

1. Write out the first 5 terms of each of the following sequences.

(a)  $a_n = n^2 - n$

$$a_1 = 0$$

$$a_2 = 2$$

$$a_3 = 9 - 3 = 6$$

$$a_4 = 16 - 4 = 12$$

$$a_5 = 25 - 5 = 20$$

(b)  $b_n = \frac{(-2)^n}{2n-1}$

$$b_1 = \frac{-2}{1}$$

$$b_2 = \frac{4}{3}$$

$$b_3 = -\frac{8}{5}$$

$$b_4 = \frac{16}{7}$$

$$b_5 = -\frac{32}{9}$$

(c)  $c_n = c_{n-1} - (c_{n-2})^2$  where  $c_1 = 1$  and  $c_2 = 2$ .

$$c_3 = 2 - 1^2 = 1$$

$$c_5 = -3 - 1^2 = -4$$

$$c_4 = 1 - 4 = -3$$

2. Given an arithmetic sequence that has terms  $a_1 = 5$  and  $a_3 = -3$ , give the general term of the sequence and find  $a_{50}$ .

$$a_1 = 5 \quad a_3 = -3$$

$$a_n = 5 - 4(n-1)$$

$$d = -\frac{8}{2} = -4$$

$$\begin{aligned} a_{50} &= 5 - 4(49) \\ &= 5 - 196 = -191 \end{aligned}$$

3. What is the 6th term of a geometric sequence where  $b_1 = 32$  and  $b_3 = 2$ .

$$r^2 = \frac{2}{32} = \frac{1}{16}$$

$$r = \frac{1}{4}$$

$$b_1 = 32$$

$$b_n = 32 \left(\frac{1}{4}\right)^{n-1}$$

$$\begin{aligned} b_6 &= 32 \left(\frac{1}{4}\right)^5 = 2 \left(\frac{1}{4}\right)^3 = 2 \left(\frac{1}{64}\right) \\ &= \frac{1}{32} \end{aligned}$$

4. Evaluate the following series.

$$(a) \sum_{k=1}^{50} (3k - 25) = 50 \left( \frac{-22 + 125}{2} \right) = 50 \left( \frac{103}{2} \right) = 2575$$

$$(b) \sum_{k=1}^8 \frac{1}{3^k} = \frac{1}{3} \left( \frac{1 - \frac{1}{3}^8}{1 - \frac{1}{3}} \right) = \frac{1}{3} \cdot \frac{3}{2} \left( 1 - \frac{1}{6561} \right) = 0.499924$$

$$(c) \sum_{k=1}^{\infty} 10 \left( \frac{4}{5} \right)^k = 8 \left( \frac{1}{1 - \frac{4}{5}} \right) = 40$$

$$(d) \sum_{k=1}^{10} \ln \left( \frac{k+1}{k} \right). \text{ (hint: use properties of logs and write out a few terms of the series.)}$$

$$= \ln \left( \frac{2}{1} \right) + \ln \left( \frac{3}{2} \right) + \ln \left( \frac{4}{3} \right) + \ln \left( \frac{5}{4} \right) + \dots + \ln \left( \frac{11}{10} \right)$$

$$= \ln 2 - \ln 1 + \ln 3 - \ln 2 + \ln 4 - \ln 3 + \dots + \ln 11 - \ln 10$$

$$= \ln 11$$

5. You will receive 5 annual payments of \$5,000 beginning 3 years from now. Assuming a constant annual discount rate of 3%, what is the present value of this series of payments?

$$PV = \frac{5000}{1.03} + \frac{5000}{1.03^2} + \frac{5000}{1.03^3} + \frac{5000}{1.03^4} + \frac{5000}{1.03^5}$$

$$PV = \frac{5000}{1.03} \left( \frac{1 - \left(\frac{1}{1.03}\right)^5}{1 - \frac{1}{1.03}} \right) = \frac{5000}{1.03} \left( \frac{0.13739}{0.029126} \right) = 22898.54$$

$$PV = 20955.40$$

6. You want to borrow \$15,000. If your loan requires equal monthly payments at an annual rate of 4% for 3 years, find the amount you owe each month for this loan. (a reminder to show all work.)

$$15000 = \frac{P}{1.04} + \frac{P}{1.04^2} + \frac{P}{1.04^3}$$

$$15000 = P \left( \frac{1}{1.04} \left( \frac{1 - \frac{1}{1.04}^3}{1 - \frac{1}{1.04}} \right) \right) = P \left( \frac{1}{1.04} \left( \frac{0.1110036}{0.03846} \right) \right)$$

$$15,000 = P(2.77509)$$

$$5,405.23 = P$$

7. Each month you deposit \$500 into an account that pays interest at a 3% annual rate compounded monthly. How much money will you have in 5 years?

$$FV = 500 \left(1 + \frac{0.03}{12}\right) + 500 \left(1 + \frac{0.03}{12}\right)^2 + \dots + 500 \left(1 + \frac{0.03}{12}\right)^{60}$$

$$FV = 500 \left(1 + \frac{0.03}{12}\right) \left( \frac{1 - \left(1 + \frac{0.03}{12}\right)^{60}}{1 - \left(1 + \frac{0.03}{12}\right)} \right) = 500 (1.0025) (64.646713)$$

$$= 32,404.16$$

8. Evaluate the series  $\sum_{n=5}^{35} (5n - 8) \quad \underline{=} \quad \sum_{n=1}^{31} 17 + 5(n-1)$

$$\sum_{n=5}^{35} = 35 \left( \frac{-3 + 167}{2} \right) = 35 \left( \frac{164}{2} \right) = 35 (82)$$

$$\sum_{n=1}^{31} = 4 \left( \frac{-3 + 12}{2} \right) = 18$$

$$\begin{array}{r} 82 \\ \times 35 \\ \hline 416 \\ 246 \\ \hline 2870 \end{array} \quad 2870 - 18 = 2852$$

$$\begin{array}{r} 92 \\ \times 31 \\ \hline 92 \end{array} \quad = 31 \left( \frac{17 + 167}{2} \right)$$

$$\begin{array}{r} 92 \\ \times 31 \\ \hline 276 \\ \hline 2852 \end{array} \quad = 31 \left( \frac{184}{2} \right) = 31 (92) = 2852$$

Please write out the following statement and sign your name to it as testament to its truth. 'I have worked on this assignment for at most 60 minutes and I have neither given nor received any unauthorized help on this work'