Jashaina Thomas

A class of distributions is in the exponential family if it can be written in the form: $p(y;\eta) = b(\eta)\exp(\eta^T T(y) - a(\eta))$, η: natural parameter

$T(y)$: sufficient statistic

$a(\eta)$: log partition function

$b(\eta)$: normalize factor to make sure $\int p(y,\eta) = 1$

$y \sim \text{Poisson}(\lambda)$

$$p(y|\lambda) = \frac{\lambda^y e^{-y}}{y!}, \quad y = 0, 1, \ldots$$

$$= \frac{1}{y!}\exp\{\log(\lambda)y - \lambda\}$$

$$= b(\eta)\exp\{\eta^T T(y) - a(\eta)\}, \quad b(\eta) = \frac{1}{y!}, \ \eta = \log(\lambda), \ T(y) = y$$

Canonical link: $\eta = \log(\lambda)$

$a(\eta) = \lambda = e^{\eta}$