permutations (order matters)

Q: How many rearrangements of n-things?

A: $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

<u>ex</u> If I have 6 things how many ways can I line up 2 of them?

$$\underline{6}\cdot\underline{5} = 30$$

$$_6P_2 = 6\cdot 5 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 6\cdot 5$$

$$_nP_k = \frac{n!}{(n-k)!}$$

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an aside.

$$\frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)(n-1)!}{(n-1)!} = (n+1)n$$

BREAD How many 3 letter "words" can you form?

$$_5P_3 = \frac{5!}{2!} = 60$$

Combinations (order does not matter
ie: divide out by all rearrangements)

ex 18 students.

How many ways can we form a governing 3-person council?

$$\frac{{}_{18}P_3}{3!} = {}_{18}C_3 = \frac{18!}{3!(18-3)!}$$

$${}_nC_k = \frac{n!}{k!(n-k)!}$$

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20 students.

$${}_{20}C_4 = \frac{20 \cdot 19 \cdot 18 \cdot 17}{4!}$$

$$= \frac{5 \cdot 19 \cdot 6 \cdot 17}{2}$$

$$= 5 \cdot 19 \cdot 3 \cdot 17$$

binomial theorem: $\boxed{(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^n b^{n-k}}$

note: $\binom{n}{k} = {}_nC_k$

${}_6C_0 = 1$ $\quad {}_6C_1 = 6$ $\quad {}_6C_2 = 15$

${}_6C_5 = 6$ $\quad {}_6C_4 = 15$

${}_6C_3 = \frac{6 \cdot 5 \cdot 4}{3!} = 20$

${}_3C_0 = 1$ $\quad {}_3C_1 = 3$ $\quad {}_3C_2 = \frac{3 \cdot 2}{2!} = 3$

${}_3C_3 = 1$

Pascal's Δ

$$
\begin{array}{ccccccccccccc}
&&&&&& 1 &&&&&& \\
&&&&& 1 && 1 &&&&& \\
&&&& 1 && 2 && 1 &&&& \\
&&& 1 && 3 && 3 && 1 &&& \\
&& 1 && 4 && 6 && 4 && 1 && \\
& 1 && 5 && 10 && 10 && 5 && 1 & \\
1 && 6 && 15 && 20 && 15 && 6 && 1
\end{array}
$$

$(a+b)^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$