Review of Sequences and Series

1 a $a_n = 2n - 4$

$a_5 = 10 - 4 = 6$

b $a_n = (-1)^n \dfrac{3}{2^n}$

$a_5 = -1\left(\dfrac{3}{32}\right) = \dfrac{-3}{32}$

c $a_5 = 5(4)(3)(2)/6$

$= 20$

d $a_3 = 2$

$a_n = a_{n-1}^2 + n a_{n-1}$

$a_4 = 2^2 + 4(2)$

$= 4 + 8 = 12$

$a_5 = 12^2 + 5(12)$

$= 144 + 60$

$= 204$

2 $b_3 = 12$

$b_8 = -3$

$\dfrac{b_8 - b_3}{8 - 3} = \dfrac{-3 - 12}{5} = -\dfrac{15}{5} = -3 = d$

$b_n = 18 + -3(n-1)$

$b_{111} = 18 + -3(111-1) = 18 - 330 = -312$

3 $c_2 = 3 \qquad c_5 = \dfrac{3}{8}$

$c_2 r = c_3$

$c_2 r^2 = c_4$

$c_2 r^3 = c_5$

so $\dfrac{c_5}{c_2} = r^3$

$\dfrac{\frac{3}{8}}{3} = \dfrac{1}{8} = r^3$

$\dfrac{1}{2} = \sqrt[3]{\dfrac{1}{8}} = r$

$c_n = 6\left(\dfrac{1}{2}\right)^{n-1}$

$c_{10} = 6\left(\dfrac{1}{2}\right)^9 = \dfrac{6}{512} = \dfrac{3}{256}$

4 $d = 12 \quad a_2 = 10 \quad a_1 = -2 \quad a_n = -2 + 12(n-1)$

$$a_{50} = -2 + 12(50-1) = -2 + 12(49)$$
$$= -2 + 588$$
$$= 586$$

5 $r = \frac{1}{3} \quad b_3 = 8 \quad b_2 = 24 \quad b_1 = 72$

$$b_n = 72\left(\tfrac{1}{3}\right)^{n-1} \qquad b_9 = 72\left(\tfrac{1}{3}\right)^8 = 8 \cdot 9\left(\tfrac{1}{3}\right)^2\left(\tfrac{1}{3}\right)^6$$
$$= \frac{8}{3^6}$$

6 a $\quad 2+3+4+\cdots+98+99 = \sum_{i=2}^{99} i$

b $\quad -3+6-12+24-48+\cdots-768 = \sum_{i=1}^{9} (-1)^i \, 3(2)^{i-1}$

$a_n = (-1)^i \, 3 \cdot 2^{i-1}$

$256 = 2^8$

c $\quad 5+1-3-7-11-15-\cdots-195 = \sum_{i=1}^{51} \left(5 - 4[n-1]\right)$

$a_n = 5 - 4(n-1)$

$-195 = 5 - 4(n-1)$

$-200 = -4(n-1)$

$50 = n-1$

$51 = n$

8 $1000\left(1+\frac{0.06}{2}\right)\left(1+\frac{0.06}{2}\right) + 1000(1.03)^4 + \cdots + 1000(1.03)^{20}$

last installment one year of interest

2nd to last 2 years of interest

1st payment compounds for 10 years.

$$= 1000(1.03)^2\left[1 + 1.03^2 + (1.03)^4 + \cdots (1.03)^{18}\right]$$

$$= 1000(1.03)^2\left[\frac{(1-1.03^{10})}{1-1.03}\right] = 1060.9\left[11.463879\right] =$$

$$= 12,162.03$$

9

$$\frac{5000}{1.03} + \frac{5000}{1.03^2} + \cdots + \frac{5000}{1.03^{10}}$$

$$= \frac{5000}{1.03}\left(1 + \frac{1}{1.03} + \cdots + \frac{1}{1.03^9}\right)$$

$$= \frac{5000}{1.03}\left(\frac{\left(1-\frac{1}{1.03^{10}}\right)}{1-\frac{1}{1.03}}\right) = 42,651.01$$

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$$\frac{5000}{1.04} + \cdots + \frac{5000}{1.04^{10}} = \frac{5000}{1.04}\left(\frac{\left(1-\frac{1}{1.04^{10}}\right)}{\left(1-\frac{1}{1.04}\right)}\right) = 40,554.48$$

then $PV = \frac{40,554.48}{1.04^3} = 36,052.79$

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$$100,000 = \frac{P}{1.04} + \frac{P}{1.04^2} + \cdots \frac{P}{1.04^{12}} = \frac{P}{1.04}\left(1 + \frac{1}{1.04} + \frac{1}{1.04^2} + \cdots \frac{1}{1.04^{11}}\right)$$

$$100,000 = \frac{P}{1.04}\left(\frac{1-\frac{1}{1.04^{12}}}{1-\frac{1}{1.04}}\right) = 9.38507\,P$$

$$10,655.22 = P$$

12 months

$$250{,}000 = \frac{P}{\left(1+\frac{0.0375}{12}\right)} + \cdots + \frac{P}{\left(1+\frac{0.0375}{12}\right)^{180}}$$

$$250{,}000 = \frac{P}{1.003125}\left(1 + \frac{1}{1.003125} + \frac{1}{1.003125^2} + \cdots + \frac{1}{1.003125^{179}}\right)$$

$$250{,}000 = \frac{P}{1.003125}\left(\frac{1-\left(\frac{1}{1.003125}\right)^{180}}{1-\frac{1}{1.003125}}\right) = 137.509P$$

$\$1{,}818.06 = P$ per month.