City Semester
Problem Set #4
Show all work for full credit.

Name:__________________________

1. The U.S. census and population growth. In 1790 the population of the U.S. was 3,929,214, whereas in 2010 it had grown to 308,745,538.

   (a) Give a model for the population growth over this time where the population grew by the same amount each year. What would the yearly growth be?

   (b) What does your model predict for the year 1900?

   (c) The actual population of the U.S. in 1900 was 76,212,168. Did your model over estimate or under estimate? What was the percentage error?

   (d) Let's look at census data by the decade in the following table:

Year	Population	Difference	Ratio
1790	3,929,214		
1800	5,308,483		
1810	7,239,881		
1820	9,683,453		
1830	12,866,020		
1840	17,069,452		
1850	23,191,876		
1860	31,443,321		

   Fill out the chart by finding the difference in population from one decade to the previous decade, and the ratio of the population from one decade to the previous decade.

   (e) Give a function that approximately models the population data above.

   (f) According to the model what should the population be in 1960?

   (g) What is wrong here?

2. Without a calculator give a sketch of the following, noting salient features such as the y-intercept, and the horizontal asymptote.

   (a) $f(x) = -3^x$

   (b) $g(x) = -3^{-x}$

   (c) $h(x) = 4 + (\frac{1}{2})^x$

   (d) $j(x) = 1 - e^x$

   (e) $k(x) = e^{-x} - 1$

3. The amount of substances such as medical drugs or caffeine in the bloodstream decays or is metabolized over time. This can be modeled by an exponential decay function. Given the function $D(t) = 50e^{-0.2t}$, which measures the milligrams left in the blood stream after t hours, how large was the original dosage? How much is left after 3 hours?

4. Suppose you have a type of bacteria that divide every minute. Suppose at noon a single bacterium of this type colonizes a garbage can. The bacterium and all his descendants never have to worry about food and continue to divide every minute, but they fear when their home, the garbage can, will be full of bacteria.

   (a) Give a model for the number of bacteria in the can in terms of t measured in minutes after noon.

   (b) How many bacteria are in the can at 12:05? 12:10?

   (c) When the can is half full, the president of the bacteria colony reassures its constituents that doomsday (the day the can is full) is far away. After all there is as much room left in the can as has been used in the entire previous history of the colony. Is the president correct? How much time is left until doomsday?

   (d) If the can is full at 1pm, when is the can half full? When is it a quarter full?

5. You are offered the choice between two investments. One will pay you 7% interest compounded quarterly and the other will pay 6.8% compounded continuously. Which gives the the higher return? If you invest \$1500 today, how much will you have the year you graduate from college(assume it takes you 4 years)?

6. Alice and Bob both offer to paint two rooms of your apartment. Bob wants to be paid \$650. Alice noticing a 6x6 kenken puzzle on your table wants to be paid in pennies. One penny on the lower right square in the kenken puzzle and double that for the square adjacent. Then keep doubling the amount of pennies for each adjacent square in the kenken puzzle until all squares have a stack of pennies on them. Assuming you have a large tub of pennies and that you have a machine that will create stacks of pennies of whatever height you want, who will you hire? Explain.

$*$ 7. Let's explore the statement: Exponentials grow faster than polynomials.

(a) First let's look at $f(x) = e^x$ and review limit notation. What is $\lim\limits_{x\to-\infty} f(x)$? This is the horizontal asymptote of f.

(b) We say that $\lim\limits_{x\to\infty} e^x = \infty$ because as x gets big e^x increases without bound (meaning there is no cap to how big the y values get).

(c) Recall how polynomials behave when $|x|$ is big. Let $g(x) = x^2$, $h(x) = x^3$, $j(x) = x^4$ and $k(x) = x^5$. Describe what happens for each as you look at $\lim_{x\to\infty}$ and $\lim_{x\to-\infty}$. Do the functions go towards a finite number, or are they unbounded? If unbounded, do they go towards positive infinity or negative infinity?

(d) Generalize the pattern you see in the previous question to polynomials of degree n.

(e) If exponentials grow faster than polynomials than no matter what value they have at $x = 0$ we would expect for some x the exponential will have a higher y value. For $f(x) = e^x - 2$ and $g(x) = x^2 + 2$ give an x that makes $f(x) > g(x)$?

(f) Repeat the previous exercise but replace $g(x)$ with $g(x) = x^3 + 2$.

(g) Repeat the previous exercise but with $g(x) = 100x^4 + 100$. Can you find the exact value of x where $f(x)$ becomes greater than $g(x)$?