Dynamic General Equilibrium Model for Climate Resilient Economic Development

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Introduction to Dynare

DGE-CRED Model

Model Simulation and Calibration

What is Dynare?

- dynare is an open-source program for dynamic general equilibrium modeling
- mainly a collection of different functions written for Matlab
- preprocessor translates mod files into matlab code.

Structure of a Mod File

- var block declares endogenous variables
- params block

Declaration of endogenous variables

```
var
k $k$ (long_name = 'capital'),
c $c$ (long_name = 'consumption'),
h $h$ (long_name = 'hours worked'),
epsil $\epsilon$ (long_name = 'tfp shock')
;
```

Declaration of endogenous variables

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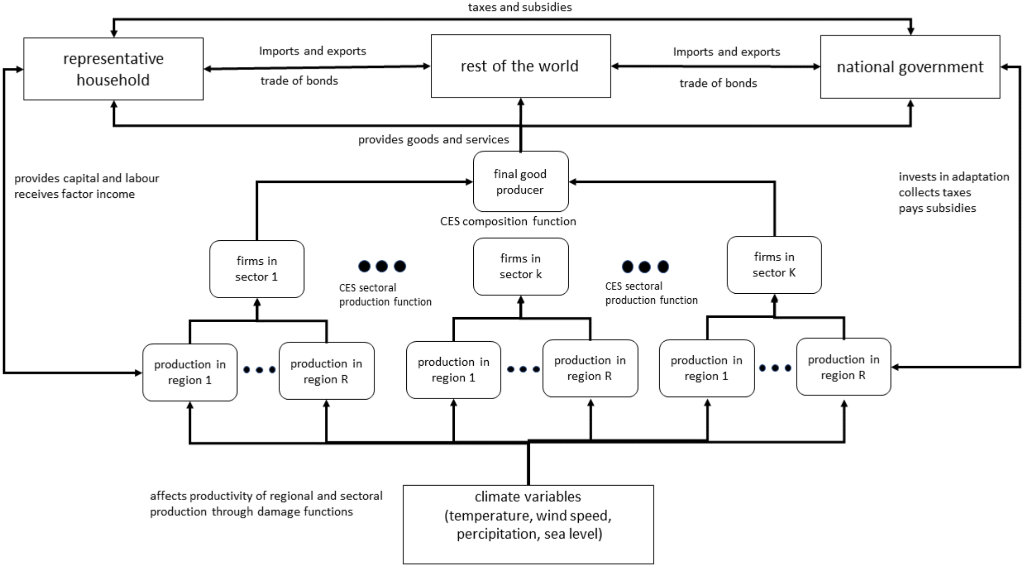
```

Introduction

- A dynamic general equilibrium model with optimizing agents
- We differentiate between regions and economic activities.
- Our model is implemented in the open source environment Dynare and can be run using Matlab or Octave.
  - Sectors in the model correspond to economic activities and the classification by the General Statistical Office (GSO).
  - Regions are based on the statistical regions.
- We extend the approach by Nordhaus 1993 to model the impact of climate change through damage functions.

Model Structure



Households

- representative households h providing labour N and capital K to domestic firms f
- maximize discounted utility over an infinite horizon by choosing consumption $C_t(h)$, capital $K_{k,r,t+1}(h)$, investments $I_{k,r,t}(h)$, labour $N_{k,r,t}(h)$ and foreign net wealth B_{t+1}
- the optimization problem of the representative household is

$$\max_{C_t(h),\, K_{k,r,t+1}(h),\, I_{k,r,t}(h),,\, N_{k,r,t}(h),\, B_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(h)^{1-\sigma^C}}{1-\sigma^C} - \sum_{k=1}^{K} \sum_{r=1}^{R} A^N_{k,r,t}\, \phi^L_{k,r} \frac{N_{k,r,t}(h)^{1+\sigma^L}}{1+\sigma^L} \right)$$

$$\text{s.t.} P_t\, C_t(h)\,(1+\tau^C) + \sum_{k=1}^{K} \sum_{r=1}^{R} P_{k,r,t} I_{k,r,t}(h) + B_{t+1}(h) =$$

$$\sum_{k=1}^{K} \sum_{r=1}^{R} (1-\tau^N)\, W_{k,r,t} N_{k,r,t}(h) + \sum_{k=1}^{K} \sum_{r=1}^{R} P_{k,r,t}\, r_{k,r,t}\,(1-\tau^K)\, K_{k,r,t}(h) + S^f_t\, \phi^B_t\, (1+r^f_t)\, B_t(h)$$

Households Lagrangian

- We set-up the Lagrangian for the optimization problem to derive the first order conditions.

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\beta^{t}\Bigg[\Bigg(\frac{C_t(h)^{1-\sigma^C}}{1-\sigma^C}-\sum_{k=1}^{K}\sum_{r=1}^{R}A^N_{k,r,t}\,\phi^L_{k,r}\,\frac{N_{k,r,t}(h)^{1+\sigma^L}}{1+\sigma^L}\Bigg)\\
&-\lambda_t(h)\Big(P_t\,C_t(h)\,(1+\tau^C)+\sum_{k=1}^{K}\sum_{r=1}^{R}P_{k,r,t}I_{k,r,t}(h)+B_{t+1}(h)-\sum_{k=1}^{K}\sum_{r=1}^{R}(1-\tau^N)\,W_{k,r,t}N_{k,r,t}(h)\\
&-\sum_{k=1}^{K}\sum_{r=1}^{R}P_{k,r,t}\,r_{k,r,t}\,(1-\tau^K)\,K_{k,r,t}(h)-S^f_t\,\phi^B_t\,(1+r^f_t)\,B_t(h)\Big)\\
&-\sum_{k=1}^{K}\sum_{r=1}^{R}\lambda_t(h)\omega^I_{k,r,t}(h)\left\{K_{k,r,t+1}-(1-\delta-D^K_{k,r,t})\,K_{k,r,t}-I_{k,r,t}\,\Gamma\left(\frac{I_{k,r,t}}{I_{k,r,t-1}}\right)\right\}\Bigg].
\end{aligned}
$$

Households First Order Conditions - Intratemporal

- Marginal utility of consumption

$$\lambda_t = \frac{C_t(h)^{-\sigma^C}}{P_t\,(1+\tau^C)}$$

- Labour supply curve

$$\phi^L_{k,r}\,A^N_{k,r,t}\,N_{k,r,t}(h)^{\sigma^L} = \lambda_t(h)\,W_{k,r,t}\,(1-\tau^N)$$

Households First Order Conditions - Intertemporal

- Euler equation for foreign bonds

$$\lambda_{t+1}\,\beta\,S^f_{t+1}\,\phi^B_{t+1}\left(1+{r^f}_{t+1}\right)=\lambda_t$$

- Euler equation for capital

$$\lambda_{t+1}(h)\,\beta\,\left(P_{k,r,t+1}\,r_{k,r,t+1}\,(1-\tau^K)+(1-\delta-D^K_{k,r,t+1})\,\omega^I_{k,r,t+1}\right)=\lambda_t(h)\,\omega^I_{k,r,t}.$$

- Euler equation for investment

$$P_{k,r,t}\,\lambda_t(h)=\lambda_t(h)\,\omega^I_{k,r,t}\left(\Gamma(\frac{I_{k,r,t}}{I_{k,r,t-1}})+\frac{\partial\Gamma(\frac{I_{k,r,t}}{I_{k,r,t-1}})}{\partial(\frac{I_{k,r,t}}{I_{k,r,t-1}})}\,\frac{I_{k,r,t}}{I_{k,r,t-1}}\right)-\beta\lambda_{t+1}(h)\,\omega^I_{k,r,t+1}\,\frac{\partial\Gamma(\frac{I_{k,r,t+1}}{I_{k,r,t}})}{\partial(\frac{I_{k,r,t+1}}{I_{k,r,t}})}\left(\frac{I_{k,r,t+1}}{I_{k,r,t}}\right)^2$$

- Investment adjustment cost

$$\Gamma(\frac{I_{k,r,t}}{I_{k,r,t-1}})=3-exp\left\{\sqrt{\phi^K/2}\left(\frac{I_{k,r,t}}{I_{k,r,t-1}}-1\right\}\right)-exp\left\{-\sqrt{\phi^K/2}\left(\frac{I_{k,r,t}}{I_{k,r,t-1}}-1\right)\right\}$$

Households First Order Conditions - Intertemporal

- Euler equation for capital

$$\lambda_{t+1}(h)\,\beta\,\left(P_{k,r,t+1}\,r_{k,r,t+1}\,(1-\tau^{K})+(1-\delta-D^{K}_{k,r,t+1})\,\omega^{I}_{k,r,t+1}\right)=\lambda_t(h)\,\omega^{I}_{k,r,t}.$$

- Euler equation for investment

$$P_{k,r,t}\,\lambda_t(h)=\lambda_t(h)\,\omega^{I}_{k,r,t}\left(\Gamma(\frac{I_{k,r,t}}{I_{k,r,t-1}})+\frac{\partial\Gamma(\frac{I_{k,r,t}}{I_{k,r,t-1}})}{\partial(\frac{I_{k,r,t}}{I_{k,r,t-1}})}\,\frac{I_{k,r,t}}{I_{k,r,t-1}}\right)-\beta\lambda_{t+1}(h)\,\omega^{I}_{k,r,t+1}\,\frac{\partial\Gamma(\frac{I_{k,r,t+1}}{I_{k,r,t}})}{\partial(\frac{I_{k,r,t+1}}{I_{k,r,t}})}\,\left(\frac{I_{k,r,t+1}}{I_{k,r,t}}\right)^{2}$$

Rest of the world

- Euler equation foreign bonds

$$\lambda_{t+1}\,\beta\,S^f_{t+1}\,\phi^B_{t+1}\left(1+{r^f}_{t+1}\right)=\lambda_t$$

- Effective exchange rate S^f and the world interest rate r^f.
- The required interest rate is above the world interest rate if the foreign debt ($B_{t+1}<0$)/ foreign claims ($B_{t+1}>0$) relative to GDP increases/decreases and future net exports relative to GDP will decrease.

$$\phi^B_{t+1}=exp\left(-\phi^B\,(S^f_{t+1}\,r^f_{t+1}\,\frac{B_{t+1}}{Y_{t+1}}+\frac{NX_{t+1}}{Y_{t+1}})\right)$$

Government Budget Constraint

- We are interested in different policy measures taken by the government to adapt to a new climate regime.
- Government behaviour is not a result of an optimization problem.

$$G_t + \sum_k^K \sum_r^R G^A_{k,r,t} + B^G_{t+1} = \sum_k^K \sum_r^R \left\{ (\tau^K + \tau^K_{r,k,t})\, P_{k,r,t}\, r_{k,r,t}\, K_{k,r,t} + (\tau^N + \tau^N_{k,r,t})\, W_{k,r,t}\, N_{k,r,t}\, Pop_t \right\}$$
$$+ (1 + r^f_t)\, S^f_t \phi^B_t\, B^G_t$$

Government Policy Instruments

- Governments can invest into adaptation capital stocks

$$K^{A,z}_{k,r,t+1} = \eta^{A,z}_{k,r,t}$$

- Evolution of adaptation capital stocks

$$K^{A,z}_{k,r,t+1} = \left(1 - \delta_{K^{A,z},k,r}\right) K^{A,z}_{k,r,t} + G^{A,z}_{k,r,t}$$

- Tax on capital expenditures paid by firms

$$\tau^{K}_{k,r,t} = \tau^{K}_{k,r,0} + \eta^{\tau^{K}}_{k,r,t}$$

- Tax rate on wage bill paid by firms

$$\tau^{N}_{k,r,t} = \tau^{N}_{k,r,0} + \eta^{\tau^{N}}_{k,r,t}$$

Resource constraint

- Households and government use domestic final goods Y_t produced by firms for consumption, investment and for exports X_t and can also use imports M_t for consumption and investment

$$Y_t = C_t + I_t + G_t + \underbrace{X_t - M_t}_{NX_t} \tag{1}$$

- The aggregation of the budget constraints of the representative households also states that positive net exports are used to increase net financial wealth to the rest of the world.

$$NX_t = B_{t+1} - (1 + r_t^f)S_t^f \phi_t^B B_t \tag{2}$$

Sectoral Decomposition

- Final domestic goods Y_t are created combining goods from different sectors $Y_{k,t}$ using a CES production function.

$$\min_{Y_{k,t}} \sum_k Y_{k,t} P_{k,t} \tag{3}$$

$$Y_t = \left(\sum_k {\omega_k^Q}^{\frac{1}{\eta^Q}} Y_{k,t}^{\frac{\eta^Q - 1}{\eta^Q}} \right)^{\frac{\eta^Q}{\eta^Q - 1}} \tag{4}$$

- Therefore, the demand for sectoral products correspond to the first order conditions of the above optimization problem.

$$\frac{P_{k,t}}{P_t} = {\omega_k^Q}^{\frac{1}{\eta^Q}} \left(\frac{Y_{k,t}}{Y_t} \right)^{\frac{-1}{\eta^Q}}$$

Regional Decomposition

- In order to model regional economic activity we further decompose the production process on a regional level.

$$\min_{Y_{k,r,t}} \sum_{k} Y_{k,r,t} \, P_{k,r,t}$$

$$Y_{k,t} = \left(\sum_{k} {\omega^{Q}_{k,r}}^{\frac{1}{\eta^{Q}_{k}}} \, Y_{k,r,t}^{\frac{\eta^{Q}_{k}-1}{\eta^{Q}_{k}}} \right)^{\frac{\eta^{Q}_{k}}{\eta^{Q}_{k}-1}}$$

- Demand for sectoral and regional products correspond to the first order conditions of the above optimization problem.

$$\frac{P_{k,r,t}}{P_{k,t}} = {\omega^{Q}_{k,r}}^{\frac{1}{\eta^{Q}_{k}}} \left(\frac{Y_{k,r,t}}{Y_{k,t}} \right)^{\frac{-1}{\eta^{Q}_{k}}}$$

Regional Production

- At the regional and sectoral level are representative firms maximizing profits using capital $K_{k,r,t}$ and labour $L_{k,r,t} = N_{k,r,t}\, Pop_t$ provided by households to produce products.
- They charge a price $P_{k,r,t}$ for their products and have to pay households wages $W_{k,r,t}$, interest on rented capital $P_{r,k,t}\, r_{r,k,t}$, taxes related to the wage bill $\tau^N_{r,k,t}$ and on capital expenditure $\tau^K_{r,k,t}$.
- Representative firms have access to a regional and sector specific constant elasticity of substitution production function.
- The productivity of capital and labour of a firm in one sector and region depends on the climate variables, and the adaption measures by the government represented by a damage function affecting total factor productivity $A_{k,r,t}$ by $D_{k,r,t} = D_{k,r}\left(T_{r,t},\ PREC_{r,t},\ WS_{r,t},\ SL_{r,t},\ CYC_{r,t},\ DRO_{r,t},\ G^A_{r,k,t}\right)$.
- Further, we explicitly differentiate between climate induced damages affecting labour productivity $D_{N,k,r,t}$ and capital depreciation $D_{K,k,r,t}$.
- As in Nordhaus 1993, we assume a polynomial functional form of the damage functions, but the damages are different across regions and sectors.

Damages on TFP

$$D_{k,r\,t} = \Big\{$$

$$\underbrace{(a_{T,1,k,r}\,T_{r\,t} + a_{T,2,k,r}\,(T_{r\,t})^{a_{T,3,k,r}})}_{\text{impact of temperature}}\;\underbrace{exp(-\phi^{G^{A,T}}_{k,r}\;K^{A,T}_{k,r,t})}_{\text{impact of adaptation}} + \underbrace{(a_{SL,1,k,r}\,SL_t + a_{SL,2,k,r}\,(SL_t)^{a_{SL,3,k,r}})}_{\text{impact of sea level}}\;\underbrace{I(SL > \frac{K^{A,SL}_{k,r,t}}{\phi^{G^{A,SL}}_{k,r}})}_{\text{impact of adaptation}}$$

$$+\underbrace{(a_{WS,1,k,r}\,WS_{r\,t} + a_{WS,2,k,r}\,(WS_{r\,t})^{a_{WS,3,k,r}})}_{\text{impact of wind speed}}\;\underbrace{exp(-\phi^{G^{A,WS}}_{k,r}\;K^{A,WS}_{k,r,t})}_{\text{impact of adaptation}}$$

$$+\underbrace{(a_{PREC,1,k,r}\,PREC_{r\,t} + a_{PREC,2,k,r}\,(PREC_{r\,t})^{a_{PREC,3,k,r}})}_{\text{impact of precipitation}}\;\underbrace{exp(-\phi^{G^{A,PREC}}_{k,r}\;K^{A,PREC}_{k,r,t})}_{\text{impact of adaptation}}$$

$$+\underbrace{(a_{CYC,1,k,r}\,CYC_{r\,t} + a_{CYC,2,k,r}\,(CYC_{r\,t})^{a_{CYC,3,k,r}})}_{\text{impact of cyclones}}\;\underbrace{exp(-\phi^{G^{A,CYC}}_{k,r}\;K^{A,CYC}_{k,r,t})}_{\text{impact of adaptation}}$$

$$+\underbrace{(a_{DRO,1,k,r}\,DRO_{r\,t} + a_{DRO,2,k,r}\,(DRO_{r\,t})^{a_{DRO,3,k,r}})}_{\text{impact of droughts}}\;\underbrace{exp(-\phi^{G^{A,DRO}}_{k,r}\;K^{A,DRO}_{k,r,t})}_{\text{impact of adaptation}}$$

$$\Big\}.$$

References I

Nordhaus, William D (1993). “Optimal greenhouse-gas reductions and tax policy in the “DICE” model”. In: American Economic Review 83.2, pp. 313–317.