Factorials

Ex. 3 colored marbles R, G, B.

$$\underline{3} \cdot \underline{2} \cdot \underline{1} = 6$$

R < G–B, B–G
G < R→B, B→R
B < R–G, G–R

Ex 4 marbles R, G, B, Y

$$4 \cdot 3 \cdot 2 = 24$$

Ex 5 marbles all different

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 = 120,$$

Reason to define $n! = n(n-1)(n-2)\cdots 2 \cdot 1$

n factorial. → counts the rearrangement of n-things.

Ex $\dfrac{10!}{8!} = \dfrac{10 \cdot 9 \cdot 8 \cdots 1}{8 \cdot 7 \cdot 6 \cdots 1} = 10 \cdot 9 = 90$

note: $y = x!$ grows very very fast.

In calculator → [math] → [PRB] → [4]

in calc anything 70! overflows

Ex $\dfrac{(n!)}{(n+2)!} = \dfrac{1}{(n+2)(n+1)}$.

Ex 10 turtles..

choose P, VP, treasure for the turtles.

$$\underline{10}, \underline{9}, \underline{8} = {}_{10}P_3 = \frac{10!}{7!}$$

Ex choose a wise council of 3.

$$\underline{10}, \underline{9}, \underline{8} = \frac{10 \cdot 9 \cdot 8}{3!} = {}_{10}C_3 = \frac{10!}{7!3!}$$

Permutation. → order matters "# of r things you choose out of n"

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combination → order does not matter " "

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

Ex 20 students.

# of ways to choose 5 to speak at graduation.

$${}_{20}C_5 = \frac{20!}{5!(15!)} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 19 \cdot 3 \cdot 17 \cdot 16 = 15504$$

Ex. 20 students choose.

6 of them to be in line for tasty lunch.

"order matters"

$$\frac{20!}{14!} = {}_{20}P_6.$$