p 194

5 $Q = 4e^{7t}$

if $Q = ab^t$ then $b = e^7$ so $Q = 4(1096.33)^t$

6 $Q = 0.3e^{0.7t}$

if $Q = ab^t$ then $b = e^{0.7}$ so $Q = 0.3(2.014)^t$

9 $Q = 12(0.9)^t$

if $Q = ae^{kt}$ then $0.9 = e^k$

$\ln 0.9 = \ln e^k = k$

$-0.105 = k$

so $Q = 12e^{-0.105t}$

10 $Q = 16(0.487)^t$ $e^k = 0.487$

$k = \ln 0.487$

$k = -0.719$

$Q = 16e^{-0.719t}$

13 $Q = 230(1.182)^t$

$a = 230$ $r = 0.182$ if $e^k = 1.182$

$k = \ln 1.182 \approx 0.167$

14 $Q = 0.181e^{0.775t}$ $a = 0.181$ $k = 0.775$

$e^{0.775} = b$

$2.171 = b$ so $r = 1.171$

21 $P = P_0 e^{0.2t}$ to find doubling time then $P = 2P_0$

$2P_0 = P_0 e^{0.2t}$

$2 = e^{0.2t}$

$\ln 2 = 0.2t$

$\frac{\ln 2}{0.2} = t \rightarrow t = 3.466$

22 26% growth per year

$P = P_0 (1.26)^t_t$ again $P = 2P_0$

$2P_0 = P_0 (1.26)^t$

$2 = 1.26^t$

$\ln 2 = t \ln 1.26$

$\frac{\ln 2}{\ln 1.26} = t$

$2.999 = t$

30

11,000 to 13,000 in 3 years

a $P = P_0 (1+r)^t$

$13{,}000 = 11{,}000 (1+r)^3$

$\frac{13}{11} = (1+r)^3$

$\sqrt[3]{\frac{13}{11}} = 1+r \rightarrow r = \sqrt[3]{\frac{13}{11}} - 1$

b $P = P_0 e^{kt}$

$\frac{13}{11} = e^{k(3)}$

$\ln \frac{13}{11} = 3k \rightarrow k = \frac{\ln 13/11}{3} = 0.0557$

c annual % change is slower than continuous % change

33 doubling time is 15 years.

$P = P_0 e^{kt}$

$2 = e^{k\,15}$

$\ln 2 = 15k$

$0.0462 = \ln 2 / 15 = k$

is the continuous growth.

$e^k = (1 + r)$

$e^{0.0462} - 1 = r$

$0.0473 = r$

35 The half life of nicotine is 2 hours

$A = A_0 e^{kt}$

$\frac{1}{2} = e^{k(2)}$

$\ln\left(\frac{1}{2}\right) = 2k$

$-0.347 = \ln\left(\frac{1}{2}\right)/2 = k$

$A = A_0 e^{-0.347t}$

-0.347

36 If 17% decay then 83% is left after 5 hours

Then when is $\frac{P}{P_0} = \frac{1}{2}$? First at $t = 5$

$\frac{P}{P_0} = 0.83$

So $0.83 = e^{k(5)}$

$\ln 0.83 = 5k$

$\ln 0.83 / 5 = k$

$-0.0373 = k$

Then $P = P_0 e^{-0.0373t}$

$\frac{1}{2} = e^{-0.0373t}$

$\ln \frac{1}{2} = -0.0373t$

$18.583 = t$

39 $P_0 = 5000$

a $m = 500$ then $P = 5000 + 500t$

if $P = 10{,}000$ then $10{,}000 = 5000 + 500t$

$5000 = 500t$

$10 = t$

if $P = 20{,}000$ then $20{,}000 = 5000 + 500t$

$15{,}000 = 500t$

$30 = t$

if $P = 40{,}000$ then $40{,}000 = 5000 + 500t$

$35{,}000 = 500t$

$70 = t$

b if growth is 5% a year

if $P = 10{,}000$ then $10{,}000 = 5000(1.05)^t$

$2 = 1.05^t$

$\ln 2 = t \ln 1.05$

$\ln 2 / \ln 1.15 = t$

$14.207 = t$

if $P = 20{,}000$ then $20{,}000 = 5000(1.05)^t$

$4 = 1.05^t$

$\ln 4 = t \ln 1.05$

$28.413 = \dfrac{\ln 4}{\ln 1.05} = t$

if $P = 40{,}000$ then $40{,}000 = 5000(1.05)^t$

$8 = 1.05^t$

$42.620 = \dfrac{\ln 8}{\ln 1.05} = t$

pgs #41-43, 45, 49, 51, 54-55, 56-57.

41 It looks like 4

b $Q = Q_0 e^{kt} \longrightarrow \frac{1}{2} = e^{4k} \Rightarrow \ln\frac{1}{2} = 4k$

$-0.173 \approx \frac{\ln\frac{1}{2}}{4} = k$

so $Q = Q_0 e^{-0.173t}$

42 $H = 70 + 120\left(\frac{1}{4}\right)^t$

a at $t=0$ $H = 190$

at $t=1$ $H = 100$

at $t=2$ $H = 77.5$

b $90 = 70 + 120\left(\frac{1}{4}\right)^t$

$\frac{1}{6} = \left(\frac{1}{4}\right)^t$

$\ln\frac{1}{6} = t\ln\left(\frac{1}{4}\right)$

$1.292 \approx \ln\left(\frac{1}{6}\right)/\ln\left(\frac{1}{4}\right) = t$

$75 = 70 + 120\left(\frac{1}{4}\right)^t$

$\frac{5}{120} = \left(\frac{1}{4}\right)^t$

$\frac{1}{24} = \left(\frac{1}{4}\right)^t$

$\ln\left(\frac{1}{24}\right) = t\ln\left(\frac{1}{4}\right)$

$\ln\left(\frac{1}{24}\right)/\ln\left(\frac{1}{4}\right) = t$

$2.292 = t$

43 Done in class

45 $P = 300(2)^{t/20}$

a $t=0$ $P=300$

$t=20$ $P=600$

b $1000 = 300(2)^{t/20}$

$\frac{10}{3} = 2^{t/20}$

$\ln\frac{10}{3} = \frac{t}{20}\ln 2$

$34.739 = 20\ln\frac{10}{3}/\ln 2 = t$

49 Technetium-99 half-life is 6 hours
initially 200mg.

a $Q = 200e^{kt}$

$\frac{1}{2} = e^{k(6)}$

$\ln(0.5) = 6k$

$\frac{\ln(0.5)}{6} = k$

$Q = 200e^{\left(\ln(0.5)/6\right)t}$

b $120 = 200e^{\left(\ln 0.5/6\right)t}$

$\frac{3}{5} = e^{\left(\ln 0.5/6\right)t}$

$\ln\left(\frac{3}{5}\right) = t\,\frac{\ln(0.5)}{6}$

$\frac{6\ln(0.6)}{\ln(0.5)} = t$

c Skip

51, 54, 55, 56 we did in class.

p215

1 $Q = 7e^{-10t}$ $\quad Q = ab^t$ so $b = e^{-10}$

$Q = 7(4.54\times10^{-5})^t$

3 $Q = 4(7)^t$ $\quad Q = ae^{kt}$ $\quad 7 = e^k$

$\ln 7 = k$

$Q = 4e^{\ln 7\, t}$

5 $Q = 4.8^{1.3t}$

$Q = 4(8^{1.3})^t$

$Q = 4e^{1.3\ln 8\, t}$

$8^{1.3} = e^k$

$1.3\ln 8 = k$

7 $1.04^t = 3$

$t\ln 1.04 = \ln 3$

$t = \ln 3 / \ln 1.04$

9 $3(1.081)^t = 14$

$1.081^t = \frac{14}{3}$

$t\ln 1.081 = \ln\left(\frac{14}{3}\right)$

$t = \frac{\ln(14/3)}{\ln(1.081)}$

11 $5(1.014)^{3t} = 12$

$(1.014)^{3t} = \frac{12}{5}$

$3t\ln 1.014 = \ln\left(\frac{12}{5}\right)$

$t = \frac{\ln(12/5)}{3\ln(1.014)}$

13 $5(1.031)^x = 8$

$1.031^x = \frac{8}{5}$

$x\ln 1.031 = \ln\left(\frac{8}{5}\right)$

$x = \frac{\ln(8/5)}{\ln 1.031}$

15 $3\log(2x+6) = 6$

$\log(2x+6) = 2$

$2x+6 = 10^2$

$2x = 94$

$x = 47$

17 $3^{4\log x} = 5$

$\log 3^{4\log x} = \log 5$

$4\log x \log 3 = \log 5$

$\log x = \frac{\log 5}{4\log 3}$

$\boxed{x = 10^{\frac{\log 5}{4\log 3}}}$

19 $13e^{0.081t} = 25e^{0.032t}$

$\frac{e^{0.081t}}{e^{0.032t}} = \frac{25}{13}$

$e^{0.049t} = \frac{25}{13}$

19 $e^{0.049t} = \frac{25}{13}$

$0.049t = \ln\left(\frac{25}{13}\right)$

$t = \frac{\ln(25/13)}{0.049}$

21 $\frac{\log x^2 + \log x^3}{\log(100x)} = 3$

$\log x^2 + \log x^3 = 3\log(100x)$

$\log(x^5) = \log(100x)^3$

$x^5 = 100^3 x^3$

$x^5 - 100^3 x^3 = 0$

$x^3(x^2 - 100^3) = 0$

$x = 0$ not a solution $\log(x)$ is not defined

$x = \pm\sqrt{10^6} = \pm 1000$

$x = -1000$ not a solution

$\boxed{x = 1000}$