Is Risk the Fuel of the Business Cycle?[1]

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Abstract

This paper develops a dynamic stochastic general equilibrium (DSGE) model with risky capital and oil as production factors. The production function of the representative firm is a nested constant elasticity of substitution function. The model is estimated using Bayesian techniques with economic data and on oil prices, production and consumption for the United States. The interaction between risk, investment decisions of firms, and the oil market are analysed, taking the short-run elasticity of substitution between oil and capital and the propagation mechanisms between risk in capital production and oil price movements into account. The model is used to reassess the contribution of the different potential drivers to the business cycle controlling for fluctuations in oil markets. Significant findings are that the contributions of financial market frictions and oil market disturbances to the US business cycle are low and that financial market disturbances mainly drove the Great Recession. The model can quantify the impact of climate change mitigation policies on the economy. Climate change mitigation policies, e.g. increasing oil taxes, to reduce crude oil consumption by 10% can cause a contraction of GDP by 1 to 2% and increases inflation. Monetary policy can stabilize inflation increasing the federal funds rate dependent on the degree of financial market imperfections by 0.15 to 0.40 percentage points annually.

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1 Introduction

Oil prices have been more volatile since the Yom Kippur war in 1973, and since then, macroeconomic research has been studying the relationship between oil prices and real economic activity. The Great Recession from 2007 to 2009 initiated a macroeconomic research agenda on the role of financial markets for the business cycle (see Christiano et al. 2014, Jermann & Quadrini 2012, Khan & Thomas 2013, Mian & Sufi 2014). We also know that oil and financial markets are interdependent (see Elder & Serletis 2009, 2010, Kilian 2008).

Suitable tools for investigating the macroeconomic role of oil (see Balke & Brown 2018, Bergholt et al. 2017, Dhawan & Jeske 2008, Milani 2009) and financial markets are general equilibrium models. A frequently used approach to model financial frictions is the so-called financial accelerator mechanism. This mechanism was introduced into a standard New-Keynesian DSGE (henceforth **NK-DSGE**) model by Bernanke et al. (1999). They showed that the accelerator could amplify small shocks, that might come from monetary policy or the oil market.

Christiano et al. (2014) (henceforth **CMR**) estimate a workhorse NK-DSGE model (see Christiano et al. 2005, Smets & Wouters 2003, 2007) (henceforth **CEE**) augmented by the financial accelerator mechanism described in Bernanke et al. (1999). Shocks to the credit market (risk shocks) can explain a majority of the US GDP growth variance, according to CMR. Quantitative financial variables (credit growth, networth) are necessary observables to achieve this result. Further, the estimated persistence in prices, wages and consumption are also important to obtain a dominant role of risk shocks for GDP growth.

Thus, CMR appear to have shown that risk is the fuel of the business cycle. However, they did not control for fluctuations in crude oil markets. Including crude oil market observables might change the estimated structural parameters. Persistence in wages and prices might be lower or higher, including oil. Estimated standard deviations of shocks are interdependent. Controlling for oil can change the contribution of other shocks to GDP growth and the business cycle.

The main objective of this paper is to study the interaction between oil and financial markets through the lens of an estimated DSGE model. This paper extends the model by CMR to include oil as production factor (henceforth **CMR–Oil**). It is essential to select a suitable benchmark model to isolate the effect of the interaction between oil markets and financial markets. This paper extends the CEE model (henceforth **CEE–Oil**). To capture the specific role of oil, one can switch from a Cobb-Douglas to a nested constant elasticity of substitution (henceforth **CES**) production function. There are two layers with the top layer combining labour and a composite production factor. The

Table 1: Overview of models

Abbreviation	Description
CEE	The workhorse model introduced by Christiano et al. (2005). It is a balanced growth model with price and wage rigidities.
CMR	The model introduced by Christiano et al. (2014) is based on Christiano et al. (2005) and includes financial frictions as described in Bernanke et al. (1999).
CEE–Oil	The CEE model with oil as production factor.
CMR–Oil	The CMR model with oil as production factor.

next layer combines oil and capital services to the composite production factor. Oil is used together with capital to produce output. In each layer, the production factors might be complements or substitutes, with the Cobb-Douglas production function as a particular case. It is standard to use Bayesian techniques to estimate the structural parameters of the model.

The results reveal that risk is not the main driver of the business cycle, but technology shocks are the main driver. However, risk shocks are an essential source for fluctuation. This result is not directly related to the inclusion of oil. The reason for a lower contribution of risk shocks to the business cycle is less persistent shocks to inflation, wages, demand and the monetary policy rule parameters.

The financial accelerator does not amplify oil market shocks in the CMR–Oil model, in contrast to the statement by Bernanke et al. (1999). Oil market shocks are essential to explain investment behaviour and less so to explain consumption. They drive changes in the permanent levels of consumption and investment, but not their growth rates. The theoretical variance decomposition for the CMR–Oil model reveals that oil explains less of the variation in investment compared to the CEE–Oil model. Oil market shocks explain about 11% of the variance without financial accelerator. With financial frictions, the contribution of oil market shocks to the variance in investment declines to almost 3%.

While a variance decomposition explains the theoretical second moments of the model variables, it does not describe specific historical episodes. Risk and oil market shocks might have been extraordinary drivers in particular episodes of the US business cycle since 1984. A historical decomposition reveals that risk shocks mainly contributed to the decline in GDP during the Great Recession. Otherwise, the contribution of risk shocks to the business cycle is low. Oil market variables have not been the leading cause of movements in GDP, investment or consumption growth. Oil market shocks moderately drive inflation. There is no remarkable difference between the historical decomposition of the variables using the CMR–Oil and the CEE–Oil model.

A striking result of the variance decomposition is that oil market variables explain less of the variance in GDP, consumption and investment with a financial accelerator. It contradicts the idea that the financial accelerator amplifies oil supply shocks. The opposite is true for monetary policy shocks. Impulse response functions to unexpected changes in the federal funds rate and unanticipated oil supply shocks support this picture. The financial accelerator mechanism amplifies the effect of monetary policy and reduces the impact of oil supply shocks.

Risk shocks, according to the historical decomposition, have been significant during the Great Recession. In contrast, oil supply shocks have not been significant during any historical episode in the last four decades. However, the US might recommit to the Paris Agreement enforced on November 4$^{\text{th}}$ 2016. A very likely consequence is the reduction of US oil consumption. Policymakers need to apply appropriate measures to reduce oil consumption to comply with the Paris Agreement. It is necessary to have adequate tools to assess the potential impact of mitigation measures on the economy. Golosov et al. (2014) use a calibrated dynamic general equilibrium model to evaluate mitigation measures and their effects on the economy. The estimated CEE–Oil and CMR–Oil model can assess the economic impact of mitigation policy. More precisely, the paper studies a reduction in oil consumption by an increase in oil taxes.

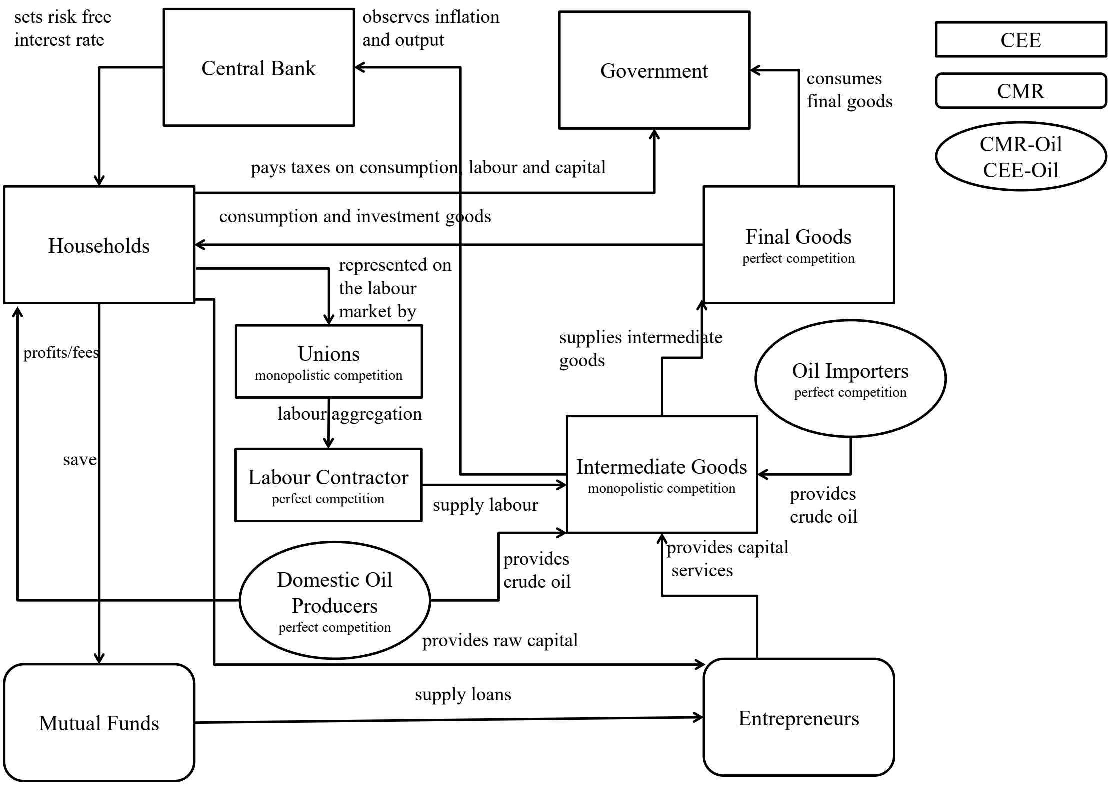
Impulse response functions derived from the structural CMR–Oil model show that a reduction in oil consumption by 10% causes a weak recession by -1 to -2%. An increase in the tax rate on oil will lead to inflation that is about 0.1 annual percentage points higher. Monetary policy may react to the rise in inflation. The federal funds rate needs to increase by 0.15 to 0.30 annual percentage points to stabilize price changes, according to the CMR–Oil model. In the CEE–Oil model an increase between 0.25 to 0.40 annual percentage points is required. Thus, more frictions in financing lead to lower changes needed in the federal funds rate to stabilize inflation.

In Section 2 I describe the CEE, CMR and the oil extended models. Section 3 describes the data and estimation procedure. Results are presented in Section 4 and discussed in Section 5. Section 6 concludes the paper.

2 The Model

This section describes the different models. Figure 1 is a graphical summary of all model versions. First, the section will non-technically discuss the CEE model. Second, the section will explain the modifications by CMR to include the financial accelerator into the CEE model. Third, the section will report the changes to fit oil as production factor into the CEE and CMR model.

Figure 1: Model overview



Source: own exhibition.
Note: The diagram illustrates relationships between the different agents in the model. Rectangles represent agents present in the CEE model, rounded rectangles represent agents present in the CMR model and ellipses represent agents present in the CEE–Oil and CMR–Oil model.

2.1 CEE

The baseline NK-DSGE model is depicted in Figure 1 and the equations are reported in Appendix C.1.[2] I generally follow the description of Christiano et al. (2014) to describe the baseline DSGE model. All households j_h provide capital services K^s and hours worked h in each period t. Households either consume C or invest I final goods into their raw capital stock $\bar{K}_{t-1}$. The raw capital stock depreciates at a constant fraction δ. Capital services $K_t^s = u_t \, \bar{K}_{t-1}$ are rented to intermediate goods producing firms. Households face utilization costs $a(u_t)$ and investment adjustment cost $\mathcal{S}(\frac{I_t}{I_{t-1}})$. Investment adjustment costs depend on the growth rate in investment. The stock of raw capital evolves according to the standard law of motion.

[2]All symbols are explained in Table 5, Table 6 and Table 7 in the Appendix.

The government charges a tax rate on consumption τ^c, labour τ^l and capital income τ^k. The government also collects taxes $Tax_{t+\kappa}$ and provides lump-sum transfers $Tr_{t+\kappa}$. Government expenditures G are financed by tax revenues. Households can purchase bonds B_t and get an interest rate R_t. Households live infinitely and maximize intertemporal discounted utility (1) subject to their budget constraint (2).

$$\max_{\substack{\bar{K}_{j_h,t+\kappa+1}, I_{j_h,t+\kappa} \\ C_{j_h,t+\kappa}, B_{j_h,t+\kappa+1}}} \mathrm{E}_0 \sum_{\kappa=0}^{\infty} \beta^{\kappa} \left[\zeta_{c,t+\kappa} \Big\{ \ln(C_{j_h,t+\kappa} - bC_{j_h,t+\kappa-1}) \Big\} - \psi_L \int_0^1 \frac{h_{j_h,j_l,t+\kappa}^{1+\sigma_L}}{1+\sigma_L} dj_l \right], \tag{1}$$

$$\text{s.t.}(1+\tau^c)P_{t+\kappa}C_{j_h,t+\kappa} + B_{j_h,t+\kappa+1} + \Big(\frac{P_{t+\kappa}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}}\Big) I_{j_h,t+\kappa} + Tax_{t+\kappa} + Q_{\bar{K},t+\kappa}(1-\delta)\bar{K}_{t+\kappa}$$
$$= (1-\tau^l)\int_0^1 W_{j_h,j_l,t+\kappa} h_{j_h,j_l,t+\kappa} dj_l + R_{t+\kappa}B_{t+\kappa} + Q_{\bar{K},t+\kappa}\bar{K}_{j_h,t+\kappa+1} + \Delta_{j_h,t+\kappa} + Tr_{j_h,t+\kappa}. \tag{2}$$

Households discount the future with the discount factor β. In each period households utility depends positively on a weighted average of the current consumption level and the change to the previous period. Habit persistence b measures how important the current change in consumption is for utility. Working is associated with disutility, where the inverse Frisch elasticity σ^L measures how sensitive labour supply is to changes in wages. Each period the budget constraint (2) is binding.

Firms j_f use capital services K^s and homogenous working hours l to produce intermediate goods $Y_{j_f,t}$. A Cobb-Douglas function combines the two primary production factors. Firms have to pay wages W_t and a rental price for capital services $\tilde{r}_t^k\, P_t$. One can derive the demand for production factors from cost minimization subject to a given amount of output. Therefore marginal costs S_t depend directly on the market prices for the primary production factors. Fixed costs ensure zero profits in steady-state and reduce the incentives for new firms to enter the market (see Christiano et al. 2010).

$$\begin{aligned} &\min_{l_{j_f,t}, K^s_{j_f,t}} W_t l_{j_f,t} + P_t \tilde{r}_t^k K^s_{j_f,t}, \\ &\text{s.t.} Y_{j_f,t} = \epsilon_t \left(\frac{K^s_{j_f,t}}{\Upsilon^{t-1}} \right)^{\alpha_K} ({\epsilon^h}_t\, z_t l_{j_f,t})^{\alpha_N} - \phi_t z_t, \\ &\qquad l_{j_f,t} > 0, K^s_{j_f,t} > 0. \end{aligned} \tag{3}$$

These intermediate goods are imperfect substitutes to produce a final good Y_t using a constant elasticity of substitution production function. Parameter λ^f determines the degree of substitutability between the different products. Profit maximization of the final goods producer (4) implies that the overall price index P_t is a weighted average over all prices set by intermediate goods producers.

$$\max_{Y_{j_f,t}} P_t Y_t - \int_0^1 P_{j_f,t} Y_{j_f,t} dj_f, \tag{4}$$
$$\text{s.t. } Y_t = \left(\int_0^1 Y_{j_f,t}^{\frac{1}{\lambda^f}} dj_f \right)^{\lambda^f}.$$

Intermediate goods-producing firms have price-setting power. They set their price $P_{j_f,t}$ to maximize expected discounted profits. Only a random fraction $1-\xi^p$ is allowed in each period to reset their price. All other intermediate firms update their prices according to an indexation rule $\tilde{\pi}_t P_{j_f,t-1}$. This two-stage production process, in combination with random price-setting, allows to model price rigidity. Further, it ensures that price inflation π_t can influence real economic variables in the model. The intertemporal expected discounted profit (5) is maximized choosing a optimal price $\tilde{P}_t$, subject to the demand for intermediate products (6).

$$\max_{\tilde{P}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta \xi^p)^\kappa \lambda_{t+\kappa} (P_{j_f,t+\kappa} Y_{j_f,t+\kappa} - S_{t+\kappa} Y_{j_f,t+\kappa}), \tag{5}$$
$$\text{s.t.} Y_{j_f,t+\kappa} = Y_{t+\kappa} \left(\frac{\tilde{\Pi}_{t,t+\kappa} \tilde{P}_t}{P_{t+\kappa}} \right)^{-\frac{\lambda^f}{\lambda^f - 1}}. \tag{6}$$

Unions represent different types of labour, j_l and sell them to a labour contractor. Labour contractors sell homogenous labour l_t to the intermediate goods producing firm. A CES aggregation function bundles different types of labour. The parameter λ^w determines the degree of substitutability between the different types of labour. Total hours worked in each year in the economy is denoted by h_t. Similar to the problem of the intermediate goods producing firm only a fraction of unions $1-\xi^w$ is allowed to reset the wage. All other unions will reset their wage according to an indexation rule $W_{j_l,t} = \tilde{\pi}_t^w W_{j_l,t-1}$. Unions reset the wage to maximize the expected discounted wage bill less the foregone utility of the household working (7), subject to the demand for the specific type of labour by labour contractors (8). Unions take into account the disutility imposed on households by supplying labour to the intermediate goods-producing firms.

$$\max_{\tilde{W}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^w)^\kappa \left[\lambda_{t+\kappa} \tilde{W}_t \tilde{\Pi}^w_{t,t+\kappa} h_{j_l,t+\kappa}(1-\tau^l_{t+\kappa}) - \psi_L \frac{h^{1+\sigma_L}_{j_l,t+\kappa}}{1+\sigma_L} \right], \tag{7}$$

$$\text{s.t.} h_{j_l,t+\kappa} = l_{t+\kappa} \left(\frac{\tilde{\Pi}^w_{t,t+\kappa} \tilde{W}_t}{W_{t+\kappa}} \right)^{\frac{\lambda^w}{1-\lambda^w}}. \tag{8}$$

Monetary policy sets the risk free interest rate for bonds according to a Taylor rule (9). Christiano et al. (2014) state in their paper the monetary policy rule as stated in (9), with expected inflation and current GDP growth instead of past values. The risk free interest rate R_t responds to deviations in previous inflation π_{t-1} from its target and in GDP growth $\frac{C_{t-1}+I_{t-1}+G_{t-1}}{C_{t-2}+I_{t-2}+G_{t-2}}$ from its potential (see Bernanke et al. 1999). Government expenditures G_t are modelled as exogenous process.

$$\frac{1+R_t}{1+\bar{R}} = \left(\frac{1+R_{t-1}}{1+\bar{R}} \right)^{\tilde{\rho}} \left\{ \left(\frac{\pi_{t-1}}{\bar{\pi}} \right)^{1+\tilde{a}_\pi} \left(\frac{\mu^z_{t-1}}{\bar{\mu^z}} \frac{c_{t-1} + \frac{i_{t-1}}{\mu^\Upsilon_{t-1}} + g_{t-1}}{c_{t-2} + \frac{i_{t-2}}{\mu^\Upsilon_{t-2}} + g_{t-2}} \right)^{\tilde{a}_{\Delta y}} \right\}^{1-\tilde{\rho}} + \frac{\sigma^{x^p}}{4} x^p_t. \tag{9}$$

The economy follows a balanced growth path. All real variables have a common stochastic trend $z_t = \mu^z_t\, z_{t-1}$. This trend reflects long-run technological change leading to economic growth. Nominal variables are scaled by the nominal price level $P_t = \pi_t\, P_{t-1}$. Capital follows the common stochastic trend and has a specific deterministic trend of Υ^t. Temporary deviations from the balanced growth path are the result of shocks hitting the economy. The standard model comprises a shock to government expenditure g_t, total factor productivity ϵ_t, labour productivity ϵ^h_t, price mark-up shocks ϵ^p_t, wage mark-up shock ϵ^w, technological growth rate μ^z_t, shocks to the relative price of investment μ^Υ, consumption preference shock ζ^c_t, and investment adjustment cost shocks ζ^i_t. All shocks follow an autoregressive moving average (henceforth **ARMA**) process. Each shock is driven by a white noise process $\eta^{j_s}, j_s \in \{g, \epsilon, \epsilon^h, \epsilon^p, \epsilon^w, \mu^z, \mu^\Upsilon, \zeta^c, \zeta^i\}$.

2.2 CMR

CMR introduces entrepreneurs j_E and mutual funds j_{MF} to the CEE model. Appendix C.2 reports different equations and modifications of the CMR model compared to the CEE model. In principle, the financial accelerator mechanism is caused by a conflict of interest between two agents (see Bernanke et al. 1999). Mutual funds use deposits (raw capital) from households to provide loans $B_{j_E,t+1}$ at the gross nominal interest rate Z_{t+1} to entrepreneurs. Mutual funds pay an interest rate R_t for households deposits. Entrepreneurs are owned by households and can either borrow or use their networth $N_{j_E,t}$ to produce effective capital $K_{j_E,t+1} = \omega_t \bar{K}_{j_E,t+1}$. Each household j_h owns a continuum of entrepreneurs j_E. All entrepreneurs experience in each period an idiosyncratic shock ω_t. This shock follows a log-normal distribution with an expectation equal to one and variance varying over time σ_t. This shock decides how much of the raw capital transforms into effective capital. Households still own raw capital, but they sell it to entrepreneurs in each period at a price $Q_{\bar{K},t-1}$. Mutual funds are operating under perfect competition to supply loans to entrepreneurs j_E using raw capital. These entrepreneurs are able to repay their loans with probability $1 - F_t(\bar{\omega}_{t+1})$, if their idiosyncratic productivity shock ω is bigger than a critical threshold $\bar{\omega}$. Entrepreneurs with an idiosyncratic productivity shock below this threshold file bankruptcy. Mutual funds need to verify whether entrepreneurs are bankrupt or not. This monitoring process is associated with costs $dcost(\bar{\omega})_t$, which are proportional by a factor μ to the earnings of the bankrupt entrepreneurs. The expected value of the assets of bankrupt entrepreneurs is given by $G_t(\bar{\omega}_{t+1})(1+R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1}$. The term $G_t(\bar{\omega}_{t+1})$ represents the expected value of ω for bankrupt entrepreneurs. Costly state verification is an agency problem. Further, it introduces a wedge between the risk-free interest rate and the total return on raw capital R^k_t. This wedge is the credit spread and is a consequence of debt financing by entrepreneurs. Entrepreneurs choose the leverage ratio $L_t = \frac{N_{j_E,t}+B_{j_E,t+1}}{N_{j_E,t}}$ to maximize their expected profits subject to the cash constraint imposed by mutual funds. Entrepreneurs solve the following optimization problem

$$\begin{aligned}
&\max_{L_t} \mathrm{E}_t \Big[\int_{\bar{\omega}_{t+1}}^{\infty} \{(1+R^k_{t+1})\omega Q_{\bar{K},t} K_{j_E,t+1} - B_{j_E,t+1}(1+Z_{t+1})\} f(\omega) d\omega \Big] \qquad (10)\\
&\text{s.t.} \{1 - F_t(\bar{\omega}_{t+1})\}(1+Z_{t+1})B_{j_E,t+1} + (1-\mu)G_t(\bar{\omega}_{t+1})(1+R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1} \ldots\\
&\qquad \geq B_{j_E,t+1}(1+R_t).
\end{aligned}$$

Entrepreneurs do not accumulate infinite wealth because of an exogenous survival rate of γ_t. Entrepreneurs receive transfers from their households W^e each period. Entrepreneurs leaving the market $1 - \gamma_t$ can consume a share Θ of their assets. Entrepreneurs transfer the remaining share of assets to households. The inclusion of entrepreneurs alternates resource constraint. The resource constraint derived from the budget constraint of households includes monitoring costs and transfers of entrepreneurs to households (see (11)). CMR include shocks to the survival rate of entrepreneurs η^γ_t and shocks to risk σ_t. These shocks are either anticipated η^s_t for $s \in [1, 8]$ or unanticipated η^σ_t.

CMR also include long-term bonds $B^L_{j_h,t}$ to control for variations in the term structure between short-term and long-term bonds. The Spread between interest rates $\frac{1+R_t^L}{1+R_t}$ is determined by a term structure shock η_t^{term}. One can use long-run government bonds that have a one-year maturity and not a ten-year maturity. The one-year maturity requires less auxiliary variables for the leads included in the model. Solving the model is less time consuming, and therefore the estimation time is faster. Further, it allows running parameter identification tests discussed in Section 3.

2.3 CEE and CMR with oil

This section describes the inclusion of oil markets into the CEE and CMR model. Oil production, consumption and prices have a deterministic trend of Υ^{O^t}, which follows the approach for raw capital in CEE and CMR. A nested CES production function is introduced rather than the particular case of a Cobb-Douglas production function. First, the subsection explains the modifications to the budget constraint of the representative household, then the behavioural equations of oil producers. Third, the subsection describes the behaviour of the intermediate representative firm.

2.3.1 The representative household

The households optimization problem is the same as in CMR except that the budget constraint features now revenues from selling allowances to extract oil to local producers O_t^d. Households provide labour $h_{j_h,j_l,t}$ of type $j_l \in [0,1]$, raw capital $\bar{K}_{j_h,t}$ at price $Q_{\bar{K},t}$, consume final goods $C_{j_h,t}$ and invest into raw capital $I_{j_h,t}$. Further, they can purchase government bonds of one-quarter maturity $B_{j_h,t+1}$ and 4-quarter maturity $B^L_{j_h,t+4}$. The budget constraint is

$$\begin{aligned}(1+\tau^c)P_tC_{j_h,t} + B_{j_h,t+1} + B^L_{j_h,t+4} + \Big(\frac{P_{t+k}}{\Upsilon^t\mu_{\Upsilon,t}}\Big)I_{j_h,t} + Q_{\bar{K},t}\bar{K}_{j_h,t+1} + Tax_{t+\kappa} \quad (11)\\ = (1-\tau^l)\int_0^1 W_{j_h,j_l,t}h_{j_h,j_l,t}dj_l + R_tB_{j_h,t} + (R_t^L)^4B^L_{j_h,t} + Q_{\bar{K},t}(1-\delta)\bar{K}_{j_h,t} + \Delta_{j_h,t}\\ +(1-\Theta)(1-\gamma_t)\{1-\Gamma_{t-1}(\bar{\omega}_t)\}R_t^kQ_{\bar{K},t-1}\bar{K}_{j_h,t} + \Gamma^d(O^d_{j_h,t}) + Tr_{t+\kappa}.\end{aligned}$$

The modification of the budget constraint implies a modification of the resource constraint as well. One can drop the index j_h for households under the assumption of representative households. Total profits of domestic firms Δ_t include expenditures for oil $P_t^O O_t$ used in the production process. Oil is the only tradable production factor. One could also assume that domestic households do not possess all active oil suppliers in the US. Further, households receive transfers from entrepreneurs $(1-\Theta)(1-\gamma_t)\{1-\Gamma_{t-1}(\bar{\omega}_t)\}R_t^kQ_{\bar{K},t-1}\bar{K}_{j_h,t}$ leaving the market, after they consumed a fraction of their assets Θ.

2.3.2 Oil producers

There exists a continuum $j_p \in [0,1]$ of domestic oil producers d and oil importers im with access to infinite oil reserves. All domestic oil producers are identical, and the same is true for all oil importers. Homogeneity of suppliers rules out market power in the crude oil market. Oil reserves are infinite in the model, which contradicts reality. Domestic intermediate goods-producing firms buy oil $O^{d,im}_{j_p,t}$ for the same price P^O_t. Oil producers need to acquire the allowance and rig services to extract a barrel of oil from their respective households. It is also possible that the government sells the allowances and rig services to the household and transfers the revenues through tax cuts or subsidies back. The price of an allowance per barrel $\Gamma^{O,d,im}\left(O^{d,im}_t\right)$ is a function of the current extraction level $O^{d,im}_t$. Firms maximize profits choosing the amount of oil to extract

$$\max_{O^{d,im}_{j_p,t}} P^O_t \left(1-\tau^O_t\right) O^{d,im}_{j_p,t} - \Gamma^{O,d,im}_t (O^{d,im}_{j_p,t}). \tag{12}$$

The model simplifies the more complex tax system for oil production in the United States by a tax rate as a share on revenues τ^o_t. The log tax rate follows an autoregressive process of order one as the other shocks.

The solution to the optimization problem is straight forward and represents the supply curve of the respective oil producers

$$\begin{aligned} P^O_t \left(1-\tau^O_t\right) &= \frac{\partial \Gamma^{O,d,im}_t (O^{d,im}_t)}{\partial O^{d,im}_t} = \frac{\partial \left(\frac{\zeta^{O,d,im}_t}{\Upsilon^{O^t} \gamma^{O,d,im}} O^{d,im}_t\right)^{1+\sigma^O}}{\partial O^{d,im}_t} \\ &= \left(\frac{\zeta^{O,d,im}_t}{\Upsilon^{O^t} \gamma^{O,d,im}}\right)^{1+\sigma^O} \left(O^{d,im}_t\right)^{\sigma^O}. \end{aligned} \tag{13}$$

Oil producers reaction to oil price fluctuations is determined by $\sigma^O > 0$ the inverse price elasticity of oil supply to an increase in oil prices. The inverse price elasticity needs to be non-negative to ensure the existence of a maximum to the profit maximization problem. It also provides an upward sloping supply curve. A lower elasticity implies a steeper supply curve resembling very inelastic oil supply. Domestic and foreign oil producers have the same price elasticities, but different cost functions. Differences in the extraction cost $\gamma^{O,d,im} > 0$ of the respective reserves drive long-run differences in the supply curve. Idiosyncratic temporary shocks $\zeta^{O,d,im}_t > 0$ allow for temporary changes in the costs to supply oil. The exploitation of oil reservoirs might entail temporary different extraction costs depending on the remaining reserves or the quality of oil extracted. Providing imported oil also requires transportation costs, which fluctuate over time.

Total oil consumption in one period is domestic production, fewer oil exports plus oil imports. Therefore, the following identity has to hold in each period.

$$O_t = O_t^d - O_t^{ex} + O_t^{im}. \tag{14}$$

How much domestic oil is exported is not the result of an optimization problem. Domestic oil exports need to be greater than zero and smaller than the total amount of domestic oil production. Therefore, the following relation is specified

$$O_t^{ex} = \zeta_t^{O,ex} O_t^d, \tag{15}$$

$$log\left(\frac{\zeta_t^{O,ex}}{\bar{\zeta}^{O,ex}}\right) = \rho^{\zeta^{O,ex}} log\left(\frac{\zeta_t^{O,ex}}{\bar{\zeta}^{O,ex}}\right) + \eta^{O,ex}, \text{ for } \zeta_t^{O,ex} \in (0,\, 1). \tag{16}$$

The exogenous process $\zeta^{O^{ex}}$ follows an autoregressive process of order one and defines the share of exported oil.

2.3.3 The representative firm

Firms (j_f) produce intermediate goods $Y_{j_f,t}$ using capital services $K^s_{j_f,t}$, hours of homogenous labour $l_{j_f,t}$ and oil $O_{j_f,t}$. The production function for gross output $X_{j_f,t} = X(M_{j_f,t}, l_{j_f,t})$ is a nested constant elasticity of substitution function. Each firm has access to the same technology and can substitute between labour and a composite production factor $M_{j_f,t} = M(O_{j_f,t}, K^s_{j_f,t})$ from capital services and oil. The production elasticity of substitution $\eta^M \in (0, \infty)$ determines how easy it is for firms to substitute labour for other production factors. The degree of substitution between oil and capital services is captured by the production elasticity of substitution $\eta^O \in (0, \infty)$ and the degree of substitutability is $\rho^O = \frac{\eta^O - 1}{\eta^O}$. I further restrict the distribution parameters $\alpha_M \in (0, 1)$ and $\alpha_O \in (0, 1)$ of the CES production function in each stage to sum up to one.

$$X(M_{j_f,t}, l_{j_f,t}) = \begin{cases} \epsilon_t \, M_{j_f,t}^{\alpha_M} \, (z_t l_{j_f,t})^{1-\alpha_M} & \text{if } \eta^M = 1, \\ \epsilon_t \left[(\alpha_M)^{\frac{1}{\eta^M}} \, M_{j_f,t}^{\rho^M} + (1-\alpha_M)^{\frac{1}{\eta^M}} (z_t l_{j_f,t})^{\rho^M} \right]^{\frac{1}{\rho^M}} & \text{otherwise,} \end{cases} \tag{17}$$

$$M(O_{j_f,t}, K_{j_f,t}^s) = \begin{cases} \left(\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon^{O\,t}} \right)^{\alpha_O} \left(\epsilon_t^K \, \frac{K_{j_f,t}^s}{\Upsilon^{t-1}} \right)^{1-\alpha_O} & \text{if } \eta^O = 1, \\ \left\{ (1-\alpha_O)^{\frac{1}{\eta^O}} \left(\epsilon_t^K \, \frac{K_{j_f,t}^s}{\Upsilon^{t-1}} \right)^{\rho^O} + (\alpha_O)^{\frac{1}{\eta^O}} \left(\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon^{O\,t}} \right)^{\rho^O} \right\}^{\frac{1}{\rho^O}} & \text{otherwise.} \end{cases} \tag{18}$$

It requires a suitable capital stock to use crude oil efficiently. The composition of the capital stock is crucial for the ability of firms and households to abandon oil consumption. The effectiveness of the workforce depends less on crude oil usage. However, it is also possible to model labour and capital in one nest and combine the composite production factor with crude oil in the final stage. Nevertheless, the model follows the approach by Balke & Brown (2018) to model oil and capital services in one CES nest.

Firms face fixed costs $\phi_t z_t$ to produce net output $Y_{j_f,t}$, where $\bar{\phi}$ is set such that there are no profits in steady-state. Fixed cost ensure that profits are zero so that no new firm enters the market in steady-state. The intermediate good producing firms minimize the costs for a given production level.

$$Y_{j_f,t} = \begin{cases} X_{j_f,t} - \phi_t z_t, & \text{if } X_{j_f,t} > \phi_t z_t, \\ 0, & \text{else.} \end{cases} \tag{19}$$

Temporary total factor productivity shocks ϵ_t, temporary capital specific factor productivity shocks ϵ_t^K, temporary oil factor productivity shocks ϵ_t^O can change production factor demand. The optimization problem is

$$\min_{l_{j_f,t}, K_{j_f,t}^s, O_{j_f,t}} W_t l_{j_f,t} + P_t \tilde{r}_t^k K_{j_f,t}^s + P_t^O O_{j_f,t}, \tag{20}$$

$$s.t. Y_{j_f,t} = X(M(O_{j_f,t}, K_{j_f,t}^s), l_{j_f,t}) - \phi_t z_t,$$

$$l_{j_f,t} > 0,\; K_{j_f,t}^s > 0,\; O_{j_f,t} > 0,\; M_{j_f,t} > 0,\; Y_{j_f,t} > 0.$$

The corresponding Lagrangian, ignoring the non-negativity constraints, of the problem is

$$\mathcal{L}_t^{\mathrm{F,min}} = W_t \, l_{j_f,t} + P_t \, \tilde{r}_t^k \, K_{j_f,t}^s + P_t^O O_{j_f,t} + S_t \{ Y_{j_f,t} - (X(M_{j_f,t}, l_{j_f,t}) - \phi z_t) \}. \tag{21}$$

The first order conditions to (21) describe the demand for production factors by the representative firms.

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial l_{j_f,t}} :0 = W_t - S_t z_t^{\frac{\eta^M-1}{\eta^M}} \epsilon_t (\alpha_N)^{\frac{1}{\eta^O}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{22}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial K_{j_f,t}^s} :0 = P_t \tilde{r}_t^k - P_t^M (1-\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{t-1})^{-\rho^O} (\epsilon^K{}_t)^{\rho^O} \left(\frac{M_{j_f,t}}{K_{j_f,t}^s}\right)^{\frac{1}{\eta_O}}, \tag{23}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial O_{j_f,t}} :0 = P_t^O - P_t^M (\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{Ot})^{-\rho^O} (\epsilon^O{}_t)^{\rho^O} \left\{\frac{M_{j_f,t}}{O_{j_f,t}}\right\}^{\frac{1}{\eta^O}}, \tag{24}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial S_t} :0 = X_{j_f,t} - X(l_{j_f,t}, M_{j_f,t}), \tag{25}$$

$$P_t^M = S_t\, z_t^{\rho^M}\, \epsilon_t\, \alpha_M^{\frac{1}{\eta^M}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}.$$

The constraint of the cost minimization is the CES production function for output. Appendix E discusses the sufficient conditions for a minimum. The shadow price of oil-capital composite goods P_t^M is equal to the marginal product $\frac{\partial X_{j_f,t}}{\partial M_{j_f,t}}$ times marginal costs S_t.

3 Estimation

This section describes the estimation procedure. It explains in detail the data used to estimate the structural model. Standard Bayesian estimation techniques are applied. Further, the section reports how priors for the structural parameters are selected. Finally, the estimated model is analysed using conventional screening tools.

The main issue with the estimation of medium-sized DSGE models is parameter identification. It is vital to obtain convergence using the Random Walk Metropolis-Hastings (RWMH) algorithm. First, one can check local parameter identification as defined in Iskrev (2010) at the prior mean before one should apply the RWMH algorithm. Further, the pairwise correlation between parameters does not exceed the upper bound of 0.99 and decrease the required number of draws for the RWMH algorithm to converge. Afterwards, a quasi-Newton with BFGS optimization routine delivers a posterior mode candidate. Parameter identification of the model is necessary at the posterior mode candidate[3]. In the next step, the scale parameter for the proposal distribution ensures an acceptance ratio for the RWMH algorithm of 0.25. It is important to note that some commonly used parameters are not estimated. Indexation parameters for inflation and wages are not estimated (ι^{π,μ^z}), and habit formation b. Including these parameters lead either to unidentified parameters at the prior mean

[3]Here, the potential point is the mode found using the CSMINWEL algorithm introduced by Sims.

or the candidate for the posterior mode. Therefore, these parameters are set to zero and excluded from the estimation. Further, I calibrate the monetary response variable to inflation ($\tilde{a}_{\pi} = 0.5$). The correlation between the monetary response variable and the monetary policy rigidity parameter ($\tilde{\rho}$) is very high. These changes make an exact replication of CMR or CEE impossible. Nevertheless, it ensures local identification of parameters by the data and model equations. It also ensures convergence of the RWMH after a reasonable amount of draws.

3.1 Data

I declare observable variables as introduced by Smets & Wouters (2003) and Christiano et al. (2005) to estimate the model. Those are GDP growth, GDP deflator as a measure for inflation, consumption growth, investment growth, hours worked, wage growth, federal funds rate and the relative price of investment (see Figure 8). The model includes additional variables to control for fluctuations in the financial market, as discussed in Christiano et al. (2014). The measure for net worth is the quarterly change in the DOW Jones Wilshire 5000 index. Credit growth is the change in loans to non-financial firms. The difference between interest rates on BAA-rated corporate bond yields and the interest rate on government bonds with a 10-year maturity measures the interest-rate spread. The observables include 1-year instead of 10-year constant maturity US government bonds to compute the term structure. This modification allows to introduce less auxiliary variables into the model and also to run identification screenings as proposed by Iskrev (2010). Figure 9 depicts the observed financial variables used to estimate the model.

The CMR–Oil model extends the set of observable variables compared to CMR by domestic crude oil production, consumption, and imports fewer changes in oil stocks growth rates. The Energy Information Administration (EIA) provides monthly historical data for crude oil field production, exports, imports and changes in the stock.[4] Further, the refinery acquisition cost of imported oil (see Kilian & Vigfusson 2013) corrected for inflation is observable for the growth in the real oil price changes. Figure 10 depicts oil market variables used to estimate the model. Growth rates in domestic field crude oil production, imported crude oil fewer changes in oil stocks and crude oil exports contain the necessary information to control for oil consumption in the US indirectly.

In Table 8, the p-values for Augmented Dickey-Fuller and Phillips-Perron tests

[4] One can download the data from `https://www.eia.gov/` under data for petroleum and other liquids. One can retrieve data for field production, exports, imports and stock changes from US crude oil supply and disposition under the subcategory summary (release date March 29th 2019). The subcategory prices (release date April 1st 2019) lists refinery acquisition costs.

are reported. The tests can not reject the null hypothesis of a unit root at the five percent significance level for hours worked using the Augmented-Dickey-Fuller test or the Phillips-Perron test. Nevertheless, hours worked is a stationary series following a standard convention in the literature. For the other variables, the test results are either not conclusive or indicate that one can reject the null hypothesis of a unit root with an error probability of less than 5%.

3.2 Steady-state

The model finds a steady-state using two different algorithms. First, one can use an algorithm to calibrate the model to estimate it. This algorithm will find the share of assets eaten up by monitoring μ using a numerical approach, the threshold productivity value separating solvent and insolvent entrepreneurs $\bar{\omega}$, and the cross-sectional dispersion of productivity σ in turning raw into effective capital. Otherwise, structural parameter values ensure to match given long-run relationships.

Second, an algorithm is applied to compute impulse response functions to permanent shocks. It requires a numerical procedure for a given set of structural parameters.

3.2.1 Calibration

Appendix D describes the procedure to calibrate the model and find the steady-state and Table 9 reports the calibrated parameters. First, the algorithm sets $r^k = 0.0525$ approximately the value reported by CMR at the posterior mode of their model. The steady-state ratio between net worth and raw capital depends on the steady-state rental rate. This value corresponds to long-run equity to debt ratio of 2 approximately the observed ratio for the period 1984-Q2 to 2018-Q4.[5]. Further, production of y equals one. Therefore, the steady-state values of consumption c, investment i and government expenditure g are easily interpretable as shares. The model without financial accelerator does not feature an external finance premium. Therefore, the risk-free interest rate of R is twice as large as in the model with a financial accelerator.

Transfers of households to entrepreneurs w^e is equal to 0.005 identical to CMR. It is necessary to find monitoring costs μ such that the first-order condition of entrepreneurs and its respective constraint is satisfied. The bankruptcy probability $F(\bar{\omega}) = 0.56\%$ corresponds to the estimated mode by Christiano et al. (2014).

The solution of first-order conditions for the entrepreneur and its corresponding constraint does not depend on the answer to other endogenous variables. Therefore,

[5]Compare with the series *Non-financial Corporate Business; Credit Market Debt as a percentage of the Market Value of Corporate Equities,%, Quarterly, Not Seasonally Adjusted* published by the Federal Reserve of St. Louis.

Table 2: **Steady-state properties, model at priors versus data**

Ratio	CEE–Oil Model	CMR–Oil Model	Sample averages
$\frac{i}{y}$	0.25	0.25	0.26
$\frac{c}{y}$	0.55	0.55	0.58
$\frac{g}{y}$	0.19	0.19	0.19
$\frac{\bar{k}-n}{n}$	–	0.5	0.5
R	0.021	0.011	0.009
$\frac{o}{y}$	0.002	0.002	0.002
$\frac{p^o o}{y}$	0.016	0.017	0.017
$\frac{o^{im}}{o}$	0.51	0.52	0.52

Notes: The sample range is 1984-Q2 to 2018-Q4. The first three ratios are computed as described in CMR. Debt to equity ratio corresponds to the inverse of the non-financial corporate business debt to equity ratio. The oil output ratio is computed for 2012 constant prices of the refinery acquisition costs and the deflator for GDP. The share of oil is the ratio between domestic oil consumption expenditures and GDP.
Sources: Own computation, Federal Reserve Bank of St. Louis, US Energy Information Administration.

it is possible to solve the remaining static equations independent of the credit market equilibrium. The procedure requires to guess a net output value y^z and to iteratively solve for all other endogenous variables. The algorithm calibrates the capital ϕ^K, the oil ϕ^O and the labour $1-\phi^M$ cost shares. Hours worked h are equal to unity in steady-state as done in Christiano et al. (2014). Different from Christiano et al. (2014), The value of the disutility to work parameter ψ^L ensures the unity of hours worked in a steady-state.

An essential modification of the routine to find the steady-state is the inclusion of a nested CES production function but also including the particular case of the Cobb-Douglas production function. Distributional parameters of the CES function α^O, α^M depend on the steady-state expenditure shares and the ratio of oil consumption and output. Elasticities of substitutions η^M, η^O determine the value of distributional parameters.

3.2.2 Permanent shocks

It is necessary for the computation of impulse response functions to permanent shocks to modify the previous routine. It allows us to consider permanent shocks. However, the routine needs the following assumptions to compute the permanent effects:

1. Long-run mark-ups are constant.

2. Long-run growth rates of prices and permanent technology shocks do not change.

3. Long-run utilization and price of raw capital are constant.

Therefore, the new long-run level of output and the associated magnitude and relative demand for production factors will change. The algorithm allows changing all included arbitrary shocks permanently. Nevertheless, the transition path for large innovations might not be computable.

The routine computes the impulse response functions for permanent and temporary shocks using a deterministic simulation framework with perfect foresight. Therefore, the impulse response functions can be non-linear. This approach, as discussed in Lindé & Trabandt (2018), is more suitable to retrieve information for policy advice compared to impulse response functions derived from log-linearised models.

3.3 Priors for structural parameters

Table 10 reports the prior distributions for all 41 parameters. It is important to note that some commonly used parameters are not estimated. Indexation parameters for inflation and wages are not estimated (ι, ι^{w,μ^z}), and habit formation b. Including these parameters lead either to not identified parameters at the prior mean or the candidate for the posterior mode[6] using the local identification analysis introduced by Iskrev (2010). Further, estimating the monetary policy parameter $\tilde{a}_\pi$ leads to a pairwise correlation with the persistence parameter $\tilde{\rho}$ above 0.99. Therefore, these parameters are excluded from the estimation and set to zero.

For the estimation of the CMR–Oil and CEE–Oil model I first obtain priors for the standard structural parameters using posterior means and standard deviations from the estimation of the baseline CEE model. For the first stage (where I estimate the CEE model), I define usual priors. The price and wage rigidity parameters follow a Beta distribution with prior mean equal to 0.5 and a prior standard deviation of 0.1. The monetary policy parameter $\tilde{a}_{\Delta y}$, which captures the response to output growth has the usual Gaussian prior distribution with a prior mean of 0.3 and a prior standard deviation of 0.05. Standard deviations of shocks follow an inverse Gamma distribution and have identical prior means and standard deviations. Persistence parameters of the exogenous disturbances with equal prior means and standard deviations follow the commonly used Beta distribution. Table 10 reports the obtained posterior mean and standard deviation for the CEE model and the prior mean and standard deviation.

In contrast to Christiano et al. (2014) I do not estimate the steady-state bankruptcy probability $F(\bar{\omega}_t)$, because it leads to non identified parameters at the prior mean. Fur-

[6] Here the posterior mode candidate is the mode found using the CSMINWEL algorithm introduced by Sims.

ther, I exclude the share of assets used to monitor bankrupt entrepreneurs μ from the set of estimated parameters, because it is calibrated to ensure that lump-sum transfers w^e of entrepreneurs to their household is equal to 0.005 as in Christiano et al. (2014). The prior distribution for signal correlation is modified to ensure that the estimated correlation is bounded between minus one and one. The signal correlation for anticipated risk shocks, is estimated indirectly through an auxiliary parameter $\sigma(\xi_s, \xi_{s+1})$. The prior distribution of the parameter follows a Beta distribution and ensures that signal correlation is zero at the prior mean. Signal correlation $Corr(\xi_t^s, \xi_t^{s+1}) = 2\,\sigma(\xi_s, \xi_{s+1}) - 1$ is zero if the auxiliary parameter is equal to its prior mean of 0.5.

The main objective of this paper is to study the interaction between oil and financial markets through the lens of a dynamic stochastic general equilibrium model. The extension compared to the model described in Christiano et al. (2014) is the inclusion of oil as a production factor. Further, the model allows for the short-run oil supply to be neither perfectly elastic (see Milani 2009) nor inelastic to the oil price. Cost functions of domestic and foreign oil producers are convex, and the inverse oil supply price elasticity is given by σ^O. The prior mean of the inverse oil supply price elasticity is 10, such as in Baumeister & Hamilton (2019). The inverse oil supply price elasticity follows a Gamma distribution with a standard deviation equal to two. The nested CES production function with oil allows defining the oil demand price elasticity η^O. The prior mean of the oil demand price elasticity from Baumeister & Hamilton (2019) equals 0.1 and also follows a Gamma distribution with a standard deviation of 0.05. The Gamma distribution and standard deviation ensure that values above and below the prior mean have similar probability, but also restricting the parameter space to positive values. Further, the set of estimated parameters contains the elasticity of substitution between hours worked and the capital oil composite production factor. The prior mean is set to one with a standard deviation of 0.2 and follows the Gamma distribution function.

3.4 Posterior mode analysis

After finding a posterior mode candidate, an optimization routine finds a scale parameter for the RWMH algorithm with an acceptance ratio of 25%.[7] A target ratio of 25% is slightly above the range Roberts et al. (1997) suggested, but well in the range of usually applied acceptance ratios. The screening Brooks & Gelman (1998) analysis assesses whether the simulations are sufficient to reach convergence, based on four RWMH chains with a total length of one million per chain. Figure 13 depicts for both models the multivariate convergence diagnosis. The Online Appendix reports single

[7]The optimization routine is implemented in Dynare using the mode compute option 6.

parameter diagnostics. A burn-in period of about 1,600,000 draws is sufficient. After 1,600,000 draws the 80% inter-quantile range based on the posterior likelihood interval and the respective second and third central moments are indistinguishable close to each other and stabilize horizontally.

4 Results

First, this section compares the estimated structural parameters for the model with and without financial accelerator. Second, the section reports the variance and historical decomposition of the business cycle for the US economy. Third, temporary and permanent impulse responses to an exogenous shock affecting the oil supply curve are depicted based on the non-linear model equations. Fourth, the section discusses the potential recessionary effect of mitigation measures to reduce oil consumption.

4.1 Structural parameters

The interaction between oil and financial markets in the model might change the estimation results for the structural parameters common to both models. Table 3 reports the posterior mean for the different model parameters. The elasticity of substitution between the capital-oil composite production factor and hours worked is above one. It indicates that labour and capital are imperfect substitutes and not complements, according to the estimation results. Further, the posterior mean for the model with financial accelerator and oil is lower than without financial accelerator. The posterior mean of the CEE–Oil model is still part of the 90% credibility interval for the CMR–Oil model. The posterior mean for the inverse supply elasticity and the credibility intervals of oil are in both models very similar. The same is true for the demand elasticity of oil. Note, that the demand elasticity is below the prior mean and the supply elasticity above the prior mean. Therefore, oil demand reacts less to price changes than the oil supply.

For the CMR–Oil model the posterior mean of the curvature parameter of investment adjustment cost is higher than for the CEE–Oil model. Both posterior means are part of the credibility interval of the other model and intervals overlap. The result indicates that changing investment levels will be less effective in the CMR–Oil model compared to the CEE–Oil model. Results for the curvature of capital utilization costs are very close in both models.

The Calvo parameter for wage stickiness is very low but is close to the one reported by CEE for the model without indexation. Including financial markets to the model leads to a decrease in the Calvo parameter for wage stickiness. Calvo parameters

Table 3: Estimation results for structural parameters

Model	CEE–Oil model	CMR–Oil model
elasticity of subsitition between energy-capital composite good and labour	1.56	1.38
η^M	[1.29, 1.87]	[1.12, 1.70]
curvature of investment adjustment cost	5.58	6.88
$\mathcal{S}''$	[3.89, 7.75]	[5.13, 8.79]
curvature of utilization cost	1.12	1.12
$\sigma^{a(u)}$	[0.97, 1.27]	[0.97, 1.27]
weight on output growth in Taylor rule	0.37	0.38
$\tilde{a}_{\Delta y}$	[0.31, 0.44]	[0.31, 0.45]
weight on inflation in Taylor rule	-	-
$\tilde{a}_{\pi}$	[-]	[-]
Calvo parameter wages	0.32	0.33
ξ^w	[0.28, 0.36]	[0.29, 0.37]
Calvo parameter prices	0.44	0.43
ξ^p	[0.40, 0.47]	[0.40, 0.47]
AR(1) coefficient for risk free interest rate	0.79	0.83
$\tilde{\rho}$	[0.77, 0.81]	[0.81, 0.85]
demand price elasticity for oil consumption	0.10	0.11
η^O	[0.08, 0.14]	[0.08, 0.15]
inverse supply price elasticity for oil production	7.46	7.60
σ^O	[5.96, 9.46]	[6.04, 9.69]

Notes: The posterior mean and the 90% highest posterior density (HPD) interval for the respective parameters in parentheses are reported.

for price rigidity are slightly below the prior mean and indicate an average one-year duration of prices. The monetary policy parameter for output is very similar between both models. However, the monetary policy instrument is more rigid in the CMR–Oil model compared to the CEE–Oil model.

Estimation results for persistence parameters are reported in Table 11 in the Appendix. In the CMR–Oil model the persistence parameter for investment adjustment costs is smaller compared to the CEE–Oil model. It implies that investment adjustment costs are less persistent in a model with a financial accelerator. All other persistence parameters present in both models are very similar. Table 12 in the Appendix reports Estimation results for standard deviations of shocks. Here the standard deviations for investment adjustment costs and price mark-ups are different across the two models. The estimated anticipated signal correlation is weak (0.08) at the posterior mean. In 90% of the draws at the posterior mode, it does not exceed a moderate (0.32) magnitude.

The comparison of structural parameters reveals no tremendous difference between both models. Therefore, results for the variance and historical decomposition are mainly driven by including the financial accelerator.

4.2 Historical and variance decomposition

Table 4 reports the theoretical variance decomposition for the national account variables for the CMR, CEE–Oil and CMR–Oil model. In contrast to the results by CMR, risk shocks only explain one fifth instead of more than half of the theoretical variance of GDP growth. The main reason for this reduction is the monetary policy rule. Christiano et al. (2014) use a log-linearised version of (9). However, the monetary policy rule in their replication code is not the log-linearised version of (9).[8] This misspecification is the main reason for the divergence between results in Table 4 and Table 5 in Christiano et al. (2014).

Results of the variance decomposition using the estimated parameters by CMR show that risk shocks contribute about 21% in total to GDP growth. The contribution of risk is between 1.7% and 5.8%. Therefore, risk shocks are only a minor driver of GDP growth rates. In addition to the Taylor rule persistence parameters for consumption, inflation and wages are responsible for the drop. A lower persistence of prices and wages affect the contribution of risk to GDP growth. Less persistent habits lead to a lower contribution of risk to consumption behaviour.

Results at the posterior distribution of both model variants state that technology shocks, especially to the long-run growth rate, explain 38% to 59% of the theoretical variance of GDP growth. The introduction of financial frictions to the baseline model with oil leads to an increase in the contribution of monetary policy shocks to the theoretical variance of GDP growth. The second most important category are demand shocks. They explain 14% to 22% of the variance in GDP growth. Risk shocks and the marginal efficiency of investment are the main drivers of the growth rate in capital formation. The external finance premium, credit and equity growth rates are mainly driven by risk shocks as reported by Table 15 in the Appendix.

A first result is that the inclusion of financial frictions slightly reduces the theoretical variance contribution of oil market variables to GDP growth at the posterior mean. However, the credibility interval of the CEE–Oil model includes the posterior mean for the CMR–Oil model. The inclusion of a financial accelerator to the model does not affect the contribution of oil to the variance of GDP growth. One main reason for the observed reduction is a lower contribution of oil market variables to investment growth. It is noteworthy that shocks from the oil market have a lower contribution to the variance of investment, consumption and wage growth rates compared to their respective levels. It is valid for both models. Oil market disturbances explain only a small fraction of the theoretical variance of the federal funds rate and inflation.

As stated in Bernanke et al. (1999), the financial accelerator mechanism can amplify

[8] The files are available under `https://www.aeaweb.org/articles?id=10.1257/aer.104.1.27`.

small shocks such as discretionary monetary policy. The theoretical variance decomposition shows that unexpected movements in the federal funds rate contribute between 11% and 16% to the theoretical variance of GDP growth for the model with a financial accelerator. The contribution ranges between 8.5% and 12% for the CEE–Oil model. Nevertheless, the results can not verify the statement that the financial accelerator mechanism amplifies oil market shocks. In contrast, for the reported aggregates oil market shocks contribute less to GDP growth, consumption and investment with a financial accelerator. Here the main reason is, that risk shocks explain more of the variance in investment and reduce the contribution previously attributed to the oil market shocks.

Table 16 in the Appendix tabulates theoretical variance decomposition for the oil market variables. Domestic and foreign oil supply shocks do not affect each other. The contribution of domestic oil demand shocks is higher for home oil supply than for foreign production. Domestic and foreign oil supply shocks equally drive crude oil prices. Further, technology innovations and unexpected changes in domestic oil demand contribute with similar shares to the theoretical variance of oil prices. Including the financial accelerator into the model, shows that risk and investment shocks become as crucial as other technology innovations explaining the variance of oil prices. Otherwise, including the financial accelerator does not qualitatively alter the variance decomposition and also only slightly in a quantitative way.

Risk and the marginal efficiency of investment shocks mainly drive investment according to the variance decomposition. Figure 2 depicts the historical contribution of the marginal efficiency of investment (m.e.i.) and risk shocks to GDP growth. The inclusion of financial frictions reduces the contribution of the marginal efficiency of investment, especially during the Great Recession (through investment growth). The historical decomposition also reveals that risk shocks are the main driver of the external finance premium and credit growth. Further, the external finance premium reached its maximum observed value during the financial crisis, and this coincides with the time risk contributed the most to GDP and investment growth. The marginal efficiency of investment on the other side has only a small impact on the external finance premium and credit growth.

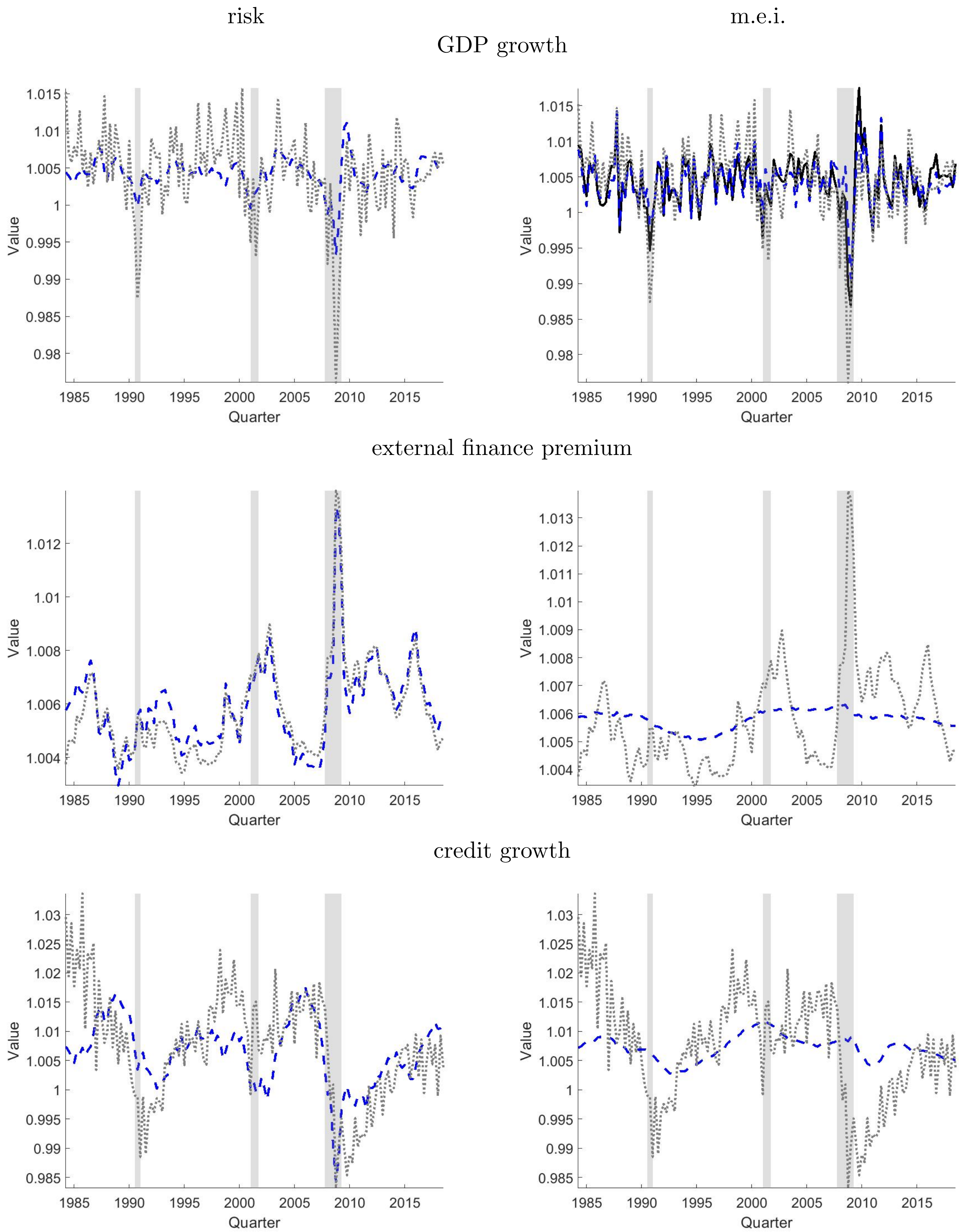
Figure 11 in the Appendix depicts oil market disturbances and their contribution to the business cycle. One can see here that the contribution of oil market disturbances to the oil price is almost identical in both models. The same result holds for GDP, investment and consumption growth. Therefore, the financial accelerator framework does not amplify the role of oil market disturbances on the US business cycle for the period 1984-Q2 to 2018-Q3. During the financial crisis, a tremendous oil price drop occurred. According to the historical decomposition at the posterior mean, the

Table 4: Variance decomposition for national account variables at the posterior distribution

Variable	risk	investment	demand	financial	M.P.	markup	technol.	oil
				GDP growth				
CMR	21.2	4.7	34.3	0.3	1.2	22.7	15.6	0.0
CEE–Oil	0.0	11.4	17.0	0.0	10.3	8.6	51.7	0.8
	[0.0, 0.0]	[8.9, 13.6]	[13.9, 19.9]	[0.0, 0.0]	[8.5, 12.1]	[6.9, 10.3]	[44.8, 58.6]	[0.5, 1.1]
CMR–Oil	3.9	10.1	18.8	2.1	13.4	7.0	43.8	0.7
	[1.7, 5.8]	[7.6, 12.3]	[15.5, 21.9]	[0.6, 3.5]	[11.1, 16.0]	[5.6, 8.3]	[37.8, 49.9]	[0.5, 1.0]
				inflation				
CMR	55.2	19.5	3.5	0.7	1.4	11.6	8.1	0.0
CEE–Oil	0.0	18.9	5.2	0.0	8.5	14.3	51.4	1.2
	[0.0, 0.0]	[14.5, 23.4]	[4.2, 6.2]	[0.0, 0.0]	[6.4, 10.8]	[10.3, 17.7]	[41.7, 59.8]	[0.8, 1.6]
CMR–Oil	11.1	10.4	5.5	5.5	12.5	11.3	42.6	1.0
	[4.7, 17.1]	[7.8, 13.3]	[4.4, 6.6]	[1.6, 9.7]	[9.8, 15.4]	[8.0, 14.6]	[34.3, 50.7]	[0.6, 1.4]
				federal funds rate				
CMR	66.1	21.6	3.9	0.8	2.4	2.8	2.3	0.0
CEE–Oil	0.0	37.0	16.1	0.0	15.8	6.5	22.7	1.1
	[0.0, 0.0]	[30.2, 43.9]	[12.8, 19.6]	[0.0, 0.0]	[12.9, 18.7]	[4.1, 8.5]	[17.4, 27.7]	[0.6, 1.7]
CMR–Oil	23.4	13.1	15.1	14.1	12.4	4.4	17.0	0.3
	[10.9, 35.2]	[8.8, 17.6]	[11.6, 18.8]	[4.3, 23.9]	[9.4, 15.4]	[2.8, 6.1]	[12.6, 21.6]	[0.2, 0.5]
				investment growth				
CMR	68.9	22.6	0.4	1.1	0.4	5.3	1.2	0.0
CEE–Oil	0.0	75.1	0.4	0.0	0.3	6.4	15.8	1.1
	[0.0, 0.0]	[66.7, 82.2]	[0.2, 0.6]	[0.0, 0.0]	[0.1, 0.5]	[3.8, 8.6]	[10.9, 20.6]	[0.5, 1.7]
CMR–Oil	26.6	48.8	0.1	15.0	1.4	3.1	4.3	0.5
	[12.9, 40.3]	[37.8, 60.6]	[0.0, 0.1]	[5.3, 24.5]	[0.8, 1.8]	[2.2, 4.0]	[3.0, 5.6]	[0.3, 0.7]
				investment				
CMR	63.9	26.1	0.4	1.4	0.2	6.5	1.5	0.0
CEE–Oil	0.0	40.2	2.8	0.0	0.1	11.1	29.7	11.0
	[0.0, 0.0]	[30.0, 51.8]	[1.3, 4.1]	[0.0, 0.0]	[0.0, 0.1]	[6.1, 15.8]	[21.8, 38.5]	[2.3, 19.8]
CMR–Oil	29.1	14.6	0.1	33.9	1.7	5.5	8.8	3.7
	[12.0, 44.1]	[7.4, 20.4]	[0.0, 0.2]	[15.1, 52.8]	[1.0, 2.4]	[2.9, 8.5]	[5.1, 13.1]	[0.6, 7.0]
				consumption growth				
CMR	46.6	20.5	19.5	0.7	0.4	8.0	4.3	0.0
CEE–Oil	0.0	5.6	28.5	0.0	17.7	6.3	41.4	0.4
	[0.0, 0.0]	[4.2, 7.0]	[23.9, 32.5]	[0.0, 0.0]	[15.0, 21.0]	[4.9, 7.7]	[34.9, 48.0]	[0.3, 0.6]
CMR–Oil	2.7	3.2	27.6	2.0	19.3	5.6	39.2	0.4
	[1.1, 4.2]	[2.3, 4.1]	[23.0, 31.4]	[0.5, 3.4]	[16.1, 22.8]	[4.3, 6.8]	[32.8, 45.3]	[0.3, 0.6]
				consumption				
CMR	52.1	21.2	5.7	1.0	0.1	11.0	8.8	0.0
CEE–Oil	0.0	8.3	34.7	0.0	1.6	7.7	40.1	5.8
	[0.0, 0.0]	[5.5, 11.2]	[26.2, 44.4]	[0.0, 0.0]	[1.1, 2.0]	[5.5, 9.8]	[31.4, 49.7]	[1.1, 10.9]
CMR–Oil	8.6	4.3	26.4	10.4	2.3	8.3	34.8	3.3
	[3.4, 13.7]	[2.7, 5.8]	[18.4, 33.9]	[2.7, 18.0]	[1.6, 2.9]	[5.9, 10.6]	[25.7, 42.4]	[0.7, 6.0]
				wage growth				
CMR	5.2	2.8	3.0	0.0	0.2	59.2	29.7	0.0
CEE–Oil	0.0	0.4	0.8	0.0	0.0	40.8	56.9	0.8
	[0.0, 0.0]	[0.2, 0.6]	[0.5, 1.0]	[0.0, 0.0]	[0.0, 0.0]	[36.1, 45.9]	[46.6, 66.4]	[0.5, 1.2]
CMR–Oil	0.2	0.4	1.0	0.2	0.0	41.3	55.5	1.1
	[0.1, 0.4]	[0.2, 0.5]	[0.6, 1.3]	[0.0, 0.3]	[0.0, 0.1]	[36.5, 46.6]	[45.2, 65.3]	[0.6, 1.5]
				wage				
CMR	37.5	28.4	1.9	1.4	0.2	21.0	9.5	0.0
CEE–Oil	0.0	2.0	4.3	0.0	0.0	28.7	55.0	7.2
	[0.0, 0.0]	[1.1, 2.9]	[2.9, 6.0]	[0.0, 0.0]	[0.0, 0.0]	[19.7, 37.3]	[42.8, 67.8]	[1.4, 13.5]
CMR–Oil	2.6	1.2	2.6	8.1	0.4	31.6	45.9	4.7
	[0.8, 4.4]	[0.6, 1.7]	[1.7, 3.7]	[2.1, 14.3]	[0.2, 0.5]	[22.3, 41.1]	[33.8, 58.0]	[0.8, 8.7]

Note: Theoretical contribution of each shock group in percent to the total variance of the respective variable is reported. Results for the CMR model are computed using the parameter values of Christiano et al. (2014) as tabulated in Table 14. The variance decomposition for the CEE–Oil and CMR–Oil model are reported for the estimated posterior distribution. Values in parentheses represent 90% HPD interval of the model parameters. The shock groups are reported in Table 13.

Figure 2: Historical contribution of risk and m.e.i. shocks



Note: The solid black line represents the historical decomposition for the CEE–Oil model, the shaded blue line for the CMR–Oil model, and the dotted gray line the observed data. Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.

results show that the change in oil price was mainly due to oil market disturbances. It is clear that at the time, mostly lower oil demand driven by lower global economic activity (see, e.g. Ratti & Vespignani 2013) caused the fall in oil prices. A more detailed historical decomposition reveals that oil domestic productivity shocks (oil demand shocks) contributed the same share to the decline in oil prices as supply shocks. One potential explanation for the contribution of oil supply shocks is the closure of less profitable drilling wells, which implies that the remaining drilling wells are less expensive to operate.[9]

4.3 Impulse response functions

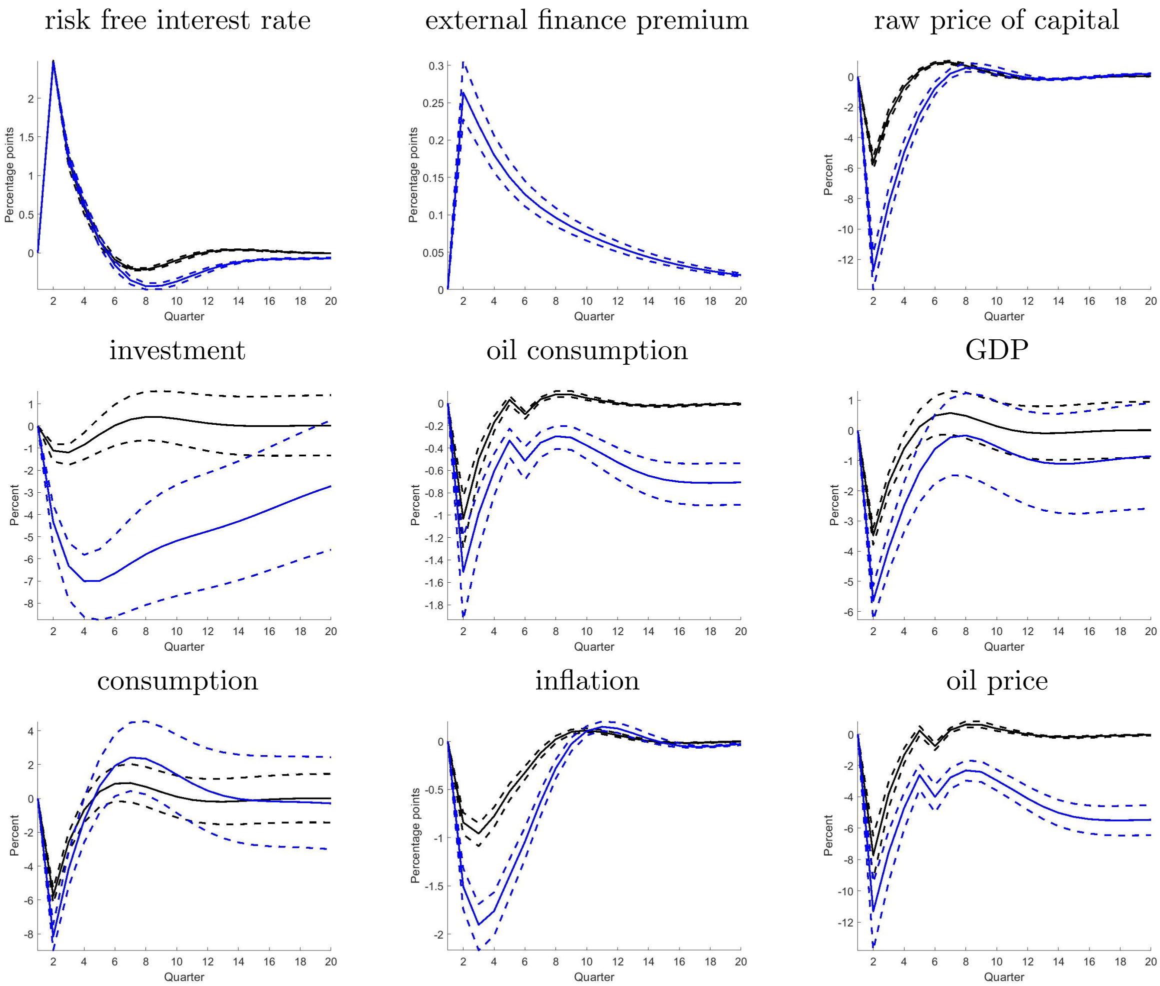
The variance and historical decomposition both mainly reveal a crowding-out of m.e.i. for risk shocks. Bernanke et al. (1999) state that the financial accelerator can amplify the impact of small shocks such as discretionary monetary policy. The variance decomposition reveals a little amplification effect for monetary policy shocks, but the opposite for oil market disturbances. Figure 3 presents impulse response functions for discretionary monetary policy shocks on different variables.[10] The monetary policy shock increases the risk-free interest rate by more than two annualized percentage points for both models. This increase leads to a rise in the external finance premium by about 0.25 annual and the bankruptcy rate by 0.5 percentage points. This increase in the probability of insolvency and the external finance premium triggers an additional reduction in investment. With the financial accelerator, investment drops about four times more compared to the model without financial accelerator. Further, the 90% HPD interval is not overlapping and suggests that this difference is unlikely a random observation. Additionally, one can see that oil consumption also declines more persistently as a response to monetary policy shocks. However, the drop in GDP growth is only slightly more significant with financial accelerator compared to the model without financial accelerator. Consumption responds similarly to a monetary policy shock in both models. The model with financial accelerator simulates a more substantial drop in inflation for the model without financial accelerator. This greater magnitude in the decline of inflation links to a more persistent plunge in oil prices as a response to the more persistent decline in oil demand.

The financial accelerator might amplify oil supply shocks. Here, domestic and foreign oil supply shocks increase the oil price simultaneously by 50%. Figure 4 depicts the response to oil supply shocks for a selected number of variables. Oil consumption drops

[9]The EIA publishes the number of US Crude Oil and Natural Gas Rotary Rigs in Operation under `https://www.eia.gov/dnav/ng/hist/e_ertrr0_xr0_nus_cM.htm`.

[10]I compute impulse response functions for the non-linear version of the model using deterministic simulations.

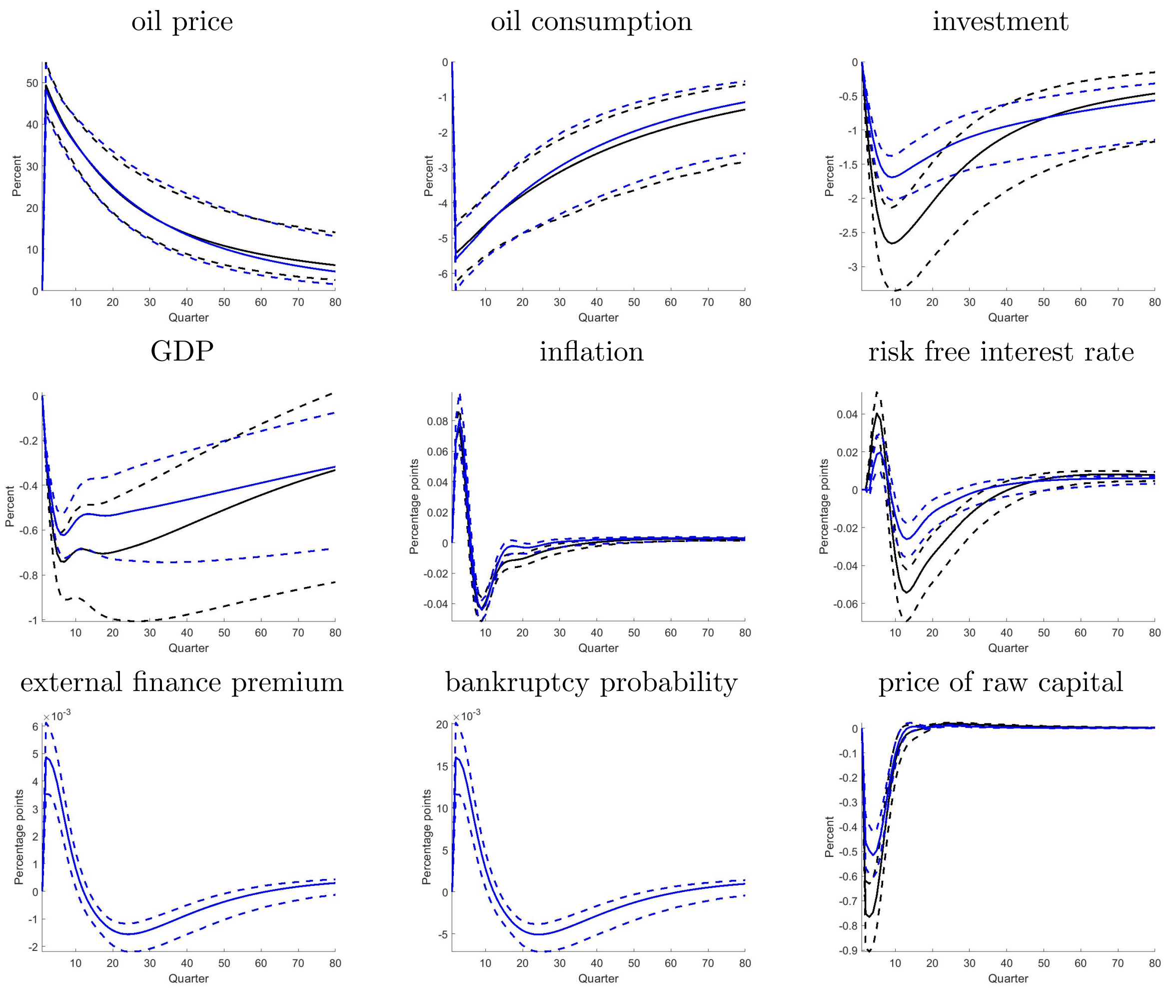
Figure 3: Impulse response functions for monetary policy shocks



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

only by roughly five percent, reflecting the low price elasticity of oil demand. Investment will fall by approximately two percent with the financial accelerator mechanism and by 2.5% without credit market frictions. The resulting drop in GDP is indistinguishable for the two model variants. Inflation will increase by the same amount with and without financial accelerator. According to the monetary policy rule, the risk-free interest rate increases to reduce observed inflation. Monetary policy amplifies the drop in GDP. The external finance premium and bankruptcy probability both increase, but the decline in investment are lower with compared to the model without financial accelerator. Raw capital prices fall less in the model with financial accelerator compared to the model without financial accelerator. Household investments react less in the CMR–Oil model compared to the CEE–Oil model. A lower drop in raw capital prices is at odds with the previous findings for the monetary policy shock. For the monetary policy shock, an increase in the risk-free interest rate triggered a rise in the external finance premium this lead to a further decrease in the raw capital price. However, for the oil price shock, the external finance premium also increases. Nevertheless, the increase in the external finance premium is not sufficient to reduce the raw capital price more compared to the model without financial accelerator. The financial accelerator mechanism also introduces rigidity for the raw capital price through the law of motion for net worth, the zero-profit condition of the mutual funds and the first-order condition of the entrepreneurs. Financial frictions have not the expected amplification effect for the oil supply shocks. In contrast, the reported impulse response functions suggest that the financial accelerator stabilizes investment compared to a model without credit market frictions.

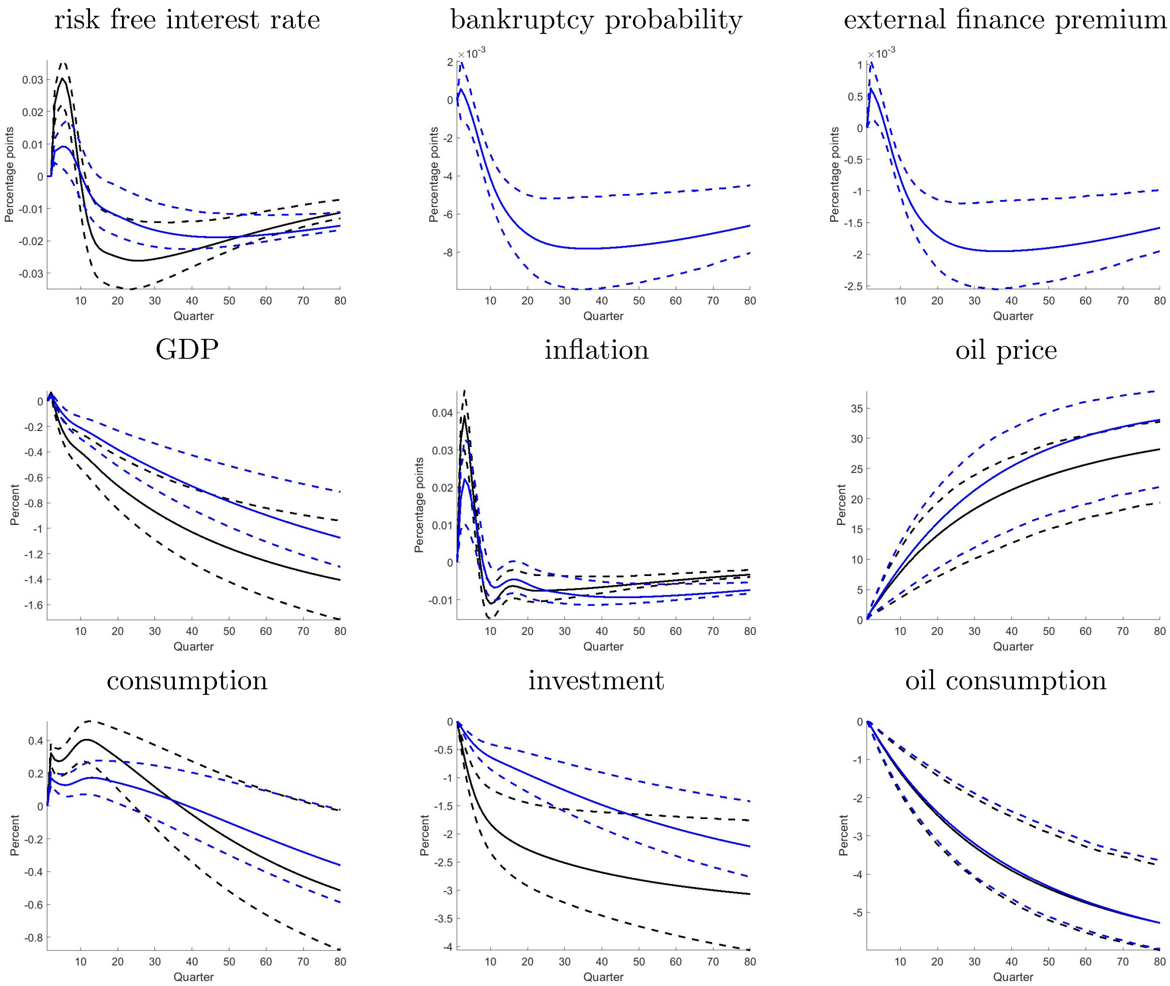
Figure 4: Impulse response functions for temporary oil supply shocks



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

Impulse response functions for permanent oil supply shock leads to a permanent increase in the price of oil and will permanently decrease oil consumption. The rise in oil prices triggers temporary initial higher inflation, an increase in GDP growth and an increase in the risk-free interest rate. Long-run stationary investment and consumption will decline, but consumption will initially increase. Here, the initial increase in consumption reflects less incentive to invest in the future capital stock, which is less productive. Therefore, households consume more disposable income. The bankruptcy probability and the external finance premium initially increase and permanently fall. This initial increase does not lead to a sharper drop in raw capital prices. In contrast, the raw capital price is more rigid and will not decline as much as without financial accelerator. Therefore, investment declines with a lower pace compared to the model without financial accelerator.

Figure 5: Impulse response functions for permanent oil supply shocks



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

4.4 Mitigation and monetary policy

The historical decomposition did not attribute recessions to oil market disturbances. However, a future reduction in oil consumption to comply with the Paris Agreement might change this. It is possible to increase the tax on oil paid by suppliers τ^o to reduce oil consumption. Here, the increase in the oil tax rate ensures that oil consumption permanently falls by 10%. Also, the impact of mitigation policy on inflation requires discretionary monetary policy to mute the effect on inflation. Therefore, computed monetary policy shocks ensure that inflation does not deviate from its target value by more than 0.01 annual percentage points. Figure 6 reports the trajectories for oil tax rate, risk free interest rate, and inflation.[11] Oil tax rate needs to increase by more than 50 percentage points to reduce oil consumption permanently by 10%. As a result, the oil price will almost double. This increase in the oil price will then trigger inflation without any intervention by monetary policy authorities. Inflation will increase by more than 0.1 annual percentage points five quarters after the oil tax rate increase. After ten quarters inflation will be at least 0.05 percentage points lower compared to its initial value. The risk-free interest rate slightly increases, but after five quarters, it falls. The initial reaction of consumption is positive for the model without financial accelerator and negative for the model with a financial accelerator. Investment drops, but for the model, without the financial accelerator, the initial investment drop exceeds the one for the model with a financial accelerator.

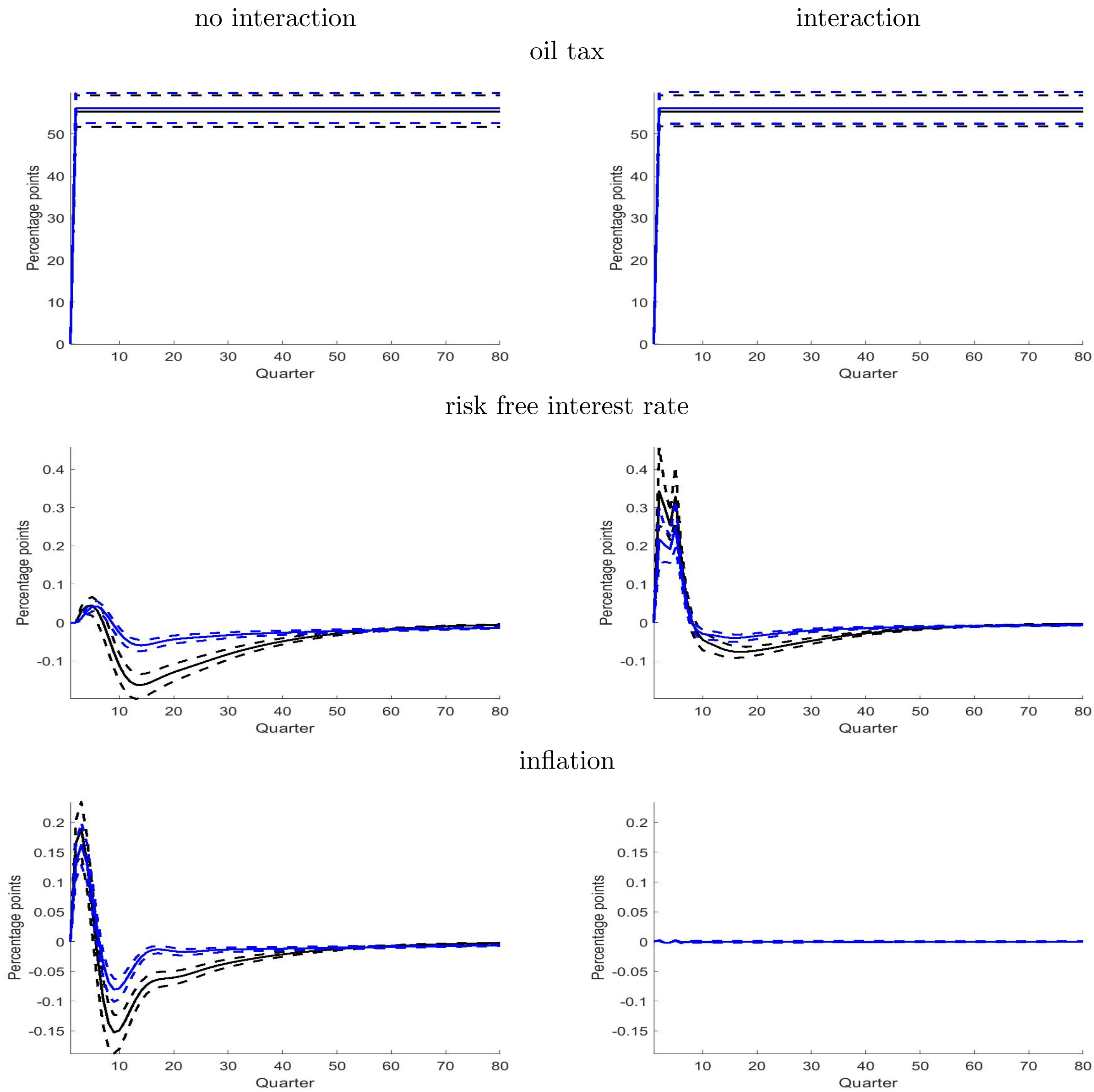
In case monetary policy wants to stabilize inflation, it needs to discretionary deviate from its monetary policy rule. The interaction panel of Figure 6 shows the required response to the risk-free interest rate to stabilize inflation. The risk-free interest rate needs to increase by at least 0.15 annual percentage points for the first seven quarters to mute the impact of the oil price increase. The required growth is by 0.1 yearly percentage points greater for the model with financial accelerator compared to the model without financial accelerator. This increase in the risk-free interest rate can stabilize inflation, but also leads to a faster decline in investment and consumption (see Figure 7). For the CEE–Oil model investment declines more severely compared to the CMR–Oil model.

5 Discussion

The comparison of the CEE–Oil and CMR–Oil model shows that the estimated structural parameters are very similar and do not reveal any credible difference at the posterior mean. Therefore differences between the models in explaining the macroeconomic

[11] The response of oil market variables are depicted in Figure 12 in the Appendix.

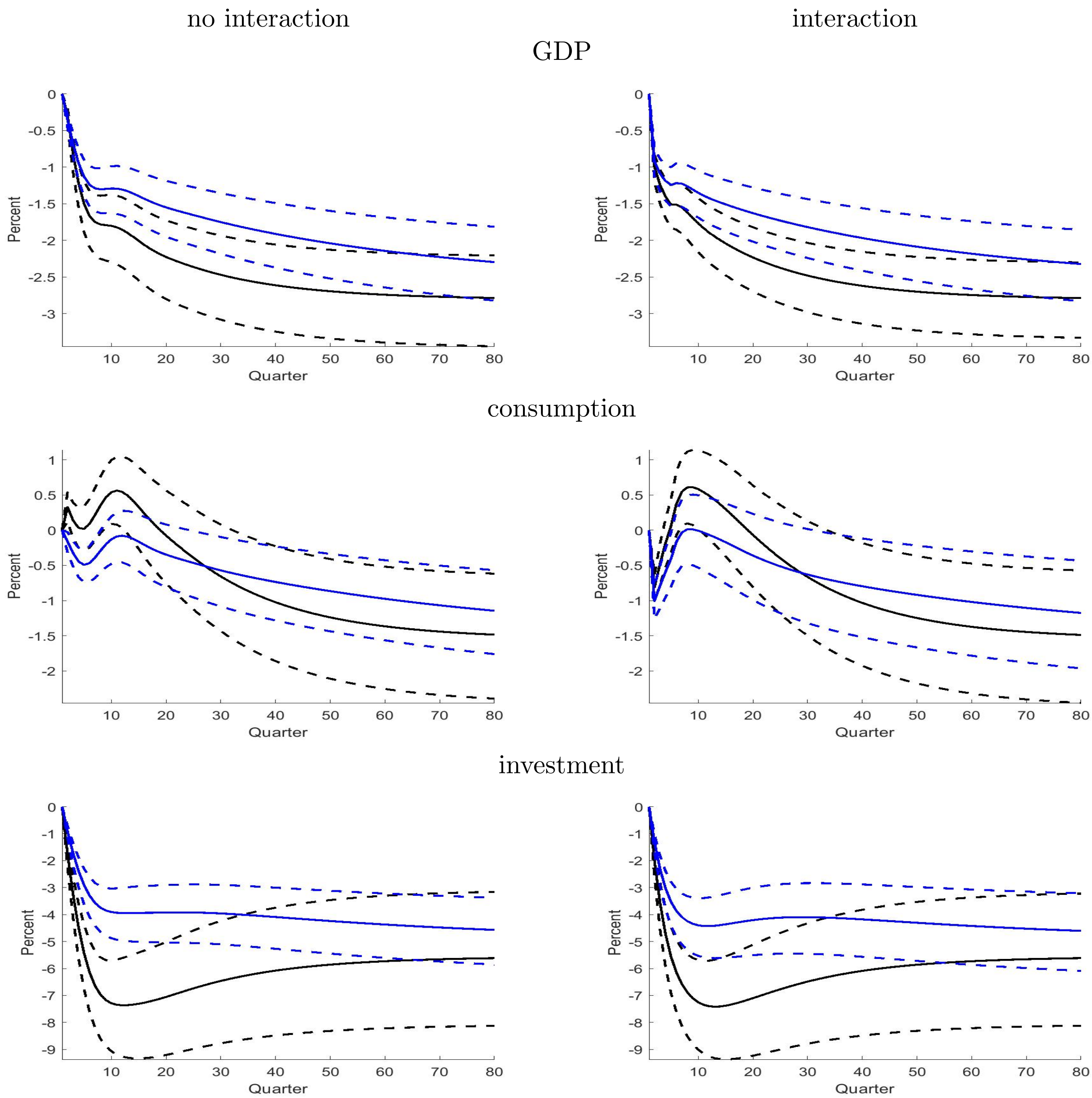
Figure 6: Trajectories for shocks to oil taxes: policy instruments and inflation



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

variables are mainly caused by the financial accelerator mechanism. The variance decomposition reveals that risk shocks are the primary driver of investment, but not of consumption. The historical decomposition shows that risk explained most of the drop in real variables during the Great Recession. However, risk shocks are not the main driver of the business cycle in the US. Oil market shocks also contribute only with roughly one percent to GDP or inflation. Technology and demand shocks contribute the most to GDP growth according to the variance decomposition. More specifically,

Figure 7: Trajectories for shocks to oil taxes: GDP growth and components



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

shocks to the long-run growth rate mainly drive GDP. For the sample period, 1984-Q2 to 2018-Q4 oil market shocks only played a minor role in the business cycle in the US.

The impulse responses reveal that monetary policy has more severe implications for investment in a model with financial accelerator compared to a model without financial accelerator. Monetary policy needs to monitor financial market imperfections to ensure that the selected policy instruments are adequate for the respective purpose. However, with the financial accelerator mechanism investment reacts less to oil supply shocks. Further, inflation is less volatile for the model with a financial accelerator.

The degree of imperfections in the credit market determines the response of inflation and investment to oil supply shocks. The monetary policy response to oil market disturbances depends on the financial frictions.

Oil market variables have not been a major driver of the business cycle in the US, but this might change with ambitious future mitigation policies. It is in line with previous studies (see Mercure et al. 2018). More precisely, a reduction of oil consumption by 10%, in the long run, can lead to a decline in consumption by 0.6 to 1.6%. Further, the models predict a permanent reduction in investment by 3 to 7%. Financial market imperfection reduces the immediate response of investment to an increase in oil taxes. Inflation will be above the target rate for about six quarters. Afterwards, inflation will be below the target rate for the same number of quarters. The monetary authority can stabilize inflation, but the risk-free interest rate needs to increase substantially above the rate determined by the monetary policy rule. This increase in the risk-free interest rate will reduce consumption. Nevertheless, the reduction in investment is almost identical to the path without interaction.

The costs of deviating from the monetary policy rule are not only captured by a further decline in consumption or investment. There are also costs not directly measurable with a DSGE model. As stated in Fischer (1990), a discretionary monetary policy might lead to a loss in confidence and further to an increase in political pressure. Therefore, it seems not recommendable to mute the rise in inflation by deviating from the monetary policy rule. The reduction in consumption will increase political pressure to stick with the monetary policy rule.

The present study considers the interaction of financial markets and oil markets in a model for the US economy. Future research should reconsider some of the underlying assumptions of the model. First, the model considers oil as production factor without a differentiation of the usage of oil in the economy. Balke & Brown (2018) differentiates between oil for transportation and consumption. Mitigation policies will target the oil used in the transportation sector. Therefore, a more elaborate model will explicitly include alternatives to oil as an input to the transportation sector. Oil as a raw material in the chemical industry is still not easy to replace by other raw materials. Mitigation policies will target the reduction of oil as an energy source mainly applied in the transportation sector.

Another issue is that oil supply is not finite in the model, and the discovery of new reserves is costless. Hansen & Gross (2018) includes limited natural resources and introduces exploration activities to increase the reserves of natural resources for a small open economy. It seems worthwhile to extend the model to include such features. Nevertheless, this extension requires additional data to estimate the model. Identical extraction costs for domestic and foreign oil producers is a testable assumption.

6 Conclusion

Is risk the fuel of the business cycle? The present study shows that disturbances from the credit market are not the main driver of the business cycle in the US. Nevertheless, they explain about one-fifth of the variance in GDP growth. Further, they are essential to explain investment behaviour. During the Great Recession, risk shocks have been the leading cause of a drop in investment and GDP.

Oil market shocks have not been a significant driver of the business cycle in the US between 1984-Q2 to 2018-Q4. These findings are based on the historical and theoretical variance decomposition of the US economy using a DSGE model with financial frictions and oil as a production factor. The impulse response functions at the posterior mean show that the financial accelerator amplifies the effect of monetary policy shocks on investment, but not for oil supply disturbances. In contrast to the statement in Bernanke et al. (1999), the response in investment to oil price shocks is not amplified compared to a model without financial accelerator.

In the future mitigation measures to reduce oil consumption can cause a recession. An increase in the oil tax rate by roughly 50 percentage points will decrease oil consumption permanently by 10%. This increase in the oil tax rate triggers higher oil prices by 50 to 90%. Inflation increases by 0.1 to 0.2 annual percentage points. Consumption permanently drops by 0.5 to 1.7% and investment by 3 to 7%. Monetary policy can stabilize inflation, but its reaction depends on financial market imperfections. The risk-free interest rate has to increase by 0.15 to 0.3 annual percentage points and without financial frictions by 0.25 to 0.45 annual percentage points, to mute the initial increase in inflation completely.

The developed model can study the interaction between financial and oil markets. Further, the model can analyse the impact of mitigation measures on the US economy. However, the discussion tackled some potential avenues for future modifications of the model. The model results are based on estimated parameters and the underlying estimation uncertainty and resemble the main contribution of the model to mitigation policy discussions.

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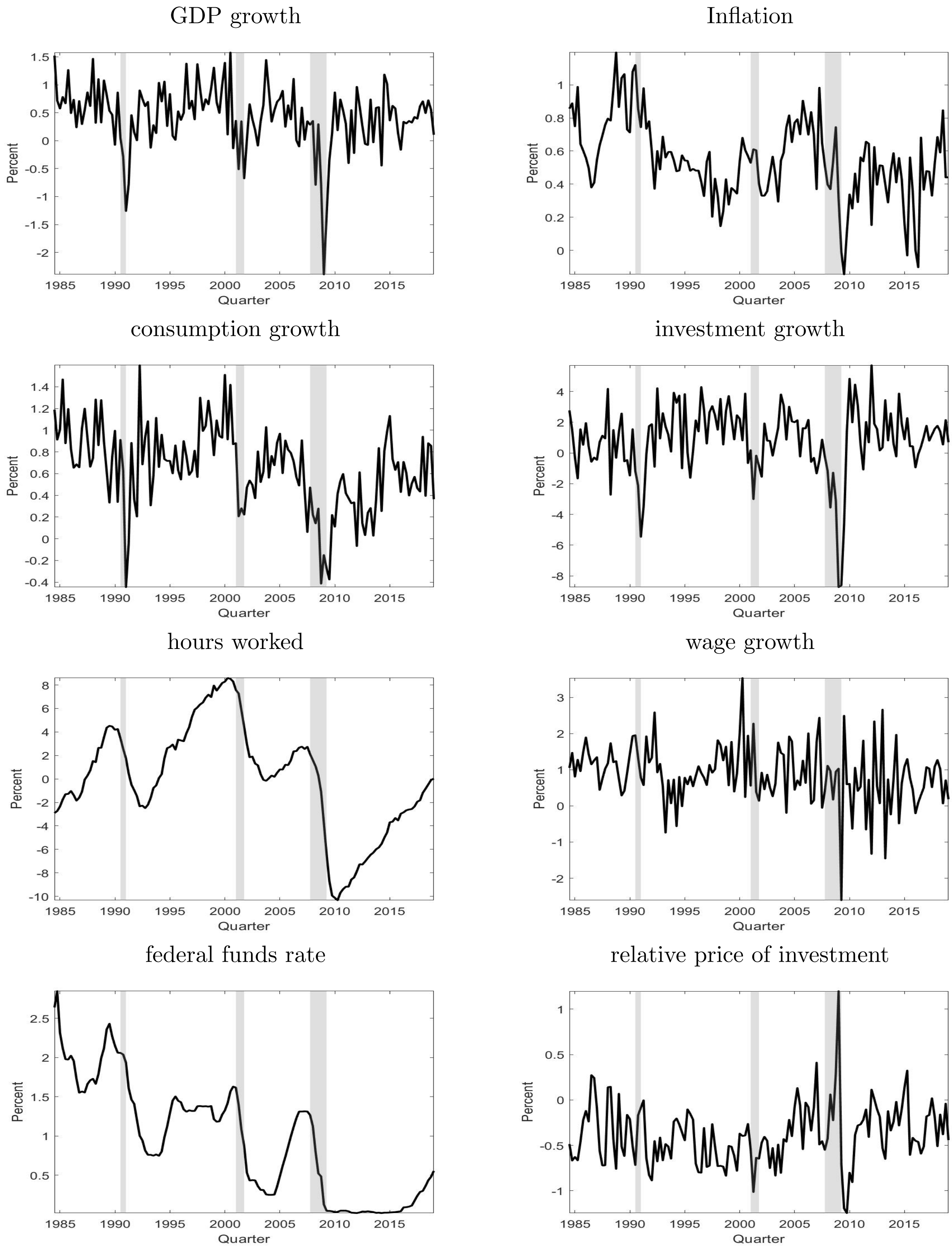
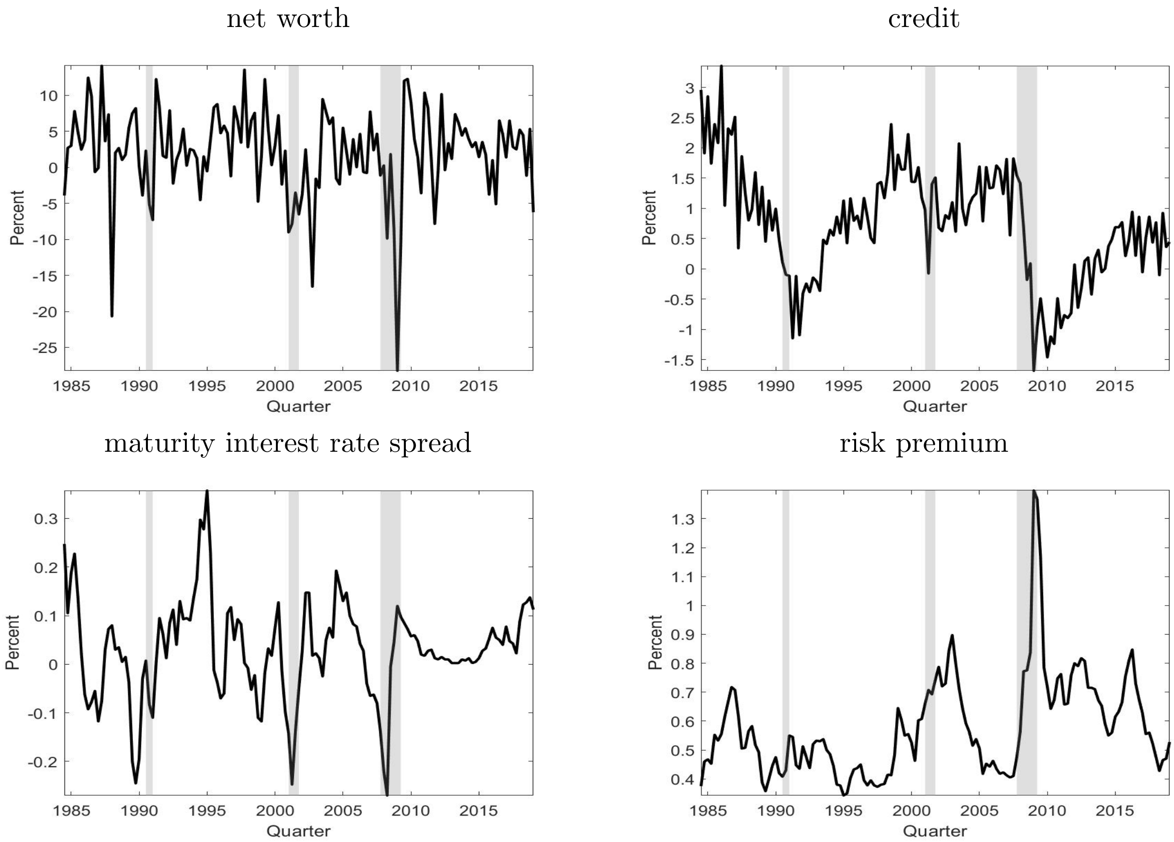
A Figures

Figure 8: Standard macroeconomic variables



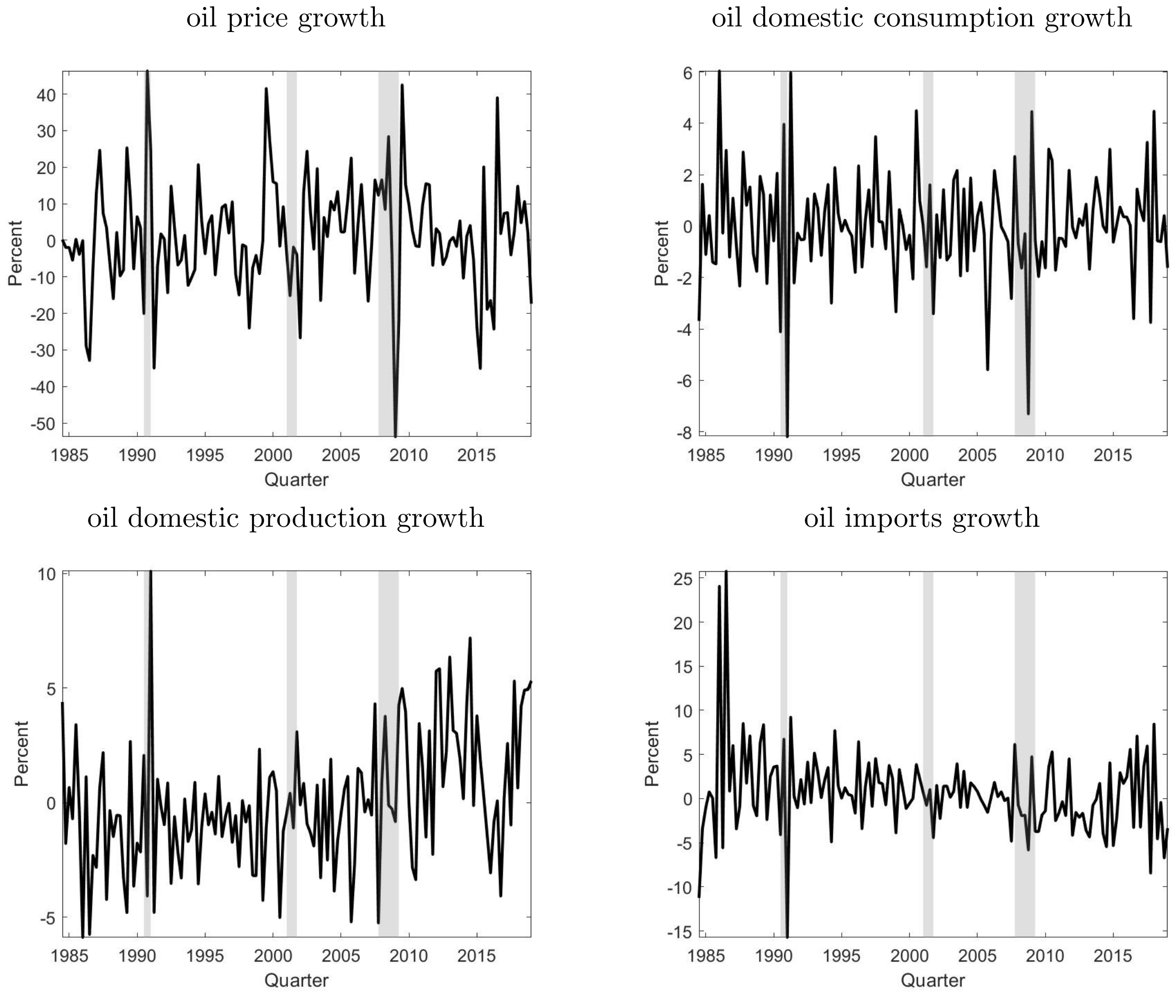
Notes: Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, Federal Reserve Bank of St. Louis.

Figure 9: Financial market variables



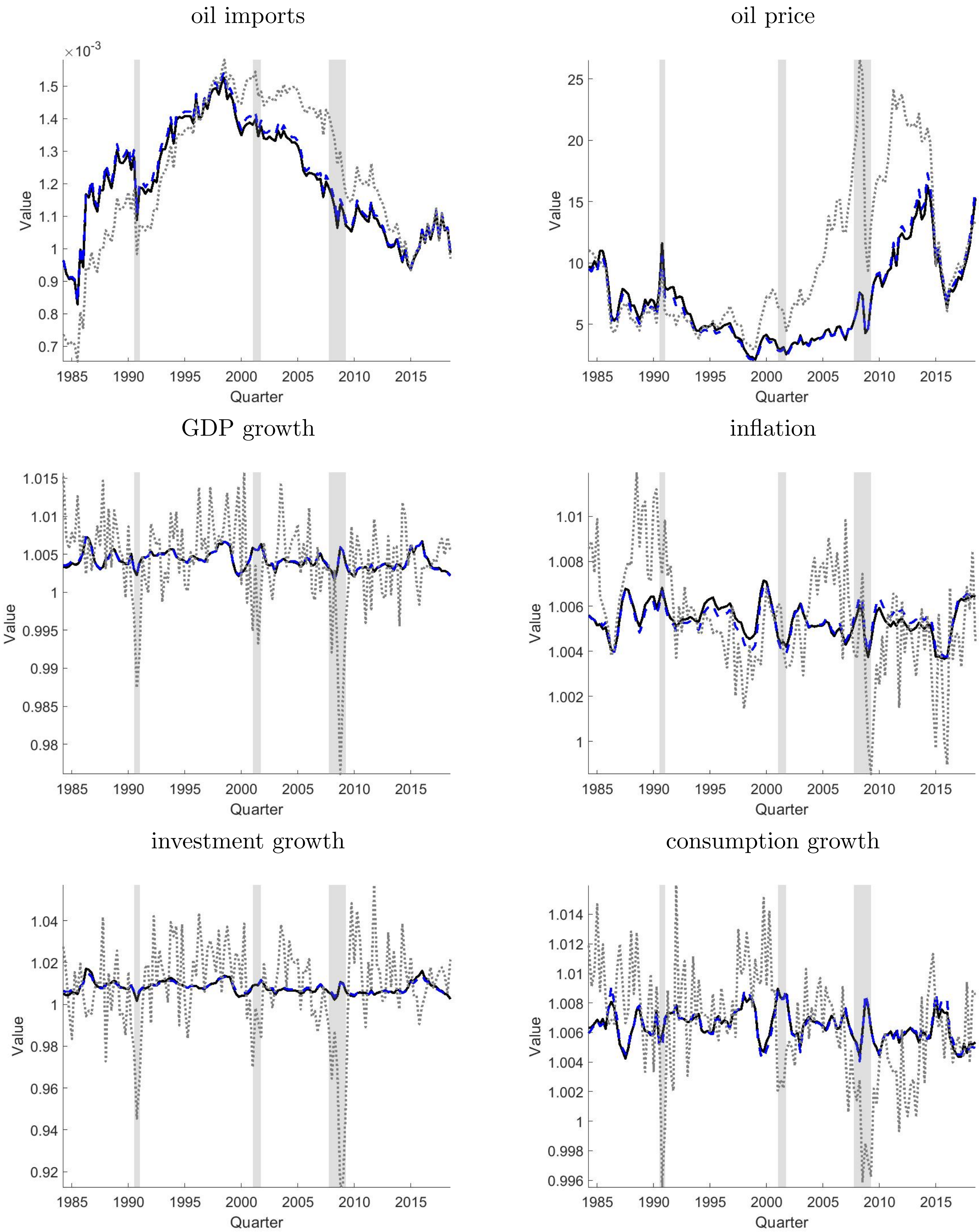
Notes: Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, Federal Reserve Bank of St. Louis.

Figure 10: Oil market variables



Notes: Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, US Energy Information Administration.

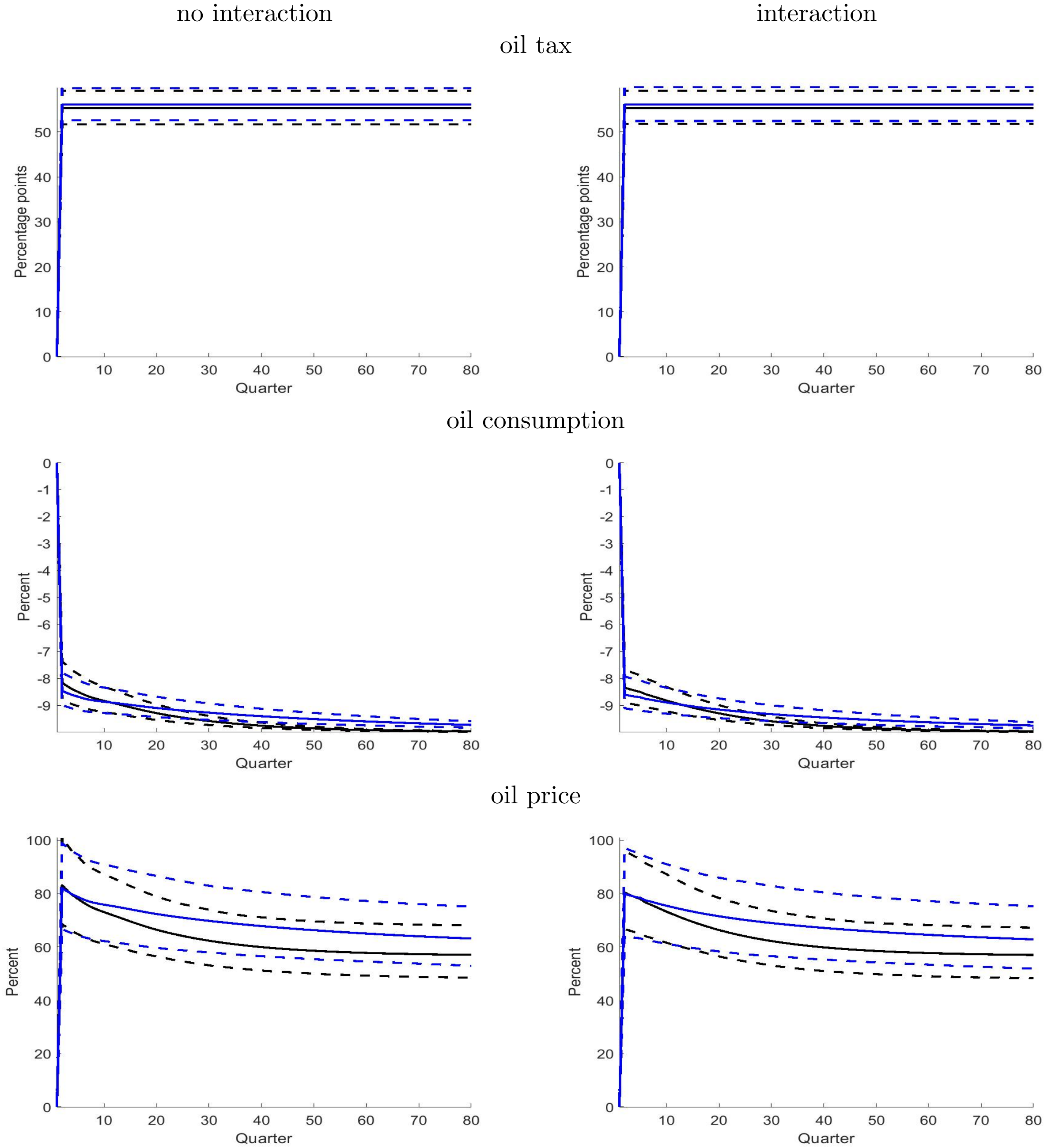
Figure 11: Historical contribution of oil market shocks



Notes: The solid black line represents the historical decomposition for the CEE–Oil model, the shaded blue line for the CMR–Oil model, and the dotted gray line the observed data. Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.

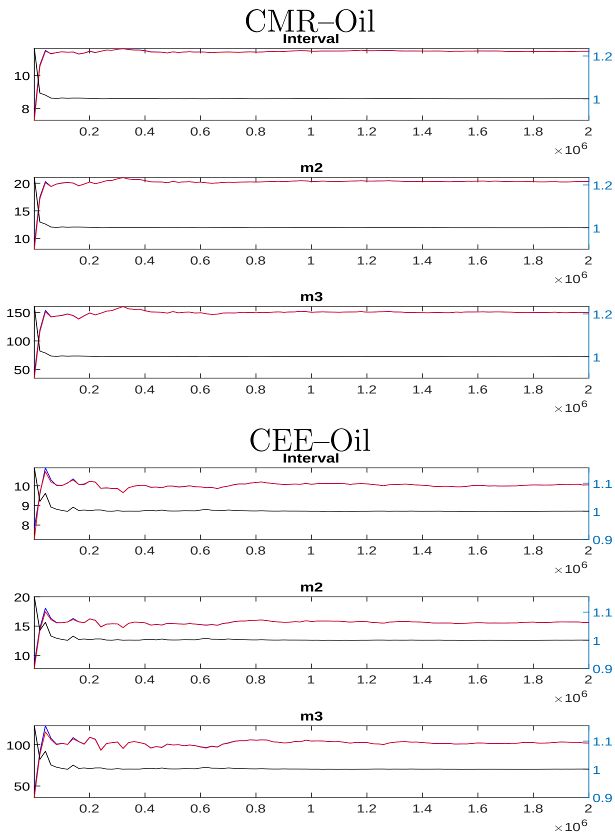
Sources: Own computation, Federal Reserve Bank of St. Louis, US Energy Information Administration.

Figure 12: Trajectories for shocks to oil taxes



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE-Oil model and the solid blue line for the CMR-Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution. Sources: Own computation, Federal Reserve Bank of St. Louis, US Energy Information Administration.

Figure 13: Multivariate parameter convergence



Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis. Here the statistics are based on the log-likelihood function.

B Tables

Table 5: Endogenous variables

Variable		Description
stationary	**non-stationary**	
	z_t	long-run unit root technology shock
ϵ^k		temporary productivity shock composite good
ϵ^o		temporary productivity shock to oil usages
p^m	P^m	price of composite good
m	M	composite good
o	O	oil consumption
o^d	O^d	oil domestic production
o^{ex}	O^{ex}	oil exports
o^{im}	O^{im}	oil imports
p^o	P^O	oil price
ζ^o		domestic oil productivity shock
$\zeta^{o,im}$		domestic oil imports shock
$\zeta^{o,ex}$		domestic oil exports shock
τ^o		oil tax
o^{obs}		observational variable for oil consumption growth rate
$p^{o,obs}$		observational variable for relative price of oil growth rate
$o^{d,obs}$		observational variable for domestic oil production growth rate
$o^{im,obs}$		observational variable for oil imports growth rate
$o^{ex,obs}$		observational variable for oil exports growth rate
R^L		long-run interest rate
R^k		return on capital
n	N	net worth
$\bar{\omega}$		threshold for idiosyncratic risk
σ		risk
γ		fraction of entrepreneurs not leaving the market
$F(\bar{\omega})$		risk of bankruptcy
$F(\bar{\omega})$		expected value of $\bar{\omega}$
$dcost(\bar{\omega})$		monitoring cost
ξ^1		news to risk 1 periods ahead
ξ^2		news to risk 2 periods ahead
ξ^3		news to risk 3 periods ahead

Table 5 – Continued

Variable		Description
stationary	**non-stationary**	
ξ^4		news to risk 4 periods ahead
ξ^5		news to risk 5 periods ahead
ξ^6		news to risk 6 periods ahead
ξ^7		news to risk 7 periods ahead
ξ^8		news to risk 8 periods ahead
ζ_{term}		term structure
b^{obs}		observational variable for credit
$R^k - R^{L^{obs}}$		observational variable for relative price of risk premium
$S^{1,obs}$		observational variable for spread
n^{obs}		observational variable for net worth
c	C	consumption
g	G	government expenditure
i	I	investment
q	Q	price of raw capital
λ^z		marginal utility of consumption
y^z	Y	net output
ϕ		fix costs
h		hours worked
$\bar{k}$	$\bar{K}$	raw capital
u		utilization rate of raw capital
r^k	$\tilde{r}^k$	rental rate of capital
w	W	wage
s	S	real marginal cost
μ^z		long-run technology growth rate
μ^Υ		long-run investment growth rate
R		risk free interest rate
F^p		auxiliary variable for optimal price
K^p		auxiliary variable for optimal price
F^w		auxiliary variable for optimal wage
K^w		auxiliary variable for optimal wage
w^*		wage dispersion index
p^*		price distortion index
π		gross inflation
$\tilde{\pi}$		gross inflation of non-optimizing firms

Table 5 – Continued

Variable		Description
stationary	**non-stationary**	
$\tilde{\pi^w}$		gross wage inflation of non-optimizing unions
π^w		gross wage inflation
ϵ		temporary TFP shock
ϵ^h		temporary productivity shocks for hours worked
ζ^i		investment adjustment cost
ζ^c		consumption preference shock
ζ^h		labour supply preference shock
ϵ^w		wage mark up shock
ϵ^p		price mark up shock
y^{obs}		observational variable for GDP growth
h^{obs}		observational variable for hours worked
i^{obs}		investment observation
w^{obs}		observational variable for wages
c^{obs}		observational variable for consumption
$p^{i,obs}$		observational variable for relative price of investment
π^{obs}		inflation observation
R^{obs}		observational variable for risk free interest rate

Table 6: Exogenous variables

Shock	Description
η^{ϵ^k}	productivity shock for capital
η^{ϵ^o}	productivity shock for capitak
η^{ζ^o}	exogenous temporary oil cost shock
$\eta^{\zeta^{o,im}}$	exogenous temporary oil import shock
$\eta^{\zeta^{o,ex}}$	exogenous temporary oil export shock
η^{τ^o}	exogenous temporary oil tax shock
η^{γ}	survival rate of entrepreneurs
η^{σ}	unanticipated risk
η^{ξ^1}	news to risk 1 periods ahead
η^{ξ^2}	news to risk 2 periods ahead
η^{ξ^3}	news to risk 3 periods ahead
η^{ξ^4}	news to risk 4 periods ahead

Table 6 – Continued

Shock	Description
η^{ξ^5}	news to risk 5 periods ahead
η^{ξ^6}	news to risk 6 periods ahead
η^{ξ^7}	news to risk 7 periods ahead
η^{ξ^8}	news to risk 8 periods ahead
η^{term}	term structure shock
η^{n}	measurement error net worth
η^{gamma}	survival rate of entrepreneurs
η^{x^p}	exogenous monetary policy shock
η^{ϵ^w}	exogenous temporary shock wage mark-up
η^{ϵ^p}	exogenous temporary shock price mark-up
η^{μ^Υ}	exogenous long-run investment shock
η^{μ^z}	exogenous long-run TFP shock
η^{ϵ}	exogenous temporary TFP shock
η^{ϵ^h}	exogenous temporary productivity shock hours
η^{ζ^h}	labour supply preference shock
η^{ζ^c}	consumption preference shock
η^{ζ^i}	marginal efficiency of investment shock
η^{g}	exogenous shock to government expenditure

Table 7: Parameters

Parameter	Description
$\tilde{a}_{\Delta p^o}$	weight on oil inflation in Taylor rule
$\bar{\epsilon^k}$	steady-state capital technology shock
σ^{ϵ^k}	standard deviation capital technology shock
ρ^{ϵ^k}	AR(1) coefficient for capital technology shock
$\bar{\epsilon^o}$	steady-state oil productivity
σ^{ϵ^o}	standard deviation oil productivity
ρ^{ϵ^o}	AR(1) coefficient for oil productivity
α^{O}	distribution parameter for oill
α^{M}	distribution parameter for composite good
ρ^{μ^o}	AR(1) coefficient for oil productivity shocks
ρ^{ζ^o}	AR(1) coefficient for oil cost shocks
$\rho^{\zeta^{o,im}}$	AR(1) coefficient for oil imports shocks
$\rho^{\zeta^{o,ex}}$	AR(1) coefficient for oil exports shocks

Table 7 – Continued

Parameter	Description
ρ^{τ^o}	AR(1) coefficient for oil tax shocks
γ^o	oil extraction cost parameter
$\gamma^{o^{ex}}$	oil exports extraction cost parameter
$\gamma^{o^{im}}$	oil imports extraction cost parameter
η^O	inverse demand price elasticity for oil consumption
σ^O	inverse supply price elasticity for oil production
τ^o	tax on oil production
ζ^o	long-run value of cost push shock
$\zeta^{o,im}$	long-run value of oil imports
$\zeta^{o,ex}$	long-run value of oil exports
ϵ^o	long-run value of oil productivity shock
σ^{μ^o}	standard deviation productivity of oil
σ^{p^o}	standard deviation measurement error refinery acquisition price
σ^{ζ^o}	standard deviation oil supply shock
$\sigma^{\zeta^{o,im}}$	standard deviation oil imports shock
$\sigma^{\zeta^{o,ex}}$	standard deviation oil exports shock
σ^{τ^o}	standard deviation oil tax shock
$\frac{o}{y}$	long-run oil output ratio
$\frac{o^d}{o}$	long-run oil domestic output to oil consumption ratio
$\frac{o^{im}}{o}$	long-run oil imports to oil consumption ratio
$\frac{o^{ex}}{o^d}$	long-run oil exports to oil domestic ratio
$o^{ex,trend,obs}$	trend in net oil exports observation
$o^{im,trend,obs}$	trend in oil imports observation
$o^{d,trend,obs}$	trend in oil domestic production observation
$o^{trend,obs}$	trend in oil consumption observation
$p^{o,trend,obs}$	trend in oil price observation
Θ	share of consumed remaining assets of leaving entrepreneurs
$F(\bar{\omega})$	steady-state bankruptcy rate
$\bar{\gamma}$	steady-state survival rate of entrepreneurs
$\frac{\bar{n}}{\bar{k}}$	steady-state equity to asset ratio
ρ^γ	AR(1) coefficient for survival rate of entrepreneurs
μ	monitoring cost
ρ^σ	AR(1) coefficient for σ
ρ^{term}	AR(1) coefficient for term structure
$\overline{R^L - R}$	steady-state term structure
$\bar{\sigma}$	steady-state risk level

Table 7 – Continued

Parameter	Description
ω^e	transfers to entrepreneurs from households
σ^σ	standard deviation unanticipated risk shock
σ^ξ	standard deviation anticipated shock
$\sigma(\xi_t, \xi_{t-1})$	signal correlation
σ^{term}	standard deviation term structure shock
σ^γ	standard deviation survival rate entrepreneurs
σ^n	standard deviation measurement error net worth
$credit^{trend,obs}$	trend in consumption observation
$n^{trend,obs}$	trend in net worth observation
$premium^{trend,obs}$	trend in premium observation
$Spread1^{trend,1,obs}$	trend in spread 1 observation
α^K	distribution parameter capital
α^N	distribution parameter labour
ϕ^G	steady-state share of government expenditure on output
ϕ^O	steady-state share of oil on output
ϕ^K	steady-state share of capital on output
λ^f	elasticity of substitution for intermediate products
λ^w	elasticity of substitution for different labour types
η^M	elasticity of substitution between energy-capital composite good and labour
β	weight on risk in Taylor rule
δ	depreciation rate of capital
$\bar{\epsilon}$	steady-state technology shock
$\bar{\epsilon^h}$	steady-state labour productivity shock
$\bar{\epsilon^w}$	steady-state wage mark-up shock
$\bar{\mu^z}$	steady-state growth rate
$\bar{\mu^\Upsilon}$	steady-state investment growth rate
ψ^L	weight on disutlity on labour
$\bar{r^k}$	steady-state rental rate on capital services
$\sigma^{a(u)}$	curvature of utilization cost
ξ^p	Calvo parameter prices
ξ^w	Calvo parameter wages
$\tilde{\rho}$	AR(1) coefficient for risk fre interest rate
$\tilde{a}_\pi$	weight on inflation in Taylor rule
$\tilde{a}_{\Delta y}$	weight on output growth in Taylor rule
$\bar{\pi}$	steady-state inflation
ι	price indexing weight of inflation target

Table 7 – Continued

Parameter	Description
ι^{μ^z}	wage indexing weight on persistent technology growth
ι^w	wage indexing weight on inflation target
$\bar{R}$	steady-state interest rate
ρ^{ϵ}	AR(1) coefficient for tfp shocks
ρ^{ϵ^h}	AR(1) coefficient for hours shocks
ρ^{ϵ^p}	AR(1) coefficient for price mark-up shock
ρ^{ϵ^w}	AR(1) coefficient for wage mark-up shock
ρ^{μ^z}	AR(1) coefficient for μ^z
ρ^{μ^Υ}	AR(1) coefficient for μ^Υ
ρ^{ζ^c}	AR(1) coefficient for ζ^c
ρ^{ζ^i}	AR(1) coefficient for ζ^i
ρ^{ζ^h}	AR(1) coefficient for ζ^h
ρ^g	AR(1) coefficient for government expenditure
ρ^s	AR(1) coefficient for marginal cost
b	habit formation parameter
τ^c	consumption tax rate
τ^k	capital income tax rate
τ^l	labour income tax rate
$\mathcal{S}''$	curvature of investment adjustment cost
σ^L	curvature for the disutility to labor
υ	mean growth rate for capital
υ^o	mean growth rate for oil consumption
$\bar{\zeta}^c$	steady-state consumption preference
$\bar{\zeta}^i$	steady-state marginal efficiency of investment
$\bar{\zeta}^h$	steady-state marginal efficiency of labour
$\bar{g}$	steady-state government expenditure
$\bar{y}$	steady-state output
σ^{ϵ}	standard deviation technology
σ^{ϵ^h}	standard deviation technology hours worked
σ^{μ^z}	standard deviation growth rate shock
σ^{μ^Υ}	standard deviation investment specific growth rate
σ^{ζ^c}	standard deviation consumption preference shock
σ^{ζ^i}	standard deviation investment specific preference shock
σ^{ζ^h}	standard deviation labour preference shock
σ^g	standard deviation government expenditure shock
σ^{ϵ^p}	standard deviation price mark-up

Table 8: Tests for stationary observable variables

	Augmented Dickey-Fuller	Phillips-Perron
π^{obs}	0.22	0.01
y^{obs}	0.01	0.01
c^{obs}	0.08	0.01
i^{obs}	0.02	0.01
h^{obs}	0.21	0.73
b^{obs}	0.34	0.01
n^{obs}	0.01	0.01
$p^{i,obs}$	0.01	0.01
$premium^{obs}$	0.06	0.03
R^{obs}	0.01	0.21
$S^{1,obs}$	0.01	0.01
w^{obs}	0.01	0.01
$o^{im,obs}$	0.01	0.01
$o^{ex,obs}$	0.01	0.01
$o^{d,obs}$	0.01	0.01
$p^{o,obs}$	0.01	0.01

Note: p-values for the tests are reported.

Table 7 – Continued

Parameter	**Description**
σ^{ϵ^w}	standard deviation wage mark-up
σ^{x^p}	standard deviation monetary policy shock
$c^{trend,obs}$	trend in consumption observation
$gdp^{trend,obs}$	trend in GDP observation
$h^{trend,obs}$	trend in hours observation
$i^{trend,obs}$	trend in investment observation
$w^{trend,obs}$	trend in wage observation
$p^{i,trend,obs}$	trend in relative price of investment observation
$R^{trend,obs}$	trend in interest rate observation
$\pi^{trend,obs}$	trend in inflation observation

Table 9: Parameter Values

Parameter	**Value**	**Description**
ζ^o	1.000	long-run value of cost push shock

Table 9 – Continued

Parameter	Value	Description
$\zeta^{o,f}$	1.000	long-run value of oil imports
ϵ^{o}	1.000	long-run value of oil productivity shock
$\frac{o}{y}$	0.002	long-run oil output ratio
$\frac{o^d}{o}$	0.474	long-run oil domestic output to oil consumption ratio
$\frac{o^f}{o}$	0.512	long-run oil imports to oil consumption ratio
$\frac{o^{ex}}{o^d}$	0.013	long-run oil exports to oil domestic ratio
Θ	0.005	share of consumed remaining assets of leaving entrepreneurs
$F(\bar{\omega})$	0.006	steady state bankruptcy rate
$\bar{\gamma}$	0.985	steady state survival rate of entrepreneurs
ω^{e}	0.005	transfers to entrpreneurs from households
ϕ^{G}	0.190	steady state share of government expenditure on output
ϕ^{O}	0.017	steady state share of oil on output
ϕ^{K}	0.400	steady state share of capital on output
λ^{f}	1.200	elasticity of subsitition for intermediate products
λ^{w}	1.050	elasticity of subsitition for different labour types
β	0.999	weight on risk in Taylor rule
δ	0.025	depreciation rate of capital
$\bar{\epsilon}$	0.516	steady state technology shock
$\bar{\epsilon^{h}}$	1.000	steady state labour productivity shock
$\bar{\epsilon^{w}}$	1.000	steady state wage mark-up shock
$\bar{\epsilon^{w}}$	1.000	steady state wage mark-up shock
$\bar{\mu^{z}}$	1.004	steady state growth rate
$\bar{\mu^{\Upsilon}}$	1.000	steady state investment growth rate
$\bar{r^{k}}$	0.052	steady state rental rate on capital services
$\tilde{a}_{\pi}$	0.500	weight on inflation in Taylor rule
$\bar{\pi}$	1.006	steady state inflation
ι	0.000	price indexing weight of inflation target
$\iota^{\mu^{z}}$	0.000	wage indexing weight on persisitent technology growth
ι^{w}	0.000	wage indexing weight on inflation target
$\bar{R}$	0.011	steady state interest rate
b	0.000	habit formation parameter
τ^{c}	0.047	consumption tax rate
τ^{k}	0.320	capital income tax rate
τ^{l}	0.241	labour income tax rate
σ^{L}	1.000	curvature for the disutility to labor

Table 9 – Continued

Parameter	**Value**	**Description**
υ	1.004	mean growth rate for capital
$\bar{\zeta}^c$	1.000	steady state consumption preference
$\bar{\zeta}^i$	1.000	steady state marginal efficiency of investment
$\bar{\zeta}^h$	1.000	steady state marginal efficiency of labour
$\bar{g}$	0.188	steady state government expenditure
$\bar{y}$	1.000	steady state output

Table 10: Prior information (parameters)

Parameter	Distribution	Mean	Std.dev.
	baseline parameters		
$\mathcal{S}''$	Gaussian	4.9844	1.7662
$\sigma^{a(u)}$	Gaussian	1.0499	0.0965
$\tilde{a}_{\Delta y}$	Gaussian	0.3422	0.0461
ξ^w	Beta	0.2989	0.0342
ξ^p	Beta	0.4634	0.0364
$\tilde{\rho}$	Beta	0.7795	0.0196
σ^{ϵ}	Inv. Gamma	0.0091	0.0006
σ^{μ^z}	Inv. Gamma	0.011	0.0008
σ^{μ^Υ}	Inv. Gamma	0.0075	0.0005
σ^{ζ^i}	Inv. Gamma	0.035	0.0057
σ^{ζ^c}	Inv. Gamma	0.0161	0.0013
σ^{g}	Inv. Gamma	0.0203	0.0013
σ^{x^p}	Inv. Gamma	0.0089	0.0006
σ^{ϵ^p}	Inv. Gamma	0.0112	0.0008
ρ^{ϵ}	Beta	0.9033	0.0163
ρ^{μ^z}	Beta	0.0897	0.0444
ρ^{μ^Υ}	Beta	0.4813	0.1491
ρ^{ζ^i}	Beta	0.6529	0.0601
ρ^{ζ^c}	Beta	0.9711	0.0078

(Continued on next page)

Table 10: (continued)

Parameter	Distribution	Mean	Std.dev.
ρ^{g}	Beta	0.9276	0.0141
$\rho^{\epsilon^{p}}$	Beta	0.8562	0.0348
	oil market		
η^{M}	Gamma	1	0.2000
η^{O}	Gamma	0.1000	0.0500
σ^{O}	Gamma	10.0000	2.0000
$\sigma^{\zeta^{o}}$	Inv. Gamma	0.1000	2.0000
$\sigma^{\zeta^{o,im}}$	Inv. Gamma	0.1000	2.0000
$\sigma^{\zeta^{o,ex}}$	Inv. Gamma	0.1000	2.0000
$\rho^{\zeta^{o}}$	Beta	0.5000	0.2000
$\rho^{\zeta^{o,ex}}$	Beta	0.5000	0.2000
$\rho^{\zeta^{o,im}}$	Beta	0.5000	0.2000
$\rho^{\epsilon^{o}}$	Beta	0.5000	0.2000
	financial accelerator		
σ^{γ}	Inv. Gamma	0.1000	2.0000
σ^{ξ}	Inv. Gamma	0.1000	2.0000
σ^{σ}	Inv. Gamma	0.1000	2.0000
σ^{term}	Inv. Gamma	0.1000	2.0000
σ^{n}	Inv. Gamma	0.1000	2.0000
$\sigma(\xi_t, \xi_{t-1})$	0.1000	2.0000	
ρ^{γ}	Beta	0.5000	0.2000
ρ^{σ}	Beta	0.5000	0.2000
ρ^{term}	Beta	0.5000	0.2000

Table 11: Estimation results for rigidity parameters

Model	CEE–Oil model	CMR–Oil model
AR(1) coefficient for TFP shocks	0.92	0.91
ρ^{ϵ}	[0.90, 0.93]	[0.89, 0.93]
AR(1) coefficient for μ^z	0.05	0.04
ρ^{μ^z}	[0.02, 0.10]	[0.01, 0.09]
AR(1) coefficient for μ^{Υ}	0.46	0.45
$\rho^{\mu^{\Upsilon}}$	[0.28, 0.64]	[0.28, 0.63]
AR(1) coefficient for ζ^i	0.63	0.57
ρ^{ζ^i}	[0.56, 0.69]	[0.49, 0.65]
AR(1) coefficient for government expenditure	0.93	0.94
ρ^{g}	[0.92, 0.95]	[0.92, 0.95]
AR(1) coefficient for price mark-up shock	0.87	0.90
ρ^{ϵ^p}	[0.83, 0.90]	[0.86, 0.92]
AR(1) coefficient for survival rate of entrepreneurs	-	0.56
ρ^{γ}	[-]	[0.31, 0.72]
AR(1) coefficient for σ	-	0.92
ρ^{σ}	[-]	[0.88, 0.94]
AR(1) coefficient for term structure	-	0.21
ρ^{term}	[-]	[0.16, 0.26]
AR(1) coefficient for oil cost shocks	0.99	0.99
ρ^{ζ^o}	[0.98, 1.00]	[0.97, 1.00]
AR(1) coefficient for oil exports shocks	-	-
$\rho^{\zeta^{o,ex}}$	[-]	[-]
AR(1) coefficient for oil imports shocks	0.96	0.96
$\rho^{\zeta^{o,f}}$	[0.93, 0.99]	[0.94, 0.99]
AR(1) coefficient for oil productivity	0.86	0.87
ρ^{ϵ^m}	[0.79, 0.92]	[0.81, 0.93]

Note: The posterior mean and the 90% highest posterior density (HPD) interval for the respective parameters are reported in parentheses.

Table 12: Estimation results for standard deviations

Model	CEE–Oil model	CMR–Oil model
standard deviation technology	0.01	0.01
σ^{ϵ}	[0.01, 0.01]	[0.01, 0.01]
standard deviation growth rate shock	0.01	0.01
σ^{μ^z}	[0.01, 0.01]	[0.01, 0.01]
standard deviation investment specific growth rate	0.01	0.01
σ^{μ^Υ}	[0.01, 0.01]	[0.01, 0.01]
standard deviation investment specific preference shock	0.03	0.03
σ^{ζ^i}	[0.03, 0.04]	[0.02, 0.03]
standard deviation consumption preference shock	0.01	0.02
σ^{ζ^c}	[0.01, 0.02]	[0.01, 0.02]
standard deviation labour preference shock	-	-
σ^{ζ^h}	[-]	[-]
standard deviation government expenditure shock	0.02	0.02
σ^{g}	[0.02, 0.02]	[0.02, 0.02]
standard deviation monetary policy shock	0.01	0.01
σ^{x^p}	[0.01, 0.01]	[0.01, 0.01]
standard deviation price mark-up	0.01	0.01
σ^{ϵ^p}	[0.01, 0.01]	[0.01, 0.01]
standard deviation oil productivity	0.03	0.03
σ^{ϵ^m}	[0.03, 0.04]	[0.03, 0.04]
standard deviation oil supply shock	0.03	0.03
σ^{ζ^o}	[0.03, 0.03]	[0.03, 0.03]
standard deviation oil imports shock	0.05	0.05
$\sigma^{\zeta^{o,f}}$	[0.04, 0.05]	[0.04, 0.05]
standard deviation oil exports shock	3.38	3.38
$\sigma^{\zeta^{o,ex}}$	[3.07, 3.74]	[3.07, 3.75]
standard deviation survival rate enetrepreneurs	-	0.01
σ^{γ}	[-]	[0.01, 0.01]
standard deviation anticipated shock	-	0.02
σ^{ξ}	[-]	[0.02, 0.02]
standard deviation unanticipated risk shock	-	0.04
σ^{σ}	[-]	[0.04, 0.05]
standard deviation term structure shock	-	0.02
σ^{term}	[-]	[0.01, 0.02]
standard deviation measurement error net worth	-	0.06
σ^{n}	[-]	[0.06, 0.07]
signal correlation	-	0.54
$\sigma(\xi_t, \xi_{t-1})$	[-]	[0.43, 0.67]

Note: The posterior mean and the 90% highest posterior density (HPD) interval for the respective parameters are reported in parentheses.

Table 13: Classification of shock groups

Group	Shocks
anticipated risk	η^{ξ^i} for $i \in \{1, \ldots, 8\}$
unanticipated risk	η^{σ}
risk	anticipated and unanticipated risk
financial	η^{γ}, η^{term}
investment	η^{ζ^i}, $\eta^{\mu^{\Upsilon}}$
monetary policy (M.P.)	η^{x^p}
fiscal policy	η^{g}
policy	fiscal policy and monetary policy
markup	η^{ϵ^p}
demand	η^{ζ^c}
domestic oil supply	$\eta^{\zeta^{o^d}}, \eta^{\zeta^{o^{ex}}}$
oil demand	η^{ϵ^o}
foreign oil supply	$\eta^{\zeta^{o^{im}}}$
oil supply	domestic and foreign oil supply
oil	oil supply and oil demand

Table 14: Parameter values for CMR replication

Description	Symbol	Value
Structural parameters		
share of consumed remaining assets of leaving entrepreneurs	Θ	0.005
steady state bankruptcy rate	$F(\bar{\omega})$	0.0056
steady state survival rate of entrepreneurs	$\bar{\gamma}$	0.985
monitoring cost	μ	0.3074
curvature of utilization cost	$\sigma^{a(u)}$	2.5356
Calvo parameter prices	ξ^p	0.7412
Calvo parameter wages	ξ^w	0.8128
AR(1) coefficient for risk free interest rate	$\tilde{\rho}$	0.8503
weight on inflation in Taylor rule	$\tilde{a}_\pi$	2.3965
weight on output growth in Taylor rule	$\tilde{a}_{\Delta y}$	0.3649
price indexing weight of inflation target	ι	0.8974
wage indexing weight on persisitent technology growth	ι^{μ^z}	0.9366
wage indexing weight on inflation target	ι^w	0.4891
habit formation parameter	b	0.7358
curvature of investment adjustment cost	S	10.78
Persistence parameters		
AR(1) coefficient for TFP shocks	ρ^ϵ	0.8089
AR(1) coefficient for hours shocks	ρ^{ϵ^h}	0.5
AR(1) coefficient for price mark-up shock	ρ^{ϵ^p}	0.9109
AR(1) coefficient for wage mark-up shock	ρ^{ϵ^p}	0.5
AR(1) coefficient for μ^z	ρ^{μ^z}	0.1459
AR(1) coefficient for μ^Υ	ρ^{μ^Υ}	0.987
AR(1) coefficient for ζ^c	ρ^{ζ^c}	0.8968
AR(1) coefficient for ζ^i	ρ^{ζ^i}	0.9087
AR(1) coefficient for ζ^h	ρ^{ζ^h}	0.5
AR(1) coefficient for government expenditure	ρ^g	0.9427
AR(1) coefficient for marginal cost	ρ^s	0.5
AR(1) coefficient for σ	ρ^σ	0.9706
AR(1) coefficient for term strucut	ρ^{term}	0.9744
Standard deviations of shocks		
standard deviation unanticipated risk shock	σ^σ	0.07
standard deviation anticipated shock	σ^ξ	0.0283
signal correlation	$\sigma(\xi_t, \xi_{t-1})$	0.6757
standard deviation term structure shock	σ^{term}	0.0016
standard deviation survival rate enetrepreneurs	σ^γ	0.0081
standard deviation measurement error net worth	σ^n	0
standard deviation technology	σ^ϵ	0.0046
standard deviation technology hours worked	σ^{ϵ^h}	0
standard deviation growth rate shock	σ^{μ^z}	0.0071
standard deviation investment specific growth rate	σ^{μ^Υ}	0.004
standard deviation consumption preference shock	σ^{ζ^c}	0.0233
standard deviation investment specific preference shock	σ^{ζ^i}	0.055
standard deviation labour preference shock	σ^{ζ^h}	0
standard deviation government expenditure shock	σ^g	0.0228
standard deviation wage mark-up	σ^{ϵ^w}	0
standard deviation price mark-up	σ^{ϵ^p}	0.011
standard deviation monetary policy shock	σ^{x^p}	0.0049

Notes: The parameter values are from Christiano et al. (2014) to compute the variance decomposition at the posterior mode as reported in Table 4.

Table 15: Variance decomposition for financial market variables at the posterior distribution

Variable	risk	investment	demand	financial	M.P.	markup	technol.	oil
				credit growth				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	19.0	1.3	0.4	64.0	3.1	2.3	9.6	0.2
	[8.8, 28.6]	[0.8, 1.8]	[0.2, 0.5]	[56.9, 72.0]	[2.3, 3.9]	[1.5, 3.1]	[6.8, 12.6]	[0.1, 0.3]
				external finance premium				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	75.8	1.5	0.4	19.9	1.5	0.1	0.9	0.0
	[45.7, 105.3]	[0.9, 2.1]	[0.3, 0.5]	[9.4, 30.7]	[1.0, 1.9]	[0.0, 0.1]	[0.5, 1.2]	[0.0, 0.0]
				equity growth				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	15.7	1.6	0.1	11.6	4.3	0.2	0.4	0.1
	[7.7, 23.1]	[1.1, 2.2]	[0.0, 0.1]	[6.2, 16.8]	[3.2, 5.2]	[0.2, 0.3]	[0.3, 0.6]	[0.0, 0.1]
				term spread				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	10.6	10.9	2.4	27.6	25.5	5.7	16.8	0.5
	[4.8, 16.5]	[8.1, 13.9]	[1.6, 3.0]	[20.7, 35.0]	[22.1, 29.3]	[3.6, 7.5]	[12.3, 21.0]	[0.2, 0.7]

Note: Contribution of each shock group in percent to the total theoretical variance of the respective variable is reported. Values in parentheses represent 90% HPD interval of the model parameters. The shock groups are reported in Table 13.

Table 16: Variance decomposition for oil market variables at the posterior distribution

Variable	risk, financial and inv.	policy, demand, markup	technol.	domestic oil supply	foreign oil supply	domestic oil demand
			domestic oil supply growth			
CEE–Oil model	0.1	1.3	2.4	66.3	15.0	14.9
	[0.1, 0.1]	[0.9, 1.7]	[1.4, 3.5]	[51.9, 80.4]	[8.8, 20.3]	[11.7, 18.4]
CMR–Oil model	0.2	1.2	2.5	67.5	13.9	14.6
	[0.1, 0.3]	[0.8, 1.7]	[1.5, 3.6]	[52.4, 82.2]	[8.4, 19.8]	[11.2, 18.2]
			domestic oil supply			
CEE–Oil model	0.2	0.5	1.1	89.6	6.7	1.9
	[0.0, 0.4]	[0.1, 1.0]	[0.2, 2.0]	[80.7, 98.9]	[0.7, 12.5]	[0.4, 3.3]
CMR–Oil model	1.4	0.8	1.3	85.5	8.4	2.6
	[0.2, 2.9]	[0.2, 1.4]	[0.3, 2.3]	[73.9, 97.6]	[1.1, 15.4]	[0.7, 4.6]
			foreign oil supply growth			
CEE–Oil model	0.1	0.8	1.4	11.2	77.4	9.1
	[0.0, 0.1]	[0.4, 1.1]	[0.9, 2.0]	[6.3, 16.5]	[69.2, 84.0]	[6.4, 12.3]
CMR–Oil model	0.1	0.8	1.5	10.4	78.5	8.8
	[0.0, 0.2]	[0.4, 1.1]	[0.9, 2.0]	[5.3, 15.4]	[70.9, 85.6]	[6.0, 11.8]
			foreign oil supply			
CEE–Oil model	0.3	0.7	1.5	11.1	83.8	2.6
	[0.1, 0.5]	[0.2, 1.3]	[0.3, 2.5]	[1.9, 21.4]	[71.7, 96.3]	[0.5, 4.2]
CMR–Oil model	1.5	0.8	1.3	8.1	85.5	2.7
	[0.1, 2.9]	[0.2, 1.5]	[0.3, 2.4]	[1.2, 15.6]	[75.3, 97.0]	[0.5, 4.5]
			oil price growth			
CEE–Oil model	0.2	2.5	4.7	34.9	28.7	29.0
	[0.1, 0.2]	[1.8, 3.1]	[3.4, 5.9]	[27.8, 42.0]	[23.3, 34.9]	[22.1, 35.8]
CMR–Oil model	0.4	2.6	4.1	34.6	28.0	30.4
	[0.2, 0.6]	[1.9, 3.2]	[3.1, 5.4]	[27.3, 41.4]	[22.7, 33.5]	[23.0, 37.6]
			oil price			
CEE–Oil model	1.2	3.1	6.2	42.7	35.7	11.1
	[0.5, 1.9]	[1.3, 5.0]	[3.0, 9.5]	[20.7, 62.4]	[17.3, 53.4]	[4.2, 17.5]
CMR–Oil model	6.4	3.6	6.0	34.4	37.0	12.6
	[1.2, 11.8]	[1.6, 5.7]	[3.1, 9.0]	[15.8, 53.2]	[18.1, 54.1]	[4.3, 19.9]

Note: Contribution of each shock group in % to the total theoretical variance of the respective variable is reported. Values in parentheses represent 90% HPD interval of the model parameters. The shock groups are reported in Table 13.

C Model equations

C.1 CEE model equations

The CEE model consists of equations (26) to (49), which describe the behaviour of endogenous variables.Here the stationary version of the model is reported. The derivation of all model equations is provided in the Online Appendix. Shocks are described by (50) to (58).

C.1.1 Households

This block contains model equations describing the behaviour of representative households in the model.

Households face investment adjustment costs. These investment adjustment costs reduce the effectiveness of investments into the raw capital stock. Investment adjustment costs depend on the curvature parameter $\mathcal{S}''$, marginal efficiency of investment adjustment shocks ζ_t^i, the change in investment $\frac{i_t}{i_{t-1}}$, the growth rate of technological change μ_t^z and investment specific trend Υ.

$$\mathcal{S}\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}}\right)=\left(exp\left(\sqrt{\frac{\mathcal{S}''}{2}}\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}}-\Upsilon\,\bar{\mu^z}\right)\right)\right.$$
$$\left.+exp\left(-\sqrt{\frac{\mathcal{S}''}{2}}\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}}-\Upsilon\,\bar{\mu^z}\right)\right)-2\right). \tag{26}$$

Raw capital evolves according to a standard law of motion. Each period a constant fraction δ of the old capital stock depreciates. Investments into the capital stock are necessary to maintain and extend the raw capital stock.

$$\bar{k}_t=\frac{(1-\delta)}{{\mu^z}_t\,\Upsilon}\,\bar{k}_{t-1}+\left(1-S\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}}\right)\right)\,i_t. \tag{27}$$

From the intertemporal optimization problem of the household the first order condition with respect to consumption is the marginal utility of consumption. The marginal utility of consumption depends on preference shocks ζ_t^c, the discount factor β, habit formation b, the tax rate on consumption and the growth rate of technological change μ_t^z.

$$\lambda^z{}_t\,\left(1+\tau^c\right)=\frac{{\mu^z}_t\,{\zeta^c}_t}{{\mu^z}_t\,c_t-b\,c_{t-1}}-\frac{\beta\,b\,{\zeta^c}_{t+1}}{c_{t+1}\,{\mu^z}_{t+1}-c_t\,b}. \tag{28}$$

Investments into the capital stock by households is a trade-off between foregone consumption today for future income. The first term in (29) represents foregone consumption today by increasing investment today. The second and third term represents the increase in potential consumption tomorrow by an increase in the capital stock.

$$0 = \frac{(-{\lambda^z}_t)}{{\mu^\Upsilon}_t} + {\lambda^z}_t\, q_t \left(1 - S\left(\frac{{\mu^z}_t\, \Upsilon\, {\zeta^i}_t\, i_t}{i_{t-1}}\right) - \frac{\partial S\left(\frac{{\mu^z}_t\, \Upsilon\, {\zeta^i}_t\, i_t}{i_{t-1}}\right)}{\partial \frac{i_t}{\Upsilon}}\right) \tag{29}$$
$$+ \frac{\beta\, {\lambda^z}_{t+1}}{\Upsilon\, {\mu^z}_{t+1}}\, q_{t+1}\, \frac{\partial S\left(\frac{{\mu^z}_{t+1}\, \Upsilon\, {\zeta^i}_{t+1}\, i_{t+1}}{i_t}\right)}{\partial \frac{i_t}{\Upsilon}} \left(\frac{\Upsilon\, {\mu^z}_{t+1}\, {\zeta^i}_{t+1}\, i_{t+1}}{i_t}\right)^2 .$$

Households provide capital services $k_t^s = u_t\, \bar{k}_{t-1}$ for a rental rate r_t^k. Utilization of raw capital u_t is associated with costs $a(u_t)$. The optimal utilization rate equates marginal costs and benefits.

$$r_t^k = \bar{r}^k\, exp\left(\sigma^{a(u)}\, (u_t - 1)\right). \tag{30}$$

In the CEE model raw capital is a control variable of households. The benefit of having one more unit of raw capital in the next period is additional discounted marginal consumption using revenues from renting capital services. This benefit equals the cost of foregone consumption today.

$$0 = \beta\, \frac{\lambda_{t+1}^z}{\mu_{t+1}^z\, \pi_{t+1}}\, r_{t+1}^k\, u_{t+1}\, (1 - \tau^k) - q_t \lambda_t^z + (1 - \delta)\beta\, q_{t+1}\, \lambda_{t+1}^z. \tag{31}$$

In addition to raw capital, households can also access short-term bonds b_t. Those bonds are purchased such that forgone consumption today λ_t^z equals potential additional consumption tomorrow. It is the Euler equation for bonds and is an implicit arbitrage condition between raw capital and bonds.

$$0 = (1 + R_t)\, \frac{\beta\, {\lambda^z}_{t+1}}{\pi_{t+1}\, {\mu^z}_{t+1}} - {\lambda^z}_t. \tag{32}$$

C.1.2 Production

The standard NK-DSGE model introduces a two layer production process of final goods. In the first stage the two primary production factors homogenous labour $l_t = h_t\, (w_t^*)^{\frac{\lambda^w}{\lambda^w - 1}}$ and capital services $\frac{u_t\, \bar{k}_{t-1}}{{\mu^z}_t\, \Upsilon}$ are used to produce intermediate goods. Homogenous labour depends on the wage dispersion index and total hours worked. Wage rigidity leads to a mismatch between the marginal product of the specific type of labour and its price. This mismatch determines the total level of homogenous labour supplied by labour contractors. Intermediate goods are transformed into a final good. The effectiveness of the transformation depends on the price dispersion index p_t^*. Therefore total final output y_t is given by

$$y_t = {p^*}_t^{\frac{\lambda^f}{\lambda^f - 1}} \, \epsilon_t \, \left(\frac{u_t \, \bar{k}_{t-1}}{{\mu^z}_t \, \Upsilon} \right)^{\alpha^K} \, \left({\epsilon^h}_t \, h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w - 1}} \right)^{\alpha^N} - \phi_t. \tag{33}$$

It is standard to include fixed costs to ensure that the no entry condition is fulfilled in steady-state. Fixed costs ϕ are set to ensure zero profits in steady-state. Further, I model fixed costs proportional to the previous year total final output.

$$\phi_t = \frac{1 - \frac{1}{\lambda^f}}{\frac{1}{\lambda^f}} \, y_{t-4}. \tag{34}$$

Intermediate goods producing firms demand capital services such that the associated relative marginal costs $\frac{{r^k}_t}{s_t}$ are equal to its marginal product.

$$\frac{{r^k}_t}{s_t} = \alpha^K \, \left(\frac{\phi_t + y_t \, p_t^{*\frac{\lambda^f}{1-\lambda^f}}}{\frac{u_t \, \bar{k}_{t-1}}{{\mu^z}_t \, \Upsilon}} \right). \tag{35}$$

Firms producing intermediate goods demand hours worked such that the marginal product of an additional unit of homogenous labour equals its marginal cost.

$$\frac{w_t}{s_t} = \alpha^N \, \left(\frac{\phi_t + y_t \, p_t^{*\frac{\lambda^f}{1-\lambda^f}}}{h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w - 1}}} \right). \tag{36}$$

C.1.3 Price setting

Intermediate goods producing firms minimize costs associated with their primary production factors. However, they also maximize expected discounted profits. The expected discounted profits of intermediate goods producing firms depend on the optimal price $\tilde{p}_t$ they set today. Firms not able to reset their price use an indexation rule. The indexation rule is a weighted average between previous inflation π_{t-1} and the inflation target $\bar{\pi}$. The weight on past inflation is ι.

$$\tilde{\pi}_t = {\pi_{t-1}}^{1-\iota} \, \bar{\pi}^{\iota}. \tag{37}$$

The share of intermediate goods-producing firms $1 - \xi^p$ able to reset their price choose all the same price. The optimal price is given by $\tilde{p}_t = \frac{K_t^p}{F_t^p}$. Here we introduce two auxiliary variables to express infinite sums recursively. The denominator

$$F^p_t = y_t \, {\lambda^z}_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda^f}} \; \beta \, \xi^p \, F^p_{t+1}. \tag{38}$$

The numerator of the optimal price is the infinite sum of discounted expected marginal costs. Further, the shock ϵ^p_t are temporary deviations to the relationship between the optimal price and marginal costs.

$$K^p_t = s_t \, y_t \, {\lambda^z}_t \, \lambda^f \, {\epsilon^p}_t + \beta \, \xi^p \, \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda^f}{1-\lambda^f}} \; K^p_{t+1}. \tag{39}$$

A relationship between numerator K^p_t and denominator F^p_t can be derived from the price index.

$$K^p_t = F^p_t \left(\frac{1 - \xi^p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda^f}}}{1 - \xi^p} \right)^{1-\lambda^f} . \tag{40}$$

Price dispersion is a consequence of the random price setting mechanism. The price dispersion index depends on the optimal price and previous price dispersion.

$$ {p^*}_t = \left((1 - \xi^p) \left(\frac{K^p_t}{F^p_t} \right)^{\frac{\lambda^f}{1-\lambda^f}} + \xi^p \left(\frac{\tilde{\pi}_t}{\pi_t} \, p^*_{t-1} \right)^{\frac{\lambda^f}{1-\lambda^f}} \right)^{\frac{1-\lambda^f}{\lambda^f}} . \tag{41}$$

C.1.4 Wage setting

Households provide different labour types $h_{j_h,j_l,t}$. Unions represent these labour types. Unions negotiate wages for each type of labour. Labour contractors use the different types of labour to provide homogenous labour l_t.

Unions can only renegotiate wages each period with a probability of $1 - \xi^w$ and otherwise reset wages according to an wage inflation indexation rule $\tilde{\pi^w}_t$. This rule depends on previous price inflation, the inflation target, the long-run growth rate of technological change and the contemporaneous growth rate of technological change.

$$\tilde{\pi^w}_t = {\pi_{t-1}}^{1-\iota^w} \, \bar{\pi}^{\iota^w} \, \bar{\mu^z}^{1-\iota^{\mu^z}} \, {\mu^z_t}^{\iota^{\mu^z}}. \tag{42}$$

Nominal wages are $W_t = z_t \, P_t \, w_t$ a product of real wages, technological change and the current price level. Wage inflation π^w is a product of current price inflation and the growth rate of technological change.

$$\pi_t^w = \mu_t^z \, \pi_t. \tag{43}$$

The wage dispersion index like the price dispersion index depends on the previous level of wage dispersion and the current optimal wage set by negotiating unions. It measures the inefficiency in the labour market caused by rigid wage setting.

$$w_t^* = \left((1-\xi^w) \left(\frac{1-\xi^w \left(\frac{\frac{\tilde{\pi^w}_t}{\pi^w_t} \, w_{t-1}}{w_t} \right)^{\frac{1}{1-\lambda^w}}}{1-\xi^w} \right)^{\lambda^w} + \xi^w \left(\frac{\frac{\tilde{\pi^w}_t}{\pi^w_t} \, w_{t-1}}{w_t} \, {w^*}_{t-1} \right)^{\frac{\lambda^w}{1-\lambda^w}} \right)^{\frac{\frac{1}{\lambda^w}}{1-\lambda^w}}. \tag{44}$$

Unions set wages $\tilde{w}_t = \frac{K_t^w \, \psi^L}{F_t^w \, w_t}$ to maximize expected discounted wage bills reduced by the implied disutility of households supplying labour. The denominator F_t^w is an auxiliary variable introduced to express an infinite sum.

$$F^w{}_t = \frac{h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}} \, \lambda^z{}_t \left(1-\tau^l\right)}{\lambda^w \, \epsilon^w{}_t} + \beta \, \xi^w \left(\frac{\tilde{\pi^w}_{t+1}}{\pi^w{}_{t+1}} \right)^{\frac{1}{1-\lambda^w}} \left(\frac{w_t}{w_{t+1}} \right)^{\frac{\lambda^w}{1-\lambda^w}} F^w{}_{t+1}. \tag{45}$$

The numerator K_t^w is like F_t^w also an auxiliary variable to express an infinite sum. This infinite sum captures the expected disutility of households to work for the optimal wage.

$$K_t^w = \left(h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}} \right)^{1+\sigma^L} + \beta \, \xi^w \left(\frac{w_t \, \frac{\tilde{\pi^w}_{t+1}}{\pi^w{}_{t+1}}}{w_{t+1}} \right)^{\frac{\lambda^w \left(1+\sigma^L\right)}{1-\lambda^w}} K_{t+1}^w. \tag{46}$$

It is possible to derive a relationship between the numerator and denominator for optimal wages using the wage index.

$$K_t^w = \frac{F^w{}_t \, w_t \left(\frac{1-\xi^w \left(\frac{\tilde{\pi^w}_t}{\pi^w{}_t} \right)^{\frac{1}{1-\lambda^w}}}{1-\xi^w} \right)^{1-\lambda^w \left(1+\sigma^L\right)}}{\zeta^h{}_t \, \psi^L}. \tag{47}$$

C.1.5 Monetary policy and resource constraint

A main objective of NK-DSGE models is the analysis of monetary policy. To model monetary policy the Taylor rule is included. The Taylor rule postulates that the risk-free interest rate is a function of deviations in previous inflation from its target value and deviations in GDP growth from it s potential. The parameters $\tilde{a}_\pi$ and $\tilde{a}_{\Delta y}$ govern the response of the monetary policy authority to the respective deviations. Further, monetary policy considers previous risk free interest rates and weights them with $\tilde{\rho}$. Potential discretionary deviations form the rule are captured by x_t^p measured in annualized terms.

$$\frac{1+R_t}{1+\bar{R}} = \left(\frac{1+R_{t-1}}{1+\bar{R}}\right)^{\tilde{\rho}} \left\{ \left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{1+\tilde{a}_\pi} \left(\frac{\mu_{t-1}^z}{\bar{\mu^z}} \frac{c_{t-1} + \frac{i_{t-1}}{\mu_{t-1}^\Upsilon} + g_{t-1}}{c_{t-2} + \frac{i_{t-2}}{\mu_{t-2}^\Upsilon} + g_{t-2}} \right)^{\tilde{a}_{\Delta y}} \right\}^{1-\tilde{\rho}} + \frac{\sigma^{x^p}}{4} x_t^p. \tag{48}$$

The resource constraint can be derived from the budget constraint of the household. Total output used in the economy is either used for investment, consumption, government expenditure or eaten up by capital utilization costs.

$$y_t = c_t + \frac{i_t}{\mu_t^\Upsilon} + g_t + \frac{\bar{k}_{t-1}}{{\mu^z}_t\,\Upsilon}\, a(u_t). \tag{49}$$

C.1.6 Shocks

Shocks in the CEE model are responsible for fluctuations of the endogenous variables around the balanced growth path. These variables do not depend on the development of endogenous variables.

The standard NK-DSGE model does not explicitly model the behaviour of fiscal policy. It is therefore assumed that government expenditure follows an autoregressive process of order one.

$$log\left(\frac{g_t}{\bar{g}}\right) = \rho^g\, log\left(\frac{g_{t-1}}{\bar{g}}\right) + \sigma^g\, \eta_t^g. \tag{50}$$

In order to capture potential fluctuations on the supply side total factor productivity shocks are introduced. These shocks capture fluctuations in the efficiency of combining primary production factors to intermediate and final goods. These shocks have no direct impact on the relative productivity of the production factors.

$$log\left(\frac{\epsilon_t}{\bar{\epsilon}}\right) = \rho^\epsilon\, log\left(\frac{\epsilon_{t-1}}{\bar{\epsilon}}\right) + \sigma^\epsilon\, {\eta^\epsilon}_t. \tag{51}$$

Labour productivity shocks only affect the productivity of labour and have direct implications for the relative productivity of both production factors.

$$log\left(\frac{\epsilon^h{}_t}{\bar{\epsilon^h}}\right) = \rho^{\epsilon^h}\, log\left(\frac{\epsilon^h{}_{t-1}}{\bar{\epsilon^h}}\right) + \sigma^{\epsilon^h}\, \eta^{\epsilon^h}{}_t. \tag{52}$$

Cost-push shocks are shocks to the desired mark-up over marginal costs and are a standard shock included in NK-DSGE models. They mainly capture variations in the markup over the business cycle.

$$log\left(\frac{\epsilon^p{}_t}{\bar{\epsilon^p}}\right) = \rho^{\epsilon^p}\, log\left(\frac{\epsilon^p{}_{t-1}}{\bar{\epsilon^p}}\right) + \sigma^{\epsilon^p}\, \eta^{\epsilon^p}{}_t. \tag{53}$$

Wage markup shocks are similar to price mark-up shocks. They mainly capture variations in the wage markup over the business cycle.

$$log\left(\frac{\epsilon^w{}_t}{\bar{\epsilon^w}}\right) = \rho^{\epsilon^p}\, log\left(\frac{\epsilon^w{}_{t-1}}{\bar{\epsilon^w}}\right) + \sigma^{\epsilon^w}\, \eta^{\epsilon^w}{}_t. \tag{54}$$

Episodes of more and less rapid technological growth require a time varying growth rate. Nevertheless, this growth rate is independent of endogenous variables in the model.

$$log\left(\frac{\mu^z{}_t}{\bar{\mu^z}}\right) = \rho^{\mu^z}\, log\left(\frac{\mu^z{}_{t-1}}{\bar{\mu^z}}\right) + \sigma^{\mu^z}\, \eta^{\mu^z}{}_t. \tag{55}$$

The relative price for investment is driven by an exogenous shock. This shock is necessary to include the relative price of investment as an observable variable for the estimation of the model.

$$log\left(\frac{\mu^\Upsilon{}_t}{\bar{\mu^\Upsilon}}\right) = \rho^{\mu^\Upsilon}\, log\left(\frac{\mu^\Upsilon{}_{t-1}}{\bar{\mu^\Upsilon}}\right) + \sigma^{\mu^\Upsilon}\, \eta^{\mu^\Upsilon}{}_t. \tag{56}$$

Households preferences to consume might fluctuate over time. This is captured by temporary shocks to consumption preferences.

$$log\left(\frac{\zeta^c{}_t}{\bar{\zeta^c}}\right) = \rho^{\zeta^c}\, log\left(\frac{\zeta^c{}_{t-1}}{\bar{\zeta^c}}\right) + \sigma^{\zeta^c}\, \eta^{\zeta^c}{}_t. \tag{57}$$

Capital formation depends on the effectiveness of investment into the capital stock. This efficiency fluctuates over time due to an exogenous process.

$$log\left(\frac{\zeta^i{}_t}{\bar{\zeta^i}}\right) = \rho^{\zeta^i}\, log\left(\frac{\zeta^i{}_{t-1}}{\bar{\zeta^i}}\right) + \sigma^{\zeta^i}\, \eta^{\zeta^i}{}_t. \tag{58}$$

C.1.7 Observational Equations

Estimating the model requires to define observational variables. Standard observational variables are the main components of GDP. It is necessary to define suitable transformations of the observed variables and the model variables. Observational variables for the CEE model are consumption growth (59), GDP growth (60), hours worked (61), investment growth (62), wage growth (63), relative price of investment (64), inflation (65), and the risk free interest rate (66).

$$c_t^{obs} = \overline{c}^{obs} \frac{{\mu^z}_t \, c_t}{\bar{\mu^z} \, c_{t-1}}, \tag{59}$$

$$y_t^{obs} = \overline{y}^{obs} \frac{{\mu^z}_t \left(c_t + \frac{i_t}{{\mu^\Upsilon}_t} + g_t \right)}{\bar{\mu^z} \left(c_{t-1} + \frac{i_{t-1}}{{\mu^\Upsilon}_{t-1}} + g_{t-1} \right)}, \tag{60}$$

$$h_t^{obs} = \overline{h}^{obs} \frac{h_t}{(\bar{h})}, \tag{61}$$

$$i_t^{obs} = \overline{i}^{obs} \frac{{\mu^z}_t \, i_t}{\bar{\mu^z} \, i_{t-1}}, \tag{62}$$

$$w_t^{obs} = \overline{w}^{obs} \frac{{\mu^z}_t \, w_t}{\bar{\mu^z} \, w_{t-1}}, \tag{63}$$

$$p_t^{i,obs} = \overline{p}^{i,obs} \frac{\bar{\mu^\Upsilon}}{{\mu^\Upsilon}_t}, \tag{64}$$

$$\pi_t^{obs} = \overline{\pi}^{obs} \frac{\pi_t}{\bar{\pi}}, \tag{65}$$

$$R_t^{obs} = \overline{R}^{obs} \, exp\left(R_t - \bar{R}\right). \tag{66}$$

C.2 CMR model equations

The CMR model uses (26) to (48). Including the financial accelerator leads to modifications of the resource constraint. Further, (67), (69), (70), (71), (72), (73) are additional model equations. These equations describe the behaviour of entrepreneurs and mutual funds. The new resource constraint is now (74) and replaces (49). Further, the financial accelerator model will introduce new shocks to the model. These shocks drive the dispersion in the idiosyncratic productivity of entrepreneurs.

C.2.1 Entrepreneurs

The main modification of CMR compared to the CEE model is to introduce entrepreneurs and mutual funds as agents. Households do not supply capital services to the intermediate goods producing firms. Entrepreneurs provide now effective capital to intermediate goods producing firms. Mutual funds grant loans to entrepreneurs. Loans need to be repaid. The probability that an entrepreneur cannot repay the loans is given by $F(\bar{\omega}_t)$. Default probability increases with the threshold $\bar{\omega}_t$. Here Φ denotes the normal distribution and σ_t is the cross-sectional dispersion of ω.

$$F(\bar{\omega}_t) = \Phi\left(\frac{log\,(\bar{\omega}_t) + \frac{{\sigma_{t-1}}^2}{2}}{\sigma_{t-1}}\right). \tag{67}$$

The value of the assets of insolvent entrepreneurs depends on the expected value of ω below the threshold $\bar{\omega}$. This expected value is required to model monitoring costs and the credit spread.

$$G(\bar{\omega}_t) = \Phi\left(\frac{log\,(\bar{\omega}_t) + \frac{{\sigma_{t-1}}^2}{2}}{\sigma_{t-1}} - \sigma_{t-1}\right). \tag{68}$$

Entrepreneurs purchase raw capital from households. Profits of entrepreneurs depend on the return on raw capital purchases. The return on raw capital depends on inflation, current and past raw capital prices, the rental rate for effective capital services and the possibility to deduct taxes on depreciated capital.

$$1 + {R^k}_t = \frac{\pi_t\,\left(\left(1-\tau^k\right)\,\left(u_t\,r_t^k - a(u_t)\right) + (1-\delta)\,q_t\right)}{\Upsilon\,q_{t-1}} + \delta\,\tau^k. \tag{69}$$

Mutual funds operate under perfect competition and free entry. This rules out profits of mutual funds. The zero profit condition determines the leverage ratio for a given credit spread.

$$0 = 1 + \frac{\left(1 + {R^k}_t\right)\,\frac{\bar{k}_{t-1}\,q_{t-1}}{n_{t-1}}\,\left(G(\bar{\omega}_t)\,(1-\mu) + \bar{\omega}_t\,(1 - F(\bar{\omega}_t))\right)}{1 + R_{t-1}} - \frac{\bar{k}_{t-1}\,q_{t-1}}{n_{t-1}}. \tag{70}$$

Entrepreneurs optimal choice of leverage defines the threshold value $\bar{\omega}$ as a nonlinear function of the credit spread, given a dispersion value in the current period. An increase in the credit spread will reduce the threshold value separating insolvent and solvent entrepreneurs.

$$\begin{aligned} 0 =& \frac{\left(1 - \left(\bar{\omega}_{t+1}\,\left(1 - F(\bar{\omega}_{t+1})\right) + G(\bar{\omega}_{t+1})\right)\right)\,\left(1 + {R^k}_{t+1}\right)}{1 + R_t} \\ &+ \frac{1 - F(\bar{\omega}_{t+1})}{1 - F(\bar{\omega}_{t+1}) - \mu\,\Phi(\frac{log(\bar{\omega}_{t+1}) + \frac{{\sigma_t}^2}{2}}{\sigma_t})\,\sigma_t}\left(\frac{1 + {R^k}_{t+1}}{1 + R_t}\left(\bar{\omega}_{t+1}\,\left(1 - F(\bar{\omega}_{t+1})\right)\ldots\right.\right. \\ &\left.\left. + (1-\mu)\,G(\bar{\omega}_{t+1})\right) - 1\right). \end{aligned} \tag{71}$$

Networth of the representative surviving entrepreneur is the sum of current profits (first term), transfers ω^e from households and previous net worth (last term) in (72).

$$n_t = q_{t-1}\,\bar{k}_{t-1}\,\frac{\gamma_t}{{\mu^z}_t\,\pi_t}\left(R^k{}_t - R_{t-1} - \left(1 + R^k{}_t\right)\ldots\right. \tag{72}$$

$$\left.\left(G(\bar{\omega}_t) + \bar{\omega}_t\,\left(1 - F(\bar{\omega}_t)\right) - \left(G(\bar{\omega})_t\,\left(1 - \mu\right) + \bar{\omega}_t\,\left(1 - F(\bar{\omega})_t\right)\right)\right)\right) + \omega^e +$$

$$\frac{n_{t-1}\,\left(1 + R_{t-1}\right)\,\gamma_t}{{\mu^z}_t\,\pi_t}.$$

Mutual funds need to monitor default entrepreneurs. Monitoring costs will eat up some of the resources in the economy. Monitoring costs are by a factor μ proportional to their value of assets. Their value of assets depends on the expected value of ω below the threshold $\bar{\omega}$ given by $G(\bar{\omega}_t)$.

$$dcost(\bar{\omega})_t = \frac{\bar{k}_{t-1}\,q_{t-1}\,\left(1 + R^k{}_t\right)\,G(\bar{\omega})_t\,\mu}{{\mu^z}_t\,\Upsilon}. \tag{73}$$

The resource constraint includes now additional terms. These additional terms are monitoring costs $dcost(\bar{\omega}_t)$ and assets used by exiting entrepreneurs $\frac{\Theta\,(1-\gamma_t)\,(n_t-\omega^e)}{\gamma_t}$.

$$y_t = dcost(\bar{\omega}_t) + c_t + \frac{i_t}{{\mu^\Upsilon}_t} + g_t + \frac{\bar{k}_{t-1}}{{\mu^z}_t\,\Upsilon}\,a(u_t) + \frac{\Theta\,\left(1 - \gamma_t\right)\,\left(n_t - \omega^e\right)}{\gamma_t}. \tag{74}$$

C.2.2 Shocks

The CMR model features additional shocks. Shocks affect directly the financial variables. The most important shock is the so-called risk shock σ_t. This shock is the dispersion in the idiosyncratic productivity of entrepreneurs. This dispersion is driven by unanticipated shocks η^σ and anticipated shocks ξ^s.

$$log\left(\frac{\sigma_t}{\bar{\sigma}}\right) = \rho^{\pi^*}\,log\left(\frac{\sigma_{t-1}}{\bar{\sigma}}\right) + \sigma^\sigma\,\eta^\sigma{}_t + \sum_{s=1}^{S} log\left(\xi^s{}_{t-s}\right). \tag{75}$$

Anticipated risk shocks are correlated $\sigma(\xi^s_t, \xi s + 1_t)$. The number of signals is a degree of freedom. CMR use 8 shocks in their baseline model.

$$log\left(\xi^s{}_t\right) = \begin{cases} \sigma^\xi\,\eta^{\xi^s}{}_t + \left(2\,\sigma(\xi^s_t, \xi s + 1_t) - 1\right)\,log\left(\xi^{s+1}{}_t\right) & \text{,if } s < S, \\ \sigma^\xi\,\eta^{\xi^s}{}_t & \text{,if } s = S. \end{cases} \tag{76}$$

The survival rate is time-varying and is also labelled equity shock. The survival rate defines how much networth from the previous period remains.

$$log\left(\frac{\gamma_t}{\bar{\gamma}}\right) = \rho^{\gamma}\, log\left(\frac{\gamma_{t-1}}{\bar{\gamma}}\right) + \sigma^{\gamma}\, \eta^{\gamma}{}_t. \tag{77}$$

Shocks to the term structure are also included. These shocks are responsible for wedges between the effective short-term risk free interest rates on short-term bonds R and long-term interest rates R^L.

$$log\left(\frac{\zeta_t^{term}}{\bar{\zeta}^{term}}\right) = \rho^{term}\, log\left(\frac{\zeta_{t-1}^{term}}{\bar{\zeta}_t^{term}}\right) + \sigma^{term}\, \eta^{term}{}_t. \tag{78}$$

C.2.3 Observational Equations

One of the main findings in CMR is that the contribution of risk shocks to the business cycle depends on the inclusion of quantitative variables describing the financial market. The CMR model is estimated in addition to the observables from the CEE model with data on extended credit growth (79), networth growth (80), credit spread (81) and the term structure (82).

$$\frac{b_t^{obs}}{\bar{b}^{obs}} = \frac{q_t\bar{k}_t - n_t}{q_{t-1}\bar{k}_{t-1} - n_{t-1}}\frac{\mu_t^z}{\mu^z}, \tag{79}$$

$$\frac{n_t^{obs}}{\bar{n}^{obs}} = \frac{n_t}{n_{t-1}}\frac{\mu_t^z}{\mu^z}, \tag{80}$$

$$\frac{premium_t^{obs}}{\overline{premium}^{obs}} = \exp\{\mu G_{t-1}(\bar{\omega}_t)\frac{q_{t-1}\bar{k}_t}{q_{t-1}\bar{k}_t - n_t} - \mu G(\bar{\omega})\frac{q\bar{k}}{q\bar{k} - n}\}, \tag{81}$$

$$\frac{S_t^{1,obs}}{\bar{S}^{1,obs}} = 1 + R_t^L - R_t. \tag{82}$$

C.3 CMR/CEE–Oil model equations

I will now outline the modifications of the CEE model and CMR model to include oil as production factor. To include oil as production factor I replace equations (33), (35), and (36) with equations (83), (84), (85), (86), (88) and (89). These equations describe the production process. It is also necessary to describe the behaviour of oil supplying firms. The behaviour of oil supplying firms is described by (90), (91), (92), and (93). It is necessary to modify the resource constraint to include oil as reported in (87). Oil market shocks are introduced with (94), (95), (96), (97), and (98). A shock for capital productivity is introduced as well (99).

C.3.1 Production

In contrast to the CEE and CMR model the production of final goods is described by a two layer CES production function. The upper layer of a nested CES production function in stationary firm and including price and wage dispersion combines capital-oil composite goods m_t and homogenous labour $l_t = h_t\, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}}$.

$$y_t = \begin{cases} {p^*}_t^{\frac{\lambda^f}{\lambda^f-1}}\, \epsilon_t \left({\alpha^M}^{\frac{1}{\eta^M}}\, {m_t}^{\frac{\eta^M-1}{\eta^M}} + {\alpha^N}^{\frac{1}{\eta^M}} \left({\epsilon^h}_t\, h_t\, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}} \right)^{\frac{\eta^M-1}{\eta^M}} \right)^{\frac{\eta^M}{\eta^M-1}} - \phi_t, & \text{if } \eta^M \neq 1, \\ {p^*}_t^{\frac{\lambda^f}{\lambda^f-1}}\, \epsilon_t\, {m_t}^{\alpha^M} \left({\epsilon^h}_t\, h_t\, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}} \right)^{\alpha^N} - \phi_t, & \text{if } \eta^M = 1. \end{cases} \tag{83}$$

The capital-oil composite production factor combines the primary production factors oil and effective capital. I include specific productivity shocks for both production factors.

$$m_t = \begin{cases} \left({\alpha^K}^{\frac{1}{\eta^O}} \left({\epsilon^m}_t\, \frac{u_t\, \bar{k}_{t-1}}{{\mu^z}_t\, \Upsilon} \right)^{\frac{\eta^O-1}{\eta^O}} + {\alpha^O}^{\frac{1}{\eta^O}} \left({\epsilon^o}_t\, o_t \right)^{\frac{\eta^O-1}{\eta^O}} \right)^{\frac{\eta^O}{\eta^O-1}}, & \text{if } \eta^O \neq 1, \\ \left({\epsilon^m}_t\, \frac{u_t\, \bar{k}_{t-1}}{{\mu^z}_t\, \Upsilon} \right)^{\alpha^K} \left({\epsilon^o}_t\, o_t \right)^{\alpha^O}, & \text{if } \eta^O \neq 1. \end{cases} \tag{84}$$

The demand for the capital composite production factor depends on the relative price $\frac{p_t^m}{s_t}$ and the marginal product represented by the right hand side of (85).

$$\frac{{p^m}_t}{s_t} = {\alpha^M}^{\frac{1}{\eta^M}}\, {\epsilon_t}^{\frac{\eta^M-1}{\eta^M}} \left(\frac{m_t}{\phi_t + y_t\, {p_t^*}^{\frac{\lambda^f}{1-\lambda^f-1}}} \right)^{\frac{(-1)}{\eta^M}}. \tag{85}$$

The demand for hours worked depends on the relative price $\frac{w_t}{s_t}$ and its marginal product represented by the right hand side of (86).

$$\frac{w_t}{s_t} = {\alpha^N}^{\frac{1}{\eta^M}}\, {\epsilon_t}^{\frac{\eta^M-1}{\eta^M}}\, {{\epsilon^h}_t}^{\frac{\eta^M-1}{\eta^M}} \left(\frac{h_t\, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}}}{\phi_t + y_t\, {p_t^*}^{\frac{\lambda^f}{1-\lambda^f}}} \right)^{\frac{(-1)}{\eta^M}}. \tag{86}$$

The resource constraint of the economy changes. It now features oil export revenues and oil import expenditures. Oil export revenues increase the funds disposable for different purposes. Oil import expenditures require goods to pay for them.

$$y_t = c_t + \frac{i_t}{\mu_t^{\Upsilon}} + g_t + \frac{\bar{k}_{t-1}\, a(u_t)}{{\mu^z}_t\, \Upsilon} - p_t^o\, \left({o^{ex}}_t - {o^f}_t\right) \tag{87}$$
$$\begin{cases} +dcost(\bar{\omega})_t + \frac{\Theta\,(1-\gamma_t)\,(n_t-\omega^e)}{\gamma_t} & \text{, for CMR–Oil model,} \\ & \text{for CEE–Oil model.} \end{cases}$$

C.3.2 Oil market

Demand for oil in the economy is given by the first order condition of representative intermediate goods producers. Intermediate goods producers demand oil as long as its marginal cost does not exceed its marginal product (right hand side of (88)).

$$\frac{p_t^o}{{p^m}_t} = {\alpha^O}^{\frac{1}{\eta^O}}\, {\epsilon^o}_t^{\frac{\eta^O-1}{\eta^O}} \left(\frac{o_t}{m_t}\right)^{\frac{(-1)}{\eta^O}}. \tag{88}$$

As for oil demand for capital is given by the first order condition of the intermediate goods producer. In case of $\eta^O = 1$ and an oil share equal to zero this equation is identical to (35).

$$\frac{{r^k}_t}{{p^m}_t} = {\alpha^K}^{\frac{1}{\eta^O}}\, {\epsilon^m}_t^{\frac{\eta^O-1}{\eta^O}} \left(\frac{\frac{u_t\,\bar{k}_{t-1}}{{\mu^z}_t\,\Upsilon}}{m_t}\right)^{\frac{(-1)}{\eta^O}}. \tag{89}$$

The previous equations represent the behaviour of the demand side for oil in the economy. Now the supply side is considered. The first order condition derived from the profit maximization problem of domestic oil producers equates the marginal product $p_t^o(1-\tau_t^o)$ with the marginal cost of providing one more unit of oil (right hand side of(90)). This is the domestic oil supply curve.

$$p_t^o\,(1-{\tau^o}_t) = \left(\frac{{\zeta^o}_t}{\gamma^o}\right)^{1+\sigma^O} {o^d}_t^{\sigma^O}. \tag{90}$$

Oil importers also supply oil according to a supply curve. This supply curve is derived from their profit maximization problem. Costs for supplying importing oil to domestic intermediate goods producers are not identical.

$$p_t^o\,(1-{\tau^o}_t) = \left(\frac{{\zeta^{o,im}}_t}{\gamma^{o^{im}}}\right)^{1+\sigma^O} {o_t^{im}}^{\sigma^O}. \tag{91}$$

In contrast to domestic oil producers and oil importers the supply o foil exports is not the result of an optimization problem. It is modelled as the share of domestically produced oil, which is not consumed domestically.

$$o_t^{ex} = {o^d}_t \, {\zeta^{o,ex}}_t. \tag{92}$$

Domestically produced oil and imported oil represent the available oil in one period. This oil supply can either be consumed or exported as stated in (93).

$$o_t + o_t^{ex} = o_t^d + o_t^{im}. \tag{93}$$

C.3.3 Shocks

Costs for providing domestic crude oil can fluctuate over time. This motivates the inclusion of a domestic oil cost shock ζ^o.

$$log\left(\frac{{\zeta^o}_t}{\zeta^o}\right) = \rho^{\zeta^o}\, log\left(\frac{{\zeta^o}_{t-1}}{\zeta^o}\right) + \sigma^{\zeta^o}\, {\eta^{\zeta^o}}_t. \tag{94}$$

The same is true for import oil. Costs for providing imported crude oil might have different short-.term developments than costs for domestically produced oil. In order to capture such differences $\zeta^{o,im}$ is included.

$$log\left(\frac{{\zeta^{o,im}}_t}{\zeta^{o,im}}\right) = \rho^{\zeta^{o,im}}\, log\left(\frac{{\zeta^{o,im}}_{t-1}}{\zeta^{o,im}}\right) + \sigma^{\zeta^{o,im}}\, {\eta^{\zeta^{o,im}}}_t. \tag{95}$$

The share of oil exported relative to overall domestic oil production is not constant. Therefore, a shock to the share of exported oil $\zeta^{o,ex}$ is included.

$$log\left(\frac{{\zeta^{o,ex}}_t}{\zeta^{o,ex}}\right) = \rho^{\zeta^{o,ex}}\, log\left(\frac{{\zeta^{o,ex}}_{t-1}}{\zeta^{o,ex}}\right) + \sigma^{\zeta^{o,ex}}\, {\eta^{\zeta^{o,ex}}}_t. \tag{96}$$

In the US different tax rates on crude oil are applied in the federal states. I model only a simplified tax system. Taxes paid by oil suppliers τ^o are modelled as an autoregressive process of order one.

$$log\left(\frac{{\tau^o}_t}{\tau^o}\right) = \rho^{\tau^o}\, log\left(\frac{{\tau^o}_{t-1}}{\tau^o}\right) + \sigma^{\tau^o}\, {\eta^{\tau^o}}_t. \tag{97}$$

Demand for oil is also driven by the efficiency of oil. The quality of crude oil can vary over time. It is also possible that extending the quantity of used oil in refineries will impact the effect of oil on the productivity of the factor.

$$log\left(\frac{{\epsilon^o}_t}{\bar{\epsilon^o}}\right) = \rho^{\epsilon^o}\, log\left(\frac{{\epsilon^o}_{t-1}}{\bar{\epsilon^o}}\right) + \sigma^{\epsilon^o}\, {\eta^{\epsilon^o}}_t. \tag{98}$$

The same reasons to include productivity shocks for oil apply to effective capital. It is possible to use ϵ^k for permanent shocks. A combination of ϵ^o and ϵ^k is interesting to study potential mitigation scenarios.

$$log\left(\frac{\epsilon^k{}_t}{\bar{\epsilon^k}}\right) = \rho^{\epsilon^k}\, log\left(\frac{\epsilon^k{}_{t-1}}{\bar{\epsilon^k}}\right) + \sigma^{\epsilon^k}\, \eta^{\epsilon^k}{}_t. \tag{99}$$

C.3.4 Observational Equations

In addition to the observables in the MCR and CEE model I introduce observables for the oil market. The oil makret obseervagbles are oil consumption growth (100), domestic oil production growth (101), oil import growth (102), oil exports growth (103) and real price of oil changes (104).

$$o^{obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}} \frac{o_t}{o_{t-1}} \bar{o}^{obs}, \tag{100}$$

$$o^{d,obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}} \frac{o^d_t}{o^d_{t-1}} \bar{o}^{d,obs}, \tag{101}$$

$$o^{im,obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}} \frac{o^{im}_t}{o^{im}_{t-1}} \bar{o}^{im,obs}, \tag{102}$$

$$o^{ex,obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}} \frac{o^{ex}_t}{o^{ex}_{t-1}} \bar{o}^{ex,obs}, \tag{103}$$

$$p^{o,obs}_t = \bar{p}^{o,obs} \frac{p^o_t}{p^o_{t-1}}. \tag{104}$$

$$\tag{105}$$

D Steady-state

D.1 Calibration

For the estimation of the model around a deterministic steady-state the following algorithm is used.

1. define the following steady-state shares:
   (a) rental rate on capital services r^k
   (b) capital expenditure share $\phi^K = \frac{r^k \frac{\bar{k}}{\mu^*_z \Upsilon}}{y}$
   (c) oil expenditure share $\phi^O = \frac{p^O o}{y}$
   (d) oil to output ratio $\frac{o}{y}$
   (e) oil and capital expenditure share $\phi^M = \phi^K + \phi^O$

(f) domestic oil share $\theta^{\frac{o^d}{o}} = \frac{o^d}{o}$

(g) oil exports share $\theta^{\frac{o^{ex}}{o^d}} = \frac{o^e x}{o^d}$

(h) oil imports share $\theta^{\frac{o^{im}}{o}} = \frac{o^{im}}{o}$

(i) oil tax rate τ^o

(j) steady-state output $\bar{y}$

2. set the following variables to pre-defined values under a flexible price equilibrium:

   (a) mark-up λ^f

   (b) long-run growth rate μ_z^*

   (c) investment specific long-run growth rate μ^{Υ}

   (d) gross inflation and inflation target π, π^*

   (e) retained earnings of entrepreneurs γ

   (f) hours worked h, capital utilization rate u, price dispersion index p^* and wage dispersion index w^* are equal to one

3. compute the following variables:

   (a) marginal cost $s = \frac{1}{\lambda^f}$

   (b) fixed cost $\phi = \frac{1-s}{s}\, y$

   (c) price of raw capital $q = \frac{1}{\mu^{\Upsilon}}$

   (d) short-run and long-run interest rate $R = R^L = \frac{\pi \mu_z^*}{\beta} - 1$

   (e) return on capital $R^K = \frac{\{(1-\tau^k)\, r^k + 1 - \delta\}\pi}{\Upsilon} + \tau^k \delta - 1$

   (f) interest rate spread $s^p = \frac{1+R^K}{1+R}$

4. compute $\phi = y\, \frac{1-s}{s}$

5. compute $\bar{k} = \frac{\phi^K y \mu^z\, \Upsilon}{r^k (1+\psi^k\, R)}$

6. compute $w = \frac{\{1-(\phi^K+\phi^O)\} y\, s}{h}$

7. compute $o = \theta^{\frac{o}{y}}\, y$

8. compute $o^{im} = \theta^{\frac{o^{im}}{o}}\, o$

9. compute $o^d = \frac{o - o^{im}}{1-\phi^{o^{ex}}}$

10. compute $o^{ex} = \theta^{o^{ex}}\, o^d$

11. compute $p^o = \phi^O \frac{s\,y}{o}$

12. if $\eta^O = 1$ do

    (a) compute $\alpha^O = \frac{\phi^O}{\phi^O+\phi^K}$

    (b) compute $\alpha^K = \frac{\phi^K}{\phi^O+\phi^K}$

    (c) compute $p^M = \left(\frac{p^O}{\alpha^O}\right)^{\alpha^O} \left(\frac{r^k}{\alpha^K}\right)^{\alpha^K}$

    (d) compute $m = \frac{y\,(\phi^O+\phi^K)}{p^M}$

13. if $\eta^O \neq 1$ do

    (a) compute $p^M = \left(\frac{\phi^O}{\phi^O+\phi^K} p^{O\,\eta^O-1} + \frac{\phi^K}{\phi^O+\phi^K} r^{k\,\eta^O-1}\right)^{\frac{1}{\eta^O-1}}$

    (b) compute $m = \frac{y\,(\phi^O+\phi^K)}{p^M}$

    (c) compute $\alpha^O = \left(\frac{p^O}{p^M}\right)^{\eta^O} \frac{o}{m}$

    (d) compute $\alpha^K = \left(\frac{r^k}{p^M}\right)^{\eta^O} \frac{\frac{u\,\bar{k}}{\Upsilon\mu^z}}{m}$

14. if $\eta^M = 1$ do

    (a) compute $\alpha^M = \phi^O + \phi^K$

    (b) compute $\alpha^N = 1 - \alpha^M$

    (c) compute $\epsilon = s^{-1} \left(\frac{p^M}{\alpha^M}\right)^{\alpha^M} \left(\frac{w}{\alpha^N}\right)^{\alpha^N}$

15. if $\eta^M \neq 1$ do

    (a) compute $\epsilon = s^{-1} \left(\phi^M p^{M\,\eta^M-1} + (1-\phi^M) w^{\eta^M-1}\right)^{\frac{1}{\eta^M-1}}$

    (b) compute $\alpha^M = \left(\frac{p^M}{s}\right)^{\eta^M} \frac{m}{y+\phi}$

    (c) compute $\alpha^N = \left(\frac{w}{s}\right)^{\eta^O} \frac{h}{y+\phi}$

16. compute $i = (1 - \frac{1-\delta}{\mu^z\,\Upsilon})\,\bar{k}$

17. compute $dcost = \frac{\mu\,G\,(1+r^k)\bar{k}}{\pi\mu^z}$

18. solve the contract problem of entrepreneurs for the monitoring cost parameter μ the bankruptcy threshold $\bar{\omega}$ and the idiosyncratic dispersion σ.
    use numerical procedure to find μ such that $|\epsilon^{\mu}| < i^{Tol}$

    (a) use numerical procedure to find $\bar{\omega}$ such that $|\epsilon^{\bar{\omega}}| < i^{Tol}$

        i. use numerical procedure to find σ such that $|\epsilon^{\sigma}| < i^{Tol}$

A. guess σ

B. define $z = \frac{log(\bar{\omega})+0.5\,\sigma^2}{\sigma}$

C. calculate $\epsilon^{\sigma} = \bar{F} - \Phi(z)$

ii. define $\Gamma = \Phi(z-\sigma) + \bar{\omega}(1-\Phi(z))$

iii. define $G = \mu\,\Phi(z-\sigma)$

iv. define $n = \frac{(w^e + \frac{\gamma}{\pi\,\mu^z}\,(r^k - R - \mu G\,(1+r^k)\,\bar{k})}{1-\gamma\,\frac{1+R}{\pi\,\mu^z}}$

v. calculate $\epsilon^{\bar{\omega}} = (1-\Gamma)\,s^p - \frac{1-\bar{F}}{1-\bar{F}-\mu\omega\varphi(z)}\,(s^p(\Gamma-\mu\,G)-1)$

(b) calculate $\epsilon^{\mu} = \frac{n}{\bar{k}} - (1 - s^p\,(\Gamma - \mu\,G))$

19. compute $c = (1-\eta^g)\,y + (o^d - o)\,\frac{p^o}{\mu^o} - d - \Theta\,\frac{1-\gamma}{\gamma}\,(n - w^e) - \frac{i}{\mu^{\Upsilon}}$

20. compute $g = \frac{\eta^g}{1-\eta^g}\,(c + \frac{i}{\mu^{\Upsilon}})$

21. compute $\lambda^z = \frac{\zeta^c}{(1+\tau^c)\,c}\,\frac{\mu^z - b\beta}{\mu^z - b}$

22. compute $\Psi^L = \frac{(1-\tau^l)\,\lambda^z\,w\,h^{-\sigma^L}}{\zeta^h\,\lambda^w}$

23. compute $F^p = \frac{\lambda^z\,y}{1-\beta\,\xi^p}$

24. compute $K^p = \frac{\lambda^z\,y\,s\,\lambda^f}{1-\beta\,\xi^p}$

25. compute $F^w = \frac{h\,(1-\tau^l)\lambda^z}{\lambda^w\,(1-\beta\,\xi^w)}$

26. compute $K^w = \frac{h^{1+\sigma^L}}{1-\beta\,\xi^w}$

D.2 Permanent shock

For the computation of impulse response functions to permanent shocks I need to modify the steady-state routine.

1. solve the oil consumption identity, the first order condition defining labour supply, the first oder condition of entrepreneuers with respect to leverage ratio and the constraint of the optimality problem of entrepreneurs

2. guess the real price of oil p^o, the capital stock $\bar{k}$, the rental rate of capital r^k and the threshold value dividing entrepreneurs into solvent and insolvent firms $\bar{\omega}$ such that $\begin{pmatrix} |\epsilon^{p^o}| \\ |\epsilon^{\bar{k}}| \\ |\epsilon^{r^k}| \\ |\epsilon^{\bar{\omega}}| \end{pmatrix} < i^{Tol}$

3. set mark-up λ^f
4. set long-run growth rate μ_z^*
5. set investment specific long-run growth rate μ^Υ
6. set gross inflation and inflation target π, π^*
7. set retained earnings of entrepreneurs γ
8. set capital utilization rate u, price dispersion index p^* and wage dispersion index w^* to one
9. compute marginal cost $s = \frac{1}{\lambda^f}$
10. compute fixed cost $\phi = \frac{1-s}{s}\, y$
11. compute price of raw capital $q = \frac{1}{\mu^\Upsilon}$
12. compute short-run and long-run interest rate $R = R^L = \frac{\pi \mu_z^*}{\beta} - 1$
13. $o = \left(\frac{p^o}{r^k}\right)^{-\eta^O} \frac{\alpha^O}{\alpha^K} \frac{\bar{k}}{\mu^z\,\Upsilon}\, \epsilon^{o\eta^O - 1}$
14. $o^{im} = \left(\frac{p^o\,(1-\tau^o)}{\left(\frac{\zeta^{o^{im}}}{\gamma^{o^{im}}}\right)^{1+\sigma^o}}\right)^{\frac{1}{\sigma^o}}$
15. $o^d = \left(\frac{p^o\,(1-\tau^o)}{\left(\frac{\gamma^o}{\zeta^o}\right)^{1+\sigma^o}}\right)^{\frac{1}{\sigma^o}}$
16. $o^{ex} = \zeta^{o^{ex}}\, o^d$
17. if $\eta^O = 1$ do
    (a) $p^m = \left(\frac{r^k}{\alpha^K}\right)^{\alpha^K} \left(\frac{p^o}{\alpha^O}\right)^{\alpha^O}$
    (b) $m = \left(\epsilon^M \frac{\bar{k}}{\mu^z\,\Upsilon}\right)^{\alpha^K} (\epsilon^o\, o)^{\alpha^O}$
18. if $\eta^O \neq 1$ do
    (a) $p^m = \left(\alpha^K \left(\frac{r^k}{\epsilon^M}\right)^{1-\eta^O} + \alpha^O \left(\frac{p^o}{\epsilon^o}\right)^{1-\eta^O}\right)^{\frac{1}{1-\eta^O}}$
    (b) $m = \left(\alpha^{K\frac{1}{\eta^O}} \left(\epsilon^M \frac{\bar{k}}{\mu^z\,\Upsilon}\right)^{\frac{\eta^O-1}{\eta^O}} + \alpha^{O\frac{1}{\eta^O}} (\epsilon^o\, o)^{\frac{\eta^O-1}{\eta^O}}\right)^{\frac{\eta^O}{\eta^O-1}}$
19. $\rho^m = \frac{\eta^M - 1}{\eta^M}$

20. if $\eta^M = 1$

    (a) $w = s\,\epsilon \left\{ \left(\frac{p^m}{\alpha^M}\right)^{\alpha^M} \right\}^{\frac{-1}{\alpha^N}} \alpha^N$

    (b) $h = m \left(\frac{w}{p^m}\right)^{-\eta^M} \frac{\alpha^N}{\alpha^M}\, \epsilon^{h \eta^M - 1}$

    (c) $y = s\,\epsilon m^{\alpha^M} (\epsilon^h\, h)^{\alpha^N}$

21. if $\eta^M \neq 1$

    (a) $w = \left((s * \epsilon)^(1 - \eta^M) - \frac{\alpha^M}{\alpha^N}\, p^{m 1 - \eta^M}\right)^{\frac{1}{1-\eta^M}} \epsilon^h$

    (b) $h = m \left(\frac{w}{p^m}\right)^{-\eta^M} \frac{\alpha^N}{\alpha^M}\, \epsilon^{h \eta^M - 1}$

    (c) $y = s\,\epsilon\, \alpha^{M \frac{1}{\eta^M}}\, m^{\rho^m} + \alpha^{N \frac{1}{\eta^M}}\, (\epsilon^h\, h)^{\rho^m})^{\frac{1}{\rho^m}}$

22. $\phi = y\, \frac{1-s}{s}$

23. $n = \frac{\bar{k}\,\gamma}{\pi\,\mu^z}\, \frac{R^k - R - \mu\, G\,(1+R^k)}{1 - \gamma\, \frac{1+R}{\pi\,\mu^z} - \frac{w^e}{(1-s^p\,(\Gamma - \mu\, G))\,\bar{k}}}$

24. compute $i = (1 - \frac{1-\delta}{\mu^z\, \Upsilon})\, \bar{k}$

25. compute $d = \frac{\mu\, G\,(1+r^k)\bar{k}}{\pi \mu^z}$

26. define $z = \frac{log(\bar{\omega}) + 0.5\,\sigma^2}{\sigma}$

27. compute $\bar{F} = \Phi(z)$

28. define $\Gamma = \Phi(z - \sigma) + \bar{\omega}(1 - \Phi(z))$

29. compute $G = \mu\, \Phi(z - \sigma)$

30. define $n = \frac{(w^e + \frac{\gamma}{\pi\,\mu^z}\,(r^k - R - \mu G\,(1+r^k)\,\bar{k})}{1 - \gamma\, \frac{1+R}{\pi\,\mu^z}}$

31. compute $c = (1 - \eta^g)\, y + (o^d - o)\, \frac{p^o}{\mu^o} - d - \Theta\, \frac{1-\gamma}{\gamma}\, (n - w^e) - \frac{i}{\mu^\Upsilon}$

32. compute $g = \frac{\eta^g}{1-\eta^g}\, (c + \frac{i}{\mu^\Upsilon})$

33. compute $\lambda^z = \frac{\zeta^c}{(1+\tau^c)\, c}\, \frac{\mu^z - b\beta}{\mu^z - b}$

34. compute $F^p = \frac{\lambda^z\, y}{1 - \beta\, \xi^p}$

35. compute $K^p = \frac{\lambda^z\, y\, s\, \lambda^f}{1 - \beta\, \xi^p}$

36. compute $F^w = \frac{h\,(1-\tau^l)\lambda^z}{\lambda^w\,(1-\beta\,\xi^w)}$

37. compute $K^w = \frac{h^{1+\sigma^L}}{1-\beta\,\xi^w}$

38. compute residuals for the following model equations

   (a) compute $\epsilon^{\bar{k}} = \Psi^L - \frac{(1-\tau^l)\,\lambda^z\,w\,h^{-\sigma^L}}{\zeta^h\,\lambda^w}$

   (b) compute $\epsilon^{p^o} = o - (o^d - o^{ex} + o^{im})$

   (c) compute $\epsilon^{r^k} = w^e - \left(1 - \frac{\gamma}{\pi\,\mu^z}\right)\,(R^k - R - \mu\,G\,(1+R^k))\,\bar{k} - \frac{\gamma\,(1+R)}{\pi\,\mu^z}\,n$

   (d) compute $\epsilon^{\bar{\omega}} = (1-\Gamma)\,s^p - \frac{1-\bar{F}}{1-\bar{F}-\mu\omega\varphi(z)}\,(s^p(\Gamma - \mu\,G) - 1)$

E Sufficient conditions for a minimum of the cost minimization problem

I will now discuss sufficient conditions for a minimum of the intermediate goods producing firm's cost minimization problem. Intermediate goods producing firms use homogenous labour $l_{j_f,t}$, capital services $K^s_{j_f,t}$ and crude oil $O_{j_f,t}$. These production factors are combined in a two layer nested CES function to produce intermediate goods $Y_{j_f,t}$. The cost minimization problem of the firm is

$$\begin{aligned} \min_{l_{j_f,t},K^s_{j_f,t},O_{j_f,t}} \quad & W_t l_{j_f,t} + P_t\tilde{r}^k_t K^s_{j_f,t} + P^O_t O_{j_f,t}, \\ s.t. Y_{j_f,t} &= X(M(O_{j_f,t}, K^s_{j_f,t}), l_{j_f,t}) - \phi_t z_t, \\ l_{j_f,t} > 0,\ K^s_{j_f,t} > 0,\ & O_{j_f,t} > 0,\ M_{j_f,t} > 0,\ Y_{j_f,t} > 0. \end{aligned} \tag{106}$$

Intermediate goods producers pay wages $W_{j_f,t}$, rental rates on capital $P_t\tilde{r}^k_t$, and a price for crude oil P^O_t. In addition to variable costs firms also have fixed costs $z_t\phi_t$. The production functions for total output $X(M(O_{j_f,t}, K^s_{j_f,t}), h_{j_f,t})$ and the capital-oil composite production factor $M_{j_f,t} = M(O_{j_f,t}, K^s_{j_f,t})$ are given by

$$X(M_{j_f,t}, l_{j_f,t}) = \begin{cases} \epsilon_t\, M_{j_f,t}^{\alpha_M}\,(z_t l_{j_f,t})^{1-\alpha_M}, \text{if } \eta^M = 1, \\ \epsilon_t \left[(\alpha_M)^{\frac{1}{\eta^M}}\, M_{j_f,t}^{\rho^M} + (1-\alpha_M)^{\frac{1}{\eta^M}}\,(z_t l_{j_f,t})^{\rho^M}\right]^{\frac{1}{\rho^M}} \text{ otherwise.} \end{cases} \tag{107}$$

$$M(K^s_{j_f,t}, O_{j_f,t}) = \begin{cases} \left(\epsilon^O_t\,\frac{O_{j_f,t}}{\Upsilon^{O^t}}\right)^{\alpha_O}\,\left(\epsilon^K_t\,\frac{K^s_{j_f,t}}{\Upsilon^{t-1}}\right)^{1-\alpha_O}, \text{if } \eta^O = 1, \\ \left\{(1-\alpha_O)^{\frac{1}{\eta^O}}\,\left(\epsilon^K_t\,\frac{K^s_{j_f,t}}{\Upsilon^{t-1}}\right)^{\rho^O} + (\alpha_O)^{\frac{1}{\eta^O}}\,\left(\epsilon^O_t\,\frac{O_{j_f,t}}{\Upsilon^{O^t}}\right)^{\rho^O}\right\}^{\frac{1}{\rho^O}} \text{ otherwise.} \end{cases} \tag{108}$$

The corresponding Lagrangian, ignoring the non-negativity constraints, of the problem is

$$\mathcal{L}_t^{\text{F,min}} = W_t\, l_{j_f,t} + P_t\, \tilde{r}_t^k\, K_{j_f,t}^s + P_t^O O_{j_f,t} + S_t\{Y_{j_f,t} - (X(M(O_{j_f,t}, K_{j_f,t}^s), l_{j_f,t}) - \phi z_t)\}. \tag{109}$$

The necessary conditions for a stationary point of (109) are

$$\frac{\partial \mathcal{L}_t^{\text{F,min}}}{\partial l_{j_f,t}} : 0 = \quad W_t - X_{l,j_f,t} = W_t - S_t z_t^{\frac{\eta^M - 1}{\eta^M}}\, \epsilon_t (\alpha_N)^{\frac{1}{\eta^M}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{110}$$

$$\frac{\partial \mathcal{L}_t^{\text{F,min}}}{\partial K_{j_f,t}^s} : 0 = \quad P_t \tilde{r}_t^k - X_{K^s,j_f,t} = P_t \tilde{r}_t^k - P_t^M\, (1 - \alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{t-1})^{-\rho^O}\, (\epsilon^K{}_t)^{\rho^O} \left(\frac{M_{j_f,t}}{K_{j_f,t}^s}\right)^{\frac{1}{\eta^O}}, \tag{111}$$

$$\frac{\partial \mathcal{L}_t^{\text{F,min}}}{\partial O_{j_f,t}} : 0 = \quad P_t^O - X_{O,j_f,t} = P_t^O - P_t^M (\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{O^t})^{-\rho^O}\, (\epsilon^O{}_t)^{\rho^O} \left\{\frac{M_{j_f,t}}{O_{j_f,t}}\right\}^{\frac{1}{\eta^O}}, \tag{112}$$

$$\frac{\partial \mathcal{L}_t^{\text{F,min}}}{\partial S_t} : 0 = \quad X_{j_f,t} - X(l_{j_f,t}, M_{j_f,t}). \tag{113}$$

I define an auxiliary variable $P_t^M = S_t\, X_{M,j_f,t} = S_t\, z_t^{\rho^M}\, \epsilon_t\, \alpha_M^{\frac{1}{\eta^M}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}$ to define the partial derivative of total output $X_{j_f,t}$ with respect to $M_{j_f,t}$ times marginal costs. For the following analysis I will drop the time index t and the index for firms j_f.

I apply Theorem 1.14 in De la Fuente (2000) to check whether the solution to (110), (111), (112), (113). is indeed a minimizer of the cost function. The optimization problem consists of choice variables $\mathbf{x}_t = [O_{j_f,t}, K_{j_f,t}^s, h_{j_f,t}]^\top$, an objective function $\mathcal{F}(\mathrm{x}_t) = W_t\, h_{j_f,t} + P_t^O\, O_{j_f,t} + P_t\, r_t^k\, K_{j_f,t}^s$ and one constraint $\mathcal{G}(\mathbf{x}_t) = X(M_{j_f,t}, h_{j_f,t}) - X_{j_f,t}$. The optimization problem in compact notation is $\{\min_{\mathbf{x_t}} \mathcal{F}(\mathrm{x}_t); \mathcal{G}(\mathbf{x}_t) = 0\}$. According to Theorem 1.14 the objective function $\mathcal{F}$ needs to be pseudo-convex and all constraints quasi-concave $\mathcal{G}^j$ for a minimum. $\mathcal{F}$ is pseudo-convex for positive factor prices. All constraints need to be quasi-concave. If all constraints are concave they are also quasi-concave. I show that the CES production function is concave if $\rho^{M,O} \in (0, 1)$ and $\alpha^{M,O} > 0$. The Hessian matrix of $X(M_{j_f,t}, h_{j_f,t})$ is denoted by $\mathcal{H}^X$.

$$\mathcal{H}^X = \begin{pmatrix} \frac{\partial^2 X}{\partial O^2} & \frac{\partial^2 X}{\partial O \partial K^s} & \frac{\partial^2 X}{\partial O \partial l} \\ \frac{\partial^2 X}{\partial O \partial K^s} & \frac{\partial^2 X}{\partial K^{s2}} & \frac{\partial^2 X}{\partial K^s \partial l} \\ \frac{\partial^2 X}{\partial O \partial l} & \frac{\partial^2 X}{\partial K^s \partial l} & \frac{\partial^2 X}{\partial l^2} \end{pmatrix} = \begin{pmatrix} X_{OO} & X_{OK^s} & X_{Ol} \\ X_{K^s O} & X_{K^s K^s} & X_{K^s l} \\ X_{lO} & X_{lK^s} & X_{K^s K^s} \end{pmatrix}.$$

$$X_{OO} = -\frac{1}{\eta^M} X_M \left(\frac{1}{M} - \frac{X_M}{X} \right) M_O^2 - \frac{1}{\eta^O} \left(\frac{1}{O} - \frac{M_O}{M}) M_O X_M \right).$$

$$X_{K^s K^s} = -\frac{1}{\eta^M} X_M \left(\frac{1}{M} - \frac{X_M}{X} \right) M_{K^s}^2 - \frac{1}{\eta^O} \left(\frac{1}{O} - \frac{M_K^s}{M}) M_{K^s} X_M \right).$$

$$X_{ll} = -\frac{1}{\eta^M} X_l \left(\frac{1}{l} - \frac{X_l}{X} \right).$$

$$X_{OK^s} = -\frac{1}{\eta^M} X_M \left(\frac{1}{M} - \frac{X_M}{X} \right) M_{K^s} M_O + \frac{1}{\eta^O} M_O \frac{M_{K^s}}{M} X_M.$$

$$X_{Ol} = \frac{1}{\eta^M} X_l X_M M_O \frac{1}{X}.$$

In order to check that the matrices are negative semidefinite, the first principal minor needs to be harmful, and the sign of the principal minors are alternating. The leading principal minors $\kappa_{1,2,3}^{minor}$ are

$$\kappa_1^{minor} = X_{OO}, \tag{114}$$

$$\kappa_2^{minor} = X_{OO} X_{K^s K^s} - {X_{OK^s}}^2, \tag{115}$$

$$\begin{aligned} \kappa_3^{minor} = {} & X_{OO} X_{K^s K^s} X_{ll} + X_{OK^s} X_{Ol} X_{lO} + X_{Ol} X_{K^s O} X_{lK^s} \\ & - X_{OK^s} X_{OK^s} X_{ll} - X_{OO} X_{Kl} X_{lK} - X_{Ol} X_{K^s K^s} X_{lO}. \end{aligned} \tag{116}$$

The first principal minor is the second derivative of output with respect to oil. This term is negative, if $\alpha^{M,O,N,K} > 0$ and $\eta^{M,O} > 0$. Further, note that $X(M, l)$ and $M(K^s, O)$ are both homogenous of degree one. This implies that the following identities hold

$$M = M_{K^s} K^s + M_O O, \tag{117}$$

$$X = X_M M + X_l l. \tag{118}$$

It is now necessary to show that the second principal minor is positive. Under the parameter restrictions this indeed is the case.

$$\kappa_2^{minor} = X_{OO}\, X_{K^sK^s} - {X_{OK^s}}^2,$$
$$\kappa_2^{minor} = \left(a\,\frac{K^s}{O} + b\,\frac{M_O}{M_{K^s}}\right)\left(a\,\frac{O}{K^s} + b\,\frac{M_{K^s}}{M_O}\right) - (a-b)^2,$$
$$\kappa_2^{minor} = \left(2 + \frac{M_O O}{M_{K^s} K^s} + \frac{M_{K^s} K^s}{M_O O}\right) a\, b > 0,$$
$$a = \frac{M_{K^s}\, M_O\, X_M}{M\, \eta^O} > 0,$$
$$b = \frac{M_{K^s}\, M_O\, X_M\, X_l\, l}{M\, X\, \eta^M} > 0.$$

The third principal minor is the determinant of the Hessian matrix $\mathcal{H}^X$. The production function is homogenous of degree one, and the determinant of the Hessian matrix is zero. One can derive the following expression for the determinant.

$$\begin{aligned}\kappa_3^{minor} &= X_{OO}\, X_{K^sK^s}\, X_{ll} + X_{OK^s}\, X_{Ol}\, X_{lO} + X_{Ol}\, X_{K^sO}\, X_{lK^s}\\ &\quad - X_{OK^s}\, X_{OK^s}\, X_{ll} - X_{OO}\, X_{Kl}\, X_{lK} - X_{Ol}\, X_{K^sK^s}\, X_{lO},\end{aligned}$$
$$\kappa_3^{minor} = -\frac{a\, b\, c\, d}{e} + \frac{a\, b\, c\, d}{e},$$
$$a = M_{K^s}^4\, M_O^4\, X_l^5\, X_M^6\, (X - X_l\, l),$$
$$b = M_{K^s}^2\, X_M\left(\frac{X_M}{X} - \frac{1}{M}\right)\frac{1}{\eta^M} + M_{K^s}\, X_M\left(\frac{M_{K^s}}{M} - \frac{1}{K^s}\right)\frac{1}{\eta^O},$$
$$c = M_O^2\, X_M\left(\frac{X_M}{X} - \frac{1}{M}\right)\frac{1}{\eta^M} + M_O\, X_M\left(\frac{M_O}{M} - \frac{1}{O}\right)\frac{1}{\eta^O},$$
$$d = (X\, \eta^M - X\, \eta^O + M\, X_M\, \eta^O)^2,$$
$$e = M^2\, X^7\, {\eta^M}^7\, {\eta^O}^2\, l.$$

The determinant is zero, and this implies that the Hessian matrix is negative semidefinite. The optimization problem satisfies the conditions for Theorem 1.14 to apply.

F Online Appendix

F.1 Model derivation

F.1.1 Scaling and observational equations

I will now explicitly state the scaling of the different variables to transform the non-stationary model to a stationary model. The following scaling is applied:

$$q_t = \frac{Q_{\bar{K},t}\Upsilon^t}{P_t}, \quad y_{z,t} = \frac{Y_t}{z_t}, \quad i_t = \frac{I_t}{z_t\Upsilon^t}, \quad w_t = \frac{W_t}{z_t P_t}, \quad \lambda_{z,t} = P_t z_t \lambda_t,$$
$$k_t = \frac{\bar{K}_t}{z_{t-1}\Upsilon^{t-1}}, \quad \mu_t^z = \mu^z \frac{z_t}{z_{t-1}}, \quad c_t = \frac{C_t}{z_t}, \quad n_{t+1} = \frac{N_{t+1}}{P_t\, z_t},$$
$$r_t^k = \frac{\tilde{r}_t^k}{\Upsilon^t} \quad o_t^{d,im,ex} = \frac{O_t^{d,im,ex} z_t}{\Upsilon^{O^t}}, \quad p_t^o = \frac{P_t^o}{P_t}\Upsilon^{O^t}.$$

I have 17 observational equations, linking the model variables to the observed variables. The sample average of arbitrary variable x_t is denoted by $\bar{x}_t$.

$$\frac{y_t^{obs}}{\overline{y}^{obs}} = \frac{c_t + \frac{i_t}{\mu_{\Upsilon,t}} + g_t}{c_{t-1} + \frac{i_{t-1}}{\mu_{\Upsilon,t-1}} + g_{t-1}} \frac{\mu_t^z}{\mu^z}.$$

$$\frac{c_t^{obs}}{\overline{c}^{obs}} = \frac{c_t}{c_{t-1}} \frac{\mu_t^z}{\mu^z}.$$

$$\frac{i_t^{obs}}{\overline{i}^{obs}} = \frac{i_t}{i_{t-1}} \frac{\mu_t^z}{\mu^z}.$$

$$\frac{b_t^{obs}}{\overline{b}^{obs}} = \frac{q_t\bar{k}_t - n_t}{q_{t-1}\bar{k}_{t-1} - n_{t-1}} \frac{\mu_t^z}{\mu^z}.$$

$$\frac{n_t^{obs}}{\overline{n}^{obs}} = \frac{n_t}{n_{t-1}} \frac{\mu_t^z}{\mu^z}.$$

$$\frac{premium_t^{obs}}{\overline{premium}^{obs}} = \exp\{\mu G_{t-1}(\bar{\omega}_t)\frac{q_{t-1}\bar{k}_t}{q_{t-1}\bar{k}_t - n_t} - \mu G(\bar{\omega})\frac{q\bar{k}}{q\bar{k} - n}\}.$$

$$\frac{w_t^{obs}}{\overline{w}^{obs}} = \frac{w_t}{w_{t-1}} \frac{\mu_t^z}{\mu^z}.$$

$$\frac{S_t^{1,obs}}{\overline{S}^{1,obs}} = 1 + R_t^L - R_t.$$

$$\frac{h_t^{obs}}{\overline{h}^{obs}} = \frac{h_t}{h}.$$

$$\frac{p_t^{i,obs}}{\overline{p}^{i,obs}} = \frac{1}{\mu_{\Upsilon,t}}.$$

$$\frac{R_t^{obs}}{\overline{R}^{obs}} = \exp(R_t - R).$$

$$\frac{\Pi_t^{obs}}{\overline{\Pi}^{obs}} = \frac{\Pi_t}{\Pi}.$$

$$\frac{p_t^{O,obs}}{\overline{p}^{O,obs}} = \frac{p_t^O}{p_{t-1}^O}.$$

$$\frac{o_t^{d,obs}}{\overline{o}^{d,obs}} = \frac{o_t^d}{o_{t-1}^d} \frac{\mu_t^z}{\mu^z}.$$

I demean the observed variables by their respective sample means. This approach allows to deal with different growth rates of oil market quantities in the sample. Sample means also include the deterministic trends Υ^O.

F.1.2 Final goods producers

The firms producing homogeneous output Y_t from $Y_{j_f,t}$ solve

$$\max_{Y_{j_f,t}} P_t Y_t - \int_0^1 P_{j_f,t} Y_{j_f,t} dj_f, \tag{119}$$
$$\text{s.t. } Y_t = \left(\int_0^1 Y_{j_f,t}^{\frac{1}{\lambda^f}} dj_f \right)^{\lambda^f}.$$

The firms are facing perfect competition and can not set their prices and have no influence on the input prices. Therefore the FOC w.r.t. $Y_{j_f,t}$ can be derived with the envelope theorem,

$$\begin{aligned} P_t \frac{dY_t}{dY_{j_f,t}} - P_{j_f,t} = & \quad 0, \\ P_t \left(\frac{Y_t}{Y_{j_f,t}} \right)^{\frac{\lambda^f - 1}{\lambda^f}} - P_{j_f,t} = & \quad 0. \end{aligned} \tag{120}$$

Solve for $Y_{j_f,t}$ and set back in definition for Y_t to get a relationship between P_t and $P_{j_f,t}$.

$$\begin{aligned} Y_t = & \left(\int_0^1 Y_{j_f,t}^{\frac{1}{\lambda^f}} dj_f \right)^{\lambda^f}, \\ Y_t = & \left[\int_0^1 \left\{ \left(\frac{P_{j_f,t}}{P_t} \right)^{-\frac{\lambda^f}{\lambda^f - 1}} Y_t \right\}^{\frac{1}{\lambda^f}} dj_f \right]^{\lambda^f}, \\ Y_t = & \left\{ \int_0^1 (P_{j_f,t}^{\frac{\lambda^f}{1-\lambda^f}})^{\frac{1}{\lambda^f}} dj_f \right\}^{\lambda^f} Y_t P_t^{\frac{\lambda^f}{\lambda^f - 1}}, \\ P_t = & \left(\int_0^1 P_{j_f,t}^{\frac{1}{1-\lambda^f}} \right)^{1-\lambda^f}. \end{aligned} \tag{121}$$

I need to express total output by firms $Y_t = \int_0^1 Y_{j_f,t} dj_f$ by total demand for output. Remember, that prices for production factors in the model are identical for all firms. Under the assumption of identical production functions, all firms use the same production factor ratios.

$$\int_0^1 Y_{j_f,t} dj_f = \epsilon_t \begin{cases} \epsilon_t \left(\frac{M_{j_f,t}}{z_t\, l_{j_f,t}}\right)^{\alpha_M} \; z_t \int_{j_f=0}^1 l_{j_f,t} dj_f - \phi_t z_t & \text{if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M \left(\frac{M_{j_f,t}}{z_t\, l_{j_f,t}}\right)^{\frac{\eta^M-1}{\eta^M}} + (1-\alpha_M)(1)^{\frac{\eta^M-1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M-1}} z_t h_t - \phi_t z_t & \text{else.} \end{cases} \tag{122}$$

$$\int_0^1 Y_{j_f,t} dj_f = \begin{cases} \epsilon_t\, M_t^{\alpha_M} \; (z_t l_t)^{1-\alpha_M} - \phi_t z_t & \text{if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M^{\frac{1}{\eta^M}} \; M_t^{\frac{\eta^M-1}{\eta^M}} + (1-\alpha_M)^{\frac{1}{\eta^M}} (z_t\, l_t)^{\frac{\eta^M-1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{else.} \end{cases} \tag{123}$$

Using the demand for individual products of intermediate goods-producing firms I derive

$$\int_0^1 Y_{j_f,t} dj_f = Y_t \int_{j_f=0}^1 \left(\frac{P_{j_f,t}}{P_t}\right)^{\frac{\lambda^f}{1-\lambda^f}} dj_f. \tag{124}$$

$$Y_t = \left(\int_0^1 Y_{j_f,t} dj_f\right) \int_{j_f=0}^1 \left(\frac{P_{j_f,t}}{P_t}\right)^{\frac{\lambda^f}{\lambda^f-1}} dj_f. \tag{125}$$

I can write the current price dispersion level P_t^* as a function of prices set optimally in t and the ones which have to stick with the old prices. Due to the Calvo assumption only a share of $1-\xi^p$ can reset their prices, the others have to stick to the old prices.

$$P_t^* = \left(\int_0^1 P_{j_f,t}^{\frac{\lambda^f}{1-\lambda^f}}\right)^{\frac{1-\lambda^f}{\lambda^f}}, \tag{126}$$

$$P_t^* = \left\{\xi^p \left(\tilde{\Pi}_t P_{t-1}^*\right)^{\frac{\lambda^f}{1-\lambda^f}} + (1-\xi^p)\tilde{P}_t^{\frac{\lambda^f}{1-\lambda^f}}\right\}^{\frac{1-\lambda^f}{\lambda^f}},$$

$$p_t^* = \left\{\xi^p \left(\frac{\tilde{\Pi}_t}{\Pi_t} p_{t-1}^*\right)^{\frac{\lambda^f}{1-\lambda^f}} + (1-\xi^p)\tilde{p}_t^{\frac{\lambda^f}{1-\lambda^f}}\right\}^{\frac{1-\lambda^f}{\lambda^f}}. \tag{127}$$

Here I define $p_t^* = \frac{P_t^*}{P_t}$ and $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$ and use again the homogeneity of degree one. The current price level is a weighted average over optimally set prices and the price level of the past.

Note that $\frac{P_{t-1}^*}{P_t} = \frac{1}{\Pi_t} p_{t-1}^*$. The firms which can not optimize their prices, set them according to $P_t = \tilde{\Pi}_t P_{t-1}$, where $\tilde{\Pi}_t = (\Pi_t^*)^{\iota} \Pi_{t-1}^{1-\iota}$.

Now I can use the price dispersion index derived above to express total demand for the final good as a function of the price dispersion and the production factors.

$$Y_t = p_t^{*\frac{\lambda^f}{\lambda^f-1}} \begin{cases} \epsilon_t\, M_t^{\alpha_M}\ (z_t l_t)^{1-\alpha_M} - \phi_t z_t & \text{if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M^{\frac{1}{\eta^M}}\ M_t^{\frac{\eta^M-1}{\eta^M}} + (1-\alpha_M)^{\frac{1}{\eta^M}} (z_t\, l_t)^{\frac{\eta^M-1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{else.} \end{cases}$$

F.1.3 Intermediate goods producers

Let us turn to the optimization problem of the firms facing monopolistic competition. They seek to maximize

$$\max_{\tilde{P}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} (P_{j_f,t+\kappa} Y_{j_f,t+\kappa} - S_{t+\kappa} Y_{j_f,t+\kappa}), \tag{128}$$

$$\text{s.t.} Y_{j_f,t+\kappa} = Y_{t+\kappa} \left(\frac{P_{j_f,t+\kappa}}{P_{t+\kappa}}\right)^{-\frac{\lambda^f}{\lambda^f-1}}, \tag{129}$$

$$P_{j_f,t+\kappa} = \tilde{\Pi}_{t,t+\kappa} \tilde{P}_t. \tag{130}$$

Firms optimizing their prices consider future states in which they are not able to reset their prices. Therefore they take into account that an optimal price set today $\tilde{P}_t$ might be effective forever.

Consider the fraction of prices $\frac{P_{j_f,t+\kappa}}{P_{t+\kappa}}$, I can plug in (130) in this expression to obtain

$$\frac{P_{j_f,t+\kappa}}{P_{t+\kappa}} = \frac{\tilde{\Pi}_{t,t+\kappa}\tilde{P}_t}{P_{t+\kappa}}.$$

Furthermore use $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$ and manipulate the price fraction such that

$$\left(\tilde{\Pi}_{t,t+\kappa}\right) \tilde{p}_t \frac{P_t}{P_{t+\kappa}} = \frac{\tilde{\Pi}_{t,t+\kappa}}{\Pi_{t,t+\kappa}} \tilde{p}_t. \tag{131}$$

Note that $\frac{P_{t+\kappa}}{P_t} = \Pi_{t,t+\kappa} = \prod_{h_\kappa=0} \Pi_{t+h_\kappa}$. For the following define $X_{t,t+\kappa} = \frac{\tilde{\Pi}_{t,t+\kappa}}{\Pi_{t,t+\kappa}}$. Now take the first derivative of (128) w.r.t $\tilde{P}_t$ set it to zero and make use of the envelope theorem.

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \left\{ \left(\tilde{\Pi}_{t,t+\kappa}\tilde{P}_t - S_{t+\kappa}\right) \frac{dY_{j_f,t+\kappa}}{d\tilde{P}_t} + \tilde{\Pi}_{t,t+\kappa} Y_{j_f,t+\kappa} \right\}. \tag{132}$$

I know that only $Y_{j_f,t+\kappa}$ and $P_{j_f,t+\kappa}$ depend on $\tilde{P}_t$. It is therefore necessary to find the first derivative for these variables w.r.t. $\tilde{P}_t$. For $\frac{dP_{j_f,t+\kappa}}{d\tilde{P}_t}$ this is trivial and equals $\tilde{\Pi}_{t,t+\kappa}$.

The first derivative is

$$
\begin{aligned}
\frac{dY_{j_f,t+\kappa}}{d\tilde{P}_t} &= Y_{t+\kappa}P_{t+\kappa}^{\frac{\lambda^f}{\lambda^f-1}}\frac{-\lambda^f}{\lambda^f-1}\tilde{P}_t^{\frac{-\lambda^f}{\lambda^f-1}-1}, \\
\frac{dY_{j_f,t+\kappa}}{d\tilde{P}_t} &= \frac{-\lambda^f}{\lambda^f-1}\frac{Y_{j_f,t+\kappa}}{\tilde{P}_t}. \qquad (133)
\end{aligned}
$$

Now plug in (133) into (132) to obtain

$$
\begin{aligned}
0 &= \mathrm{E}_t\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}\left\{\tilde{\Pi}_{t,t+\kappa}\frac{-1}{\lambda^f-1}Y_{j_f,t+\kappa}+\frac{\lambda^f}{\lambda^f-1}\frac{Y_{j_f,t+\kappa}}{\tilde{P}_t}S_{t+\kappa}\right\}, \\
0 &= \mathrm{E}_t\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}\left\{\tilde{\Pi}_{t,t+\kappa}Y_{j_f,t+\kappa}-\lambda^f\frac{Y_{j_f,t+\kappa}}{\tilde{P}_t}S_{t+\kappa}\right\}. \qquad (134)
\end{aligned}
$$

Use (129) to rearrange (134).

$$
0 = \mathrm{E}_t\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}\left\{\tilde{\Pi}_{t,t+\kappa}\frac{-1}{\lambda^f-1}Y_{j_f,t+\kappa}+\frac{\lambda^f}{\lambda^f-1}\frac{Y_{j_f,t+\kappa}}{\tilde{P}_t}S_{t+\kappa}\right\}, \qquad (135)
$$

$$
0 = \mathrm{E}_t\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}\left\{\tilde{\Pi}_{t,t+\kappa}Y_{j_f,t+\kappa}-\lambda^f\frac{Y_{j_f,t+\kappa}}{\tilde{P}_t}S_{t+\kappa}\right\}, \qquad (136)
$$

$$
0 = \mathrm{E}_t\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}Y_{t+\kappa}P_{t+\kappa}\left\{(X_{t,t+\kappa}\tilde{p}_t)^{\frac{-1}{\lambda^f-1}}-\lambda^f s_{t+\kappa}(X_{t,t+\kappa}\tilde{p}_t)^{\frac{-\lambda^f}{\lambda^f-1}}\right\}. \qquad (137)
$$

In the above derivation I made use of several simplifications to obtain the last align. To get from (137) to (135) use the demand constraint and take $P_{t+\kappa}$ and $Y_{t+\kappa}$ out of the parentheses. Therefore you get the real marginal cost $s_{t+\kappa}=S_{t+\kappa}/P_{t+\kappa}$. For the first part of the sum I use (131). Now solve for $\tilde{p}_t$ to obtain the following fraction

$$
\tilde{p}_t = \mathrm{E}_t\frac{\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}Y_{t+\kappa}P_{t+\kappa}\lambda^f s_{t+\kappa}(X_{t,t+\kappa})^{\frac{-\lambda^f}{\lambda^f-1}}}{\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa\lambda_{t+\kappa}Y_{t+\kappa}P_{t+\kappa}(X_{t,t+\kappa})^{\frac{-1}{\lambda^f-1}}}.
$$

Define auxiliary expressions for the numerator $K_{p,t}$ and denominator $F_{p,t}$ of (138). Derive the law of motions for these two. For the auxiliary expression $F_{p,t}$ the law of motion is derived by

$$F_{p,t} = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} Y_{t+\kappa} P_{t+\kappa} (X_{t,t+\kappa})^{\frac{-1}{\lambda^f - 1}}, \tag{138}$$

$$F_{p,t} = \lambda_t Y_t P_t + \mathrm{E}_t\, \beta\xi^p \,(X_{t,1})^{\frac{1}{1-\lambda^f}}\, F_{p,t+1}, \tag{139}$$

$$F_{p,t} = \lambda_t Y_t P_t + \mathrm{E}_t\, \beta\xi^p \left(\frac{\tilde{\Pi}_{t+1}}{\Pi_{t+1}}\right)^{\frac{1}{1-\lambda^f}} F_{p,t+1}. \tag{140}$$

Analogously the law of motion for $K_{p,t}$ is

$$K_{p,t} = \lambda_t Y_t P_t s_t \lambda^f + \mathrm{E}_t\, \beta\xi^p \left(\frac{\tilde{\Pi}_{t+1}}{\Pi_{t+1}}\right)^{\frac{\lambda^f}{1-\lambda^f}} K_{p,t+1}. \tag{141}$$

These two law of motions are used for the simulation and estimation of the model in Dynare. Therefore (127), (140) and (141) are entering the equilibrium conditions of the model.

Further, I know that the price index is a weighted average of optimal prices and not reset prices. I can derive the following relationship between numerator and denominator.

$$K_t^p = \left\{ 1 - \xi^p \left(\frac{\tilde{\Pi}_t}{\Pi_t}\right)^{1-\lambda^f} \right\}^{\frac{1}{1-\lambda^f}} F_t^p. \tag{142}$$

In contrast to Christiano et al. (2014), one can differentiate between the mark-up charged by a firm $\lambda_{f,t}$ over marginal cost and the elasticity of substitution between intermediate goods to produce final goods λ_f. This modification affects the law of motion of the price dispersion index. It is only possible to reformulate the price dispersion index recursively, assuming time-invariant elasticities of substitution.

The inflation adjustment rule (127), the law of motion for the denominator of the optimal price (140), the law of motion for the numerator of the optimal price (141), the relationship between numerator and denominator (142), and the price dispersion index (127) enter the model.

F.1.4 Labour contractor

After the unions negotiated for each type of labour $h_{j_l,t}$ the wages $W_{j_l,t}$, the labour contractor has to decide how much labour is supplied $l_t = (\int_0^1 h_{j_l,t}^{\frac{1}{\lambda^w}} dj_l)^{\lambda^w}$. Therefore a similar problem as for the final goods producer has to be solved. Here the optimization problem is the following

$$\max_{h_{j_l,t}} W_t l_t - \int_0^1 W_{t,j_l} h_{t,j_l} dj_l,$$

$$\max_{h_{j_l,t}} W_t \left(\int_0^1 h_{j_l,t}^{\frac{1}{\lambda^w}} dj_l \right)^{\lambda^w} - \int_0^1 W_{t,j_l} h_{j_l,t} dj_l. \tag{143}$$

(143) is a typical static profit optimization problem. The FOC condition is

$$0 = W_t l_t^{\frac{\lambda^w - 1}{\lambda^w}} h_{j_l,t}^{\frac{1-\lambda^w}{\lambda^w}} - W_{j_l,t}. \tag{144}$$

Now I can obtain an expression for the demanded labour $h_{j_l,t}$ of the different types relative to the total supplied labour h_t. Therefore solve (144) for $h_{j_l,t}$ to obtain

$$h_{j_l,t} = l_t \left(\frac{W_{j_l,t}}{W_t} \right)^{\frac{\lambda^w}{1-\lambda^w}}. \tag{145}$$

This labour demand function for each type can be used to express the current wage level W_t as a function of the different wages for the different labour types $W_{j_l,t}$. Plug (145) in $l_t = (\int_0^1 h_{j_l,t}^{\lambda^w} dj_l)^{\frac{1}{\lambda^w}}$ to obtain

$$l_t = \left[\int_0^1 \left\{ \left(\frac{W_{j_l,t}}{W_t} \right)^{\frac{\lambda^w}{1-\lambda^w}} l_t \right\}^{\frac{1}{\lambda^w}} dj_l \right]^{\lambda^w}, \tag{146}$$

$$W_t^{\frac{\lambda^w}{1-\lambda^w}} = \int_0^1 W_{j_l,t}^{\frac{\lambda^w}{1-\lambda^w}} dj_l, \tag{147}$$

$$W_t = \left(\int_0^1 W_{j_l,t}^{\frac{1}{1-\lambda^w}} dj_l \right)^{1-\lambda^w}. \tag{148}$$

Now one can derive an expression for the aggregate wage level depending on the different wages for the different labour types. Analogously to the price-setting problem only a fraction of unions ξ^w is allowed to reset their prices in period t. If they reset their prices in period t, all unions set their prices to the optimal wage $\tilde{W}_t$. The share $1 - \xi^w$ of unions have to reset their wages according to the following rule

$$W_{j_l,t} = \tilde{\Pi}_{w,t} (\mu_{z,t})^{\iota_\mu} (\mu_z)^{1-\iota_\mu} W_{t-1},$$

$$\tilde{\Pi}_{w,t} = (\Pi_t^{target})^{\iota_w} (\Pi_{t-1})^{1-\iota_w} {\mu_{t-1}^z}^{\iota_\mu} \mu^{z \iota_\mu}, \tag{149}$$

$$\Pi_t^w = \Pi_t \, \mu_t^z. \tag{150}$$

Define $\tilde{\Pi}_{t,t+\kappa}^w = \prod_{h=0}^{\kappa} \tilde{\Pi}_{t,t+h_\kappa} (\mu_{z,t+h_\kappa})^{\iota_\mu} (\mu_z)^{1-\iota_\mu}$ for further computations this will be useful. As for the intermediate firms wee need to derive a relationship between total homogenous hours supplied l_t and total hours worked $h_t = \int_0^1 h_{j_l,t} dj_l$. As for the intermediate firms unions can optimize their wages with probability ξ^w. Therefore the current wage dispersion level can be expressed as

$$W_t^* = \left[(1-\xi^w)\tilde{W}_t^{\frac{\lambda^w}{1-\lambda^w}} + \xi^w \left\{ x_t^w W_{t-1}^* \right\}^{\frac{\lambda^w}{1-\lambda^w}} \right]^{\frac{1-\lambda^w}{\lambda^w}} . \tag{151}$$

Now divide the whole expression (151) by W_t and I get

$$w_t^* = \left[(1-\xi^w)\tilde{w}_t^{\frac{\lambda^w}{1-\lambda^w}} + \xi^w \left\{ \frac{x_t^w}{\pi_t^w} w_{t-1}^* \right\}^{\frac{\lambda^w}{1-\lambda^w}} \right]^{\frac{1-\lambda^w}{\lambda^w}} . \tag{152}$$

Here I define $w_t^* = W_t^*/W_t$ and $\tilde{w}_t = \tilde{W}_t/W_t$.

F.1.5 Unions

Now one can turn to the optimization problem of the unions. They face similar to the intermediate goods producers monopolistic competition. Nevertheless, the unions are representing the households. Therefore they maximize the wage bill less the associated disutility to work. Their objective is

$$\max_{\tilde{W}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^w)^\kappa \left[\lambda_{t+\kappa} \tilde{W}_t \tilde{\Pi}_{t,t+\kappa}^w h_{j_l,t+\kappa}(1-\tau_{t+\kappa}^l) - \psi_L \frac{h_{j_l,t+\kappa}^{1+\sigma_L}}{1+\sigma_L} \right], \tag{153}$$

$$\text{s.t.} h_{j_l,t+\kappa} = l_{t+\kappa} \left(\frac{\tilde{\Pi}_{t,t+\kappa}^w \tilde{W}_t}{W_{t+\kappa}} \right)^{\frac{\lambda^w}{1-\lambda^w}} . \tag{154}$$

The objective function (153) is the maximization of the wage bill and minimizing the disutility to work. Here the discounted net wage bill $\tilde{W}_t x_{t,t+\kappa}^w h_{j_l,t+\kappa}(1-\tau_{t+\kappa}^l)$ expressed in utility terms $\lambda_{t+\kappa}$ is the revenue and the costs are the dis-utility to labour. Similar to the intermediate goods producer the unions have to consider the demand for their labour captured by the constraint (153).

Lets derive the FOC of the above optimization problem. This is done analogously as for the intermediate good producer. The FOC reads

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \Bigg[\lambda_{t+\kappa} x^w_{t,t+\kappa}\, h_{jl,t+\kappa}\,(1-\tau^l_{t+\kappa}) + \frac{\lambda^w}{1-\lambda^w}\,\lambda_{t+\kappa}\, x^w_{t,t+\kappa} h_{jl,t+\kappa}\,(1-\tau^l_{t+\kappa}) \ldots \\ -\psi_L \frac{\lambda^w}{1-\lambda^w} \frac{h^{1+\sigma_L}_{jl,t+\kappa}}{\tilde{W}_t} \Bigg], \tag{155}$$

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \Bigg[\lambda_{t+\kappa} x^w_{t,t+\kappa} \tilde{W}_t \left(\frac{x^w_{t,t+\kappa}\tilde{W}_t}{W_{t+\kappa}} \right)^{\frac{\lambda^w}{1-\lambda^w}} l_{t+\kappa}(1-\tau^l_{t+\kappa}) \ldots \\ -\lambda^w \psi_L \left(\frac{x^w_{t,t+\kappa}\tilde{W}_t}{W_{t+\kappa}} \right)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l^{1+\sigma_L}_{t+\kappa} \Bigg], \tag{156}$$

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \Bigg[\lambda_{t+\kappa} \Pi^w_{t,t+\kappa} W_t (X^w_{t,t+\kappa}\tilde{w}_t)^{\frac{1}{1-\lambda^w}} l_{t+\kappa}(1-\tau^l_{t+\kappa}) \\ -\lambda^w \psi_L (X^w_{t,t+\kappa}\tilde{w}_t)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l^{1+\sigma_L}_{t+\kappa} \Bigg]. \tag{157}$$

The FOC (157) is obtained by plugging in the demand constraint (154) in (155), rescale by W_t and define $X^w_{t,t+\kappa} = \frac{x^w_{t,t+\kappa}}{\prod_{h=0}^{\kappa} \Pi^w_{t+h}}$. I can now solve for $\tilde{w}_t$. Therefore divide (157) by $\tilde{w}_t^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)}$ and obtain

$$\tilde{w}_t^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} = \mathrm{E}_t \frac{\sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda^w \psi_L (X^w_{t,t+\kappa})^{\frac{\lambda^w}{1-\lambda^w}} l_{t+\kappa}(1+\sigma_L)}{\sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \Pi^w_{t,t+\kappa} W_t (X^w_{t,t+\kappa})^{\frac{1}{1-\lambda^w}} l_{t+\kappa}(1-\tau^l_{t+\kappa})}, \tag{158}$$

$$\tilde{w}_t = \mathrm{E}_t \left\{ \frac{\sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda^w \psi_L\, (X^w_{t,t+\kappa})^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l^{1+\sigma_L}_{t+\kappa}}{\sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \Pi^w_{t,t+\kappa} W_t (X^w_{t,t+\kappa})^{\frac{1}{1-\lambda^w}} l_{t+\kappa}(1-\tau^l_{t+\kappa})} \right\}^{\frac{1-\lambda^w}{1-\lambda^w(1+\sigma_L)}}. \tag{159}$$

Now the fraction is split again in numerator K^w_t and denominator F^w_t. The law of motions is then derived analogously to the price equations. I can rewrite (159) as

$$\tilde{w}_t = \mathrm{E}_t \left(\frac{\psi_L K_{w,t}}{W_t F_{w,t}} \right)^{\frac{1-\lambda^w}{1-\lambda^w(1+\sigma_L)}}, \tag{160}$$

$$F_t^w = \frac{\lambda_t l_t (1-\tau_t^l)}{\lambda^w} + \mathrm{E}_t\, \beta \xi^w \Pi_{t+1}^w (X_{t,1}^w)^{\frac{1}{1-\lambda^w}}, \tag{161}$$

$$K_t^w = l_t^{1+\sigma_L} + \mathrm{E}_t\, \beta \xi^w (X_{t,1}^w)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l_{t+1}^{1+\sigma_L}. \tag{162}$$

The wage index in each period states an implicit relationship between the numerator K_t^w and denominator F_t^w. One can use the wage index w_t to derive.

$$1 = \left[(1-\xi^w) \tilde{w}_t^{\frac{1}{1-\lambda^w}} + \xi^w \left\{ \frac{\tilde{\Pi}_t^w}{\Pi_t^w} \right\}^{\frac{1}{1-\lambda^w}} \right]^{1-\lambda^w},$$

$$\tilde{w}_t = \left[\frac{1 - \xi^w \left\{ \frac{\tilde{\Pi}_t^w}{\Pi_t^w} \right\}^{\frac{1}{1-\lambda^w}}}{1-\xi^w} \right]^{1-\lambda^w},$$

$$K_t^w = \frac{w_t F_t^w}{\psi^L} \left[\frac{1 - \xi^w \left\{ \frac{\tilde{\Pi}_t^w}{\Pi_t^w} \right\}^{\frac{1}{1-\lambda^w}}}{1-\xi^w} \right]^{1-\lambda^w}. \tag{163}$$

Consider again the aggregated labour input $h_t = \int_0^1 h_{j_l,t} dj_l$ can also be expressed as function of homogenous labour supply l_t. I know that

$$\begin{aligned} h_t &= \int_0^1 h_{j_l,t}, \\ &= \int_0^1 l_t \left(\frac{W_{j_l,t}}{W_t}\right)^{\frac{\lambda^w}{1-\lambda^w}} dj_l, \\ &= l_t (w_t^*)^{\frac{\lambda^w}{1-\lambda^w}}. \end{aligned} \tag{164}$$

One can solve (164) and solve for l_t and plug it back in (161) and (162).

For the model the wage block consists of (149), (150) (152), (161), (162) and (163).

F.1.6 Production

Firms produce intermediate goods $Y_{j_f,t}$ using capital services $K_{j_f,t}^s$, hours of labour $h_{j_f,t}$ and oil $O_{j_f,t}$ The production function is a nested constant elasticity of substitution function. Each firm has access to the same technology and can substitute between labour and a composite production factor $M_{j_f,t}$ from capital services and oil. The production elasticity of substitution η^M determines how easy it is for firms to substitute labour for other production factors. The degree of substitution between oil and capital services is captured by the production elasticity of substitution η^O. One can further restrict the distribution parameters α_M and α_O of the CES production function in each stage, to sum up to one in contrast to the paper by Cantore et al. (2015).

$$Y_{j_f,t} = \begin{cases} \epsilon_t \, M_{j_f,t}^{\alpha_M} \, (z_t l_{j_f,t})^{\alpha_N} - \phi_t z_t & \text{, if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M{}^{\frac{1}{\eta^M}} \, M_{j_f,t}^{\frac{\eta^M-1}{\eta^M}} + \alpha_N^{\frac{1}{\eta^M}} \, (z_t \, l_{j_f,t})^{\frac{\eta^M-1}{\eta^M}} \right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{, else.} \end{cases} \tag{165}$$

$$M_{j_f,t} = \epsilon_t^M \begin{cases} \left(\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon_t^O} \right)^{\alpha_O} \left(\epsilon_t^M \, \frac{K_{j_f,t}^s}{\Upsilon^{t-1}} \right)^{\alpha_K} & \text{, if } \eta^O = 1, \\ \left\{ \alpha_K^{\frac{1}{\eta^O}} \, (\epsilon_t^M \, \frac{K_{j_f,t}^s}{\Upsilon^{t-1}})^{\frac{\eta^O-1}{\eta^O}} + \alpha_O{}^{\frac{1}{\eta^M}} \, (\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon_t^O})^{\frac{\eta^O-1}{\eta^O}} \right\}^{\frac{\eta^O}{\eta^O-1}} & \text{, else.} \end{cases} \tag{166}$$

$$\phi_t z_t = (\lambda^f - 1) \, Y_{j_f,t-4}. \tag{167}$$

$$\begin{aligned} \min_{l_{j_f,t}, K_{j_f,t}^s, O_{j_f,t}, M_{j_f,t}} \quad & W_t l_{j_f,t} + P_t \tilde{r}_t^k K_{j_f,t}^s + P_t^O O_{j_f,t}, \\ & s.t. (165), (166), \\ & \quad l_{j_f,t} > 0, K_{j_f,t}^s > 0, O_{j_f,t} > 0, M_{j_f,t} > 0. \end{aligned} \tag{168}$$

The corresponding Lagrangian of the problem is

$$\begin{aligned} \mathcal{L}_t^{\mathrm{F,min}} = & W_t \, l_{j_f,t} + P_t \tilde{r}_t^k K_{j_f,t}^s + P_t^O O_{j_f,t} + S_t \{ Y_{j_f,t} - (\mathcal{X}(M_{j_f,t}, l_{j_f,t}) - \phi z_t) \} \ldots \\ & + P_t^M \{ M_{j_f,t} - \mathcal{M}(O_{j_f,t}, K_{j_f,t}^s) \}. \end{aligned} \tag{169}$$

It is straightforward to solve (169). The FOCs are

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{l_{j_f,t}} :0 = \quad W_t - S_t z_t^{\frac{\eta^M-1}{\eta^M}} \epsilon_t (\alpha_N)^{\frac{1}{\eta^O}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{170}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{M_{j_f,t}} :0 = \quad P_t^M - (\alpha_M)^{\frac{1}{\eta^M}} (\Upsilon^{t-1})^{\rho^M} ({\epsilon^K}_t)^{\rho^M} \left(\frac{X_{j_f,t}}{M_{j_f,t}}\right)^{\frac{1}{\eta_M}}, \tag{171}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{K_{j_f,t}^s} :0 = \quad P_t \tilde{r}_t^k - P_t^M (1-\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{t-1})^{\frac{1-\eta^O}{\eta^O}} ({\epsilon^K}_t)^{\frac{\eta^O-1}{\eta^O}} \left(\frac{M_{j_f,t}}{K_{j_f,t}^s}\right)^{\frac{1}{\eta_O}}, \tag{172}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{O_{j_f,t}} :0 = \quad P_t^O - P_t^M (\alpha_O)^{\frac{1}{\eta^O}} ({\Upsilon^O}^t)^{\frac{1-\eta^O}{\eta^O}} ({\epsilon^O}_t)^{\frac{\eta^O-1}{\eta^O}} \left\{\frac{M_{j_f,t}}{O_{j_f,t}}\right\}^{\frac{1}{\eta^O}}, \tag{173}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{P_t^M} :0 = \quad M_{j_f,t} - \mathcal{M}(O_{j_f,t}, K_{j_f,t}^s), \tag{174}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{S_t} :0 = \quad X_{j_f,t} - \mathcal{X}(l_{j_f,t}, M_{j_f,t}). \tag{175}$$

I can transform the equations into stationary versions, I need to divide them with z_t and P_t.

$$w_t P_t z_t = \quad s_t P_t z_t^{\frac{\eta^M-1}{\eta^M}} \epsilon_t^{\frac{\eta^M-1}{\eta^M}} \alpha_N^{\frac{1}{\eta^M}} \left(\frac{y_{j_f,t} z_t + \phi_t\, z_t}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{176}$$

$$p_t^M P_t = \quad s_t P_t \epsilon_t^{\frac{\eta^M-1}{\eta^M}} \alpha_M^{\frac{1}{\eta^M}} \left(\frac{y_{j_f,t} z_t + \phi_t\, z_t}{m_{j_f,t} z_t}\right)^{\frac{1}{\eta^M}}, \tag{177}$$

$$\frac{r_t^k}{\Upsilon^t} = \quad p_t^M P_t {\epsilon_t^M}^{\frac{\eta^O-1}{\eta^O}} \left(\frac{1}{\Upsilon^t}\right)^{\frac{\eta^O-1}{\eta^O}} \alpha_K^{\frac{1}{\eta^O}} \left(\frac{m_{j_f,t} z_t}{\frac{u_{j_f,t}\, \bar{k}_{j_f,t} z_{t-1} \Upsilon^{t-1}}{\Upsilon^t}}\right)^{\frac{1}{\eta^O}}, \tag{178}$$

$$P_t \frac{p_t^O}{{\Upsilon^O}^t} = \quad p_t^M\, P_t \left(\frac{\epsilon_t^O}{{\Upsilon^O}^t}\right)^{\frac{\eta^O-1}{\eta^O}} \alpha_O^{\frac{1}{\eta^O}} \left\{\frac{m_{j_f,t}\, z_t}{o_{j_f,t}\, {\Upsilon^O}^t\, z_t}\right\}^{\frac{1}{\eta_O}}. \tag{179}$$

Now I need to consider the results from the previous subsections regarding the representation of $y_{j_f,t}$ and $l_{j_f,t}$ as a function of aggregate production y_t and total hours worked h_t.

$$y_t = p_t^{*\,\frac{\lambda^f}{\lambda^f-1}} \begin{cases} \epsilon_t\, m_t^{\alpha_M}\,(h_t\, w_t^{*\,\frac{\lambda^w}{\lambda^w-1}})^{\alpha_N} - \phi_t & \text{, if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M{}^{\frac{1}{\eta^M}}\, m_t^{\frac{\eta^M-1}{\eta^M}} + \alpha_N^{\frac{1}{\eta^M}}\,(h_t\, w_t^{*\,\frac{\lambda^w}{\lambda^w-1}})^{\frac{\eta^M-1}{\eta^M}} \right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{, else,} \end{cases} \tag{180}$$

$$m_t = \epsilon_t^M \begin{cases} \left((\epsilon_t^O\, o_t\right)^{\alpha_O}\; \frac{u_t \bar{k}_t}{\Upsilon \mu_t^z}^{\alpha_K} & \text{, if } \eta^O = 1, \\ \left\{ \alpha_K^{\frac{1}{\eta^O}}\,(\frac{K^s_{j_f,t}}{\Upsilon^{t-1}})^{\frac{\eta^O-1}{\eta^O}} + \alpha_O{}^{\frac{1}{\eta^M}}\,(\epsilon_t^O\, \frac{O_{j_f,t}}{\Upsilon_t^O})^{\frac{\eta^O-1}{\eta^O}} \right\}^{\frac{\eta^O}{\eta^O-1}} & \text{, else,} \end{cases} \tag{181}$$

$$\phi_t = (\lambda^f - 1)\, y_{t-4}, \tag{182}$$

$$w_t = s_t \epsilon_t{}^{\frac{\eta^M-1}{\eta^M}}\, \alpha_N{}^{\frac{1}{\eta^M}} \left(\frac{y_t\, p_t^{*\,\frac{\lambda^f}{1-\lambda^f}} + \phi_t}{h_t\, w_t^{*\,\frac{\lambda^w}{\lambda^w-1}}} \right)^{\frac{1}{\eta^M}}, \tag{183}$$

$$p_t^M = s_t \epsilon_t{}^{\frac{\eta^M-1}{\eta^M}}\, \alpha_M{}^{\frac{1}{\eta^M}} \left(\frac{y_t\, p_t^{*\,\frac{\lambda^f}{1-\lambda^f}} + \phi_t}{m_{j_f,t}} \right)^{\frac{1}{\eta^M}}, \tag{184}$$

$$r_t^k = p_t^M \epsilon_t^{M\,\frac{\eta^O-1}{\eta^O}}\, \alpha_K^{\frac{1}{\eta^O}} \left(\frac{m_t}{\frac{u_t \bar{k}_t}{\mu_t^z\, \Upsilon}} \right)^{\frac{1}{\eta^O}}, \tag{185}$$

$$p_t^O = p_t^M\, \epsilon_t^{M\,\frac{\eta^O-1}{\eta^O}}\, \left(\epsilon_t^O\right)^{\frac{\eta^O-1}{\eta^O}}\, \alpha_O^{\frac{1}{\eta^O}} \left\{ \frac{m_t}{o_t} \right\}^{\frac{1}{\eta^O}}. \tag{186}$$

Equations (180), (181), (182), (183), (184), (185), (181) and (186) are part of the model.

F.1.7 Households

Households face a typical dynamic problem to maximize their discounted present utility. They have to find optimal level of consumption $C_{j_h,t+\kappa}$. Furthermore, they can either purchase short term risk-free bonds $B_{j_h,t+\kappa}$ used by mutual funds or long term risk-free bonds $B^L_{j_h,t+\kappa}$. Households are also able to invest in capital $I_{j_h,t+\kappa}$. The dynamic optimization problem for a representative household is

$$\max_{C_{j_h,t+\kappa},B_{j_h,t+\kappa+1},B^L_{j_h,t+\kappa+4},I_{j_h,t+\kappa},I_{j_h,t+\kappa+1}} \mathrm{E}_t \sum_{\kappa=0}^{\infty} \beta^{\kappa} \Big[\Big\{ \zeta_{c,t+\kappa} \ln(C_{j_h,t+\kappa} - bC_{j_h,t+\kappa-1}) \Big\} \ldots \tag{187}$$

$$- \psi_L \int_0^1 \frac{h_{j_h,t+\kappa}(j_l)^{1+\sigma_L}}{1+\sigma_L} dj_l \Big],$$

$$\begin{aligned}
\text{s.t.}&(1+\tau^c)P_{t+\kappa}C_{j_h,t+\kappa} + B_{j_h,t+\kappa+1} + B^L_{j_h,t+\kappa+4} + \Big(\frac{P_{t+k}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}}\Big) I_{j_h,t+\kappa} + Tax_{t+\kappa} \\
=&\Delta^{O,j^d}_{t+\kappa} + \Gamma(O^{j^d}_{t+\kappa}) + (1-\tau^l)\int_0^1 W_{j_h,t+\kappa}(j_l)h_{j_h,t+\kappa}(j_l)dj_l + R_{t+\kappa}B_{t+\kappa} + (R^L_{t+\kappa})^4 B^L_{j_h,t+\kappa} \\
&+ Q_{\bar{K},t+\kappa}\bar{K}_{j_h,t+\kappa+1} - Q_{\bar{K},t+\kappa}(1-\delta)\bar{K}_{t+\kappa} + \Delta_{j_h,t+\kappa} \\
&+ (1-\Theta)(1-\gamma_{t+\kappa})\{1-\Gamma_{t+\kappa-1}(\bar{\omega}_{t+\kappa})\}R^k_{t+\kappa}Q_{\bar{K},t+\kappa-1}\bar{K}_{j_h,t+\kappa} + Tr_{j_h,t+\kappa}.
\end{aligned} \tag{188}$$

The raw capital stock evolves according to a standard law of motion. This law of motion for capital features proportional depreciations and investment adjustment costs.

$$\bar{K}_{t+\kappa+1} = (1-\delta)\bar{K}_{t+\kappa} + \{1-\mathcal{S}(\zeta_{i,t+\kappa}I_{t+\kappa}/I_{t+\kappa-1})\}I_{t+\kappa}. \tag{189}$$

At every point in time, a household maximizes utility for each variable to optimize. It is necessary to set up a Lagrangian to solve this problem. The following Lagrangian has to be solved.

$$\begin{aligned}
L^H_t =& \mathrm{E}_t \sum_{k=0}^{\infty} \beta^{\kappa} \Big[\zeta_{c,t+\kappa} \ln(C_{j_h,t+\kappa} - bC_{j_h,t+\kappa-1}) - \psi_L \int_0^1 \frac{h_{j_h,t+\kappa}(j_l)^{1+\sigma_L}}{1+\sigma_L} dj_l \Big] \\
&- \lambda_{j_h,t+\kappa}\Big\{ (1+\tau^c)P_{t+\kappa}C_{j_h,t+\kappa} + B_{j_h,t+\kappa+1} + B^L_{j_h,t+\kappa+4} + \left(\frac{P_{t+k}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}}\right) I_{j_h,t+\kappa} \\
&+ Tax_{j_h,t+\kappa} - (1-\tau^l)\int_0^1 W_{j_h,t}(j_l)h_{j_h,t+\kappa}(j_l)dj_l - R_{t+\kappa}B_{j_h,t+\kappa} - (R^L_{t+\kappa})^4 B^L_{j_h,t+\kappa} \ldots \\
&+ Q_{\bar{K},t+\kappa}\bar{K}_{j_h,t+\kappa+1} - Q_{\bar{K},t+\kappa}(1-\delta)\bar{K}_{j_h,t+\kappa} - \Delta^{O,d}_{j_h,t+\kappa} + \Gamma(O^d_t) \ldots \\
&- (1-\Theta)(1-\gamma_{t+\kappa})\{1-\Gamma_{t+\kappa-1}(\bar{\omega}_{t+\kappa})\}R^k_{t+\kappa}Q_{\bar{K},t+\kappa-1}\bar{K}_{j_h,t+\kappa} - Tr_{j_h,t+\kappa}\Big\}\Big].
\end{aligned} \tag{190}$$

One can use the standard law of motion for raw capital $\bar{K}_{t+1}$ as a function of non depreciated previous capital and former investment I_t. Here δ is the standard depreciation rate of capital and $\mathcal{S}(\zeta_{i,t+\kappa}I_{t+\kappa}/I_{t+\kappa-1})$ is a convex adjustment cost function. This function punishes either to high investment today or to low investment in the past. Consider the Lagrangian and the law of motion for capital (189). To find the optimal level of investment in each period the effect of $I_{t+\kappa}$ on $\bar{K}_{t+\kappa+1}$ and $\bar{K}_{t+\kappa+2}$ has to be considered. The FOC w.r.t to $I_{t+\kappa}$ reads

$$\frac{dL_t^H}{dI_{t+\kappa}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} \frac{P_{t+\kappa}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}} + \lambda_{t+\kappa} Q_{\bar{K},t+\kappa} \frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} - \beta\lambda_{t+\kappa+1}\left(1-\delta\right) Q_{\bar{K},t+\kappa+1} \frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} \tag{191}$$

$$+ \beta\lambda_{t+\kappa+1} Q_{\bar{K},t+\kappa+1} \frac{d\bar{K}_{t+\kappa+2}}{dI_{t+\kappa}} \Big\}.$$

The first term reflects the marginal cost for investment expressed in expected utility terms today. The second term reflects the increase in raw capital revenue in the next period, while the third term mirrors the decrease in purchase costs for raw capital two periods ahead by decreasing adjustment costs. To see this more clearly it is necessary to look at the FOCs for $\bar{K}_{t+\kappa+1}$ and $\bar{K}_{t+\kappa+2}$,

$$\frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} = 1 - S(\zeta_{i,t+\kappa} I_{t+\kappa}/I_{t+\kappa-1}) - S^{'}(\zeta_{i,t+\kappa} I_{t+\kappa}/I_{t+\kappa-1}) \frac{\zeta_{i,t+\kappa} I_{t+\kappa}}{I_{t+\kappa-1}}, \tag{192}$$

$$\frac{d\bar{K}_{t+\kappa+2}}{dI_{t+\kappa}} = \left(1-\delta\right) \frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} + S^{'}(\zeta_{i,t+\kappa+1} I_{t+\kappa+1}/I_{t+\kappa}) \zeta_{i,t+\kappa+1} \Big(\frac{I_{t+\kappa+1}}{I_{t+\kappa}} \Big)^2. \tag{193}$$

One can insert (192) and (193) in (191) to obtain the final FOC. For the final expression I use $x^I_{t+\kappa} = \frac{I_{t+\kappa}}{I_{t+\kappa-1}}$. In the following all FOCs are reported, which are then used in the model.

$$\frac{dL_t^H}{dC_{t+\kappa}} = \mathrm{E}_t \Big\{ \frac{\zeta_{c,t+\kappa}}{C_{t+\kappa} - bC_{t+\kappa-1}} - \frac{\zeta_{c,t+\kappa+1} b}{C_{t+\kappa+1} - bC_{t+\kappa}} - \lambda_{t+\kappa}(1+\tau^c) P_{t+\kappa} \Big\}, \tag{194}$$

$$\frac{dL_t^H}{dB_{t+\kappa+1}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} + \beta\lambda_{t+\kappa+1} R_{t+\kappa+1} \Big\}, \tag{195}$$

$$\frac{dL_t^H}{dB^L_{t+\kappa+4}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} + \beta^4 (\prod_{s=1}^{4} \zeta_{term,t+\kappa+s}) \lambda_{t+\kappa+4} (R^L_{t+\kappa+4})^4 \Big\}, \tag{196}$$

$$\frac{dL_t^H}{dI_{t+\kappa}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} \frac{P_{t+\kappa}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}} + \lambda_{t+\kappa} Q_{\bar{K},t+\kappa} (1 - S(\zeta_{i,t+\kappa} x^I_{t+\kappa}) \tag{197}$$

$$- \mathcal{S}^{'}(\zeta_{i,t+\kappa} x^I_{t+\kappa})) \zeta_{i,t+\kappa} x^I_{t+\kappa} + \beta\lambda_{t+\kappa+1} Q_{\bar{K},t+\kappa+1} \mathcal{S}^{'}(\zeta_{i,t+\kappa+1} x^I_{t+\kappa+1}) \zeta_{i,t+\kappa+1} (x^I_{t+\kappa+1})^2,$$

$$\frac{dL_t^H}{d\bar{K}_{t+\kappa+1}} = \mathrm{E}_t \Big\{ \lambda_{t+\kappa} Q_{\bar{K},t+\kappa} - \beta\lambda_{t+\kappa+1} Q_{\bar{K},t+\kappa+1} (1-\delta) \Big\}. \tag{198}$$

Equations (194), (195), (196), and (197) are used in all model versions. The FOC for capital (198) is not used in the risk shock model with entrepreneurs.

F.1.8 Entrepreneurs

Christiano et al. (2014) is the main source for this section. The key modification of the risk shock model compared to the classic NK-DSGE model is the introduction of entrepreneurs. To each household belongs a large number of entrepreneurs of different types. Entrepreneurs j_E purchase raw capital $\bar{K}_{t+1}$ from different households for the price $Q_{\bar{K},t-1}$. To finance these purchases, each entrepreneur has its net worth $N_{j_E,t}$ and access to loans $B_{j_E,t+1}$ from mutual funds. They purchase loans after production took place in the period t. $N_{j_E,t}$ introduces heterogeneity to entrepreneurs. One can assume that net worth $N_{j_E,t}$ in all periods satisfies the following conditions

- $N_{j_E,t} \geq 0 \quad \forall j_E,\, t,$
- $N_{j_E,t}$ has the density function $f_t(N_{j_E,t})$,
- $N_{j_E,t+1} = \int_0^\infty N_{j_E} f_t(N_{j_E}) dj_E.$

Let us consider the actions of an entrepreneur during one period. Each entrepreneur does the following actions during one period.

1. The entrepreneur purchases raw capital with the loans from mutual funds and its net worth. This leads to the following condition for each period t,

$$Q_{\bar{K},t}\bar{K}_{j_E,t+1} = N_{j_E,t} + B_{j_E,t+1}. \tag{199}$$

2. After raw capital is purchased an idiosyncratic shock hits each entrepreneur ω. This shock transforms raw capital to effective capital $K_{j_E,t+1} = \omega\bar{K}_{j_E,t+1}$. I assume that the idiosyncratic shock follows a log-normal distribution with an expectation equal to one and variance varying over time.
   - $\mathrm{E}(\omega) = 1$ and $\mathrm{Var}(\omega) = \sigma_t^2$,
   - $\log\omega \sim N(-\frac{\sigma^2}{2}, \sigma^2)$.

3. The entrepreneur has to decide how much capital services $u_{t+1}\omega\bar{K}_{j_E,t+1}$ she wants to provide at a competitive market rental rate r_{t+1}^k. Here the variable $u_{j_E,t+1}$ is the utilization rate for effective capital. The utilization of effective capital will produce costs $a(u_{t+1})$. Therefore the net revenues by capital services can be expressed as

$$\{u_{t+1}r_{t+1}^k - \tau^o a(u_{t+1})\}\omega\bar{K}_{j_E,t+1}\frac{P_{t+1}}{\Upsilon^{t+1}}(1-\tau^k). \tag{200}$$

I find the optimal level of utilization by taking the first derivative with respect to u_{t+1}. Therefore the rental rate for capital is given by

$$r^k_{t+1} = a'(u_{t+1}). \tag{201}$$

This FOC implies that optimal utilization rates are independent of the type of entrepreneur. All entrepreneurs face the same utilization costs and the same return on capital services. The utilization costs come later.

4. In $t+1$ the entrepreneurs will sell the non depreciated effective capital $(1-\delta)\omega\bar{K}_{j_E,t+1}$ to the households at price $Q_{\bar{K},t+1}$. Furthermore, it is assumed that entrepreneurs can deduct depreciated effective capital by $\delta\tau^k$ at historical costs $Q_{\bar{K}',t}$.

With the information from above, it is possible to determine the total return to effective capital in one period. Therefore one can set up the profit function for an N_{j_E} type entrepreneur. The costs for an entrepreneur purchasing raw capital from households are given by

$$C(\omega\bar{K}_{j_E,t+1}) = Q_{\bar{K},t}\omega\bar{K}_{j_E,t+1}.$$

The Revenues are given by

$$R(\omega\bar{K}_{j_E,t+1}) = (u_{t+1}r^k_{t+1} - \tau^o a(u_{t+1}))\omega\bar{K}_{j_E,t+1}\frac{P_{t+1}}{\Upsilon^{t+1}}(1-\tau^k)\omega\bar{K}_{j_E,t+1} + (1-\delta)Q_{\bar{K},t+1}\omega\bar{K}_{j_E,t+1} \\ +\delta\tau^k Q_{\bar{K}',t}\omega.$$

The total return of effective capital $1+R^k_{t+1}$ can be derived by dividing the revenues by the costs.

$$1+R^k_{t+1} = \frac{(u_{t+1}r^k_{t+1} - \tau^o a(u_{t+1}))\frac{P_{t+1}}{\Upsilon^{t+1}}(1-\tau^k) + (1-\delta)Q_{\bar{K},t+1} + \delta\tau^k Q_{\bar{K}',t}}{Q_{\bar{K},t}}. \tag{202}$$

In (202) there is no variable depending on the type of the entrepreneur. This is caused by the fact that all entrepreneurs will choose the same level of utilization. The return for raw capital is for each entrepreneur uncertain, because of the realization of ω and the return to raw capital is given by $\omega(1+R^k_{t+1})$.

The most crucial decision of an entrepreneur is about its leverage $L_t = \frac{N_{j_E,t}+B_{j_E,t+1}}{N_{j_E,t}}$. This variable expresses the expenditures for raw capital relative to the net worth of an entrepreneur. Mutual funds lend loans $B_{j_E,t+1}$ to entrepreneurs at the gross nominal rate of interest Z_{t+1}. Therefore an entrepreneur has to repay $B_{j_E,t+1}(1+Z_{t+1})$. Whether the entrepreneur is able to pay this amount depends on ω. Let $\bar{\omega}_{t+1}$ denote the threshold for the value of ω which separates entrepreneurs in insolvent and solvent ones. The total returns of effective capital are just enough to cover the loan costs, which translate into

$$(1 + R^k_{t+1})\bar{\omega}_{t+1}Q_{\bar{K},t}K_{j_E,t+1} = B_{j_E,t+1}(1 + Z_{t+1}). \tag{203}$$

It is assumed that entrepreneurs evaluate debt contracts according to their expected net worth in period $t + 1$. They will maximize

$$\begin{aligned}
&\mathrm{E}_t\Big[\int_{\bar{\omega}_{t+1}}^{\infty}\{(1 + R^k_{t+1})\omega Q_{\bar{K},t}K_{j_E,t+1} - B_{j_E,t+1}(1 + Z_{t+1})\}f(\omega)d\omega\Big] = \dots &(204)\\
&\mathrm{E}_t\{1 - \Gamma_t(\bar{\omega}_{t+1})\}(1 + R^k_{t+1})L_tN_{j_E,t},\\
&\Gamma_t(\bar{\omega}_{t+1}) = \{1 - F_t(\bar{\omega}_{t+1})\}\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1}), &(205)\\
&F_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} f_t(\omega)d\omega, &(206)\\
&G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega f_t(\omega)d\omega, &(207)\\
&L_t = \frac{Q_{\bar{K},t}\bar{K}_{j_E,t+1}}{N_{j_E,t}}.
\end{aligned}$$

I define $\Gamma_t(\bar{\omega}_{t+1})$ and $G_t(\bar{\omega}_{t+1})$ for notational purposes. To obtain the right hand side of (204) insert (203) into the left hand side. I then obtain the following

$$\mathrm{E}_t\int_{\bar{\omega}_{t+1}}^{\infty}(\omega - \bar{\omega}_{t+1})f_t(\omega,\sigma_t)d\omega(1 + R^k_{t+1})Q_{\bar{K},t}K_{j_E,t+1}\frac{N_{j_E,t}}{N_{j_E,t}} = \dots$$
$$\mathrm{E}_t\int_{\bar{\omega}_{t+1}}^{\infty}(\omega - \bar{\omega}_{t+1})f_t(\omega,\sigma_t)d\omega(1 + R^k_{t+1})L_tN_{j_E,t},$$
$$\int_{\bar{\omega}_{t+1}}^{\infty}(\omega - \bar{\omega}_{t+1})f_t(\omega,\sigma_t)d\omega = 1 - G_t(\bar{\omega}_{t+1}) - \{1 - F(\bar{\omega}_{t+1})\}\bar{\omega}_{t+1} = \{1 - \Gamma_t(\bar{\omega}_{t+1})\}.$$

Here I use the fact that $\lim_{\bar{\omega}_{t+1}\to\infty} G_t(\bar{\omega}_{t+1}) = \int_0^{\infty} f(\omega)\omega d\omega = \mathrm{E}\,\omega = 1$ and that I can arbitrary split the integral. This implies $\int_{\bar{\omega}_{t+1}}^{\infty} f(\omega)\omega d\omega = \int_0^{\infty} f(\omega)\omega d\omega - \int_0^{\bar{\omega}_{t+1}} f(\omega)\omega d\omega = 1 - G_t(\bar{\omega}_{t+1})$. Note that $1 - \Gamma_t(\bar{\omega}_{t+1})$ is the share of average entrepreneurial earnings. Here $\int_{\bar{\omega}_{t+1}}^{\infty} f(\omega)\omega d\omega$ denotes the expected value of ω conditional that $\omega \geq \bar{\omega}_{t+1}$. On the other side $\bar{\omega}_{t+1}\int_{\bar{\omega}_{t+1}}^{\infty} f(\omega)d\omega$ weights $\bar{\omega}_{t+1}$ with the probability that $\Pr(\omega \geq \bar{\omega}_{t+1})$. Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are ignored, because their net worth in $t+1$ is zero. These entrepreneurs will go bankrupt, because they are not able to repay their obligations to the mutual funds.

This raises the question how mutual funds decide how much loan they grant to entrepreneurs. It is assumed that each mutual fund holds perfectly diversified loans of portfolios to entrepreneurs with different $N_{j_E,t}$. Mutual funds get deposits from households and they have to pay them back the principal times R_t. Therefore the opportunity costs of extending loans to entrepreneurs at rate Z_{t+1} is reflected by loans granted to households. If an entrepreneur goes bankrupt the mutual fund obtains $(1-\mu)\omega(1+R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1}$. Here μ is the fraction of monitoring costs a mutual fund has to pay for knowing whether the entrepreneur is bankrupt or not. In this case the mutual fund gets all the effective capital of this entrepreneur. From solvent firms they get the promised $(1+Z_{t+1})B_{j_E,t+1}$. Due to the fact that they hold perfectly diversified portfolio and they are not allowed to discriminate a priori, they have to provide loans to every entrepreneur at the same rate of interest. A mutual fund extends loans to entrepreneurs according to

$$\{1-F_t(\bar{\omega}_{t+1})\}(1+Z_{t+1})B_{j_E,t+1}+(1-\mu)G_t(\bar{\omega}_{t+1})(1+R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1} \geq B_{j_E,t+1}(1+R_t). \tag{208}$$

(208) is the cash constraint, stating that expected earnings from lending loans to entrepreneurs must be greater or equal to the amount, which mutual funds have to repay to the households. New mutual funds do not face entry costs. It is therefore not possible for a mutual fund to make expected nonzero profits. The inequality is equality under free entry. I can simplify this expression by inserting (203) and use $\frac{B_{j_E,t+1}}{Q_{\bar{K},t}\bar{K}_{j_E,t+1}} = \frac{L_t-1}{L_t}$. The cash constraint is given by

$$\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1}) = \frac{L_t-1}{L_t}\frac{1+R_t}{1+R^k_{t+1}}. \tag{209}$$

An entrepreneur has to choose the optimal level of leverage given the realized ω according to the menu of contracts supplied by mutual funds (209). One can now set up the Lagrangian for the entrepreneur's optimization problems. The objective is given by (204) and the constraint is (209). One can obtain the following Lagrangian

$$\begin{aligned} L^E_t &= \mathrm{E}_t[\{1-\Gamma_t(\bar{\omega}_{t+1})\}(1+R^k_{t+1})L_tN_{j_E,t}\ldots \\ &+\mu^E_t\{\Gamma_t(\bar{\omega}_{t+1})-\mu\, G_t(\bar{\omega}_{t+1})-\frac{L_t-1}{L_t}\frac{1+R_t}{1+R^k_{t+1}}\}]. \end{aligned} \tag{210}$$

Here it is important to know that the entrepreneur knows $\bar{\omega}_{t+1}$ before they optimize L_t. The FOC associated with $\bar{\omega}_{t+1}$ determines the value of the Lagrange multiplier.

This multiplier is derived by

$$\frac{\partial L^E}{\partial \bar{\omega}_{t+1}} = \mathrm{E}_t[-\Gamma'_t(\bar{\omega}_{t+1})(1+R^k_{t+1})L_t N_{j_E,t} + \mu^E_t\{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})\}],$$

$$\mu^E_t = \mathrm{E}_t \frac{\Gamma'_t(\bar{\omega}_{t+1})(1+R^k_{t+1})L_t N_{j_E,t}}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}. \tag{211}$$

Now plug (211) into (210) and take the FOC w.r.t. L_t to get

$$\frac{dL^E}{dL_t} = \mathrm{E}_t\left[\{1-\Gamma_t(\bar{\omega}_{t+1})\}\frac{1+R^k_{t+1}}{1+R_t} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}\left[\frac{(1+R^k_{t+1})}{(1+R_t)}\{\Gamma_t(\bar{\omega}_{t+1})\dots\right.\right. \tag{212}$$

$$\left.\left. -\mu G_t(\bar{\omega}_{t+1})\} - 1\right]\right].$$

The standard debt contract is independent of the specific $N_{j_E,t}$ of an entrepreneur.

Note, that revenues of mutual funds $B_{t+1}\,Z_{t+1}$ are equal to $\Gamma_t(\bar{\omega}_{t+1})\,(1+R^k_{t+1})\,Q_{\bar{K},t}K_{j_E,t+1}$. Further, the cash constraint can be solved for the risk free interest rate R_t. Therefore, it is possible to express the credit spread by

$$Z_t - R_t = [\Gamma_{t-1}(\bar{\omega}_t) - \{\Gamma_{t-1}(\bar{\omega}_t) - \mu\, G_{t-1}(\bar{\omega}_t)\}]\ Q_{\bar{K},t-1}K_{j_E,t}, \tag{213}$$

$$Z_t - R_t = \mu\, G_{t-1}(\bar{\omega}_t)\, Q_{\bar{K},t-1}K_{j_E,t}.$$

This expression is used for the observational equation to estimate the model.

It is also necessary to derive the law of motion for N_{t+1}. One can define

$$V_t = \{1-\Gamma_{t-1}(\bar{\omega}_t)\}(1+R^k_t)Q_{\bar{K},t-1}\bar{K}_t, \tag{214}$$

$$V_t = [1 - \{1-F_{t-1}(\bar{\omega}_t)\}\bar{\omega}_t - G_{t-1}(\bar{\omega}_t)](1+R^k_t)Q_{\bar{K},t-1}, \tag{215}$$

$$V_t = (1+R^k_t)Q_{\bar{K},t-1} - [\{1-F_{t-1}(\bar{\omega}_t)\}\bar{\omega}_t$$

$$+(1-\mu)G_{t-1}(\bar{\omega}_t)](1+R^k_t)Q_{\bar{K},t-1} - \mu G_{t-1}(\bar{\omega}_t)(1+R^k_t)Q_{\bar{K},t-1}, \tag{216}$$

$$V_t = \{R^k_t - R_{t-1} - \mu G_{t-1}(\bar{\omega}_t)(1+R^k_t)Q_{\bar{K},t-1}\}Q_{\bar{K},t-1}\bar{K}_t + (1+R_t)N_t. \tag{217}$$

Here V_t in (214) represents the net worth of an entrepreneur minus lump sum transfers W^e_t from households and the transfers from entrepreneurs to households $1-\gamma_t$. The average share of entrepreneurial earnings received by entrepreneurs is $\{1-\Gamma_{t-1}(\bar{\omega}_t)\}$, which is multiplied by the initial amount of investment $Q_{\bar{K},t-1}\bar{K}_t$ and the total return to capital $1+R^k_t$. I plug in (205) into (214) to obtain (215). Afterwards I use the fact that mutual funds earnings are equal to the second addend in (216) or $(1+R_t)\,(Q_{\bar{K},t-1}\bar{K}_t - N_t)$, which follows from (203) and (208). Now you just rearrange terms in (216) to get to (217).

Now one can multiply this expression by the share of earnings not transferred to households γ_t and add lump-sum transfers W_t^e to get

$$N_{t+1} = \gamma_t\{R_t^k - R_{t-1} - \mu G_{t-1}(\bar{\omega}_t)(1+R_t^k)\}Q_{\bar{K},t-1}\bar{K}_t + \gamma_t(1+R_t)N_t + W^e. \quad (218)$$

Lets now take a look at the aggregates of the model. The aggregate, raw capital stock, capital services and loans extended are given by

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N)dN, \quad (219)$$

$$K_t^s = \int_0^\infty \int_0^\infty u_t \omega \bar{K}_t^N f_{t-1}(N) f(\omega, \sigma_t) d\omega dN = u_t \bar{K}_t, \quad (220)$$

$$B_{t+1} = \int_0^1 B_{t+1}^N f_t(N)dN = \int_0^1 (Q_{\bar{K},t}\bar{K}_{t+1}^N - N) f_t(N)dN = Q_{\bar{K},t}\bar{K}_{t+1} - N_{t+1}. \quad (221)$$

The following equations are used in the model: (202), (206), (207), (212), (209) and (218).

F.1.9 Monetary policy

Risk free interest rates for short-term bonds B_t are determined by the central bank. The central bank or the monetary authority is assumed to set R_t according to an interest rate rule,

$$\frac{1+R_t}{1+\bar{R}} = \left(\frac{1+R_{t-1}}{1+\bar{R}}\right)^{\tilde{\rho}} \left\{ \left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{1+\tilde{a}_\pi} \left(\frac{\mu_t^z}{\bar{\mu}^z} \frac{c_{t-1} + \frac{i_{t-1}}{\mu_{t-1}^\Upsilon} + g_{t-1}}{c_{t-2} + \frac{i_{t-2}}{\mu_{t-2}^\Upsilon} + g_{t-2}} \right)^{\tilde{a}_{\Delta y}} \right\}^{1-\tilde{\rho}} + \frac{\sigma^{x^p}}{4} x_t^p. \quad (222)$$

The specification is the same as the one used by Christiano et al. (2014), with an annual monetary policy shock x_t^p.

F.1.10 Resource constraint

The whole economy produces aggregate real output Y_t. This aggregate output consists of private real consumption C_t, government real consumption G_t, real monitoring costs by mutual funds D_t and real costs for providing capital services $a(u_t)\Upsilon^{-t}\bar{K}_t$. I can derive the following resource constraint from the budget constraint of the representative household

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + \tau_t^o a(u_t) \frac{\bar{K}_t}{\Upsilon^t} + \Theta \frac{1-\gamma_t}{\gamma_t P_t}(N_{t+1} - W^e) - P_t^O (O_t^{ex} - O_t^{im}). \tag{223}$$

Here one can use the fact that government expenditure is the sum of all lump-sum taxes Tax_t, taxes on capital, taxes on labour income, taxes on oil and less lump-sum transfers Tr_t to households and deductible taxes on capital depreciation. Profits of intermediate goods-producing firms are $\Delta_t = P_t Y_t - W_t h_t - \tilde{r}_t^k P_t u_t \bar{K}_{t-1} - P_t^O O_t$. Domestic oil-producing firms transfer profits $P_t^O O_t^d - \Gamma(O_t^d)$ to households. One can use the identity for oil consumption to replace domestic oil production O^d by domestic oil consumption O, oil exports O^{ex} and oil imports O^{im}. Domestic oil consumption expenditures will cancel out, but oil exports and imports remain in the resource constraint. In order to have monitoring costs by mutual funds and the share of net worth consumed by existing entrepreneurs, one can modify the following expressions from the budget constraint

$$B_{t+1} + Q_{\bar{K},t}(1-\delta)\bar{K}_t + W^e = (1+R_{t-1})B_t + \underbrace{Q_{\bar{K},t}\bar{K}_{t+1}}_{B_{t+1}+N_{t+1}} \dots \tag{224}$$

$$+ (1-\gamma_t)(1-\Theta)[1-\Gamma(\bar{\omega}_t)](1+R_t^k)Q_{\bar{K},t-1}\bar{K}_t \dots$$

$$+ \{r_t^K u_t - a(u_t)\} P_t \Upsilon^{-t} \bar{K}_t + (1-\delta) Q_{\bar{K},t}\bar{K}_t - (1+R_t^k) Q_{\bar{K},t-1}\bar{K}_t,$$

$$a(u_t) P_t \Upsilon^{-t}\bar{K}_t = r_t^K u_t P_t \Upsilon^{-t}\bar{K}_t + N_{t+1} - W^e + (1-\gamma_t)(1-\Theta)\frac{N_{t+1}-W^e}{\gamma_t} \tag{225}$$

$$- \underbrace{((1+R_t^k) Q_{\bar{K},t-1}\bar{K}_t - (1+R_{t-1})B_t)}_{\frac{1}{\gamma_t}(N_{t+1}-W^e)},$$

$$a(u_t) P_t \Upsilon^{-t}\bar{K}_t = N_{t+1} - W^e + r_t^K u_t P_t \Upsilon^{-t}\bar{K}_t + (1-\gamma_t)(1-\Theta)\frac{N_{t+1}-W^e}{\gamma_t} \tag{226}$$

$$- \underbrace{((1+R_t^k) Q_{\bar{K},t-1}\bar{K}_t - (1+R_{t-1})B_t)}_{\frac{1}{\gamma_t}(N_{t+1}-W^e)+D_t} + \Theta(1-\gamma_t)\frac{N_{t+1}-W^e}{\gamma_t}.$$

The real monitoring costs D_t is the share of earnings of entrepreneurs spent for monitoring relative to the present price level,

$$D_t = \mu G_{t-1}(\bar{\omega}_t)(1+R_t^k)\frac{Q_{\bar{K},t-1}\bar{K}_t}{P_t}. \tag{227}$$

I assume that government expenditures is the product of z_t and g_t,

$$G_t = z_t g_t. \tag{228}$$

Further I know that oil imports and oil exports have a different trend and I assume that government expenditures is the product of z_t and g_t,

$$O_t = z_t \Upsilon^{O^t} o_t, \tag{229}$$

$$O_t^d = z_t \Upsilon^{O^t} o_t^d. \tag{230}$$

F.1.11 Utilization costs

I assume the following cost function for the utilization of effective capital into capital services $a(u_t)$. This function is given by

$$a(u_t) = r^k \{\exp(\sigma_a(u-1)) - 1\} \frac{1}{\sigma_a}, \tag{231}$$

where $\sigma_a > 0$ and r^k is the steady-state rental rate of capital. In steady-state $u = 1$ by the definition of $a(u)$. To see this just consider the first derivative of (232) set to zero. Here I get

$$a^{'}(u_t) = r^k \{\exp(\sigma_a(u-1))\} = 0 \iff u = 1. \tag{232}$$

The steady-state level of u is independent of r^k.

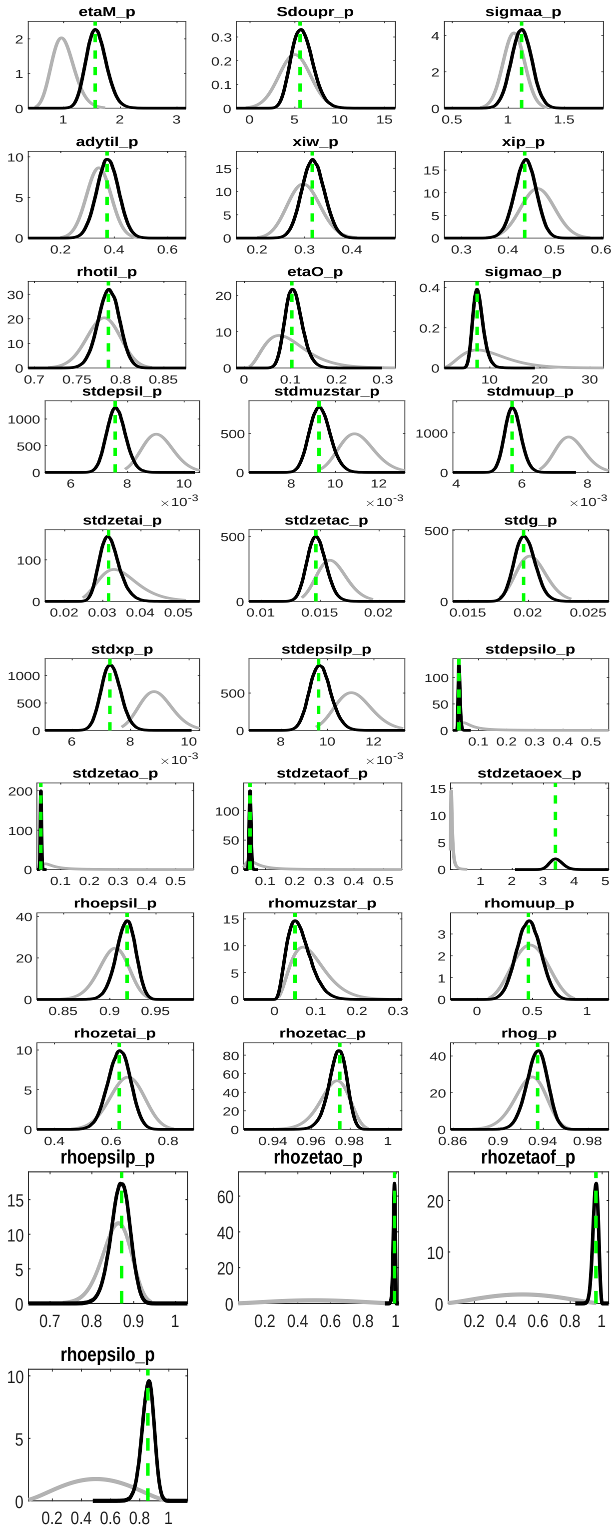
F.1.12 Investments adjustment costs

One can model the adjustment costs for investment such that the global minimum appears if investment today is equal to investment from yesterday. If this ratio is greater or smaller than in steady-state, the adjustment costs will increase. I therefore formulate the following adjustment cost function

$$S(\zeta_{I,t} x_t^I) = \frac{1}{2}[\exp\{\sqrt{S''}(\zeta_{I,t} x_t^I - \zeta_I x^I)\} + \exp\{-\sqrt{S''}(\zeta_{I,t} x_t^I - \zeta_I x^I)\} - 2]. \tag{233}$$

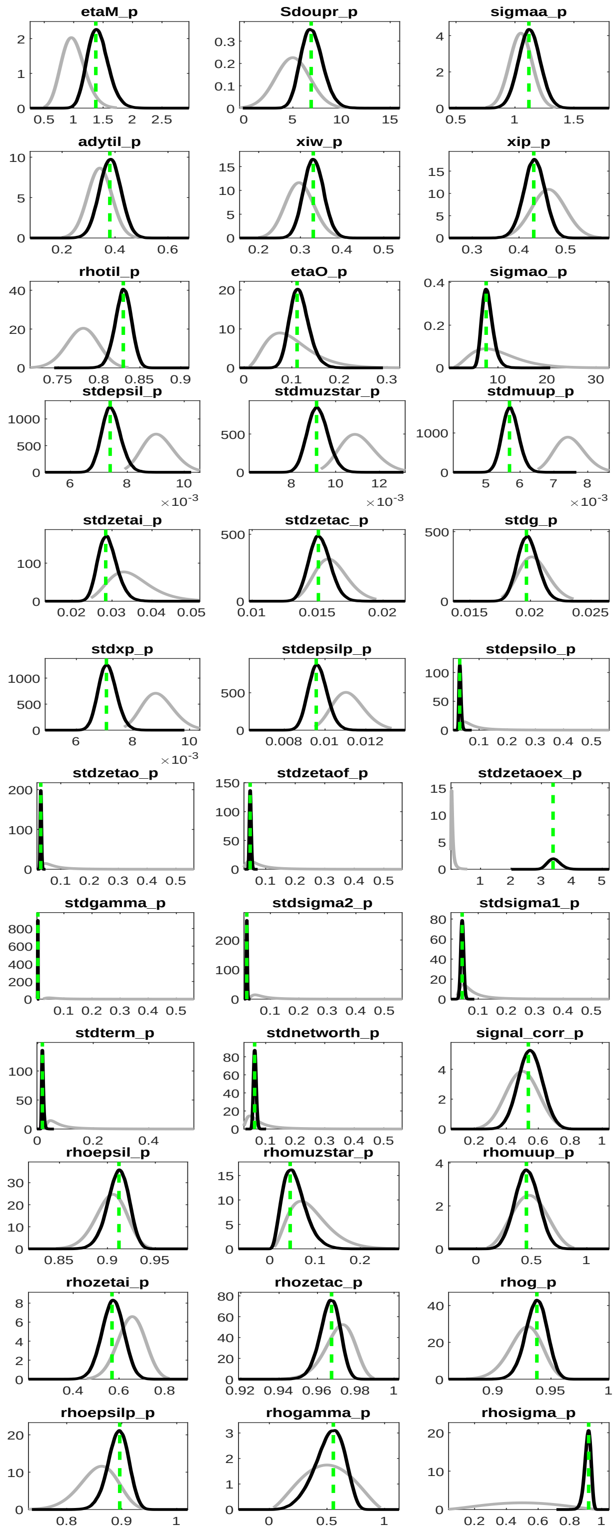
F.2 Figures

Figure 14: Priors and posteriors CEE-Oil



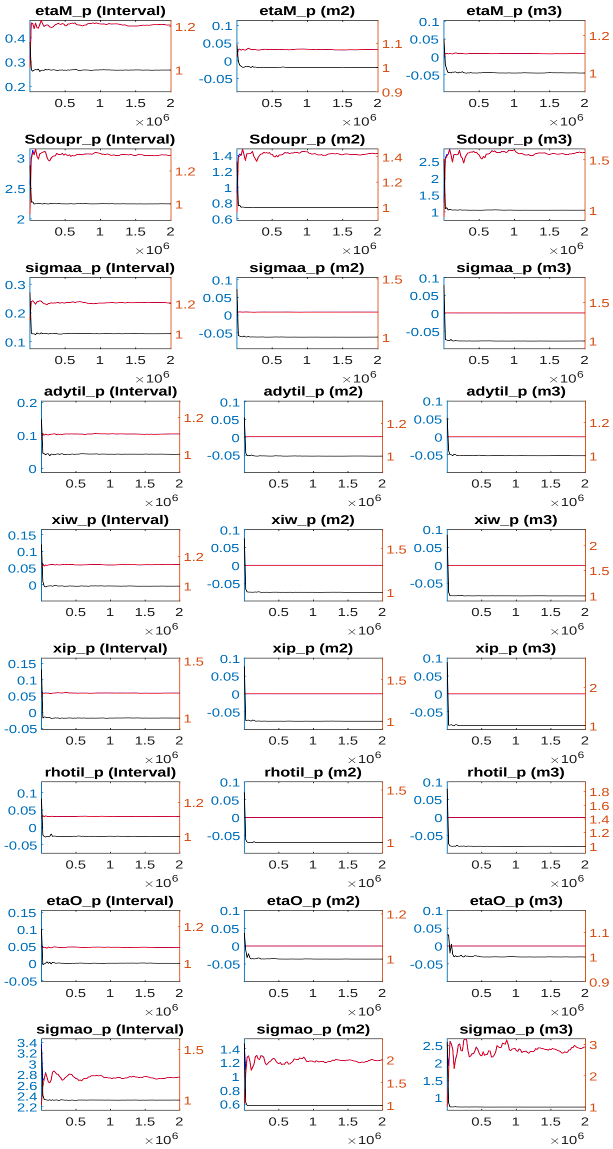
Notes: The grey line depicts the prior density and the black line the posterior density. The posterior mode is depicted by the green dashed line.

Figure 15: Priors and posteriors CMR-Oil



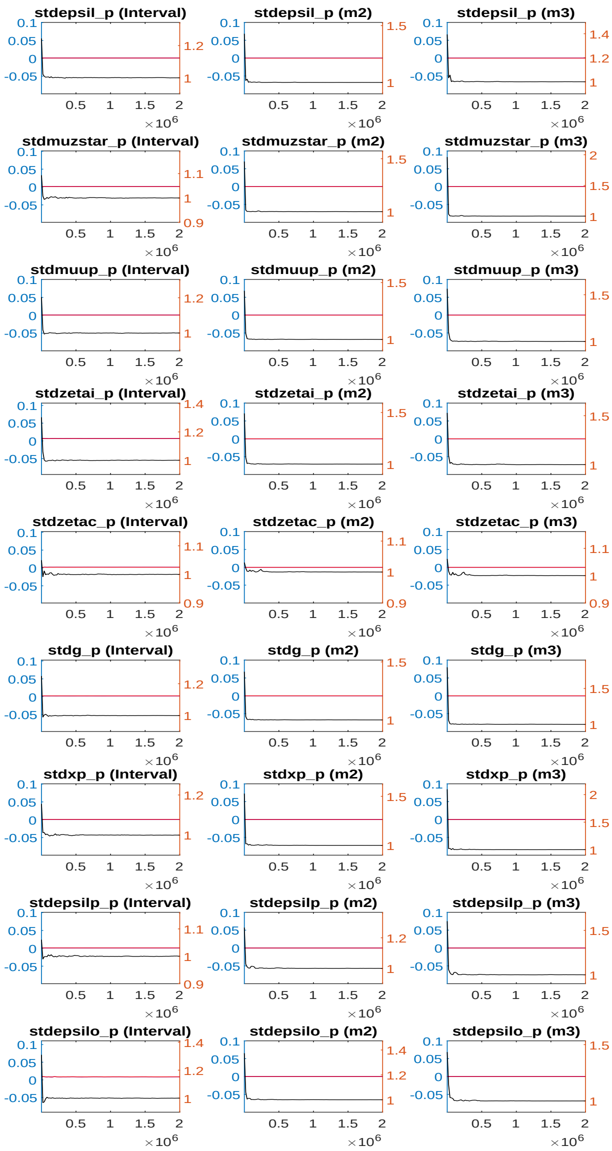
Notes: The grey line depicts the prior density and the black line the posterior density. The posterior mode is depicted by the green dashed line.

Figure 16: Parameter convergence CEE-Oil



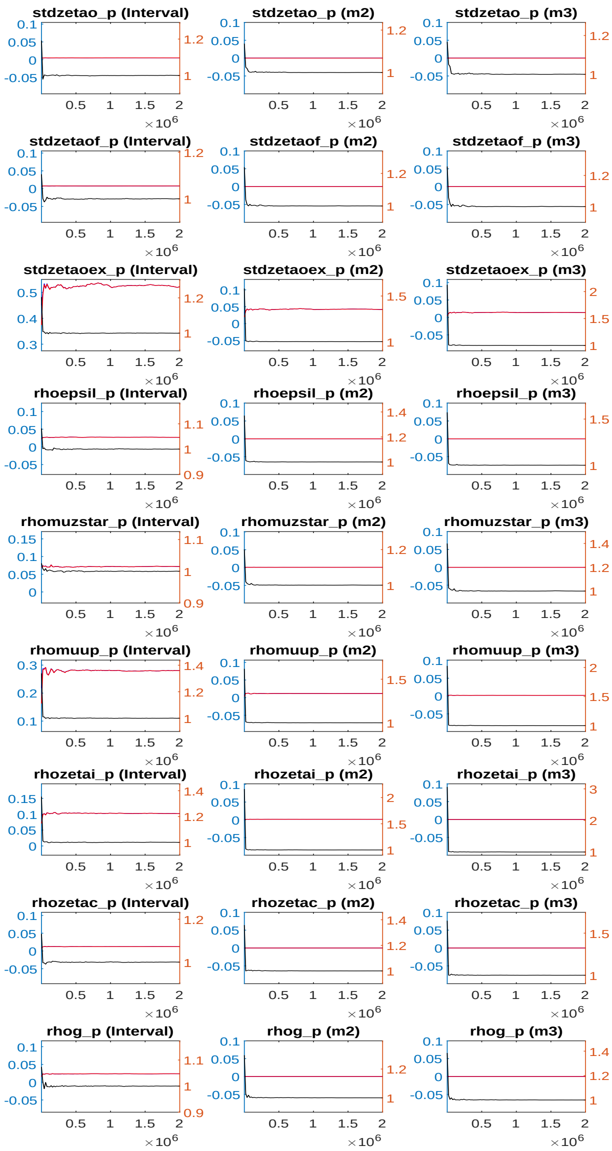
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 17: Parameter convergence CEE-Oil II



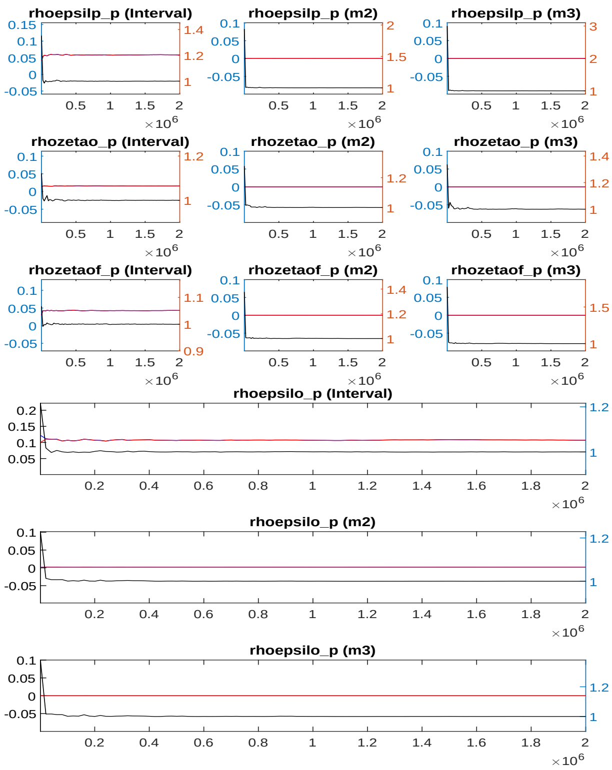
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 18: Parameter convergence CEE-Oil III



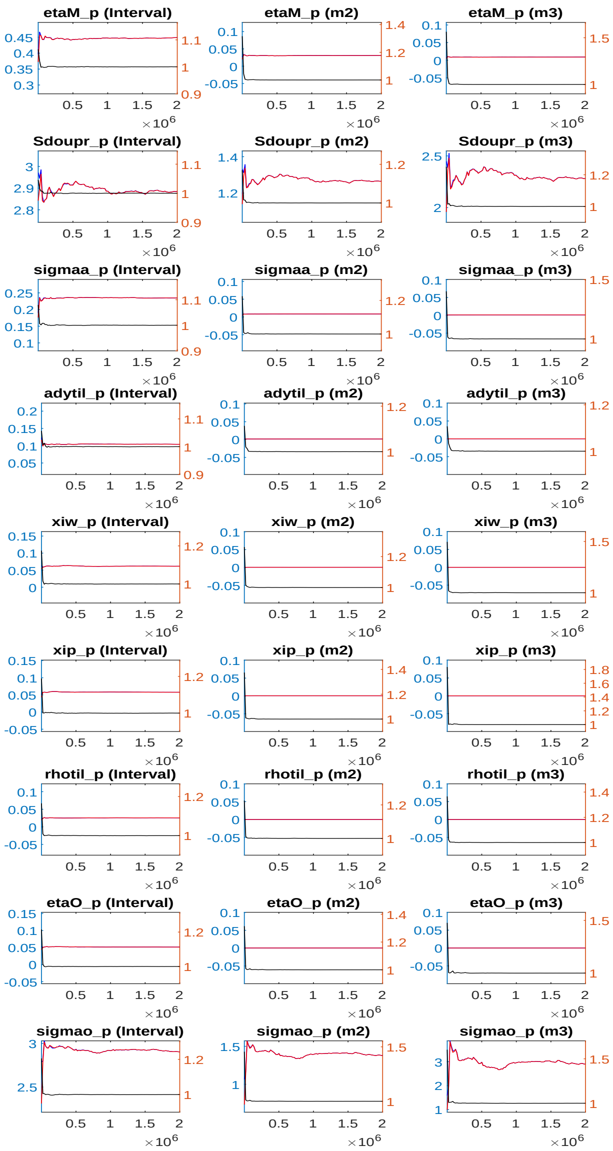
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 19: Parameter convergence CEE-Oil IV



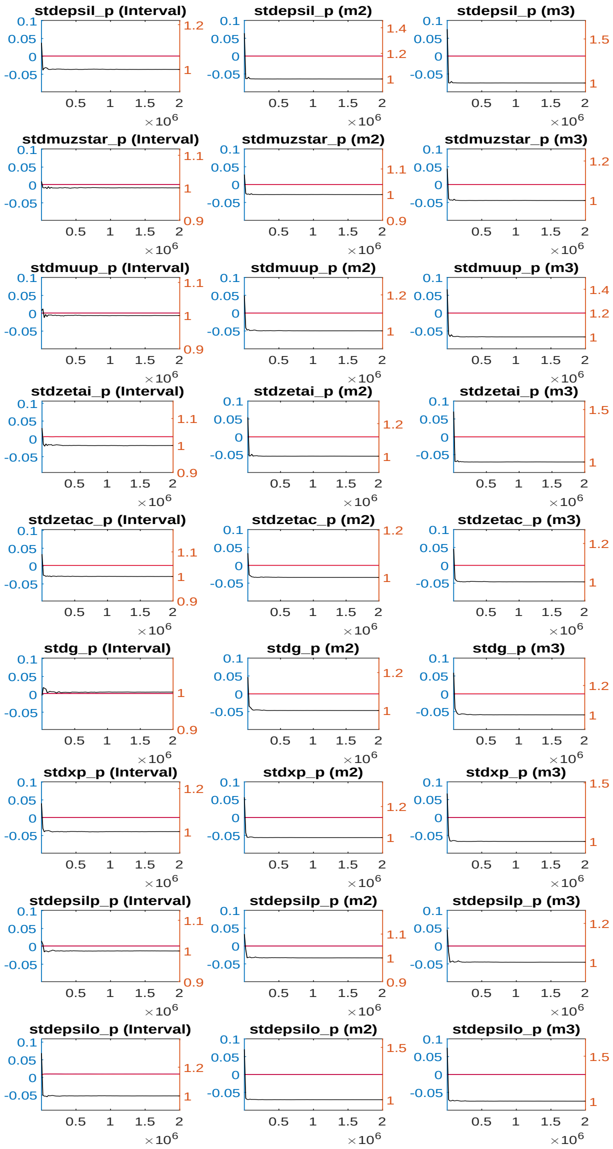
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 20: Parameter convergence CMR-Oil



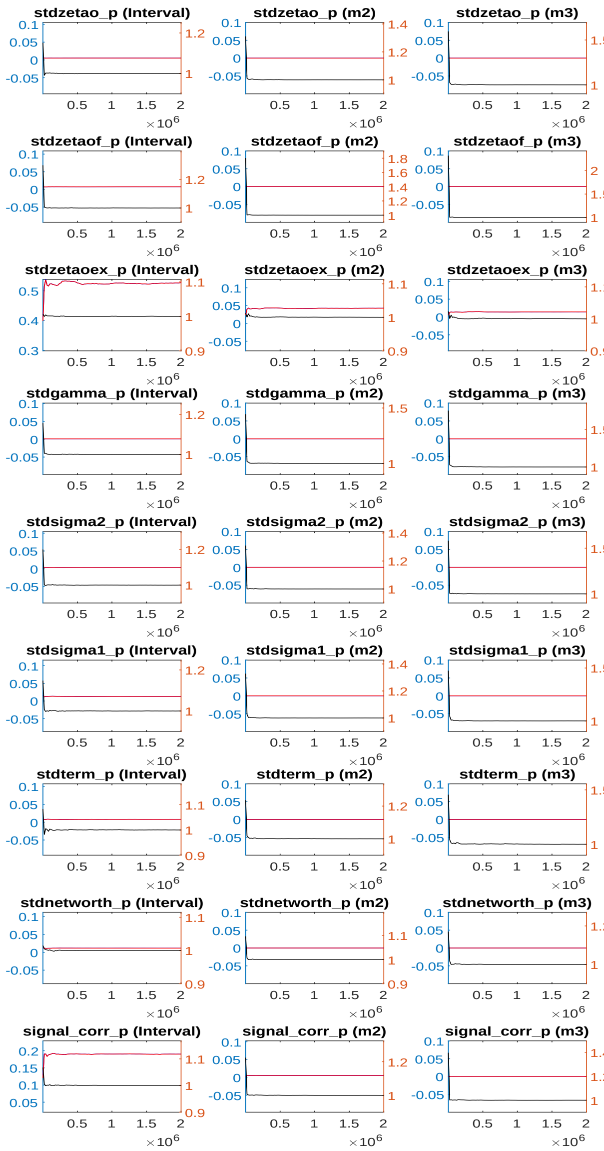
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 21: Parameter convergence CMR-Oil II



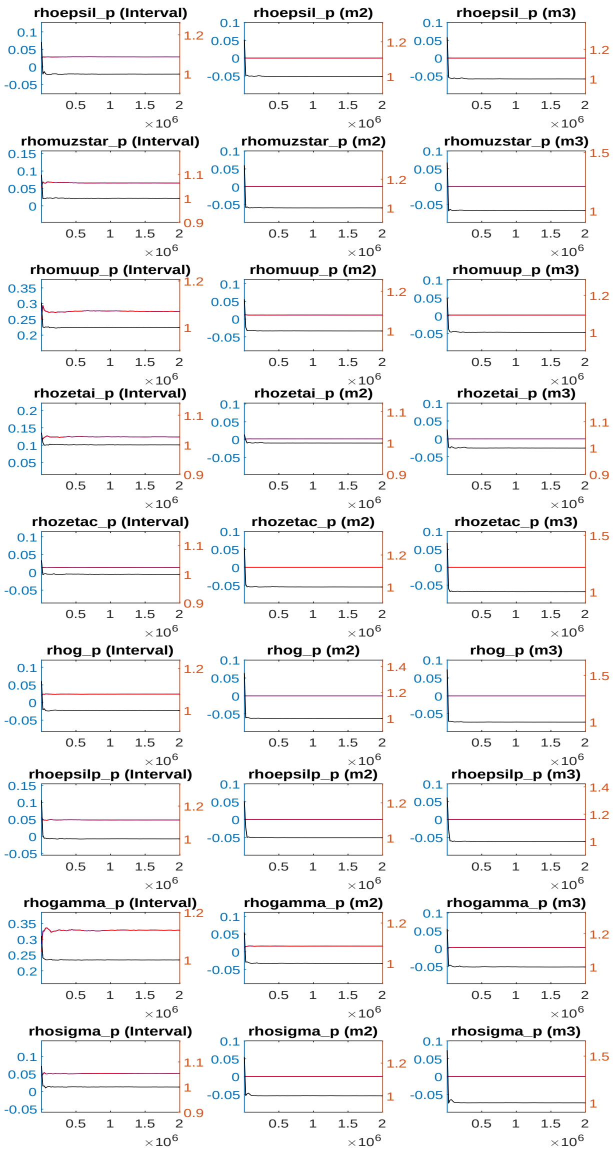
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 22: Parameter convergence CMR-Oil III



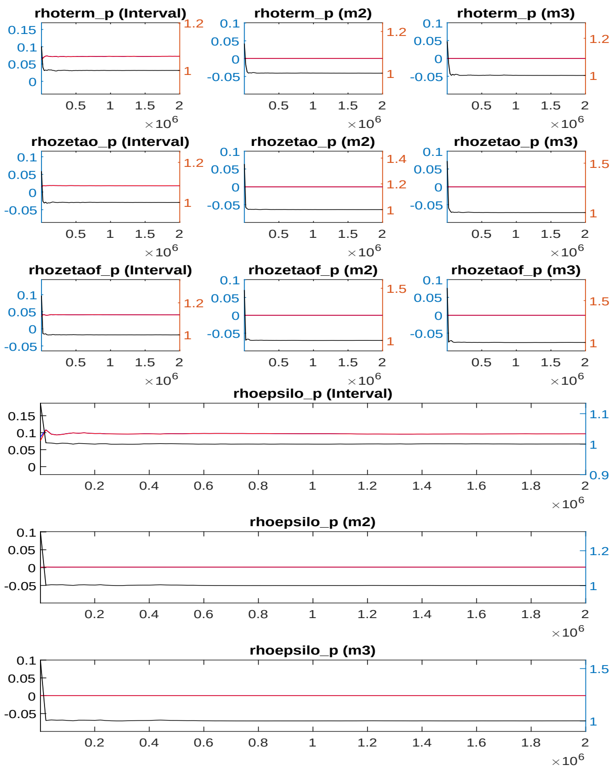
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 23: Parameter convergence CMR-Oil IV



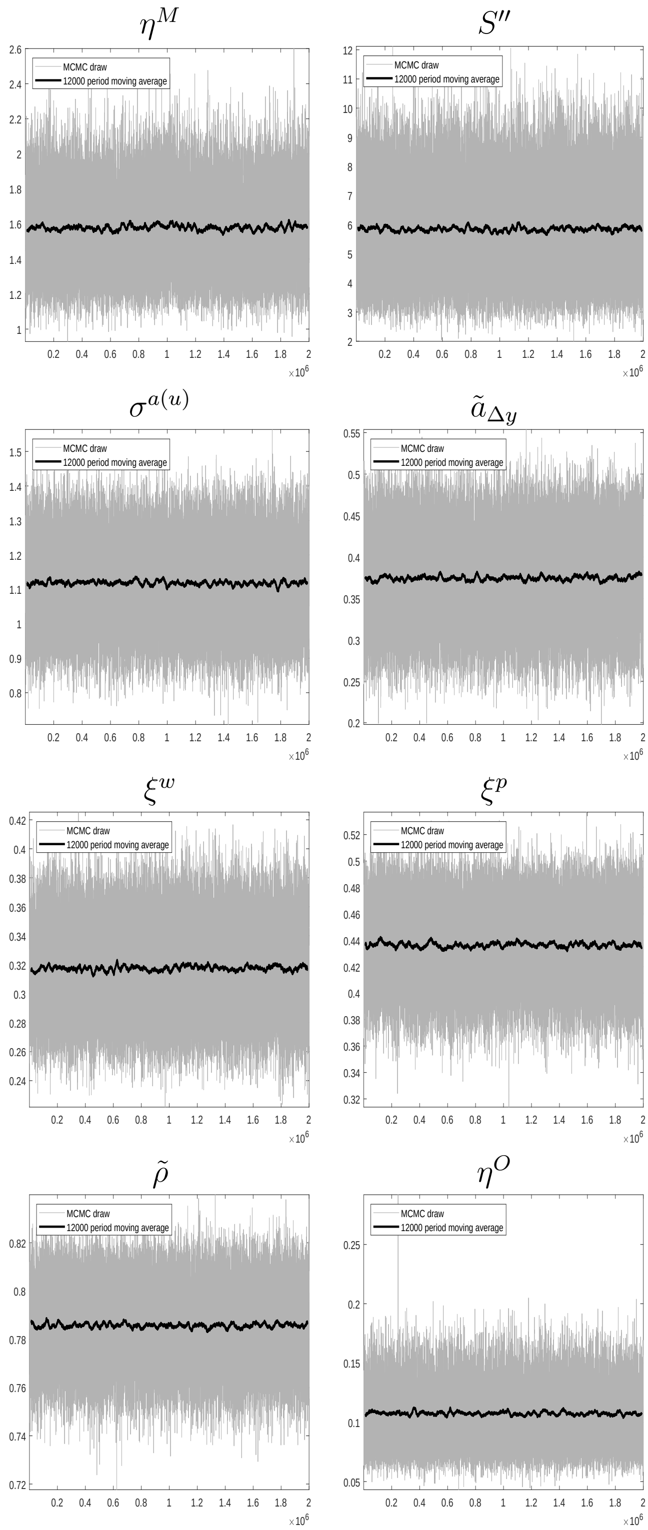
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 24: Parameter convergence CMR-Oil V



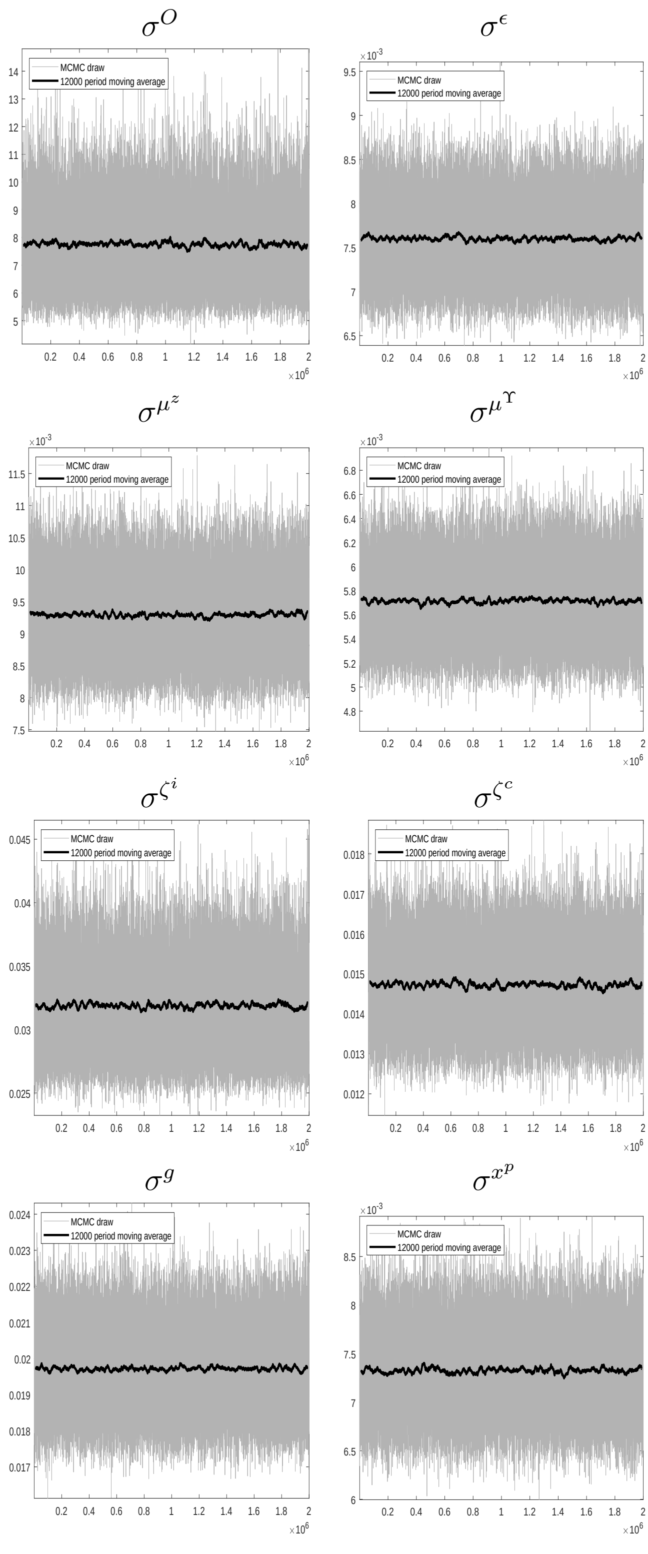
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 25: Trace plots for chain 1 CEE–Oil I



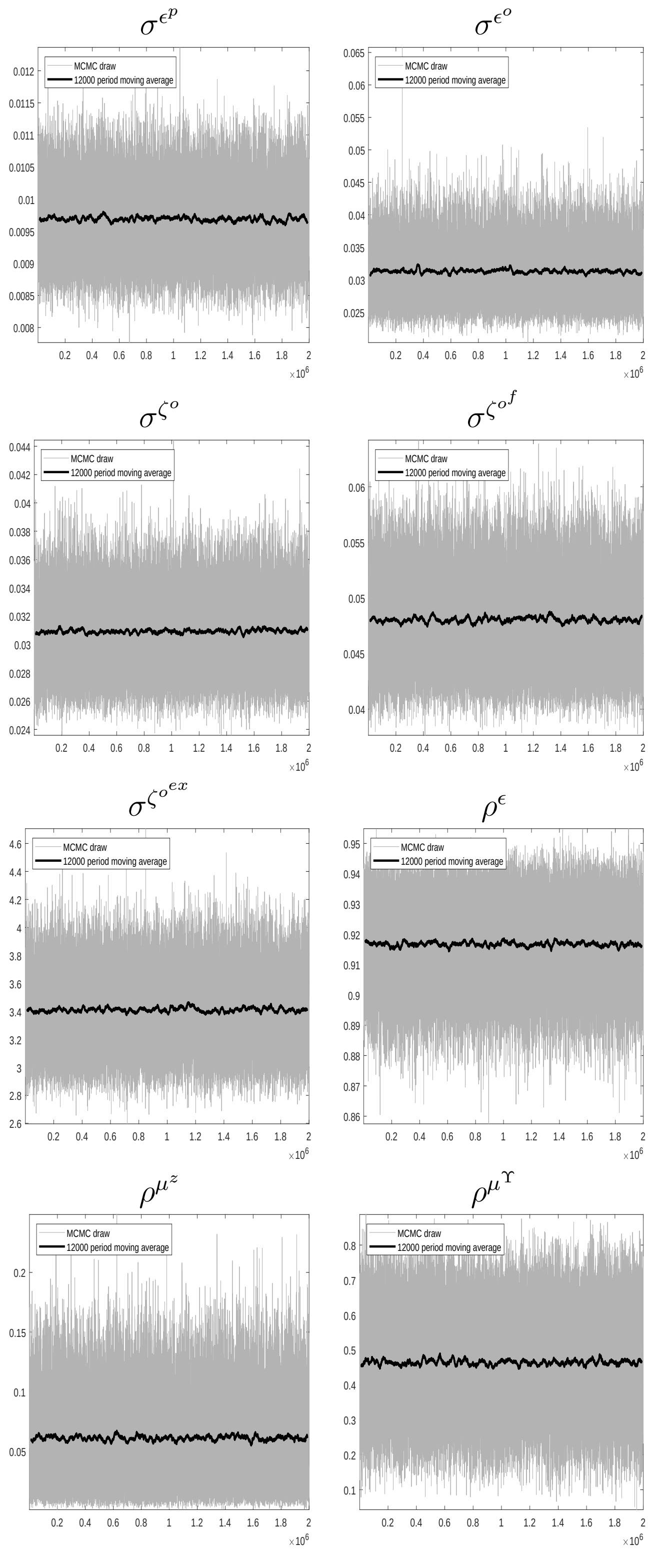
Notes: The grey line depicts parameter values and the back line the moving average.

Figure 26: Trace plots for chain 1 CEE–Oil II



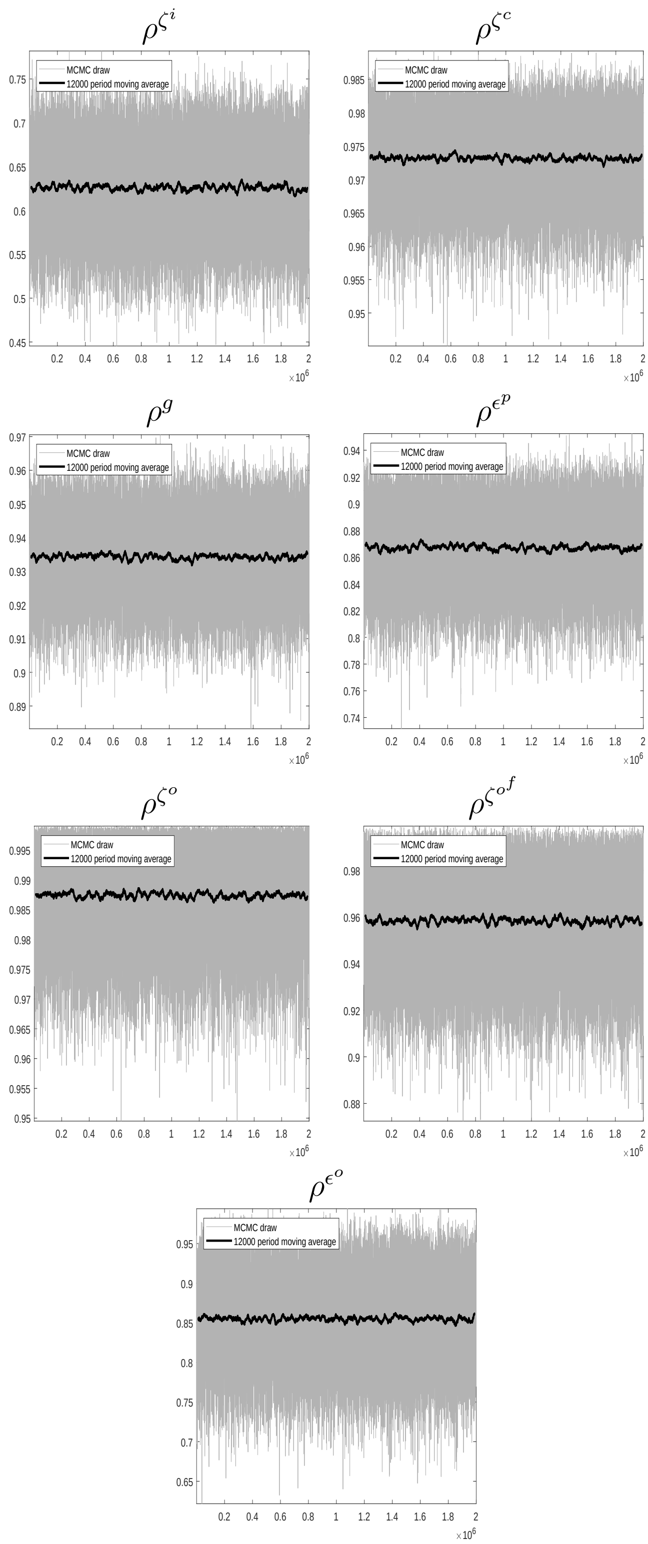
Notes: The grey line depicts parameter values and the back line the moving average.

Figure 27: Trace plots for chain 1 CEE–Oil III



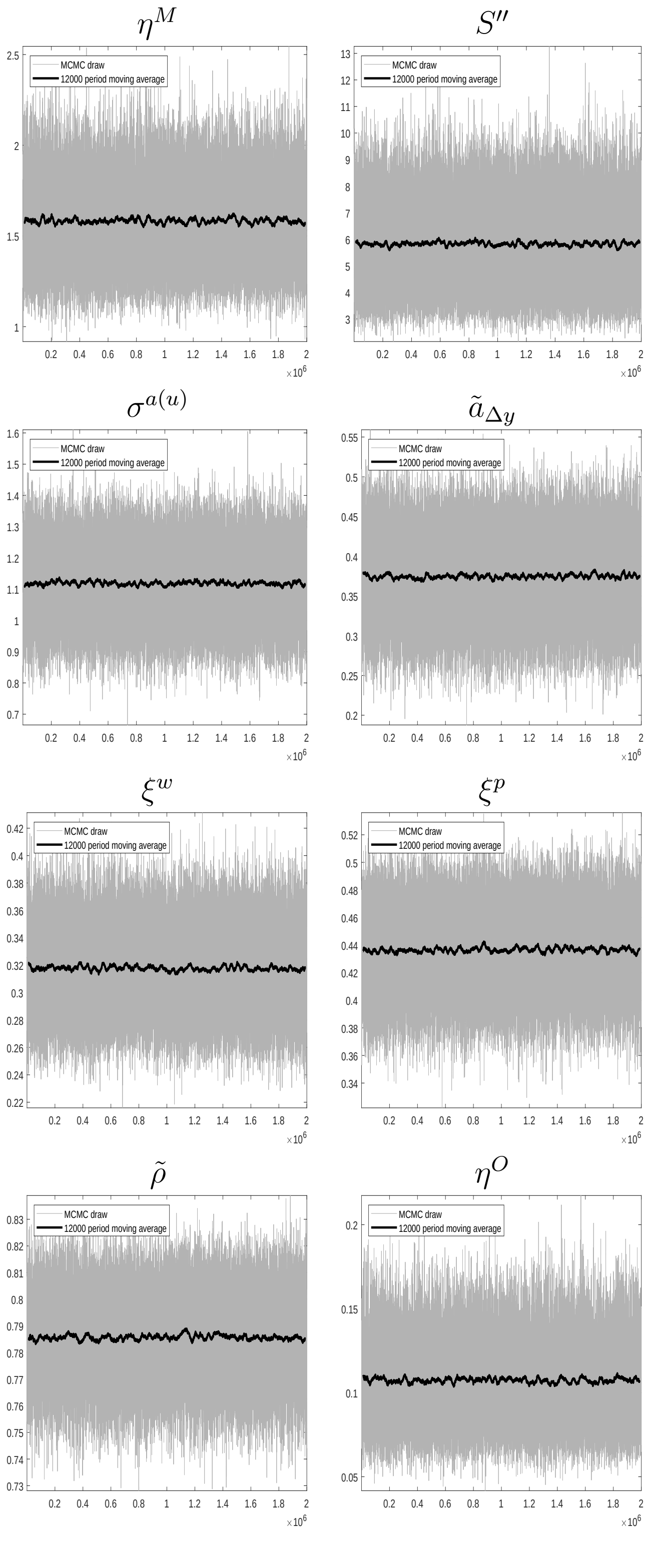
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 28: Trace plots for chain 1 CEE–Oil IV



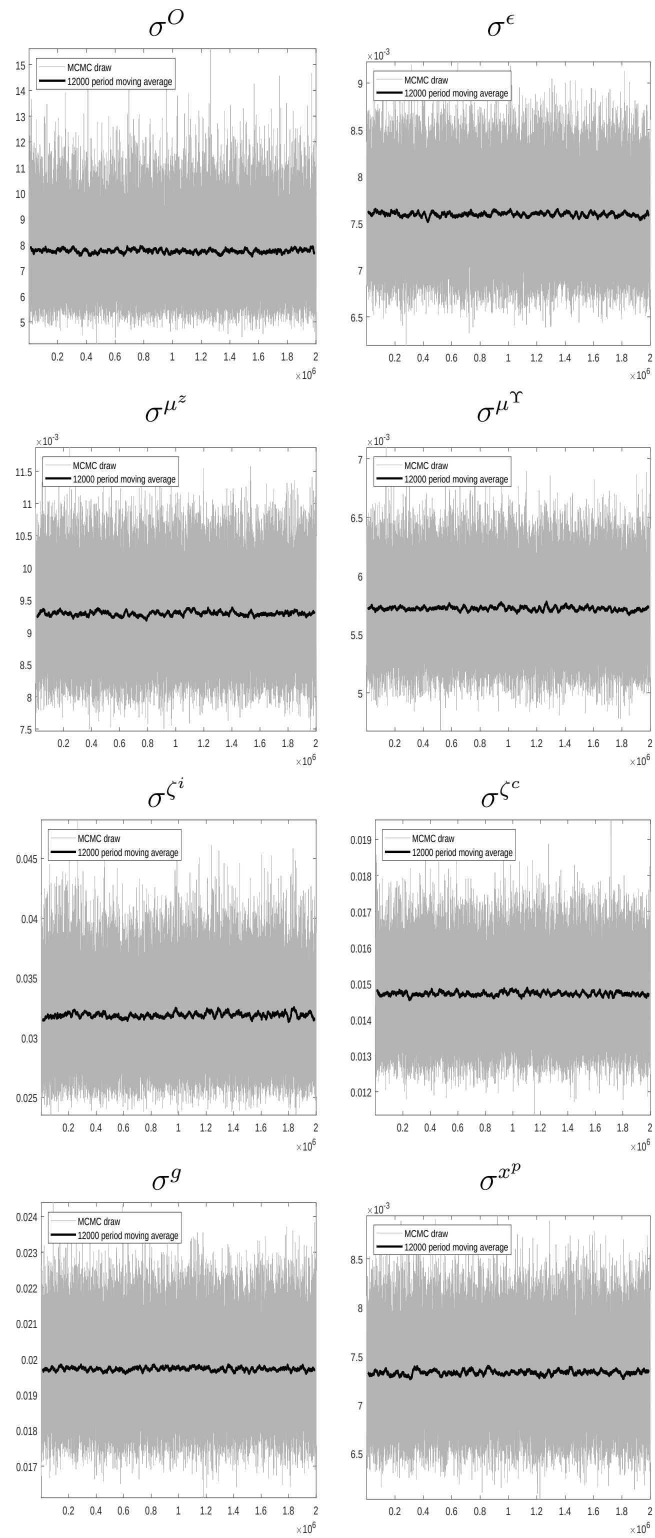
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 29: Trace plots for chain 2 CEE–Oil I



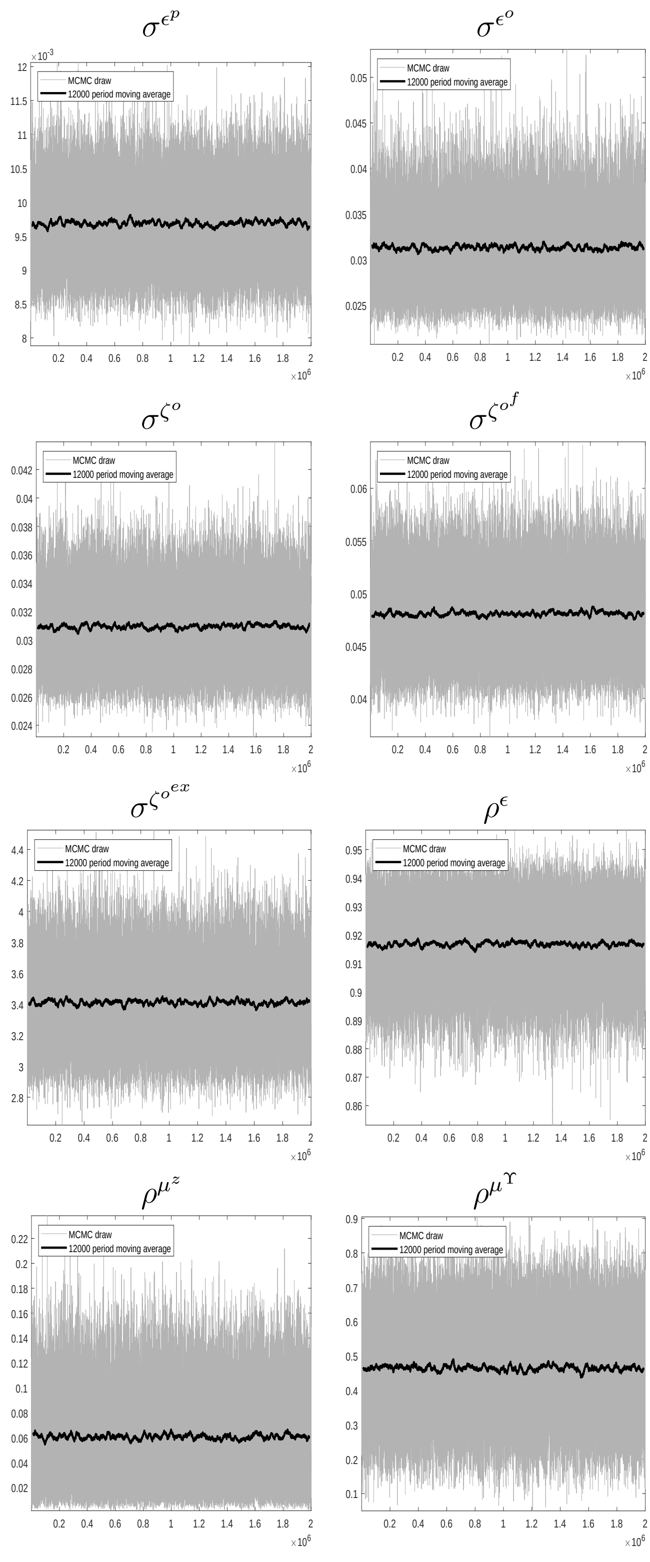
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 30: Trace plots for chain 2 CEE–Oil II



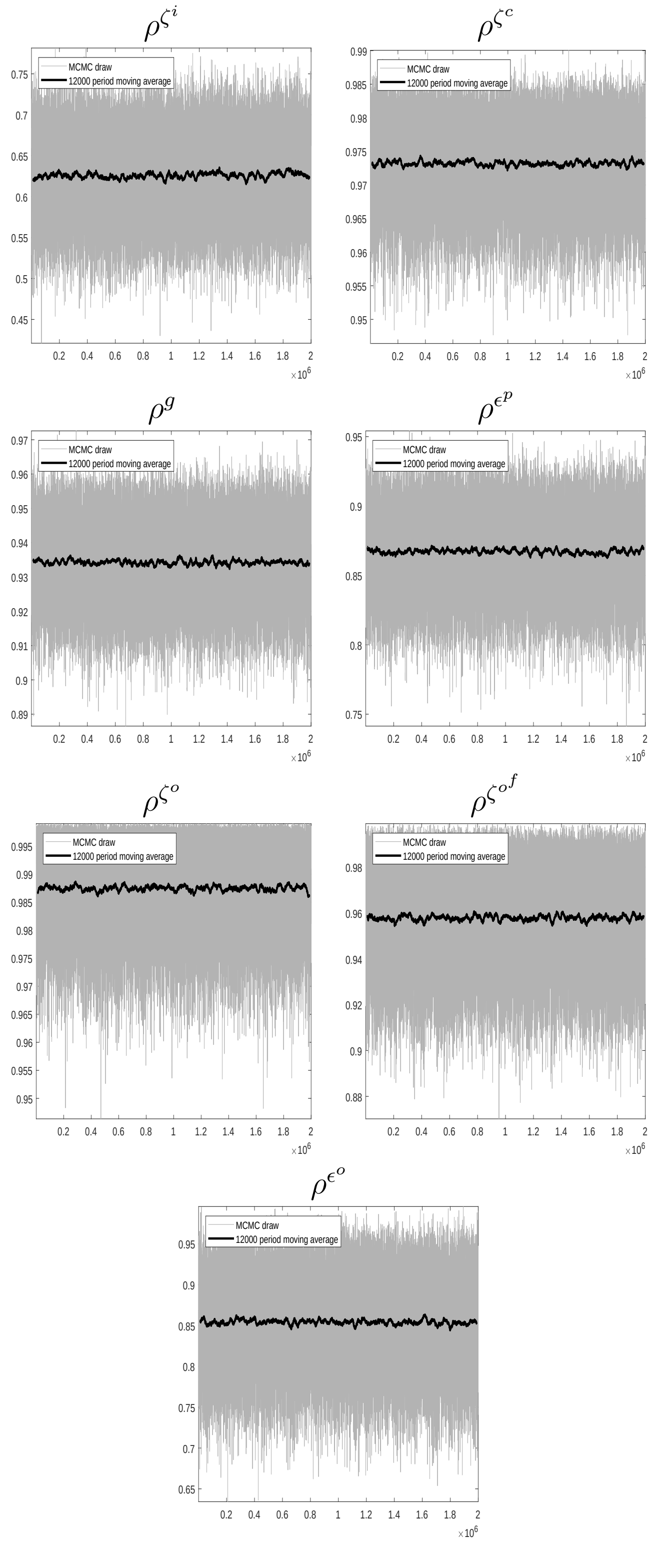
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 31: Trace plots for chain 2 CEE–Oil III



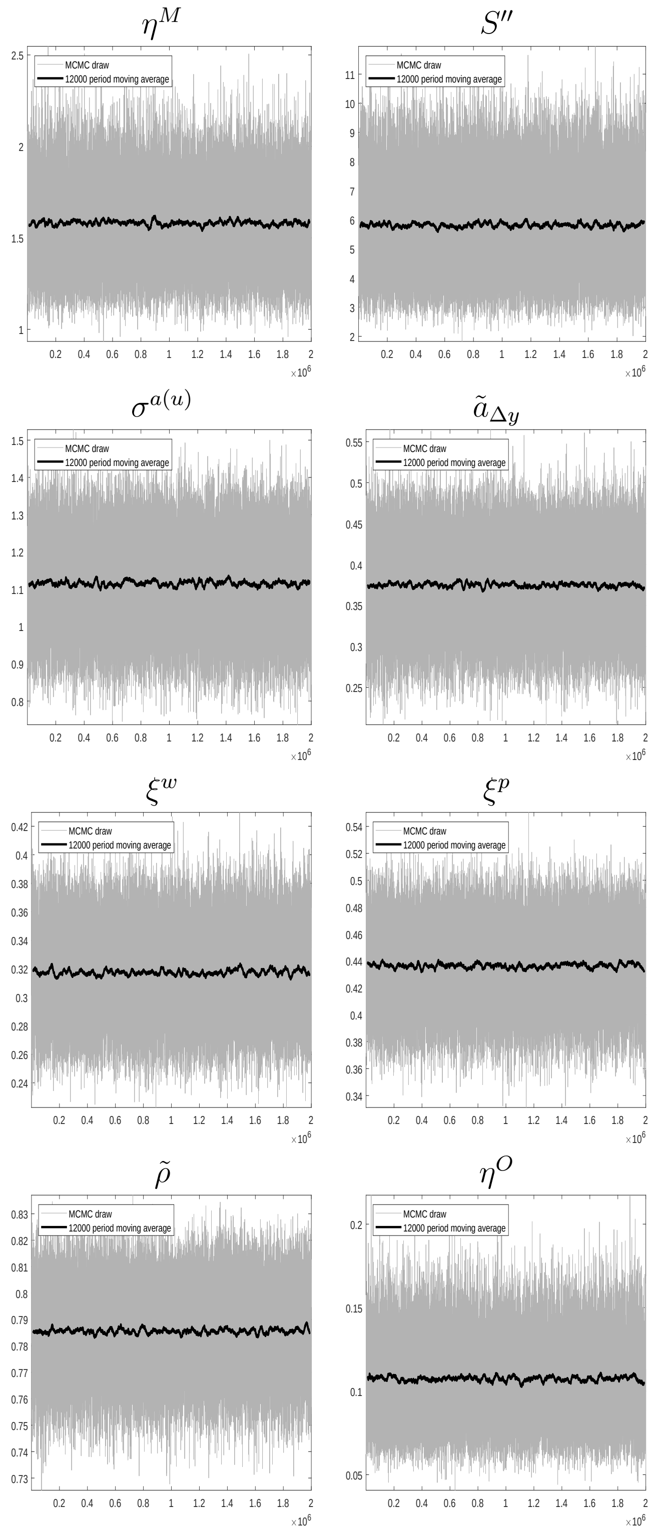
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 32: Trace plots for chain 2 CEE–Oil IV



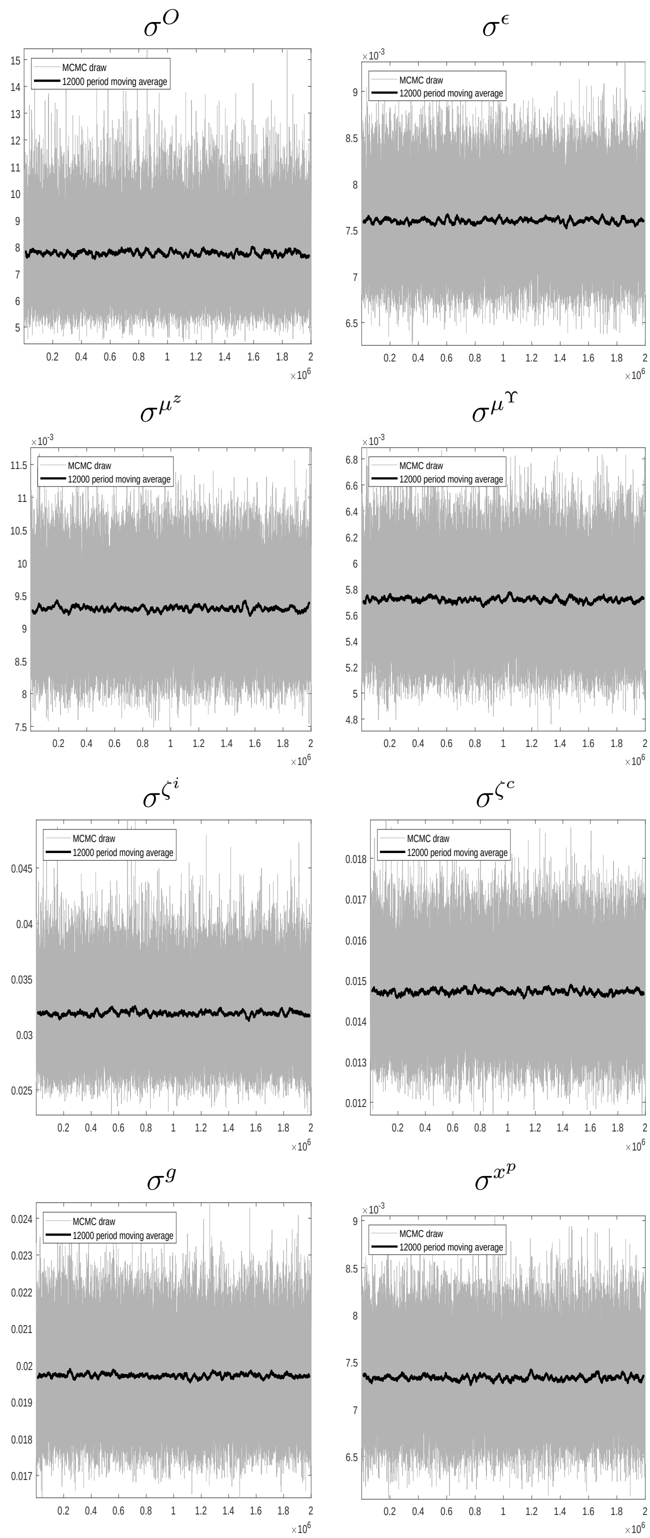
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 33: Trace plots for chain 3 CEE–Oil I



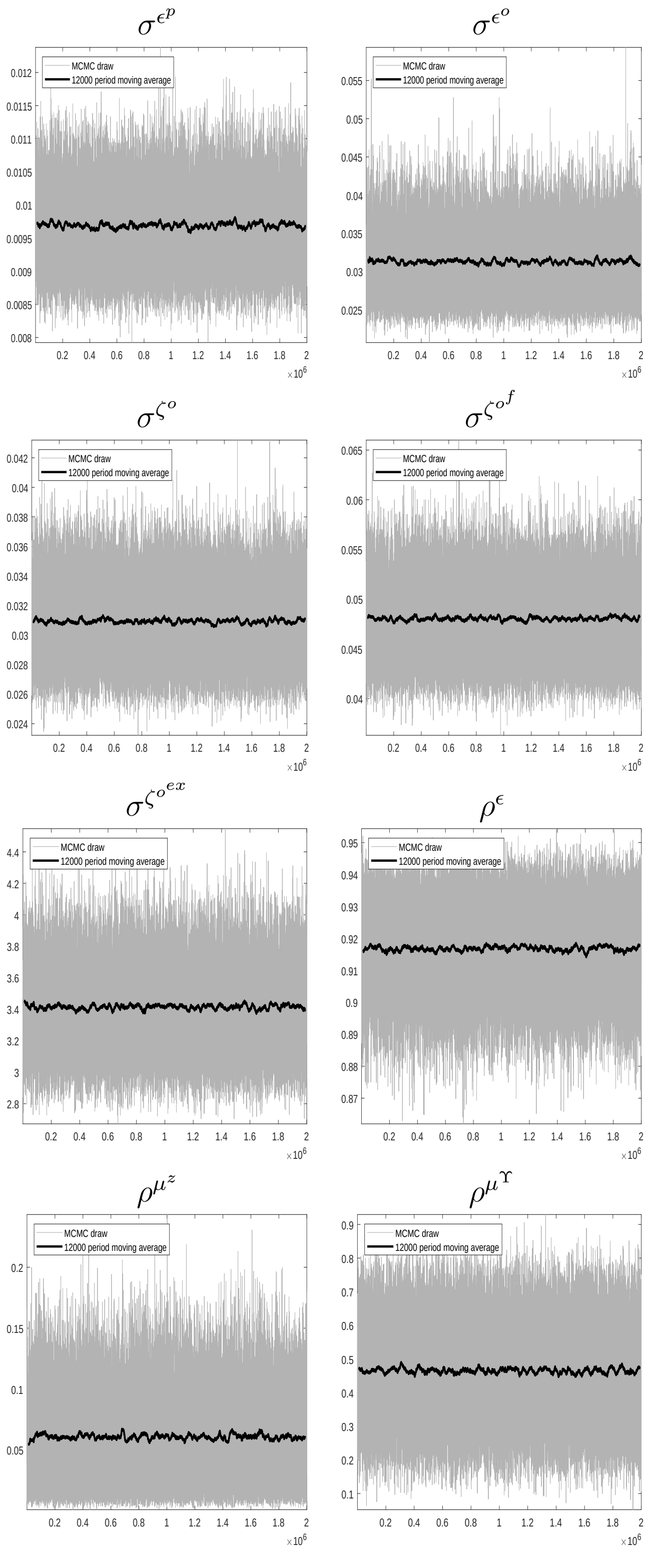
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 34: Trace plots for chain 3 CEE–Oil II



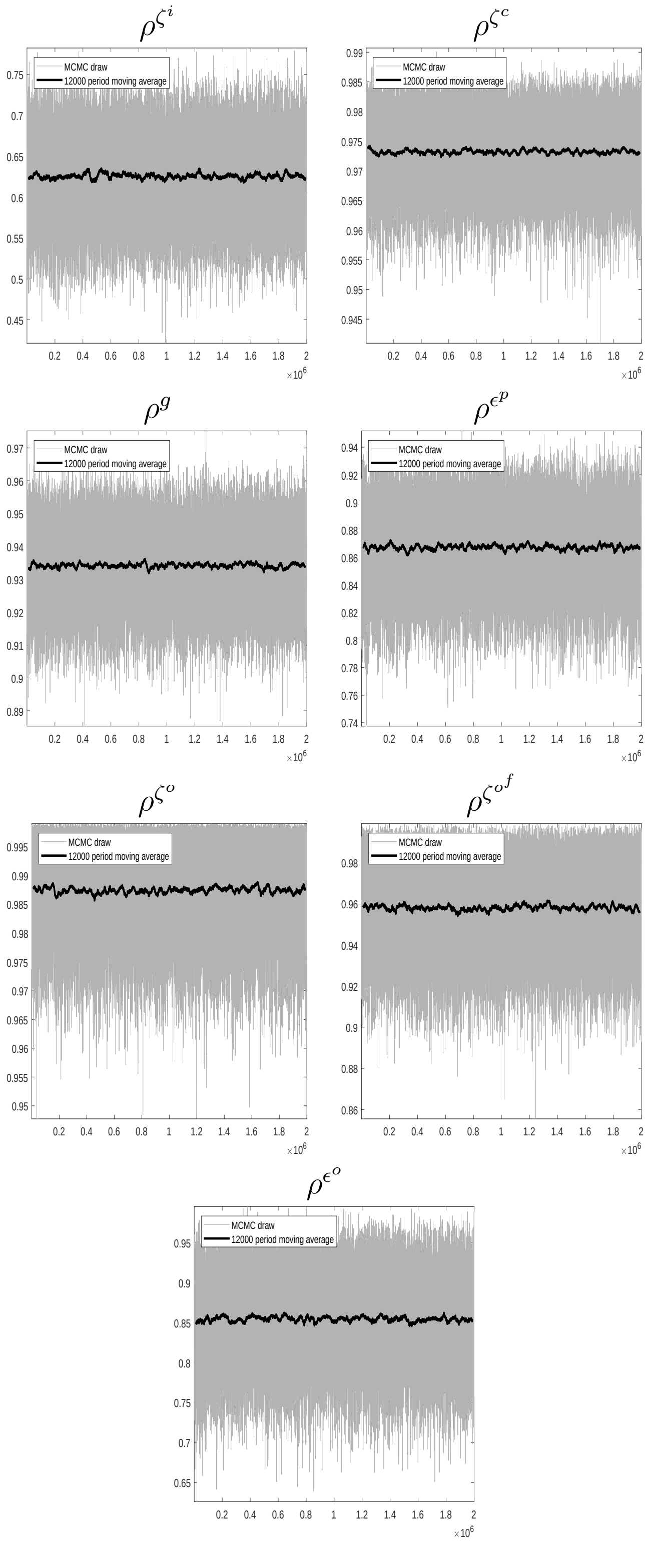
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 35: Trace plots for chain 3 CEE–Oil III



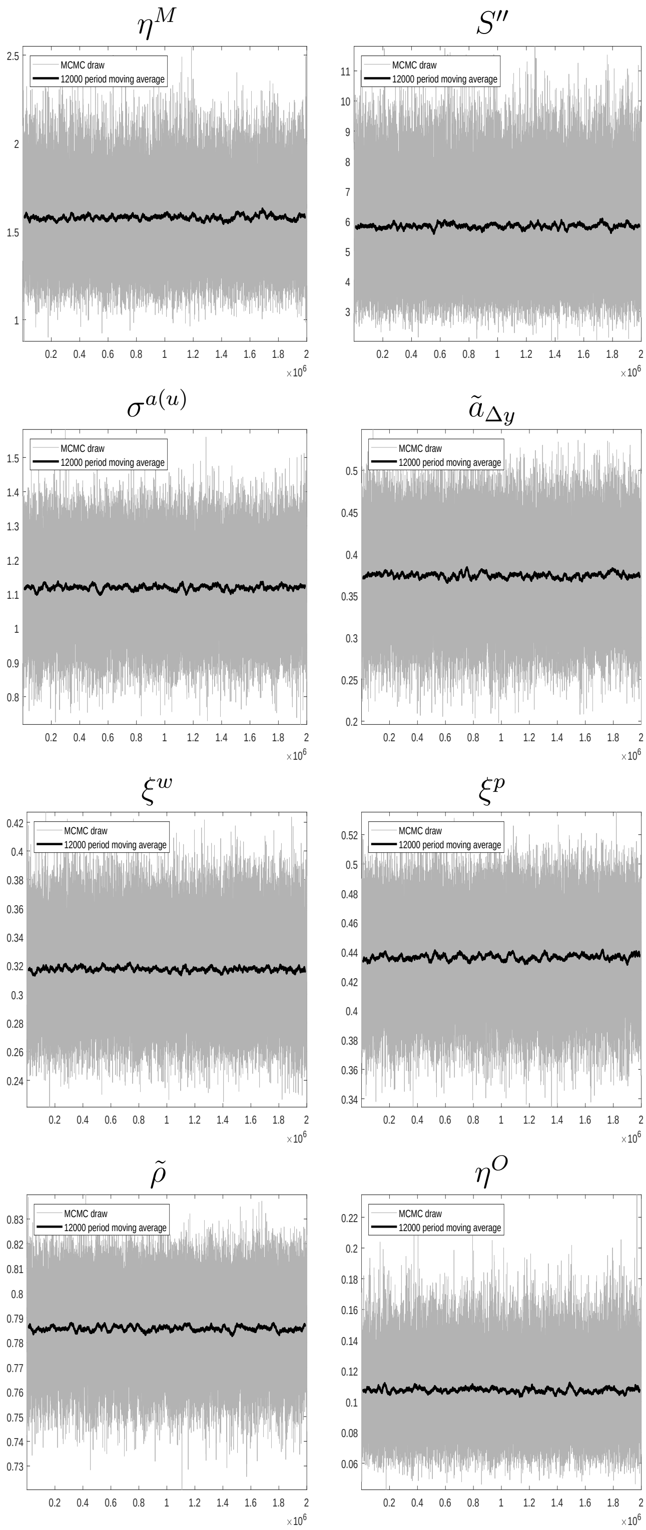
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 36: Trace plots for chain 3 CEE–Oil IV



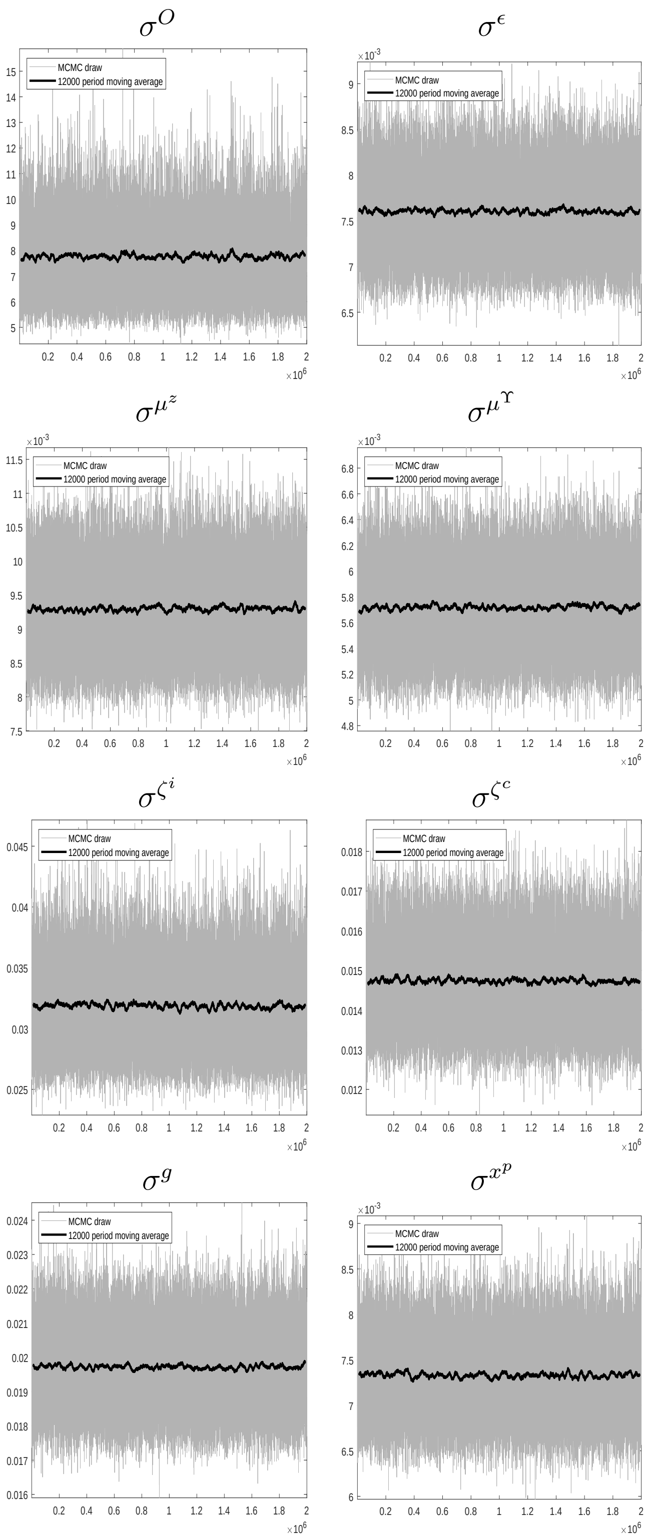
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 37: Trace plots for chain 4 CEE–Oil I



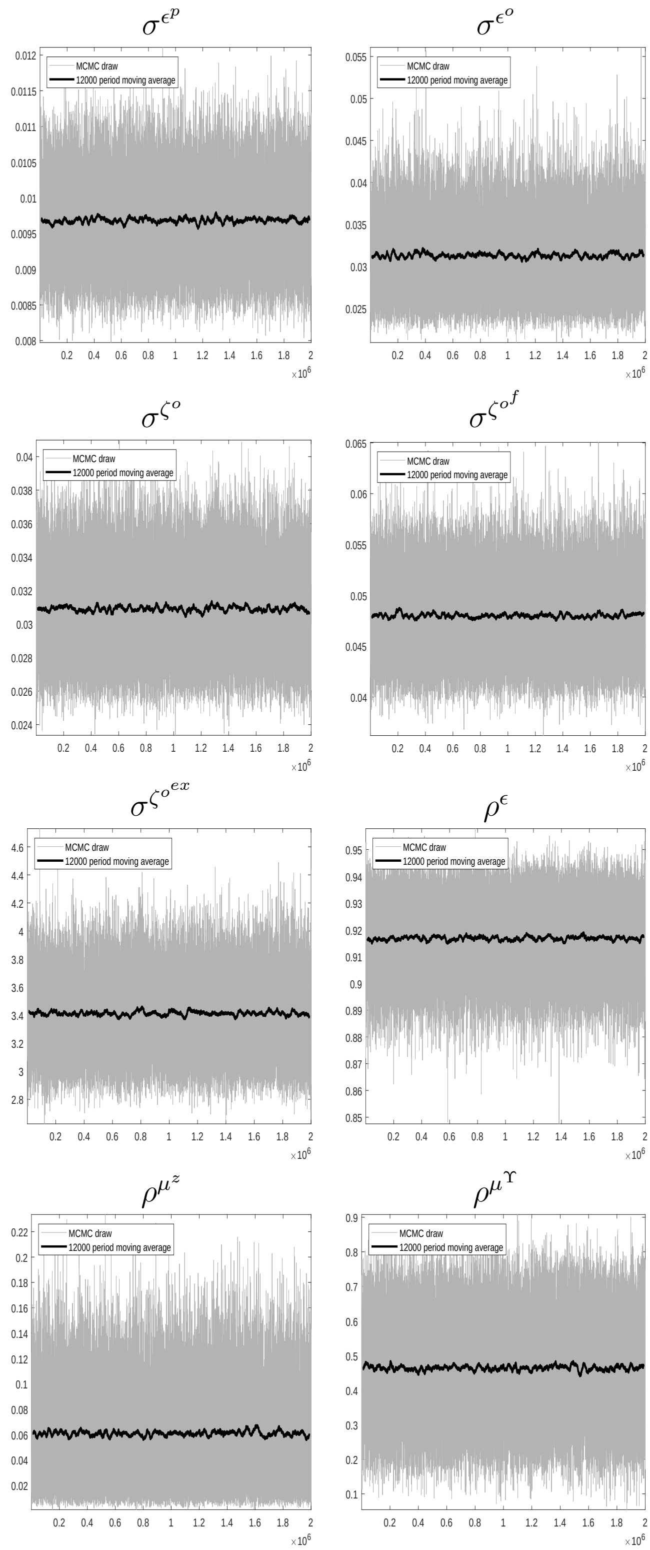
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 38: Trace plots for chain 4 CEE–Oil II



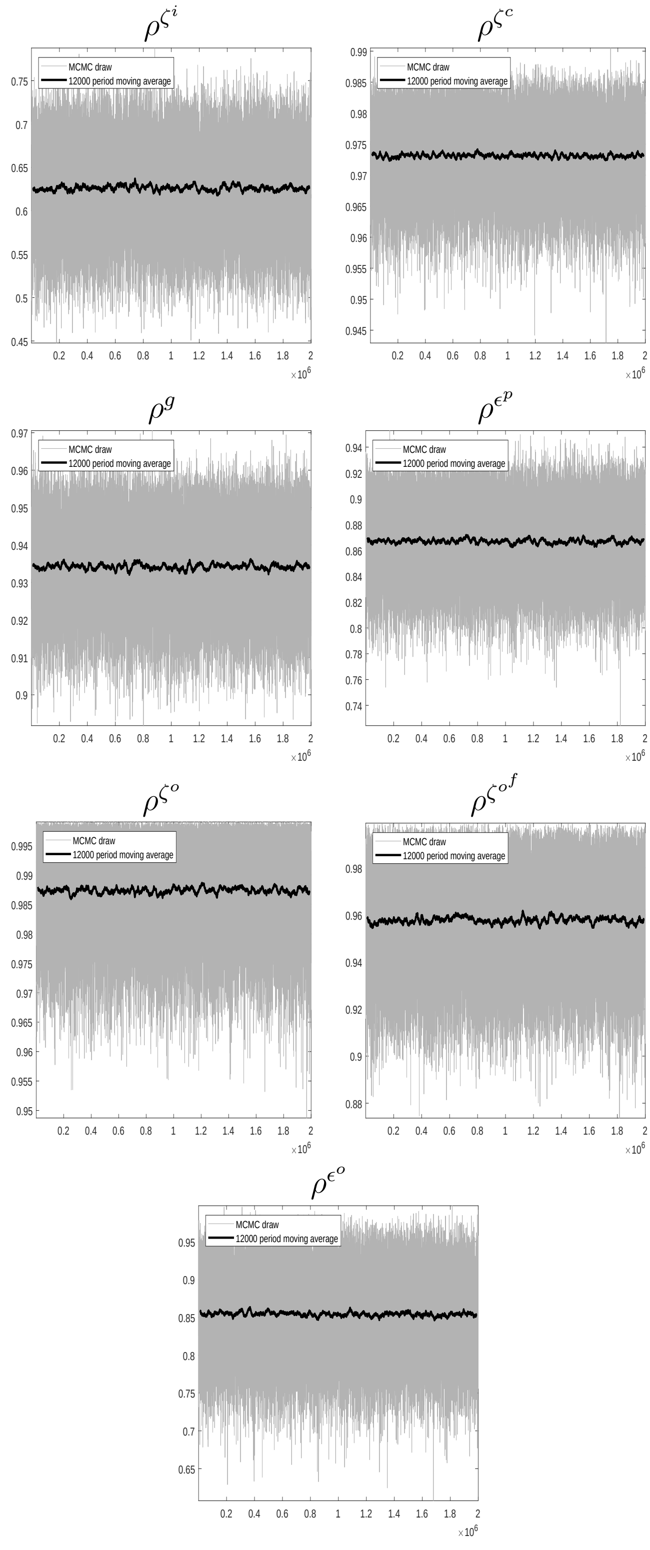
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 39: Trace plots for chain 4 CEE–Oil III



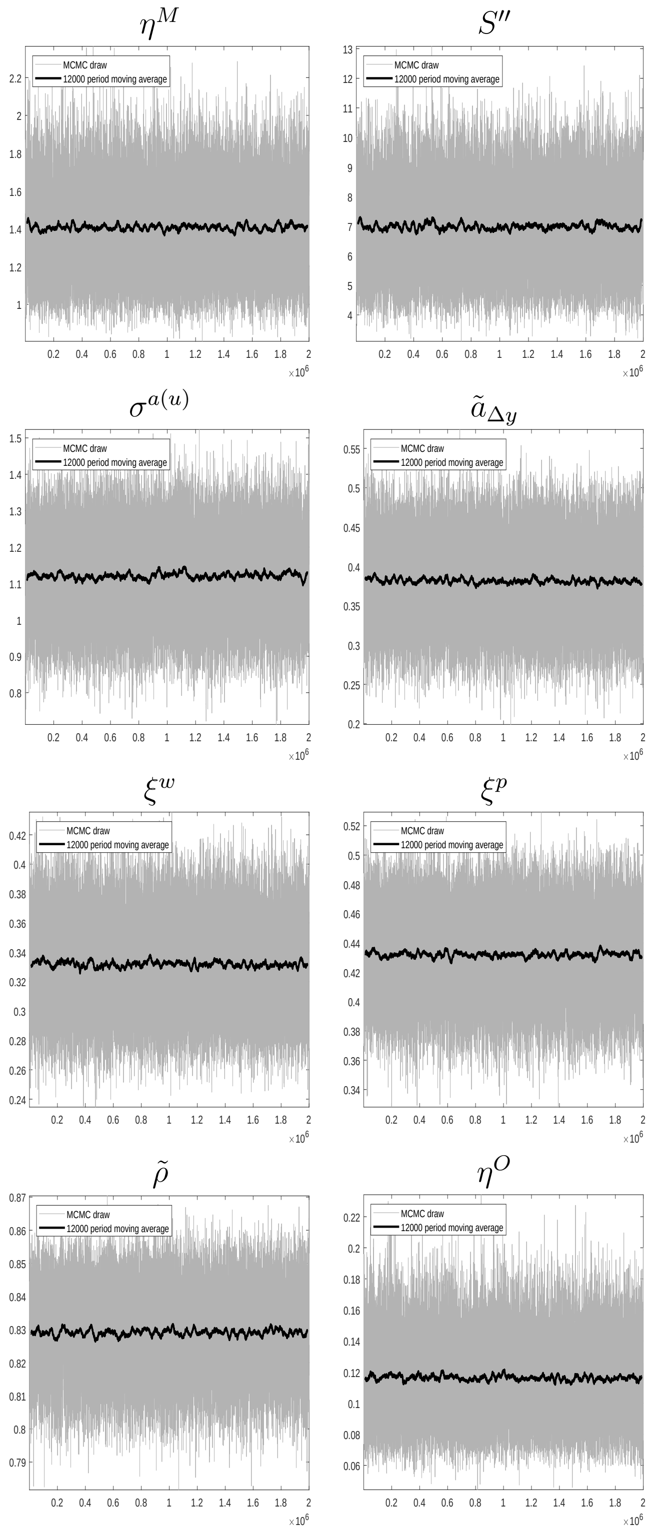
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 40: Trace plots for chain 4 CEE–Oil IV



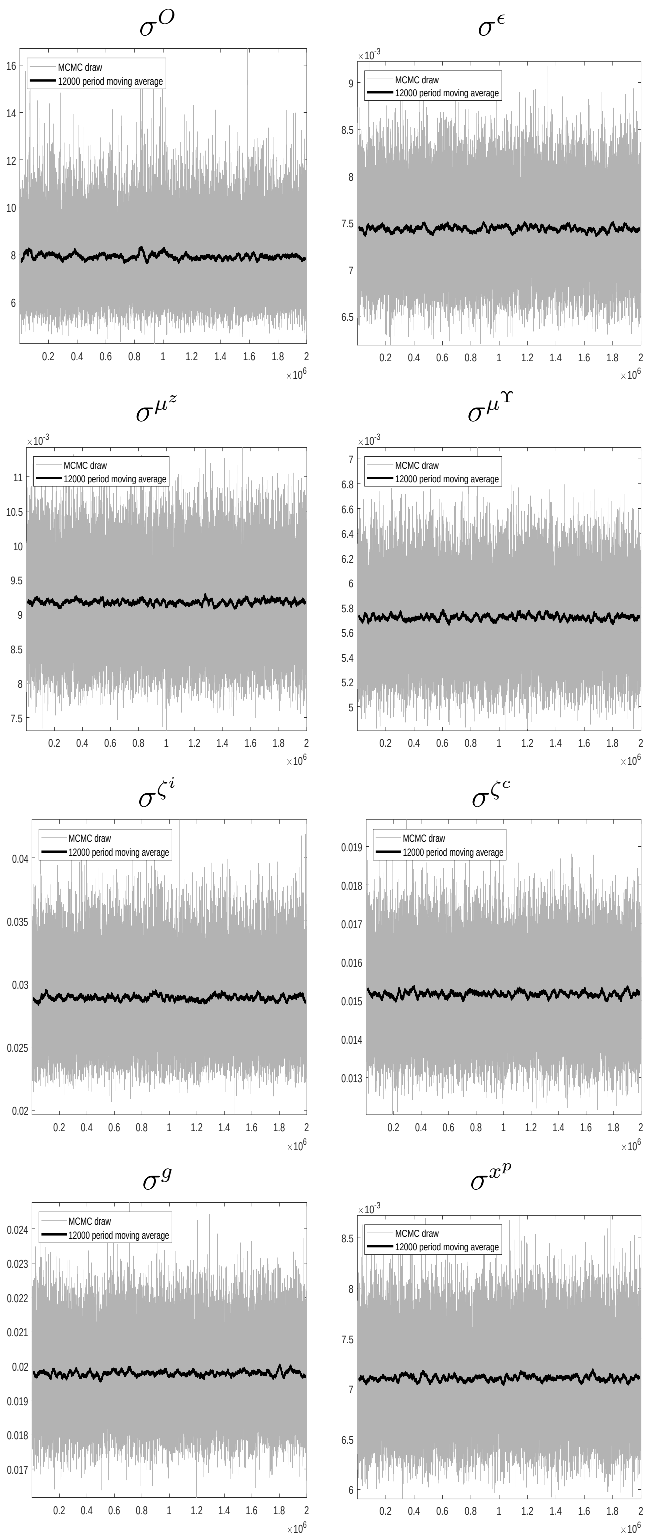
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 41: Trace plots for chain 1 CMR–Oil I



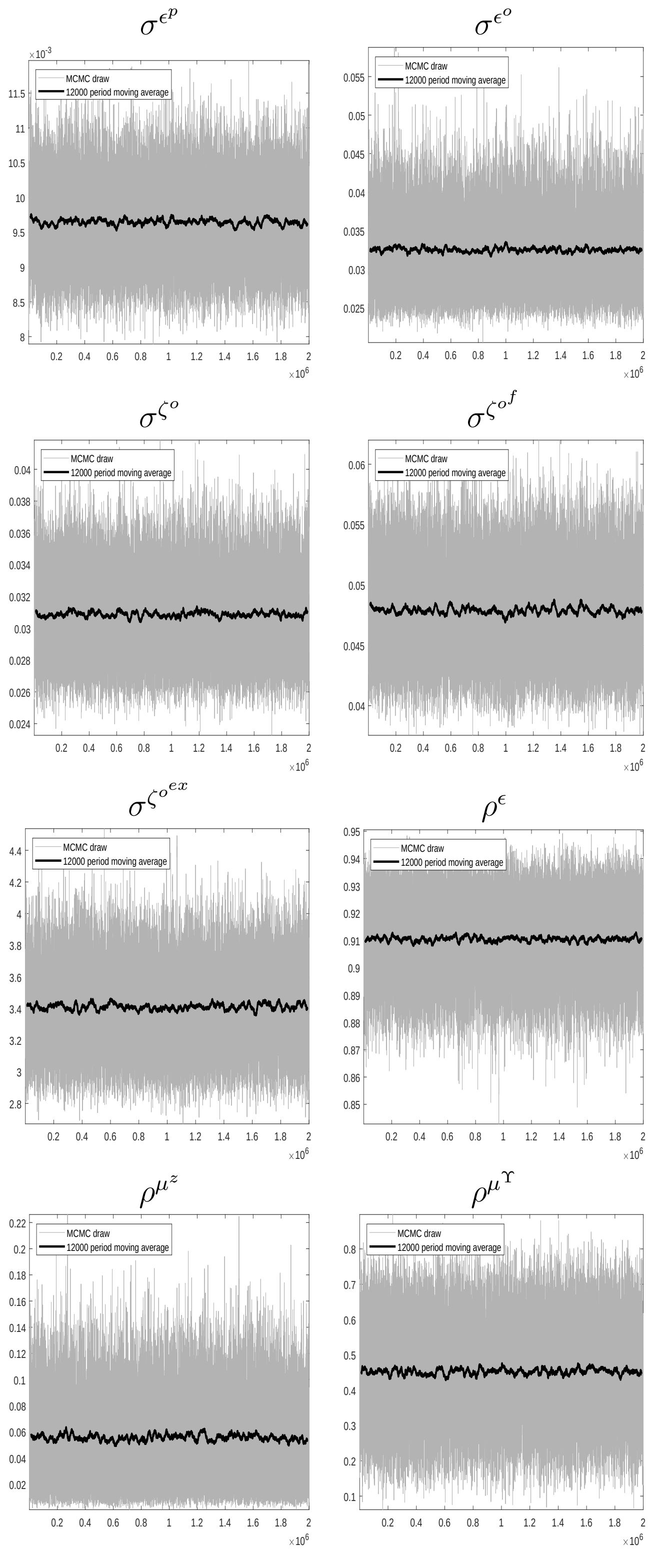
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 42: Trace plots for chain 1 CMR–Oil II



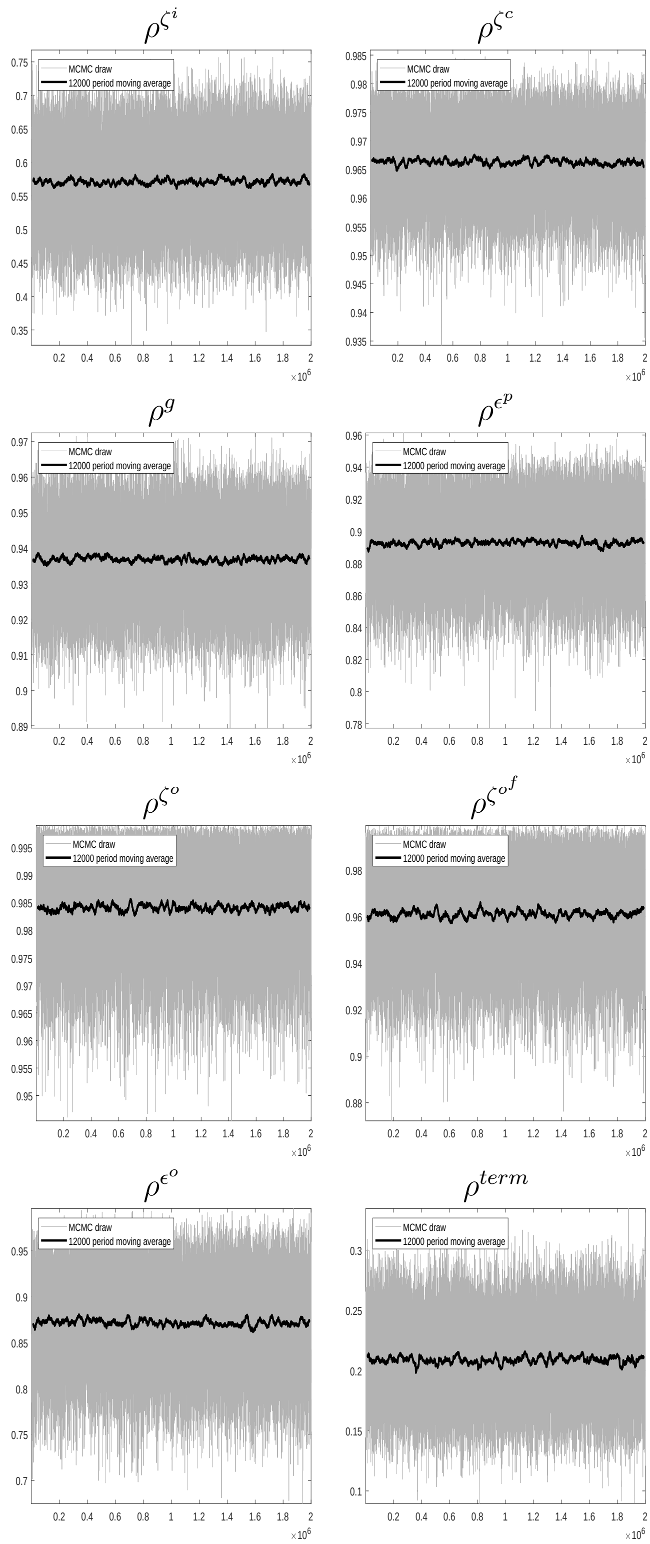
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 43: Trace plots for chain 1 CMR–Oil III



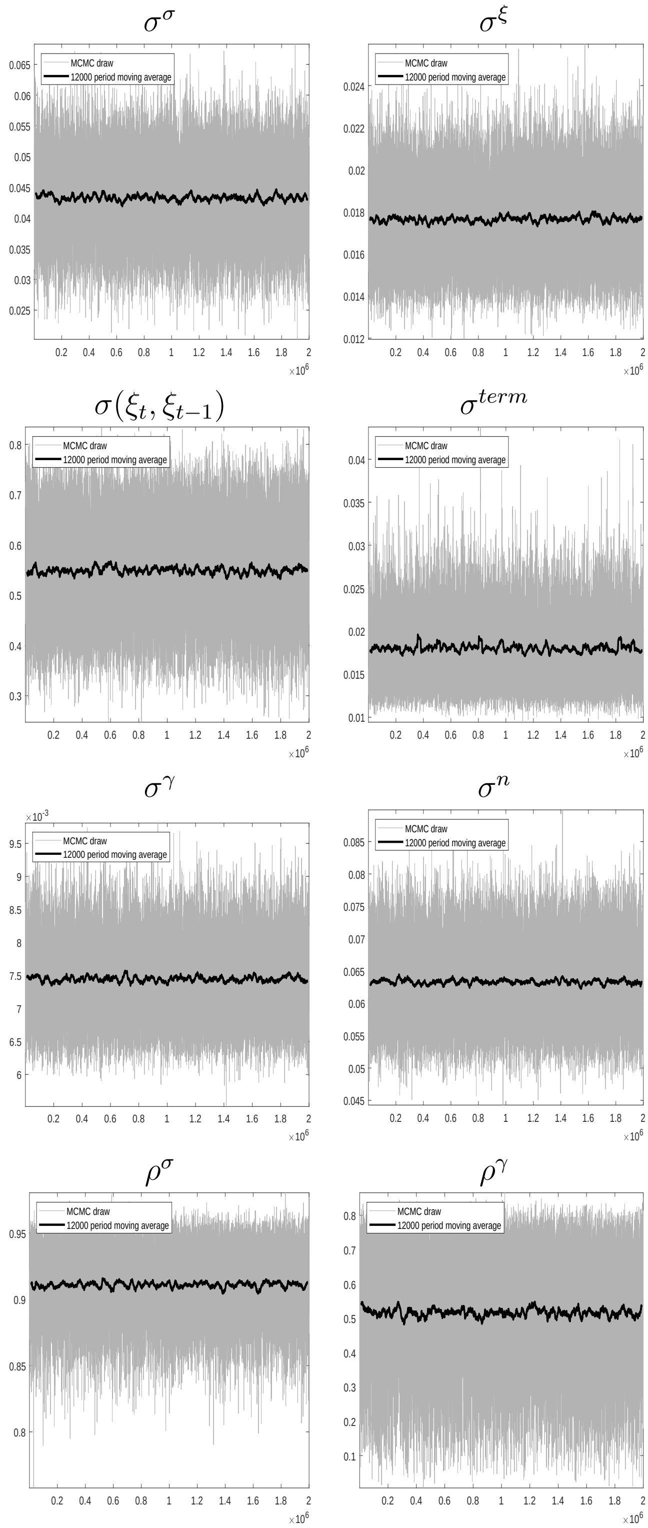
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 44: Trace plots for chain 1 CMR–Oil IV



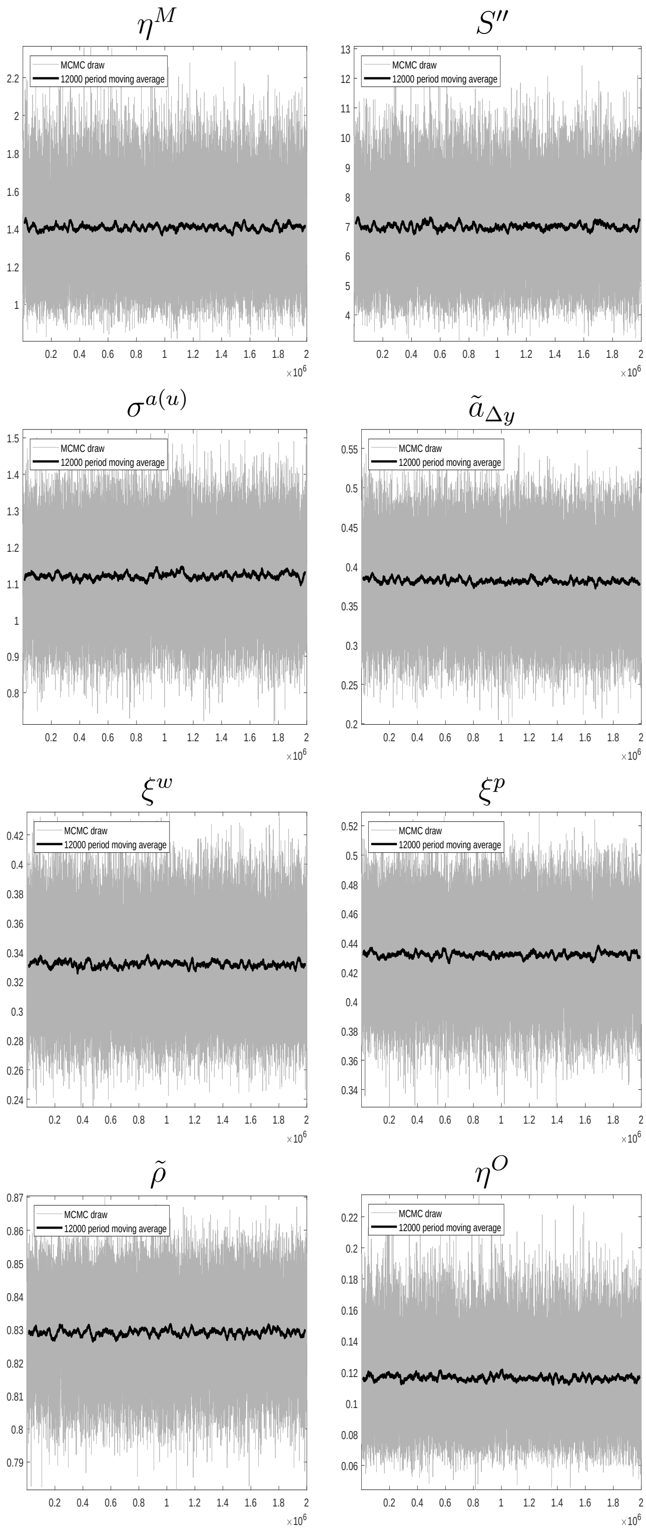
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 45: Trace plots for chain 1 CMR–Oil V



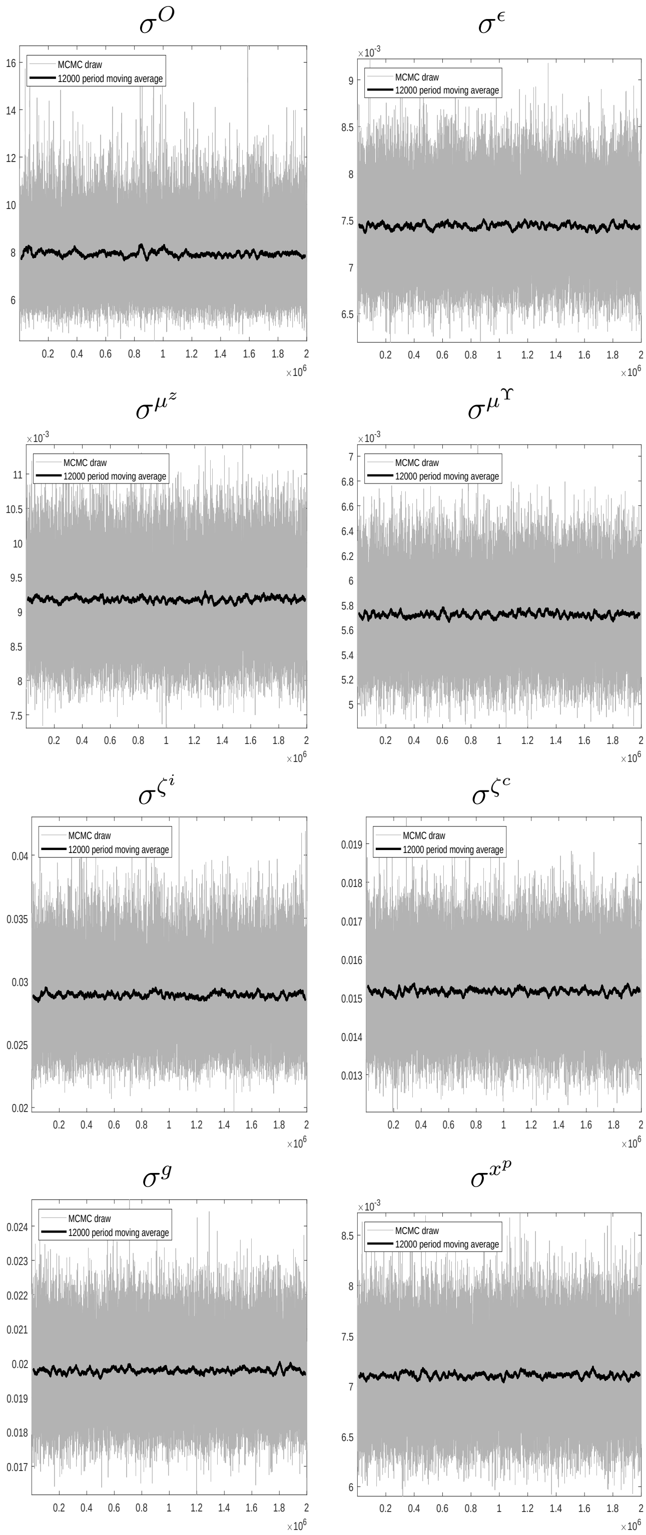
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 46: Trace plots for chain 2 CMR–Oil I



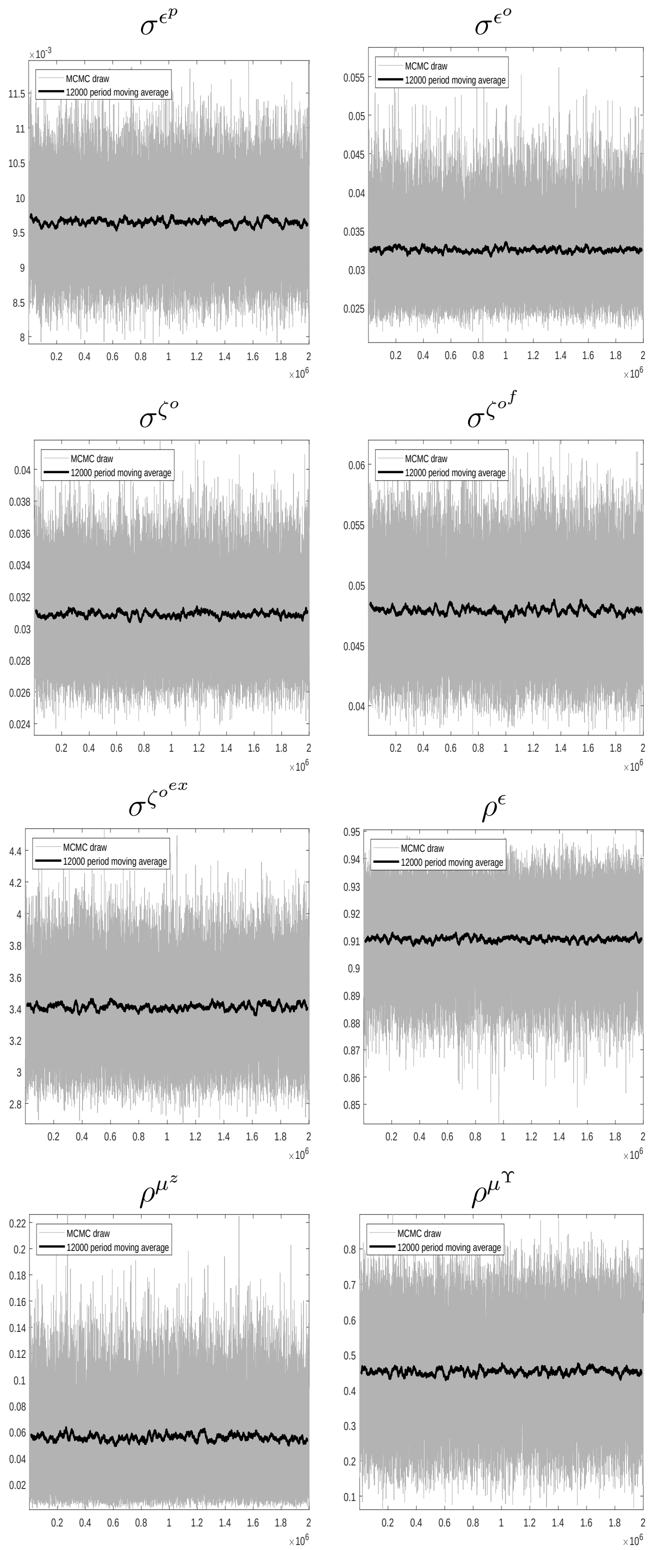
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 47: Trace plots for chain 2 CMR–Oil II



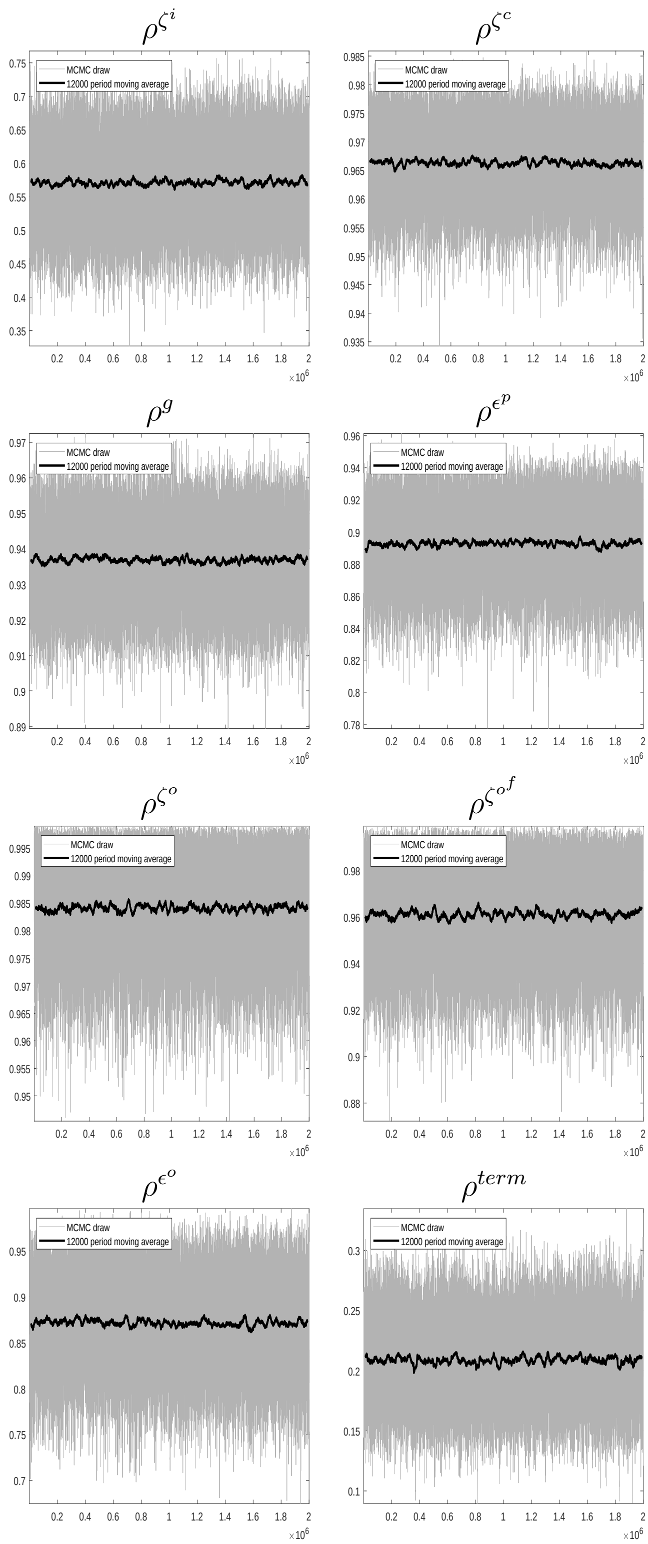
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 48: Trace plots for chain 2 CMR–Oil III



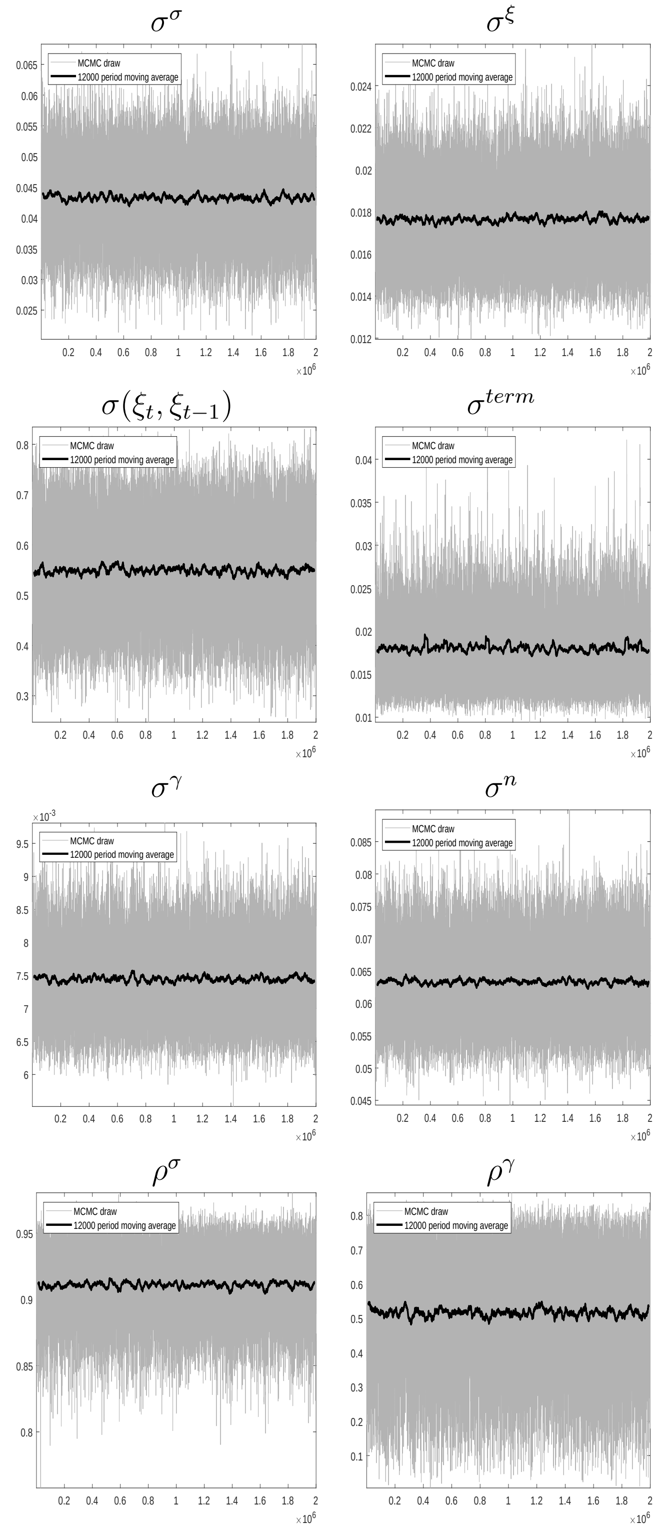
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 49: Trace plots for chain 2 CMR–Oil IV



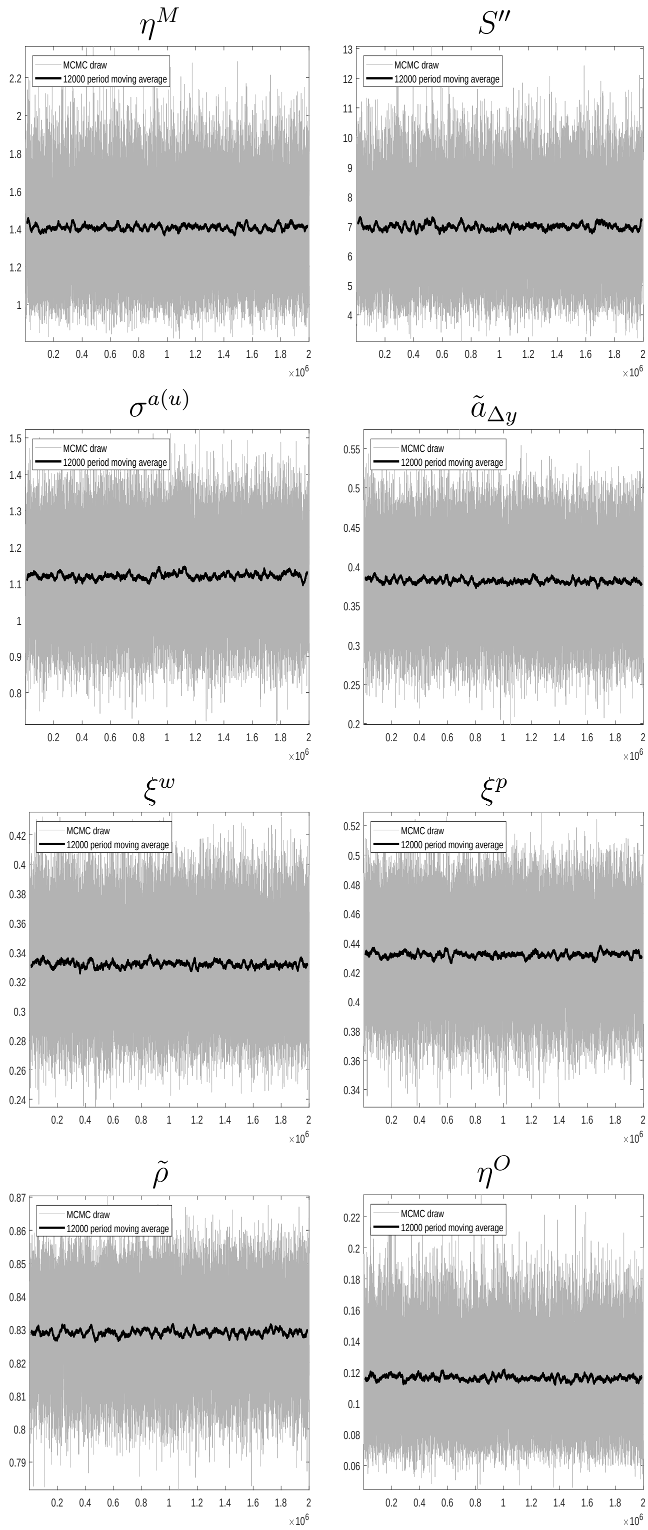
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 50: Trace plots for chain 2 CMR–Oil V



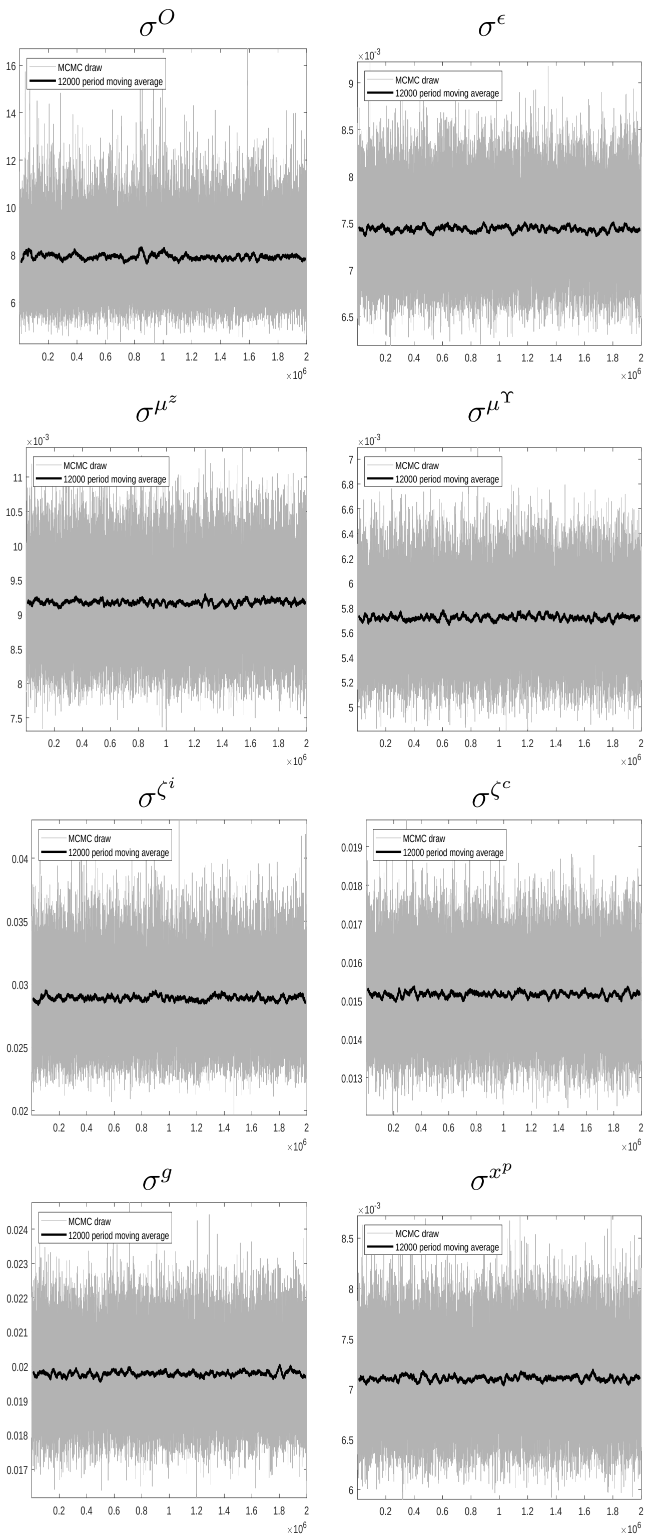
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 51: Trace plots for chain 3 CMR–Oil I



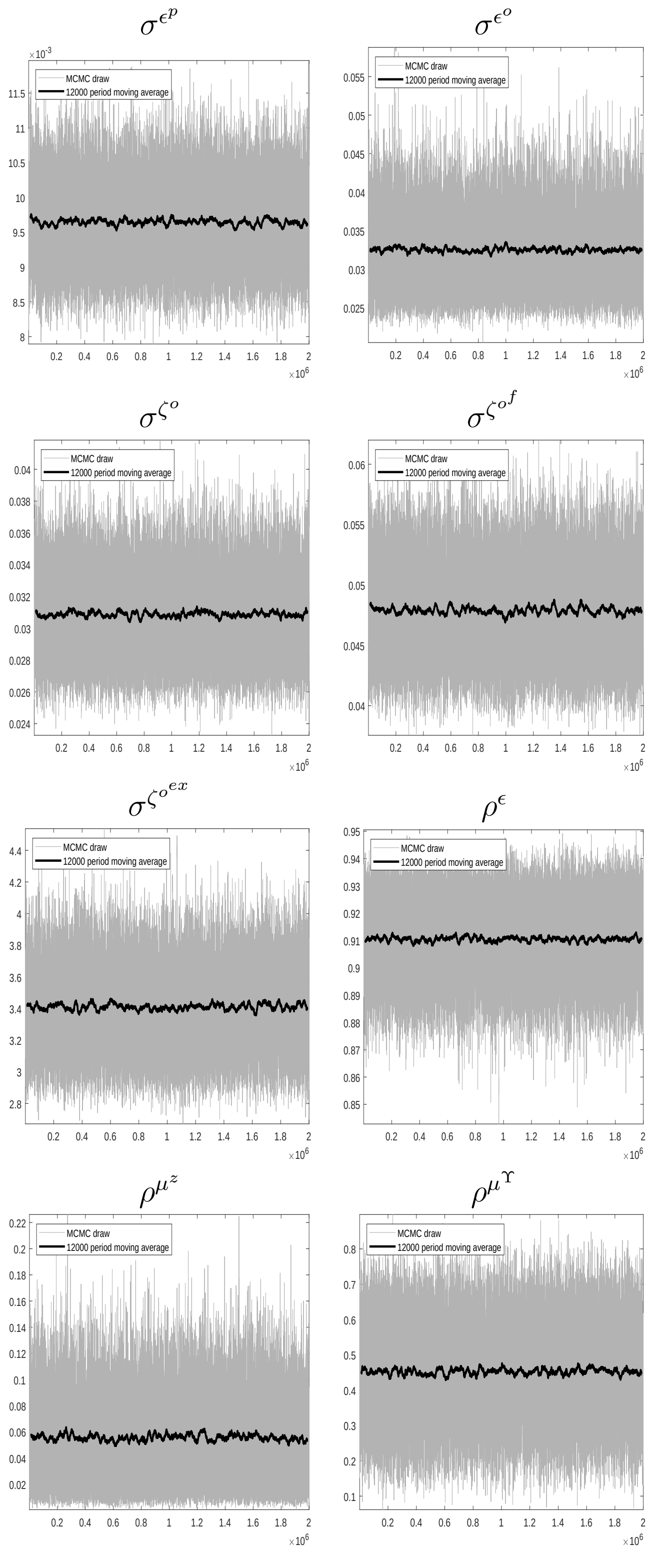
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 52: Trace plots for chain 3 CMR–Oil II



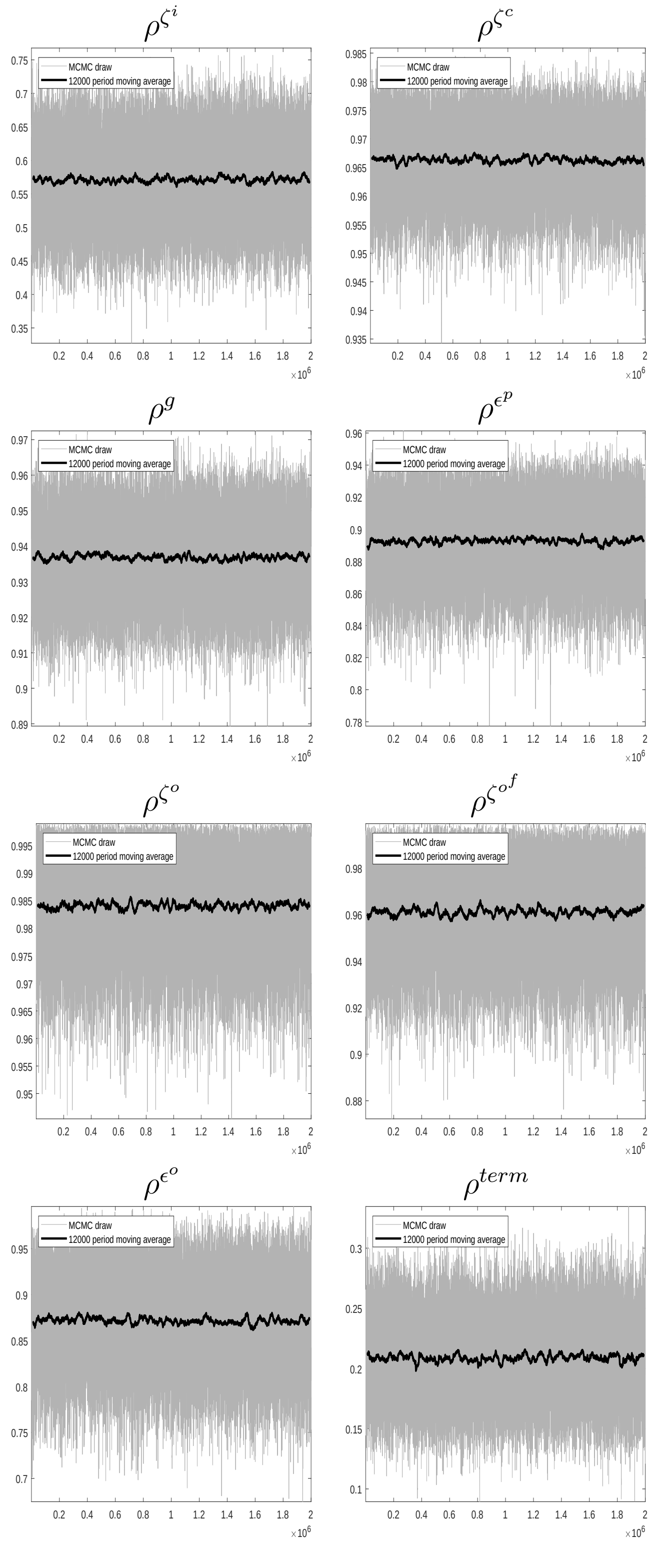
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 53: Trace plots for chain 3 CMR–Oil III



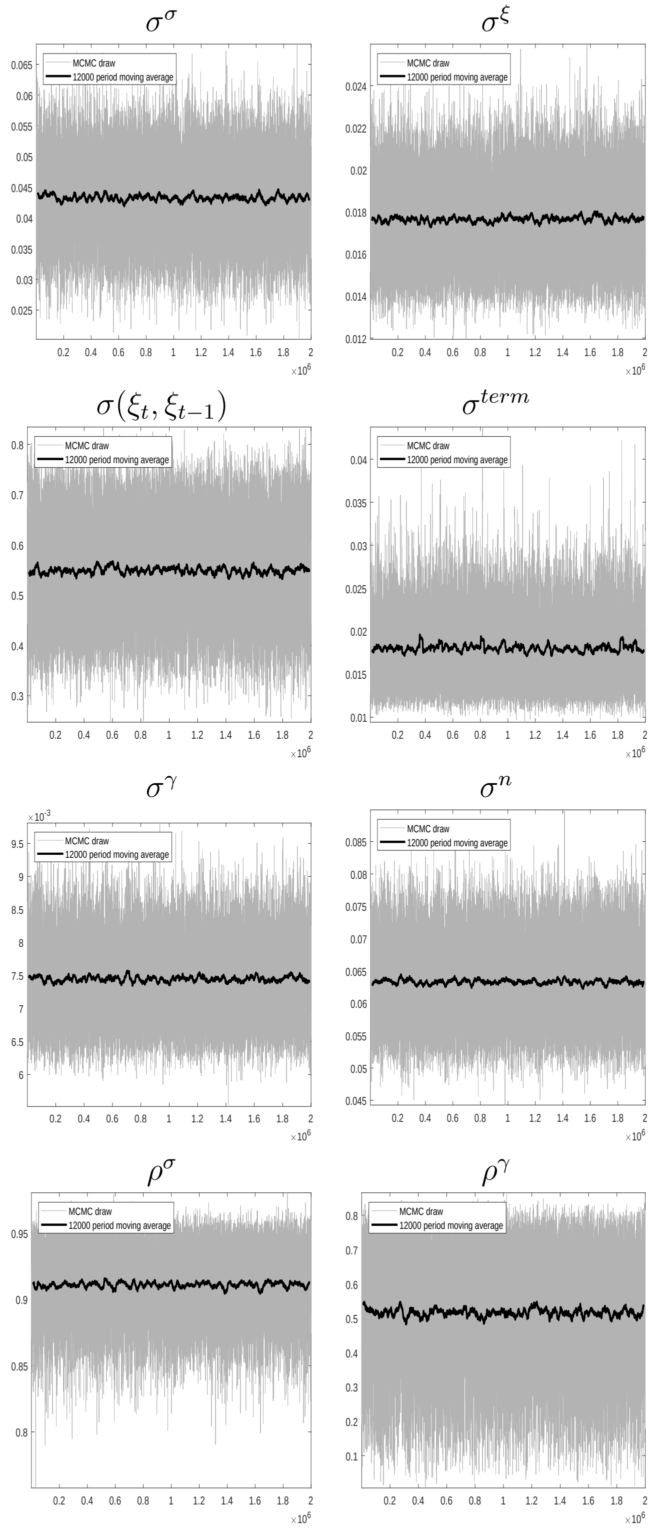
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 54: Trace plots for chain 3 CMR–Oil IV



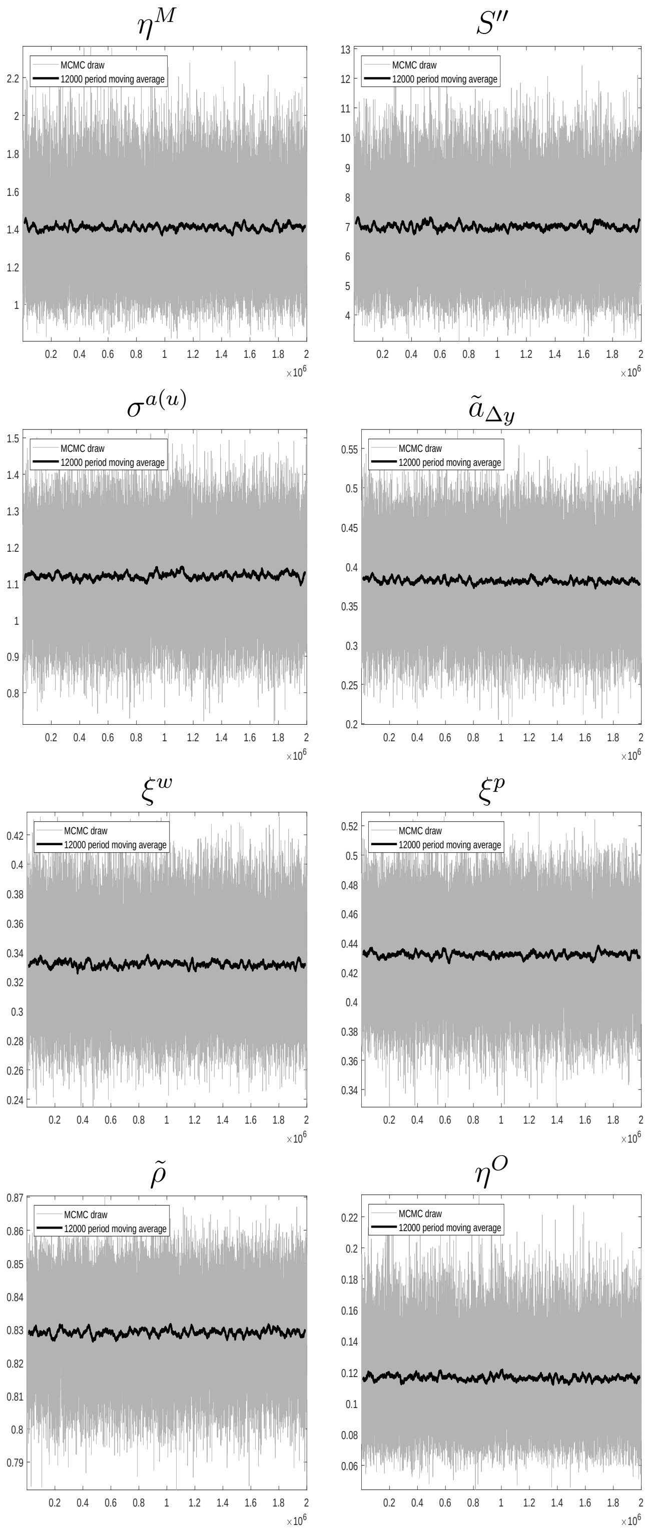
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 55: Trace plots for chain 3 CMR–Oil V



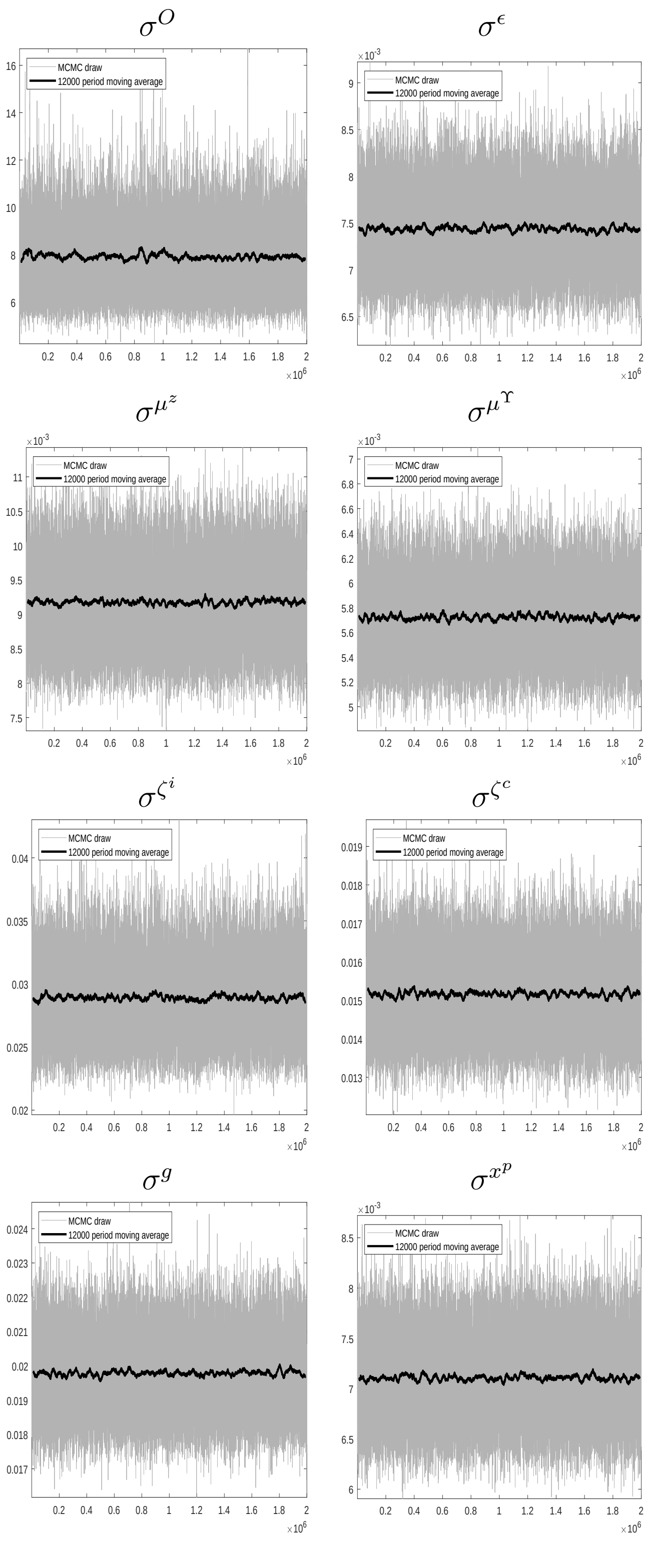
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 56: Trace plots for chain 4 CMR–Oil I



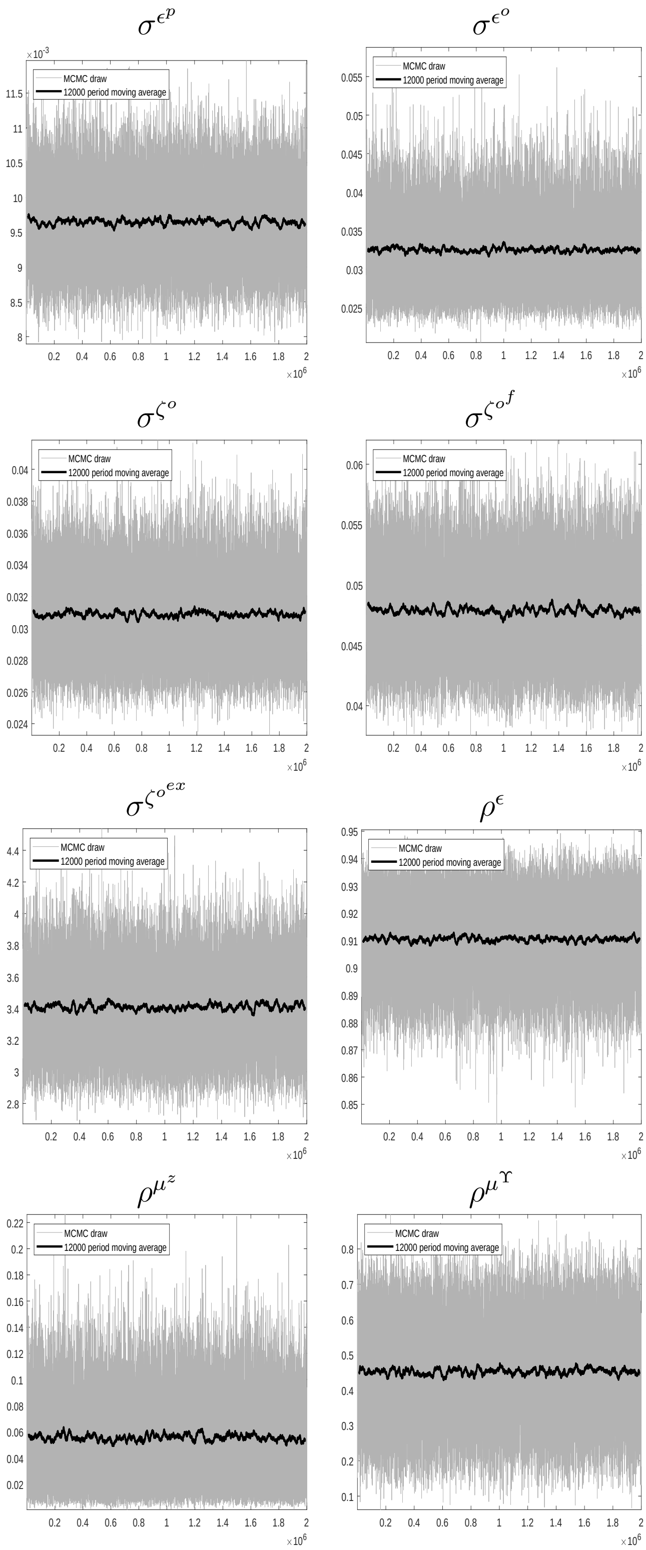
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 57: Trace plots for chain 4 CMR–Oil II



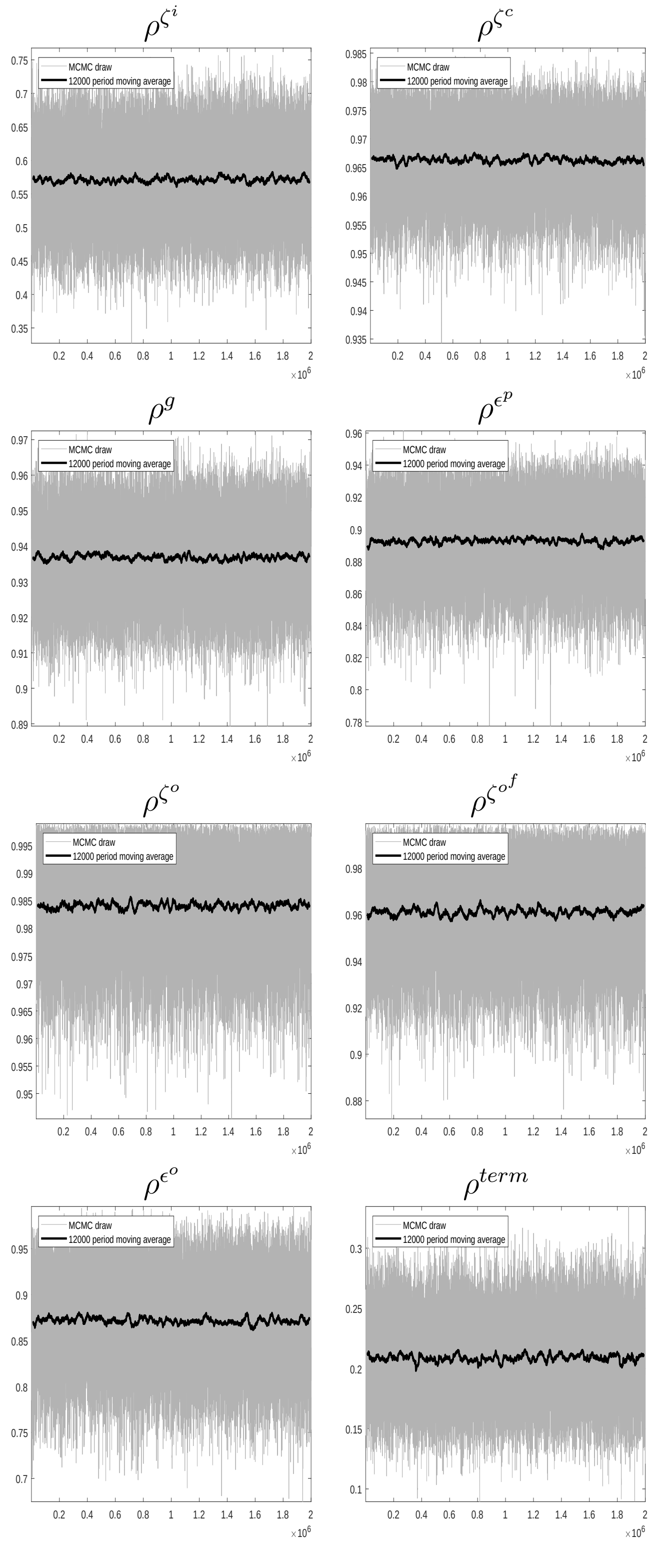
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 58: Trace plots for chain 4 CMR–Oil III



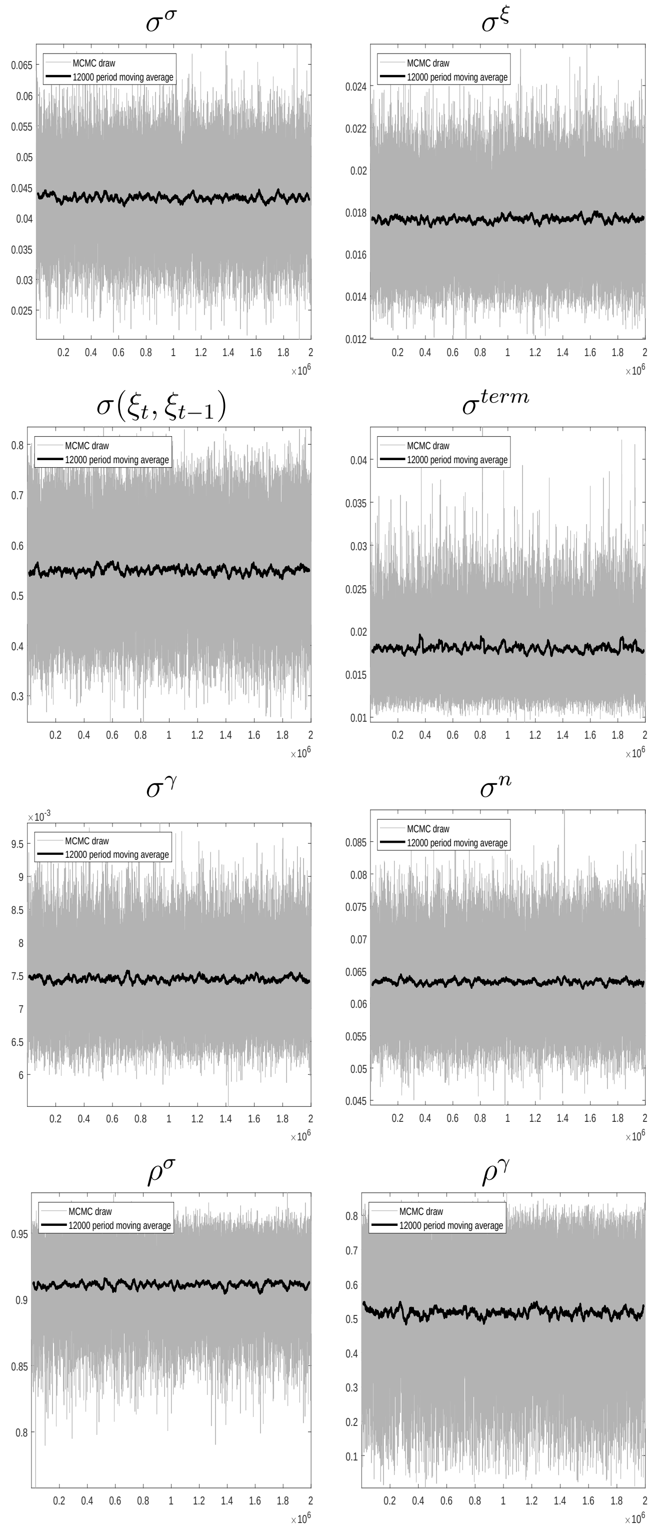
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 59: Trace plots for chain 4 CMR–Oil IV



Notes: The grey line depicts parameter values and the black line the moving average.

Figure 60: Trace plots for chain 4 CMR–Oil IV



Notes: The grey line depicts parameter values and the black line the moving average.