

Poisson distribution into exponential factor.

Poisson distribution: $f(k; \lambda) = \Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$

Write into exponential family:

$$\begin{aligned} f(k; \lambda) &= \frac{1}{k!} \cdot \exp \log_e (\lambda^k e^{-\lambda}) \\ &= \frac{1}{k!} \cdot (\exp \log_e \lambda^k) \cdot e^{-\lambda} \\ &= \frac{1}{k!} e^{k \log_e \lambda - \lambda} \end{aligned}$$

Let $T(y) = k$, $\eta = \log_e \lambda$, $a(\eta) = \lambda$, $b(\eta) = \frac{1}{k!}$

Then we have $P(y, \eta) = b(\eta) \exp(\eta T(y) - a(\eta))$
and $a(\eta) = \lambda = \exp(\log_e \lambda) = e^\eta$