

# Exponential Function

ex A City has a population of 100 thousand  
 it grows 5% a year.  
 What is its population after 1 year?  
 after 10 years?

Year 1 105, because 5% of 100 is 5

$$\hookrightarrow 100 + 100(0.05)$$

$$= 100(1 + 0.05)$$

$$= \boxed{100(1.05)}$$

Year 2  $100(1.05)(1.05)$

Year 10  $100(1.05)(1.05)\dots\dots(1.05)$

$$= \boxed{100(1.05)^{10}}$$

↓                      t

$y = 100(1.05)^t$   
 an exponential function!

ex

\$20,000 car depreciates

at 12% a year.

$$W = 20,000 (1 - 0.12)^t$$

$$W = 20,000 (0.88)^t$$

$$f(t) = a b^t$$

original  
amount

(represents change)  
constant %  
base of the exponential

$$f(0) = a b^0$$

$$f(0) = a$$

ex  $P(t) = 100(1.02)^t$   $t$  is in years.

choose  $\hat{t}$  to be in decades.

$$P(10) = 100(1.02)^{10} = 100(1.219)$$

over 10 years grows by 21.9%

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same city but now in decades.

$$\hat{P}(\hat{t}) = 100(1.219)^{\hat{t}}$$

$$\hat{P}(\hat{t}) = 100(1.02^{10})^{\hat{t}} = 100(1.02)^{10\hat{t}}$$


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ex      2% growth every year.  
 initial population 100.

Q1: write model for pop when  $t$  is  
 measured in 6 mo intervals

$$P(t) = 100(1.02)^{\frac{t}{2}}$$

Q2: write this same model, but  $t$  is in days,

$$P(t) = 100(1.02)^{\frac{t}{365}}$$

Q3: if 2% a day and  $t$  is in years.

$$P(t) = 100(1.02)^{365t}$$

Q1 over a decade your stocks grew by 45%.

what is the constant annual % change that gives this?



$$b^{10} = 1.45$$

$$b = 1.45^{1/10}$$

alternative.

$$f(t) = 100 (1.45)^{t/10}$$