DGE–CRED Practice Session 4 and 5: Implementing the DGE–CRED Model

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IWH Halle Institute for Economic Research
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Outline II

Outline III

Outline

1. Task 1

Task 1: Try to run the `DGE_CRED_Model.mod` model stored in the folder `DGE_CRED_Model_Training`.

- Make sure that the number of sectors and regions is 3, respectively.
- Execute the `RunSimulations.m` file
- Why do you receive an error?
- How can you resolve the error?

Solution Task 1: Try to run the `DGE_CRED_Model.mod` model stored in the folder `DGE_CRED_Model_Training`.

- `DGE_CRED_Model_Equations.mod` model is not in the correct subfolder `ModFiles`
- You need to copy the respective file in the correct subfolder.

```
@# include "ModFiles/DGE_CRED_Model_AuxiliaryVariables.mod"
@# include "ModFiles/DGE_CRED_Model_Declaration.mod"
@# include "ModFiles/DGE_CRED_Model_Equations.mod"
```

Outline

Task 2: Try to run the `DGE_CRED_Model.mod` model stored in the folder `DGE_CRED_Model_Training`.

- Make sure that the number of sectors and regions is 3, respectively.
- Execute the `RunSimulations.m` file
- Why do you receive an error?
- How can you resolve the error?

Solution Task 2: Try to run the `DGE_CRED_Model.mod` model stored in the folder `DGE_CRED_Model_Training`.

- You need to make sure that the declaration file is included.
- You just need to remove the comment command in the mod file in line 42.

```
@# include "ModFiles/DGE_CRED_Model_AuxiliaryVariables.mod"
@# include "ModFiles/DGE_CRED_Model_Declaration.mod"
@# include "ModFiles/DGE_CRED_Model_Equations.mod"
```

Outline

3 Task 3

Task 3: Enter the FOC of households with respect to capital.

- Missing Equations are represented by $0 = 0$ in the `DGE_CRED_Model_Equations.mod` file.

$$\frac{\left(\frac{P_{t+1}\, C_{t+1}}{Pop_{t+1}}\right)^{(-\sigma^C)}}{\left(1+\tau_t^C\right)\, P_{t+1}}\, \beta\, P_{k,r\,t+1}\, r_{k,r\,t+1}\, \left(1-\tau_{t+1}^{K,H}\right) + \beta\, \omega^I_{k,r\,t+1}\, \left(1-\delta-D^K_{k,r,t+1}\right) = \omega^I_{k,r\,t} \tag{1}$$

Solution Task 3: Enter the FOC of households with respect to capital.

```
[name = 'HH FOC capital',mcp = 'K_@{sec}_@{reg}>0']
(C(+1)/PoP(+1))^(-sigmaC_p)/(P(+1) * (1 + tauC(+1))) * beta_p * r_@{sec}_@{reg}(+1) * P_@{sec}_@{reg}(+1) * (1
      - tauKH(+1)) + (C(+1)/PoP(+1))^(-sigmaC_p)/(P(+1)*(1 + tauC(+1))) * omegal_@{sec}_@{reg}(+1) * beta_p *
     (1 - delta_p - D_K_@{sec}_@{reg}(+1)) = omegal_@{sec}_@{reg} * (C/PoP)^(-sigmaC_p)/(P*(1 + tauC));
```

Outline

4. Task 4

Task 4: Enter the law of motion for capital.

- Missing Equations are represented by $0 = 0$ in the `DGE_CRED_Model_Equations.mod` file.

$$K_{k,r_{t+1}} = K_{k,r_t}\left(1 - \delta - D^K_{k,r,t}\right) + I_{k,r,t}\,\Gamma\left(\frac{I_{k,r_t}}{I_{k,r_{t-1}}}\right) \tag{2}$$

Solution Task 4: Enter the FOC of households with respect to capital.

```
[name = 'LOM capital',mcp = 'I_@{sec}_@{reg} > 0']
K_@{sec}_@{reg} = (1 - delta_p - D_K_@{sec}_@{reg}) * K_@{sec}_@{reg}(-1) + I_@{sec}_@{reg} * (1 - (exp(sqrt(
      phiK_p / 2)*(I_@{sec}_@{reg}/I_@{sec}_@{reg}(-1)-1)) + exp(-sqrt(phiK_p / 2) * (I_@{sec}_@{reg}/I_@{sec}
      _@{reg}(-1)-1)) - 2));
```

Outline

5 Task 5

Task 5: Enter the FOC of households with respect to labour.

- Missing Equations are represented by $0 = 0$ in the `DGE_CRED_Model_Equations.mod` file.

$$\frac{W_{k,r\,t}\left(1-\tau_t^{N,H}\right)\left(\frac{C_t}{Pop_t}\right)^{\left(-\sigma^C\right)}}{\left(1+\tau_t^C\right)\,P_t} = \phi^L\,{N_{k\,t}}^{\sigma^L} \tag{3}$$

Solution Task 5: Enter the FOC of households with respect to labour.

```
[name = 'HH FOC labour',mcp = 'N_@{sec}_@{reg}>0']
(1 - tauNH) * W_@{sec}_@{reg} * (C/PoP)^(-sigmaC_p) / (P * (1 + tauC)) = A_N_@{sec}_@{reg} * phiL_@{sec}_@{reg
    }_p * (N_@{sec}_@{reg})^(sigmaL_p);
```

Outline

6 Task 6

Task 6: Enter the Euler equation for foreign assets.

- Missing Equations are represented by $0 = 0$ in the `DGE_CRED_Model_Equations.mod` file.

$$\frac{\left(\frac{C_{t+1}}{Pop_{t+1}}\right)^{(-\sigma^C)}}{\left(1+\tau_t^C\right) P_{t+1}} \beta \, S^f{}_{t+1} \exp\left(-\phi^B \left(\frac{B_{t+1} \, S^f{}_{t+1} \, r^f{}_{t+1}}{Y_{t+1}} + \frac{NX_{t+1}}{Y_{t+1}}\right)\right) \left(1 + r^f{}_{t+1}\right) = \ldots \quad (4)$$

$$\frac{\left(\frac{C_t}{Pop_t}\right)^{(-\sigma^C)}}{P_t \left(1+\tau_t^C\right)}$$

Solution Task 6: Enter the Euler equation for foreign assets.

```
[name = 'Foreign Assets']
(C(+1)/PoP(+1))^(-sigmaC_p)/(P(+1)*(1 + tauC(+1))) * beta_p * Sf(+1) * exp(-phiB_p*((Sf(+1)*rf(+1)*B/Y(+1)+NX
      (+1)/Y(+1)))) * (1 + rf(+1)) = (C/PoP)^(-sigmaC_p)/(P*(1 + tauC(+1)));
```

Outline

7 Task 7

Task 7: Enter the Euler equation for foreign assets.

- Missing Equations are represented by $0 = 0$ in the `DGE_CRED_Model_Equations.mod` file.

$$\frac{\left(\frac{C_{t+1}}{Pop_{t+1}}\right)^{(-\sigma^C)}}{\left(1+\tau_t^C\right) P_{t+1}} \beta \, S^f{}_{t+1} \exp\left(-\phi^B \left(\frac{B_{t+1} \, S^f{}_{t+1} \, r^f{}_{t+1}}{Y_{t+1}} + \frac{NX_{t+1}}{Y_{t+1}}\right)\right) \left(1 + r^f{}_{t+1}\right) = \ldots \quad (5)$$

$$\ldots \frac{\left(\frac{C_t}{Pop_t}\right)^{(-\sigma^C)}}{P_t \left(1+\tau_t^C\right)}$$

Solution Task 7: Enter the Euler equation for foreign assets.

```
[name = 'Foreign Assets']
(C(+1)/PoP(+1))^(-sigmaC_p)/(P(+1)*(1 + tauC(+1))) * beta_p * Sf(+1) * exp(-phiB_p*((Sf(+1)*rf(+1)*B/Y(+1)+NX
      (+1)/Y(+1)))) * (1 + rf(+1)) = (C/PoP)^(-sigmaC_p)/(P*(1 + tauC(+1)));
```

Outline

8 Task 8

Task 8: Complete the government budget constraint.

$$P_t\, G_t + \sum_{r}^{R} \sum_{k}^{K} P_t \sum_{z} G_{k,r,t}^{A,z} + P_t\, B_{t+1}^{G} = P_t\, S_t^{f}\ \left(1 + r^{f}{}_{t}\right)\ B_t^{G} \ldots \qquad (6)$$

$$+ C_t\, P_t\, \tau_t^{C} + \sum_{k}^{K} \sum_{r}^{R} N_{k,r,t}\, W_{k,r,t} \left(\tau_t^{N,H} + \tau_{k,r,t}^{N,F}\right) + K_{k,r_t}\, r_{k,r_t}\, P_{k,r_t} \left(\tau_t^{K,H} + \tau_{k,r,t}^{K,F}\right)$$

Solution Task 8: Complete the government budget constraint.

```
[name = 'Government Budget Constraint']
G + BG
@# for sec in 1:Sectors
    @# for reg in 1:Regions
        @# for z in ClimateVars
            + G_A_@{z}_@{sec}_@{reg}
        @# endfor
    @# endfor
@# endfor
= (1 + rf) * Sf * exp(-phiB_p*((Sf*rf*B(-1)/Y+NX/Y))) * BG(-1) + tauC * C
@# for sec in 1:Sectors
    @# for reg in 1:Regions
        + (tauNH + tauNF_@{sec}_@{reg}) * W_@{sec}_@{reg} * N_@{sec}_@{reg} * PoP/P + (tauKH + tauKF_@{sec}_@{
            reg}) * P_@{sec}_@{reg} /P * r_@{sec}_@{reg} * K_@{sec}_@{reg}
    @# endfor
@# endfor
;
```

Outline

9 Task 9

Task 9: Enter government policy instruments.

- tax rates on capital expenditure firms

$$\tau^{K,F}_{k,r,t} = \tau^{K,F}_{k,r,0} + \eta^{\tau^{K,F}}_{k,r,t} \tag{7}$$

- tax rates on labour compensation firms

$$\tau^{N,F}_{k,r,t} = \tau^{N,F}_{k,r,0} + \eta^{\tau^{N,F}}_{k,r,t} \tag{8}$$

- tax rates on capital expenditure households

$$\tau^{K,H}_{k,t} = \tau^{K,H}_{r,0} + \eta^{\tau^{K,H}}_{r,t} \tag{9}$$

- tax rates on labour compensation households

$$\tau^{N,H}_{r,t} = \tau^{N,H}_{r,0} + \eta^{\tau^{N,H}}_{r,t} \tag{10}$$

Solution Task 9: Enter the government policy instruments.

```
[name = 'sector specific corporate tax rate paid by firms']
tauKF_@{sec}_@{reg} = tauKF_@{sec}_@{reg}_p + exo_tauKF_@{sec}_@{reg};

[name = 'sector specific labour tax rate paid by firms']
tauNF_@{sec}_@{reg} = tauNF_@{sec}_@{reg}_p + exo_tauNF_@{sec}_@{reg};
```

```
[name = 'taxes on household labour income']
tauNH = tauNH_p + exo_tauNH;

[name = 'taxes on household capital income']
tauKH = tauKH_p + exo_tauKH;

[name = 'taxes on consumption']
tauC = tauC_p + exo_tauC;
```

Outline

10 Task 10

Task 10: Enter the resource constraint.

$$Y_t = C_t + I_t + G_t + \sum_{k}^{K} \sum_{r}^{R} G^{A}_{k,r,t} + NX_t \tag{11}$$

Solution Task 10: Enter the resource constraint.

```
[name = 'Resource Constraint']
Y = C + I + G + NX
@# for sec in 1:Sectors
    @# for reg in 1:Regions
        @# for z in ClimateVars
            + G_A_@{z}_@{sec}_@{reg}
        @# endfor
    @# endfor
@# endfor
;
```

Outline

Task 11: Enter the demand equation for sectoral output.

$$\frac{P_{kt}}{P_t} = \omega_k^{Q\,\frac{1}{\eta^Q}} \left(\frac{Y_{kt}}{Y_t}\right)^{\frac{(-1)}{\eta^Q}} \tag{12}$$

Solution Task 11: Enter the resource constraint.

```
[name = 'demand for sector output']
P_@{sec} / P = omegaQ_@{sec}_p^(1/etaQ_p) * (Y_@{sec}/Y)^(-1/etaQ_p);
```

Outline

Task 12: Enter the sector aggregate specific production function.

$$Y_{k,t} = \left(\sum_{r}^{R} \omega_{k,r}^{Q}{}^{\frac{1}{\eta_k^Q}} \; Y_{k,r_t}{}^{\frac{\eta_k^Q - 1}{\eta_k^Q}} \right)^{\frac{\eta_k^Q}{\eta_k^Q - 1}} \tag{13}$$

Solution Task 12: Enter the sector aggregate specific production function.

```
[name = 'sector aggregate specific output']
Y_@{sec} = (
@# for reg in 1:Regions
    + omegaQ_@{sec}_@{reg}_p^(1/etaQ_@{sec}_p) * ((Y_@{sec}_@{reg}))^((etaQ_@{sec}_p - 1) / etaQ_@{sec}_p)
@# endfor
)^(etaQ_@{sec}_p / (etaQ_@{sec}_p - 1));
```

Outline

13 Task 13

Task 13: Enter damage function on TFP.

$$
\begin{aligned}
D_{k,r_t} = \Big\{ & \left(a_{T,1,k,r}\, T_{rt} + a_{T,2,k,r}\, (T_{rt})^{a_{T,3,k,r}}\right) exp\left(-\phi^{G^{A,T}_{k,r}}\, G^{A,T}_{k,r,t}\right) \\
& + \left(a_{SL,1,k,r}\, SL_t + a_{SL,2,k,r}\, (SL_t)^{a_{SL,3,k,r}}\right) I\left(SL_t > \frac{K^{A,SL}_{k,r,t}}{\phi^{G^{A,SL}_{k,r}}}\right) \\
& + \left(a_{W,1,k,r}\, WS_{rt} + a_{W,2,k,r}\, (WS_{rt})^{a_{W,3,k,r}}\right) exp\left(-\phi^{G^{A,WS}_{k,r}}\, G^{A,WS}_{k,r,t}\right) \\
& + \left(a_{P,1,k,r}\, PREC_{rt} + a_{P,2,k,r}\, (PREC_{rt})^{a_{P,3,k,r}}\right) exp\left(-\phi^{G^{A,PREC}_{k,r}}\, G^{A,PREC}_{k,r,t}\right) \\
& + \left(a_{C,1,k,r}\, CYC_{rt} + a_{C,2,k,r}\, (CYC_{rt})^{a_{C,3,k,r}}\right) exp\left(-\phi^{G^{A,CYC}_{k,r}}\, G^{A,CYC}_{k,r,t}\right) \\
& + \left(a_{D,1,k,r}\, DRO_{rt} + a_{D,2,k,r}\, (DRO_{rt})^{a_{DRO,3,k,r}}\right) exp\left(-\phi^{G^{A,DRO}_{k,r}}\, G^{A,DRO}_{k,r,t}\right) \Big\}
\end{aligned}
\tag{14}
$$

Solution Task 13: Enter damage function on TFP.

```
[name = 'sector specific damage function']
D_@{sec}_@{reg} = min(0.7,
                    @# for z in ClimateVars
                       @# if z == "SL"
                          + (a_@{z}_1_@{sec}_@{reg}_p * @{z} + a_@{z}_2_@{sec}_@{reg}_p * @{z}^(a_@{z}_3_@{sec}
                              _@{reg}_p)) * (SL > (K_A_SL_@{sec}_@{reg}(-1) / phiGASL_@{sec}_@{reg}_p))
                       @# else
                          + (a_@{z}_1_@{sec}_@{reg}_p * @{z}_@{reg} + a_@{z}_2_@{sec}_@{reg}_p * @{z}_@{reg}^(
                              a_@{z}_3_@{sec}_@{reg}_p)) * exp(-phiGA@{z}_@{sec}_@{reg}_p*K_A_@{z}_@{sec}_@{reg
                              }(-1))
                       @# endif
                    @# endfor
);
```

Outline

14 Task 14

Task 14: Enter regional sectoral production function and demand for production factors.

- production function

$$Y_{k,r_t} = A_{k,r_t}\left(1 - D_{k,r_t}\right)\left(\alpha^{K}_{k,r}{}^{\frac{1}{\eta^{N,K}_{k,r}}}\left(K_{k,r_t}\right)^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}} + \alpha^{N}_{k,r}{}^{\frac{1}{\eta^{N,K}_{k,r}}}\left(A^{N}_{k,r_t}\left(1 - D^{N}_{k,r,t}\right) Pop_t\, N_{k,r_t}\right)^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}}\right)^{\frac{\eta^{N,K}_{k,r}}{\eta^{N,K}_{k,r}-1}} \tag{15}$$

- firms FOC capital

$$r_{k,r_t}\left(1 + \tau^{K,F}_{k,r,t}\right) = \alpha^{K}_{k,r}{}^{\frac{1}{\eta^{N,K}_{k,r}}}\left(A_{k,r_t}\left(1 - D_{k,r,t}\right)\right)^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}}\left(\frac{K_{k,r_t}}{Y_{k,r_t}}\right)^{\frac{-1}{\eta^{N,K}_{k,r}}} \tag{16}$$

- firms FOC labour

$$\frac{W_{k,r_t}\left(1 + \tau^{N,F}_{k,r,t}\right)}{P_{k,r_t}} = \alpha^{N}_{k,r}{}^{\frac{1}{\eta^{N,K}_{k,r}}}\left(A_{k,r,t}\left(1 - D_{k,r,t}\right) A^{N}_{k,r,t}\left(1 - D^{N}_{k,r,t}\right)\right)^{\frac{\eta^{N,K}_{k,r}-1}{\eta^{N,K}_{k,r}}}\left(\frac{Pop_t\, N_{k,r_t}}{Y_{k,r_t}}\right)^{\frac{-1}{\eta^{N,K}_{k,r}}} \tag{17}$$

Solution Task 14: Enter regional sectoral production function and demand for production factors.

```
[name = 'sector specific output']
Y_@{sec}_@{reg} = (1 - D_@{sec}_@{reg}) * A_@{sec}_@{reg} * (alphaK_@{sec}_@{reg}_p^(1/etaNK_@{sec}_@{reg}_p)
      * (A_K_@{sec}_@{reg} * K_@{sec}_@{reg}(-1))^((etaNK_@{sec}_@{reg}_p-1)/etaNK_@{sec}_@{reg}_p) + (alphaN_@
      {sec}_@{reg}_p)^(1/etaNK_@{sec}_@{reg}_p) * ((1 - D_N_@{sec}_@{reg}) * A_N_@{sec}_@{reg} * PoP * N_@{sec}
      _@{reg})^((etaNK_@{sec}_@{reg}_p-1)/etaNK_@{sec}_@{reg}_p))^(etaNK_@{sec}_@{reg}_p/(etaNK_@{sec}_@{reg}_p
      - 1));

[name = 'Firms FOC capital',mcp = 'K_@{sec}_@{reg} > 0']
r_@{sec}_@{reg} * (1 + tauKF_@{sec}_@{reg}) = alphaK_@{sec}_@{reg}_p^(1/etaNK_@{sec}_@{reg}_p) * ((1 - D_@{sec
      }_@{reg}) * A_@{sec}_@{reg})^((etaNK_@{sec}_@{reg}_p-1)/(etaNK_@{sec}_@{reg}_p)) * A_K_@{sec}_@{reg}^((
      etaNK_@{sec}_@{reg}_p-1)/(etaNK_@{sec}_@{reg}_p)) * (K_@{sec}_@{reg}(-1) / Y_@{sec}_@{reg})^(-1/etaNK_@{
      sec}_@{reg}_p);

[name = 'Firms FOC labour',mcp = 'N_@{sec}_@{reg} > 0']
W_@{sec}_@{reg} * (1 + tauNF_@{sec}_@{reg})/P_@{sec}_@{reg} = alphaN_@{sec}_@{reg}_p^(1/etaNK_@{sec}_@{reg}_p)
       * ((1 - D_N_@{sec}_@{reg}) * A_N_@{sec}_@{reg} * (1 - D_@{sec}_@{reg}) * A_@{sec}_@{reg})^((etaNK_@{sec}
      _@{reg}_p-1)/(etaNK_@{sec}_@{reg}_p)) * ((PoP * N_@{sec}_@{reg}) / Y_@{sec}_@{reg})^(-1/etaNK_@{sec}_@{
      reg}_p);
```