p 146 #26 $(0,20)$ $(5,75)$

if linear then

$$m = \frac{75-20}{5} = 11$$

$$\boxed{y = 11x + 20}$$

if exponential then

$$20\,b^5 = 75$$

$$b^5 = \frac{75}{20} = \frac{15}{4} \qquad b = \left(\frac{15}{4}\right)^{\frac{1}{5}}$$

$$y = 20\left(\frac{15}{4}\right)^{\frac{1}{5}x}$$

#29 $(0,20)$ $(10,40)$ for P

$(10,40)$ $(15,20)$ for g

For P $\quad 20b^{10} = 40$

$$b = (2)^{\frac{1}{10}} \longrightarrow P(x) = 20\left(2^{\frac{1}{10}}\right)^x$$

for g $\quad 40\,b^5 = 20 \longrightarrow g(x) = a\left(\frac{1}{2}^{\frac{1}{5}}\right)^x$

$$b^5 = \frac{1}{2}$$

$$b = \left(\frac{1}{2}\right)^{\frac{1}{5}}$$

use $(10,40)$ to get value of a

$$40 = g(10) = a\left(\frac{1}{2}\right)^{\frac{10}{5}}$$

$$40 = a\,\frac{1}{4} \longrightarrow g(x) = 160\left(\frac{1}{2}\right)^{\frac{1}{5}x}$$

$$160 = a$$

or $g(x) = 160\left(2^{-\frac{1}{5}}\right)^x$

#34 Penalty A $\quad P_A(t) = 1\times10^6 + 10\times10^6\,t$

Penalty B $\quad P_B(t) = 0.01(2)^t$

a on 8/31 29 days have elapsed.

$$P_A(29) = 1\times10^6 + 290\times10^6$$
$$= 291\times10^6$$

$$P_B(29) = 0.01(2)^{29} \approx 5.369\times10^6$$

b already done

c Graph $P_A(t) - P_B(t)$ in your calculator then look for where the graph crosses the x-axis. (ie. $P_A(t) - P_B(t) = 0$)

around day 35.

#36 look at the exponential part

$(0, 80)$ to $(10, 15)$ $\quad 80b^{10} = 15$

$$b^{10} = \frac{15}{80} = \frac{3}{16}$$

$$y = 80\left(\frac{3}{16}\right)^{\frac{1}{10}x}$$

so $V(t) = \begin{cases} 80 & t \le 0 \\ 80\left(\frac{3}{16}\right)^{\frac{1}{10}t} & t > 0 \end{cases}$

b again graphing on the calculator

around $t = 40$

p 153

#30 Let $f(x) = 5 + 3(0.9)^x$

as $x \to \infty$ $f(x) \to 5$ because

$3(0.9)^x \to 0$ as $x \to \infty$ leaving 5.

#31 a which constants are definitely positive?

p, c, a, q, d, b

b which are in (0,1)?

only b for sure. (decaying)

c which could be in (0,1)?

b for sure. possibly p, c, a

d which are definitely equal?

a and c

e which pairs could be equal?

q and d could be the same.

#40 decay by 10% / year

a $y(t) = 10{,}000(0.9)^t$

b $y(5) = 10{,}000(0.9)^5 = 5904.9$

c Again by graphing around 22 years.

#42 $P_0 = 10$ rabbits
$P_5 = 340$ rabbits

$10b^5 = 340$
$b^5 = 34$
$b = 34^{\frac{1}{5}}$

$P(t) = 10(34)^{\frac{t}{5}}$

b Again by graphing around 6.6 years from the start.

p158 #15
3% growth per year for 10 years.

$(1.03)^{10} \approx 1.343916 \rightarrow 34.39\%$ increase.

#16 30% growth over 5 years means

$(1+r)^5 = 1.30$
$1+r = (1.30)^{\frac{1}{5}}$
$r = (1.30)^{\frac{1}{5}} - 1 \approx 5.3874\%$ annual growth.

#21 \$300 is an initial deposit at a bank

a $B(t) = 300(1.2)^t$ 20% annual growth (i)

c $B(t) = 300(1.06)^{2t} = 300\left(1 + \frac{0.12}{2}\right)^{2t}$ 12% nominal compounded semi-annually (ii) b $B(t) = 300(1.12)^t$ 12% annual growth (i)

d $B(t) = 300(1.06)^{t/2}$

$= 300\left(1 + \frac{0.03}{\frac{1}{2}}\right)^{\frac{1}{2}t}$ 3% nominal compounded every 2 years (iv)

e $B(t) = 300(1.03)^{4t} = 300\left(1 + \frac{0.12}{4}\right)^{4t}$ 12% nominal compounded quarterly

(v)

p165

#23 a (i) 2% annual growth

$P(t) = 3.2(1.02)^t$

$P(100) = 3.2(1.02)^{100} \approx 23.183$ MM

(ii) a continuous growth of 2%

$\hat{P}(t) = 3.2\,e^{0.02t}$

$\hat{P}(100) = 3.2\,e^2 \approx 23.645$ MM

b e^{rt} grows faster than $(1+r)^t$

for $r > 0$

42 to convert $P(t) = P_0 e^{0.053t}$ to an effective annual rate we calculate $e^{0.053} \approx 1.05443$

so $P(t) = P_0 (1.05443)^t$ we can now read the eff annual rate as

$5.443\% < 5.5\%$ so go with

$\hat{P}(t) = P_0 (1.055)^t$ has eff annual yea of 5.5%.