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The poisson distribution is belongs to exponential family. The probability mass function (i.e. the density respect to continuing counting measure) of a Poisson random variable is given as follows:

$$P(x \mid \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Rewriting this expression we obtain:

$$P(x \mid \lambda) = \frac{1}{x!} \exp \{x \log \lambda - \lambda\}$$

Thus the Poisson distribution is an exponential family distribution, with:

$$\eta = \log \lambda$$
$$T(x) = x$$
$$A(\eta) = \lambda = e^{\eta}$$
$$h(x) = \frac{1}{x!}$$

Moreover, we can obviously invert the relationship between η and λ: $\quad \lambda = e^{\eta}$