Solution
(a) Since the fine increased each day by a factor of 2, the fine grew exponentially with growth factor $b = 2$. To find the percent growth rate, we set $b = 1 + r = 2$, from which we find $r = 1$, or 100%. Thus the daily percent growth rate is 100%. This makes sense because when a quantity increases by 100%, it doubles in size.

(b) If t is the number of days since August 2, the formula for the fine, P in dollars, is

$$P = 100 \cdot 2^t.$$

(c) After 30 days, the fine is $P = 100 \cdot 2^{30} \approx 1.074 \cdot 10^{11}$ dollars, or \$107,374,182,400.

Exercises and Problems for Section 4.1

Skill Refresher

In Exercises S1–S2, express the percentages in decimal form.   In Exercises S3–S4, express the decimals as a percent.

S1. 6% **S2.** 0.6% **S3.** 0.0012 **S4.** 1.23

Exercises

Are the functions in Exercises 1–9 exponential? If so, write the function in the form $f(t) = ab^t$.

1. $g(w) = 2\left(2^{-w}\right)$ **2.** $m(t) = (2 \cdot 3^t)^3$

3. $f(x) = \dfrac{3^{2x}}{4}$ **4.** $G(t) = 3(t)^t$

5. $q(r) = \dfrac{-4}{3^r}$ **6.** $j(x) = 2^x 3^x$

7. $Q(t) = 8^{t/3}$ **8.** $K(x) = \dfrac{2^x}{3 \cdot 3^x}$

9. $p(r) = 2^r + 3^r$

What is the growth factor in Exercises 10–13? Assume time is measured in the units given.

10. Water usage is increasing by 3% per year.

11. A city grows by 28% per decade.

12. A diamond mine is depleted by 1% per day.

13. A forest shrinks 80% per century.

In Exercises 14–17, give the starting value a, the growth factor b, and the growth rate r if $Q = ab^t = a(1 + r)^t$.

14. $Q = 1750(1.593)^t$ **15.** $Q = 34.3(0.788)^t$

16. $Q = 79.2(1.002)^t$ **17.** $Q = 0.0022(2.31)^{-3t}$

Problems

18. The populations, P, of six towns with time t in years are given by

(i) $P = 1000(1.08)^t$ (ii) $P = 600(1.12)^t$
(iii) $P = 2500(0.9)^t$ (iv) $P = 1200(1.185)^t$
(v) $P = 800(0.78)^t$ (vi) $P = 2000(0.99)^t$

(a) Which towns are growing in size? Which are shrinking?
(b) Which town is growing the fastest? What is the annual percent growth rate for that town?
(c) Which town is shrinking the fastest? What is the annual percent "decay" rate for that town?

(d) Which town has the largest initial population (at $t = 0$)? Which town has the smallest?

19. The value, V, of a \$100,000 investment that earns 3% annual interest is given by $V = f(t)$ where t is in years. How much is the investment worth in 3 years?

20. An investment decreases by 5% per year for 4 years. By what total percent does it decrease?

21. Without a calculator, match each of the formulas to one of the graphs in Figure 4.6.

(a) $y = 0.8^t$ (b) $y = 5(3)^t$
(c) $y = -6(1.03)^t$ (d) $y = 15(3)^{-t}$