

Dynamic General Equilibrium Model for Climate Resilient Economic Development

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On behalf of:



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for the Environment, Nature Conservation
and Nuclear Safety

of the Federal Republic of Germany

Introduction to Dynare

DGE-CRED Model

Model Simulation and Calibration

What is Dynare?

- dynare is an open-source program for dynamic general equilibrium modeling
- mainly a collection of different functions written for Matlab
- preprocessor translates mod files into matlab code.

Structure of a Mod File

- var block declares endogenous variables
- params block

Declaration of endogenous variables

```
var  
k $k$ (long_name = 'capital') ,  
c $c$ (long_name = 'consumption') ,  
h $h$ (long_name = 'hours worked') ,  
epsil $\epsilon$ (long_name = 'tfp shock')  
;
```

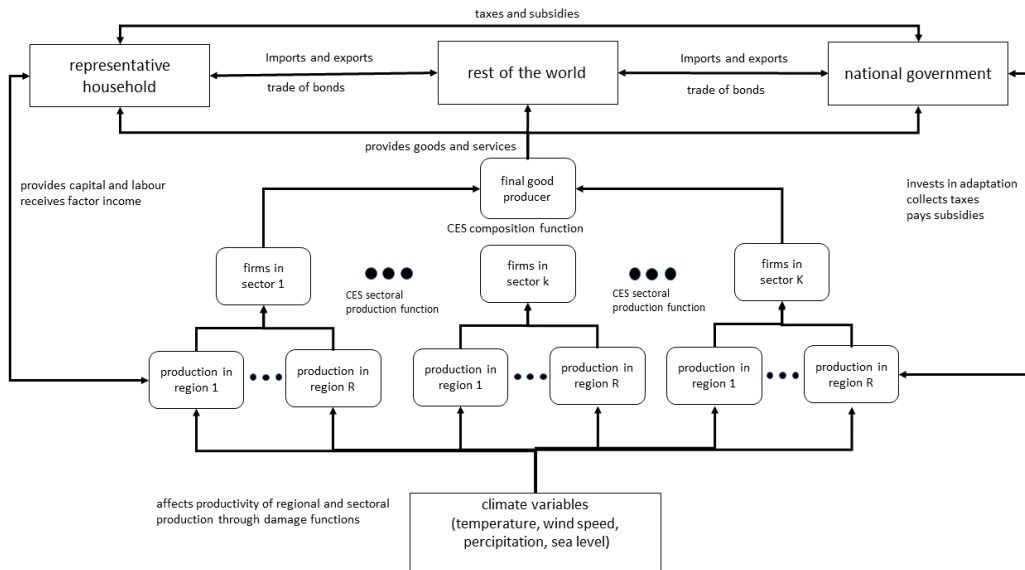
Declaration of endogenous variables

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Introduction

- A dynamic general equilibrium model with optimizing agents
- We differentiate between regions and economic activities.
- Our model is implemented in the open source environment Dynare and can be run using Matlab or Octave.
 - ▶ Sectors in the model correspond to economic activities and the classification by the General Statistical Office (GSO).
 - ▶ Regions are based on the statistical regions.
- We extend the approach by Nordhaus 1993 to model the impact of climate change through damage functions.

Model Structure



Households

- representative households h providing labour N and capital K to domestic firms f
- maximize discounted utility over an infinite horizon by choosing consumption $C_t(h)$, capital $K_{k,r,t+1}(h)$, investments $I_{k,r,t}(h)$, labour $N_{k,r,t}(h)$ and foreign net wealth B_{t+1}
- the optimization problem of the representative household is

$$\begin{aligned}
 & \max_{C_t(h), K_{k,r,t+1}(h), I_{k,r,t}(h), N_{k,r,t}(h), B_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(h)^{1-\sigma^C}}{1-\sigma^C} - \sum_{k=1}^K \sum_{r=1}^R A_{k,r,t}^N \phi_{k,r}^L \frac{N_{k,r,t}(h)^{1+\sigma^L}}{1+\sigma^L} \right) \\
 & \text{s.t. } P_t C_t(h) (1 + \tau^C) + \sum_{k=1}^K \sum_{r=1}^R P_{k,r,t} I_{k,r,t}(h) + B_{t+1}(h) = \\
 & \sum_{k=1}^K \sum_{r=1}^R (1 - \tau^N) W_{k,r,t} N_{k,r,t}(h) + \sum_{k=1}^K \sum_{r=1}^R P_{k,r,t} r_{k,r,t} (1 - \tau^K) K_{k,r,t}(h) + S_t^f \phi_t^B (1 + r_t^f) B_t(h)
 \end{aligned}$$

Households Lagrangian

- We set-up the Lagrangian for the optimization problem to derive the first order conditions.

$$\begin{aligned}
 & \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{C_t(h)^{1-\sigma^C}}{1-\sigma^C} - \sum_{k=1}^K \sum_{r=1}^R A_{k,r,t}^N \phi_{k,r}^L \frac{N_{k,r,t}(h)^{1+\sigma^L}}{1+\sigma^L} \right) \right. \\
 & - \lambda_t(h) \left(P_t C_t(h) (1 + \tau^C) + \sum_{k=1}^K \sum_{r=1}^R P_{k,r,t} l_{k,r,t}(h) + B_{t+1}(h) - \sum_{k=1}^K \sum_{r=1}^R (1 - \tau^N) W_{k,r,t} N_{k,r,t}(h) \right. \\
 & - \sum_{k=1}^K \sum_{r=1}^R P_{k,r,t} r_{k,r,t} (1 - \tau^K) K_{k,r,t}(h) - S_t^f \phi_t^B (1 + r_t^f) B_t(h) \Big) \\
 & \left. - \sum_{k=1}^K \sum_{r=1}^R \lambda_t(h) \omega_{k,r,t}^l(h) \left\{ K_{k,r,t+1} - (1 - \delta - D_{k,r,t}^K) K_{k,r,t} - l_{k,r,t} \Gamma \left(\frac{l_{k,r,t}}{l_{k,r,t-1}} \right) \right\} \right].
 \end{aligned}$$

Households First Order Conditions - Intratemporal

- Marginal utility of consumption

$$\lambda_t = \frac{C_t(h)^{-\sigma^C}}{P_t(1 + \tau^C)}$$

- Labour supply curve

$$\phi_{k,r}^L A_{k,r,t}^N N_{k,r,t}(h)^{\sigma^L} = \lambda_t(h) W_{k,r,t} (1 - \tau^N)$$

Households First Order Conditions - Intertemporal

- Euler equation for foreign bonds

$$\lambda_{t+1} \beta S_{t+1}^f \phi_{t+1}^B (1 + r_{t+1}^f) = \lambda_t$$

- Euler equation for capital

$$\lambda_{t+1}(h) \beta \left(P_{k,r,t+1} r_{k,r,t+1} (1 - \tau^K) + (1 - \delta - D_{k,r,t+1}^K) \omega_{k,r,t+1}^l \right) = \lambda_t(h) \omega_{k,r,t}^l.$$

- Euler equation for investment

$$P_{k,r,t} \lambda_t(h) = \lambda_t(h) \omega_{k,r,t}^l \left(\Gamma\left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right) + \frac{\partial \Gamma\left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right)}{\partial \left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right)} \frac{l_{k,r,t}}{l_{k,r,t-1}} \right) - \beta \lambda_{t+1}(h) \omega_{k,r,t+1}^l \frac{\partial \Gamma\left(\frac{l_{k,r,t+1}}{l_{k,r,t}}\right)}{\partial \left(\frac{l_{k,r,t+1}}{l_{k,r,t}}\right)} \left(\frac{l_{k,r,t+1}}{l_{k,r,t}} \right)^2$$

- Investment adjustment cost

$$\Gamma\left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right) = 3 - \exp \left\{ \sqrt{\phi^K/2} \left(\frac{l_{k,r,t}}{l_{k,r,t-1}} - 1 \right) \right\} - \exp \left\{ -\sqrt{\phi^K/2} \left(\frac{l_{k,r,t}}{l_{k,r,t-1}} - 1 \right) \right\}$$

Households First Order Conditions - Intertemporal

■ Euler equation for capital

$$\lambda_{t+1}(h) \beta \left(P_{k,r,t+1} r_{k,r,t+1} (1 - \tau^K) + (1 - \delta - D_{k,r,t+1}^K) \omega_{k,r,t+1}^l \right) = \lambda_t(h) \omega_{k,r,t}^l.$$

■ Euler equation for investment

$$P_{k,r,t} \lambda_t(h) = \lambda_t(h) \omega_{k,r,t}^l \left(\Gamma\left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right) + \frac{\partial \Gamma\left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right)}{\partial \left(\frac{l_{k,r,t}}{l_{k,r,t-1}}\right)} \frac{l_{k,r,t}}{l_{k,r,t-1}} \right) - \beta \lambda_{t+1}(h) \omega_{k,r,t+1}^l \frac{\partial \Gamma\left(\frac{l_{k,r,t+1}}{l_{k,r,t}}\right)}{\partial \left(\frac{l_{k,r,t+1}}{l_{k,r,t}}\right)} \left(\frac{l_{k,r,t+1}}{l_{k,r,t}} \right)^2$$

Rest of the world

- Euler equation foreign bonds

$$\lambda_{t+1} \beta S_{t+1}^f \phi_{t+1}^B (1 + r_{t+1}^f) = \lambda_t$$

- Effective exchange rate S^f and the world interest rate r^f .
- The required interest rate is above the world interest rate if the foreign debt ($B_{t+1} < 0$) / foreign claims ($B_{t+1} > 0$) relative to GDP increases/decreases and future net exports relative to GDP will decrease.

$$\phi_{t+1}^B = \exp \left(-\phi^B (S_{t+1}^f r_{t+1}^f \frac{B_{t+1}}{Y_{t+1}} + \frac{NX_{t+1}}{Y_{t+1}}) \right)$$

Government Budget Constraint

- We are interested in different policy measures taken by the government to adapt to a new climate regime.
- Government behaviour is not a result of an optimization problem.

$$G_t + \sum_k^K \sum_r^R G_{k,r,t}^A + B_{t+1}^G = \sum_k^K \sum_r^R \left\{ (\tau^K + \tau_{r,k,t}^K) P_{k,r,t} r_{k,r,t} K_{k,r,t} + (\tau^N + \tau_{k,r,t}^N) W_{k,r,t} N_{k,r,t} Pop_t \right\} \\ + (1 + r_t^f) S_t^f \phi_t^B B_t^G$$

Government Policy Instruments

- Governments can invest into adaptation capital stocks

$$K_{k,r,t+1}^{A,z} = \eta_{k,r,t}^{A,z}$$

- Evolution of adaptation capital stocks

$$K_{k,r,t+1}^{A,z} = (1 - \delta_{K^{A,z},k,r}) K_{k,r,t}^{A,z} + G_{k,r,t}^{A,z}$$

- Tax on capital expenditures paid by firms

$$\tau_{k,r,t}^K = \tau_{k,r,0}^K + \eta_{k,r,t}^{\tau^K}$$

- Tax rate on wage bill paid by firms

$$\tau_{k,r,t}^N = \tau_{k,r,0}^N + \eta_{k,r,t}^{\tau^N}$$

Resource constraint

- Households and government use domestic final goods Y_t produced by firms for consumption, investment and for exports X_t and can also use imports M_t for consumption and investment

$$Y_t = C_t + I_t + G_t + \underbrace{X_t - M_t}_{NX_t} \quad (1)$$

- The aggregation of the budget constraints of the representative households also states that positive net exports are used to increase net financial wealth to the rest of the world.

$$NX_t = B_{t+1} - (1 + r_t^f) S_t^f \phi_t^B B_t \quad (2)$$

References I



Nordhaus, William D (1993). “Optimal greenhouse-gas reductions and tax policy in the “DICE” model”. In: [American Economic Review](#) 83.2, pp. 313–317.