# Stats Library

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## Definition 1.1 Mean

This finds the mean of a sample:

$$\bar{y} = \sum_{i=1}^{n} y_i$$

## Definition 1.2 Variance

This finds the variance of a sample:

$$s^2 = \frac{1}{n-1} \sum_{i=0}^{n} (y_1 - \bar{y})^2$$

## Definition 1.3: Standard Deviation

This finds the standard deviation of the sample:

$$s = \sqrt{\frac{\sum_{i=0}^{n} (x - x)^2}{n-1}}$$

This sample finds the standard deviation of the population

$$s = \sqrt{\frac{\sum_{i=0}^{n} (x - m)^2}{n}}$$

## Theorem 2.1: MxN Rule

With $m$ elements $a1, a2, ..., am$ and $n$ elements $b1, b2, ..., bn$, it is possible to form $mn = mn$ pairs containing one element from each group.

## Definition 2.2: Permutations

This finds the permutations of $n$ distinct objects taken $r$ times:

$$P_r^n = \frac{n!}{(n-r)!}$$

## Theorem 2.3 Partition

The number of ways partitioning $n$ distinct objects into $k$ distinct groups where each object appears only once:

$$N = \frac{n!}{n_1!n_2!...n_k!}$$

## Definition 2.8 Combinations

This finds the number of combinations of $n$ objects taken of $r$ amount:

$$P_r^n = \frac{n!}{r!\,(n-r)!}$$

## Definition 2.9: Conditional Probability

This finds the conditional probability of event A, given that event B has occurred:

$$P\,(A|B) = \frac{P\,(A \wedge B)}{P\,(B)}$$

## Definition 2.10: Independent Probability

Events $A$ and $B$ are independent if any of the following holds:

$$P\,(A|B) = P\,(A)$$

$$P\,(B|A) = P\,(B)$$

$$P\,(A \wedge B) = P\,(A)\,P\,(B)$$

Otherwise, the events are dependent.

## Theorem 2.8: Theorem of Total Probability

Theorem of Total Probability:

$$P\,(A) = \sum_{i=1}^{n} P\,(A|B_i)\,P\,(B_i)$$

Assuming $P\,(B_i) > 0\,for\,i = 1, ..., n$

## Theorem 2.9: Baye's Rule

Baye's Rule:
Assume that $B1, B2, ..., Bk$ is a partition of $S$ (see Definition 2.11) such that $P(Bi) > 0$, for $i = 1, 2, ..., k$. Then

$$P\left(P_j|A\right) = \frac{P\left(A|B_j\right) P\left(B_j\right)}{\sum_{i=1}^{k} P\left(A|B_i\right) P\left(B_i\right)}$$

## Definition 3.4: Discrete Random Variable

Let Y be a discrete random variable with the probability function $p(y)$. Then the expected value of $Y$ , $E(Y)$, is defined to be:

$$E\left(Y\right) = \sum_{y} yp\left(y\right)$$

## Definition 3.5: Expected Value and Standard Deviation of Discrete Random Variable

If $Y$ is a random variable with mean $E(Y) = \mu$, the variance of a random variable $Y$ is defined to be the expected value of $(Y - \mu)2$. That is,

$$V\left(Y\right) = E\left[\left(Y - \mu\right)^2\right]$$

The standard deviation of $Y$ is the positive square root of $V\left(Y\right)$

## Definition 3.7: Binomial Probability Distribution

A random variable $Y$ is said to have a binomial probability distribution based on $n$ trials with success probability $p$ if and only if:

$$p\left(y\right) = \binom{n}{y} p^y q^{n-y}, \;\; y = 0, 1, 2, ..., n \, and \, 0 \leq p \leq 1.$$

## Theorem 3.7: Expected Value and Variance of Binomial Probability Distribution

Let $Y$ be a binomial random variable based on $n$ trials and success probability $p$. Then:

Expected Value: $\mu = E\left(Y\right) = np$
Variance: $\sigma^2 = V\left(Y\right) = npq$

## Definition 3.8: Geometric Probability Distribution

A random variable $Y$ is said to have a geometric probability distribution if and only if:

$$p\left(y\right)=q^{y-1}p,\quad y=1,2,3,...,\ 0\leq p\leq 1.$$

## Theorem 3.8: Expected Value and Variance of Geometric Probability Distribution

If $Y$ is a random variable with a geometric distribution,

Expected Value: $\mu=E\left(Y\right)=\frac{1}{p}$
Variance: $\sigma^2=V\left(Y\right)=\frac{1-p}{p^2}$

## Definition 3.10: Hypergeometric Probability Distribution

A random variable $Y$ is said to have a hypergeometric probability distribution if and only if

$$\frac{\binom{p}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

## Theorem 3.10: Expected Value and Variance of Hypergeometric Distribution

If $Y$ is a random variable with a hypergeometric distribution,

Expected Value: $\mu=E\left(Y\right)=\frac{nr}{N}$
Variance: $\sigma^2=V\left(Y\right)=n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$

## Definition 3.9: Negative Binomial Probability Distribution

Finds the negative binomial probability distribution

$$p\left(y\right)=\binom{y-1}{r-1}p^r q^{y-r}$$

## Theorem 3.9: Expected Value and Variance of Negative Binomial Probability Distribution

If Y is a random variable with a negative binomial distribution,

$$\text{Expected Value: } \mu = E\left(Y\right) = \tfrac{r}{p}$$
$$\text{Variance: } \sigma^2 = V\left(Y\right) = \tfrac{r(1-p)}{\frac{r}{p}}$$

## Definition 3.11: Poisson Probability Distribution

A random variable $Y$ is said to have a Poisson probability distribution if and only if

$$p\left(Y\right) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Where $\lambda =$ average number of successes in a given interval

## Theorem 3.11: Expected Value and Variance of Poisson Probability Distribution

$$\text{Expected Value: } \mu = E\left(Y\right) = \lambda$$
$$\text{Variance: } \sigma^2 = V\left(Y\right) = \lambda$$

## Theorem 3.14: Tchebysheff's Theorem

Let $Y$ be a random variable with mean $\mu$ and finite variance $\sigma^2$. Then, for any constant $k > 0$,

$$P\left(|Y - \mu| < k\sigma\right) \geq 1 - \frac{1}{k^2}$$

or

$$\left(|Y - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}$$

## Definition 4.1: Distribution Function of Continuous Random Variable

Let $Y$ denote any random variable. The $distribution\ function$ of $Y$, denoted by $F(y)$, is such that $F(y) = P(Y \leq y)$ for $-\infty < y < \infty$s.

## Definition 4.3: Probability Density Function of Continuous Random Variable

Let $F(y)$ be the distribution function for a continuous random variable $Y$. Then $f(y)$, given by

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

wherever the derivative exists, is called the probability density function for the random variable $Y$.

## Theorem 4.3: Probability of Density Function in a Range

If the random variable $Y$ has density function $f(y)$ and $a < b$, then the probability that $Y$ falls in the interval $[a, b]$ is

$$P(a \leq Y \leq b) = \int_a^b f(y)\,dy$$

## Definition 4.5: Expected Value and Variance of Continuous Random Variable

The expected value of a continuous random variable $Y$ is

$$\mu = E(Y) = \int_{-\infty}^{\infty} y f(y)\,dy,$$

The variance of a continuous random variable $Y$ is

$$\sigma^2 = V(Y) = E\left(Y^2\right) - [Y]^2$$

## Definition 4.6: Continuous Uniform Probability Distribution

If $\theta_1 < \theta_2$, a random variable $Y$ is said to have a continuous uniform probability distribution on the interval $(\theta_1, \theta_2)$ if and only if the density function of $Y$ is

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1} \theta_1 \leq y \leq \theta_2 \\ \cdot 0, \; elsewhere, \end{cases}$$

## Theorem 4.6: Expected Value and Variance of Uniform Probability Distribution

If $\theta_1 < \theta_2$ and $Y$ is a random variable uniformly distributed on the interval $(\theta_1, \theta 2)$, then

$$\text{Expected Value: } \mu = E\left(Y\right) = \frac{\theta_1+\theta_2}{2}$$

$$\text{Variance: } \sigma^2 = V\left(Y\right) = \frac{(\theta_2-\theta_1)^2}{12}$$

## Theorem 4.11: Exponential Distribution

The gamma density function in which $\alpha = 1$ is called the exponential density function.

A random variable $Y$ is said to have an exponential distribution with parameter $\beta > 0$ if and only if the density function of $Y$ is

$$f\left(y\right) = \begin{cases} \frac{1}{\beta}e^{\frac{-y}{\beta}} & ,0\leq y<\infty \\ \cdot 0, & elsewhere, \end{cases}$$

## Expected Value and Variance of Right Skewed Distribution

$$\text{Expected Value: } \mu = E\left(Y\right) = \alpha\beta$$

$$\text{Variance: } \sigma^2 = V\left(Y\right) = \alpha\beta^2$$

## Theorem 4.10: Expected Value and Variance of Exponential Distribution Distribution

If $Y$ is an exponential random variable with parameter $\beta$, then

$$\text{Expected Value: } \mu = E\left(Y\right) = \beta$$

$$\text{Variance: } \sigma^2 = V\left(Y\right) = \beta^2$$

The proof follows directly from Theorem 4.8 with $\alpha = 1$.

## Definition 5.1: Joint Probability Function of Discrete Variable

Let Y1 and Y2 be discrete random variables. The joint (or bivariate) probability function for $Y_1$ and $Y_2$ is given by

$$p\left(y_1, y_2\right) = P\left(Y_1 = y_1, Y_2 = y_2\right), \quad -\infty < y < \infty, -\infty < y_2 < \infty$$

## Definition 5.2: Joint Probability Function of Random Variable

For any random variables $Y_1$ and $Y_2$, the joint (bivariate) distribution function $F(y1, y2)$ is

$$F\left(y_1, y_2\right) = P\left(Y_1 \leq y_1, Y_2 \leq y_2\right), \quad -\infty < y < \infty, -\infty < y_2 < \infty$$

## Definition 5.4: Marginal Probability Function

Let $Y_1$ and $Y_2$ be jointly discrete random variables with probability function $p(y_1, y_2)$. Then the marginal probability functions of $Y_1$ and $Y_2$, respectively, are given by

$$p_1(y_1) = \sum_{all\ y_2} p(y_1, y_2) \quad and \quad p_2(y_2) = \sum_{all\ y_1} p(y_1, y_2).$$

## Definition 5.4: Marginal Density Function

Let $Y_1$ and $Y_2$ be jointly continuous random variables with joint density function $f(y_1, y_2)$. Then the marginal density functions of $Y_1$ and $Y_2$, respectively, are given by

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_2 \quad and \quad f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_1$$

## Definition 5.5: Conditional Discrete Probability Function

If $Y_1$ and $Y_2$ are jointly discrete random variables with joint probability function $p(y_1, y_2)$ and marginal probability functions $p1(y_1)$ and $p_2(y_2)$, respectively, then the conditional discrete probability function of $Y_1$ given $Y_2$ is

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1 = y_1, Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(y_1, y_2)}{p_2(y_2)}$$

## Definition 5.6: Conditional Distribution Function

If $Y_1$ and $Y_2$ are jointly continuous random variables with joint density function $f(y_1, y_2)$ , then the conditional distribution function of $Y_1$ given$Y_2 = y_2$ is

$$F(y_1|y_2) = P(Y_1 \leq y_1|Y_2 = y_2)$$

## Definition 5.7

Let $Y_1$ and $Y_2$ be jointly continuous random variables with joint density $f(Y_1, Y_2)$ and marginal densities $f_1(y_1)$ and $f_2(y_2)$, respectively. For any $y_2$ such that $f_2(y_2) > 0$, the conditional density of $Y_1$ given $Y_2 = y_2$is given by

$$f(y_1|y_2 = \frac{f(y_1, y_2)}{f_2(y_2)}$$

and, for any $y_1$ such that $f_1(y_1) > 0$, the conditional density of $Y_1$ given $Y_1 = y_1$ is given by

$$f(y_2|y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$$

## Definition 5.8: Independence of Distribution Function

Let $Y_1$ have distribution function $F_1(y_1)$, $Y_2$ have distribution function $F_2(y_2)$, and $Y_1$ and $Y_2$ have joint distribution function $F(y_1, y_2)$. Then $Y_1$ and $Y_2$ are said to be independent if and only if

$$F(y_1, y_2) = F_1(y_1)F_2(y_2)$$

for every pair of real numbers $(y_1, y_2)$. If $Y_1$ and $Y_2$ are not independent, they are said to be dependent.