City Semester
Test #4
Show all work for full credit.

Name:____________________________

1. Write out the first 5 terms of each of the following sequences.

   (a) $a_n = n^2 - n$

   $a_1 = 0$

   $a_2 = 2$

   $a_3 = 9 - 3 = 6$

   $a_4 = 16 - 4 = 12$

   $a_5 = 25 - 5 = 20$

   (b) $b_n = \dfrac{(-2)^n}{2n-1}$

   $b_1 = \dfrac{-2}{1}$

   $b_2 = \dfrac{4}{3}$

   $b_3 = -\dfrac{8}{5}$

   $b_4 = \dfrac{16}{7}$

   $b_5 = -\dfrac{32}{9}$

   (c) $c_n = c_{n-1} - (c_{n-2})^2$ where $c_1 = 1$ and $c_2 = 2$.

   $c_3 = 2 - 1^2 = 1$

   $c_4 = 1 - 4 = -3$

   $c_5 = -3 - 1^2 = -4$

2. Given an arithmetic sequence that has terms $a_1 = 5$ and $a_3 = -3$, give the general term of the sequence and find a_{50}.

   $a_1 = 5 \quad a_3 = -3$

   $d = -\dfrac{8}{2} = -4$

   $a_n = 5 - 4(n-1)$

   $a_{50} = 5 \cdot 4(49)$

   $= 5 - 196 = -191$

3. What is the 6th term of a geometric sequence where $b_1 = 32$ and $b_3 = 2$.

   $r^2 = \dfrac{2}{32} = \dfrac{1}{16}$

   $r = \dfrac{1}{4}$

   $b_1 = 32$

   $b_n = 32\left(\frac{1}{4}\right)^{n-1}$

   $b_6 = 32\left(\frac{1}{4}\right)^5 = 2\left(\frac{1}{4}\right)^3 = 2\left(\frac{1}{64}\right)$

   $= \dfrac{1}{32}$

4. Evaluate the following series.

(a) $\displaystyle\sum_{k=1}^{50}(3k-25)$ $= 50\left(\frac{-22+125}{2}\right) = 50\left(\frac{103}{2}\right) = 25(103)$

$= 2575$

(b) $\displaystyle\sum_{k=1}^{8}\frac{1}{3^k}$ $= \frac{1}{3}\left(\frac{1-\frac{1}{3}^8}{1-\frac{1}{3}}\right) = \frac{1}{3}\cdot\frac{3}{2}\left(1-\frac{1}{6561}\right) = 0.499924$

(c) $\displaystyle\sum_{k=1}^{\infty}10\left(\frac{4}{5}\right)^k$ $= 8\left(\frac{1}{1-\frac{4}{5}}\right) = 40$

(d) $\displaystyle\sum_{k=1}^{10}\ln\left(\frac{k+1}{k}\right)$. (hint: use properties of logs and write out a few terms of the series.)

$= \ln\left(\frac{2}{1}\right) + \ln\left(\frac{3}{2}\right) + \ln\left(\frac{4}{3}\right) + \ln\left(\frac{5}{4}\right) + \cdots + \ln\left(\frac{11}{10}\right)$

$= \ln 2 - \ln 1 + \ln 3 - \ln 2 + \ln 4 - \ln 3 + \cdots + \ln 11 - \ln 10$

$= \ln 11$

5. You will receive 5 annual payments of \$5,000 beginning 3 years from now. Assuming a constant annual discount rate of 3%, what is the present value of this series of payments?

$$PV = \frac{5000}{1.03} + \frac{5000}{1.03^2} + \frac{5000}{1.03^3} + \frac{5000}{1.03^4} + \frac{5000}{1.03^5}$$

$$PV = \frac{5000}{1.03}\left(\frac{1-\left(\frac{1}{1.03}\right)^5}{1-\frac{1}{1.03}}\right) = \frac{5000}{1.03}\left(\frac{0.13739}{0.029126}\right) = 22898.54$$

$$PV = 20,955.40$$

6. You want to borrow \$15,000. If your loan requires equal monthly payments at an annual rate of 4% for 3 years, find the amount you owe each month for this loan. (a reminder to show all work.)

$$15000 = \frac{P}{1.04} + \frac{P}{1.04^2} + \frac{P}{1.04^3}$$

$$15000 = P\left(\frac{1}{1.04}\left(\frac{1-\frac{1}{1.04}^3}{1-\frac{1}{1.04}}\right)\right)$$

$$P$$

7. Each month you deposit \$500 into an account that pays interest at a 3% annual rate compounded monthly. How much money will you have in 5 years?

$$FV = 500\left(1+\frac{0.03}{12}\right) + 500\left(1+\frac{0.03}{12}\right)^2 + \cdots + 500\left(1+\frac{0.03}{12}\right)^{60}$$

$$FV = 500\left(1+\frac{0.03}{12}\right)\left(\frac{1-\left(1+\frac{0.03}{12}\right)^{60}}{1-\left(1+\frac{0.03}{12}\right)}\right)$$

$$=$$

8. Evaluate the series $\sum_{n=5}^{35}(5n-8)$ $= \sum_{n=1}^{31} 17+5(n-1)$

$$S_{35} = 35\left(\frac{-3+167}{2}\right) = 35\left(\frac{164}{2}\right) = 35(82)$$

$$S_4 = 4\left(\frac{-3+12}{2}\right) = 18$$

$$\begin{array}{r} 82 \\ \times 35 \\ \hline 410 \\ 246 \\ \hline 2870 \end{array}$$

$$2870 - 18 = 2852$$

$$\begin{array}{r} 92 \\ \times 31 \\ \hline 92 \\ 276 \\ \hline 2852 \end{array}$$

$$= 31\left(\frac{17+167}{2}\right)$$

$$= 31\left(\frac{184}{2}\right) = 31(92) = 2852$$

Please write out the following statement and sign your name to it as testament to its truth. 'I have worked on this assignment for at most 60 minutes and I have neither given nor received any unauthorized help on this work'