HW #5

$\frac{1}{125}, \frac{1}{25}, \frac{1}{5}, 1, 5 \ldots 625$

$625 \downarrow 5^4$

$a_n = \frac{1}{125}(5)^{n-1}$

$625 = \frac{1}{125}(5)^{n-1}$

$\sum_{n=1}^{8} \frac{1}{125}(5)^{n-1}$

$= \frac{1}{125}\left(\frac{1-5^8}{1-5}\right)$

15.

$32 - 16 + 8 - 4 + 2 - 1$

$a_n = 32\left(\frac{-1}{2}\right)^{n-1}$

$\sum_{k=1}^{6} 32\left(\frac{-1}{2}\right)^{k-1}$ or $\sum_{n=1}^{6} (-1)^{n-1}\, 32\left(\frac{1}{2}\right)^{n-1}$

22. 50mg / day. half-life 6.3 hours

$\rightarrow Q = 50\left(\frac{1}{2}\right)^{t/6.3}$

$Q_{24} = 50\left(\frac{1}{2}\right)^{24/6.3}$

$Q_0 = 50$

$Q_1 = 3.566 + 50 = 50(0.713) + 50$

$Q_2 = 50 + 50(0.713) + 50(0.713)^2$

$\vdots$

if $Q = 50\left(\frac{1}{2}\right)^{t/6.3} \rightarrow Q = 50e^{-0.11t}$

$Q = 50e^{kt}$

note: $\left(\frac{1}{2}\right)^{1/6.3} = e^{k}$

$\ln\left(\frac{1}{2}\right)^{1/6.3} = k$

Infinite Geometric Series:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 2$$

$$a_n = 1\left(\frac{1}{2}\right)^{n-1}$$

If we just sum k terms.

$$\sum_{n=1}^{k} 1\left(\frac{1}{2}\right)^{n-1} = 1\left(\frac{1-\frac{1}{2}^k}{1-\frac{1}{2}}\right)$$

$\vdots$

let $k \to \infty$ $\left(\frac{1}{2}\right)^k \to 0$

$$\sum_{n=1}^{\infty} 1\left(\frac{1}{2}\right)^{n-1} = 1\left(\frac{1-0}{\frac{1}{2}}\right) = 2$$

Any geometric series.

$$\sum_{n=1}^{K} a_n = a_1\left(\frac{1-r^K}{1-r}\right)$$

let $K \to \infty$ when will $\sum_{n=1}^{\infty} a_n$ be finite?

Ex $a_n = 3(2)^{n-1}$

$$\sum_{n=1}^{K} a_n = 3\frac{(1-2^K)}{1-2}$$

what happens when $K \to \infty$?

$2^K \to \infty$

Theorem. $\sum_{n=1}^{\infty} a_n$ is finite when $|r|<1$
for a_n geometric (the series converges)

Ex $\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$ converges $\frac{1}{3}<1$

Ex $\sum_{n=1}^{\infty} 10(3)^{n-1}$ diverges $3>1$

$$= 2\left(\frac{1-\frac{1}{3}^K}{1-\frac{1}{3}}\right)$$

$$= 2\left(\frac{1}{1-\frac{1}{3}}\right) = 3$$

Theorem if $a_n = a_1 r^{n-1}$
then $\boxed{\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r}}$

Ex. $\sum_{n=1}^{\infty} 3(1)^{n-1} = 3+3+3+3+3+\cdots$
diverges

Ex $0.44\overline{4}$

$$= 0.4 + 0.04 + 0.004 + 0.0004 + \cdots\cdots$$

$$= 4\left(\frac{1}{10}\right) + 4\left(\frac{1}{100}\right) + 4\left(\frac{1}{1000}\right) + \cdots\cdots$$

$$= 4\left[\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots\cdots\right]$$

$$= 4\left[\frac{\frac{1}{10}}{1-\frac{1}{10}}\right] = 4\left[\frac{1}{10}\cdot\frac{10}{9}\right] = \frac{4}{9}$$

Ex $0.\overline{356} = \frac{9}{6} = \frac{89}{250}$

$$= \frac{356}{1000} + \frac{356}{1\times10^6} + \frac{356}{1\times10^9} + \cdots$$