City Semester — Name:______________________

Unit 2: Exponentials

1 Exponents

Before learning about exponential functions it is helpful to be savvy with manipulating expressions with exponents.

1.1 Integer Exponents

Exercises Simplify using properties of exponents

1. x^3x^5
2. 2^32^5
3. $(2x^2y^5)^3$
4. $x^5(x^2+3x)^3$
5. $((x^2+y^2)^3)^2$
6. $(a^2)^3$
7. a^{2^3}
8. $\dfrac{x^2y^3}{x^3y}$
9. $\dfrac{x^2(xy^2)^3}{yx^{-2}}$
10. $\left(\dfrac{(xy)^3y^{-1}}{(3x^2y^3)^2}\right)^{-1}$

Sometimes (particularly for next year) it is nicer to work with x^{-1} rather than $\frac{1}{x}$. For each of the following simplify expressions so that exponents are in the numerator, if possible.

1. $\dfrac{1}{x}+\dfrac{6}{x^2}$
2. $\dfrac{2x}{yx^3}-\dfrac{3y}{y^3x}$
3. $\dfrac{1-\dfrac{y}{x+y}}{\dfrac{x}{y}-\dfrac{y}{x}}$

Problems

1. Using the fact that $x^3 = x\cdot x\cdot x$ and $x^4 = x\cdot x\cdot x\cdot x$, explain why $x^3\cdot x^4$ is generally not equal to $(x^3)^4$. For what x's are they the same?
2. $32, 16, 8, 4\ldots$ Looking at the previous sequence, what would you expect the next two terms of the sequence to be?
3. From the previous problem if we notice that each term can be expressed as a power of two and rewrite the sequence we have $2^5 = 32$, $2^4 = 16$, $2^3 = 8$ and $2^2 = 4$. Then we would guess that $2^1 = \underline{?}$ and $2^0 = \underline{?}$ and $2^{-1} = \underline{?}$.
4. We could repeat the above with any number, thus it makes sense to let a^1 and a^0 be defined as what values for any rational number a? Does this really work for any rational a?

5. Using properties of exponents what is $2^{\frac{1}{2}}2^{\frac{1}{2}} =$? What is $\sqrt{2}\sqrt{2} =$?
6. What is $a^{\frac{1}{2}}a^{\frac{1}{2}} =$? And what is $\sqrt{a}\sqrt{a} =$? So from this we can see that $x^{1/2}$ is another way to write what?
7. Using properties of exponents what is $x^{1/3} \cdot x^{1/3} \cdot x^{1/3} =$? Using this what is another way to write the expression $x^{1/3}$?
8. We have seen that many radical expressions can also be rewritten as fractional exponents and vise-versa. How would you rewrite $\sqrt[5]{y^2}$ in radical notation?

$*$ 9. Is there a problem with giving a value to 0^0? Look at $y = 0^x$. What happens as $x \to 0$? Now look at $y = x^0$. What happens as $x \to 0$? Explain why these two examples are relevant to the original question.

$*$ 10. Solve for u that satisfies $\dfrac{3}{u^2} + \dfrac{7}{u} = 6$.

1.2 Rational Exponents

Having worked with integer exponents we will now see that fractional exponents can be defined, and they are related to radical expressions you may have already worked with: $\sqrt[q]{x^p} = x^{p/q}$. Keep in mind we will assume $x > 0$.

Exercises Simplify each expression by using the properties of exponents you already learned when the exponents were integers. Nothing changes.

1. $x^{1/2} \cdot x^{1/3}$
2. $(4x)^{1/2} \cdot (27x)^{1/3}$
3. $(x \cdot x^{1/4})^2$
4. $(a^{2/3} \cdot b^{1/2})^{1/2}$
5. $\dfrac{(ab)^{1/2}}{a^2b^{-1}}$
6. $\dfrac{a}{b} \cdot a^{1/3} \cdot b^{-1/2}$

Rewrite each number using rational exponents, and then simplify if possible.

1. $\sqrt[3]{9^2}$
2. $\sqrt[4]{16}$
3. $\sqrt[4]{32}$
4. $\sqrt{\dfrac{27}{8}}$

Simplify each expression so that exponents are in the numerator when possible. Rational exponents will help.

1. $\dfrac{3x}{\sqrt{x}} + \dfrac{\sqrt{x}}{x^3}$
2. $\dfrac{(3x)^2}{27\sqrt{x}} + \dfrac{3}{18\sqrt[4]{x^3}}$