International Review of Economics Education 23 (2016) 65–81

Contents lists available at ScienceDirect

International Review of Economics Education

journal homepage: www.elsevier.com/locate/iree

A simple climate-Solow model for introducing the economics of climate change to undergraduate students

Panagiotis Tsigaris, Joel Wood*

Thompson Rivers University, 900 McGill Road, Kamloops, BC V2C 0C8, Canada

ARTICLE INFO

Article history:
Received 18 February 2016
Received in revised form 24 May 2016
Accepted 14 June 2016
Available online 1 July 2016

JEL classification:
A22
O44
Q54

Keywords:
Integrated assessment models
Climate change
Solow growth model
Teaching economics

ABSTRACT

In this paper the simplest integrated assessment model is developed in order to illustrate to undergraduate students the economic issues associated with climate change. The growth model developed in this paper is an extension of the basic Solow model and includes a simple climate model. Even though the model is very simple it is very powerful in its predictions. Students use the model to explore various scenarios illustrating how economic activity today will inflict damages from higher temperatures on future generations. But students also observe that future generations will be richer than today's generation due to productivity growth and population stabilization. Hence, the richer future generations will not be as rich as they would be without climate change. Since the cost of action is absorbed by the current generation and the benefits of action accrue to future generations students can conduct a cost-benefit analysis and explore the importance of the discount rate. The appendix provides step-by-step instructions for students to setup the model in MS Excel and to conduct simulations.

1. Introduction

> "Greenhouse gas (GHG) emissions are externalities and represent the biggest market failure the world has seen"—Sir Nicholas Stern (2007)

Climate change caused by the Greenhouse Gas (GHG) emissions released by the burning of fossil fuels and land use changes imposes damages to future generations.[1] GHG emissions trap heat and affect the future climate resulting in damages from increased temperatures. For example, increased temperatures are expected to cause sea level rise, increased floods, increased droughts and heat waves, and possibly even increased human conflict. The current generation benefits from using fossil fuels, but does not internalize these external costs. As a result, climate change is what economists call a negative externality. Covert et al. (2016) examine historical data on fossil fuel production and consumption and conclude that neither supply (e.g., Peak Oil) nor demand (e.g., development of low-carbon technologies) factors will sufficiently reduce GHG emissions. Without government intervention, humans will overproduce GHG emissions.

The climate change problem is further complicated as being a global externality rather than a local one. Even though each nation emits a different amount of greenhouse gases (GHG), the marginal impact of a tonne of GHG is independent of where it is emitted (Stern, 2007); whereas, the effects of smog in a city are local and heterogeneous depending on the geography

* Corresponding author.
E-mail addresses: ptsigaris@tru.ca (P. Tsigaris), jwood@tru.ca (J. Wood).

[1] For scientific consensus on the issue see Oreskes (2004).

http://dx.doi.org/10.1016/j.iree.2016.06.002
1477-3880/

and demographics of the city. Furthermore, GHGs accumulate in the atmosphere and stay a long time, i.e., carbon dioxide has an average atmospheric life of over a century (Archer et al., 2009). The impact is persistent and long term, whereas the effects of smog in a city are relatively immediate following exposure.

Due to the persistence of GHGs in the atmosphere, the climate change problem is characterized by the issue of inter-generational equity: The current generation is imposing external costs on future generations and would have to forego some economic growth to limit those costs. How the costs of action of the current generation varies relative to the benefits, in terms of reduced damages, to future generations depends heavily on the discount rate used. The discount rate in turn depends on the social rate of time preference, risk aversion and, per capita economic growth. Discounting at normal discount rates does not put too much value on what happens 100 or 200 years from now; however, a very low discount rate, such as that used in the Stern Review (Stern, 2007), places much more weight on future damages. Discounting plays a significant role as to whether it is optimal, from an inter-generational perspective, to undertake strong emission reduction action immediately or to start reducing emissions more slowly and to follow an increasingly stringent climate policy (a ramp up climate policy).[2]

In addition to the issue of inter-generational equity, climate change is also characterized by issues of intra-generational equity. For example, rich nations which are relatively GHG intensive are located in temperate climates and have the funds and strong institutions to more easily adapt to climate change; whereas, poorer nations, say in sub-Saharan Africa, are expected to be hit relatively harder by the negative impacts of higher temperatures.[3]

Complicating the problem is the fact that uncertainty and risk are significant. Damages from climate change could be potentially large and irreversible (Weitzman, 2009, 2011). Furthermore, the continuous disposal of carbon into the atmosphere, oceans, and land could thus result in the tragedy of the commons (Broome, 2012). Finally, reducing GHG emissions can be characterized as a public good in that the benefits of mitigation are non-rival and non-exclusionary resulting in a free-rider problem and the under-provision of mitigation policy.[4] This free-rider problem can provide insights into the failure of the Kyoto Protocol and subsequent annual meetings. It is no wonder that Sir Nicholas Stern considers this issue the biggest market failure the world has ever seen.

One of the most common approaches to evaluate the impact of climate change is to use an Integrated Assessment Model (IAM). These models integrate a model of the world economy with a representation of the global climate system. The models assess different scenarios from these complex systems and are used by governments when evaluating the impact of climate polices (e.g., estimating the Social Cost of Carbon) and informing the general public (Schwanitz, 2013).[5]

In spite of these significant issues and all the research being undertaken to study the economics of climate change, not much has been formally done to introduce IAMs to undergraduate students. Tol (2014) is a notable exception, as a text on climate economics suitable for a full course in climate economics with a specific focus on the IAM at the masters' or advanced undergraduate levels. Yet there is little available to introduce undergraduate students to IAMs for a portion of a climate economics course or for courses in macroeconomic growth theory, development economics or environmental economics. The existing IAMs are overly complex for teaching the economics of climate change to undergraduate students. For example, the Dynamic Integrated Climate Economics (DICE) model is based on the Ramsey growth model that many economics students do not encounter until graduate school.[6] Our approach adjusts the simple Solow growth model that undergraduate economics students are familiar with.[7] Furthermore, the existing IAMs include a complex representation of the climate system that takes a significant amount of time to explain to undergraduate students. Our model replaces the complex climate system with a simple linear relationship between atmospheric carbon accumulation and expected temperature change demonstrated by Matthews et al. (2009, 2012). This paper is aimed at making the simple IAM model available to instructors and undergraduate students in order to explore the economics of climate change. The model is available in two possible formats both accompanying this article: an MS Excel workbook or an R code version. Throughout the paper figures and key points are provided for instructors to highlight to students and to use as starting points to motivate in-class discussion.

The step-by-step instructions to replicate the simple IAM (included in an accompanying appendix) and the exercises provided throughout the paper allow students to learn the economic issues surrounding climate change in a hands-on way. Learning-by-doing, rather than watching only the instructor's lectures, is a more effective way to absorb and understand the material (Findley, 2014; Dalton et al., 2015). After being exposed to the topic by the instructor, students learn more when they can use the simple IAM to make and graph the projections and to explain the results. Visually seeing the pattern students created themselves is a powerful teaching tool (Watts and Becker, 2008). Psychological studies show that visuals improve learning outcomes and learning-by-doing increases knowledge retention and also becomes a more enjoyable experience to

[2] This issue will be explored in more detail in Section 4.

[3] For a critical review of inter-generational and intra-generational climate justice see Forsyth (2014).

[4] Recently, Nordhaus (2015) has proposed the formation of climate clubs to solve the free rider problem.

[5] Because of the large amount of uncertainty with respect to climate change and climate damages Pindyck (2013, 2015) concludes that IAMs are not very useful for guiding policy;

[6] The closest economic models to the one we have constructed are Nordhaus' DICE model, Brock and Taylor (2010), and Taylor (2014). None of these three closely related works are aimed at educating undergraduate students about the economics of climate change.

[7] In case students are not exposed to the Solow model, more time can be spent explaining the basics of the Solow model and the concept of steady state levels.

students (Vazquez and Chiang, 2014). This paper (and accompanying appendix) guides instructors and students to create visuals of future trajectories of the standard of living of the world economy with and without climate change under different scenarios.

Section 2 describes the basic climate-Solow model for the world economy. Section 3 alters the model to examine damages which are more severe. This section provides direction for instructors to use the model to illustrate the impact of climate change when damages are more severe at higher temperatures, and when temperature increases affect the depreciation of capital and productivity growth. Section 4 uses the model developed in Section 2 to illustrate the costs and benefits of emission reductions by conducting a simple Benefit-Cost Analysis for the 2° target. Finally, concluding remarks and other possible classroom extensions are mentioned. The appendix provides step-by-step instructions for students to create and run the base case version of the simple IAM outlined in the paper following an approach similar to Elmslie and Tebaldi (2010). Students can construct, on their own, the income per capita trajectories with and without damages, the Environmental Kuznets curve, and the time paths of other variables over 200 years. Exercises are provided throughout the main text.

2. The simple climate-Solow model

2.1. Economic growth & climate impacts

The economic growth component of the model is a variation on the standard Solow Growth model. In the standard undergraduate treatment of the Solow model, output is produced by the combination of capital, K_t labor, L_t and technology, A_t according to the Cobb-Douglas production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, which can be rearranged in terms of output per worker as

$$y_t = A_t k_t^{\alpha}.$$

This is the standard Solow Growth model that students should be already familiar with. For the purposes of studying climate change, the effect of increased temperatures is added to the model in a similar way as by Nordhaus (2008) and Fankhauser and Tol (2005). This is a standard assumption in most IAMS. The production function in the model is slightly altered to be the following

$$y_t = D_t A_t k_t^{\alpha},$$

where $D_t = 1/\left(1 + \theta_1 T_t^{\theta_2}\right) \leq 1$ is the damage function and T_t is the temperature anomaly in year t. The production function looks the same as the standard Cobb-Douglas production function, except output per worker is now reduced by increased temperatures, i.e., the higher is T_t, the lower is y_t *ceteris paribus*.

The savings rate, s is constant, leading to investment per worker in period t of sy_t. Capital depreciates at a constant rate, δ_K. To reflect recent UN population projections that predict global population will plateau around 10.5 billion, total population and the labor force grow at a decreasing rate over time, $g_{L,t} = g_{L,0}/(1+\delta_L)^t$ determined by the parameter $\delta_L > 0$ which reduces the degree of population growth over time. The term $g_{L,0}$ is the population growth rate in the base year of 2010. Total factor productivity, A_t also grows at a decreasing rate over time: $g_{A,t} = g_{A,0}/(1+\delta_A)^t$.[8] This leads to the following difference equation to describe the transitional dynamics in the model:

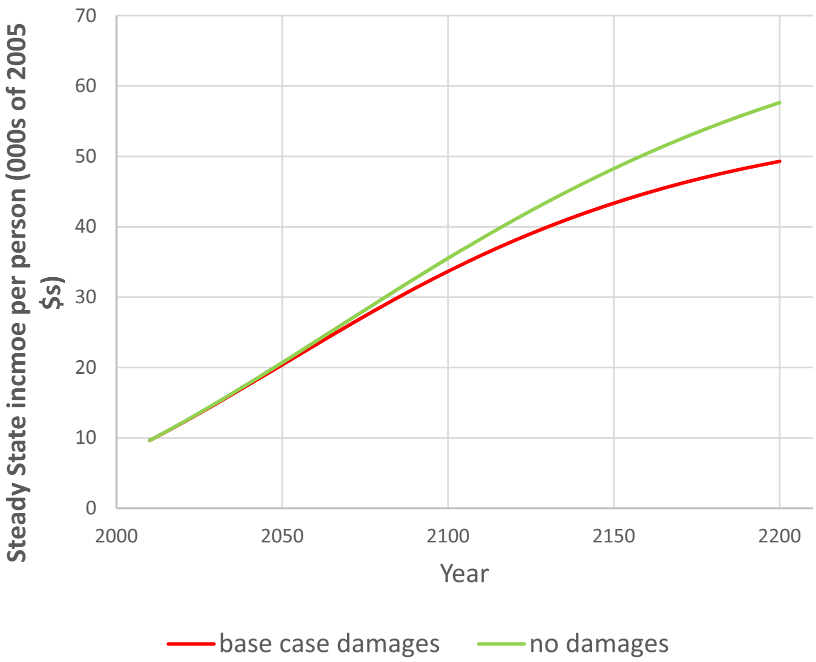
$$k_{t+1} - k_t = sy_t - (\delta_K + g_{L,t})k_t.$$

Given this equation it is easy to show convergence to a balanced growth stable steady state capital labour ratio $k_{ss,t} = \left[\frac{sA_tD_t}{\delta_K + g_{n,t}}\right]^{1/(1-\alpha)}$ for a given time period t.[9] Due to population growth declining and technology advancing, the balanced growth steady state capital labor ratio will increase over time (offset by damages). Along the balanced growth path, output per worker, $y_{ss,t} = D_t A_t k_{ss,t}^{\alpha}$ grows at a rate dependent on changes in temperature (outlined in Section 2.3), the growth rate of total factor productivity, $g_{A,t}$ (which grows at a declining rate) and the growth rate of the capital labour ratio which is weighted by the income share of capital, α. It can be easily seen that in the absence of climate damages (i.e., $D_t = 1$), y_t grows at a faster rate.

To identify the impact of Business-As-Usual (BAU) in the model, a simple comparison of $D_t = 1$ for all t (i.e., no climate damages) to $D_t < 1$ (i.e., with climate damages) is required. This comparison is shown in Fig. 1 for the parameter values displayed in the appendix. The figure is very useful to highlight to students the central trade-off involved in the climate change problem. There are two important aspects of this figure to highlight. First, that the model, consistent with other IAMs,

[8] The assumption of a declining growth rates of total factor productivity and population growth as shown above are also used in Nordhaus and Sztorc (2013). Most undergraduate students will be familiar with the Solow model with constant rates of population and technology growth; therefore, the diminishing growth rates used here may appear more complicated at first glance to the students. However, this change has little effect on how an instructor would traditionally introduce the dynamics of the Solow model.

[9] Simulations can also conducted using transitional dynamics but this is a possible extension. The differences between the two paths is not significant and this path will converge to the same unique steady state values when technology is constant and population growth is constant.



Source: Authors' calculations.

Fig. 1. The Solow Model with and without Climate Impacts.

predicts that future generations are better off despite climate damages. Second, that the climate change problem is intergenerational in nature; the damages of climate change, as represented by the wedge between the two lines, are imposed mainly on future generations.[10] Combined, these two aspects highlight that the climate change problem can be encapsulated by the following trade-off: A relatively poorer current generation is imposing damages (costs) on relatively richer future generations. This is of course only true in the base case of the model, and altering either the damage function or where damages enter the model can lead to future generations being made worse off; which is a useful exercise for instructors to do for their class using our provided Excel workbook or R code.

Classroom discussion: The base model predicts that future generations will be worse off because of climate change but that they will still be richer than the current generation. What are the implications for the climate policy decisions being made by politicians in the current generation?

2.2. Carbon emissions

Carbon emissions, E_t are generated in the model by the production process based on a variable, σ_t that specifies how emissions intensive (i.e., how dirty or clean) the production technology is at time t. The emissions intensity variable defines how much emissions are released per unit of output. Carbon emissions in year t are calculated by multiplying the emissions intensity in year t by the output in year t

$$E_t = \sigma_t Y_t.$$

[10] Damages by 2100 are 5.5% (as a % of the income per person without climate change) and increase to 17% by 2200. These damages are within the range found in the literature (See Tol, 2015).

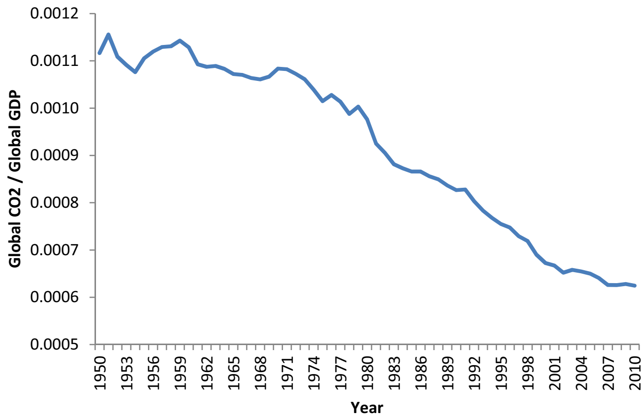


Fig. 2. Global Emissions Intensity, 1950–2010.
Source: CDIAC, 2015; Maddison Project, 2013; authors' calculations.

where E_t is tonnes of carbon released and Y_t is total output. For modelling purposes, emissions intensity is computed by assuming a level in the base year and then specifying the growth rate of emissions intensity over time into the future. Fig. 2 shows that global emissions intensity has steadily declined between 1950 and 2010. This decline has occurred for many reasons. Sectors that have been growing most rapidly, like information technology or health care, are generally less energy intensive than the sectors that are growing more slowly or stagnating. Also, the advance of technology improves the efficiency of production, so that it now takes less energy to produce the same product. There has also been a general shift in the composition of the sources of energy away from coal and towards natural gas, nuclear, hydroelectricity, and others. Future declines in emissions intensity take the following relationship

$$g_{\sigma,t} = g_{\sigma,t-1}/(1+\delta_\sigma),$$

where $g_{\sigma,t} < 0$ is the growth rate of emissions intensity between periods t and t-1 and $\delta_\sigma < 0$. The value of emissions intensity in year t can then be calculated as[11]

$$\sigma_t = \sigma_{t-1}\left(1+g_{\sigma,t}\right).$$

This formula can also be expressed in terms of the base year

$$\sigma_t = \sigma_0 \prod_{i=1}^{n=t}\left[1+g_{a,0}/(1+\delta_\sigma)^i\right].$$

This information is provided for the benefit of instructors and can be given to especially interested students; however, the important thing to highlight to students is that emissions intensity of output is assumed to decline at an increasing rate into the future (consistent with past history)

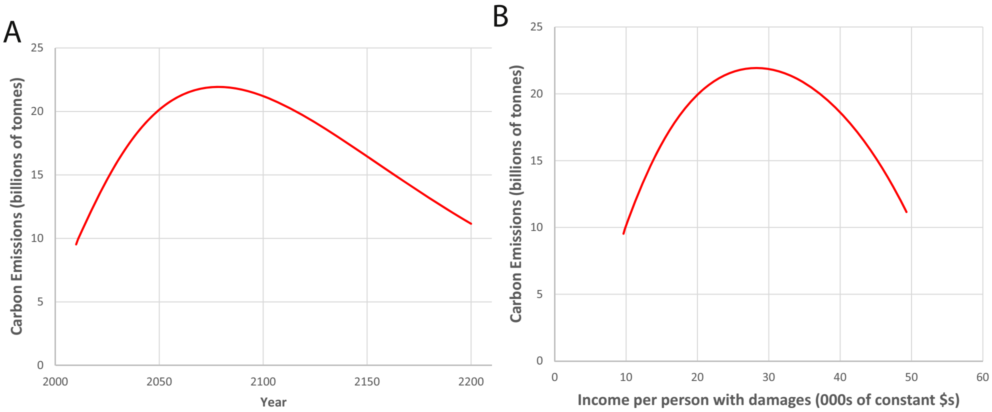
The carbon emissions predicted by the model follow an inverse-u shape consistent with the Environmental Kuznets' Curve hypothesis and are displayed in Fig. 3A and B. As income per capita increases emissions initially increase, peak in the later part of this century when income per capita reaches approximately twenty eight thousand dollars and then emissions start declining. Along a steady state, emissions initially grow because output grows faster than the rate at which intensity falls but after a certain period the latter becomes stronger than the former causing emissions to fall. This can be seen as follows (See also for a similar expression):

$$g_{E,t} = g_{\sigma,t} + g_{Y,t}.$$

This relationship is important as it indicates to students how difficult it is to reduce emissions in an economy that is growing along a steady state due to population growth, total factor productivity growth and capital per worker growth.[12]

This relationship can also be connected to the IPAT equation when expressed in growth rates. The IPAT equation is used by the IPCC for setting future emission targets. It links environmental impact (I) to population (P_t), affluence ($\frac{Y_t}{P_t}$) and technology ($\frac{E_t}{Y_t}$). In our experience, students find the IPAT equation easy to understand even though it is an identity.[13] The IPAT equation

[11] Similar assumptions about emissions intensity were made by Nordhaus and Sztorc (2013). For details see http://www.econ.yale.edu/~nordhaus/homepage/documentS/DICE_Manual_103113r2.pdf.
[12] This is offset partially by the growth rate of the damage that occurs with increasing temperature.
[13] Students can download yearly data from Gapminder.org to explore this relationship for individual countries.



Source: Authors' calculations.

Fig. 3. (A) Predicted Global Carbon Emissions, 2010–2200. (B) Environmental Kuznet's Curve.

for carbon emissions is usually expressed as follows:

$$E_t \equiv P_t \frac{Y_t E_t}{P_t Y_t}.$$

Carbon dioxide emissions at time t (i.e., E_t) are proportional to population multiplied by affluence as measured by output per capita at time t and technology as measured by carbon emissions per dollar of output (recall that in the model $E_t/Y_t = \sigma_t$). In growth rates, after cancelling out the growth of population, this identity becomes:

$$g_{E,t} \equiv g_{\sigma,t} + g_{Y,t}$$

which is identical to the growth rate of emissions from the model. The difference now is that the Solow model provides a theory that explains why output grows. Emissions grow because affluence grows along a steady state that in the Solow model is due to population growth, growth in total factor productivity and growth of capital per worker offset by the impact on growth from damages growing over time. The emissions growth rate is also affected by the emissions intensity falling over time (i.e., $g_{\sigma,t} < 0$). Hence the IPAT equation in growth rates arises from the long run properties of the Solow model and can explain why the model produces an inverse u-shaped emissions path over time (as displayed in Fig. 3A). At first, $-g_{\sigma,t} < g_{Y,t}$ but over time the growth rate of output slows down (due to the assumed diminishing TFP and population growth) and eventually $-g_{\sigma,t} > g_{Y,t}$ producing negative emissions growth (i.e., $g_{E,t} < 0$).

Classroom exercise and discussion: Use the IPAT equation

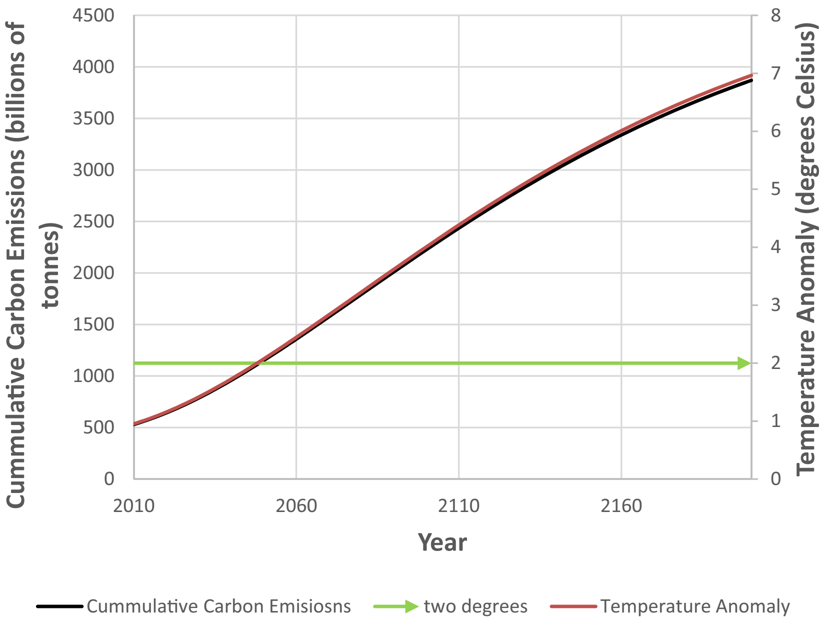
$$E_t \equiv P_t \frac{Y_t}{P_t} \frac{E_t}{Y_t}$$

to find what it takes in terms of technology to reduce emissions in 2050 by 50% below 2010 levels with an assumed population growth of 1.5 percent and growth of affluence as measured by income per person by 2.5% per year.

2.3. Carbon accumulation & temperature change

One of the aspects that make this model so useful for teaching is the simplicity of how the climate system is modelled.[14] The simple proportional stable linear relationship between carbon accumulation and global temperature change found by Matthews et al. (2012) is used in the model. They found that temperature increases by approximately 1.8 Celsius per 1000

[14] It is important to give students a basic understanding of the science of climate change before exposing them to the modelling of temperature anomaly. Basics understanding of climate change can be found at the U.S. EPA http://www.epa.gov/climatechange/basics/ or showing students the IPCC AR5 short video on the physical science basis at https://www.youtube.com/watch?v=6yiTZm0y1YA. For students that want to go beyond the basics on the science of climate change, Professor Archer's video lectures are recommended: http://forecast.uchicago.edu/lectures.html.



Source: authors' calculations.

Fig. 4. Predicted Cumulative Carbon Emissions and temperature anomaly.

billion tons of carbon (i.e., 1000 PgC) emitted with a 95 percent confidence band between 1 and 2.5 °C. This relationship is found to be independent of both time and the level of stabilization of atmospheric carbon concentration (i.e., the emissions scenario). Using this scientifically based relationship avoids modelling much of the complexity of the climate system done by other IAM models.[15] The following relationship shows the cumulative emissions from pre-industrial levels to 2010. The cumulative emissions from the pre-industrial levels to 2010 (the base year for the simulations) are labelled as C_0, i.e., these are the sum of past emission releases. The global temperature change relationship to carbon accumulation into the future is:

$$T_t = \beta\left[C_0 + \sum_{i=1}^{t} E_i\right],$$

where $t \geq 1$. The first term, βC_0, is the impact on global temperature change relative to pre-industrial levels due to the accumulated carbon emissions that were released prior to 2010 (i.e., there are 530 billion tons already accumulated). The second term, $\beta\sum_{i=1}^{t} E_i$, is the impact on global temperature at any time t in the future due to the additional emissions accumulated since 2010. Because of a growing economy, as shown in the previous section, emissions will continue to accumulate resulting in a higher temperature change.

Note that the above relationship is independent of the emissions pathway selected. What matters in terms of temperature change anomaly is the cumulative carbon emissions and the targeted budget. For example, to keep global temperature anomaly below 2 °C relative to pre-industrial levels then cumulative emissions should not increase more than approximately 1110 billion tons (i.e., the budget). If they increase by 470 billion tons over the next 50 years which is within the current BAU pathway (See Fig. 3A and B) they will reach 1000 billion tons. This will result in a temperature increase of 1.8 °C relative to pre-industrial level given that Matthew et al. found β to be 0.0018 per 1 billion tons of cumulative carbon emitted. Fig. 4 shows the path of cumulative carbon emissions starting from 530 billion tons. Fig. 4 also shows the corresponding temperature increases as well as the 2 °C target. With business as usual, 2 °C will be reached just before 2050 and surpass 2000 billion tons by 2100 leading to a temperature increase of 4 °C which is considered dangerous climate change.[16]

[15] This complexity arises because there is uncertainty associated with the path of carbon emissions towards affecting the atmospheric concentration level, through carbon sensitivity. Also there is uncertainty as to the impact of the concentration level of carbon to temperature anomaly change via the climate sensitivity parameter.

[16] There is an estimated 6000 PgC that can be accumulated given the fossil fuels available. Recently, the relationship has been found to be stable within 5000 PgC (Tokarska et al., 2016).

Classroom exercise and discussion: Ask students to find different paths, using Figure 3A, in order to keep the accumulation of carbon below 1000 billion tones by 2100. Note that each box in Figure 3A is 250 billion tonnes of carbon and that 530 billion tonnes since 2010 have already been accumulated. Can emissions increase in the short run? Does stabilizing emissions reduce the concentration? What are the implications of the alternative paths?

3. Additions to the base model for discussion

Damages in the base model enter multiplicatively in the production function as in Nordhaus and Sztorc (2013). It is assumed that climate change causes losses to production in the same period only via the damage function. Temperature increases are assumed not to affect the depreciation of physical capital nor any other form of capital such as environmental, social and organizational capital. In addition, climate change is assumed not to impact the factors of production individually nor the growth rate of total factor productivity. Also, the damage function used in the base model has been calibrated for losses when temperature increases to 2.5–3 °C but it does not apply for higher temperature changes which are a real possibility under BAU (Stern, 2013). Furthermore, catastrophic damages are not incorporated into the base model (See Pindyck, 2013; Weitzman, 2011). Below some of these additions are incorporated into the base model. This enriches the simple model in terms of illustrating impacts to students.[17]

First consider the depreciation rate of physical capital. It is easy to conceive that increased temperatures and more severe weather will lead to capital having a shorter life span. It was mentioned as a possibility by Fankhauser and Tol (2005) and by Stern (2013). Recently, it has been incorporated into the DICE model by Dietz and Stern (2015) as well as Moore and Diaz (2015). Climate change can affect the durability and the longevity of the stock of capital, for example, increased temperatures cause increased frequency of storms, more extreme weather, rising sea levels, and many other impacts. Such events can cause permanent damage to capital infrastructure.[18] Capital will require more maintenance to keep it from further wear and tear due to temperature rising. Capital could even be stranded if people move far away from the ocean shores due to sea level rising. Extremely powerful storms could destroy capital which then needs replacement. With temperature increasing, a larger fraction of investment spending will be allocated towards depreciation (and to adaptation measures) than to new investment which is the engine of economic growth. This increased spending on necessary investment, to keep the capital labour ratio constant, reduces the steady state capital per person and hence the steady state income per capita. A simple way to introduce the impact of temperature on the depreciation rate is as follows[19]:

$$\delta_K = \delta_0 + \delta_1 T_t.$$

For the simulations we suggest that the depreciation rate increase by 1% per 1 °C temperature increase (i.e., $\delta_1 = 0.01$). Currently, on average, capital is replaced after 10 years assuming the depreciation rate is at the base rate of 0.1. If temperature increases to 2 °C (5 °C) then capital will need to be replaced on average every 8.3 years (6.7 years) as it wears out faster.

Second, temperature increases could affect the growth rate of total factor productivity (Moyer et al., 2013; Dietz and Stern, 2015; Moore and Diaz, 2015). The growth rate of total factor productivity could be impacted negatively because resources will be diverted away from R&D and instead used for climate adaptation and for the reconstruction of capital due to climate damages. Furthermore, output per hour of input (labour or capital) could decline if inputs need more hours to produce the same output level due to a different climate environment. Also, a warmer climate will increase the likelihood of human conflict (Hsiang et al., 2013). This in turn negatively impacts the institutions that protect property rights causing a possible reduction in the growth rate of total factor productivity. There is also evidence that economic growth is lower with higher temperatures (Dell et al., 2012). Total factor productivity is modelled along the following lines[20]:

$$g_{A,t} = \frac{g_{A,0}}{(1+\delta_A)^t} - \gamma T_t.$$

For the simulations we set $\gamma = 0.001$. It reduces the growth rate of total factor productivity by 0.001 for every 1 °C increase. Although this might seem like a small effect, it accumulates over time into a significant impact as it directly affects current production and the future growth rate of output per person. Note that total factor productivity is assumed to decline

[17] Stern (2013) suggests four alterations to the basic model to make it more relevant. First, damages to social, organizational and environmental capital. Secondly damages to the stock of capital and land. Third, damages to overall factor productivity. Finally, damages to learning and endogenous growth.

[18] Stern states: "Climate events such as storms or inundation can do permanent or long term damages to capital and land. If it is necessary to abandon certain areas, capital, infrastructure and land have zero use value and are essentially lost. This could be incorporated via a permanent damage or a reduction in capital occurring in period t as a result of temperature and events in that period." p. 849.

[19] Stern (2013) suggested an equation along these lines (See page 850).

[20] This formulation is similar to Dell et al. (2012).

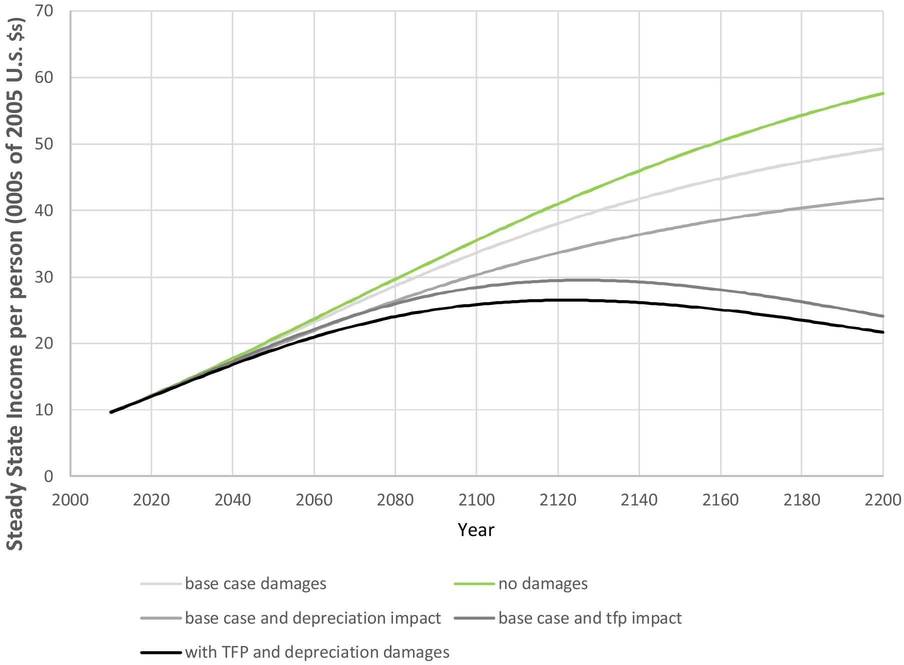


Fig. 5. The Solow Model with climate impacts on depreciation and TFP growth.

Source: authors' calculations.

over time even without climate change but remains positive.[21] But with temperature rising beyond 3 °C, the impact from the warmer temperature can offset the exogenous growth rate of total factor productivity.

Fig. 5 shows the path of steady state output under base case damages (slight gray path), base case and depreciation impact (darker gray path), base case and impact on the growth rate of total factor productivity (darkest gray path), and the black path under all damages. It is clear that adding only the depreciation effect does not cause a large reduction in steady state income per person. Income per person is rising throughout but is always at a lower level relative to the base case. Steady state income per person in the year 2200 increases to $42,000 rather than to $50,000. Adding the impact of climate change on the growth rate of total factor productivity amplifies the damages relative to the base case. In this case, steady state income increases initially, reaches a maximum after 2100, and then starts falling. Given that the growth rate of total factor productivity is assumed to decline even without climate change eventually there will be a temperature level after which the growth rate of total factor productivity will be negative and this will cause the reduction in steady state output per person. As seen from the path, the losses to output per person are significant if temperature increases affect the growth rate of total factor productivity. Taken all together the damages approach 60 percent of income by 2200 relative to no climate change and about 27 percent lower by 2100.[22] Still income per person is higher, at approximately $21,670, than the current income level of approximately $10,000. Note that the slower growth in output per person and the decline in income per person from 2100 helps in slowing down emissions and hence temperature increases. The temperature increases to 5.5 °C by 2200.

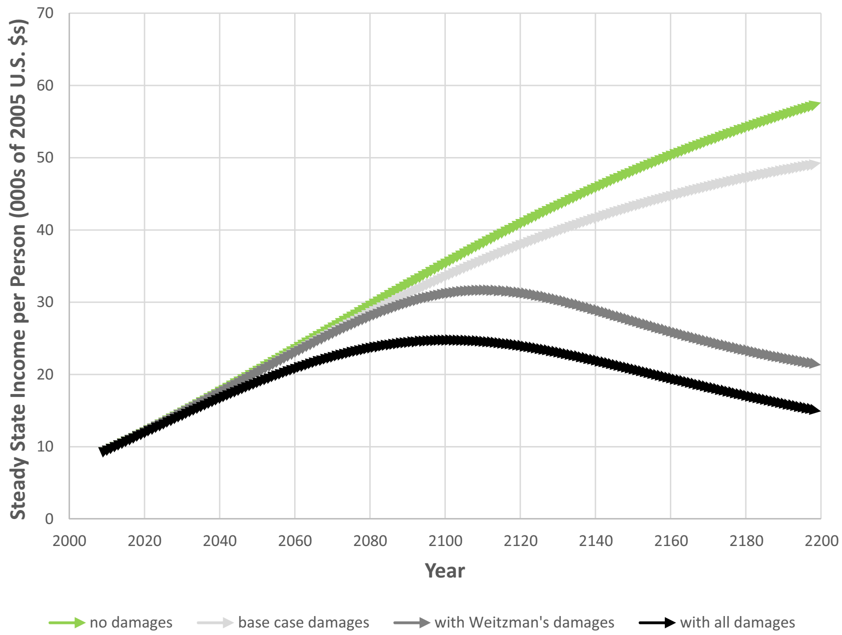
Another extension that can be incorporated into the simple model is associated with the type of damage function. The standard damage function in the DICE model represented in the base model has parameter values that have been calibrated for temperature increases not exceeding 3 °C relative to pre-industrial levels.[23] Higher temperature levels than 3 Celsius are highly likely with BAU as seen from Fig. 4. Fig. 1 shows very low damages at temperatures which reach 6–7 °C by 2200 with BAU. But such changes in the climate have not been observed for millions of years and could have profound impacts on the planet. Weitzman (2012) states that "six degrees of extra warming is about the upper limit of what the human mind can envision for how the planet might change" (p. 226). He speculates on further temperature increases

> A temperature change of ≈12 °C therefore represents an extreme threat to human civilization and global ecology as we know it, even if, conceivably, it might not necessarily mean the end of *Homo sapiens* as a species (p. 232).

[21] Dietz and Stern argued that this is due to depreciation of productivity (i.e., displacement of skills and know how) being stronger than institutional innovations that promote growth in productivity.

[22] Burke et al. (2015) estimate a 23% decrease in average global incomes by 2100 relative to world income without climate change.

[23] More recently Nordhaus (2008), and Nordhaus and Sztorc (2013) have altered the damage function to be $D_t = 1/\left(1 + \pi_1 T_t + \pi_2 T_t^2\right)$ but calibrated the parameters to yield similar results to the damage function used in the base model which was the Nordhaus, (1994) damage function.



Source: authors' calculations.
Notes: The lines referred to as no damages and base case damage function are as displayed in Figure 5.

Fig. 6. The Solow Model with expanded climate impacts and new damage function.

He envisions an increase of 18° Celsius as the "death temperature." Weitzman calibrates θ_1 to be 0.00238 so that it conforms to the Nordhaus (2008) DICE model. Nordhaus' model results in an eight percent loss from a temperature rise to 6 Celsius relative to pre-industrial levels and only a 26% loss with temperature increasing to 12 °C. According to Weitzman this type of temperature increase (12 C) was observed during the Eocene epoch, 55–34 million years ago with an ice free planet and alligators living near the North Pole. While for low temperature increases the damages are not high, and the Nordhaus parameters are fine, he argues that temperature increases of 6 ºC can result in at least a 50% loss of output and that a 12 C increase will result in a 99% loss of output. He recommends to capture this with the damage function along the following lines:

$$D_t = 1/\left(1 + \theta_1 T_t^{\theta_2} + \theta_3 T_t^{\theta_4}\right)$$

where the parameters assigned to θ_1 and θ_2 are as the Nordhaus damage function but θ_3 = 0.507E-05 and θ_4 = 6.754 as given in Table A1 of the Appendix. A simple way to show students the fundamental difference between the Nordhaus and Weitzman damage functions is to use a traditional, static marginal damage graph with marginal damages on the vertical axis and emissions on the horizontal axis. Environmental economics students will be readily familiar with this type of graph. In this space, the Nordhaus marginal damage curve is linear and upwards sloping, whereas the Weitzman marginal damage curve is convex.

Fig. 6 illustrates the different cases in the model. The path without climate change is the green trajectory, followed by the slight gray color path with Nordhaus type of damages, followed by Weitzman's path (darker grey path) which closely follows the Nordhaus path until temperature increases reach 3 ºC and then the last term in the damage function takes on a more important role and the two income paths start diverging. This can be considered a tipping point. When temperature reaches 4 °C steady state income per person starts declining reaching $21,000 by 2200. Note that the current generation is poorer than the generation living in 2200 but the 2200 generation is poorer than a generation living around the year 2100.[24] Adding in possible damage impacts on the depreciation rate and the growth rate of TFP, δ_1 = 0.01 and $\gamma = 0.001$, the steady state income per person follows the black path. Steady state income per capita increases initially reaches a maximum of approximately $25,000 (i.e., a 30% damage relative to no climate change) and then drops. This drop is due to the new damage function and the impact climate change has on the growth rate of total factor productivity. Steady state income per person declines and reaches $15,000 by 2200 (i.e., a 74% damage).

[24] With different parameter values it could be possible that the current generation is richer than one living in 2200.

Classroom discussion: What would the consequences be if damages negatively affected the growth rate of population?

Classroom exercise: Using the instructions from the Appendix ask students to reproduce the results in table 2 and to conduct a sensitivity analysis by changing the parameters as well as Figure 1.

Classroom discussion: What would the economic consequences be if there was a one-time impact arising from climate change such as a sudden destruction of the capital stock and/or population due to a tipping point occurring after 4 degrees Celsius? Would the steady state path be relevant to follow or would the transitional dynamic path be more appropriate in this case?

Classroom exercise: Using the transitional dynamic path in lieu of the balanced growth steady state path reproduce the income per person path when all damages are present. Can the future generation be worse off than the current generation?

4. Emissions mitigation, benefit-cost analysis & the 2° target

At COP16 in Cancun in 2010, attending nations agreed to a long-term goal of limiting the increase in average global temperatures to below 2 °C (UNFCC, 2011). In a recent meeting, the leaders of the G7 countries reiterated their commitment to this target (Carrel and Martin, 2015). To achieve this goal in the model, global cumulative carbon emissions should be limited to 1.111 trillion tons. Given that 530 billion tons have already been emitted by 2010, this leaves a carbon budget from 2010 on of 581 billion tons. If this budget is exceeded, temperature anomaly will have more than a 50% chance of exceeding 2 °C.

The simple model can be used to teach students about the costs and the benefits of the proposed target relative to business-as-usual as well as the importance of the choice of the discount rate for evaluating climate policy. The model is very well suited to incorporating the 2° target and its carbon budget because temperature change is based directly on cumulative emissions. Capping cumulative emissions is therefore relatively straightforward.

We choose an emissions reduction path for which government emissions regulation (the control rate, M_t) increases at a constant growth rate, m. Assume that in 2010, the emissions control rate is 9% and grows annually at a 4.267% growth rate (i.e., $M_t = M_{t-1}(1+m)$). The choice of emission control path here is relatively arbitrary and certainly not universally optimal.[25] The following equation describes how emissions control enters the model:

$$E_t = (1 - M_t)\sigma_t Y_t$$

Given the assumed parameter values ($M_0 = 0.09$ and $m = 0.04267$), the annual emissions path is displayed in Fig. 7. Emissions continue to increase but peak around 2035 and decline to zero by 2068. The emissions peak to reach the two degree target is almost half of the peak in the BAU scenario displayed in Fig. 3A.

Reducing emissions to zero and limiting temperature increase to 2° reduces temperature damages on future generations; however, reducing emissions is not costless. To reflect the immediate cost of reducing emissions, use the convex abatement cost function of Nordhaus and Sztorc (2013),

$$AC_t = \Omega_t M_t^2,$$

where $\Omega_0 = 0.06$ is an abatement cost coefficient that declines over time at the rate at which TFP grows (i.e., Ω_t declines at the rate$-g_{A,t}$). Total income per capita net of abatement cost in year t is

$$y_t = (1 - AC_t)D_t A_t k_t.$$

[25] The path is optimal (maximizes net present value) among all control paths that follow exponential growth assuming a 5% discount rate. Changing the discount rate affects which path is optimal.

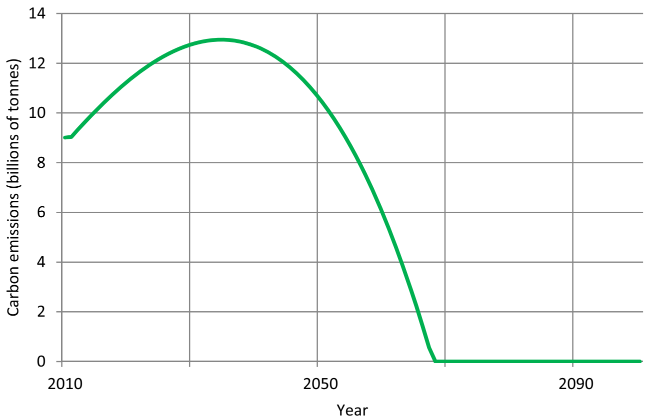
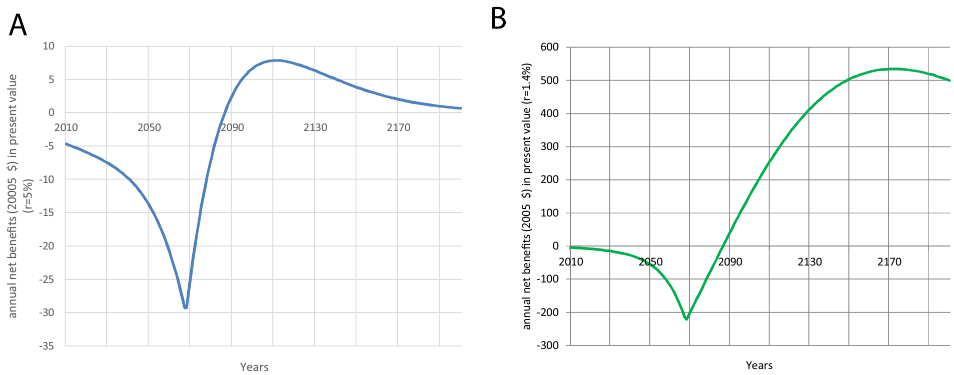


Fig. 7. Predicted emissions path to achieve 2° Limit.

Source: authors' calculations.

The reduction in per capita income of reducing emissions is difficult to show by just plotting the income path of BAU and the income path when the 2 ° limit is imposed. Instead Fig. 8A shows the annual net benefits in present value of the 2 ° limit versus BAU over time (2010–2200) assuming a 5% discount rate. The present value of total net benefits with a 5% discount rate is −$449 per capita. However, the net present value depends critically on the choice of discount rate. The annual net benefits of the 2 ° mitigation policy are displayed in Fig. 8B, but in present values using the 1.4% discount rate selected by the Stern Review. In this case, the future annual net benefits do not limit to zero by 2200 and the present value of total net benefits is positive ($40,665 per capita). The Internal Rate of Return for limiting to 2 ° is 4.08%. Another exercise we have used in the past to highlight the importance of the choice of social discount rate is to have students replicate the simple benefit-cost analysis done by Arrow (2007).

What is interesting to highlight to students between the two figures, is that the discount rate determines if the 2 °C mitigation target is a potential Pareto improvement, but not a pure Pareto improvement. For both discount rate values, the annual net benefits still show the inter-generational trade-off in which the costs of emissions reductions are imposed on early generations and the benefits of lower temperature increases accrue to later generations. Unlike with intra-generational policy evaluation, in this case there is no potential mechanism for a future generation to compensate earlier generations to make them at least as well off. Mitigation policy is not asking the current generation to incur costs to benefit their children; it is asking them to incur costs to benefit their great- great-grandchildren.



Source: authors' calculations.

Fig. 8. (A) Annual Net Benefits in Present Value using 5% Discount Rate. (B) Annual Net Benefits in Present Value using 1.4% Discount Rate.

Classroom discussion: What factors determine the social discount rate? Does it make sense to have the social discount rate decline over time?

Classroom exercise: Read the article by Arrow (2007) and using an Excel spreadsheet verify Arrow's cost benefit analysis on page 4-5. Discuss the implications of this finding.

Classroom exercise: Increase carbon emissions per person by 1 tonne in 2020 only and compare the net present value of income per person with the additional tonne and without. Repeat the exercise by adding an additional tonne of carbon in 2050 only. Discuss your findings.

5. Extensions and concluding remarks

This paper presents an extension of the Solow model that includes a simple climate model. To the best of our knowledge, we have developed the simplest Integrated Assessment Model. The simplicity of the model serves well for introducing the economics of climate change, which relies heavily on IAMs, to undergraduate students. Even though the model is very basic its predictions match well with Nordhaus' more complex DICE model.

In addition to introducing the model to students and highlighting the central inter-generational trade-off inherent in the climate change problem, two other classroom applications are outlined. The model can be used to teach the academic controversy over how damages from increased temperatures should enter the model and the implications of changing the standard assumptions on damages. The model is also very useful for teaching students about how economists approach evaluating the 2 °C target and the importance of the discount rate.

Although only two specific applications of the model have been highlighted, many other learning applications are possible both in and out of the classroom. For example, students can be presented with the basic model and then asked to do their own simulations and to write up an essay on the economics of climate change. Students will observe that climate change will affect the standard of living of the world economy in the future and that climate change caused by humans is a global externality and as a result requires coordinated, corrective action by governments. They will also observe the inter-generational trade-offs involved, i.e., economic activity today inflicts a cost on future generation and that future generations will be richer than today's generation due to increases in productivity and the stabilization of global population. However, they will also observe that the richer future generations will not be as rich as they would have been without climate change. Since cost of action is absorbed by the current generation and the benefits of action accrue to future generations students can conduct a cost-benefit analysis and explore the importance of the discount rate. But future generations could be poorer than the current generation if temperature anomaly exceeds 3 Celsius and under a different damage function; different model parameters; when climate change affects the depreciation of capital and especially when temperature affects total factor productivity. In this case, the benefits of action relative to the costs in present value terms increase and stronger action is needed.

There are many extensions students can explore with the model beyond what is mentioned above (i.e., different damage impacts and a cost-benefit analysis). One example would be for students to conduct a sensitivity analysis by changing the parameters of the model and observing the impact on the standard of living. Secondly, in this paper the balanced growth path is used; however, simulations can be conducted according to the transitional growth path given by $k_{t+1} = k_t + sy_t - (\delta_K + g_{n,t})k_t$. Namely, the capital stock (per person) next period is equal to the capital stock per capita this period plus the difference between actual investment per person and the necessary investment that is needed to maintain the capital labor ratio constant. The new stock of capital is then channeled into the actual path of output per person through the production function $y_t = D_t A_t k_t^{\alpha}$. Using this approach students and instructors can explore the impact of sudden destructions of the capital stock or population due to tipping points. In the balanced growth model, the impact of sudden shocks are only for one period as the system returns to the steady state in the next period, but with the introduction of transitional dynamics the impact could last for over a decade until the system reaches a new steady state. This can be either done in-class led by the instructor or be given as an assignment so that students can compare and contrast the different paths of the economy.

Another option is to use the model to compute the social cost of carbon. We have found that students easily grasp the general idea of the social cost of carbon, but have difficulty understanding how the various estimates are calculated. Although the true social cost of carbon may include non-market values not best reflected in a damage function that is multiplicative on production; the model is still useful to give students a hands-on demonstration of how the social cost of carbon is actually calculated using integrated assessment models. This is done easily by comparing the change in the net present value of steady state income per person under the base case carbon emissions path relative to a path when 1 additional tonne of carbon per person is added in a particular year. Students will see that the social cost of carbon increases if the tonne of carbon is released further into the future. Students will also immediately see why the choice of discount rate matters so crucially for the resulting social cost of carbon value.

A further application is to ask students to find parameters that are relevant to a particular nation and to examine the impact climate change will have on that particular nation. They can also examine the impact on economies that are emerging and growing faster relative to the industrial nations. For more advanced courses they could also change the economic model to an endogenous growth model and examine the impact of climate change.

Appendix.

Instructions for students: excel spreadsheet

Below are set of instructions for you (the student) to setup the Excel spreadsheet of the simple Climate-Solow IAM. Setting up the model will help you examine different future trajectories of the economic system under the base model (as outlined in Section 2). Once set up, you can then use the model to explore other more complicated scenarios as assigned by your instructor.

To set up the model, you will use Excel with two worksheets initially. One sheet will contain the parameter values of the model. The other sheet will be used to compute the steady state values, such as income per person, over time. We assume you have a basic understanding of Excel.

STEP 1: the parameters worksheet

The first step is to setup the first worksheet called "parameters" similar to Table A1 below. You should enter the information as reported in Table A1. The worksheet will have:

1. **Column A** contains descriptions of the parameters
2. **Column B** contains the symbols of the parameters. (Note: This is not necessary and can be skipped.)
3. **Column C** contains the assigned values

Table A1
Variables, symbols and values used for base case. "Parameters" sheet.

		Column		
		A	B	C
Row	1	Description	Symbol	Value
	2	Capital's share of income	α	0.3
	3	Savings rate	s	0.25
	4	Depreciation rate	δ_0	0.1
	5	Impact of temperature on depreciation rate	δ_1	0.01
	6			
	7	Initial 2010 population (in billions)	L_0	6.838
	8	Initial 2010 population growth rate	$g_{L,0}$	0.023
	9	Parameter affecting population growth	δ_L	0.052
	10			
	11	Initial 2010 total factor productivity	A_0	3.955
	12	Initial productivity growth rate	$g_{A,0}$	0.015
	13	Parameter affecting productivity growth	δ_A	0.011
	14	Temperature impact on productivity growth		0.001
	15			
	16	Initial world GDP (trillions of 2005 US $)	Y_0	63.69
	17	Initial world capital (trillion of 2005 US $s)	K_0	135
	18			
	19	Initial emission intensity	σ_0	0.549
	20	Initial 2010 growth of emissions intensity	$g_{\sigma,0}$	−0.01
	21	Parameter affecting emissions intensity growth	δ_σ	−0.0002
	23	Damage parameter	θ_1	0.002384
	24	Damage parameter	θ_2	2
	25	Damage parameter	θ_3	0.00000507
	26	Damage parameter	θ_4	6.754
	27			
	28	CCR per trillion tonnes	β	0.0018
	29	Initial Carbon (billions of tonnes)		530

This is very easy to setup but very important as all other sheets used will reference this worksheet named "parameters".

STEP 2: the model worksheet

This step requires the creation of a new worksheet named "model 1". In this sheet you will compute the values of the various variables of the model over 200 years into the future (See Table A2). The first row will contain the variable symbols or names. In order to create this sheet you will need to create the data for the next two rows. The second row will contain the initial values for each variable and the third row contains the formulas that calculate the value of the variable for the next year. Once these two rows are completed, students can copy and paste the third row until the year 2200 (row 192).

1. **Column A:** This column lists the years from 2010 to 2200. These cells correspond to the subscript t in the model's mathematical equations. Enter 2010 in cell A2 and then in cell A3 enter the formula = A2 + 1.
2. **Column B:** To compute the growth rate of population over time, $g_{L,t}$, the first value in cell B2 is the initial population growth rate of 2.3 percent in the sheet titled "parameters" under cell $C$8. The formula to enter in cell B2 is: = parameters!$C$8. The value in cell B3 is calculated by using the population growth formula, $g_{L,t} = g_{L,t-1}/(1+\delta_L)$. The formula to enter in cell B3 is: = B2/(1 + parameters!$C$9).
3. **Column C:** To compute the world population level over the years, starting from the initial value of 6.838 billion obtained from parameters!$C$7 and entered in cell C2 with the formula = parameters!$C$7. In cell C3 the new population level in 2011 is computed by multiplying the value in cell C2 by the growth rate of that same year in column B3. The formula to enter is: =C2*(1 + B3).
4. **Column D:** To compute the growth rate of carbon intensity, the first value of the growth rate of carbon intensity is taken from Table A1 as −0.01 in cell parameters!$C$20. The formula to enter in cell D2 is = parameters!$C$20. In cell D3 the value of this variable is generated using the formula $g_{\sigma,t} = \frac{g_{\sigma,t-1}}{1+\delta_\sigma}$ where δ_σ is from Table A1 cell parameters!$C$21, while $g_{\sigma,t-1}$ is the previous value of the growth rate of carbon intensity. For cell D3 the formula is = D2/(1 + parameters!$C$21).
5. **Column E:** To compute the carbon intensity, the formula to enter in E2 is = parameters!$C$19. The value in E3 is found using: $\sigma_t = \sigma_{t-1}(1+g_{\sigma,t})$ where $g_{\sigma,t}$ is from column D and σ_{t-1} is the previous carbon intensity value. In cell E3 the formula is = E2*(1 + D3).
6. **Column F:** This column shows world output/income per capita lagged one year (y_{t-1}) and is used to calculate emissions per capita. The formula to enter in cell F2 is: = parameters!$C$16/parameters!$C$7. This value should be aligned as closely as possible with the steady state value in 2010. Cell F3 is computed as = P2. There will be no value in F3 as column P has not been constructed yet but will appear once there is a value in P2.
7. **Column G:** Carbon dioxide emissions per person is computed by multiplying carbon intensity in column E with output per person in column F. The formula to enter in cell G2 is =E2*F2. For cell G3 enter = E3*F2. Note that column F does not have values yet, but will be computed soon.
8. **Column H:** To compute total annual carbon emissions, multiply carbon dioxide emissions per person (column G) by the total population (column C) and divide by 3.67 (the conversion factor between carbon and carbon dioxide). The formula to enter in cell H2 is =G2*C2/3.67. For cell H3 the formula is =G3*C3/3.67.
9. **Column I:** Cumulative carbon emissions start in 2010 as 530 billion tonnes of carbon found in $C$29 of the parameters sheet. This is entered in cell I2 as = parameters!$C$29. Cell I3 requires the formula = I2 + H2.
10. **Column J:** To compute the temperature anomaly ($T_t = \beta\left[C_0 + \sum_{i=1}^{t} E_i\right]$), multiply the cumulative carbon emissions in column I by the cell parameters!$C$28 (this is the selected value for β). The formula to enter in cell J2 is = I2*parameters!$C$28. For cell J3 the formula is = I3*parameters!$C$28.
11. **Column K:** This column computes the growth rate of total factor productivity, $g_{A,t} = \frac{g_{A,0}}{(1+\delta_A)^t}$. The first value in cell K2 is computed using the formula = parameters!$C$12 (this is the initial value $g_{A,t}$). The other value in cell K3 is computed according to the formula: = $K$2/((1 + parameters!$C$13)^ (A3-$A$2)).
12. **Column L:** To calculate total factor productivity, A_t, the initial value in cell L2 is taken from $C$11 in the parameters sheet. The next value grows at the rate calculated in column K, the formula in cell L3 is = L2*(1 + K3).
13. **Column M:** This column computes the depreciation rate, δ_K. For all cells in this column the formula is the same: = parameters!$C$4. **Column N:** This column computes the damage function for the base model for each year ($D_t = 1/\left(1+\theta_1 T_t^{\theta_2}\right)$). It is obtained using the values for the damage coefficients in the parameters sheet and the corresponding temperature anomaly (column J). In cell N2 the following formula is entered: = 1/(1 + parameters!$C$23*(J3^parameters!$C$24)). Copy and paste this formula in cell N3.
14. **Column O:** The steady state level of capital per person for a particular year is then computed as $k_{ss,t} = \left[\frac{sA_tD_t}{\delta_K+g_{n,t}}\right]^{1/(1-\alpha)}$. The formula is entered in cell O2 as follows: = ((parameters!$C$3*L2*N2)/(M2 + B2))^(1/(1- parameters!$C$2)). Copy and paste this formula in cell O3. **Column P:** The steady state output/income per person for a particular year is computed using the formula $y_{ss,t} = D_tA_tk_{ss,t}^\alpha$. The formula is entered in cell P2 as =N2*L2*O2^parameters!$C$2. Copy and paste this formula in cell P3.

Table A2
The path of the variables from 2010 to 2200.

	1	Column A Year	B $g_{L,t}$	C L_t	D $g_{s,t}$	E σ_t	F y_{t-1}	G CO_2/L_t	H E_t	I $C_0+\sum_{t=1}^{t} E_i$	J T_t	K $g_{A,t}$	L A_t	N δ_k	O D_t	P k_t	Q y_t
Row	2	2010	0.023	6.838	−0.010	0.549	9.314	5.113	9.527	530.000	0.954	0.015	3.955	0.100	0.998	19.578	9.632
	3	2011	0.022	6.988	−0.010	0.544	9.632	5.235	9.968	539.527	0.971	0.015	4.014	0.100	0.998	20.259	9.875
	4	2012	0.021	7.133	−0.010	0.538	9.875	5.314	10.327	549.495	0.989	0.015	4.073	0.100	0.998	20.948	10.120
	5	2013	0.020	7.274	−0.010	0.533	10.120	5.391	10.685	559.822	1.008	0.015	4.132	0.100	0.998	21.643	10.368
	6	2014	0.019	7.410	−0.010	0.527	10.368	5.467	11.039	570.507	1.027	0.014	4.191	0.100	0.997	22.345	10.617
	7	2015	0.018	7.542	−0.010	0.522	10.617	5.543	11.391	581.546	1.047	0.014	4.251	0.100	0.997	23.054	10.868
	8	2016	0.017	7.670	−0.010	0.517	10.868	5.617	11.740	592.938	1.067	0.014	4.310	0.100	0.997	23.768	11.120
	9	2017	0.016	7.794	−0.010	0.512	11.120	5.690	12.084	604.677	1.088	0.014	4.370	0.100	0.997	24.488	11.375
	10	2018	0.015	7.914	−0.010	0.507	11.375	5.762	12.425	616.762	1.110	0.014	4.430	0.100	0.997	25.213	11.632
	11	2019	0.015	8.029	−0.010	0.501	11.632	5.833	12.761	629.186	1.133	0.014	4.490	0.100	0.997	25.943	11.890
	12	2020	0.014	8.140	−0.010	0.496	11.890	5.903	13.092	641.947	1.156	0.013	4.551	0.100	0.997	26.677	12.149
		...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
		2197	0.000	10.615	−0.010	0.081	48.931	3.957	11.444	3835.483	6.904	0.002	12.902	0.100	0.898	122.558	49.024
		2198	0.000	10.616	−0.010	0.080	49.024	3.923	11.347	3846.928	6.924	0.002	12.926	0.100	0.897	122.787	49.115
		2199	0.000	10.616	−0.010	0.079	49.115	3.889	11.250	3858.275	6.945	0.002	12.951	0.100	0.897	123.013	49.206
	192	2200	0.000	10.616	−0.010	0.078	49.206	3.856	11.154	3869.525	6.965	0.002	12.975	0.100	0.896	123.237	49.296

15. **FINAL STEP:** Copy and paste all row 3 cells to row 4 until row 192. Check that all values in your worksheet correspond with Table A2. Congrats, you have created an Integrated Assessment Model!

Exercise: graph the path of income per person with damages and without

In order to graph the path of output per person with and without climate damages (i.e., reproduce Fig. 1), and be available for other simulations, create a new worksheet labelled “graph data”. It will utilize three columns. The first row in the worksheet will indicate the name of each variable: Years, Income per person with damages, and Income per person without damages.

1. **Column A:** This will be the column indicating Years, it is the same as column A of the “model 1” sheet. In cell A2 enter = model 1!A2. Copy and paste the formula from cells A3 to A192.

2. **Column B:** This column is labeled Income per person with damages. You have already calculated the values for this column in column P of the model 1 worksheet. In cell B2 enter the following: = model 1!P2. Copy and paste this formula in cells B3 to B192.

3. For income per person without damages you will need to enter two new columns in the model 1 worksheet:

Column R: Compute capital per person without damages as $k_{ss,t} = \left[\frac{sA_t}{\delta_K + g_{n,t}}\right]^{1/(1-\alpha)}$. This formula entered into cell R2 is as follows: = ((parameters!$C$3*L2)/(M2 + B2))^(1/(1- parameters!$C$2)).

Copy and paste this formula in cells R3 to R192.

Column S: Compute income per person without damages using the formula $y_{ss,t} = A_t k_{ss,t}^{\alpha}$. This formula entered in cell S2 is = K2*P2^parameters!$C$2. Copy and paste this formula in cells S3 to S192.

Now return to the “graph data” worksheet to fill in column C.

Column C: This column displays income per person without climate damages. In cell C2 enter the formula = model 1!S2. Copy and paste the formula in cells C3 to C192.

4. Finally use Excel to graph the two variables over time.

Note: Other figures can be similarly created. Careful when including damages for depreciation and total factor productivity as these will affect the no climate scenario.

References

Archer, David, Eby, Michael, Brovkin, Victor, Ridgwell, Andy, Cao, Long, Mikolajewicz, Uwe, Caldeira, Ken, et al., 2009. Atmospheric lifetime of fossil fuel carbon dioxide. Annu. Rev. Earth Planet. Sci. 37 (1), 117.

Arrow, Kenneth J., 2007. Global climate change: a challenge to policy. Econ. Voice 4 (3) .

Brock, William A., Taylor, M. Scott, 2010. The green solow model. J. Econ. Growth 15 (2), 127–153.

Broome, John., 2012. Climate Matters Ethics in a Warming World. W.W. Norton and Company, New York.

Burke, Marshall, Hsiang, Solomon M., Miguel, Edward, 2015. Global non-linear effect of temperature on economic production. Nature .

Carrel, Paul, Michelle Martin. G7 leaders bid Auf Wiedersehen to carbon fuels. Reuters, 2015. http://in.reuters.com/article/2015/06/08/g7-summit-idINKBN0OO25120150608, (accessed 19.06.15.

Covert, Thomas, Greenstone, Michael, Knittel, Christopher R., 2016. Will we ever stop using fossil fuels? J. Econ. Perspect. 30 (1), 117–138.

Dalton, Thomas R., Morris Coats, R., Andrew Luccasen, R., 2015. Exploring easter island economics with excel. Int. Rev. Econ. Educ. 18, 1–10.

Dell, Melissa, Jones, Benjamin F., Olken, Benjamin A., 2012. Temperature shocks and economic growth: evidence from the last half century. Am. Econ. J.: Macroecon. 4 (3), 66–95.
Dietz, Simon, Stern, Nicholas, 2015. Endogenous growth, convexity of damages and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. Econ. J. 125 (583), 574–620.
Elmslie, Bruce, Tebaldi, Edinaldo, 2010. Teaching economic growth theory with data. J. Econ. Educ. 41 (2), 110–124.
Fankhauser, Samuel, Tol, Richard S.J., 2005. On climate change and economic growth. Resour. Energy Econ. 27 (1), 1–17.
Findley, T. Scott, 2014. Using MS excel to solve and simulate the life-cycle/permanent-Income model of consumption and saving. Int. Rev. Econ. Educ. 16 (B), 129–146.
Forsyth, Tim, 2014. Climate justice is not just ice. Geoforum 54, 230–232.
Hsiang, Solomon M., Burke, Marshall, Miguel, Edward, 2013. Quantifying the influence of climate on human conflict. Science 341 (6151), 1235367.
Matthews, H. Damon, et al., 2009. The proportionality of global warming to cumulative carbon emissions. Nature 459 (7248), 829–832.
Matthews, H. Damon, Solomon, Susan, Pierrehumbert, Raymond, 2012. Cumulative carbon as a policy framework for achieving climate stabilization. Philos. Trans. R. Soc. A: Math. Phys. Eng. Sci. 370 (1974), 4365–4379.
Moore, Frances C., Diaz, Delavane B., 2015. Temperature impacts on economic growth warrant stringent mitigation policy. Nat. Clim. Change 5, 127–131.
Moyer, Elisabeth J., 2013. Climate Impacts on Economic Growth as Drivers of Uncertainty in the Social Cost of Carbon, vol. 652. University of Chicago Coase-Sandor Institute for Law & Economics Research Paper.
Nordhaus, William D., and Paul Sztorc Dice 2013R: Introduction and User's Manual, Second edition, 2013.
Nordhaus, William D., 1994. Managing the Global Commons: The Economics of Climate Change. MIT Press, Cambridge, MA.
Nordhaus, William D., 2008. A question of balance: weighing the options on global warming policies. Yale University Press, New Haven, CT.
Nordhaus, William, 2015. Climate clubs: overcoming free-riding in international climate policy. Am. Econ. Rev. 105 (4), 1339–1370.
Oreskes, Naomi, 2004. The scientific consensus on climate change. Science 306 (5702), 1686.
Pindyck, Robert S., 2013. Climate change policy: what do the models tell us? J. Econ. Lit. 51 (3), 860–872.
Pindyck, Robert S., 2015. The Use and Misuse of Models for Climate Policy. No. w21097. National Bureau of Economic Research.
Schwanitz, Valeria Jana, 2013. Evaluating integrated assessment models of global climate change. Environ. Model. Softw. 50, 120–131.
Stern, Nicholas, 2007. The Economics of Climate Change: the Stern Review. Cambridge University press.
Stern, Nicholas, 2013. The structure of economic modeling of the potential impacts of climate change: grafting gross underestimation of risk onto already narrow science models. J. Econ. Lit. 51 (3), 838–859.
Taylor, M. Scott, 2014. Can green power save us from climate change? Swiss J. Econ. Stat. 150 (1), 23–31.
Tokarska, Katarzyna B., Gillett, Nathan P., Weaver, Andrew J., Arora, Vivek K., Eby, Michael, 2016. How much would five trillion tonnes of carbon warm the climate? Nat. Clim. Change doi:http://dx.doi.org/10.1038/nclimate3036.
Tol, Richard S.J., 2014. Climate Economics Economic Analysis of Climate, Climate Change and Climate Policy. Edward Elgar Publishing.
Tol, Richard S.J., 2015. Bootstraps for meta-analysis with an application to the impact of climate change. Comput. Econ. 46 (2), 287–303.
UNFCC, Report of the conference of the parties in its sixteenth session held in Cancun from 29 november to 10 December 2010. Part Two: Action taken by the Conference of the Parties at its sixteenth session. United Nations, 2011. http://unfccc.int/resource/docs/2010/cop16/eng/07a01.pdf#page=2, (accessed 19.06.15.).
Vazquez, Jose J., Chiang, Eric P., 2014. A picture is worth a thousand words (at least): the effective use of visuals in the economics classroom. Int. Rev. Econ. Educ. 17, 109–119.
Watts, Michael, Becker, William E., 2008. A little more than chalk and talk: results from a third national survey of teaching methods in undergraduate economics courses. J. Econ. Educ. 39 (3), 273–286.
Weitzman, Martin L., 2009. On modeling and interpreting the economics of catastrophic climate change. Rev. Econ. Stat. 91 (1), 1–19.
Weitzman, Martin L., 2011. Fat-tailed uncertainty in the economics of catastrophic climate change. Rev. Environ. Econ. Policy 5 (2), 275–292.
Weitzman, Martin L., 2012. GHG targets as insurance against catastrophic climate damages. J. Public Econ. Theory 14 (2), 221–244.