Solution

(a) Since the fine increased each day by a factor of 2, the fine grew exponentially with growth factor $b = 2$. To find the percent growth rate, we set $b = 1 + r = 2$, from which we find $r = 1$, or 100%. Thus the daily percent growth rate is 100%. This makes sense because when a quantity increases by 100%, it doubles in size.

(b) If t is the number of days since August 2, the formula for the fine, P in dollars, is

$$P = 100 \cdot 2^t.$$

(c) After 30 days, the fine is $P = 100 \cdot 2^{30} \approx 1.074 \cdot 10^{11}$ dollars, or \$107,374,182,400.

Exercises and Problems for Section 4.1

Skill Refresher

In Exercises S1–S2, express the percentages in decimal form.

S1. 6%

S2. 0.6%

In Exercises S3–S4, express the decimals as a percent.

S3. 0.0012

S4. 1.23

Exercises

Are the functions in Exercises 1–9 exponential? If so, write the function in the form $f(t) = ab^t$.

1. $g(w) = 2\left(2^{-w}\right)$

2. $m(t) = (2 \cdot 3^t)^3$

3. $f(x) = \dfrac{3^{2x}}{4}$

4. $G(t) = 3(t)^t$

5. $q(r) = \dfrac{-4}{3^r}$

6. $j(x) = 2^x 3^x$

7. $Q(t) = 8^{t/3}$

8. $K(x) = \dfrac{2^x}{3 \cdot 3^x}$

9. $p(r) = 2^r + 3^r$

What is the growth factor in Exercises 10–13? Assume time is measured in the units given.

10. Water usage is increasing by 3% per year.

11. A city grows by 28% per decade.

12. A diamond mine is depleted by 1% per day.

13. A forest shrinks 80% per century.

In Exercises 14–17, give the starting value a, the growth factor b, and the growth rate r if $Q = ab^t = a(1+r)^t$.

14. $Q = 1750(1.593)^t$

15. $Q = 34.3(0.788)^t$

16. $Q = 79.2(1.002)^t$

17. $Q = 0.0022(2.31)^{-3t}$

Problems

18. The populations, P, of six towns with time t in years are given by

(i) $P = 1000(1.08)^t$ (ii) $P = 600(1.12)^t$
(iii) $P = 2500(0.9)^t$ (iv) $P = 1200(1.185)^t$
(v) $P = 800(0.78)^t$ (vi) $P = 2000(0.99)^t$

(a) Which towns are growing in size? Which are shrinking?
(b) Which town is growing the fastest? What is the annual percent growth rate for that town?
(c) Which town is shrinking the fastest? What is the annual percent "decay" rate for that town?
(d) Which town has the largest initial population (at $t = 0$)? Which town has the smallest?

19. The value, V, of a \$100,000 investment that earns 3% annual interest is given by $V = f(t)$ where t is in years. How much is the investment worth in 3 years?

20. An investment decreases by 5% per year for 4 years. By what total percent does it decrease?

21. Without a calculator, match each of the formulas to one of the graphs in Figure 4.6.

(a) $y = 0.8^t$ **(b)** $y = 5(3)^t$
(c) $y = -6(1.03)^t$ **(d)** $y = 15(3)^{-t}$

Malthus believed that populations increase exponentially while food production increases linearly. The last example explains his gloomy predictions: Malthus believed that any population eventually outstrips its food supply, leading to famine and war.

Exercises and Problems for Section 4.2

Skill Refresher

In Exercises S1–S4, write each of the following with single positive exponents.

S1. $b^4 \cdot b^6$

S2. $8g^3 \cdot (-4g)^2$

S3. $\dfrac{18a^{10}b^6}{6a^3b^{-4}}$

S4. $\dfrac{(2a^3b^2)^3}{(4ab^{-4})^2}$

In Exercises S5–S6, evaluate the functions for $t = 0$ and $t = 3$.

S5. $f(t) = 5.6(1.043)^t$

S6. $g(t) = 12{,}837(0.84)^t$

In Exercises S7–S10, solve for x.

S7. $4x^3 = 20$

S8. $\dfrac{5x^3}{x^5} = 125$

S9. $\dfrac{4x^8}{3x^3} = 7$

S10. $\sqrt{4x^3} = 5$

Exercises

1. Write a formula for the price p of a gallon of gas in t days if the price is \$2.50 on day $t = 0$ and the price is:

(a) Increasing by \$0.03 per day.
(b) Decreasing by \$0.07 per day.
(c) Increasing by 2% per day.
(d) Decreasing by 4% per day.

2. A population has size 5000 at time $t = 0$, with t in years.

(a) If the population decreases by 100 people per year, find a formula for the population, P, at time t.
(b) If the population decreases by 8% per year, find a formula for the population, P, at time t.

3. The following formulas give the populations (in 1000s) of four different cities, A, B, C, and D, where t is in years. Which are changing exponentially? Describe in words how each of these populations is changing over time. Graph those that are exponential.

$$P_A = 200 + 1.3t, \quad P_B = 270(1.021)^t,$$
$$P_C = 150(1.045)^t, \quad P_D = 600(0.978)^t.$$

4. In an environment with unlimited resources and no predators, a population tends to grow by the same percentage each year. Should a linear or exponential function be used to model such a population? Why?

5. Find $g(t) = ab^t$ if $g(10) = 50$ and $g(30) = 25$.

6. Find a formula for $f(x)$, an exponential function such that $f(-8) = 200$ and $f(30) = 580$.

7. Suppose that $f(x)$ is exponential and that $f(-3) = 54$ and $f(2) = \frac{2}{9}$. Find a formula for $f(x)$.

8. Find a formula for $f(x)$, an exponential function such that $f(2) = 1/27$ and $f(-1) = 27$.

9. Find the equation of an exponential curve through the points $(-1, 2)$, $(1, 0.3)$.

For Exercises 10–15, find a formula for the exponential function.

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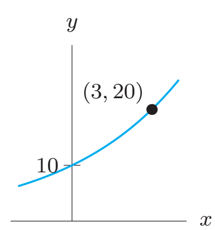

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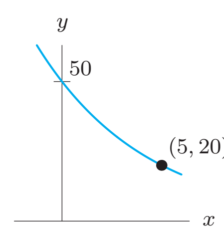

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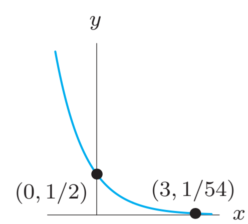

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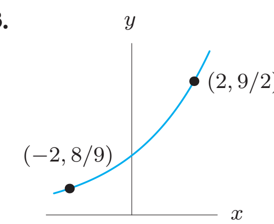

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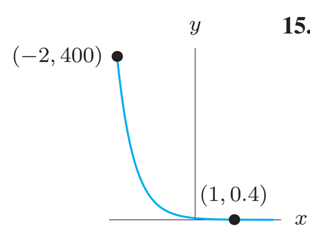

15.