Dynamic General Equilibrium Model for Climate Resilient Economic Development (DGE-CRED)

Technical Report

Christoph Schult and Andrej Drygalla
Halle Institute for Economic Research

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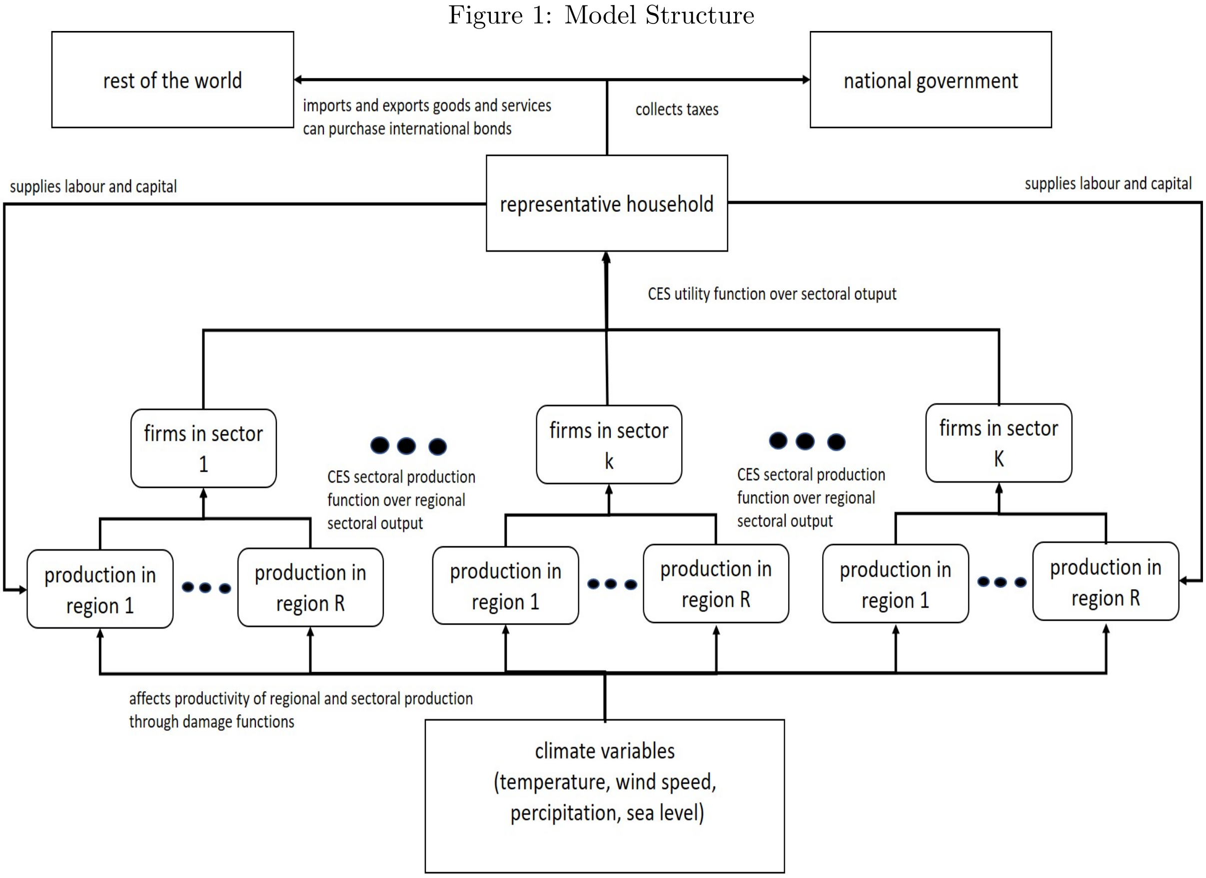
1 Introduction

This report is a guide on how to use the spatial small open economy dynamic general equilibrium model for climate change and adaptation simulations. In general the model belongs to the class of real business cycle models, because no nominal rigidities are explicitly considered. Nevertheless, it is possible to extend the model to feature also nominal rigidities. The model structure is depicted in Figure 1. Regional climate variables (precipitation, wind speed, temperature and sea level) are exogenous to

Figure 1: Model Structure



economic variables. Regional sectoral production functions depend on regional climate variables. The model is meant to reflect small open economies and therefore the climate system is unaffected by the domestic economic system.

The model consists of an arbitrary number of regions and sectors. Regional differentiation is only provided on the supply side and not on the demand side. Representative households consume sectoral goods and supply capital and labour to the firms in the regions. Households also demand goods and services from the rest of the world. Firms use capital and labour to produce sectoral goods with sectoral and regional specific constant elasticity of substitution production functions.

The government collects taxes, consumes and can use its funds to finance adaptation measures for specific regions and sectors. So far, adaptation measures will reduce overall damage by all climate variables at the same time. The effectiveness of government expenditure in one specific region and sector can vary.

One can use the model to conduct scenario simulations to evaluate the costs and benefits for different adaptation measures. It is important to understand that the model is not meant to produce explicit forecasts for an economy. The model is meant to simulate long-run developments considering the impact of potential changes in climate variables and their effect on the supply side of the economy. the user is able to define scenarios for different climate variables and adaptation measures. Therefore, it is possible to disentangle the effect of specific climate variable changes on the economy. Further, the model is able to quantify upper limits for costs of adaptation measures to reduce damages by climate change. E.g., it is possible to evaluate the impact of temperature increases on different sectors and the overall impact on total gross value added. The discounted cumulative difference between a scenario without a temperature increase and with temperature increase can be used to determine the upper bound for the costs to reduce the damage caused by a temperature increase.

In the following Section 2 the derivation of the model equations is explicitly described. Readers who are interested in using the model can skip the model section and can directly go to Section 3.

2 Model

2.1 Climate variables

In order to capture the effect of climate change on the economy it is necessary to include climate variables into the model. A small open economy model does not need to include the impact of domestic economic activity on climate variables. Therefore, in contrast to Nordhaus (1993) we do not need to model the interaction between economic activity and climate change. Climate variables are independent of other endogenous variables in the model. We explicitly model the regional average annual temperature $T_{r,t}$, the average precipitation $PREC_{r,t}$, the average annual wind speed $WS_{r,t}$, and the sea level SL_t.

$$\begin{aligned} T_{r,t} &= T_{r,0} + \eta_{T,r,t} \\ PREC_{r,t} &= PREC_{r,0} + \eta_{PREC,r,t} \\ W^S_{r,t} &= W^S_{r,0} + \eta_{W^S,r,t} \\ SL_t &= SL_0 + \eta_{SL,r,0} \end{aligned} \tag{1}$$

The approach in eq. 1 allows to specify the evolution of climate variables according to the projections by metreolgical models (Stocker et al. 2013, e.g.).

2.2 Demand

2.2.1 Households

As depicted in Figure 1 the demand side is represented by representative households h providing labour N and capital K to domestic firms f. Households maximize discounted utility over an infinite horizon by choosing consumption $C_t(h)$, capital $K_{k,r,t+1}(h)$, investments $I_{k,r,t}(h)$, labour $N_{k,r,t}(h)$ and foreign net bond holdings B_{t+1} to maximize utility constrained by the budget constraint and the law of motion for sectoral and regional capital. Therefore, the Lagrangian eq. 2 of the representative household is

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \Bigg[& \left(\frac{C_t(h)^{1-\sigma^C}}{1-\sigma^C} - \sum_{k=1}^{K} \sum_{r=1}^{R} \phi^L_{k,r} \frac{N_{k,r,t}(h)^{1+\sigma^L}}{1+\sigma^L} \right) \\ & - \lambda_t(h) \Bigg(P_t \, C_t(h) \, (1+\tau^C) + \sum_{k=1}^{K} \sum_{r=1}^{R} P_{k,r,t} I_{k,r,t}(h) + S^f_t \, \phi^B_t \, (1+r^f_t) \, B_t(h) \\ & - \sum_{k=1}^{K} \sum_{r=1}^{R} (1-\tau^N) \, W_{k,r,t} N_{k,r,t}(h) - \sum_{k=1}^{K} \sum_{r=1}^{R} P_{k,r,t} \, r_{k,r,t} \, (1-\tau^K) \, K_{k,r,t}(h) - B_{t+1}(h) \Bigg) \\ & - \sum_{k=1}^{K} \sum_{r=1}^{R} \lambda_t(h) \omega^I_{k,r,t}(h) \left\{ K_{k,r,t+1} - (1-\delta) \, K_{k,r,t} - I_{k,r,t} \, S\left(\frac{I_{k,r,t}}{I_{k,r,t-1}} \right) \right\} \Bigg]. \end{aligned} \tag{2}$$

Households receive utility by consuming goods, where the inter temporal elasticity of consumption is defined by σ^C. Dis-utility from labour is sector and region specific $\phi^L_{k,r}$, the inverse Frisch elasticity σ^L is identical for all sectors and regions. Households spent money either on consumption goods $P_t \, C_t(h) \, (1+\tau^C)$, regional and sector specific investment $P_{k,r,t} I_{k,r,t}(h)$ and need to repay foreign bonds $B_{t+1}(h)$. They receive income from labour $W_{k,r,t} \, N_{k,r,t}(h) \, (1-\tau^L)$, capital renting $P_{k,r,t} \, r_{k,r,t} \, K_{k,r,t}(h) \, (1-\tau^K)$ and can use their borrowed money from the foreign economy $B_t(h)$. The first order conditions to the problem are the behavioral equations. As is standard in teh literature we replace the Lagrange multiplier λ_t by the marginal utility of consumption $\frac{C_t(h)^{-\sigma^C}}{P_t \, (1+\tau^C)}$ derived from the first order condition (FOC) of the above problem with respect to (w.r.t.) consumption. Households supply labour according to the FOC w.r.t. labour eq. 3 for each sector and region depending on the wage $W_{k,r,t}$ and the marginal dis-utility of labour for the specific sector and region

$$\phi^L_{k,r} N_{k,r,t}(h)^{\sigma^L} = \lambda_t(h) \, W_{k,r,t} \, (1-\tau^N). \tag{3}$$

The household also needs to decide how much of its income it wants to consume or invest into capital. The famous Euler equation eq. 4 is obtained by taking the first derivative of the Lagrangian w.r.t. sector and region specific capital

$$\lambda_{t+1}(h)\,\beta\,\left(P_{k,r,t+1}\,r_{k,r,t+1} + (1-\delta)\,\omega^I_{k,r,t+1}\right) = \lambda_t(h)\,\omega^I_{k,r,t}. \tag{4}$$

Further, the household also faces investment adjustment cost $S(\frac{I_{k,r,t}}{I_{k,r,t-1}}) = 3 - exp\left\{\sqrt{\phi^K/2}\left(\frac{I_{k,r,t}}{I_{k,r,t-1}} - 1\right\}\right) - exp\left\{-\sqrt{\phi^K/2}\left(\frac{I_{k,r,t}}{I_{k,r,t-1}} - 1\right)\right\}$, which are sector and region specific. The specification of the investment adjustment cost function is the same as proposed and estimated by Christiano et al. (2014) for the US. The marginal value of sectoral and regional investment $\omega^I_{k,r,t}$ is determined by

$$\begin{aligned} &P_{k,r,t}\,\lambda_t(h) \\ &= \lambda_t(h)\,\omega^I_{k,r,t}\left(S(\frac{I_{k,r,t}}{I_{k,r,t-1}}) - \frac{\partial S(\frac{I_{k,r,t}}{I_{k,r,t-1}})}{\partial I_{k,r,t}}\,\frac{I_{k,r,t}}{I_{k,r,t-1}}\right) + \beta\lambda_{t+1}(h)\,\omega^I_{k,r,t+1}\,\frac{\partial S(\frac{I_{k,r,t+1}}{I_{k,r,t}})}{\partial I_{k,r,t}}\,\left(\frac{I_{k,r,t+1}}{I_{k,r,t}}\right)^2 \end{aligned} \tag{5}$$

Households have access to the international financial market to purchase and sell internationally traded bonds. We only consider net foreign positions.

$$\lambda_{t+1}\,\beta\,S^f_{t+1}\,\phi^B_{t+1}\left(1 + {r^f}_{t+1}\right) = \lambda_t \tag{6}$$

The required interest rate will increase if the foreign debt relative to GDP increases and current net exports relative to GDP will decrease.

$$\phi^B_{t+1} = exp\left(-\phi^B\,(S^f_{t+1}\,r^f_{t+1}\,\frac{B_t}{Y_{t+1}} + \frac{NX_t}{Y_t})\right) \tag{7}$$

2.2.2 Government

We are interested in different policy measures taken by the government to adapt to a new climate regime. Government behavior is not a result of an optimization problem. The Government collects taxes from consumption $\tau^C\,C_t$, labour income $\sum_k^K\sum_r^R(\tau^N + \tau^N_{k,r,t})\,W_{k,r,t}\,N_{k,r,t}\,PoP_t$ and capital income $\sum_k^K\sum_r^R(\tau^K + \tau^K_{r,k,t})\,P_{k,r,t}\,r_{k,r,t}\,K_{k,r,t}$. In order to finance its activities the government can also get loans from the rest of the world B^G_{t+1} and has to repay loans and interest from the previous period denominated in foreign currency $(1 + r^f_t)$ identical to the household. The government budget constraint boils down to eq. 8.

$$\begin{aligned} G_t + \sum_k^K\sum_r^R G^A_{k,r,t} + B^G_t = &\sum_k^K\sum_r^R\left\{(\tau^K + \tau^K_{r,k,t})\,P_{k,r,t}\,r_{k,r,t}\,K_{k,r,t} + (\tau^N + \tau^N_{k,r,t})\,W_{k,r,t}\,N_{k,r,t}\,PoP_t\right\} \\ &+ (1 + r^f_t)\,S^f_t\,\phi^B_t\,B^G_{t-1} \end{aligned} \tag{8}$$

Government expenditures can be used to finance adaptation measures in specific sectors and regions $G^A_{k,n,t}$. Government expenditures on adaptation measures, taxes on regional and sectoral capital expenditure, and government debt are independent of other variables or to formulate it differently are discretionary. This allows us to evaluate different policy paths for the future and to model the variables by exogenous processes as stated in eq. 9.

$$\begin{aligned} G^A_{k,r,t} &= G^A_{k,r,0} + \eta^A_{k,r,t} \\ \tau^K_{k,r,t} &= \tau^K_{k,r,0} + \eta^{\tau^K}_{k,r,t} \\ \tau^N_{k,r,t} &= \tau^N_{k,r,0} + \eta^{\tau^N}_{k,r,t} \\ B^G_t &= B^G_0 + \eta^{B^G}_t \end{aligned} \tag{9}$$

2.2.3 Resource constraint

Households and the Government use domestic final goods Y_t produced by firms for consumption, investment and for exports X_t and can also use imports M_t for consumption and investment. This gives rise to the well known resource constraint or the expenditure approach to define GDP

$$Y_t = C_t + I_t + G_t + \underbrace{X_t - M_t}_{NX_t} \tag{10}$$

2.3 Production

Households demand final domestic goods Y_t combining goods from different sectors $Y_{k,t}$ using a CES composition function. They minimize expenditures subject to the composition function

$$\underset{Y_{k,t}}{min} \sum_k Y_{k,t}\, P_{k,t} \tag{11}$$

$$Y_t = \left(\sum_k \omega_k^{Q\,\frac{1}{\eta^Q}} Y_{k,t}^{\frac{\eta^Q - 1}{\eta^Q}} \right)^{\frac{\eta^Q}{\eta^Q - 1}} \tag{12}$$

Therefore demand for sectoral products correspond to the first order conditions of the above optimization problem. The Lagrange multiplier is the price level P_t of domestic products.

$$\frac{P_{k,t}}{P_t} = \omega_k^{Q\,\frac{1}{\eta^Q}} \left(\frac{Y_{k,t}}{Y_t} \right)^{\frac{-1}{\eta^Q}} \tag{13}$$

In order to model regional economic activity we further decompose the production process on a regional level. One can either think about this approach as modeling the optimization problem of a representative firm operating in one sector on a national level allocating production activity across the nation. Another way is to consider that households make direct purchases from regional operating firms in one sector. In this case the following optimization problem would be part of the above optimization problem.

$$\underset{Y_{k,r,t}}{min} \sum_k Y_{k,r,t}\, P_{k,r,t} \tag{14}$$

$$Y_{k,t} = \left(\sum_k \omega_{k,r}^{Q\,\frac{1}{\eta_k^Q}} Y_{k,r,t}^{\frac{\eta_k^Q - 1}{\eta_k^Q}} \right)^{\frac{\eta_k^Q}{\eta_k^Q - 1}} \tag{15}$$

Demand for sectoral and regional products correspond to the first order conditions of the above optimization problem. The Lagrange multiplier is the sectoral price level $P_{k,t}$ of domestic products.

$$\frac{P_{k,r,t}}{P_{k,t}} = \omega_{k,r}^{Q\,\frac{1}{\eta_k^Q}} \left(\frac{Y_{k,r,t}}{Y_{k,t}} \right)^{\frac{-1}{\eta_k^Q}} \tag{16}$$

At the regional and sectoral level are representative firms maximizing profits using capital $K_{k,r,t}$ and labour $L_{k,r,t} = N_{k,r,t}\, PoP_t$ provided by households to produce products. They charge a price $P_{k,r,t}$ for their products and have to pay households wages $W_{k,r,t}$, interest on rented capital $P_{r,k,t}\, r_{r,k,t}$, taxes related to the wage bill $\tau^N_{r,k,t}$ and on capital expenditure $\tau^K_{r,k,t}$. Representative firms have access to a regional and sector specific constatn elasticity of substitution production function. The productivity of capital and labour of a firm in one sector and region depends on the climate variables, and the adaption measures by the government represented by a damage function $D_{k,r,t} = D_{k,r}\left(T_{r,t},\, PREC_{r,t},\, W^S_{r,t},\, SL_{r,t}\, G^A_{r,k,t}\right)$ and the exogenous level of productivity $A_{k,r,t}$. As in Nordhaus (1993) we assume a polynomial functional form of the damage functions, but the damages are different across regions and sectors eq. 17.

$$
\begin{aligned}
D_{k,r_t} =& exp\left(-\phi^{G^A_{k,r}}\, G^A_{k,r,t}\right) \Bigg(\\
& \underbrace{a_{T,1,k,r}\, T_{rt} + a_{T,2,k,r}\ (T_{rt})^{a_{T,3,k,r}}}_{\text{impact of temperature}} + \underbrace{a_{SL,1,k,r}\, SL_t + a_{SL,2,k,r}\ (SL_t)^{a_{SL,3,k,r}}}_{\text{impact of sea level}} \\
& + \underbrace{a_{W^S,1,k,r}\, W^S_{r\ t} + a_{W^S,2,k,r}\ (W^S_{r\ t})^{a_{W^S,3,k,r}}}_{\text{impact of wind speed}} + \underbrace{a_{PREC,1,k,r}\, PREC_{rt} + a_{PREC,2,k,r}\ (PREC_{rt})^{a_{PREC,3,k,r}}}_{\text{impact of precipitation}} \\
& \Bigg).
\end{aligned} \tag{17}
$$

Firms in each region and sector have access to a constant elasticity of substitution production function with production factors labour and capital. Eq. 18 states the optimization problem of the firm.

$$
\begin{aligned}
& \underset{Y_{k,r,t},N_{k,r,t},K_{k,r,t}}{max}\ P_{k,r,t}\, Y_{k,r,t} - W_{k,r,t}\, N_{k,r,t}\, PoP_t - r_{k,r,t} P_{k,r,t} K_{k,r,t} \\
\text{s.t.}\ Y_{k,r,t} = A_{k,r,t}(1 - D_{k,r,t}) & \left[{\alpha^N_{k,r}}^{\frac{1}{\eta^{NK}_{k,r}}}\ (PoP_t\, N_{k,r,t})^{\rho_{k,r}} + {\alpha^K_{k,r}}^{\frac{1}{\eta^{NK}_{k,r}}}\ (K_{k,r,t})^{\rho_{k,r}} \right]^{\frac{1}{\rho_{k,r}}}, \\
& \text{where}\ \rho_{k,r} = \frac{\eta^{NK}_k - 1}{\eta^{NK}_k}.
\end{aligned} \tag{18}
$$

Demand for production factors are given by the first order condition of the above optimization problem. The Lagrange multiplier is equal to the price charged by companies.

$$
\frac{W_{k,r,t}}{P_{k,r,t}} = {\alpha^N_{k,r}}^{\frac{1}{\eta^{NK}_{k,r}}} \left(\frac{PoP_t N_{k,r,t}}{Y_{k,r,t}} \right)^{-\frac{1}{\eta^{NK}_{k,r}}}
$$

$$
r_{k,r,t} = {\alpha^K_{k,r}}^{\frac{1}{\eta^{NK}_{k,r}}} \left(\frac{K_{k,r,t}}{Y_{k,r,t}} \right)^{-\frac{1}{\eta^{NK}_{k,r}}} \tag{19}
$$

$$
\tag{20}
$$

We use the more general case of the CES production function rather than the more commonly used Cobb-Douglas production function. The parameter $\eta^{NK}_{k,r}$ allows us to control the response of capital and labour demand to temporary productivity shocks. Temporary productivity shocks are in our set-up also weather extremes. We will discuss the problem later.

2.4 Rest of the world

The demand for domestic exports and foreign imports is not explicitly modeled in this version of the model. We assume that net exports follow an auto-regressive process of order one and that the long run value of net exports depend on the long-run development of gross domestic product. We therefore assume that imports and exports will grow at the same speed as GDP. Sluggish adjusmtents in export and import behavior of companies is captured by an auto-regressive process.

$$
NX_t = \rho^{NX}\, NX_{t-1} + (1 - \rho^{NX})\omega^{NX}\, P_t\, Y_t \tag{21}
$$

The effective exchange rate S^f_t and the world interest rate r^f_t determine how much governments and households have to pay back in domestic currency as net lender or how much they receive as net borrower to the rest of the world. Here the world interest rate is independent of domestic developments and only the effective exchange rate adjusts according to eq. 6.

3 Scenario Analysis

4 How to use the model?

4.1 Usage

1. In order to use the model you need to install Dynare (at least version 4.6.1) and Matlab (at least 2018b) or Octave on your computing machine. For Octave you need to have the version 5.2.0 as reported by the Dynare team.

2. You need to download the repository from Github.

3. Open Octave or Matlab GUI and browse to the location of the folder in your computer. You have the right folder if the command `pwd()` returns `YourPath/DGE-CRED/DGE_CRED_Model`.

4. The script `RunSimulations.m` has to be executed in order to run simulations for different scenarios. Make sure that the scenarios and model parameters are defined in the file `ModelSimulationandCalibrationKSectorsandRRegions.xlsx`. We need to adopt the number of sectors and regions in the file `DGE_CRED_Model.mod`.

5. The simulation results are stored in the file `ResultsScenariosKSectorsandRRegions.xlsx`.

5 Folder structure

1. The main file containing all necessary mod files is `DGE_CRED_Model.mod`. This file includes the following files stored in the `ModFiles` folder:

   (a) `DGE_CRED_Model_Declarations.mod` declares all endogenous and exogenous variables if the model and structural parameters.

   (b) `DGE_CRED_Model_Parameters.mod` assigns values to the structural parameters of the model.

   (c) `DGE_CRED_Model_Equations.mod` contains the equations of the model.

   (d) `DGE_CRED_Model_LatexOutput.mod` produces latex output for documentation of the declared variables and model equations.

   (e) `DGE_CRED_Model_SteadyState.mod` computes initial and terminal condition for the dynamic simulation.

   (f) `DGE_CRED_Model_Simulations.mod` starts the dynamic simulation.

2. Subroutines responsible for finding the initial and terminal conditions are located in the subfolder `Functions`:

   (a) `Calibration.mat` finds the initial conditions to reflect a specific year of the economy.

   (b) `FindA.mat` looks for exogenous productivity shocks across sectors and regions to meet the terminal conditions.

   (c) `FindK.mat` looks for a capital allocation across sectors and regions to fulfill the static equations of the model.

   (d) `rng.mat` random number generator function necessary for Octave users.

   (e) `LoadExogenous.mat` reads exogenous variables for different scenarios.

3. To define scenarios and structural parameters one needs to create an Excel workbook located in the subfolder `ExcelFiles`:

   (a) `ModelSimulationandCalibrationforKSectorsandRregions.xlsx` has multiple sheets:

      i. initial `Start`

      ii. terminal `Terminal`

      iii. parameters to define rigidity parameters `Dynamics`

      iv. elasticity parameters and tax rates `Structural Parameters`

      v. coefficients for regional and sector specific damage functions `Climate Damage Functions`

      vi. `Baseline` scenario and other optional scenario sheets defining paths for exogenous varibales

   (b) `ResultsScenariosKSectorsandRregions.xlsx` has as many sheets as Scenarios defined in the previous Excel file.

4. The latex files produced by `DGE_CRED_Model_LatexOutput.mod` are stored in `LatexFiles`.

   (a) the system of dynamic equations as implemented in Matlab `DGE_CRED_Model_Dynamic`, `DGE_CRED_Model_Dynam`

   (b) names of endogenous, exogenous variables and parameters `DGE_CRED_Model_latex_definitions`

   (c) the system of dynamic equations in original form without auxiliary variables for leads and lags `DGE_CRED_Model_original`, `DGE_CRED_Model_original_content`

5. The file to run different simulations is `RunSimulations.m`.

6. A Matlab function to find solutions to the static system of equations is `DGE_CRED_Model_steady_state.m`.

References

Christiano, L. J., Motto, R. & Rostagno, M. (2014), 'Risk shocks', *American Economic Review* **104**(1), 27–65.

Nordhaus, W. D. (1993), 'Optimal greenhouse-gas reductions and tax policy in the" dice" model', *The American Economic Review* **83**(2), 313–317.

Stocker, T. F., Qin, D., Plattner, G.-K., Tignor, M., Allen, S. K., Boschung, J., Nauels, A., Xia, Y., Bex, V., Midgley, P. M. et al. (2013), 'Climate change 2013: The physical science basis', *Contribution of working group I to the fifth assessment report of the intergovernmental panel on climate change* **1535**.

A Model Equations

A.1 Regional Industries

Damage function

$$D_{k,r_t} = exp\left(-\phi^{G^A_{k,r}}\, G^A_{k,r,t}\right)\,\left(a_{1,k,r}\, T_{rt} + a_{2,k,r}\,(T_{rt})^{a_{3,k,r}} + a_{1,k,r}\, SL_t + a_{2,k,r}\,(SL_t)^{a_{3,k,r}} + a_{1,k,r}\, W^S_{r\ t} + a_{2,k,r}\,\left(W^S_{r\ t}\right)^{a_{3,k,r}} + a_{1,k,r}\, PREC_{rt} + a_{2,k,r}\,(PREC_{rt})^{a_{3,k,r}}\right) \tag{22}$$

Government expenditure for adaptation measures

$$G^A_{k,r,t} = \eta_{G^A,k,r,t} \tag{23}$$

TFP

$$A_{k,r,t} = A_{k,r,0}\, exp\left(\eta_{A,k,r,t}\right) \tag{24}$$

capital specific productivity

$$A^K_{k,r,t} = A^K_{k,r,0}\, exp\left(\eta_{A^K,k,r,t}\right) \tag{25}$$

labour specific productivity

$$A^N_{k,r,t} = A^N_{k,r,0}\, exp\left(\eta_{A^N,k,r,t}\right) \tag{26}$$

Price of regional sectoral goods

$$\frac{P_{k,r_t}}{P_{k_t}} = {\omega^Q_{k,r}}^{\frac{1}{\eta^Q_k}}\,\left(\frac{Y_{k,r_t}}{Y_{k_t}}\right)^{\frac{(-1)}{\eta^Q_k}} \tag{27}$$

Production function

$$Y_{k,r_t} = A_{k,r_t}\,\left(1 - D_{k,r_t}\right)\left({\alpha^K_{k,r}}^{\frac{1}{\eta^{N,K}_{k,r}}}\,\left(A^K_{k,r_t}\, K_{k,r_{t-1}}\right)^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}} + {\alpha^N_{k,r}}^{\frac{1}{\eta^{N,K}_{k,r}}}\,\left(A^N_{k,r_t}\, PoP_t\, N_{k,r_t}\right)^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}}\right)^{\frac{\eta^{N,K}_{k,r}}{\eta^{N,K}_{k,r}-1}} \tag{28}$$

Firms FOC capital

$$r_{k,r_t}\,\left(1 + \tau^K_{k,r,t}\right) = {\alpha^K_{k,r}}^{\frac{1}{\eta^{N,K}_{k,r}}}\,{A^K_{k,r_t}}^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}}\,\left(\frac{K_{k,r_{t-1}}}{Y_{k,r_t}}\right)^{\frac{-1}{\eta^{N,K}_{k,r}}} \tag{29}$$

Firms FOC labour

$$\frac{W_{k,r_t}\left(1 + \tau^N_{k,r,t}\right)}{P_{k,r_t}} = {\alpha^N_{k,r}}^{\frac{1}{\eta^{N,K}_{k,r}}}\,\left(\frac{A^N_{k,r_t}\, PoP_t\, N_{k,r_t}}{Y_{k,r_t}}\right)^{\frac{-1}{\eta^{N,K}_{k,r}}} \tag{30}$$

A.2 Retailer for Aggregation

Relative price of sectoral output

$$\frac{P_{k_t}}{P_t} = {\omega^Q_k}^{\frac{1}{\eta^Q}}\,\left(\frac{Y_{k_t}}{Y_t}\right)^{\frac{(-1)}{\eta^Q}} \tag{31}$$

Sectoral CES aggregation

$$Y_k, t = \left(\sum_r^R {\omega^Q_{k,r}}^{\frac{1}{\eta^Q_k}}\, {Y_{k,r_t}}^{\frac{\eta^Q_k-1}{\eta^Q_k}}\right)^{\frac{\eta^Q_k}{\eta^Q_k-1}} \tag{32}$$

A.3 Households

Households FOC labour

$$\frac{W_{k,r_t}\,\left(1 - \tau^N\right)\,\left(\frac{C_t}{PoP_t}\right)^{\left(-\sigma^C\right)}}{\left(1 + \tau^C\right)\, P_t} = \phi^L\, {N_{k_t}}^{\sigma^L} \tag{33}$$

Households FOC capital

$$\frac{\left(\frac{P_{t+1}\, C_{t+1}}{PoP_{t+1}}\right)^{\left(-\sigma^C\right)}}{\left(1 + \tau^C\right)\, P_{t+1}}\,\beta\, P_{k,r_{t+1}}\, r_{k,r_{t+1}}\,\left(1 - \tau^K\right) + \beta\,\omega^I_{k,r_{t+1}}\,(1 - \delta) = \omega^I_{k,r_t} \tag{34}$$

Households FOC investment

$$P_{k,r,t}\,\frac{\left(\frac{C_t}{PoP_t}\right)^{\left(-\sigma^C\right)}}{P_t\,\left(1 + \tau^C\right)} = \omega^I_{k,r_t}\,\frac{\left(\frac{C_t}{PoP_t}\right)^{\left(-\sigma^C\right)}}{P_t\,\left(1 + \tau^C\right)}\,\left(S\left(\frac{I_{k,r_t}}{I_{k,r_{t-1}}}\right) - S'\left(\frac{I_{k,r_t}}{I_{k,r_{t-1}}}\right)\,\left(\frac{I_{k,r_t}}{I_{k,r_{t-1}}}\right)\right) + \omega^I_{k,r_{t+1}}\,\frac{\left(\frac{C_{t+1}}{PoP_{t+1}}\right)^{\left(-\sigma^C\right)}\,\beta}{\left(1 + \tau^C\right)\, P_{t+1}}\, S'\left(\frac{I_{k,r_{t+1}}}{I_{k,r_t}}\right)\,\frac{{I_{k,r_{t+1}}}^2}{{I_{k,r_t}}^2} \tag{35}$$

Households LOM capital

$$K_{k,r_t} = K_{k,r_{t-1}}\, (1-\delta) + I_{k,r,t}\, S\left(\frac{I_{k,r_t}}{I_{k,r_{t-1}}}\right) \tag{36}$$

Households FOC foreign bonds

$$\frac{\left(\frac{C_{t+1}}{PoP_{t+1}}\right)^{(-\sigma^C)}}{(1+\tau^C)\, P_{t+1}}\, \beta\, S^f{}_{t+1}\, \exp\left(-\phi^B\, \left(\frac{B_t\, S^f{}_{t+1}\, r^f{}_{t+1}}{Y_{t+1}} + \frac{NX_t}{Y_t}\right)\right) \left(1 + r^f{}_{t+1}\right) = \frac{\left(\frac{C_t}{PoP_t}\right)^{(-\sigma^C)}}{P_t\, (1+\tau^C)} \tag{37}$$

A.4 Climate Variables

Temperature

$$T_{rt} = T_{0,r} + \eta_{T,r_t} \tag{38}$$

Wind speed

$$W^S_{r\ t} = W^S_{0,r} + \eta_{W^S,r_t} \tag{39}$$

PRECipitation

$$PREC_{rt} = PREC_{0,r} + \eta_{PREC,r_t} \tag{40}$$

Sea level

$$SL_t = SL_0 + \eta_{SL\,t} \tag{41}$$

A.5 Trade

Trade balance

$$NX_t = -\left(B_t - \left(1 + r^f{}_t\right)\, S^f_t\, B_{t-1}\right) \tag{42}$$

Net exports

$$NX_t = \rho^{NX}\, NX_{t-1} + Y_t\, \left(1 - \rho^{NX}\right)\, exp\left(\eta_{NX\,t}\right)\, \omega^{NX} \tag{43}$$

foreign interest rates

$$r^f_t = \bar{r}^f \tag{44}$$

A.6 Government

Budget constraint

$$\begin{aligned} P_t\, G_t + \sum_r^R \sum_k^K P_t\, G^A_{k,r,t} + P_t\, S^f_t\, \left(1 + r^f{}_t\right)\, BG_{t-1} = P_t\, BG_t + C_t\, P_t\, \tau^C + \sum_k^K \sum_r^R N_{k,r,t}\, W_{k,r,t}\, \left(\tau^N + \tau^N_{k,r,t}\right) \\ + K_{k,r_t}\, r_{k,r_t}\, P_{k,r_t}\, \left(\tau^K + \tau^K_{k,r,t}\right) \end{aligned} \tag{45}$$

Government foreign debt

$$BG_t = \eta_{BG\,t} \tag{46}$$

Tax rates on capital expenditure

$$\tau^K_{k,r,t} = \tau^K_{k,r,0} + \eta^{\tau^K}_{k,r,t} \tag{47}$$

Tax rates on labour compensation

$$\tau^N_{k,r,t} = \tau^N_{k,r,0} + \eta^{\tau^N}_{k,r,t} \tag{48}$$

A.7 Aggregates

National price level

$$P_t = exp\left(\eta_{P\,t}\right) \tag{49}$$

National population

$$PoP_t = \rho^{PoP}\, PoP_{t-1} + \left(1 - \rho^{PoP}\right)\, PoP_0\, exp\left(\eta_{PoP\,t}\right) \tag{50}$$

Resource constraint

$$Y_t = C_t + I_t + G_t + \sum_k^K \sum_r^R G^A_{k,r,t} - NX_t \tag{51}$$

Sector labour

$$N_{k\,t} = \sum_r^R N_{k,r_t} \tag{52}$$

Sector wage bill

$$N_{k\,t}\, W_{k\,t} = \sum_r^R N_{k,r_t}\, W_{k,r_t} \tag{53}$$

Sector investment

$$P_{k\,t}\,I_{k\,t} = \sum_{r}^{R} P_{k,r\,t}\,I_{k,r\,t} \tag{54}$$

Sector capital stock

$$P_{k\,t}\,K_{k\,t} = \sum_{r}^{R} P_{k,r\,t}\,K_{k,r\,t} \tag{55}$$

National investment

$$P_t\,I_t = \sum_{k}^{K} P_{k\,t}\,I_{k\,t} \tag{56}$$

National capital

$$P_t\,K_t = \sum_{k}^{K} P_{k\,t}\,K_{k\,t-1} \tag{57}$$

National output

$$P_t\,Y_t = \sum_{k}^{K} P_{k\,t}\,Y_{k\,t} \tag{58}$$

National labour share

$$N_t = \sum_{k}^{K} N_{k\,t} \tag{59}$$

Table 1: Endogenous

Variable	LaTeX	Description
P	P	price level
K	K	capital stock
C	C	consumption
PoP	PoP	population
B	B	international traded bonds
Sf	S^f	effective exchange rate with the rest of the world
BG	BG	government debt
NX	NX	net exports
rf	r^f	foreign interest rate
G	G	government expenditure
I	I	private investment
Y	Y	GDP
N	N	labour
SL	SL	sea level
PREC_1	$PREC_r$	regional PRECipitation
T_1	T_r	regional temperature
WS_1	W_r^S	regional wind speed
Y_1	Y_k	sector GDP
K_1	K_k	sector capital
N_1	N_k	sector employment
I_1	I_k	sector private investment
P_1	P_k	sector price index
W_1	W_k	sector wage index
Y_1_1	$Y_{k,n}$	regional sector GDP
D_1_1	$D_{k,n}$	regional sector damages
K_1_1	$K_{k,n}$	regional sector capital
N_1_1	$N_{k,n}$	regional sector employment
W_1_1	$W_{k,n}$	regional sector wages
A_1_1	$A_{k,n}$	regional sector TFP
G_A_1_1	$G_{k,n}^A$	regional sector adaptation government expenditure
gA_1_1	$g_{k,n}^A$	regional growth rate of sector TFP
A_N_1_1	$A_{k,n}^N$	regional sector labour specific TFP
A_K_1_1	$A_{k,n}^K$	regional sector capital specific TFP
I_1_1	$I_{k,n}$	regional sector private investment
P_1_1	$P_{k,n}$	regional sector price index
omegaI_1_1	$\omega_{k,n}^I$	shadow value of regional private sector investment
r_1_1	$r_{k,n}$	regional sector rental rate on capital
tauK_1_1	$\tau_{k,n}^K$	regional sector corporate tax rate on capital
tauN_1_1	$\tau_{k,n}^N$	regional sector labour tax rate on capital

Table 2: Exogenous

Variable	LaTeX	Description
exo_P	η_P	exogeneous price index evolution
exo_PoP	η_{PoP}	exogeneous population
exo_SL	η_{SL}	exogeneous sea level
exo_NX	η_{NX}	exogenous net exports
exo_BG	η_{BG}	exogenous structural balance
exo_tauK_1_1	$\eta_{\tau^K,k,n}$	exogenous sector and region corporate tax rate
exo_tauN_1_1	$\eta_{\tau^N,k,n}$	exogenous sector and region labour tax rate
exo_1_1	$\eta_{A,k,n}$	exogenus TFP
exo_N_1_1	$\eta_{A^N,k,n}$	exogenous labour specific TFP
exo_K_1_1	$\eta_{A^K,k,n}$	exogenous capital specific TFP
exo_GA_1_1	$\eta_{G^A,k,n}$	exogenous sector adaptation expenditure
exo_T_1	$\eta_{T,n}$	exogenus regional temperature
exo_PREC_1	$\eta_{PREC,n}$	exogenus regional PRECipitation
exo_WS_1	$\eta_{W^S,n}$	exogenus regional wind speed

Table 3: Parameters

Variable	LaTeX	Description
omegaQ_1_p	$\omega^{Q}_{k,}$	distribution parameter for output from one sector
etaQ_1_p	η^{C}_{k}	elasticity of substitution between regional production
tauK_1_1_p	$\tau^{K}_{k,n}$	region and sector specific tax rate on capital
tauN_1_1_p	$\tau^{N}_{k,n}$	region and sector specific tax rate on labour
rhoA_1_1_p	$\rho^{A}_{k,n}$	persistence productivity shock
rhoA_N_1_1_p	$\rho^{A,N}_{k,n}$	persistence labour specific productivity shock
rhoA_K_1_1_p	$\rho^{A,K}_{k,n}$	persistence capital specific productivity shock
phiY_1_1_p	$\frac{P_{k,n,0}\,Y_{k,n,0}}{P_0\,Y_0}$	long-run share of regional and sectoral output
phiN_1_1_p	$N_{k,n,0}$	long run share of regional and sectoral employment
phiY0_1_1_p	$\frac{P_{k,n,0}\,Y_{k,n,0}}{P_0\,Y_0}$	terminal share of regional and sectoral output
phiN0_1_1_p	$N_{k,n,0}$	initial share of regional and sectoral employment
phiYT_1_1_p	$\frac{P_{k,n,0}\,Y_{k,n,0}}{P_0\,Y_0}$	terminal share of regional and sectoral output
phiNT_1_1_p	$N_{k,n,0}$	terminal share of regional and sectoral employment
phiW_1_1_p	$\frac{W_{k,n,0}\,N_{k,n,0}}{P_{k,n,0}\,Y_{k,n,0}}$	share of regional and sectoral employment
phiP_1_1_p	$\frac{P_{k,n,0}}{P_0}$	share of regional and sectoral employment
phiL_1_1_p	$\phi^{L}_{k,n}$	coefficient of disutility to work
omegaQ_1_1_p	$\omega^{Q}_{k,n}$	distribution parameter for regional production
alphaK_1_1_p	$\alpha^{K}_{k,n}$	distribution parameter capital share
alphaN_1_1_p	$\alpha^{N}_{k,n}$	distribution parameter labour share
etaNK_1_1_p	$\eta^{N,K}_{k,n}$	elasticity of substitution between labour and capital
A_1_1_p	$A_{k,n}$	sector long-run TFP
GAT_1_1_p	$G^{A}_{T,k,n}$	sector region specific government expenditure for adaptation
phiGA_1_1_p	$\phi^{G^{A}}_{k,n}$	coefficient of effectiveness of government expenditure on adaptation measures in a specific region and se
gY0_1_1_p	$\frac{Y_{2,k,n}}{Y_{1,k,n}}$	initial sector growth
gN0_1_1_p	$\frac{\frac{N_{2,k,n}}{N_2}}{\frac{N_{1,k,n}}{N_1}}$	initial sector labour growth
omegaA_1_1_p	$\omega^{A}_{k,n}$	exponent for productivity growth
A_N_1_1_p	$A^{N}_{k,n}$	sector labour specific TFP
A_K_1_1_p	$A^{K}_{k,n}$	sector capital specific TFP
a_T_1_1_1_p	$a_{1,k,n}$	intercept of damage function temperature
a_T_2_1_1_p	$a_{2,k,n}$	slope of damage function temperature
a_T_3_1_1_p	$a_{3,k,n}$	exponent of damage function temperature
a_P_1_1_1_p	$a_{1,k,n}$	intercept of damage function PRECipitation
a_P_2_1_1_p	$a_{2,k,n}$	slope of damage function PRECipitation
a_P_3_1_1_p	$a_{3,k,n}$	exponent of damage function PRECipitation
a_W_1_1_1_p	$a_{1,k,n}$	intercept of damage function wind speed
a_W_2_1_1_p	$a_{2,k,n}$	slope of damage function wind speed
a_W_3_1_1_p	$a_{3,k,n}$	exponent of damage function wind speed
a_SL_1_1_1_p	$a_{1,k,n}$	intercept of damage function sea level
a_SL_2_1_1_p	$a_{2,k,n}$	slope of damage function sea level
a_SL_3_1_1_p	$a_{3,k,n}$	exponent of damage function sea level
beta_p	β	discount factor
delta_p	δ	capital depreciation rate
sigmaL_p	σ^{L}	inverse Frisch elasticity
sigmaC_p	σ^{C}	intertemporal elasticity of substitution
etaQ_p	η^{Q}	elasticity of substitution between sectoral production
phiB_p	ϕ^{B}	coefficient of foreign adjustment cost
phiK_p	ϕ^{K}	coefficient of investment adjustment cost
tauC_p	τ^{C}	consumption tax
tauN_p	τ^{N}	labour tax
tauK_p	τ^{K}	capital tax
omegaNX_p	ω^{NX}	share of net exports relative to domestic GDP
omegaNX0_p	$\omega^{NX,0}$	initial share of net exports relative to domestic GDP
omegaNXT_p	$\omega^{NX,T}$	terminal share of net exports relative to domestic GDP
rhoNX_p	ρ^{NX}	persistency in net exports
rhoA_p	ρ^{A}	persistency in TFP
rhoPoP_p	ρ^{PoP}	persistency in population
rhoSL_p	ρ^{SL}	persistency in sea level
rhoT_p	ρ^{T}	persistency in temperature
rhoWS_p	ρ^{T}	persistency in wind speed

Table 3 – Continued

Variable	LaTeX	Description
rhoPREC_p	ρ^T	persistency in PRECipitation
inbsectors_p	K	number of sectors
inbregions_p	R	number of regions
lCalibration_p	l^{Calib}	logical indiactor whether model is calibrated or not
T0_1_p	$T_{0,n}$	initial regional temperature
PREC0_1_p	$PREC_{0,n}$	initial regional PRECipitation
WS0_1_p	$W^S_{0,n}$	initiial regional wind speed
TT_1_p	$T_{T,n}$	terminal regional temperature
PRECT_1_p	$PREC_{T,n}$	terminal regional PRECipitation
WST_1_p	$W^S_{T,n}$	terminal regional wind speed
SL0_p	SL_0	initial sea level
PoP0_p	POP_0	initial population
Y0_p	Y_0	initial output
P0_p	P_0	initial price level
N0_p	Y_0	initial employment
SLT_p	SL_0	terminal sea level
PoPT_p	PoP_0	terminal population
YT_p	Y_0	terminal output
NT_p	Y_0	terminal employment