3.5 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. An exponential growth model has the form ________ and an exponential decay model has the form ________.
2. A logarithmic model has the form ________ or ________.
3. Gaussian models are commonly used in probability and statistics to represent populations that are ________ ________.
4. The graph of a Gaussian model is ________ shaped, where the ________ ________ is the maximum y-value of the graph.
5. A logistic growth model has the form ________.
6. A logistic curve is also called a ________ curve.

SKILLS AND APPLICATIONS

In Exercises 7–12, match the function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

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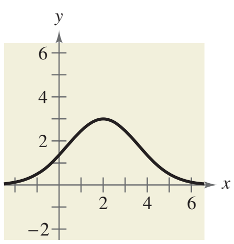

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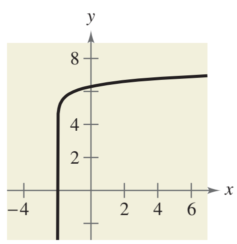

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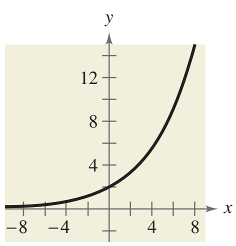

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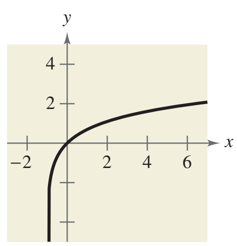

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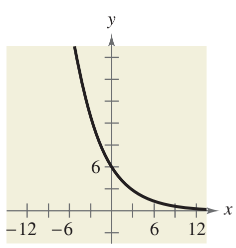

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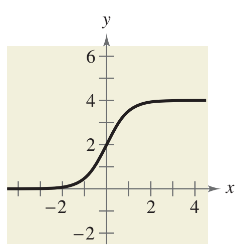


7. $y = 2e^{x/4}$
8. $y = 6e^{-x/4}$
9. $y = 6 + \log(x + 2)$
10. $y = 3e^{-(x-2)^2/5}$
11. $y = \ln(x + 1)$
12. $y = \dfrac{4}{1 + e^{-2x}}$

In Exercises 13 and 14, (a) solve for P and (b) solve for t.

13. $A = Pe^{rt}$
14. $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

COMPOUND INTEREST In Exercises 15–22, complete the table for a savings account in which interest is compounded continuously.

	Initial Investment	*Annual % Rate*	*Time to Double*	*Amount After 10 Years*
15.	\$1000	3.5%		
16.	\$750	$10\frac{1}{2}\%$		
17.	\$750		$7\frac{3}{4}$ yr	
18.	\$10,000		12 yr	
19.	\$500			\$1505.00
20.	\$600			\$19,205.00
21.		4.5%		\$10,000.00
22.		2%		\$2000.00

COMPOUND INTEREST In Exercises 23 and 24, determine the principal P that must be invested at rate r, compounded monthly, so that \$500,000 will be available for retirement in t years.

23. $r = 5\%, t = 10$
24. $r = 3\frac{1}{2}\%, t = 15$

COMPOUND INTEREST In Exercises 25 and 26, determine the time necessary for \$1000 to double if it is invested at interest rate r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.

25. $r = 10\%$
26. $r = 6.5\%$

27. **COMPOUND INTEREST** Complete the table for the time t (in years) necessary for P dollars to triple if interest is compounded continuously at rate r.

r	2%	4%	6%	8%	10%	12%
t						

28. **MODELING DATA** Draw a scatter plot of the data in Exercise 27. Use the *regression* feature of a graphing utility to find a model for the data.

29. COMPOUND INTEREST Complete the table for the time t (in years) necessary for P dollars to triple if interest is compounded annually at rate r.

r	2%	4%	6%	8%	10%	12%
t						

30. MODELING DATA Draw a scatter plot of the data in Exercise 29. Use the *regression* feature of a graphing utility to find a model for the data.

31. COMPARING MODELS If \$1 is invested in an account over a 10-year period, the amount in the account, where t represents the time in years, is given by $A = 1 + 0.075[\![t]\!]$ or $A = e^{0.07t}$ depending on whether the account pays simple interest at $7\frac{1}{2}\%$ or continuous compound interest at 7%. Graph each function on the same set of axes. Which grows at a higher rate? (Remember that $[\![t]\!]$ is the greatest integer function discussed in Section 1.6.)

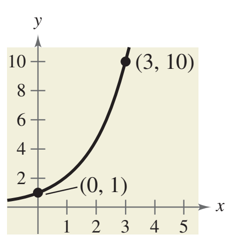
32. COMPARING MODELS If \$1 is invested in an account over a 10-year period, the amount in the account, where t represents the time in years, is given by $A = 1 + 0.06[\![t]\!]$ or $A = [1 + (0.055/365)]^{[\![365t]\!]}$ depending on whether the account pays simple interest at 6% or compound interest at $5\frac{1}{2}\%$ compounded daily. Use a graphing utility to graph each function in the same viewing window. Which grows at a higher rate?

RADIOACTIVE DECAY In Exercises 33–38, complete the table for the radioactive isotope.

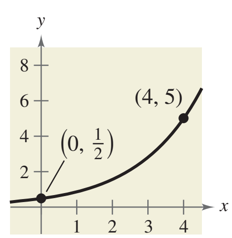
	Isotope	Half-life (years)	Initial Quantity	Amount After 1000 Years
33.	^{226}Ra	1599	10 g	
34.	^{14}C	5715	6.5 g	
35.	^{239}Pu	24,100	2.1g	
36.	^{226}Ra	1599		2 g
37.	^{14}C	5715		2 g
38.	^{239}Pu	24,100		0.4 g

In Exercises 39–42, find the exponential model $y = ae^{bx}$ that fits the points shown in the graph or table.

39.



40.



41.

x	0	4
y	5	1

42.

x	0	3
y	1	$\frac{1}{4}$

43. POPULATION The populations P (in thousands) of Horry County, South Carolina from 1970 through 2007 can be modeled by

$$P = -18.5 + 92.2e^{0.0282t}$$

where t represents the year, with $t = 0$ corresponding to 1970. (Source: U.S. Census Bureau)

(a) Use the model to complete the table.

Year	1970	1980	1990	2000	2007
Population					

(b) According to the model, when will the population of Horry County reach 300,000?

(c) Do you think the model is valid for long-term predictions of the population? Explain.

44. POPULATION The table shows the populations (in millions) of five countries in 2000 and the projected populations (in millions) for the year 2015. (Source: U.S. Census Bureau)

Country	2000	2015
Bulgaria	7.8	6.9
Canada	31.1	35.1
China	1268.9	1393.4
United Kingdom	59.5	62.2
United States	282.2	325.5

(a) Find the exponential growth or decay model $y = ae^{bt}$ or $y = ae^{-bt}$ for the population of each country by letting $t = 0$ correspond to 2000. Use the model to predict the population of each country in 2030.

(b) You can see that the populations of the United States and the United Kingdom are growing at different rates. What constant in the equation $y = ae^{bt}$ is determined by these different growth rates? Discuss the relationship between the different growth rates and the magnitude of the constant.

(c) You can see that the population of China is increasing while the population of Bulgaria is decreasing. What constant in the equation $y = ae^{bt}$ reflects this difference? Explain.

45. WEBSITE GROWTH The number y of hits a new search-engine website receives each month can be modeled by $y = 4080e^{kt}$, where t represents the number of months the website has been operating. In the website's third month, there were 10,000 hits. Find the value of k, and use this value to predict the number of hits the website will receive after 24 months.

46. VALUE OF A PAINTING The value V (in millions of dollars) of a famous painting can be modeled by $V = 10e^{kt}$, where t represents the year, with $t = 0$ corresponding to 2000. In 2008, the same painting was sold for \$65 million. Find the value of k, and use this value to predict the value of the painting in 2014.

47. POPULATION The populations P (in thousands) of Reno, Nevada from 2000 through 2007 can be modeled by $P = 346.8e^{kt}$, where t represents the year, with $t = 0$ corresponding to 2000. In 2005, the population of Reno was about 395,000. (Source: U.S. Census Bureau)

(a) Find the value of k. Is the population increasing or decreasing? Explain.

(b) Use the model to find the populations of Reno in 2010 and 2015. Are the results reasonable? Explain.

(c) According to the model, during what year will the population reach 500,000?

48. POPULATION The populations P (in thousands) of Orlando, Florida from 2000 through 2007 can be modeled by $P = 1656.2e^{kt}$, where t represents the year, with $t = 0$ corresponding to 2000. In 2005, the population of Orlando was about 1,940,000. (Source: U.S. Census Bureau)

(a) Find the value of k. Is the population increasing or decreasing? Explain.

(b) Use the model to find the populations of Orlando in 2010 and 2015. Are the results reasonable? Explain.

(c) According to the model, during what year will the population reach 2.2 million?

49. BACTERIA GROWTH The number of bacteria in a culture is increasing according to the law of exponential growth. After 3 hours, there are 100 bacteria, and after 5 hours, there are 400 bacteria. How many bacteria will there be after 6 hours?

50. BACTERIA GROWTH The number of bacteria in a culture is increasing according to the law of exponential growth. The initial population is 250 bacteria, and the population after 10 hours is double the population after 1 hour. How many bacteria will there be after 6 hours?

51. CARBON DATING

(a) The ratio of carbon 14 to carbon 12 in a piece of wood discovered in a cave is $R = 1/8^{14}$. Estimate the age of the piece of wood.

(b) The ratio of carbon 14 to carbon 12 in a piece of paper buried in a tomb is $R = 1/13^{11}$. Estimate the age of the piece of paper.

52. RADIOACTIVE DECAY Carbon 14 dating assumes that the carbon dioxide on Earth today has the same radioactive content as it did centuries ago. If this is true, the amount of ^{14}C absorbed by a tree that grew several centuries ago should be the same as the amount of ^{14}C absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal if the half-life of ^{14}C is 5715 years?

53. DEPRECIATION A sport utility vehicle that costs \$23,300 new has a book value of \$12,500 after 2 years.

(a) Find the linear model $V = mt + b$.

(b) Find the exponential model $V = ae^{kt}$.

(c) Use a graphing utility to graph the two models in the same viewing window. Which model depreciates faster in the first 2 years?

(d) Find the book values of the vehicle after 1 year and after 3 years using each model.

(e) Explain the advantages and disadvantages of using each model to a buyer and a seller.

54. DEPRECIATION A laptop computer that costs \$1150 new has a book value of \$550 after 2 years.

(a) Find the linear model $V = mt + b$.

(b) Find the exponential model $V = ae^{kt}$.

(c) Use a graphing utility to graph the two models in the same viewing window. Which model depreciates faster in the first 2 years?

(d) Find the book values of the computer after 1 year and after 3 years using each model.

(e) Explain the advantages and disadvantages of using each model to a buyer and a seller.

55. SALES The sales S (in thousands of units) of a new CD burner after it has been on the market for t years are modeled by $S(t) = 100(1 - e^{kt})$. Fifteen thousand units of the new product were sold the first year.

(a) Complete the model by solving for k.

(b) Sketch the graph of the model.

(c) Use the model to estimate the number of units sold after 5 years.

56. LEARNING CURVE The management at a plastics factory has found that the maximum number of units a worker can produce in a day is 30. The learning curve for the number N of units produced per day after a new employee has worked t days is modeled by $N = 30(1 - e^{kt})$. After 20 days on the job, a new employee produces 19 units.

(a) Find the learning curve for this employee (first, find the value of k).

(b) How many days should pass before this employee is producing 25 units per day?

57. IQ SCORES The IQ scores for a sample of a class of returning adult students at a small northeastern college roughly follow the normal distribution $y = 0.0266e^{-(x-100)^2/450}$, $70 \le x \le 115$, where x is the IQ score.

(a) Use a graphing utility to graph the function.

(b) From the graph in part (a), estimate the average IQ score of an adult student.

58. EDUCATION The amount of time (in hours per week) a student utilizes a math-tutoring center roughly follows the normal distribution $y = 0.7979e^{-(x-5.4)^2/0.5}$, $4 \le x \le 7$, where x is the number of hours.

(a) Use a graphing utility to graph the function.

(b) From the graph in part (a), estimate the average number of hours per week a student uses the tutoring center.

59. CELL SITES A cell site is a site where electronic communications equipment is placed in a cellular network for the use of mobile phones. The numbers y of cell sites from 1985 through 2008 can be modeled by

$$y = \frac{237{,}101}{1 + 1950e^{-0.355t}}$$

where t represents the year, with $t = 5$ corresponding to 1985. (Source: CTIA-The Wireless Association)

(a) Use the model to find the numbers of cell sites in the years 1985, 2000, and 2006.

(b) Use a graphing utility to graph the function.

(c) Use the graph to determine the year in which the number of cell sites will reach 235,000.

(d) Confirm your answer to part (c) algebraically.

60. POPULATION The populations P (in thousands) of Pittsburgh, Pennsylvania from 2000 through 2007 can be modeled by

$$P = \frac{2632}{1 + 0.083e^{0.0500t}}$$

where t represents the year, with $t = 0$ corresponding to 2000. (Source: U.S. Census Bureau)

(a) Use the model to find the populations of Pittsburgh in the years 2000, 2005, and 2007.

(b) Use a graphing utility to graph the function.

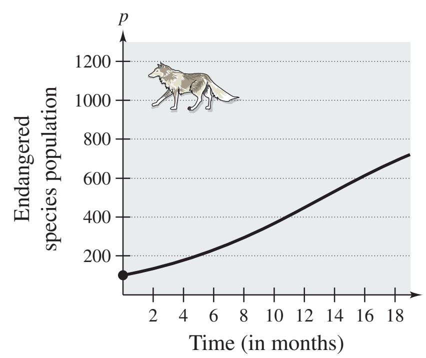
(c) Use the graph to determine the year in which the population will reach 2.2 million.

(d) Confirm your answer to part (c) algebraically.

61. POPULATION GROWTH A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1000 animals and that the growth of the pack will be modeled by the logistic curve

$$p(t) = \frac{1000}{1 + 9e^{-0.1656t}}$$

where t is measured in months (see figure).



(a) Estimate the population after 5 months.

(b) After how many months will the population be 500?

(c) Use a graphing utility to graph the function. Use the graph to determine the horizontal asymptotes, and interpret the meaning of the asymptotes in the context of the problem.

62. SALES After discontinuing all advertising for a tool kit in 2004, the manufacturer noted that sales began to drop according to the model

$$S = \frac{500{,}000}{1 + 0.4e^{kt}}$$

where S represents the number of units sold and $t = 4$ represents 2004. In 2008, the company sold 300,000 units.

(a) Complete the model by solving for k.

(b) Estimate sales in 2012.

GEOLOGY In Exercises 63 and 64, use the Richter scale

$$R = \log \frac{I}{I_0}$$

for measuring the magnitudes of earthquakes.

63. Find the intensity I of an earthquake measuring R on the Richter scale (let $I_0 = 1$).

(a) Southern Sumatra, Indonesia in 2007, $R = 8.5$

(b) Illinois in 2008, $R = 5.4$

(c) Costa Rica in 2009, $R = 6.1$

64. Find the magnitude R of each earthquake of intensity I (let $I_0 = 1$).

(a) $I = 199{,}500{,}000$ (b) $I = 48{,}275{,}000$

(c) $I = 17{,}000$

INTENSITY OF SOUND In Exercises 65–68, use the following information for determining sound intensity. The level of sound β, in decibels, with an intensity of I, is given by $\beta = 10 \log(I/I_0)$, where I_0 is an intensity of 10^{-12} watt per square meter, corresponding roughly to the faintest sound that can be heard by the human ear. In Exercises 65 and 66, find the level of sound β.

65. (a) $I = 10^{-10}$ watt per m^2 (quiet room)

(b) $I = 10^{-5}$ watt per m^2 (busy street corner)

(c) $I = 10^{-8}$ watt per m^2 (quiet radio)

(d) $I = 10^{0}$ watt per m^2 (threshold of pain)

66. (a) $I = 10^{-11}$ watt per m^2 (rustle of leaves)

(b) $I = 10^{2}$ watt per m^2 (jet at 30 meters)

(c) $I = 10^{-4}$ watt per m^2 (door slamming)

(d) $I = 10^{-2}$ watt per m^2 (siren at 30 meters)

67. Due to the installation of noise suppression materials, the noise level in an auditorium was reduced from 93 to 80 decibels. Find the percent decrease in the intensity level of the noise as a result of the installation of these materials.

68. Due to the installation of a muffler, the noise level of an engine was reduced from 88 to 72 decibels. Find the percent decrease in the intensity level of the noise as a result of the installation of the muffler.

pH LEVELS In Exercises 69–74, use the acidity model given by $\text{pH} = -\log[H^+]$, where acidity (pH) is a measure of the hydrogen ion concentration $[H^+]$ (measured in moles of hydrogen per liter) of a solution.

69. Find the pH if $[H^+] = 2.3 \times 10^{-5}$.

70. Find the pH if $[H^+] = 1.13 \times 10^{-5}$.

71. Compute $[H^+]$ for a solution in which $\text{pH} = 5.8$.

72. Compute $[H^+]$ for a solution in which $\text{pH} = 3.2$.

73. Apple juice has a pH of 2.9 and drinking water has a pH of 8.0. The hydrogen ion concentration of the apple juice is how many times the concentration of drinking water?

74. The pH of a solution is decreased by one unit. The hydrogen ion concentration is increased by what factor?

75. FORENSICS At 8:30 A.M., a coroner was called to the home of a person who had died during the night. In order to estimate the time of death, the coroner took the person's temperature twice. At 9:00 A.M. the temperature was 85.7°F, and at 11:00 A.M. the temperature was 82.8°F. From these two temperatures, the coroner was able to determine that the time elapsed since death and the body temperature were related by the formula

$$t = -10 \ln \frac{T - 70}{98.6 - 70}$$

where t is the time in hours elapsed since the person died and T is the temperature (in degrees Fahrenheit) of the person's body. (This formula is derived from a general cooling principle called *Newton's Law of Cooling*. It uses the assumptions that the person had a normal body temperature of 98.6°F at death, and that the room temperature was a constant 70°F.) Use the formula to estimate the time of death of the person.

76. HOME MORTGAGE A \$120,000 home mortgage for 30 years at $7\frac{1}{2}\%$ has a monthly payment of \$839.06. Part of the monthly payment is paid toward the interest charge on the unpaid balance, and the remainder of the payment is used to reduce the principal. The amount that is paid toward the interest is

$$u = M - \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}$$

and the amount that is paid toward the reduction of the principal is

$$v = \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}.$$

In these formulas, P is the size of the mortgage, r is the interest rate, M is the monthly payment, and t is the time (in years).

(a) Use a graphing utility to graph each function in the same viewing window. (The viewing window should show all 30 years of mortgage payments.)

(b) In the early years of the mortgage, is the larger part of the monthly payment paid toward the interest or the principal? Approximate the time when the monthly payment is evenly divided between interest and principal reduction.

(c) Repeat parts (a) and (b) for a repayment period of 20 years ($M = \$966.71$). What can you conclude?

77. HOME MORTGAGE The total interest u paid on a home mortgage of P dollars at interest rate r for t years is

$$u = P\left[\frac{rt}{1 - \left(\dfrac{1}{1 + r/12}\right)^{12t}} - 1\right].$$

Consider a \$120,000 home mortgage at $7\frac{1}{2}\%$.

(a) Use a graphing utility to graph the total interest function.

(b) Approximate the length of the mortgage for which the total interest paid is the same as the size of the mortgage. Is it possible that some people are paying twice as much in interest charges as the size of the mortgage?

78. DATA ANALYSIS The table shows the time t (in seconds) required for a car to attain a speed of s miles per hour from a standing start.

Speed, s	Time, t
30	3.4
40	5.0
50	7.0
60	9.3
70	12.0
80	15.8
90	20.0

Two models for these data are as follows.

$$t_1 = 40.757 + 0.556s - 15.817 \ln s$$

$$t_2 = 1.2259 + 0.0023s^2$$

(a) Use the *regression* feature of a graphing utility to find a linear model t_3 and an exponential model t_4 for the data.

(b) Use a graphing utility to graph the data and each model in the same viewing window.

(c) Create a table comparing the data with estimates obtained from each model.

(d) Use the results of part (c) to find the sum of the absolute values of the differences between the data and the estimated values given by each model. Based on the four sums, which model do you think best fits the data? Explain.

EXPLORATION

TRUE OR FALSE? In Exercises 79–82, determine whether the statement is true or false. Justify your answer.

79. The domain of a logistic growth function cannot be the set of real numbers.

80. A logistic growth function will always have an x-intercept.

81. The graph of $f(x) = \dfrac{4}{1 + 6e^{-2x}} + 5$ is the graph of $g(x) = \dfrac{4}{1 + 6e^{-2x}}$ shifted to the right five units.

82. The graph of a Gaussian model will never have an x-intercept.

83. WRITING Use your school's library, the Internet, or some other reference source to write a paper describing John Napier's work with logarithms.

84. CAPSTONE Identify each model as exponential, Gaussian, linear, logarithmic, logistic, quadratic, or none of the above. Explain your reasoning.

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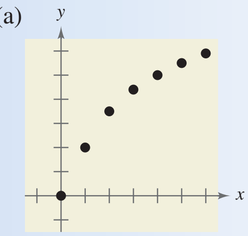

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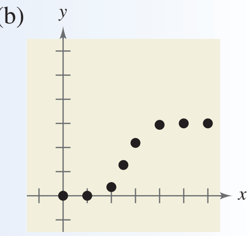

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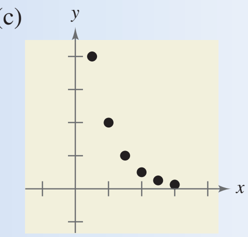

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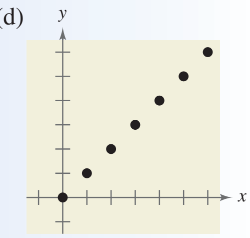

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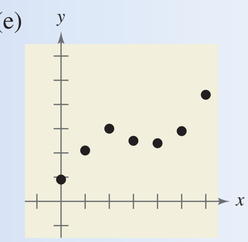

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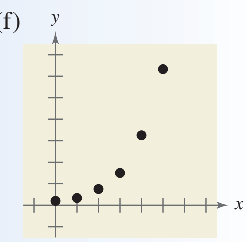

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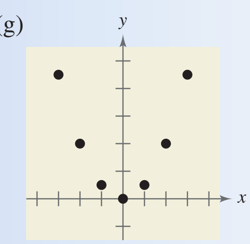

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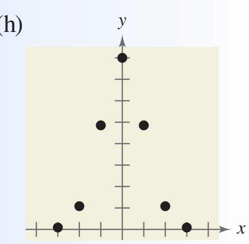

PROJECT: SALES PER SHARE To work an extended application analyzing the sales per share for Kohl's Corporation from 1992 through 2007, visit this text's website at *academic.cengage.com*. (Data Source: Kohl's Corporation)