

$$\begin{aligned}
 A_1 &= 250 \\
 A_2 &= 250(0.04) + 250 \\
 A_3 &= 250 + 250(0.04) + 250(0.04)^2 \\
 A_n &= \sum_{n=1}^n 250(0.04)^{n-1} \\
 A_{40} &= 250 \left(\frac{1 - 0.04^{40}}{1 - 0.04} \right) = 260.416 \\
 A_{\infty} &= 250 \left(\frac{1}{1 - 0.04} \right) = 260.416 \\
 \frac{x^3}{x^2} &= \frac{(1-x)^3}{(1-x)^2}
 \end{aligned}$$

25

$$\begin{aligned}
 P_1 &= 0 \\
 P_2 &= 250(0.04) \\
 P_3 &= 250(0.04) + 250(0.04)^2
 \end{aligned}$$

Time Value of Money

money is worth more in your pocket today

- ① today you can invest.
- ② uncertainty in the future (inflation, disaster)

In order to value money you receive in the future you need to determine its present value.

Ex for 10 years you will invest \$100 annually at a guaranteed 5%
Future Value, FV is just a compounding exercise.

FV = ?

$$\begin{aligned}
 Q_1 &= 100 \\
 Q_2 &= 100(1.05) + 100 \\
 Q_3 &= 100(1.05)^2 + 100(1.05) + 100 \\
 Q_{10} &= \sum_{n=1}^{10} 100(1.05)^{n-1} = 100 \left(\frac{1 - 1.05^{10}}{1 - 1.05} \right) \\
 FV &\approx 1257.79
 \end{aligned}$$

Present Valuing (ie discounting) is the "inverse" of FV.
instead of $100(1.05) \rightarrow 1\text{yr}$

$$\frac{100}{1.05} \leftarrow 1\text{yr}$$

Ex Lotto. Jackpot \$50 million.

① money NOW! \$20 million.

② 20 payments of \$1,020,000 over 20 years.

Value option ②

after year 1 $Q_1 = 1.02$ $Q_2 = 1.02 / 1.07$ let's say we can get 7%.

$$Q_3 = 1.02 / (1.07)^2$$

$$Q_4 = 1.02 / (1.07)^3$$

$$Q_{20} = 1.02 / (1.07)^{19}$$

$$\begin{aligned}
 \sum_{n=1}^{20} Q_n &= 1.02 \left(\frac{1 - \left(\frac{1}{1.07} \right)^{20}}{1 - \frac{1}{1.07}} \right) \\
 &= 1.02 \left(\frac{1 - 0.2584}{1 - 0.9358} \right) \\
 &\approx 1.02 \left(\frac{0.74158}{0.0658} \right) \\
 &\approx 11.5623
 \end{aligned}$$