

NOTE: THIS PAGE UNDER CONSTRUCTION

Basis-set related quantities, general definitions

- \vec{v} : "Sample values of physical field $v(r)$ " - values of field at sample points r_p ,
 $[\vec{v}]_p \equiv v_p \equiv v(r_p)$
- \hat{v} : "Expansion coefficients for field $v(r) = \sum_a \hat{v}_a b_a(r)$ " - $[\hat{v}]_a = \hat{v}_a$
- O : "Overlap operator" - overlaps of basis functions $O_{\beta\alpha} \equiv \int d^3r b_\beta^*(r) b_\alpha(r)$
- L : "Laplacian operator" - matrix elements in the basis set of the Laplacian operator
 $L_{\beta\alpha} \equiv \int d^3r b_\beta^*(r) \nabla^2 b_\alpha(r)$
- I : "Forward transform" - transforms from expansion coefficient representation to real space sample representation $I_{p\alpha} \equiv b_\alpha(r_p)$
- $Idag$: "Dual forward transform" - $Idag \equiv I^\dagger$
- J : "Inverse transform" - transforms from real space sample representation to expansion coefficient representation - $J = I^{-1}$
- $Jdag$: "Dual inverse transform" - $Jdag \equiv J^\dagger = I^{-\dagger}$

Basis-set related quantities, plane wave basis

For periodic boundary conditions with lattice vectors $R \equiv [\vec{R}_0, \vec{R}_1, \vec{R}_2]$ on a grid of dimensions $\vec{S} = [S_0, S_1, S_2]$, final quantities will be on per unit cell ...

- $O = (\det R) \cdot 1$
- $L = (-\det R) \text{Diag} [|\vec{G}_0|^2; |\vec{G}_1|^2; |\vec{G}_2|^2 \dots]$
- $I = F$, where F is the standard kernel for the discrete (fast) Fourier transform with a positive sign in the exponent. Note the following facts about this standard kernel:
 $F^T = F$, $F^\dagger = F^*$, $F^* F = \text{prod } \vec{S}$, $F^{-1} = \frac{1}{\text{prod } \vec{S}} F^*$, where F^* is just the standard kernel with a negative sign in the exponent.
- $I^\dagger = F^*$
- $J = \frac{1}{\text{prod } \vec{S}} F^*$
- $J^\dagger = \frac{1}{\text{prod } \vec{S}} F$

Physical quantities in density-functional theory

- \hat{C} : "Expansion coefficients for orthonormal wave functions" - $\psi_i(r) = \sum_a \hat{C}_{ai} b_a(r)$
- \hat{P} : "Basis representation of density matrix" - $\hat{P} = f \hat{C} \hat{C}^\dagger$
- P : "Density matrix" - $P_{pq} = \rho(r_p, r_q)$, $P = I \hat{P} I^\dagger$
- \vec{n} : "Sample values of electron density" - $\vec{n} = \text{diag } P$
- $\vec{\varphi}$: "Mean field (Hartree) potential from electrons" - $\vec{\varphi} = (-4\pi) I L^{-1} O J \vec{n}$
- \hat{W} : "Expansion coefficients for unconstrained wave functions" - $\hat{C} = \hat{W} U^{-1/2}$
- U : "Overlaps between (unconstrained) wave functions" - $U_{ij} = \langle W_i(r) | W_j(r) \rangle$
- $E(\hat{W})$: "DFT energy functional" - $E(\hat{W}) = -\frac{1}{2} \hat{L} \hat{P} + \vec{n}^\dagger J^\dagger O J \vec{V}_{nuc} + \frac{1}{2} \vec{n}^\dagger J^\dagger O J \vec{n} + \vec{n}^\dagger J^\dagger O J \varepsilon_{xc}(\vec{n})$