XLAB Science Camp August 12th – 15th 2024

Exercise

Numerical simulation of physical systems:

Exercise 1 (Partial Integration / Boundary Conditions)
We recall the product rule of differentiation:

$$f(x) = g(x) \cdot h(x) \quad \Longrightarrow \quad f'(x) = g'(x)h(x) + g(x)h'(x)$$

Therefore, with the Fundamental theorem of Calculus, we have

$$[f(x)]_a^b = \int_a^b f'(x)dx = \int_a^b \left(g'(x)h(x) + g(x)h'(x)\right) dx = \int_a^b g'(x)h(x)dx + \int_a^b g(x)h'(x)dx. \tag{1}$$

(a) Convert the equation

$$\int_a^b -k\frac{d^2u}{dx^2}(x)v(x)\ dx = \int_a^b q_Q(x)v(x)\ dx$$

using (1) into

$$\Longrightarrow \int_a^b k\frac{du}{dx}(x)\frac{dv}{dx}(x)\ dx - \left[k\frac{du}{dx}(x)v(x)\right]_a^b = \int_a^b q_Q(x)v(x)\ dx \tag{2}$$

(b) We consider two types of boundary conditions.

- "Robin" type boundary conditions, where the (outgoing) heat flux at the edge $q_S(a) = k\frac{du}{dx}(a)$ or $q_S(b) = -k\frac{du}{dx}(b)$ is modeled with:

$$q_S(x) = \alpha(u(x) - T^{\text{ref}}),$$

here $\alpha \in \mathbb{R}$ is the heat transfer coefficient and $T^{\text{ref}} \in \mathbb{R}$ is a reference temperature outside the interval.

- "Neumann" type boundary conditions, where the heat flux is directly prescribed:

$$q_S(x) = q_S^{\text{ref}} \in \mathbb{R}.$$

Formulate (2) for both cases in such a way that on the left side of the equations all terms with u and derivatives of u stand, and all other terms appear on the right side.