SKILLS REFRESHER FOR CHAPTER 4: EXPONENTS

We list the definition and properties that are used to manipulate exponents.

Definition of Zero, Negative, and Fractional Exponents
If m and n are positive integers:[34]

- $a^0 = 1$
- $a^{-n} = \dfrac{1}{a^n}$
- $a^{1/n} = \sqrt[n]{a}$, the n^{th} root of a
- $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Properties of Exponents

- $a^m \cdot a^n = a^{m+n}$ For example, $2^4 \cdot 2^3 = (2\cdot2\cdot2\cdot2)\cdot(2\cdot2\cdot2) = 2^7$.
- $\dfrac{a^m}{a^n} = a^{m-n}$, $a \neq 0$ For example, $\dfrac{2^4}{2^3} = \dfrac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2} = 2^1$.
- $(a^m)^n = a^{mn}$ For example, $(2^3)^2 = 2^3 \cdot 2^3 = 2^6$.
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$, $b \neq 0$

Be aware of the following notational conventions:

$$ab^n = a(b^n), \qquad \text{but } ab^n \neq (ab)^n,$$
$$-b^n = -(b^n), \qquad \text{but } -b^n \neq (-b)^n,$$
$$-ab^n = (-a)(b^n).$$

For example, $-2^4 = -(2^4) = -16$, but $(-2)^4 = (-2)(-2)(-2)(-2) = +16$. Also, be sure to realize that for $n \neq 1$,

$$(a+b)^n \neq a^n + b^n \qquad \text{Power of a sum} \neq \text{Sum of powers.}$$

Example 1 Evaluate without a calculator:

(a) $(27)^{2/3}$ (b) $(4)^{-3/2}$ (c) $8^{1/3} - 1^{1/3}$

Solution

(a) We have $(27)^{2/3} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$, or, equivalently, $(27)^{2/3} = \left(27^{1/3}\right)^2 = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$.

(b) We have $(4)^{-3/2} = (2)^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$.

(c) We have $8^{1/3} - 1^{1/3} = 2 - 1 = 1$.

[34] We assume that the base is restricted to the values for which the power is defined.

Example 2 Use the rules of exponents to simplify the following:

(a) $\dfrac{100x^2y^4}{5x^3y^2}$ (b) $\dfrac{y^4(x^3y^{-2})^2}{2x^{-1}}$ (c) $\sqrt[3]{-8x^6}$ (d) $\left(\dfrac{M^{1/5}}{3N^{-1/2}}\right)^2$

Solution (a) We have

$$\frac{100x^2y^4}{5x^3y^2} = 20(x^{2-3})(y^{4-2}) = 20x^{-1}y^2 = \frac{20y^2}{x}.$$

(b) We have

$$\frac{y^4\left(x^3y^{-2}\right)^2}{2x^{-1}} = \frac{y^4x^6y^{-4}}{2x^{-1}} = \frac{y^{(4-4)}x^{(6-(-1))}}{2} = \frac{y^0x^7}{2} = \frac{x^7}{2}.$$

(c) We have

$$\sqrt[3]{-8x^6} = \sqrt[3]{-8}\cdot\sqrt[3]{x^6} = -2x^2.$$

(d) We have

$$\left(\frac{M^{1/5}}{3N^{-1/2}}\right)^2 = \frac{\left(M^{1/5}\right)^2}{\left(3N^{-1/2}\right)^2} = \frac{M^{2/5}}{3^2N^{-1}} = \frac{M^{2/5}N}{9}.$$

Example 3 Solve for x:

(a) $\dfrac{10x^7}{4x^2} = 37$ (b) $\dfrac{x^2}{3x^5} = 10$ (c) $\sqrt{9x^5} = 10$

Solution (a) We have

$$\begin{aligned}\frac{10x^7}{4x^2} &= 37\\ 2.5x^5 &= 37\\ x^5 &= 14.8\\ x &= (14.8)^{1/5} = 1.714.\end{aligned}$$

(b) We have

$$\begin{aligned}\frac{x^2}{3x^5} &= 10\\ \frac{1}{3}x^{-3} &= 10\\ \frac{1}{x^3} &= 30\\ x^3 &= \frac{1}{30}\\ x &= \left(\frac{1}{30}\right)^{1/3} = 0.322.\end{aligned}$$

(c) We have

$$\begin{aligned}\sqrt{9x^5} &= 10\\ 3x^{5/2} &= 10\\ x^{5/2} &= \frac{10}{3}\\ x &= \left(\frac{10}{3}\right)^{2/5} = 1.619.\end{aligned}$$

Exercises to Skills for Chapter 4

For Exercises 1–33, evaluate without a calculator.

1. $(-5)^2$ **2.** 11^2 **3.** 10^4

4. $(-1)^{13}$ **5.** $\frac{5^3}{5^2}$ **6.** $\frac{10^8}{10^5}$

7. $\frac{6^4}{6^4}$ **8.** $\sqrt{4}$ **9.** $\sqrt{4^2}$

10. $\sqrt{4^4}$ **11.** $\sqrt{(-4)^2}$ **12.** $\frac{1}{7^{-2}}$

13. $\frac{2^7}{2^3}$ **14.** $(-1)^{445}$ **15.** -11^2

16. $\left(5^0\right)^3$ **17.** $2.1\left(10^3\right)$ **18.** $16^{1/2}$

19. $16^{1/4}$ **20.** $16^{3/4}$ **21.** $16^{5/4}$

22. $16^{5/2}$ **23.** $100^{5/2}$ **24.** $\sqrt{(-4)^2}$

25. $(-1)^3\sqrt{36}$ **26.** $(0.04)^{1/2}$ **27.** $(-8)^{2/3}$

28. 3^{-1} **29.** $3^{-3/2}$ **30.** 25^{-1}

31. 25^{-2} **32.** $(1/27)^{-1/3}$ **33.** $(0.125)^{1/3}$

Simplify the expressions in Exercises 34–55 and leave without radicals if possible. Assume all variables are positive.

34. $\sqrt{x^4}$ **35.** $\sqrt{y^8}$

36. $\sqrt{w^8z^4}$ **37.** $\sqrt{x^5y^4}$

38. $\sqrt{49w^9}$ **39.** $\sqrt{25x^3z^4}$

40. $\sqrt{r^2}$ **41.** $\sqrt{r^3}$

42. $\sqrt{r^4}$ **43.** $\sqrt{64s^7}$

44. $\sqrt{50x^4y^6}$ **45.** $\sqrt{48u^{10}v^{12}y^5}$

46. $\sqrt{6s^2t^3v^5}\sqrt{6st^5v^3}$ **47.** $\left(S\sqrt{16xt^2}\right)^2$

48. $\sqrt{e^{2x}}$ **49.** $(3AB)^{-1}\left(A^2B^{-1}\right)^2$

50. $e^{kt}\cdot e^3\cdot e$ **51.** $\sqrt{M+2}(2+M)^{3/2}$

52. $\left(y^{-2}e^y\right)^2$ **53.** $\frac{a^{n+1}3^{n+1}}{a^n3^n}$

54. $\left(a^{-1}+b^{-1}\right)^{-1}$

55. $\left(\frac{35(2b+1)^9}{7(2b+1)^{-1}}\right)^2$ (Do not expand $(2b+1)^9$.)

If possible, evaluate the quantities in Exercises 56–64. Check your answers with a calculator.

56. $(-32)^{3/5}$ **57.** $-32^{3/5}$ **58.** $-625^{3/4}$

59. $(-625)^{3/4}$ **60.** $(-1728)^{4/3}$ **61.** $64^{-3/2}$

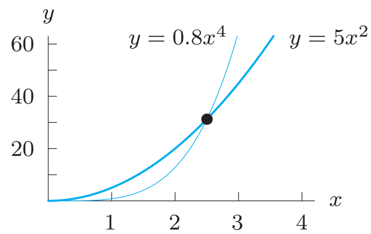
62. $-64^{3/2}$ **63.** $(-64)^{3/2}$ **64.** $81^{5/4}$

In Exercises 65–66, solve for x.

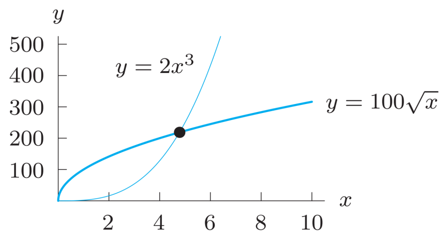
65. $7x^4 = 20x^2$ **66.** $2(x+2)^3 = 100$

In Exercises 67–68, use algebra to find the point of intersection.

67.



68.



Are the statements in Exercises 69–74 true or false?

69. $x^2y^5 = (xy)^{10}$

70. $5u^2 + 5u^3 = 10u^5$

71. $(3r)^2 9s^2 = 81r^2s^2$

72. $\sqrt[3]{-64b^3c^6} = -4bc^2$

73. $-4w^2 - 3w^3 = -w^2(4 + 3w)$

74. $(u + v)^{-1} = \dfrac{1}{u} + \dfrac{1}{v}$

Solve the equations in Exercises 75–76 in terms of r and s, given that

$$2^r = 5 \qquad \text{and} \qquad 2^s = 7.$$

75. $2^x = 35.$

76. $2^x = 140.$

Let $2^a = 5$ and $2^b = 7$. Using exponent rules, solve the equations in Exercises 77–82 in terms of a and b.

77. $5^x = 32$

78. $7^x = \dfrac{1}{8}$

79. $25^x = 64$

80. $14^x = 16$

81. $5^x = 7$

82. $0.4^x = 49$