17)

60% decline over 12 yrs.

$$0.40 = 1(1+r)^{12}$$

$$(0.40)^{1/12} = 1+r$$

$$(0.40^{1/12}) - 1 = \quad = \quad r$$

$$1.6 = b^{12}$$

19 $850 \longrightarrow 1000$ over 10 years

compounded quarterly

$$1000 = 850\left(1+\frac{r}{4}\right)^{40}$$

$$\frac{100}{85} = \left(1+\frac{r}{4}\right)^{40}$$

$$\left(\frac{20}{17}\right)^{1/40} = \left(1+\frac{r}{4}\right)$$

$$\left(\frac{20}{17}\right)^{1/40} - 1 = \frac{r}{4}$$

$$4\left[\left(\frac{20}{17}\right)^{1/40} - 1\right] = r$$

$$.016285 \approx r$$

Introduce the number e.

ex Start with \$1 & grows at 100% annually. for one year.

a compound it annually $= 1(1+1) = 2$

b quarterly $= 2.44 = \left(1+\frac{1}{4}\right)^4$

c weekly $= \left(1+\frac{1}{52}\right)^{52} \approx 2.6925$

d daily $\approx \left(1+\frac{1}{365}\right)^{365} \approx 2.7146$

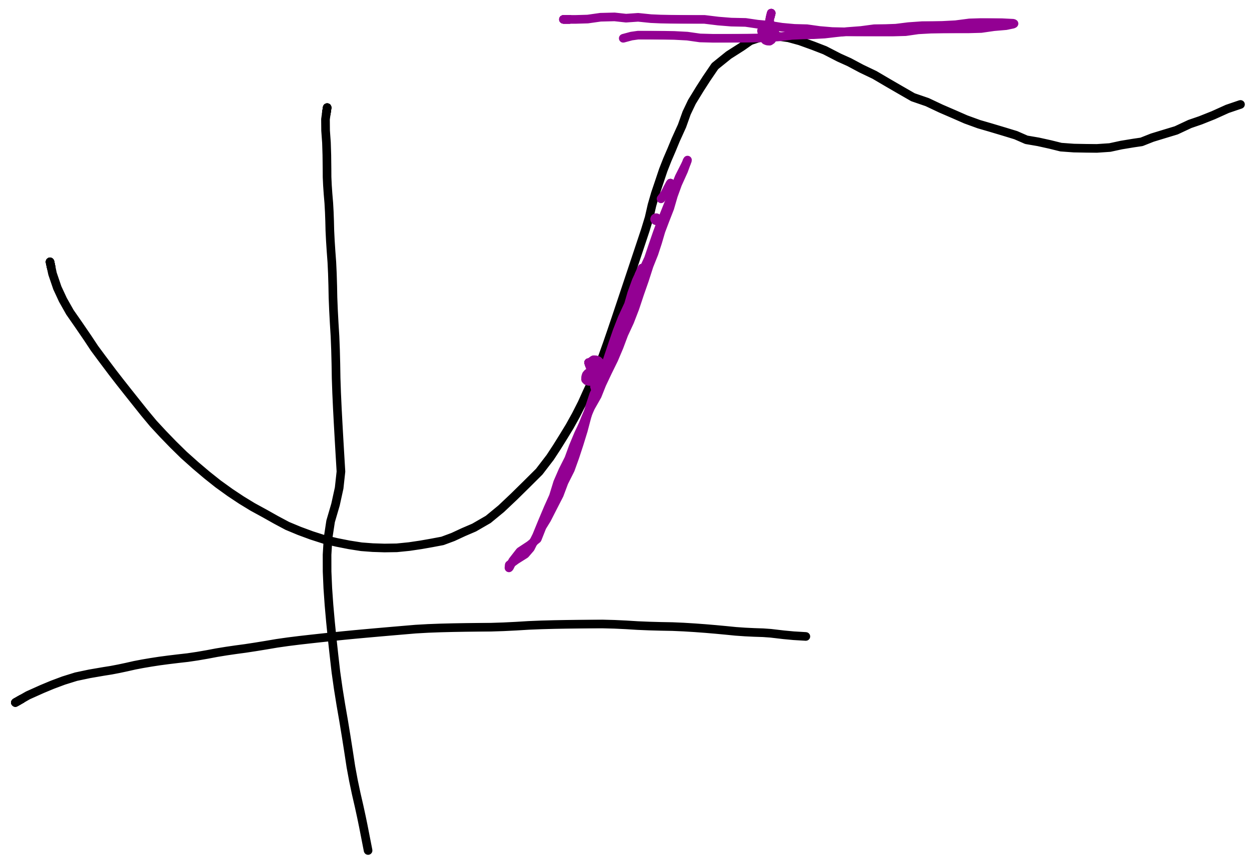
e every seconds $= \left(1+\frac{1}{31MM}\right)^{31MM} \approx 2.71828$

as $n \to \infty$ $\left(1+\frac{1}{n}\right)^n \to e$

$\hookrightarrow$ compounded continuously

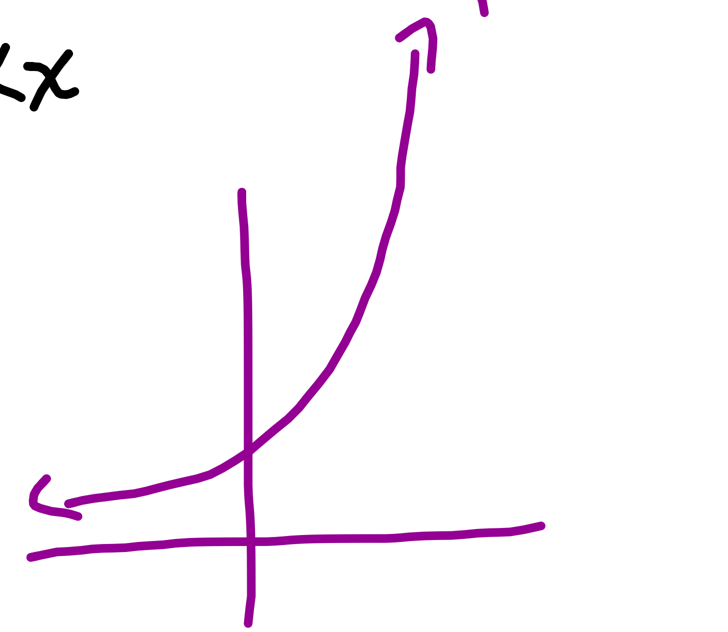
$$f(x) = \left(1+\frac{1}{x}\right)^x$$

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{60} + \frac{1}{120} + \frac{1}{720} +$$



Before : $f(x) = A_0 b^x$

Continuous growth/decay : $f(x) = A_0 e^{kx}$



Q: what is the purpose of k?

$f(x) = A_0 b^x$ $\qquad$ $f(x) = A_0 e^{kx}$

$= A_0 (e^k)^x$

$\boxed{b = e^k}$

$k \to$ continuous growth rate.

ex

$$f(x) = 20e^{0.15x}$$

initial amount is 20

continuous growth rate 15%.

$$f(x) = Ab^x$$

$$f(x) = 20b^x \to b = e^{0.15}$$

$$b \approx 1.1618 \to f(x) = 20(1.1618)^x$$

annual growth of 16.18%

ex

$f(x) = 10e^{-0.20x}$ continuous 20% decay.

$f(x) = 10b^x \rightarrow b = e^{-0.20}$

$b \approx 0.8187$

annual decay 18.13%

$f(x) = 10\left(1+\frac{r}{4}\right)^{4x}$

$\left(1+\frac{r}{4}\right)^4 = e^{-0.20}$

$f(x) = 1000e^{0.15x}$

given 15%
~~continuous~~
growth.

start and
$1000 \to 1200$.

$1200 = 1000e^{0.15x}$

$\boxed{1.2 = e^{0.15x}}$

↳ graph: $y = 1.2$
$y = e^{0.15x}$

$f(x) = A_0 e^{kx}$.

k is the continuous growth/decay rate.