Sequences (and Series)

Def. a sequence $\{a_n\}$ is a list of numbers indexed by the counting numbers.

Ex $\{a_n\} = 2, 4, 6, 8, 10, 12, \ldots$

general term of the sequence $\rightarrow a_n = 2n$ (formula to generate all the terms)

$a_{100} = 200$
$a_{11} = 22$
$a_8 = 16$

Ex $b_n = 2n-1 = 1, 3, 5, 7, 9, \ldots$

Ex $3, 6, 12, 24, 48, 96, \ldots$

$C_n = 3(2)^{n-1}$ [exponential] call it geometric sequence.

Ex $4, 9, 14, 19, 24, 29, \ldots$

$d_n = 4 + 5(n-1)$ [linear] call it arithmetic.

$d_1 = 4$
$d_2 = 9$

Ex $e_n = (-1)^n \left(\frac{1}{2}\right)^{n-1}$

$e_1 = -1 \quad e_2 = \frac{1}{2} \quad e_3 = -\frac{1}{4} \quad e_4 = \frac{1}{8}, \quad e_5 = -\frac{1}{16}$

Ex $1, 11, 21, 1211, \underline{111221}, 312211,$

$13112221, 1113213211, \ldots$

Conway sequence

Other ways to define sequences.

- define the n^{th} term of a sequence in terms of previous terms.

This is called recursively defining your sequence

$$\boxed{x_1 = 1 \quad x_2 = 1 \quad x_{n+2} = x_{n+1} + x_n}$$

$x_3 = 2$
$x_4 = 3$
$x_5 = 5$

$x_6 = 8$
$x_7 = 13$

let $n=3$
$x_5 = x_4 + x_3$

Fibonacci.

Ex $a_n = 3(2)^{n-1}$ define it recursively.

$a_1 = 3$
$a_2 = 6$
$a_3 = 12$

$a_{n+1} = 2a_n$

let $n=1$
$a_2 = 2a_1$
let $n=2$
$a_3 = 2a_2$

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note: If you know the general term, you don't want to use a recursive definition

Ex $\boxed{\begin{array}{l} b_1 = 5 \quad b_2 = 3 \\ b_{n+2} = 2b_{n+1} - b_n \end{array}}$ This defines a sequence recursively.

$b_3 = 1$ $\quad b_6 = -5$

$b_4 = -1$ $\quad b_7 = -7.$

$b_5 = -3$

main types of sequences we will cover are arithmetic and geometric

So how can you tell the difference.

$a_n = 7, 9, 11, 13, 15, \ldots$

there is a constant difference

thus arithmetic.

$b_n = 5, 15, 45, 135, \ldots$

there is a constant ratio

hence geometric.

$b_1, b_2, b_3 \ldots$

$a_n = 7 + 2(n-1)$	$b_n = 5(3)^{n-1}$
if or a_0 is your first term then.	or b_0 is your start
$a_n = 7 + 2n$	$b_n = 5(3)^n$.
Generally	Generally
$a_n = a_1 + d(n-1)$	$b_n = b_1(r)^{n-1}$
or	
$a_n = a_0 + d(n)$	$b_n = b_0(r)^n$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$