Probability

sample space / event space: $\Omega = \{$ set of all possible events (outcomes) of a random experiment $\}$

Ex flipping a coin. $\Omega = \{H, T\}$

events, $A, B, C, \ldots$ are subsets of Ω

Ex rolling a 4-sided die.

$\Omega = \{1, 2, 3, 4\}$

$A = \{$rolling an even #$\}$

We will assign a probability function P: an event in $\Omega \longrightarrow$ a number $0 \le x \le 1$

Ex Flipping a fair coin twice.

$\Omega = \{HH, TT, HT, TH\}$

$A = \{HH\}$

for this P: all outcomes equally likely

$P(A) = \frac{1}{4}$

$P(HH \text{ or } TT) = \frac{1}{4} + \frac{1}{4}$

$P(HH \underbrace{\text{and}}_{\text{intersection of the 2 events}} TT) = 0$ because intersection is ϕ

Ex rolling a fair 6 sided die

$\Omega = \{1, 2, 3, 4, 5, 6\}$

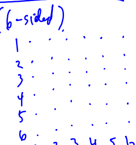
fairness $\to$ P: assigning equal likelihood to each outcome

$P(\text{rolling an even \#}) = \frac{3}{6} = \frac{1}{2}$

$P(\text{rolling a multiple of } 3) = \frac{2}{6} = \frac{1}{3}$

def. $P(A) = \dfrac{|A|}{|\Omega|}$ note: assumes all outcomes are equally likely.

Ex rolling 2 dice (6-sided)



$P(\text{roll sum even}) = P(2) + P(4) + P(6) + P(8) + P(10) + P(12)$

note: $\{\text{roll sum even}\} = (\text{roll a } 2) \cup (\text{roll a } 4) \cup \cdots \cup (\text{roll a } 12)$

$P(\text{roll sum even}) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{5}{36} + \frac{3}{36} + \frac{1}{36}$
$= \frac{1}{2}$

Ex $P(\underset{A}{\text{sum is prime}} \underset{\text{or}}{\text{or}} \underset{B}{\text{even}})$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{15}{36} + \frac{18}{36} - \frac{1}{36} = \frac{32}{36}$
$= \frac{8}{9}$

Let $C = \overline{A \cup B} = \{\text{rolling } 9\}$

$P(C) = \frac{4}{36} = \frac{1}{9}$

$P(A \cup B) = 1 - P(C) = 1 - \frac{1}{9}$
$= \frac{8}{9}.$

Simple properties of P probability function

Let A be a subset of Ω

1. $P(\Omega) = 1$
2. $P(A) \geq 0$
3. $P(A) \leq 1$
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
5. we say 2 events A, B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex repeated flipping of a coin each toss is independent.

$$P(HHHHH) = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \frac{1}{32}$$

Ex not independent (dependent events)

picking 2 hearts from a well shuffled deck,

$$\left(\frac{1}{4}\right) \cdot \left(\frac{12}{51}\right)$$

$$P(\heartsuit) = \frac{1}{4}$$