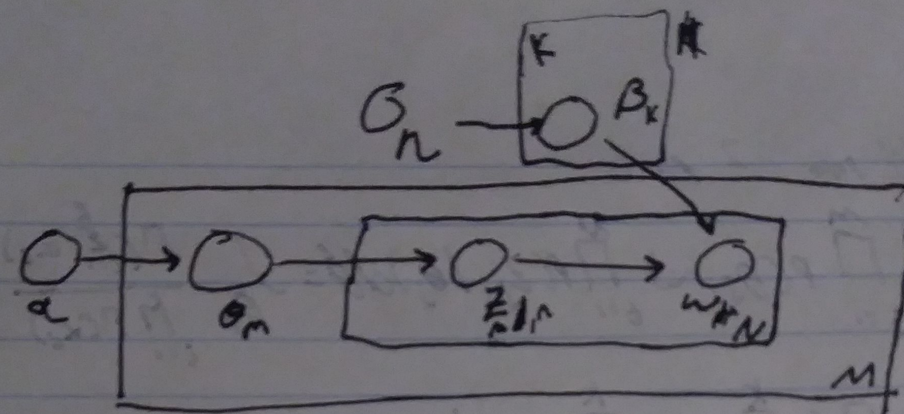

α = parameter of dirichlet prior on the per doc topic distr.

β = parameter of dirichlet prior on the per topic word distribution

η : topic paramater

θ_i = topic distr for doc i

ϕ_k = word distr for topic k

z_{ij} = topic for j^{th} word in doc i

w_{ij} = specified word

M = # docs

N = # words in given doc

K = # topics

$$\prod_{i=1}^{K} p(\beta_i|\eta)\prod_{d=1}^{D} p(\theta_d|\alpha)\left(\prod_{i=1}^{N} p(z_{m,n}|\theta_m)p(w_{m,n}|\beta_{1:K}, z_{m,n})\right)$$

$\therefore$ $\theta_i \sim Dir(\alpha)$ $\quad\phi_K \sim Dir(\beta)$ $\quad z_{ij} \sim Multinomial(\theta_i)$

$w_{ij} \sim Multinomial(\phi_{z_{ij}})$

Can use Gibbs Sampler so that

$$P(W,Z,\theta,\phi;\alpha,\beta) = \prod_{i=1}^{K} P(\phi_i;\beta)\prod_{j=1}^{M} P(\theta_j;\alpha)\prod_{t=1}^{N} P(Z_{j,t}|\theta_j)P(W_{j,t}|\phi_{Z_{j,t}})$$

$$P(Z,W;\alpha,\beta) = \int_\theta\int_\phi P(W,Z,\theta,\phi;\alpha,\beta)\,d\phi\,d\theta$$

$$= \int_\phi \prod_{i=1}^{K} P(\phi_i;\beta)\prod_{j=1}^{M}\prod_{t=1}^{N} P(W_{j,t}|\phi_{z_{j,t}})\,d\phi \int_\theta \prod_{j=1}^{M} P(\theta_j;\alpha)\prod_{t=1}^{N} P(Z_{j,t}|\theta_j)$$

Split into 2 parts

$$\int_{\theta} \prod_{j=1}^{M} P(\theta_j ; \alpha) \prod_{t=1}^{N} P(Z_{j,t} \mid \theta_j) \, d\theta = \int_{\theta_j} \frac{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} \theta_j^{\alpha_i - 1} \prod_{t=1}^{N} [illegible]$$

$$= \prod_{j=1}^{M} \frac{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right) \prod_{i=1}^{K} \Gamma(n_j^i + \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i) \, \Gamma\left(\sum_{i=1}^{K} n_j^i + \alpha_i\right)} \qquad \star^1$$

$$\int_{\phi} \prod_{i=1}^{K} P(\phi_i ; \beta) \prod_{j=1}^{M} \prod_{t=1}^{N} P(W_{j,t} \mid \phi \, Z_{j,t}) \, d\phi = \prod_{i=1}^{K} \int_{\phi_i} P(\phi_i ; \beta) \prod_{j=1}^{M} \prod_{t=1}^{N} P(W \mid \phi_i) \, d\phi$$

$$= \prod_{i=1}^{K} \int_{\phi_i} \frac{\Gamma\left(\sum_{r=1}^{V} \beta_r\right)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \prod_{r=1}^{V} \phi_{i,r}^{n + \beta_r - 1} \, d\phi_i$$

$$= \prod_{i=1}^{K} \frac{\Gamma\left(\sum_{r=1}^{V} \beta_r\right)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \cdot \frac{\prod_{r=1}^{V} \Gamma(n_r^i + \beta_r)}{\Gamma\left(\sum_{r=1}^{V} n_r^i + \beta_r\right)} \qquad \star^2$$

$$\Rightarrow P(Z, W ; \alpha, \beta) = \star^1 \cdot \star^2$$