

$$y = \log x \longleftrightarrow \textcircled{10^y = x} \quad \begin{array}{l} \text{swapped} \\ \text{inputs} \\ \text{and} \\ \text{outputs.} \end{array}$$
$$y = \ln x \longleftrightarrow e^y = x$$
$$y = \log_2 x \longleftrightarrow 2^y = x$$

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$$\underline{1)} \quad \log(AB) = \log A + \log B$$

$$A, B > 0$$

$$\text{let } A = 10^x, B = 10^y \rightarrow x = \log A, y = \log B$$

$$\log(AB) = \log(10^x \cdot 10^y) = \log(10^{x+y})$$

$$= x + y = \log A + \log B$$

$$\underline{2)} \quad \log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\log\left(\frac{10^x}{10^y}\right) = \log(10^{x-y}) = x - y$$

$$= \log A - \log B$$

aside:  $\log 10^9 = 9$   $\log_2 2^9 = 9$

$$10^{\log 5} = 5$$

$$2^{\log_2 5} = 5$$

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$$\ln e^t = t$$

$$e^{\ln t} = t$$

3]

$$\log(A^t) = t \log A$$

let  $A = 10^x \longleftrightarrow x = \log A$

$$A^t = 10^{tx}$$

$$\log(A^t) = \log(10^{tx})$$

$$\log(A^t) = tx$$

xx

$$\log(A^t) = t \log A$$

$$2^x = 10$$

$$\ln 2^x = \ln 10$$

$$x \ln 2 = \ln 10$$

$$x = \frac{\ln 10}{\ln 2} = 3.32$$

$\Sigma_v$

$$10^x = 33$$

$$\log 10^x = \log 33$$

$$x = \log 33 \approx 1.518$$

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$\Sigma_x$

$$2^x = x$$

$$\log 2^x = \log x$$

$$x \log 2 = \log x$$

$$x \log 2 = \log x$$

$$10^{x \log 2} = 10^{\log x}$$

$$2^x = x$$

C-14

half life 5700 years

$$A(t) = A_0 \left( \frac{1}{2} \right)^{t/5700}$$

20% of C<sup>14</sup> has decayed. how much time has elapsed.

$$0.80 = 1 \left( \frac{1}{2} \right)^{t/5700}$$

$$\log(.80) = \log \left( \frac{1}{2} \right)^{t/5700}$$

$$\log(0.80) = \frac{t}{5700} \log \left( \frac{1}{2} \right)$$

$$1834 \approx \frac{5700 \log(0.80)}{\log \left( \frac{1}{2} \right)} = t$$

years

ex   Expand    $\log(1000x^3) = \log 1000 + \log x^3$   
 $= 3 + 3\log x$   
 $= 3(1 + \log x)$

Condense

$$3 \log x - 2 \log y + 10 \log z$$

$$= \log x^3 - \log y^2 + \log z^{10}$$

$$= \log \left( \frac{x^3}{y^2} \right) + \log z^{10}$$

$$= \log \left( \frac{x^3 z^{10}}{y^2} \right)$$

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$$\frac{1}{2} \log y = \log y^{1/2} = \log \sqrt{y}$$