## Properties of Logs

① $y = \log_b x$ iff $b^y = x$

aside: $(x^2)^5 = x^{10}$ vs. $x^{2^5} = x^{32}$

② $\log_b(1) = 0$ and $\log_b(b) = 1$

ex. $\log_3(\sqrt{3}) = \frac{1}{2}$

③ $\log(AB) = \log A + \log B$

note: common error: $\log(A+B) \neq \log A + \log B$ is not true.

④ $\log A^n = n \log A$

⑤ let $n = -1$ $\log\left(\frac{1}{A}\right) = \log A^{-1} = -\log A$

$\log\left(\frac{A}{B}\right) = \log A + \log\left(\frac{1}{B}\right) = \log A - \log B$

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## Applications

Recall. $f(x) = 5000(1.03)^x$

$g(x) = 5000(1.09)^x$

f $70{,}000 = 5000(1.03)^x \to 89$ years.

g $70{,}000 = 5000(1.09)^x \to 2.14$ years $\to 30.6$ years

$\frac{70}{5} = 1.09^x$

$\ln 14 = \ln 1.09^x = x \ln 1.09$

$30.6 \approx \frac{\ln 14}{\ln 1.09} = x$

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Note: now you can solve for $x$ given a $y$ in exponentials $y = ab^x$

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Also swith between annual growth and continuous growth.

$y = ab^t$ vs. $y = ae^{kt}$

$b = 1 + r$, r is the rate of growth.

k is the continuous growth rate

Before. continuous to annual.

ie. $y = e^{0.2t} \to y = (1.22)^t$

Now we can reverse this.

ie $y = (1.09)^t \to y = e^{kt} = e^{\ln 1.09 t} \doteq e^{0.086t}$

$e^k = 1.09$

$\ln e^k = \ln 1.09$

$k = \ln 1.09$

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Ex convert to continuous growth

$y = 10\boxed{(1.07)}^t \longleftrightarrow y = 10\boxed{e^k}^t$

$1.07 = e^k$

$\ln 1.07 = \ln e^k$

$\boxed{\ln 1.07 = k}$

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Doubling Time.

Ex 500 bacteria, the pop. doubles in 10 minutes

$P(t) = 500(2)^{10t}$ if t is in minutes

one way $P(t) = 500(e^k)^t$

$2^{10} = e^k$

$\ln 2^{10} = k$ $\ln 1024 = 10\ln 2$

$6.932 \approx 10 \ln 2 = k$

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Half Life

Nicotine has a half-life in bloodstream of about 1.5 hours.

Doubling Time.

$P(t) = P_0 e^{kt}$ k constant of growth

find the tripling time....

$\frac{P(t)}{P_0} = e^{kt}$