right)P\left(B_i\right)$$

Assuming $P\left(B_i\right) > 0\, for\, i = 1, ..., n$

Theorem 2.9: Baye's Rule

Baye's Rule:
Assume that $B1, B2, ..., Bk$ is a partition of S (see Definition 2.11) such that $P(Bi) > 0$, for $i = 1, 2, ..., k$. Then

$$P\left(P_j|A\right) = \frac{P\left(A|B_j\right) P\left(B_j\right)}{\sum_{i=1}^{k} P\left(A|B_i\right) P\left(B_i\right)}$$

Definition 3.4: Discrete Random Variable

Let Y be a discrete random variable with the probability function $p(y)$. Then the expected value of Y , $E(Y)$, is defined to be:

$$E\left(Y\right) = \sum_{y} yp\left(y\right)$$

Definition 3.5: Expected Value and Standard Deviation of Discrete Random Variable

If Y is a random variable with mean $E(Y) = \mu$, the variance of a random variable Y is defined to be the expected value of $(Y - \mu)2$. That is,

$$V\left(Y\right) = E\left[\left(Y - \mu\right)^2\right]$$

The standard deviation of Y is the positive square root of $V\left(Y\right)$

Definition 3.7: Binomial Probability Distribution

A random variable Y is said to have a binomial probability distribution based on n trials with success probability p if and only if:

$$p\left(y\right) = \tbinom{n}{y} p^y q^{n-y},\ \ y = 0, 1, 2, ..., n \, and \, 0 \leq p \leq 1.$$

Theorem 3.7: Expected Value and Variance of Binomial Probability Distribution

Let Y be a binomial random variable based on n trials and success probability p. Then:

Expected Value: $\mu = E\left(Y\right) = np$
Variance: $\sigma^2 = V\left(Y\right) = npq$

Definition 3.8: Geometric Probability Distribution

A random variable Y is said to have a geometric probability distribution if and only if:

$$p(y) = q^{y-1}p, \quad y = 1, 2, 3, ..., \; 0 \leq p \leq 1.$$

Theorem 3.8: Expected Value and Variance of Geometric Probability Distribution

If Y is a random variable with a geometric distribution,

$$\text{Expected Value: } \mu = E(Y) = \tfrac{1}{p}$$
$$\text{Variance: } \sigma^2 = V(Y) = \tfrac{1-p}{p^2}$$

Definition 3.10: Hypergeometric Probability Distribution

A random variable Y is said to have a hypergeometric probability distribution if and only if

$$\frac{\binom{p}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

Theorem 3.10: Expected Value and Variance of Hypergeometric Distribution

If Y is a random variable with a hypergeometric distribution,

$$\text{Expected Value: } \mu = E(Y) = \tfrac{nr}{N}$$
$$\text{Variance: } \sigma^2 = V(Y) = n\left(\tfrac{r}{N}\right)\left(\tfrac{N-r}{N}\right)\left(\tfrac{N-n}{N-1}\right)$$

Definition 3.9: Negative Binomial Probability Distribution

Finds the negative binomial probability distribution

$$p(y) = \binom{y-1}{r-1} p^r q^{y-r}$$

Theorem 3.9: Expected Value and Variance of Negative Binomial Probability Distribution

If Y is a random variable with a negative binomial distribution,

$$\text{Expected Value: } \mu = E(Y) = \frac{r}{p}$$
$$\text{Variance: } \sigma^2 = V(Y) = \frac{r(1-p)}{\frac{r}{p}}$$