Warm up

$$\sum_{n=0}^{20} (4n+6) = \#\text{terms}\,\frac{(a_0 + a_{20})}{2}$$

$$= 21\,\frac{(6+86)}{2}$$

$$= 21(46) = 966.$$

$$\sum_{n=1}^{K} n = \frac{K(K+1)}{2}$$

Geometric Series.

a geometric sequence (exponential)

$$\boxed{a_n = a_1 r^{n-1}}$$

r is the "base" of the exponential

$a_n = a_0 r^n$

$$S_K = \sum_{n=1}^{K} a_1 r^{n-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{K-1}$$

$$S_K = a_1 \left(1 + r + r^2 + r^3 + \dots + r^{K-1}\right)$$

$$(1-r) S_K = a_1 \left(1 - r^K\right)$$

$$S_K = \frac{a_1 \left(1 - r^K\right)}{1-r}$$

ex $\quad a_n = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots \right.$

$$a_n = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$$

$S_1 = \frac{1}{2}$

$S_2 = \frac{3}{4}$

$S_3 = 7/8$

$$S_{100} = \sum_{n=1}^{100} \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{\frac{1}{2}\left(1 - \frac{1}{2}^{100}\right)}{1 - \frac{1}{2}}$$

$$= \frac{\frac{1}{2}\left(1 - \frac{1}{2}^{100}\right)}{\frac{1}{2}}$$

$$= 1 - \frac{1}{2}^{100}$$

$$= 1 - 7.8 \times 10^{-31}$$

Max Planck

$\hbar$

ex Roth IRA (retirement acct)

at most \$5500

30 years to save. every year.

I put in \$5500

you make 9% each year.

$$Y_1 = 5500$$

$$Y_2 = 5500 + 5500(1.09)$$

$$Y_3 = 5500 + 5500(1.09) + 5500(1.09)^2$$

$$Y_4 = 5500 + 5500(1.09) + 5500(1.09)^2 + 5500(1.09)^3$$

$$Y_5 = \sum_{n=1}^{5} 5500(1.09)^{n-1}$$

$$Y_{30} = \sum_{n=1}^{30} 5500(1.09)^{n-1}$$

$$= \frac{5500\left(1-(1.09)^{30}\right)}{1-1.09}$$

$$= 749{,}691.46.$$

an aside: $(1-r)(1+r) = 1-r^2$

$$(1-r)(1+r+r^2) = 1+r+r^2$$
$$-r-r^2-r^3$$
$$= 1-r^3$$

$$(1-r)(1+r+r^2+r^3) = 1+r+r^2+r^3$$
$$-r-r^2-r^3-r^4$$
$$= 1-r^4$$

April 27, 2016