

6. d is diff. of arithmetic seq.

recall $a_n = a_1 + dn - d$.

$$a_n = a_1 + d(n-1) \quad \top$$

$$\stackrel{\text{or}}{=} a_n - a_{n-1} = a_1 + dn - d - [a_1 + d(n-1) - d] = d$$

Alternating sequence: $(-1)^n$ will be factor.

13 $\sum_{i=1}^n 3 = 3n \quad \top$

25 \$2000 annual earn 5% annually

$$2000 \sum_{n=1}^{20} (1.05)^{n-1} = 2000 \left[\frac{1 - (1.05)^{20}}{1 - 1.05} \right] = 61,000$$

$$\sum_{n=1}^k a_1 r^{n-1} = a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{k-1}$$

$$(-1) \sum_{n=1}^k a_1 r^{n-1} = a_1 - a_1 r^k$$

$$\sum_{n=1}^k a_1 r^{n-1} = \frac{a_1 (1 - r^k)}{1 - r}$$

$$(1-r) [a_1 + a_1 r + \dots + a_1 r^{k-1}]$$

$$= a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{k-1}$$

$$- a_1 r - a_1 r^2 - a_1 r^3 - \dots - a_1 r^k$$

$$= a_1 - a_1 r^k$$

Counting

Experiment \Rightarrow roll 2 dice. $\underline{6} \cdot \underline{6}$

count the outcomes.

36 outcomes

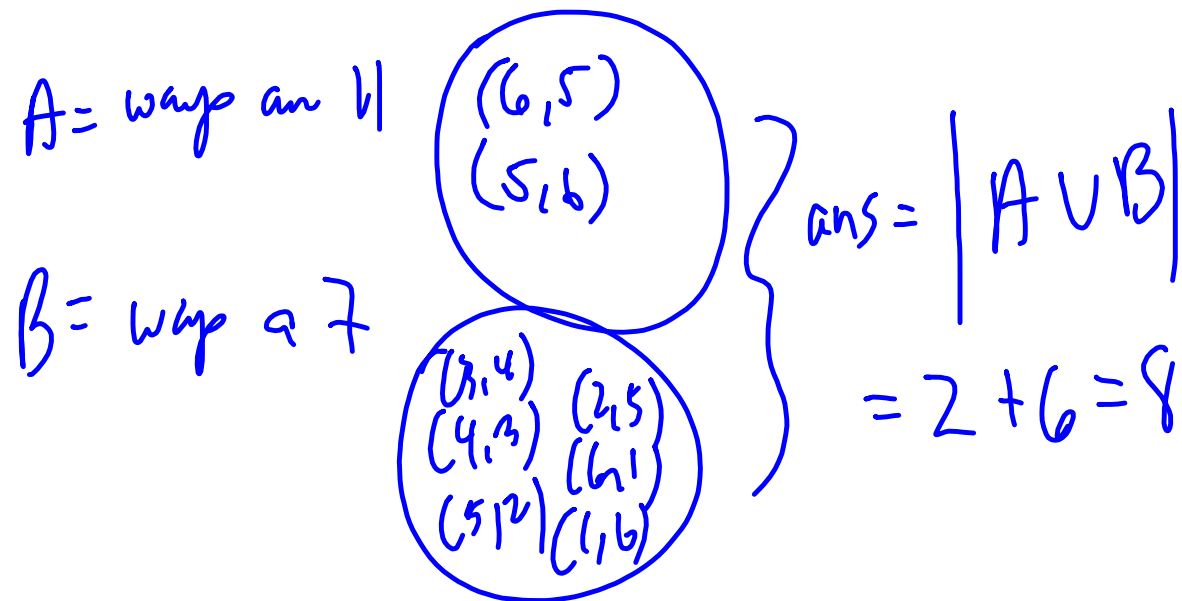
$(1,1)$	$(1,2)$...	$(1,6)$
$(2,1)$	$(2,2)$		
$(3,1)$	$(3,2)$		
$(6,1)$	$(6,2)$		$(6,6)$

if you don't distinguish 21 outcomes

if you only sum dice 11 outcomes

Addition principle.

Ex # ways I can roll an 11 or 7.



of ways to do 2 mutually exclusive things (non overlapping sets). is just the sum of ways to do each.

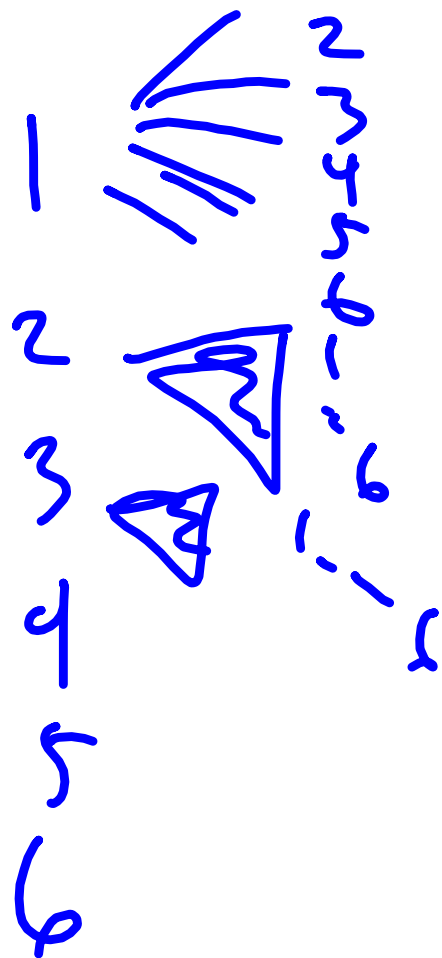
non Ex | die. roll an even or a prime.

rolling an even $A = \{2, 4, 6\}$ rolling a prime $B = \{2, 3, 5\}$

Q: What happens when you count
ways to "do" things successively

Ex.

2 dice



$$\underline{6}, \underline{6} = 36 \text{ possibilities}$$

Ex. ice cream cones.

4 cones, 15 flavours, 12 topping.

$$\frac{4}{1}, \frac{15}{1}, \frac{12}{1}, = 720$$

2

3

4

Complements

note: $\bar{A} = \{\text{everything } \underline{\text{not}} \text{ in } A\}$

Ex $A =$ Find the # of 4 digit #'s
with at least one 3.

$\bar{A} =$ 4 digit #'s with no 3's.

$$\bar{A} = \underline{8} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9} = 5832.$$

$$|A| = |U| - |\bar{A}| = 9000 - 5832 = 3168$$

$$|U| = 9 \cdot 10 \cdot 10 \cdot 10 = 9000$$

Ex # of ways to choose a President, VP of CS. when we have at least 1 boy.

14 8C³
6 ♀

$$8, 13 = 104$$

~~P~~, ~~VP~~

$$\underset{P}{6}, \underset{VP}{8} = 48$$

$$\text{total diff} = 14 \cdot 13 = 182$$

$$\text{just girls} = 6 \cdot 5 = 30$$

$$\text{Ans} = 182 - 30 = 152.$$

