DÚ 2. 6. Roland Schulz xschul06

$f(x) = 1 - \frac{1}{x-2}$, $g(x) = 1 + \frac{1}{1-x^2}$: $f \circ g$? $g \circ f$?

$f(g(x))$ $g(f(x))$

$f(g(x)) : \quad 1 - \frac{1}{g(x)-2} = 1 - \frac{1}{(1+\frac{1}{1-x^2})-2} = 1 - \frac{1}{\frac{1-1+x^2}{1-x^2}} = 1 - \frac{1}{\frac{x^2}{1-x^2}} =$

$= 1 - \frac{1-x^2}{x^2}$

$g(f(x)) : \quad 1 + \frac{1}{1-f(x)^2} = 1 + \frac{1}{1-(1-\frac{1}{x-2})^2} = 1 + \frac{1}{1-(\frac{x-3}{x-2})^2} =$

$= 1 + \frac{1}{\frac{(x-2)^2}{(x-2)^2} - \frac{(x-3)^2}{(x-2)^2}} = 1 + \frac{1}{\frac{(x-2)^2-(x-3)^2}{(x-2)^2}} = 1 + \frac{(x-2)^2}{(x-2)^2-(x-3)^2} =$

$= 1 + \frac{(x-2)^2}{2x-5}$

$f(x) = 1 - \frac{1}{x-2}$

$x \neq 2$

$D(f) = \mathbb{R} \setminus \{2\}$

$H(f) = \mathbb{R} \setminus \{1\}$

$\frac{1}{x-2} \neq 0 \dots 1 - \frac{1}{x-2} \neq 1$

$g(x) = 1 + \frac{1}{1-x^2}$

$x \neq \pm 1$

$D(g) = \mathbb{R} \setminus \{1, -1\}$

$H(g) = (-\infty, 1) \cup \langle 2, +\infty)$

$x^2 \geq 0 \dots 1 - x^2 \leq 1$

$\frac{1}{1-x^2} > 0 \dots 1 + \frac{1}{1-x^2} \geq 2$

$\frac{1}{1-x^2} < 0 \dots 1 + \frac{1}{1-x^2} < 1$

$g(\langle -1, 1 \rangle) = g(\langle 0, 1 \rangle) = \langle 2, \infty)$

$g^{-1}(\langle 2, 3 \rangle) = \langle 0, \sqrt{0,5} \rangle$

$g(0) = 2$

$g(\sqrt{0,5}) = 1 + \frac{1}{1-\sqrt{0,5}^2} = 1 + \frac{1}{0,5} =$

$= 1 + 2 = 3$