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1B (a) $\det(A-\lambda I)=0$ $\quad \det\begin{pmatrix} 1-\lambda & 0 & -4 \\ 0 & 5-\lambda & 4 \\ -4 & 4 & 3-\lambda \end{pmatrix} = 0$

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ew: $\lambda_1 = 3$ $\quad \lambda_2 = 9$ $\quad \lambda_3 = -3$

for $\lambda_1 = 3$ $\quad \begin{pmatrix} -2 & 0 & -4 \\ 0 & 2 & 4 \\ -4 & 4 & 0 \end{pmatrix} v = 0$ $\quad v_1 = \begin{pmatrix} 1 \\ -1 \\ -\frac{1}{2} \end{pmatrix}$

Similar: ev_2 for ew $\lambda_2 = 9$ is $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

ev_3 for ew $\lambda_3 = -3$ is $\begin{pmatrix} 1 \\ -\frac{1}{2} \\ 1 \end{pmatrix}$

(b) $A = A^T$ $\therefore$ $P = \begin{pmatrix} | & | & | \\ ev_1 & ev_2 & ev_3 \\ | & | & | \end{pmatrix}$

$D = P^{T}AP = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & -3 \end{pmatrix} = \begin{pmatrix} ew_1 & & 0 \\ & ew_2 & \\ 0 & & ew_3 \end{pmatrix}$

(c) ew of $A^{-1} = \frac{1}{ew}$ $\therefore$ $\lambda = \frac{1}{3}, \frac{1}{9}, -\frac{1}{3}$

(d) $A^2 v$ $\cancel{= \lambda_2 v_2}$

$= AAv = \lambda A v = \lambda^2 v$

$\therefore$ ev is same, ew is squared. i.e. 9, 81, 9

3A (a) $\|u\| = \sqrt{2}$ $\quad \|v\| = \sqrt{2}$.

$\hat{u} = \left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)^T$

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(b) $\|v-w\| = \sqrt{1+9+6} = 4$

(c) angle $= \frac{u\cdot v}{\|u\|\cdot\|v\|} = \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2}$

This is cosine of the angle. -1

(d) $v\cdot w = 2 - 2 + 0 = 0$ $\therefore$ v and w are ortho.

4A (a) ① $\langle f,f\rangle = \int_{-\pi}^{\pi} f^2 dt \geq 0$ and equals 0 iff f = 0. -1

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② $\langle f,g\rangle = \int_{-\pi}^{\pi} fg\,dt = \int g f\,dt = \langle g,f\rangle$

③ $\langle f+g,h\rangle = \int (f+g)h\,dt = \int fh+gh\,dt = \langle f,h\rangle + \langle g,h\rangle$

<rf,g> =r <f,g>. -1

(b) $\langle 1, \cos mt\rangle = \int \cos mt = 0$

$\langle 1, \sin nt\rangle = \int \sin nt = 0$

also $\langle \cos mt, \cos nt\rangle = 0$ for $m \neq n$

$\langle \cos mt, \sin nt\rangle = 0$ for $m \neq n$

$\therefore$ the set is an orthogonal set.

show work. Also not complete. -4

(c) $1 \to 1$

$\|\cos nt\|^2 = \int \frac{1+\cos 2nt}{2}\, dt = \pi$

$\|\sin nt\|^2 = \int \frac{1-\cos 2nt}{2}\, dt = \pi$.

$\therefore$ ~~each~~ other elements divide by $\sqrt{\pi}$

5. ~~Ans~~

(a) $A = A^T$ $\quad \det(A-\lambda I)=0 \quad \lambda = -4, -4, 5$

$v_1 = (-1, 1, 0)^T/\sqrt{2} \quad v_2 = (-1, 0, 1)^T/\sqrt{2}$

$v_3 = (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})^T$

You need to convert these into an orthogonal basis for P to be orthogonal. -2

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$\therefore P = \begin{pmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{pmatrix} \quad D = \begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

(b) $\cancel{A\lambda v = \lambda v} \ \lambda\|v\|^2 = \langle Av, v\rangle = \cancel{\langle v, Av\rangle} \ \bar{\lambda}\|v\|^2$ $\quad \cancel{\lambda = \bar{\lambda}}$

$\cancel{\langle Av, v\rangle} \ \cancel{(v^T A v)^*} \ \langle Av, v\rangle = \langle v, Av\rangle$

$\therefore \lambda = \bar{\lambda}$ $\qquad = \bar{\lambda}\|v\|^2$

(c) $\lambda_1(u, v) = \langle \lambda_1 u, v\rangle = \langle Au, v\rangle = \langle u, Av\rangle = \langle u, \lambda_2 v\rangle$

$= \lambda_2 (u, v)$

$\lambda_1 \neq \lambda_2 \quad \therefore \langle u, v\rangle = 0$

(d) $x^T A x > 0 \quad A = A^T$

if A is positive definite, $A = Q \Sigma Q^T \quad \Sigma : \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{pmatrix}$

$= Q\sqrt{\Sigma}\sqrt{\Sigma}Q^T$

$= P^T P$

conversely : $x^T A x = x^T P^T P x = \|Px\|^2 > 0$

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6E : A ~~are~~ $\lambda = 1, 1, 4$, all positive $\Rightarrow A$ is positive definite

B : $\lambda = 4 \pm \sqrt{5}$, all positive definite.

C : $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$v^T C v = x^2 + 4xy + 2y^2 + 5xz + 6yz + 3z^2$

not ~~all~~ always > 0

~~D~~ E : $\lambda = 3 \pm \sqrt{13}$ $\quad 3 - \sqrt{13} < 0$ $\therefore$ not positive definite