Numerical simulation of microwaves

C.Lehrenfeld

1 The PDE model

We start from Maxwell's equations in 3D for the electric and the magnetic field density:

$$\operatorname{curl} E = -\tfrac{\partial}{\partial t} B \tag{1.1a}$$
$$\operatorname{curl} B = \mu_0 \varepsilon_0 \tfrac{\partial}{\partial t} E \tag{1.1b}$$

which yields

$$\operatorname{curl}\operatorname{curl} B = -\mu_0\varepsilon_0 \frac{\partial^2}{\partial^2 t} B.$$

With the 2D assumption $B(x,y,z) = (0,0,u(x,y))$, $E(x,y,z) = (e_1(x,y), e_2(x,y), 0)$, i.e. magnetic field and electric field don't vary in z-direction and hence, the electric field is in the plane while the magnetic field is orthogonal to it, we have

$$(0,0,-\Delta u) = -\mu_0\varepsilon_0 (0,0,\frac{\partial^2}{\partial^2 t} u)$$

or

$$-\Delta u + \mu_0\varepsilon_0 \frac{\partial^2}{\partial^2 t} u = 0.$$

Additionally we add the assumption of a time-harmonic signal $u(x,y,t) = \hat{u}(x,y)e^{i2\pi f t}$ for a given frequency f $(\omega = 2\pi f)$ which yields:

$$(-\Delta - \mu_0\varepsilon_0\omega^2 \frac{\partial^2}{\partial^2 t})\hat{u}(x,y)e^{i2\pi f t} = 0$$

and hence the Helmholtz equation $(\omega = 2\pi f)$

$$-\Delta\hat{u} - \mu_0\varepsilon_0\omega^2 \frac{\partial^2}{\partial^2 t}\hat{u} = 0.$$

With $\mu_0\varepsilon_0 = c^{-2}$ and $k = \omega/c = 2\pi f/c$ we may also write

$$-\Delta\hat{u} - k^2 \frac{\partial^2}{\partial^2 t}\hat{u} = 0.$$

This is our PDE for the 2D domains where c depends on the material and k is the microwave frequency with $k = \frac{2\pi f}{c} = \frac{2\pi}{l}$ where l is the (micro wave) wave length.

Note that this derivation has been for vacuum with $\mu_0 \varepsilon_0 = c^{-2}$. For the wave propagation in material, we have that μ_r and ε_r will not be 1 and will have a change in the material parameter k which is proportional to the refrection index.

In the example configuration we have (in vacuum) $k = 10^2/2.8m^{-1}$.

2 Boundary conditions

2.1 Outer boundary conditions

On the outer boundary we typically want "outgoing", non-reflecting boundary conditions, which does not translate directly to a simple PDE boundary condition. We solve the problem by taking a bounding box around the area of interest which is large enough and surround it by a PML boundary layer. PML – perfectly matched layers – is an artificial material that absorbs waves all incoming waves via an artificial coordinate transformation into the complex plane (no details here – COMSOL should be able to do it magically).

2.2 Outgoing waves

The microwave sender is modeled simply by prescribing Dirichlet boundary conditions: $\hat{u}_D = e^{i(\vec{k}\cdot\vec{x})}$ where $\vec{k} = k \cdot \vec{e}$ where $\vec{e}$ is the unit direction of the sender. This models that a wave is emitted in the direction $\vec{e}$ with the frequency corresponding to k. The amplitude is normalized to 1 here.

2.3 Material boundaries

I don't know how to properly model material boundaries. A perfect reflection corresponds to Dirichlet boundary conditions. This may be the right conditions for the outer boundary of the horn antenna but even if not this should hopefully not have a big influence to the remainder anyway...

For all the other material (e.g. also the cone of the antenna) I suggest that you model them only by a different material and not by a boundary conditions.