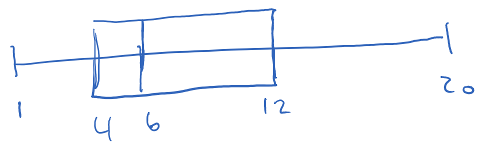
Exam review

Wednesday, June 1, 2016 9:23 PM

1 data: 1, 3, 4, 4, 6, 8, 12, 13, 20

a Median: 6
1Q: 4
3Q: 12
min: 1
max: 20



skew right

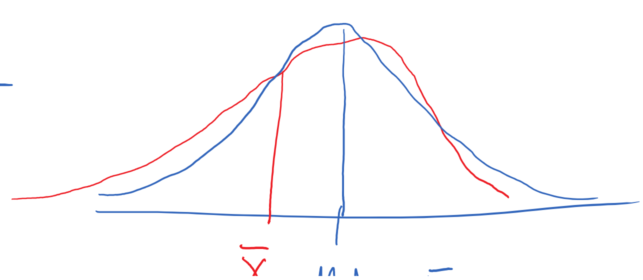
b $\bar{x} = \frac{\sum_{i=1}^{9} x_i}{9} = \frac{71}{9} = 7.\overline{8}$

$\sigma = 6.071335$

c center is measured by the median and the mean. Often the median is a better measure of center for skew distributions as the mean is heavily influenced by outliers.

d spread is measured by σ and IQR. Again σ is more affected by outliers so $IQR = 3Q - 1Q$ is a better measure of spread for skew data.

2



X Median, $\bar{x}$

3 A scatterplot helps to visualize a collection of paired data (x_i, y_i) to see if there is a relationship between x and y.

4 $\hat{y} = 4x + 3$

a $\hat{y} = 4(10) + 3 = 43$

b $\hat{y} = 4(4) + 3 = 19$ $y_{obs} = 20$

residual $y_{obs} - y_{predicted} = 20 - 19 = 1$

c Patterns in the residual plot may suggest that a linear model is not the best model to fit the data.

5 It's possible that there exists a connection between video games and math. Up to a certain threshold playing video games may signify an interest in tech/math, but this is probably a weak correlation and definitely does not imply causality.

6 r measures the strength of the correlation.
r^2 measures how good a fit a linear model is for the data.

§ 2

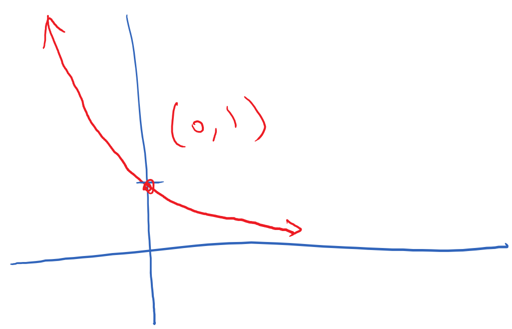
1 linear data has a constant slope $\frac{\Delta y}{\Delta x} = \text{constant}$

exponential data has a constant ratio

for equally spaced $x_1, x_2, x_3 \rightarrow \frac{f(x_3)}{f(x_2)} = \frac{f(x_2)}{f(x_1)}$

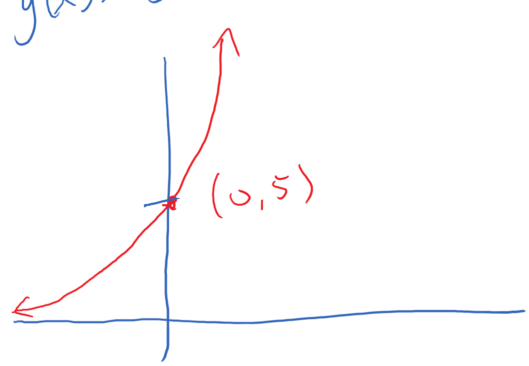
2 $P(t) = P_0(1.07)^t$ where P_0 is the initial population of moles.

3 a



a horizontal asymptote at $y=0$

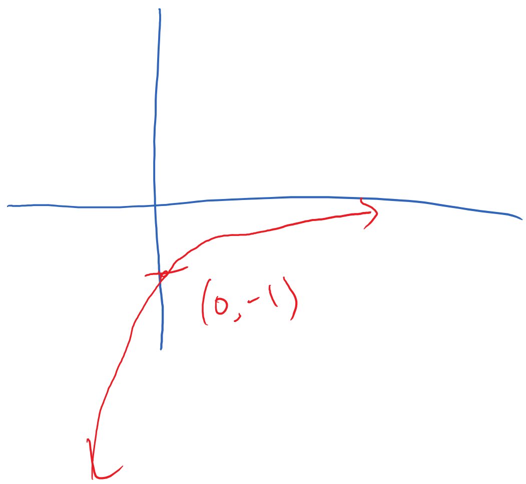
b $g(x) = 5e^x$



a horizontal asymptote at $y=0$

c. $h(x) = -\frac{1}{2}^{x}$

c $h(x) = -\frac{1}{3}^{x}$



a horizontal asymptote at $y=0$

4 let $f(x) = 2^{x} - 4$

a range $= \{ y \mid y > -4 \}$

b domain $= \{ \mathbb{R} \}$

c $y = -4$

5 $A(t) = 500 \left(\frac{1}{2}\right)^{t/10}$ where t is in hours.

midnight is $t = 12$

$$A(12) = 500\left(\frac{1}{2}\right)^{12/10} = 500\left(\frac{1}{2}\right)^{1.2} \approx 217.638 \text{ mg}$$

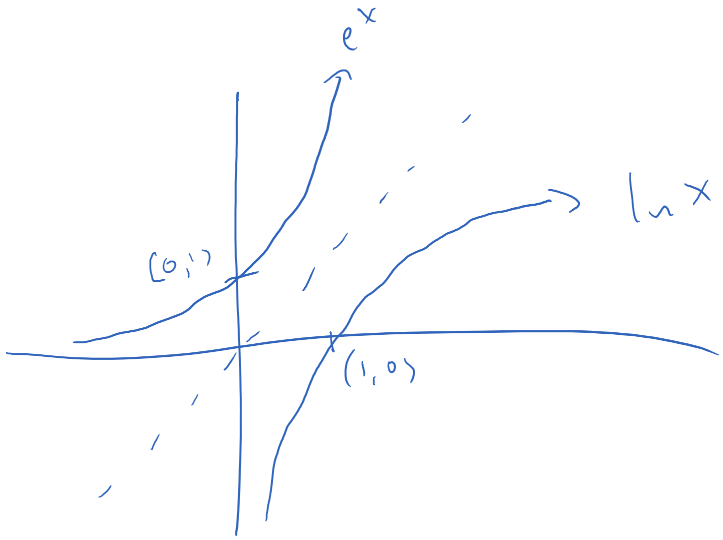
6 a $\log_2 32 = \log_2 (2^5) = 5$

b $\log 1000 = \log(10^3) = 3$

c $\ln(\sqrt{e^2}) = \ln(e) = 1$

d $\log\left(\frac{1}{100}\right) = \log(10^{-2}) = -2$

7



They are reflections of each other across the line $y = x$ hence they are inverses!

8 $f(x) = \log(x-2)$

a Domain $= \{x \mid x > 2\}$

b Range $= \{\mathbb{R}\}$

c vertical asymptote at $x = 2$

Remember $\log(0)$ is undefined!

9 $P(t) = 200(1.05)^t$

$$1200 = 200(1.05)^t$$

$$6 = (1.05)^t$$

$$\ln 6 = \ln (1.05)^t$$

$$\ln 6 = t \ln (1.05)$$

$$\frac{\ln 6}{\ln(1.05)} = t$$

$$36.724 = t$$

years.

Review of Combinatorics & Probability

1 a $\frac{10!}{8!} = 10(9) = 90$ b $2!(3!) = 2(6) = 12$

c $(2\cdot3)! = 6! = 720$ d $3! + 4! = 6 + 24 = 30$

e $(3+4)! = 7! = 5040$ f $\frac{n!}{(n+1)!} = \frac{1}{n+1}$

g $\frac{n!}{2(n-1)!} = \frac{n}{2}$

2 b + c illustrate that $a!\,b! \neq (ab)!$
d + e show that $(a+b)! \neq a! + b!$

3 4 different letters or 4 different digits.

$26\cdot25\cdot24\cdot23 + 10\cdot9\cdot8\cdot7$

4 $7\cdot6\cdot5\cdot4\cdot3$

5 ${}_8C_2 = \frac{8\cdot7}{2!} = 28$

6 RRRBBYY

$\frac{7!}{3!\cdot2!\cdot2!} = \frac{7\cdot6\cdot5\cdot4}{4} = 210$