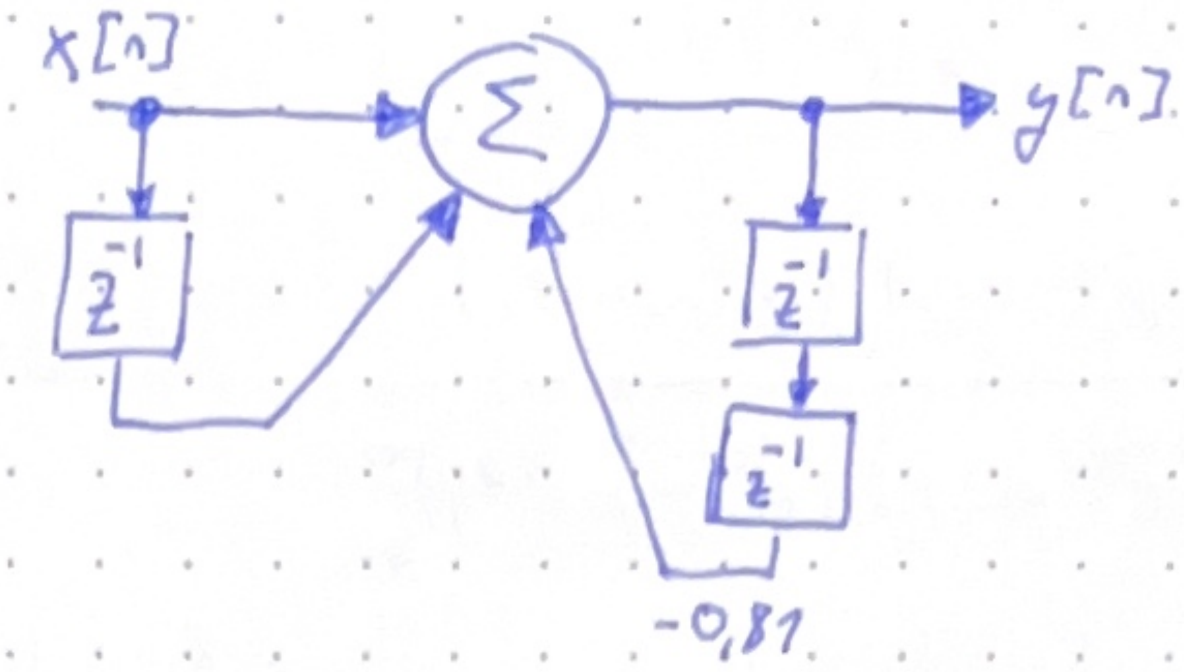
ISS Nahrazení 4. cvičení 1.-10.



1) $y[n] = x[n] + x[n-1] - 0{,}81\,y[n-2]$

2) $Y(z) = X(z) + X(z)z^{-1} - 0{,}81\,Y(z)z^{-2}$

3) $H(z) = \dfrac{B(z)}{A(z)} = \dfrac{Y(z)}{X(z)}$

$Y(z) = X(z) + X(z)z^{-1} - 0{,}81\,Y(z)z^{-2} \quad \backslash + 0{,}81\,Y(z)z^{-2}$

$Y(z) + 0{,}81\,Y(z)z^{-2} = X(z) + X(z)z^{-1} \quad \backslash$ vytknutí $Y(z)$ a $X(z)$

$Y(z)(1 + 0{,}81z^{-2}) = X(z)(1 + z^{-1}) \quad \backslash \dfrac{1}{X(z)} \quad \backslash \dfrac{1}{1+0{,}81z^{-2}}$

$\dfrac{Y(z)}{X(z)} = \dfrac{1 + z^{-1}}{1 + 0{,}81z^{-2}} = H(z)$

4) $b_0 = 1 \qquad a_0 = 1$

$b_1 = 1 \qquad a_1 = 0$

$a_2 = 0{,}81$

5) $\dfrac{1 + z^{-1}}{1 + 0{,}81z^{-2}} \quad \backslash \cdot \dfrac{z^2}{z^2}$

$\dfrac{z^2 + z}{z^2 + 0{,}81}$

6) $\mapsto z^2 + z = 0$

$z(z+1) = 0 \Rightarrow z_1 = 0, \; z_2 = -1$ nulové body

7) $\rightarrow z^2 + 0{,}81 = 0$

$D = b^2 - 4ac$

$= 0 - 4 \cdot 0{,}81$

$D = -4 \cdot 0{,}81$

$z_{1,2} = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-0 \pm \sqrt{4} \cdot \sqrt{-0{,}81}}{2} = \dfrac{\pm 2 \cdot \sqrt{-0{,}81}}{2} =$

$= \pm\sqrt{-0{,}81}$

$\Downarrow$

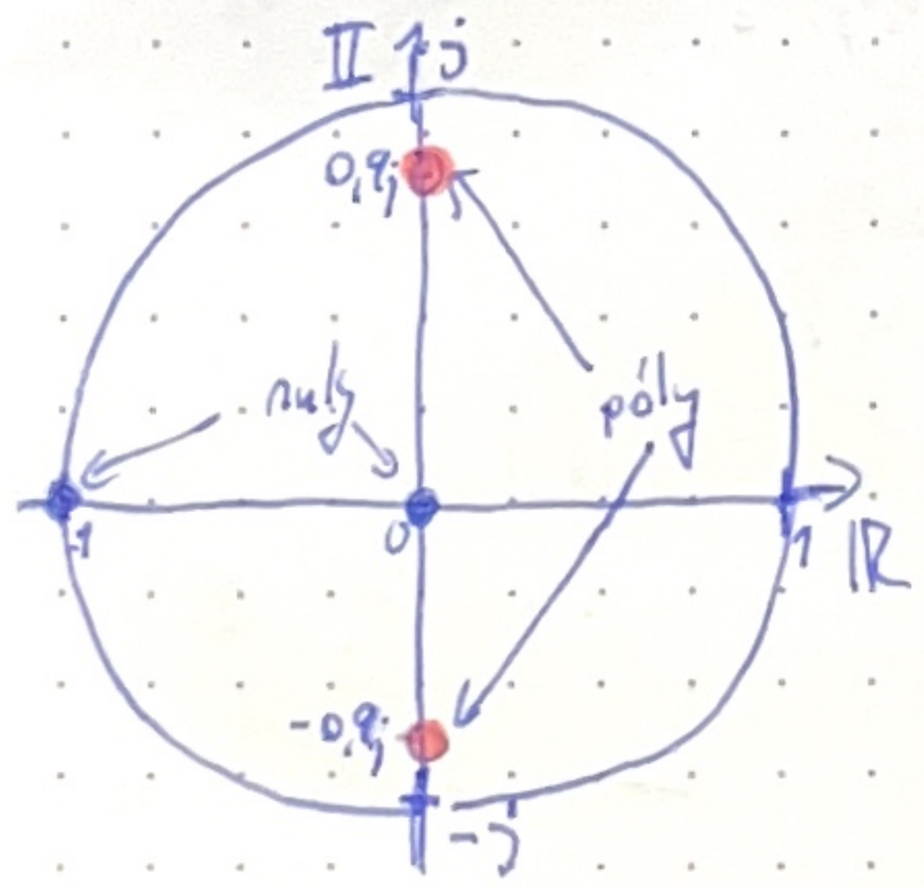
$z_1 = 0{,}9j$

$z_2 = -0{,}9j$

$\Big\}$ póly

8) $H(z) = \dfrac{(z-0)(z-(-1))}{(z-0{,}9j)(z-(-0{,}9j))}$

9)



10) jedná se o IIR filtr $\Rightarrow$ stabilita pokud póly jsou uvnitř jednotkové kružnice

$|-0{,}9j| < 1$ ✓

$|0{,}9j| < 1$ ✓

$\Rightarrow$ filtr je stabilní

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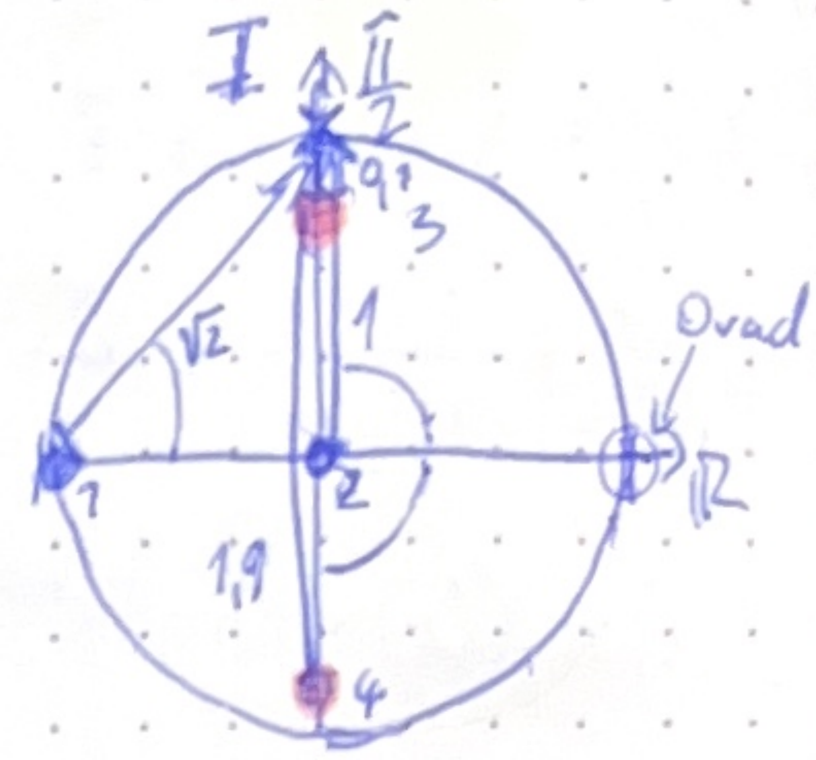
ISS nahrazení 4. cvičení 1.1. – 1.4.

1.1) $H(e^{j\omega}) = \dfrac{(e^{j\omega}-0)_2\,(e^{j\omega}+1))_1}{(e^{j\omega}-0{,}9j)_3\,(e^{j\omega}+0{,}9j)_4}$ $\qquad \omega = \frac{\pi}{2}$



$$|H(e^{j\frac{\pi}{2}})| = \left|\frac{(e^{j\frac{\pi}{2}}-0)\cdot(e^{j\frac{\pi}{2}}+1)}{(e^{j\frac{\pi}{2}}-0{,}9j)\cdot(e^{j\frac{\pi}{2}}+0{,}9j)}\right| = \frac{|e^{j\frac{\pi}{2}}|\cdot|e^{j\frac{\pi}{2}}+1|}{|e^{j\frac{\pi}{2}}-0{,}9j|\cdot|e^{j\frac{\pi}{2}}+0{,}9j|} =$$

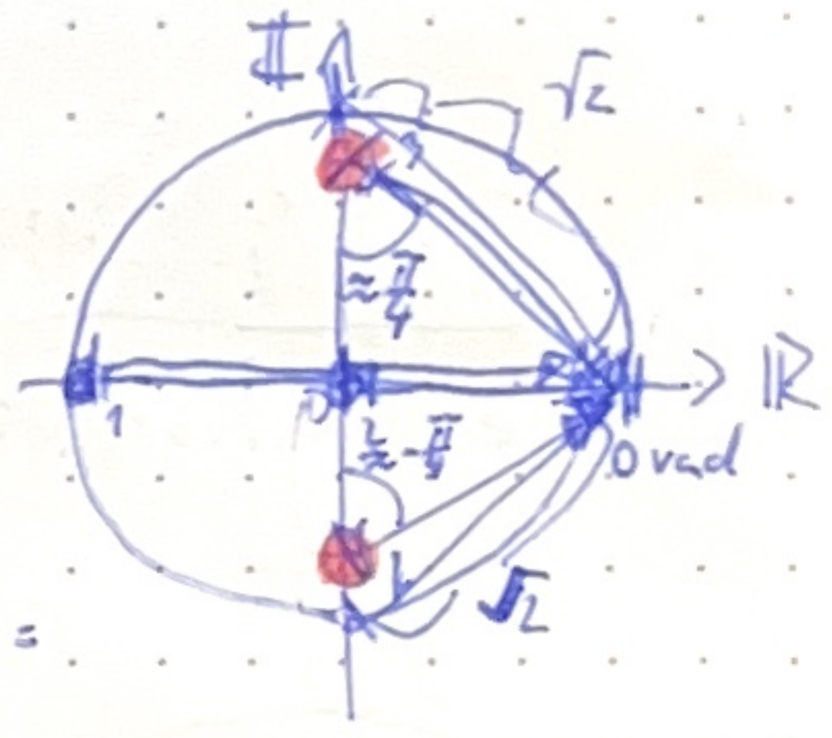
$$= \frac{1\cdot\sqrt{2}}{1{,}9\cdot 0{,}1} \doteq 7$$

$$\arg(H(e^{j\frac{\pi}{2}})) = \arg(e^{j\frac{\pi}{2}}) + \arg(e^{j\frac{\pi}{2}}+1) - \arg(e^{j\frac{\pi}{2}}-0{,}9j) - \arg(e^{j\frac{\pi}{2}}+0{,}9j) =$$

$$= \frac{\pi}{2} + \frac{\pi}{4} - \frac{\pi}{2} - \frac{\pi}{2} = -\frac{\pi}{4}$$

$\omega = \frac{\pi}{2} \rightarrow$ modul 7, argument $-\frac{\pi}{4}$

1.2) $\omega = 0\,\text{rad}$



$$|H(e^{j0})| = \frac{1\cdot 2}{\underbrace{\sqrt{2}\cdot\sqrt{2}}_{cca}} = \frac{2}{2} = 1$$

$$\arg(H(e^{j0})) = \arg(e^{j0}) + \arg(e^{j0}+1) - \arg(e^{j0}-0{,}9j) - (e^{j0}+0{,}9j) =$$

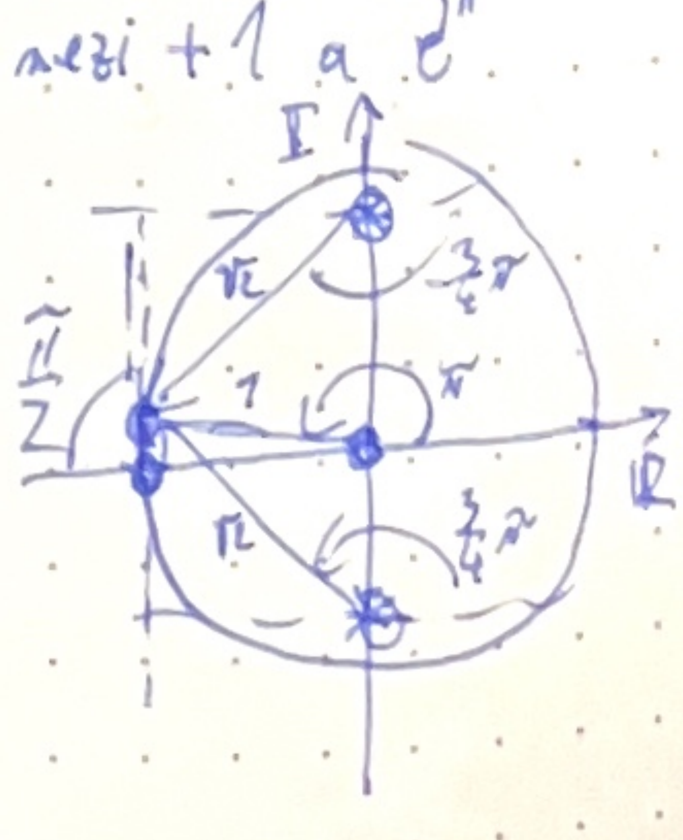
$$= 0 + 0 \underbrace{-\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)}_{cca} =$$

$$= 0$$

$\omega = 0\,\text{rad} \rightarrow$ modul 1, argument 0

1.3) řešeno v 1.1)

1.4) $\omega = 0{,}999\pi$ | nepoužijeme π, protože bychom nemohli zjistit úhel mezi $+1$ a $e^{j\pi}$



$$|H(e^{j0{,}999\pi})| = \frac{1\cdot 0}{\sqrt{2}\cdot\sqrt{2}} = 0$$

$$\arg(H(e^{j0{,}999\pi})) = \arg(e^{j0{,}999\pi}) + \arg(e^{j0{,}999\pi}+1) - \arg(e^{j0{,}999\pi}-0{,}9j) - (e^{j0{,}999\pi}+0{,}9j) =$$

$$= \pi + \frac{\pi}{2} + \frac{3}{4}\pi - \frac{3}{4}\pi = \frac{3}{2}\pi$$

$\omega = 0{,}999\pi \rightarrow$ modul 0, argument $\frac{3}{2}\pi$

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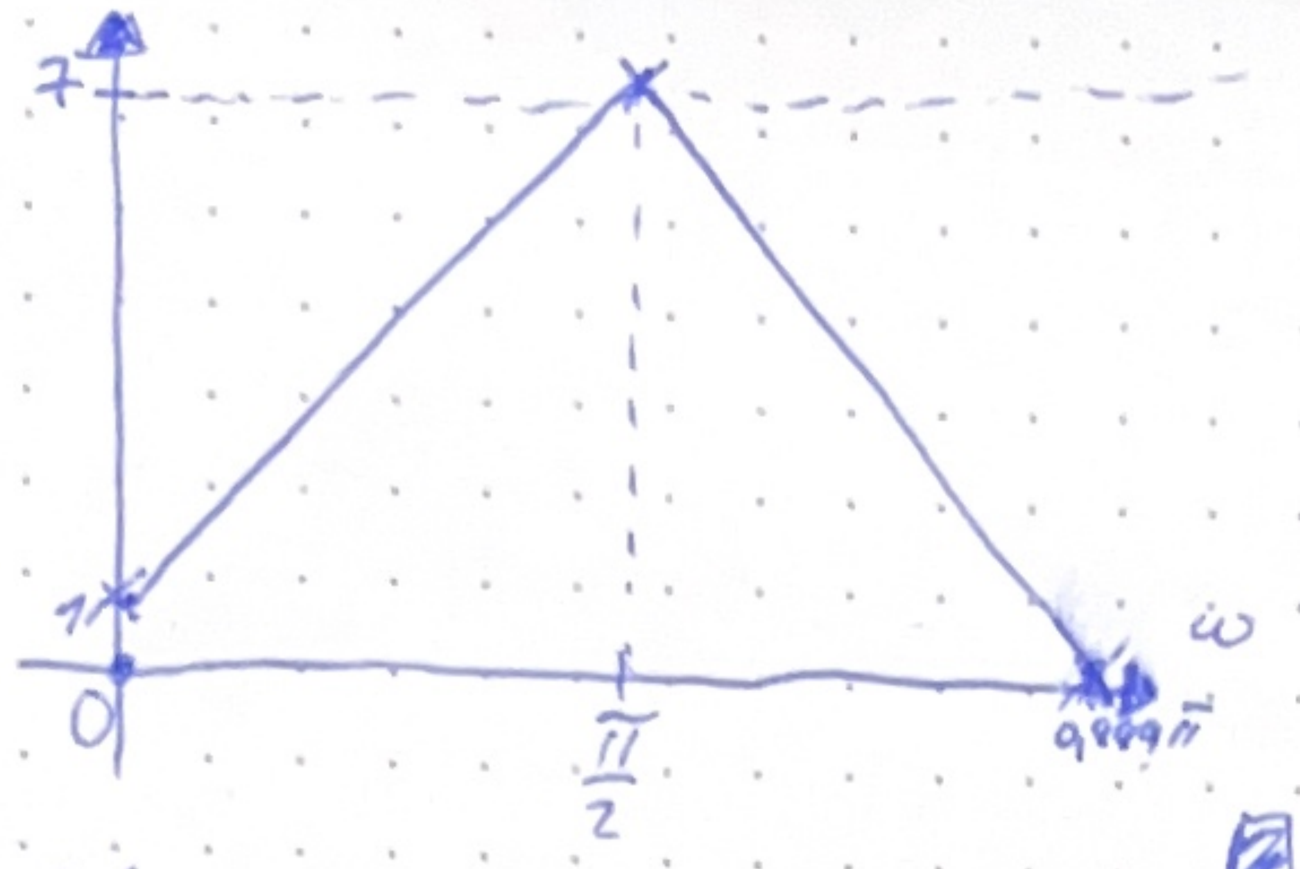
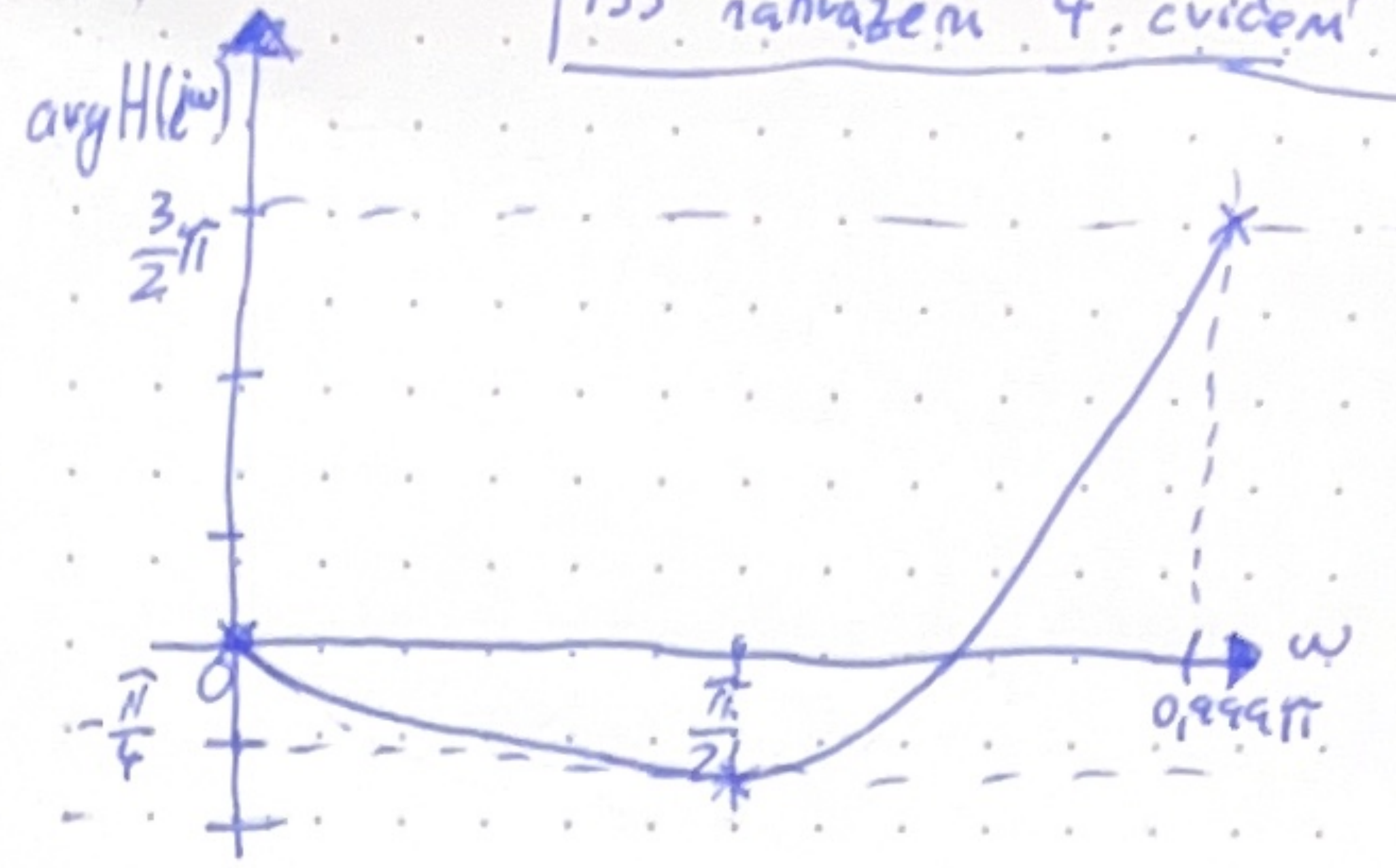
ISS nahrazení 4. cvičení 15.

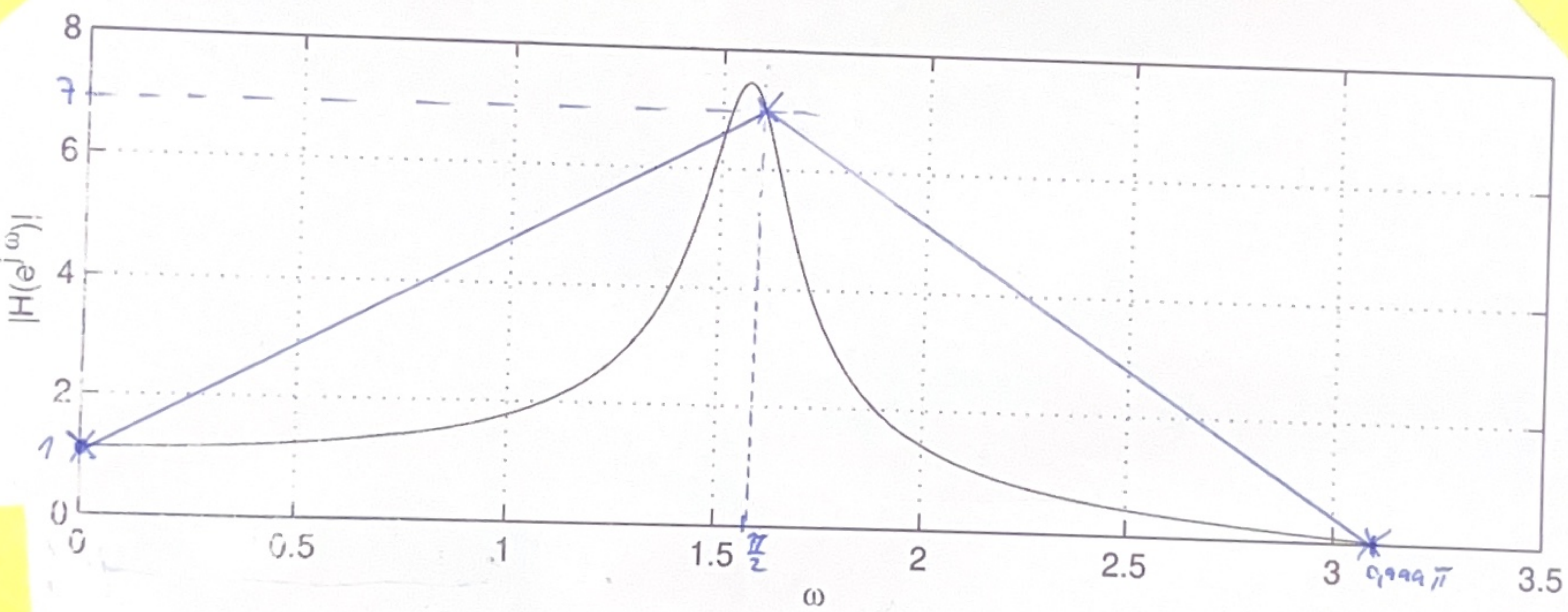
15)

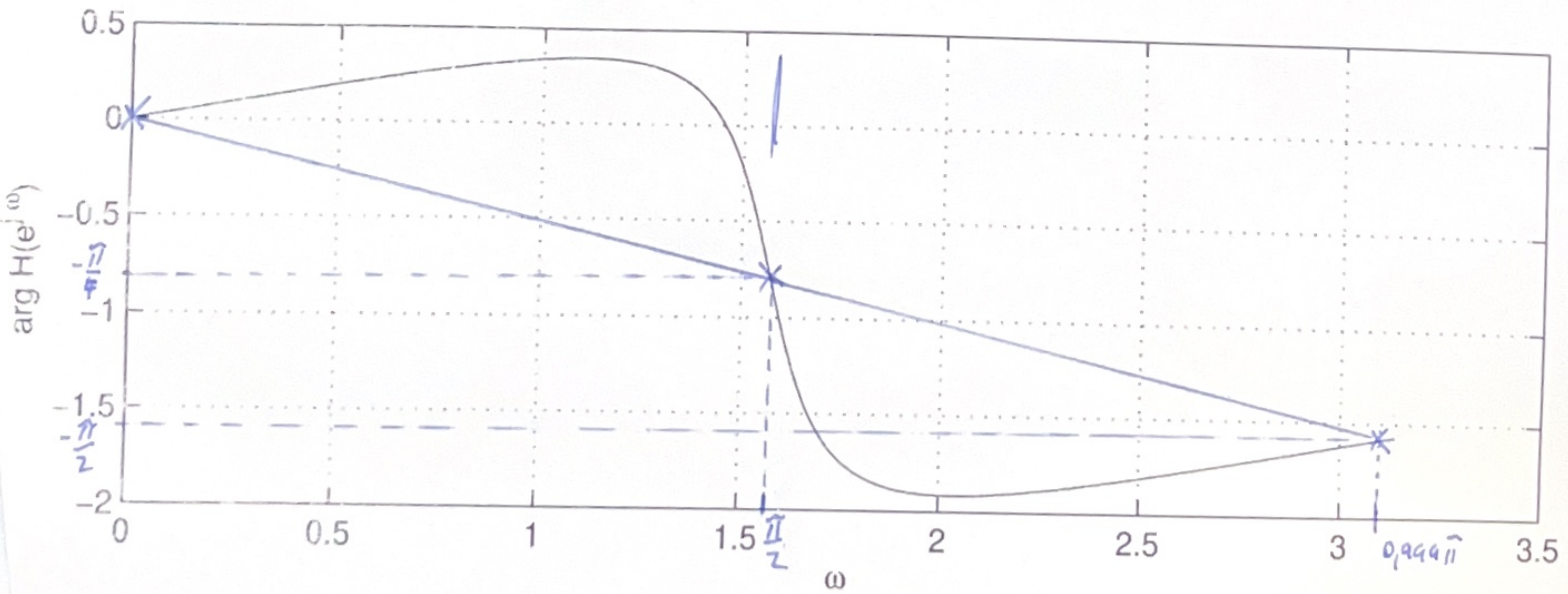




porovnání:

- graf modulu vypadá v základu velmi podobně, samozřejmě můj graf není kvůli malému množství vypočítaných hodnot zakřivený jako výsledek
- filtr realizuje band pass filtr pro spodní a střední frekvence
- graf argumentu se poměrně liší, lepší podobnost získáme vypočítáním více hodnot filtru
- u grafu argumentu třeba připomenout že otočení $-\frac{\pi}{2} = \frac{3}{2}\pi$





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