Arithmetic Sequence.

- common difference between successive terms d = common difference (slope)

$$a_n = a_1 + d(n-1)$$

$a_1 = 23 \quad d = 4$

$a_n = 23 + 4(n-1)$

$a_{101} = 23 + 4(100)$

$= 423$

geometric (exponential)

common ratio (r)

$$g_n = a_1 r^{n-1}$$

ex $g_n = \left\{ \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \ldots \right\}$

$\frac{g_2}{g_1} = \frac{2}{9} \cdot \frac{3}{2} = \frac{1}{3} \qquad r = \frac{1}{3}$

$\frac{g_3}{g_2} = \frac{2}{27} \cdot \frac{9}{2} = \frac{1}{3}$

$g_n = \frac{2}{3}\left(\frac{1}{3}\right)^{n-1}$

Series (Summation)

Let a_n be a sequence.

say I want to sume the 1st 10 terms

$a_1 + a_2 + a_3 + \cdots + a_{10}$

$$= \sum_{k=1}^{10} a_k$$

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ex $\{1, 2, 3, 4, 5, 6, 7, \ldots\}$

$a_k = k$

$a_1 + a_2 + a_3 + \cdots + a_{100}$

$$= \sum_{k=1}^{100} a_k = \sum_{k=1}^{100} k$$

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ex $\{2, 4, 6, 8, 10, 12, \ldots\}$

$a_n = 2n$

$g_n = 2^n$

sum terms 5 - 100

$$\sum_{k=5}^{100} 2k = 10 + 12 + \cdots + 200.$$

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ex $a_n = 2 = \{2, 2, 2, 2, 2, \ldots$

$$\sum_{k=1}^{150} a_k = 2(150) = 300$$

Theorem $\sum_{k=1}^{m} C = mC$ where C is a constant.

Theorem a_k is a sequence.

$$C \sum_{k=1}^{m} a_k = \sum_{k=1}^{m} C a_k$$ where C is a constant

ex $\sum_{k=1}^{m} a_k = 20$ then $\sum_{k=1}^{m} 5a_k = 5(20) = 100$

Let $a_n = n : \{1, 2, 3, 4, 5, 6 ..$

$$S_n = 1 + 2 + 3 + \dots + (m-1) + m$$

$$S_m = m + (m-1) + (m-2) + \dots + 2 + 1$$

$$2 S_m = (m+1)\, m$$

$$S_m = \frac{(m+1)m}{2}$$

$$\sum_{k=1}^{100} k = \frac{101\,(100)}{2}$$

$$= 5050 \quad \boxed{m}$$

<u>Theorem</u> if $a_n = n$

then $$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$

Let $a_n = a_1 + d(n-1)$

$$\sum_{k=1}^{m} a_k = \sum_{k=1}^{m}(a_1 + dk - d)$$

$$= \sum_{k=1}^{m} a_1 + \sum_{k=1}^{m} dk - \sum_{k=1}^{m} d$$

$$= ma_1 + d\sum_{k=1}^{m} k - md$$

$$= ma_1 + d\left[\frac{m(m+1)}{2}\right] - md$$

$$= m\left[a_1 + \frac{d(m+1)}{2} - d\right]$$

$$= \frac{m}{2}\left[2a_1 + dm + d - 2d\right]$$

$$= \frac{m}{2}\left[a_1 + a_1 + dm - d\right]$$

$$= \frac{m}{2}\left[a_1 + a_m\right] \quad \boxtimes$$

ex $\displaystyle\sum_{i=1}^{10}(2i+1) = \frac{(3+21)}{2}\cdot 10$