55

$$\log(\sqrt{uv}) = \log(uv)^{1/2}$$

$$= \frac{1}{2}\log(uv)$$

$$= \frac{1}{2}\left[\log u + \log v\right]$$

Applications of exp/log.

Malthus. - exponential growth was an early model for population growth.

$$P(t) = A_0 b^t \Leftrightarrow P(t) = A_0 e^{kt}$$

ex 1,000 bacteria to start. The population doubles every 8hrs..
Model this in hours.
How long will it take to have 15,000 bacteria?

$$P(t) = 1000(1+r)^t$$

$$P(t) = 1000(2)^{t/8} \Leftrightarrow P(t) = 1000e^{kt}$$

$$2^{1/8} = e^k \Leftrightarrow 2^{1/8} = e^k$$

$$\frac{1}{8}\ln 2 = k \qquad \log 2^{1/8} = \log e^k$$

$$0.0866 \approx k \qquad \frac{1}{8}\log 2 = k \log e$$

$$\frac{\frac{1}{8}\log 2}{\log e} = k$$

so $P(t) = 1000e^{0.0866t}$

$$15000 = 1000e^{0.0866t}$$

$$15 = e^{0.0866t}$$

$$\ln 15 = 0.0866t$$

$$\frac{\ln 15}{0.0866} = t$$

$$31.27\text{hrs} \approx t$$

$$15000 = 1000(2^{1/8})^t$$

$$15 = 2^{t/8}$$

$$\log 15 = \log 2^{t/8}$$

$$\log 15 = \frac{t}{8}\log 2$$

$$\frac{8\log 15}{\log 2} = t$$

Radioactive Decay.

U^{235} half life is 7×10^6 years.

an initial amount 100 g.

first model t in millions of years

$$A(t) = 100\, b^t \iff A(t) = 100 e^{kt}$$

$$A(t) = 100 e^{-t/\tau}$$