

Methods of Voting.

We will restrict ourselves to following situations:

1. There are more than 2 choices to vote for
2. If I prefer A to B ($A > B$) and prefer B to C ($B > C$) then I must also prefer A to C ($A > C$). My preference is transitive.
3. If I have preference $A > B > C > D$ my ranking will be valid if one of the choices is deleted. ie. if B is no longer running then $A > C > D$.

We'll look at our ballot tallies in the following format:

Number of Voters	8	4	3	2
1st choice	A	B	B	D
2nd choice	C	D	C	C
3rd choice	B	C	D	B
4th choice	D	A	A	A

total
17

majority
means
more
than
half.

≥ 9

plurality
means most
1st place votes

Plurality Method (this is the most common): The candidate with the most first place votes wins.

Note: a majority is not necessary.

Number of Voters	8	4	3	2
1st choice	A	B	B	D
2nd choice	C	D	C	C
3rd choice	B	C	D	B
4th choice	D	A	A	A

A is plurality candidate.

Might people object if A is chosen the winner since it is the plurality candidate?

Number of Voters	8	4	3	2
1st choice	A	B	B	D
2nd choice	C	D	C	C
3rd choice	B	C	D	B
4th choice	D	A	A	A

1992
Clinton
Bush
Perot.

C would be most fair

C vs A
 C 4
 A 8
 4
 3
 2
 ———
 9 8

C vs B
 C 8
 B 4
 8 4
 2 3
 ———
 10 7

C vs D
 C 8
 D 4
 8 4
 3 2
 ———
 11 6

A Condorcet candidate is preferred by a majority of voters when compared
head-to-head with each other candidate.

note: There is not always a Condorcet candidate, and with more candidates it becomes more difficult to calculate.

Number of Voters	8	4	3	2
1st choice	A	B	B	D
2nd choice	C	D	C	C
3rd choice	B	C	D	B
4th choice	D	A	A	A

The existence of a Condorcet candidate is a good jumping off point to introduce "Fairness" criteria.

To deem a voting method "fair" or "representative" of what the people want, we would like the following conditions to always hold for all possible elections:

1. If a Condorcet candidate exists, it should win the election.



2. If a majority candidate exists, it should win the election.



Plurality voting violates Condorcet fairness, and yet it is still a very common method of voting.

Number of Voters	8	4	3	2
1st choice	A	B	B	D
2nd choice	C	D	C	C
3rd choice	B	C	D	B
4th choice	D	A	A	A

plurality is A
Condorcet is C.

Some speculate that it also makes it easier to manipulate the elections, and is one of the reasons why the two party system is so entrenched. What might D supporters be tempted to do?

Borda Count Method:

Number of Voters	8	4	3	2
1st choice	A	B	B	D
2nd choice	C	D	C	C
3rd choice	B	C	D	B
4th choice	D	A	A	A

A has 8 1st choice, 9 last place $\Rightarrow 8(3) + 9(0) = 24$

B has 7 1st choice, 10 3rd place $\Rightarrow 7(3) + 10(1) = 31$

C has 13 2nd choice, 4 3rd place $\Rightarrow 13(2) + 4(1) = 30$

D has 8 last place, 4 2nd place and 3 3rd place $\Rightarrow 4(2) + 3(1) = 11$

B is 30, 1st

2(3)

17

Number of Voters	6	2	3
1st choice	A	B	C
2nd choice	B	C	D
3rd choice	C	D	B
4th choice	D	A	A

$$A \rightarrow 6(3) = 18$$

$$B \rightarrow 6(2) + 2(3) + 3(1) = 21$$

$$C \rightarrow 6(1) + 2(2) + 3(3) = 19$$

$$D \rightarrow 2(1) + 3(2) = 8$$

Who is the Borda Count winner?

B

Is that troubling?

Borda does not
always choose majority

It turns out when using the Borda count method, violations of majority and condorcet fairness are rare.

Rare enough that this is the method used to pick the:

1. Heisman Trophy winner
2. NBA rookie of the year
3. NFL MVP

Plurality-with-elimination (Hare method)

You remove the choice with the least number of 1st place votes until there is a majority candidate.

Number of Voters	5	3	5	2	1	4
1st choice	A	A	C	B	D	D
2nd choice	B	D	D	C	A	B
3rd choice	C	B	A	B	C	C
4th choice	D	C	B	A	B	A

20

First note we need 11 votes to get a majority.

A 9
B 4
C 7

A 9
C 11

✓

Who wins using plurality-with-elimination?

Number of Voters	7	8	10	4
1st choice	A	B	C	A
2nd choice	B	C	A	C
3rd choice	C	A	B	B

7	8	10	4
A	B	C	C
B	C	A	A
C	A	B	B

29 majority is 15

A	C
11	18

B	C
15	14

violates monotonicity

What if the 4 voters in the last column decided to put C>A>B? This shouldn't affect things because C is the winner.

Number of Voters	7	8	10	4
1st choice	A	B	C	A
2nd choice	B	C	A	C
3rd choice	C	A	B	B

Despite violating monotonicity and Condorcet fairness, this method is used in:

1. Many local elections
2. Choosing the host city of the Olympics

Pairwise Comparison. (Similar to finding Condorcet winner) it satisfies most fairness criteria...but

A problematic example.

Number of Voters	4	2	6	3
1st choice	A	B	C	B
2nd choice	B	C	A	A
3rd choice	C	A	B	C

$$\begin{array}{r} \textcircled{A} \text{ vs } B \\ \hline 10 \quad 5 \end{array}$$

$$\begin{array}{r} \textcircled{B} \text{ vs } C \\ \hline 9 \quad 6 \end{array}$$

$$\begin{array}{r} A \text{ vs } \textcircled{C} \\ \hline 7 \quad 8 \end{array}$$

1
2
3
4
5

Trump
Kasich
Cruz
Sanders.
Clinton

T, C_r, S, C_l, K

C_l	S
S	C_l
C_r	K
K	T
T	C_r

