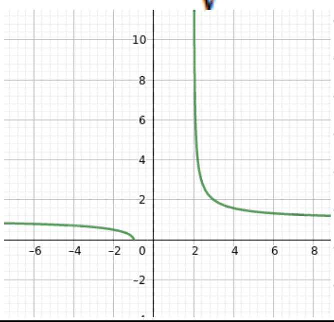
IMA1 zápisky ze cvičení

1. Cviko

1. Najděte definiční obory následujících funkcí:

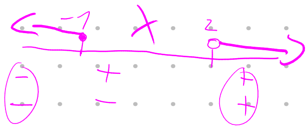
$$f_1(x) = \sqrt{\frac{x+1}{x-2}},$$



$x - 2 \neq 0 \quad | +2$

$\underline{x \neq 2}$

$\frac{x+1}{x-2} \geq 0$



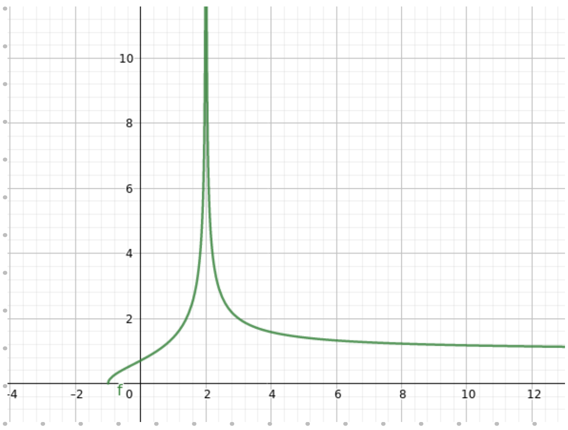
$\Rightarrow x \in (-\infty, -1\rangle \cup (2, +\infty)$

$$f_2(x) = \sqrt{\frac{x+1}{|x-2|}},$$

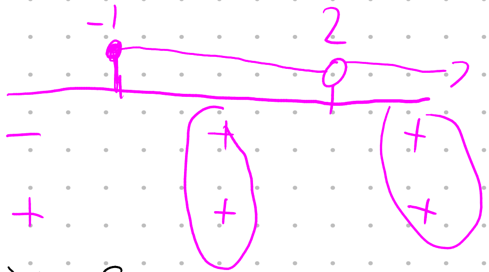
$|x-2| \neq 0$

$x - 2 \neq 0$

$\underline{x \neq 2}$



$\frac{x+1}{|x-2|} \geq 0$



$\underline{x \in \langle -1, +\infty) \setminus \{2\}}$

$$f_3(x) = \frac{\sqrt{x+1}}{\sqrt{x-2}}.$$

$x - 2 \geq 0 \quad | +2$

$\underline{x \geq 2}$

$\wedge$

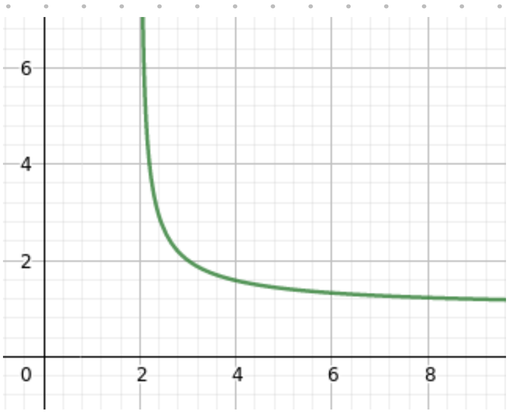
$\sqrt{x-2} \neq 0 \Rightarrow x - 2 \neq 0 \quad | +2$

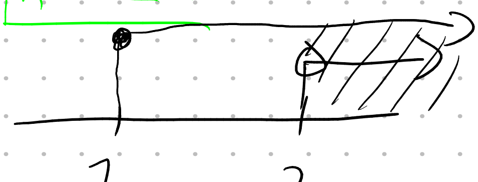
$\underline{x \neq 2}$

$\wedge$

$x + 1 \geq 0 \quad | -1$

$\underline{x \geq -1}$





$\underline{x \in (2, +\infty)}$

Příprava na test

Skupina A

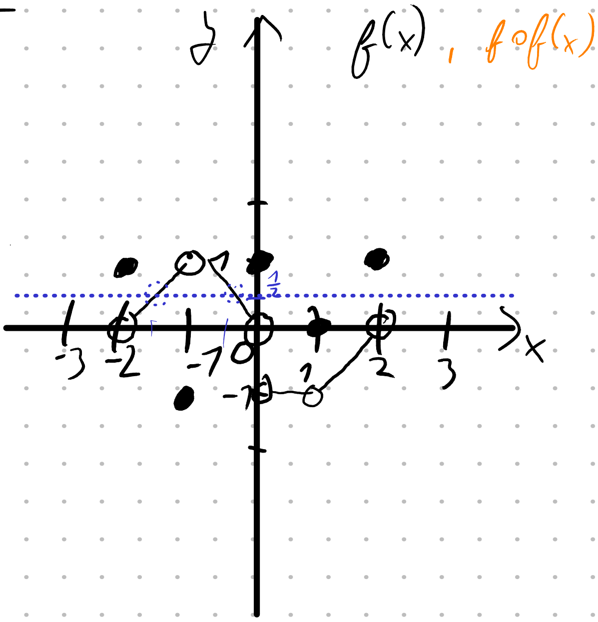
1. Nechť $f(x) = \begin{cases} 1 & x \in \{-2, 0, 2\}, \\ 0 & x \in \{1\}, \\ -1 & x \in (0,1) \cup \{-1\}, \\ x+2 & x \in (-2,-1), \\ -x & x \in (-1, 0), \\ x-2 & x \in (1, 2). \end{cases}$

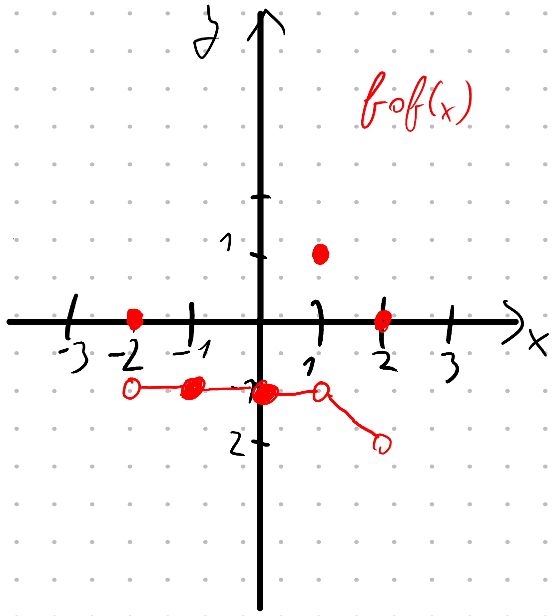
Nakreslete grafy funkcí: f, $(f \circ f)$ a určete $f^{-1}(\{\frac{1}{2}\})$.

2. Načrtnete funkci g, o které víte:

- funkce g je sudá, asymptota v $+\infty$ má rovnici: $y = 4 - x$,
- $g(0) = 1, g(2) = 2, \lim_{x\to 2^-} g(x) = 2, \lim_{x \to 2^+} g(x) = -\infty$.

1.)





$f^{-1}(\{\frac{1}{2}\}) \to f(?) = \{\frac{1}{2}\}$

$\{-1{,}5, -0{,}5\}$

$-1{,}5 + 2 = 0{,}5$

$-(-0{,}5) = 0{,}5$

$f^{-1}(\{\frac{1}{2}\}) = x$

$f^{-1}(f(x)) = x$

$f(x) = \{\frac{1}{2}\}; \; x \in \{-1{,}5; -0{,}5\}$

2.)

Načrtnete funkci g, o které víte:

- funkce g je sudá, asymptota v $+\infty$ má rovnici: $y = 4 - x$,
- $g(0) = 1, g(2) = 2, \lim_{x\to 2^-} g(x) = 2, \lim_{x \to 2^+} g(x) = -\infty$.

- sudost $f(-x) = f(x)$

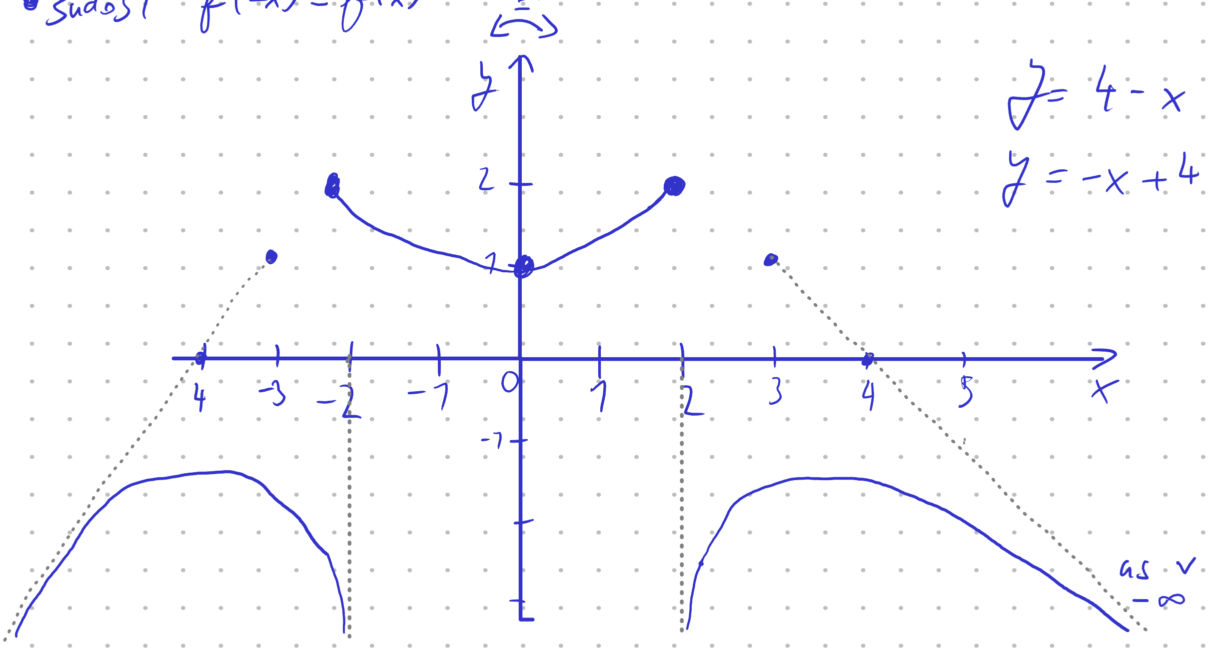
$y = 4 - x$

$y = -x + 4$

$y = ax + b$

$a = -1$

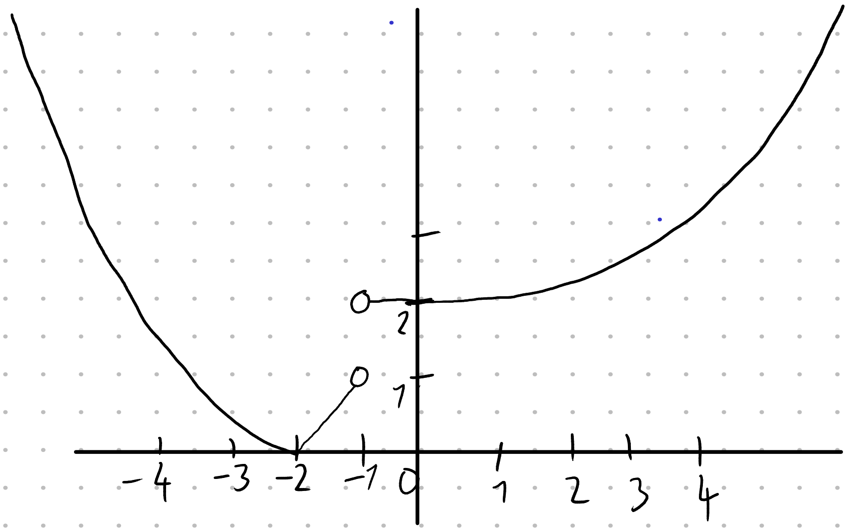
$b = 4$

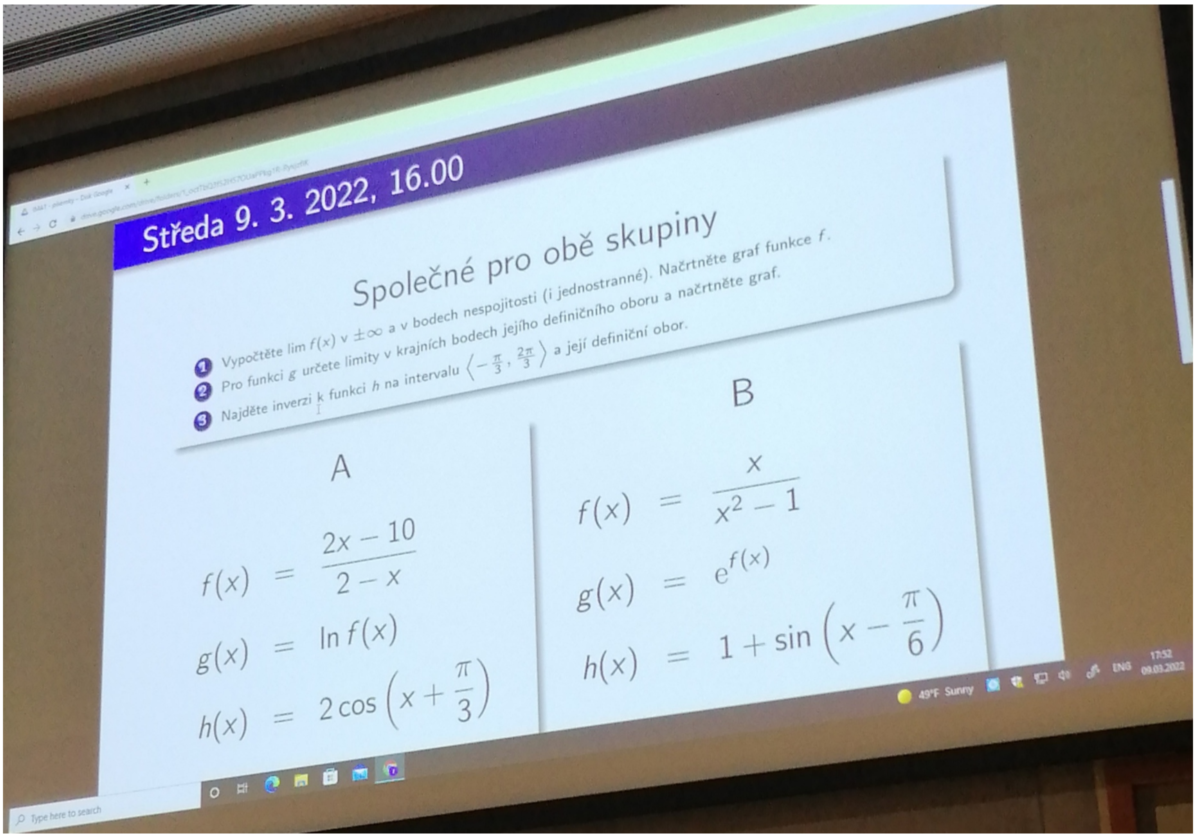


Příklad 2

Nakreslete graf funkce $f(x)$, pro kterou platí:

a) $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to-1^-} f(x) = 1$, $\lim_{x\to-1^+} f(x) = 2$, $\lim_{x\to\infty} f(x) = \infty$





A

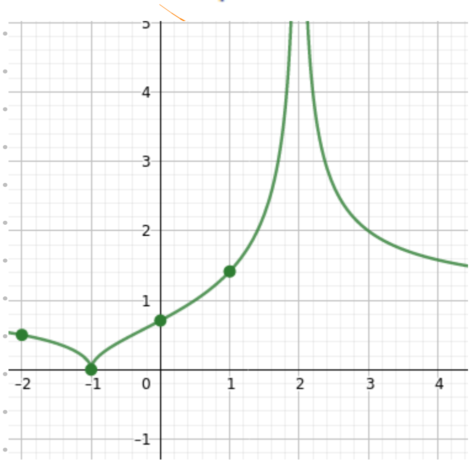
1) $\lim\limits_{x \to \pm\infty} f(x)$ + v bodech nespojitosti + graf

$\boxed{x \neq 2}$ $D_f = \mathbb{R} \setminus \{2\}$

$$f(x) = \frac{2x-10}{2-x}$$

$$\lim_{x \to +\infty} \frac{2x-10}{2-x} =$$

$$f_4(x) = \sqrt{\left|\frac{x+1}{x-2}\right|}$$



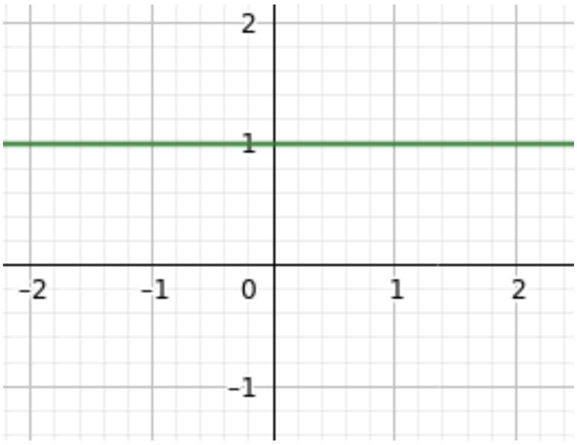
$x - 2 \neq 0 \quad | +2$

$\boxed{x \neq 2}$

$\left|\frac{x+1}{x-2}\right| \geq 0 \qquad H(\text{abs}) = \langle 0; +\infty)$

$\boxed{x \in \mathbb{R} \setminus \{2\}}$

$$f_5(x) = \sqrt{\frac{4-x^2}{4-x^2}},$$



$4 - x^2 \neq 0 \quad | -4 \quad | \cdot (-1)$

$x^2 \neq 4 \quad | \sqrt[2]{\ }$

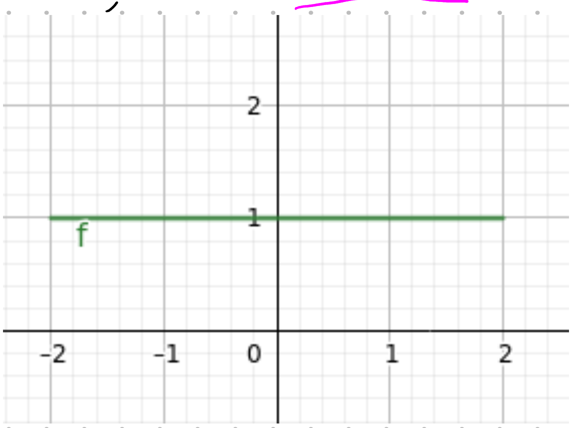
$\boxed{x \neq \pm 2}$

$\frac{4-x^2}{4-x^2} \geq 0 \qquad \boxed{\frac{a}{a} = 1}$

$\Rightarrow \quad \frac{4-x^2}{4-x^2} = 1$

$\boxed{x \in \mathbb{R} \setminus \{-2; +2\}}$

$$f_6(x) = \sqrt{\frac{4-x^2}{|4-x^2|}},$$



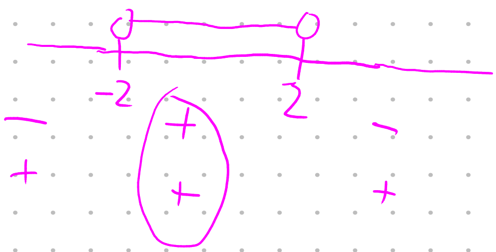
$|4 - x^2| \neq 0 \quad |$ nezáleží na $\pm$

$4 - x^2 \neq 0 \quad | -4 \quad | \cdot (-1)$

$x^2 \neq 4 \quad | \sqrt[2]{\ }$

$\boxed{x \neq \pm 2}$

$\frac{4-x^2}{|4-x^2|} \geq 0$



$\boxed{x \in (-2; 2)}$

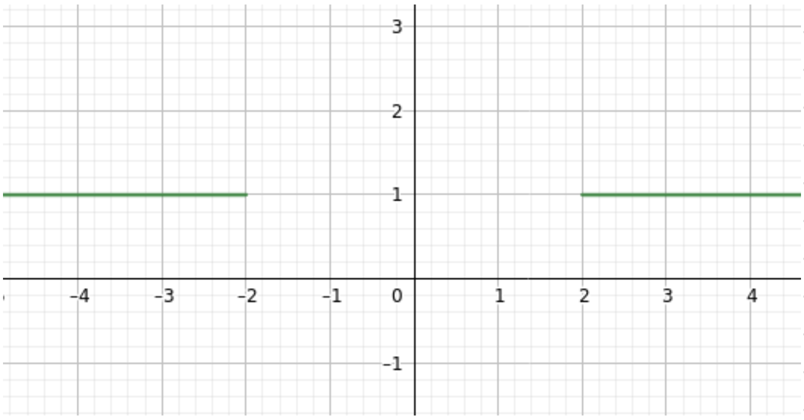
$$f_7(x) = \sqrt{\frac{x^2-4}{|4-x^2|}}$$

$|4-x^2| \neq 0 \quad \backslash$ nezáleží na $\pm$

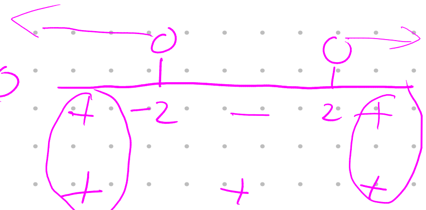
$4-x^2 \neq 0 \quad \backslash -4 \quad \backslash \cdot(-1)$

$x^2 \neq 4 \quad \backslash \sqrt{\ }$

$\boxed{x \neq \pm 2}$



$$\frac{x^2-4}{|4-x^2|} \geq 0$$

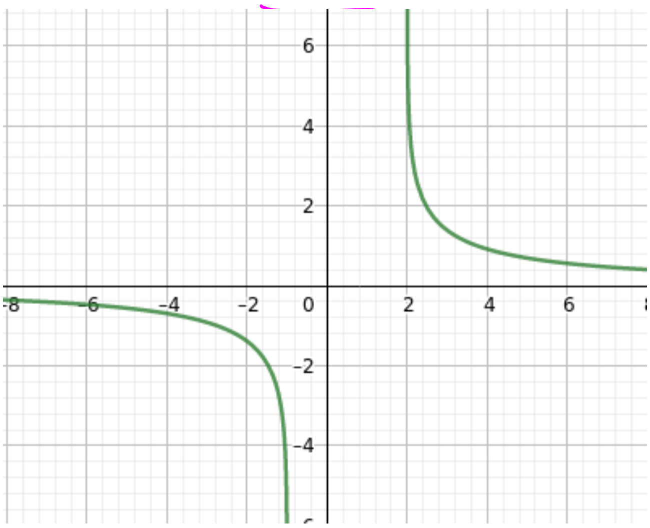


$\boxed{x \in (-\infty; -2) \cup (+2; +\infty)}$

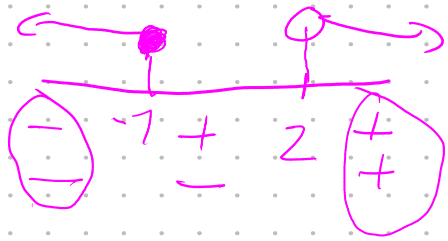
$$f_8(x) = \ln \frac{x+1}{x-2},$$

$x-2 \neq 0 \quad \backslash +2$

$\boxed{x \neq 2}$



$$\frac{x+1}{x-2} > 0$$

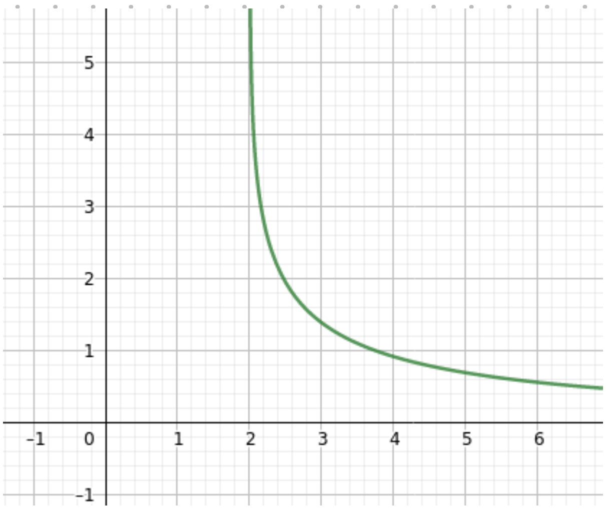


$\boxed{x \in (-\infty; -1) \cup (2; +\infty)}$

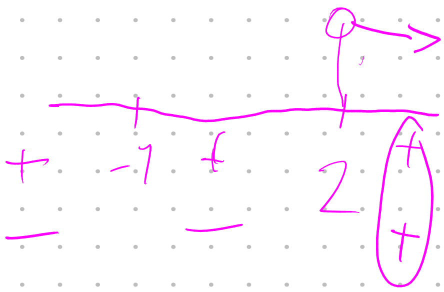
$$f_9(x) = \ln \frac{|x+1|}{x-2}$$

$x-2 \neq 0 \quad \backslash +2$

$\boxed{x \neq 2}$

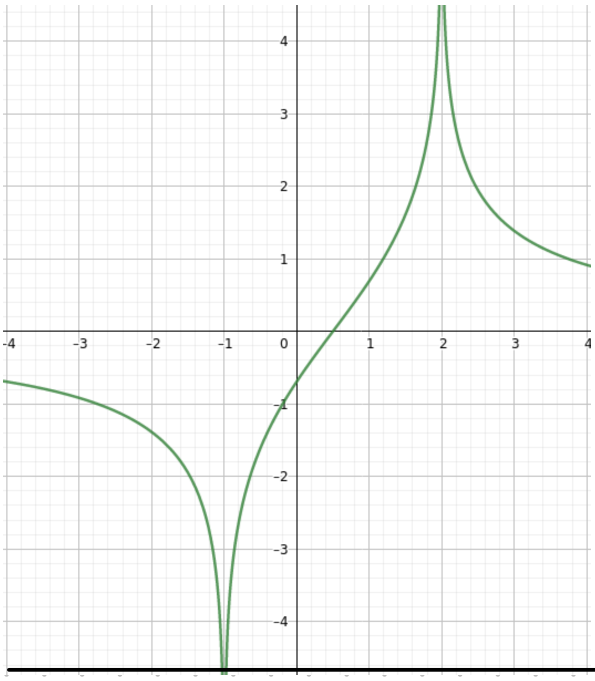


$$\frac{|x+1|}{x-2} > 0$$



$\boxed{x \in (2, +\infty)}$

$$f_{10}(x) = \ln\left|\frac{x+1}{x-2}\right|$$



$x-2 \neq 0 \quad \backslash +2$

$x \neq 2$

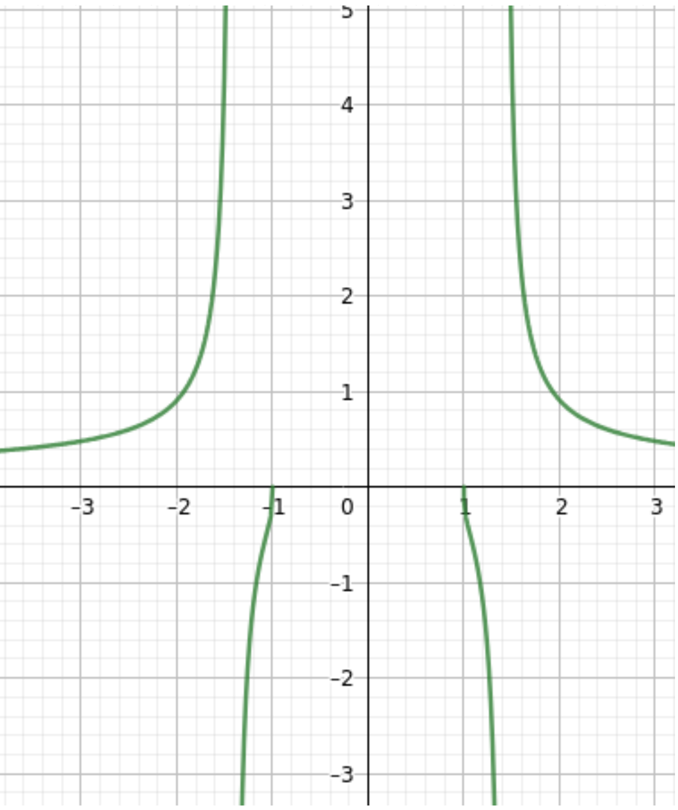
$\left|\frac{x+1}{x-2}\right| > 0$

$x+1 \neq 0 \quad \backslash -1$

$x \neq -1$

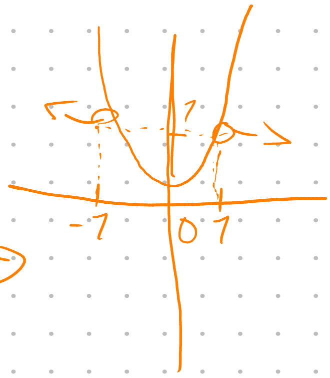
$x \in \mathbb{R} - \{-1, 2\}$

$$f_{11}(x) = \frac{1}{\ln(x^2-1)}$$



$x^2 - 1 > 0 \quad \backslash +1$

$x^2 > 1 \quad \backslash \sqrt{\ }$

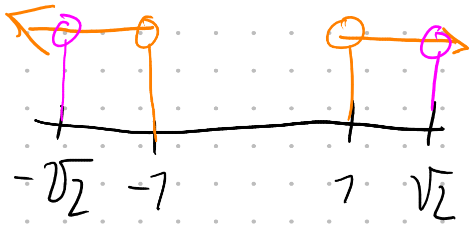


$\ln(x^2-1) \neq 0$

$\ln(x) = 0 \Rightarrow x = 1$

$x^2 - 1 \neq 1 \quad \backslash +1 \quad \backslash \sqrt{\ }$

$x \neq \pm\sqrt{2} \qquad \sqrt{2} \cong 1,414$



$x \in (-\infty; -1) \setminus \{-\sqrt{2}\} \cup$
$(1; +\infty) \setminus \{\sqrt{2}\}$

Alternativní postup:

$$-x \qquad x^2 > 1 \qquad +x$$

$x^2 > 1 \ |\sqrt[3]{\ }$

$x < 1$

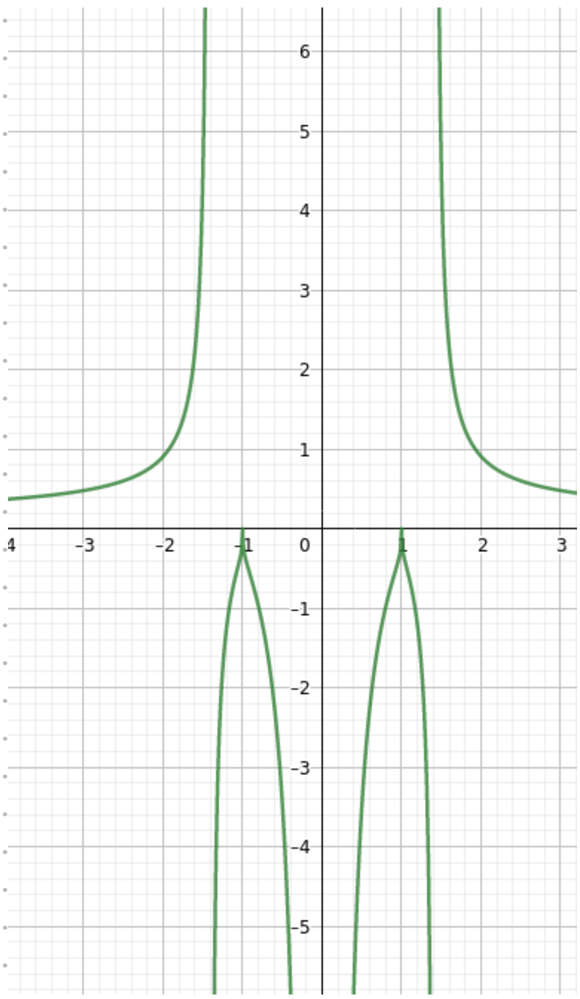
!

$x^2 > 1 \ |\sqrt[3]{\ }$

$x > 1$

Změna znaménka pokud $x \in (-\infty, 0)$

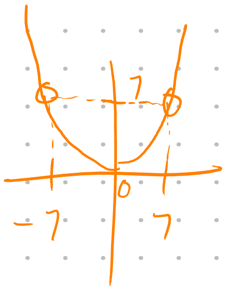
$f_{12}(x) = \dfrac{1}{\ln|x^2-1|}$



$|x^2-1| > 0$

$x^2 - 1 \neq 0 \ |+1 \ |\sqrt[3]{\ }$

$x \neq \pm 1$



$\ln|x^2-1| \neq 0$

$\ln(x) = 0 \Rightarrow x = 1$

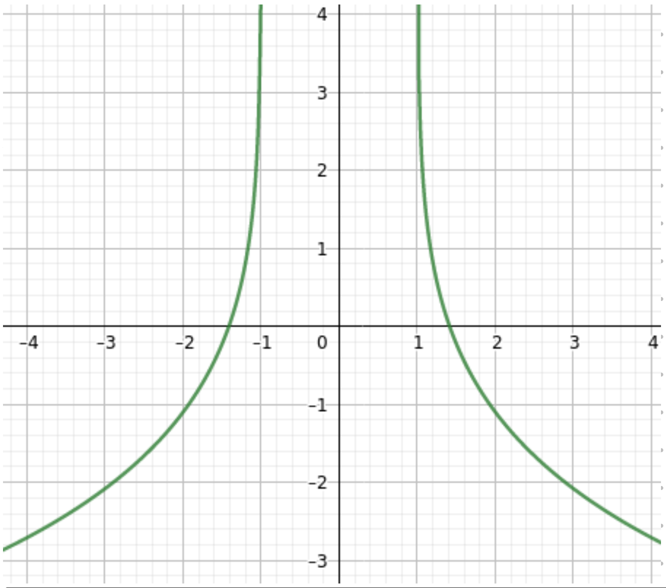
$x^2 - 1 \neq 1 \ |+1$

$x^2 \neq 2 \ |\sqrt[3]{\ }$

$x \neq \pm\sqrt{2}$

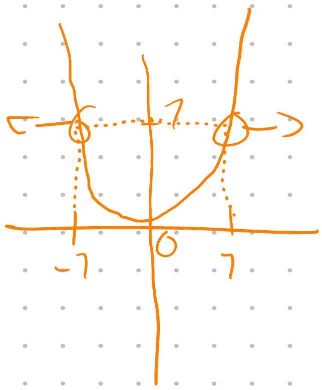
$x \in \mathbb{R} \setminus \{-\sqrt{2}, -1, 1, \sqrt{2}\}$

$f_{13}(x) = -\ln(x^2 - 1)$



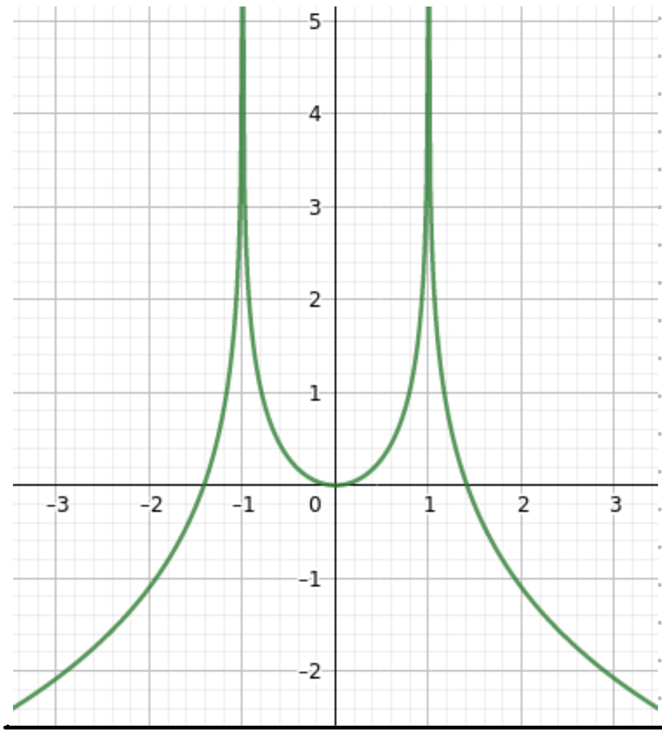
$x^2 - 1 > 0 \quad |+1$

$x^2 > 1$



$x \in (-\infty, -1) \cup (1, +\infty)$

$f_{14}(x) = -\ln|x^2 - 1|$



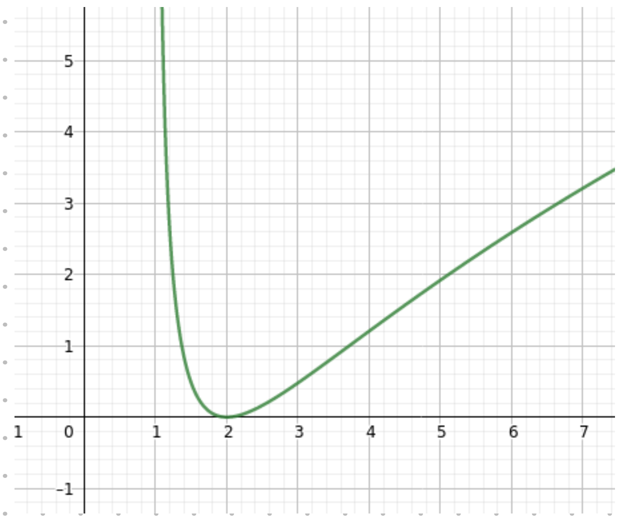
$|x^2 - 1| > 0$

$x^2 - 1 \neq 0 \quad |+1 \quad |\sqrt{\ }$

$x \neq \pm 1$

$x \in \mathbb{R} \setminus \{-1, +1\}$

$f_{15}(x) = \ln(x - 1)^2.$

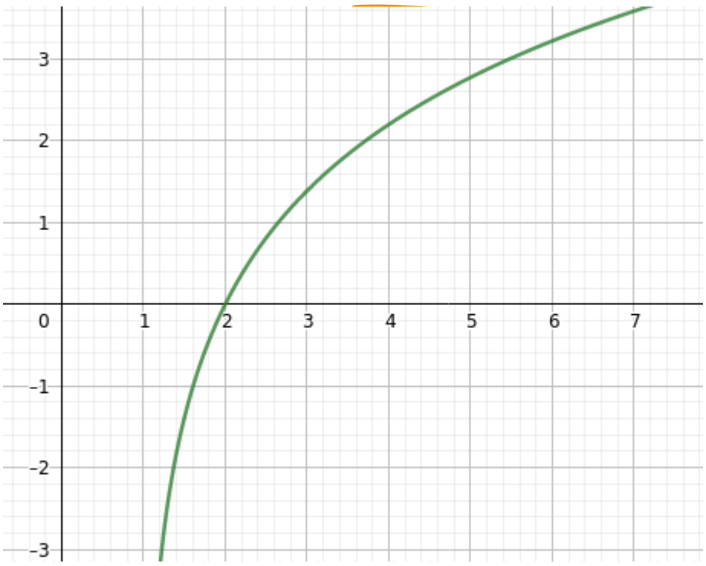


$x - 1 > 0 \quad |+1$

$x > 1$

$x \in (1; +\infty)$

$f_{16}(x) = 2\ln(x-1)$



$x - 1 < 0 \quad | +1$

$x < 1$

$x \in (1, +\infty)$

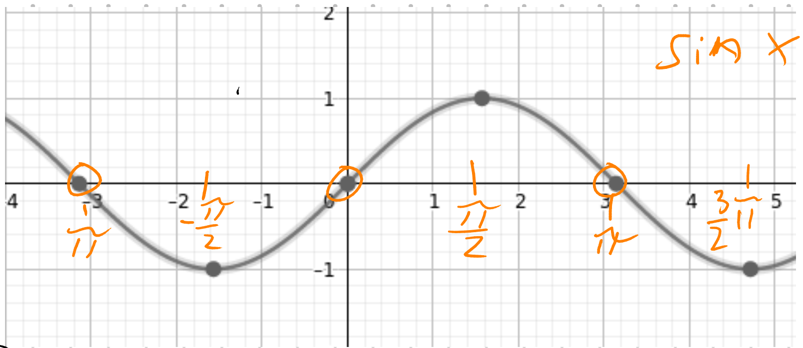
$f_{17}(x) = \sqrt{\frac{x}{\sin x}}$,

$\sin x \neq 0$

$D(\sin x) = \mathbb{R};\ H(\sin x) = \langle -1, 1\rangle$

$D(\frac{1}{x}) = \mathbb{R} \setminus \{0\}$

$D(\sqrt[2]{x}) = \langle 0, +\infty)$

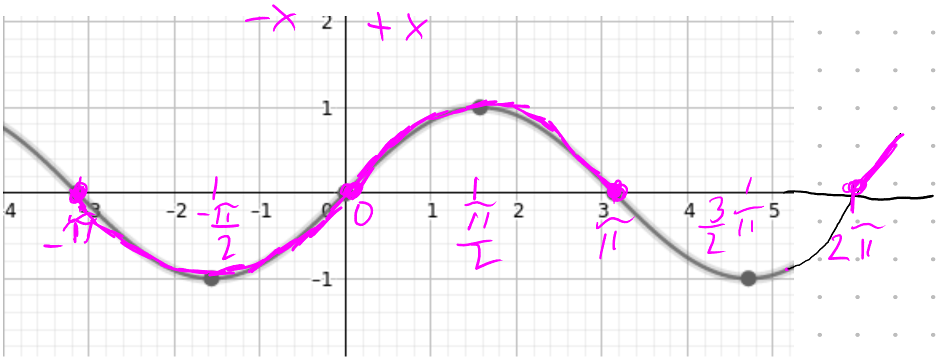


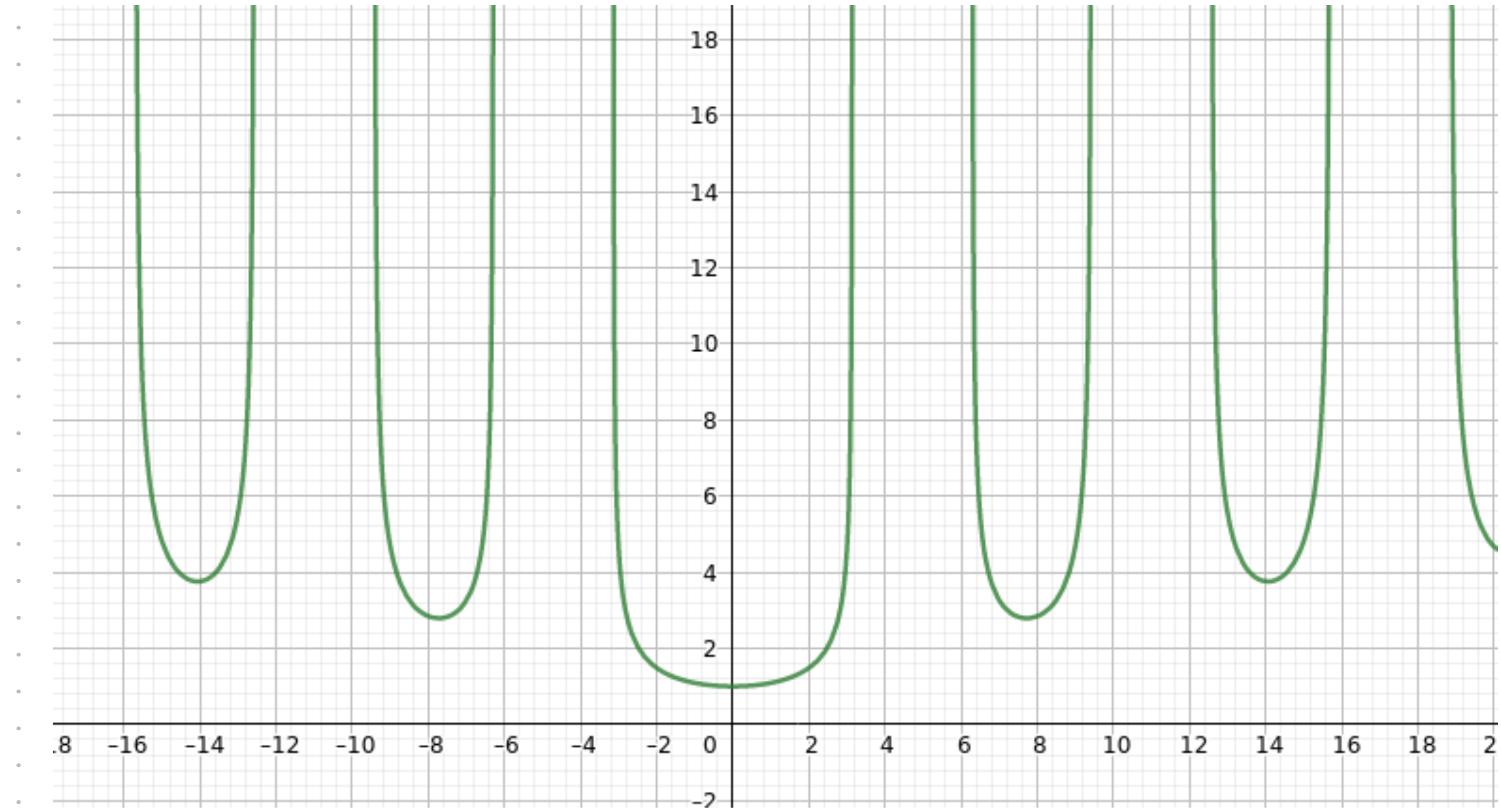
$x \in \mathbb{R} \setminus \{k\pi;\ k \in \mathbb{N}_0 \ldots k \in 0, 1, 2, \ldots\}$

$x \in \left({}^{+}\{\langle \pi k; \pi(k+1)\rangle, k \in \mathbb{N}_0\} \cup {}^{-}\{\langle \pi(-k+1); \pi\cdot(-k)\rangle; k \in \mathbb{N}_0\}\right) \setminus \{k\pi;\ k \in \mathbb{N}_0\}$

$\frac{x}{\sin x} \geq 0$

$x \in {}^{+}\{\langle \pi k; \pi(k+1)\rangle, k \in \mathbb{N}_0\} \cup {}^{-}\{\langle \pi\cdot(-(k-1)); \pi\cdot(-k)\rangle; k \in \mathbb{N}_0\}$

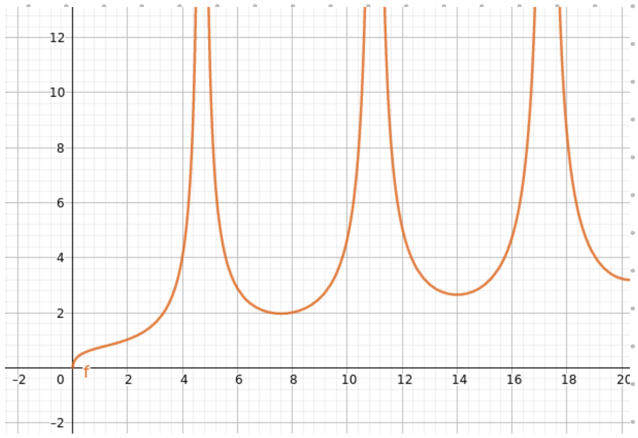


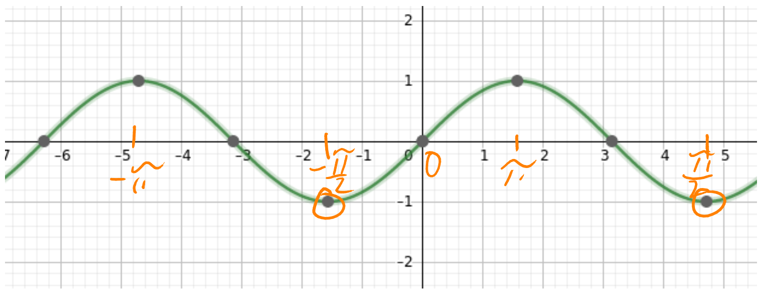


$$f_{18}(x) = \sqrt{\frac{x}{1+\sin x}}$$

$1+\sin x \neq 0 \quad \setminus -1$

$\sin x \neq -1$



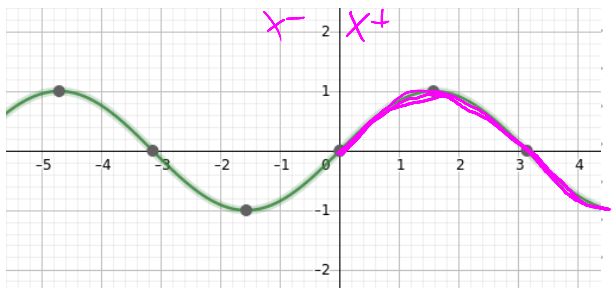


$x \in \langle 0, +\infty) \setminus \{\frac{\pi}{2} + k\pi; k \in \mathbb{Z}\}$

$x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi; k \in \mathbb{Z}$

$k \in \{\dots; -3; -2; -1; 0; 1; 2; 3; \dots\}$

$$\frac{x^{+}}{1+\sin x_{+}} \geq 0$$



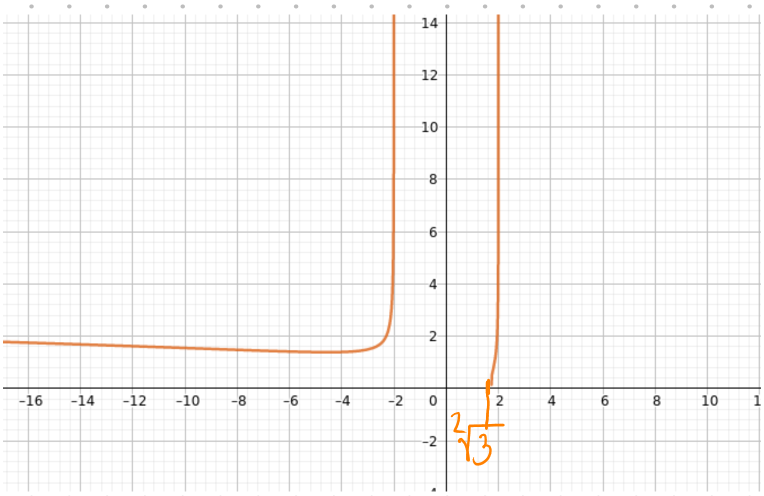
$1+\sin x$

$H(\sin x) = \langle -1, +1\rangle \;;\; H((\sin x)+1) = \langle 0, +2\rangle$

vždy kladné

$x \in \langle 0, +\infty)$

$f_{19}(x) = \sqrt{\dfrac{1-x}{\ln(x^2-3)}}$,



$x^2-3>0 \mid +3$

$+ \quad x^2>3 \quad -$

$x^2>3 \mid \sqrt[2]{\ }$ $\qquad$ $x^2>3 \mid \sqrt[2]{\ }$

$x>\sqrt{3}$ $\qquad$ $x<\sqrt{3}$

$\underline{x \in (-\infty, \sqrt{3}) \cup (\sqrt{3}; +\infty)}$

$\boxed{\sqrt[2]{3} \doteq 1,732}$

$\underline{x \in (-\infty; -2) \cup (2; +\infty)}$

$\ln(x^2-3) \neq 0$ $\qquad$ $\boxed{x^2-3 \neq e^0}$

$x^2-3 \neq 1 \mid +3 \mid \sqrt[2]{\ }$

$\underline{x \neq \pm 2}$

$\dfrac{1-x}{\ln(x^2-3)} \geq 0$

$-2 \quad -\sqrt{3} \quad 1 \quad \sqrt{3} \quad 2$

$+ \quad + \quad - \quad -$

$+ \quad - \quad - \quad +$

$$f_{20}(x) = \sqrt{\frac{x}{e+\ln x}}$$

$\boxed{x > 0}$

$e + \ln x \neq 0 \quad | -e$

$\ln x \neq -e$

$\boxed{x \neq e^{-e}}$

$\boxed{e \approx 2{,}718}$

$\ln e = 1$

$\Updownarrow$

$e^1 = e$

$\Updownarrow$

$\log_e e = 1$

$\log_e x = 1$

$$\frac{x}{e+\ln x} \geq 0$$

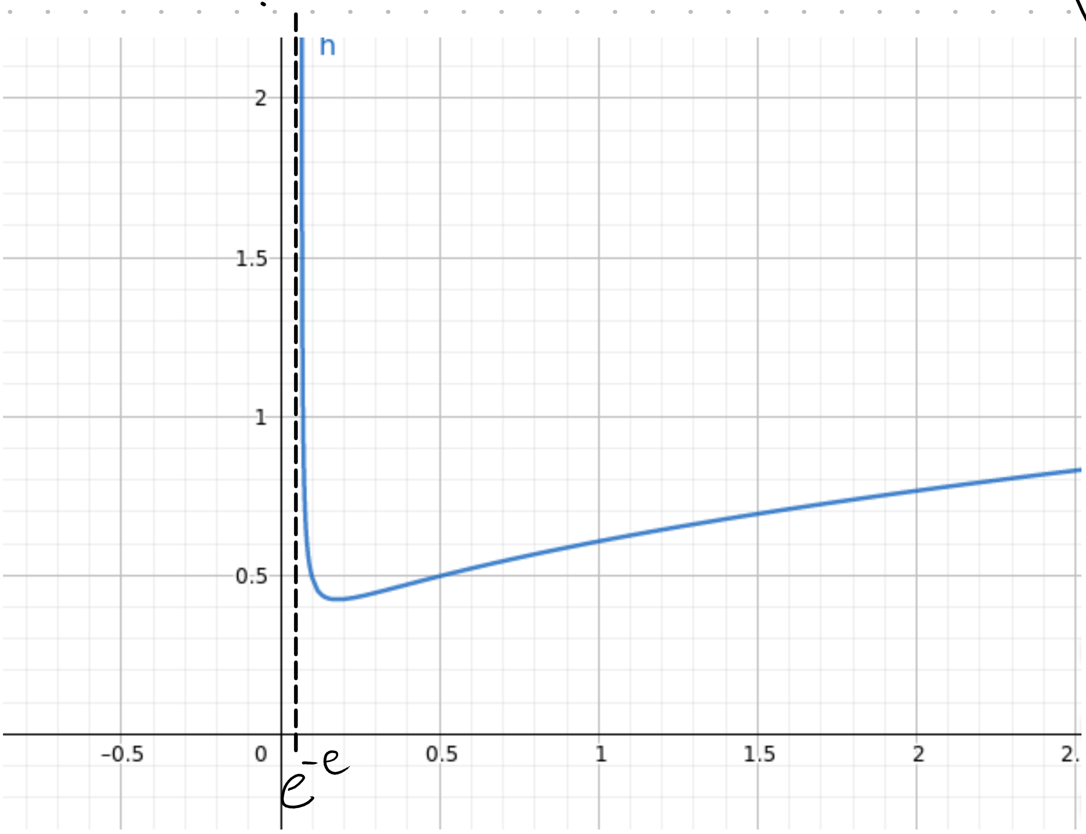
x^+

$0 \quad e^{-e} \quad 1$

$+ \quad - \quad +$

$\boxed{x \in (e^{-e}; +\infty)}$

$\ln e^{-e} = -e$



$e^{-e} \approx 0{,}06598$

2. Zjistěte, pro která $x \in \mathbb{R}$ se následující funkce rovnají:

- $h_1(x) = -1, \quad g_1(x) = \frac{|x-1|}{x-1}$

$x - 1 \neq 0 \quad | +1$

$x \neq 1$

fce f a g se rovnají když:

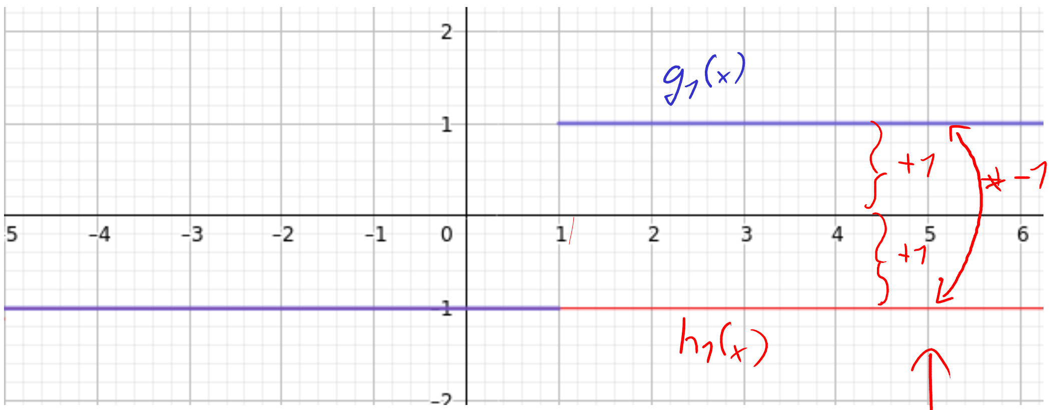
$- \; D(f) = D(g) \quad \wedge$

$- \; \forall x \in D(f) \;\; f(x) = g(x)$

$D(h_1) = \mathbb{R}$

$D(g_1) = \mathbb{R} \setminus \{1\}$

maximálně se rovnají na $\mathbb{R} \setminus \{1\}$



$h_1 \stackrel{?}{=} g_1$

$-1 = \frac{|x-1|}{x-1} \quad | \cdot (x-1)$

$-(x-1) = |x-1|$

$-x+1 = |x-1|$

$x < 1$

$-x+1 = -x+1$

$\boxed{h_1 = g_1 \text{ pro } \forall x \in (-\infty; 1)}$

$x > 1$

$-x+1 \neq x-1 \quad | +1$

$-x+2 \neq x$

$h_1 \neq g_1 \text{ pro } \forall x \in (1; +\infty)$

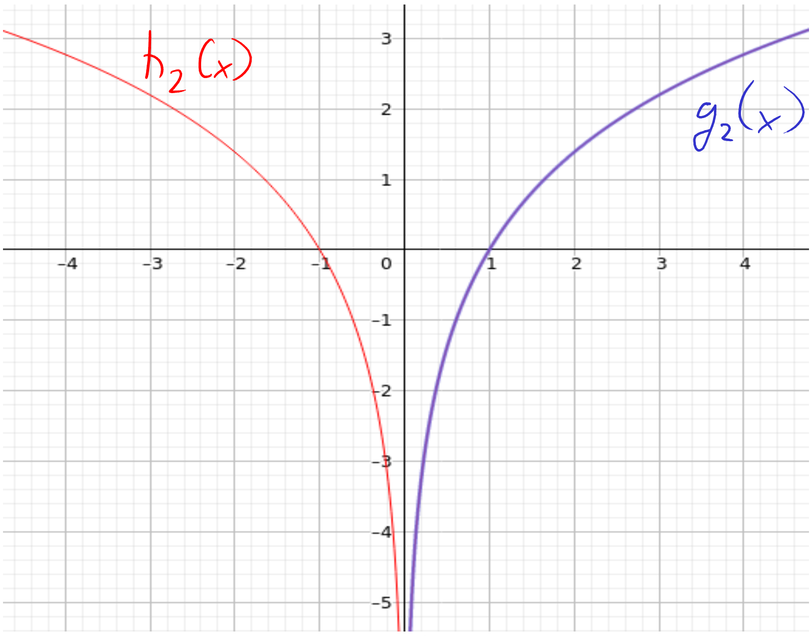
$\underset{h_1}{-x+2} = \underset{g_1}{x}$

znamená že h_1 je oproti g_1

- záporná
- ve výšce (y) o dva výš

$\boxed{\forall x \in (-\infty; 1) ; \; h_1 = g_1}$

$h_2(x) = \ln x^2, \quad g_2(x) = 2\ln x$



1) test $D(h_2) \overset{?}{=} D(g_2)$

$x^2 > 0$

$-$ $+$

$x<0 \qquad x>0$

$x \neq 0$

$x > 0$

$\forall x \in (0, +\infty)\,;\; D(h_2) = D(g_2)$

2) test ekv. na $(0, +\infty)$

$\underset{h_2}{\ln x^2} = \underset{g_2}{2\ln x}$

$2\ln x = 2\ln x \quad \backslash\, 3.$

$\forall x \in (0, +\infty)\,;\; h_2 = g_2$ [redacted]

Pravidla pro práci s ln(x)

Rule or special case	Formula
Product	$\ln(xy) = \ln(x) + \ln(y)$
Quotient	$\ln(x/y) = \ln(x) - \ln(y)$
3. Log of power	$\ln(x^y) = y\ln(x)$
Log of e	$\ln(e) = 1$
Log of one	$\ln(1) = 0$
Log reciprocal	$\ln(1/x) = -\ln(x)$

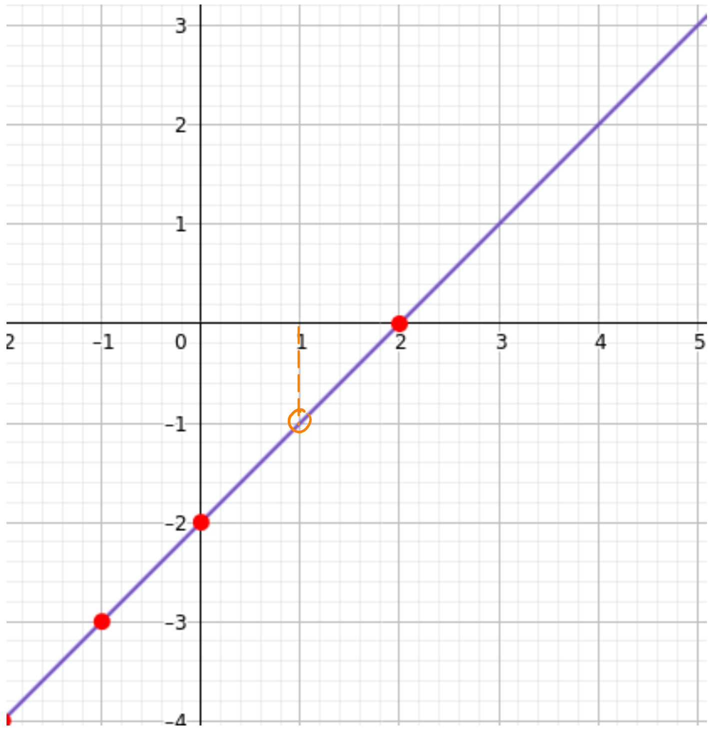
3. lepší $\ln(x^2) = 2\ln|x|$

$\ln x = \log_e x$

$\log_e e = 1 \;\rightarrow\; e^1 = e$

$\log_e 1 = 0 \;\rightarrow\; e^0 = 1$

$$h_3(x) = \frac{x^2-3x+2}{x-1}, \quad g_3(x) = x-2$$



1) $D(h_3) = \mathbb{R} \setminus \{1\}$ $\quad D(g_3) = \mathbb{R}$

$x - 1 \neq 0 \quad \backslash +1$

$\boxed{x \neq 1}$

2) $\dfrac{x^2 - 3x + 2}{x-1} = x - 2$

vysvětlení!

$(x+a)\cdot(x+b) \quad \backslash$ roznásobit

$x^2 + (x\cdot b) + (a\cdot x) + (a\cdot b)$

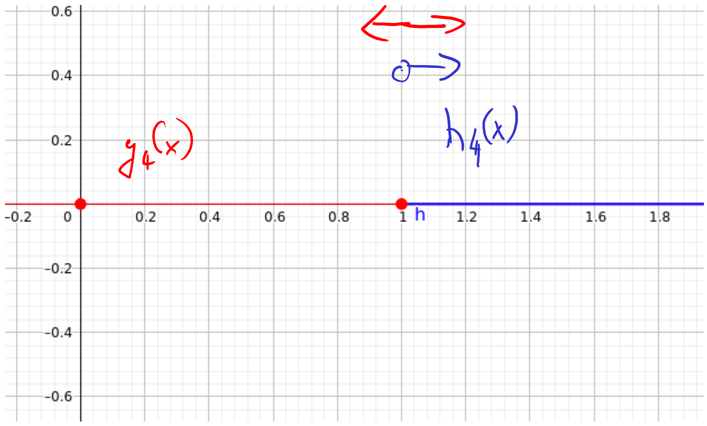
$x^2 + (a+b)\cdot x + (a\cdot b)$

$\dfrac{(x-1)\cdot(x-2)}{x-1} = x - 2 \quad \Big\backslash \quad \dfrac{a\cdot x}{a} = x$

$\boxed{x - 2 = x - 2}$

$\boxed{\forall x \in \mathbb{R}\setminus\{1\};\ h_3(x) = g_3(x)}$

$$h_4(x) = \ln\frac{x-1}{|x-1|}, \quad g_4(x) = 0$$



1) $|x-1| \neq 0 \quad \backslash$ nezáleží na $\pm$

$x - 1 \neq 0 \quad \backslash +1$

$\boxed{x \neq 1}$

$D(h_4) = \mathbb{R} \setminus \{1\}$

$D(g_4) = \mathbb{R}$

2) $\ln \dfrac{x-1}{|x-1|} = 0$ $\quad \boxed{\ln 1 = 0}$

$\dfrac{x-1}{|x-1|} = 1 \quad \backslash \cdot |x-1|$

$x - 1 = |x-1|$

$x > 1$: $\boxed{x-1 = x-1}$

$x < 1$: $x - 1 \neq -x + 1$

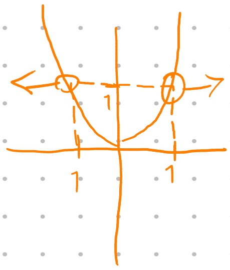
$x - 1 \neq -(x-1)$

$\boxed{\forall x \in (1; +\infty);\ h_4(x) = g_4(x)}$

$$h_5(x) = \frac{1}{\ln(x^2-1)}, \quad g_5(x) = -\ln(x^2-1)$$

1) $x^2-1 > 0 \quad \backslash +1$

$x^2 > 1$



$\boxed{x \in (-\infty,1) \cup (1,+\infty)}$

$x^2-1>0$

$\vdots$

$\boxed{x \in (-\infty,1) \cup (1,+\infty)}$

$\ln(x^2-1) \neq 0 \quad , \quad \ln 1 = 0$

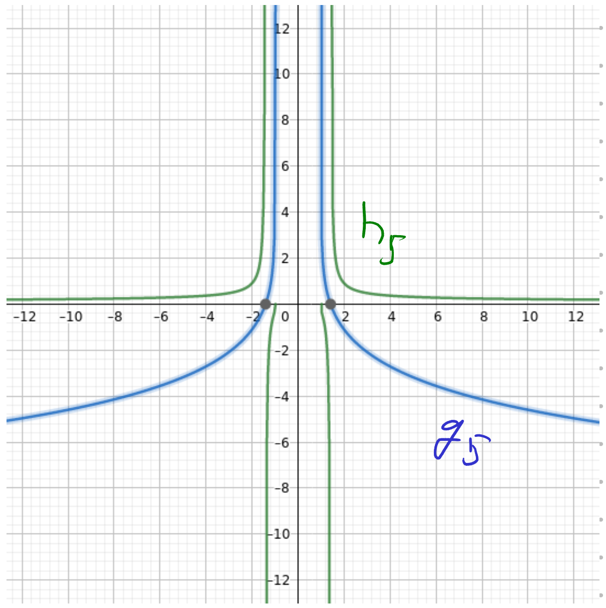
$x^2-1 \neq 1 \quad \backslash +1$

$x^2 \neq 2 \quad \backslash \sqrt[2]{\ }$

$\boxed{x \neq \pm\sqrt{2}}$

$D(h_5) = \big((-\infty,1) \cup (1;+\infty)\big) \setminus \{\pm\sqrt{2}\}$

$\underline{D(g_5) = (-\infty,1) \cup (1;+\infty)}$



2) $h_5 \qquad g_5$

$$\frac{1}{\ln(x^2-1)} = -\ln(x^2-1) \quad \backslash \cdot \ln(x^2-1)$$

$$1 = -\ln(x^2-1) \cdot \ln(x^2-1)$$

což je vlastně

$1 = -a \cdot a$

avšak

$\forall a \in \mathbb{R};\ -a \cdot a \neq 1 \implies$ $\underline{\forall x \in D(h_5):\ h_5(x) \neq g_5(x)}$

fce h_5 a g_5 se nikde nerovnají

$1 \cdot (-1) = -1$

$-1 \cdot (1) = -1$

- $h_6(x) = \sqrt{\frac{x+1}{x-2}}$ $\quad g_6(x) = \frac{\sqrt{x+1}}{\sqrt{x-2}}$

1)

$x-2 \neq 0 \quad |+2$

$\boxed{x \neq 2}$

$x-2 \geq 0 \quad |+2$

$\boxed{x \geq 2}$

$\frac{x+1}{x-2} \geq 0$

$-1 \qquad 2$

$- \quad + \quad +$

$- \quad - \quad +$

$\boxed{x \in (-\infty, -1\rangle \cup (2, +\infty)}$

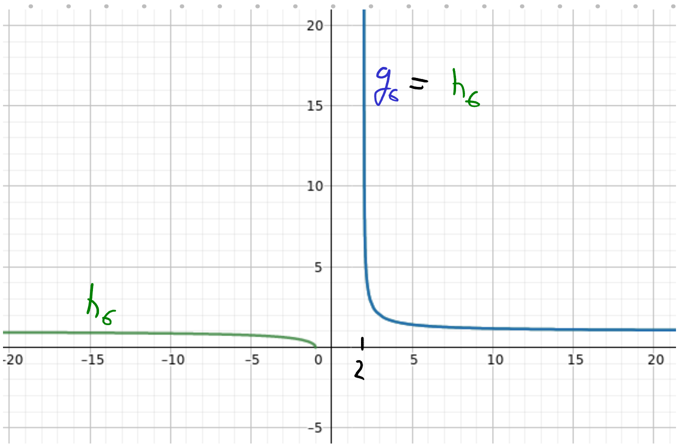
$\sqrt{x-2} \neq 0 \;, \quad \sqrt{0} = 0$

$x-2 \neq 0$

$\boxed{x \neq 2}$

$D(h_6) = (-\infty, -1\rangle \cup (2, +\infty)$

$D(g_6) = (2; +\infty)$



2)

$\sqrt{\frac{x+1}{x-2}} = \frac{\sqrt{x+1}}{\sqrt{x-2}} \qquad \Big| \quad \sqrt[2]{x} = x^{\frac{1}{2}} \quad 1.$

$\left(\frac{x+1}{x-2}\right)^{\frac{1}{2}} = \frac{(x+1)^{\frac{1}{2}}}{(x-2)^{\frac{1}{2}}} \qquad \Big| \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad 2.$

$\boxed{\frac{(x+1)^{\frac{1}{2}}}{(x-2)^{\frac{1}{2}}} = \frac{(x+1)^{\frac{1}{2}}}{(x-2)^{\frac{1}{2}}}}$

Exponent Rules For $a \neq 0, b \neq 0$		
Product Rule	$a^x \times a^y = a^{x+y}$	
Quotient Rule	$a^x \div a^y = a^{x-y}$	
Power Rule	$(a^x)^y = a^{xy}$	
Power of a Product Rule	$(ab)^x = a^x b^x$	
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	2.
Zero Exponent	$a^0 = 1$	
Negative Exponent	$a^{-x} = \frac{1}{a^x}$	
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$	1.

$\boxed{\forall x \in D(h_6) \cap D(g_6) ; \quad h_6(x) = g_6(x)} \quad \blacksquare$

$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$\left(\frac{a}{b}\right)^2 = \frac{a}{b} \cdot \frac{a}{b} = \frac{a \cdot a}{b \cdot b} = \frac{a^2}{b^2}$

$\left(\frac{a}{b}\right)^{-2} = \frac{1}{\left(\frac{a}{b}\right)^2} = \frac{1}{\frac{a^2}{b^2}} = \frac{1}{1} \cdot \frac{b^2}{a^2} = \frac{b^2}{a^2}$

$\frac{\frac{a}{b}}{\frac{a}{b}} = \frac{a}{b} \cdot \frac{b}{a} = \frac{ab \cdot ba}{ba}$

3. K daným funkcím najděte inverzní funkce (pokud existují):

- $f_1(x) = \sqrt{\frac{x+1}{x-2}}$,

$$y = \sqrt{\frac{x+1}{x-2}}$$

$$x = \sqrt{\frac{y+1}{y-2}} \quad \backslash ()^2$$

$$x^2 = \frac{y+1}{y-2} \quad \backslash (y-2)$$

$$x^2(y-2) = y+1$$

$$x^2y - 2x^2 = y+1 \quad \backslash -y$$

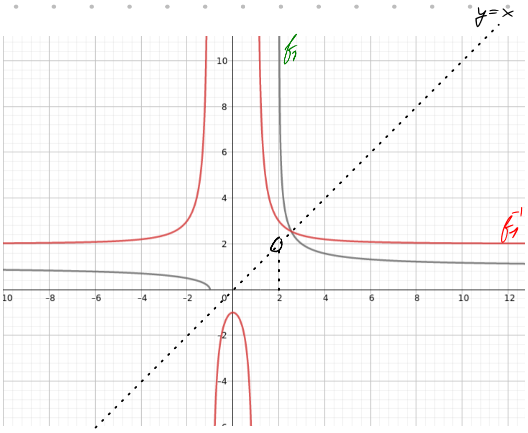
$$x^2y - 2x^2 - y = 1 \quad \backslash +2x^2$$

$$x^2y - y = 2x^2+1$$

$$y(x^2-1) = 2x^2+1 \quad \backslash \frac{1}{(x^2-1)}$$

$$\boxed{y = \frac{2x^2+1}{x^2-1}}$$

$$\boxed{f_1^{-1}(x) = \frac{2x^2+1}{x^2-1}}$$



$f_2(x) = \sqrt{\frac{x+1}{|x-2|}}$, $\quad D(f_2) = \mathbb{R} \setminus \{2\}$

$$y = \sqrt{\frac{x+1}{|x-2|}}$$

$$x = \sqrt{\frac{y+1}{|y-2|}} \quad \backslash ()^2$$

$$x^2 = \frac{y+1}{|y-2|}$$

$+$

$$x^2 = \frac{y+1}{y-2} \quad \backslash \cdot (y-2)$$

$$x^2(y-2) = y+1$$

$$x^2y - 2x^2 \quad +1 \quad \backslash -y \quad \backslash +2x^2$$

$$x^2y - y = 2x^2+1$$

$$y(x^2-1) = 2x^2+1 \quad \backslash \frac{1}{(x^2-1)}$$

$$\boxed{y = \frac{2x^2+1}{x^2-1}}$$

$-$

$$x^2 = \frac{y+1}{-y+2} \quad \backslash (-y+2)$$

$$x^2(-y+2) = y+1$$

$$x^2(-y) + 2x^2 = y+1 \quad \backslash -y \quad \backslash -2x^2$$

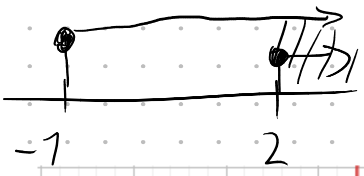
$$x^2(-y) - y = 1 - 2x^2$$

$$y(-x^2-1) = 1-2x^2 \quad \backslash \frac{1}{(-x^2-1)}$$

$$\boxed{y = \frac{1-2x^2}{-x^2-1}}$$

$$\boxed{f_2^{-1}(x) = \begin{cases} x > 0 \Rightarrow \frac{2x^2+1}{x^2-1} \\ x < 0 \Rightarrow \frac{1-2x^2}{-x^2-1} \\ 0 \Rightarrow \sqrt{\frac{1}{2}} \end{cases}}$$

$$f_3(x)=\frac{\sqrt{x+1}}{\sqrt{x-2}},$$



$D(f_3)=\langle 2,+\infty)$

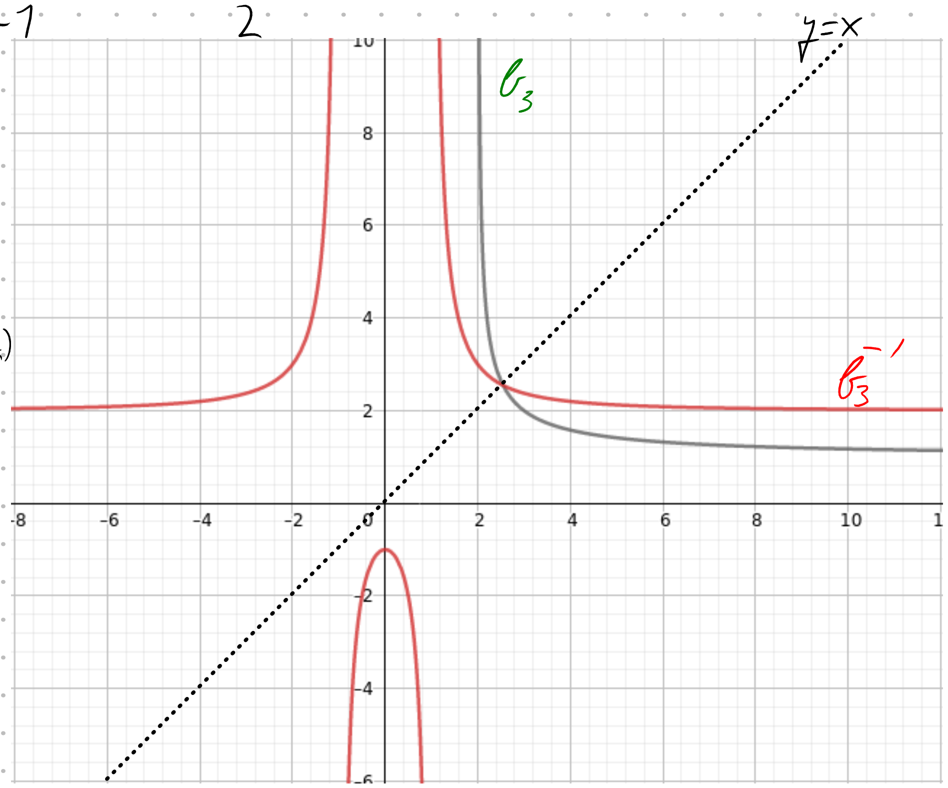
$$y=\frac{\sqrt{x+1}}{\sqrt{x-2}}$$

$$x=\frac{\sqrt{y+1}}{\sqrt{y-2}} \quad \backslash \left(\frac{a}{b}\right)^x=\frac{a^x}{b^x}$$

$$x=\sqrt{\frac{y+1}{y-2}} \quad \leftarrow \text{stejné jako u } f_1(x)$$

: aplikovat $f_1(x)$

$$\boxed{y=\frac{2x^2+1}{x^2-1} \qquad f_3^{-1}(x)=\frac{2x^2+1}{x^2-1}}$$



$$f_4(x)=\sqrt{\left|\frac{x+1}{x-2}\right|}$$

$D(f_4)=\mathbb{R}\setminus\{2\}$ $\quad \left|\frac{x+1}{x-2}\right| \overset{?}{\geq} 0$

$$x=\sqrt{\left|\frac{y+1}{y-2}\right|} \quad \backslash (\ )^2$$

$$x^2=\left|\frac{y+1}{y-2}\right|$$

$y\in\mathbb{R}\setminus\langle -1,2)$ $\quad y\in\langle -1,2)$

$+$ $\quad -$

$$x^2=\frac{y+1}{y-2} \quad |\cdot(y-2)$$

$$x^2(y-2)=y+1 \quad |-y \quad |+2x^2$$

$$x^2y-y=1+2x^2$$

$$y(x^2-1)=1+2x^2 \quad \Big|\frac{1}{(x^2-1)}$$

$$\boxed{y=\frac{1+2x^2}{x^2-1}}$$

$$x^2=-\left(\frac{y+1}{y-2}\right)$$

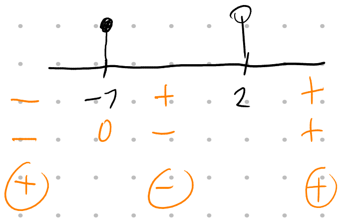
$$x^2=\frac{-y-1}{y-2} \quad |\cdot(y-2)$$

$$x^2(y-2)=-y-1 \quad \backslash +y \quad \backslash +2x^2$$

$$x^2y+y=-1+2x^2$$

$$y(x^2+1)=-1+2x^2 \quad \backslash \frac{1}{(x^2+1)}$$

$$\boxed{y=\frac{-1+2x^2}{x^2+1}}$$



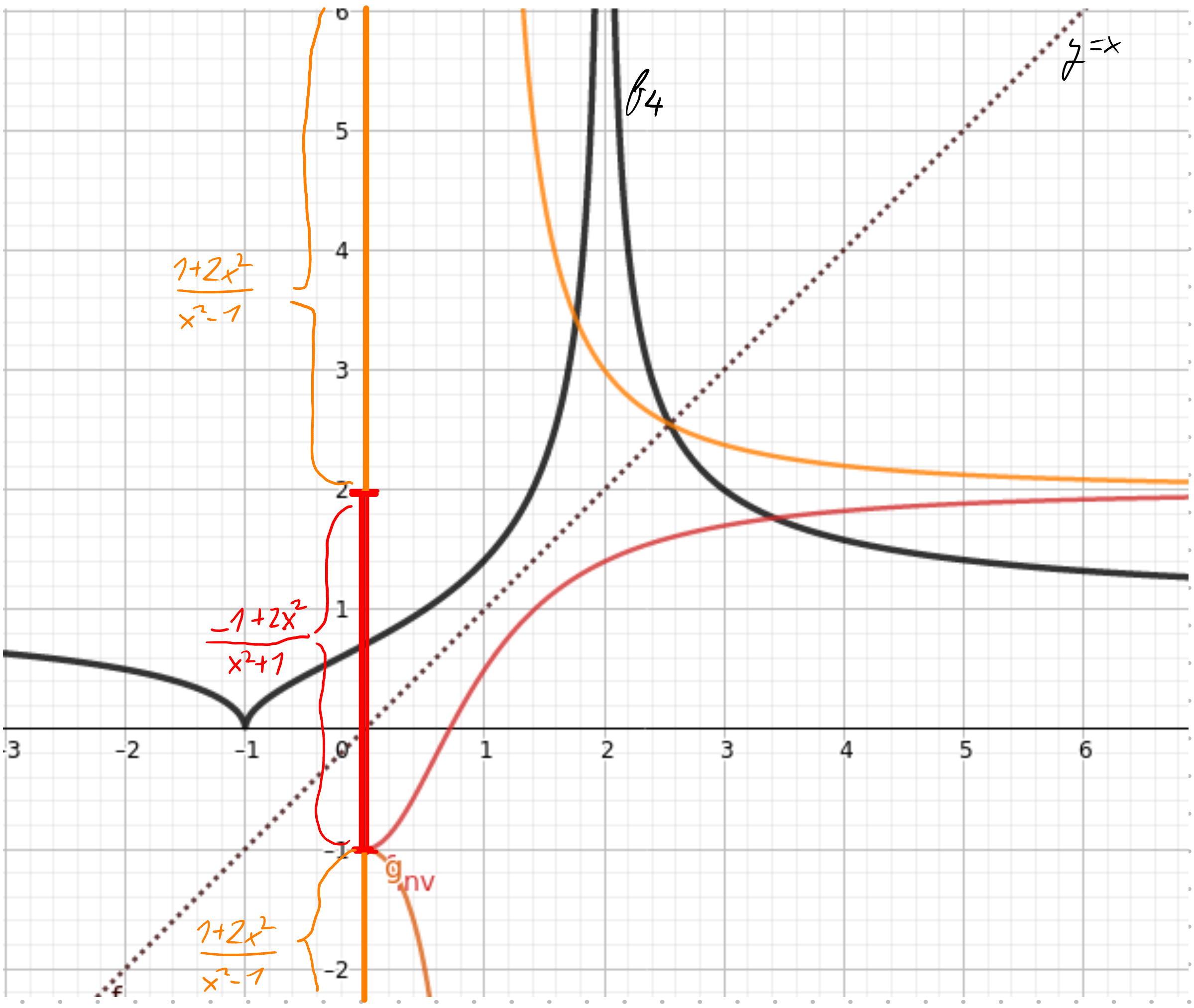
$$-1+2x^2=0 \quad \backslash +1$$

$$2x^2=1 \quad \backslash \tfrac{1}{2}$$

$$x^2=\frac{1}{2}$$

$$x=\sqrt{\frac{1}{2}}$$

$$f_4^{-1}(x)=\begin{cases}\dfrac{-1+2x^2}{x^2+1} & \langle -1,2) \\[2ex] \dfrac{1+2x^2}{x^2-1} & (-\infty,-1)\cup(2,+\infty)\end{cases}$$



y=x
f4
g_nv
$\frac{1+2x^2}{x^2-1}$
$\frac{-1+2x^2}{x^2+1}$
$\frac{1+2x^2}{x^2-1}$

Příprava na 1. test

$D(\sqrt[2n]{\ }) = \langle 0, +\infty)$

$D(\frac{1}{x}) = \mathbb{R} \setminus \{0\}$

$D(\ln x) = (0, +\infty)$, $\log_a 1 = 0$

1) $f(x) = \dfrac{\sqrt{6+x-x^2}}{x-2}$

$\boxed{D(f) = \langle -2, 3\rangle \setminus \{2\}}$

$x-2 \neq 0 \quad | +2$

$\boxed{x \neq 2}$

$6 + x - x^2 \geq 0$

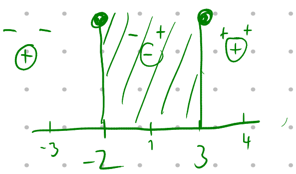
$-x^2 + x + 6 \leq 0 \quad | \cdot (-1)$

$x^2 - x - 6 \leq 0$

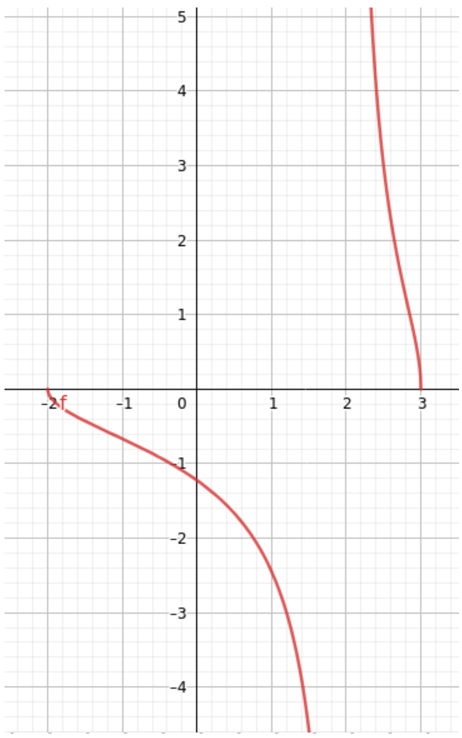
$(x-3)\cdot(x+2) \leq 0$

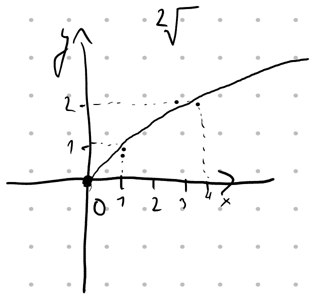
$x-3=0$ $x+2=0$

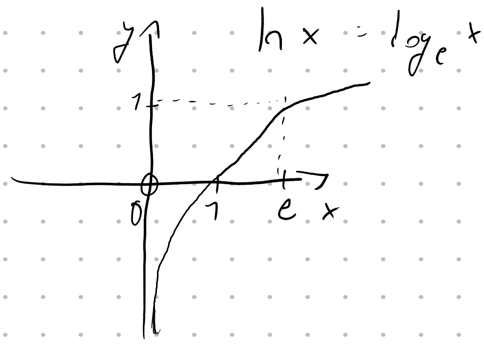
$x=3$ $x=-2$



$\boxed{x \in \langle -2, 3\rangle}$







2) $g(x) = x^3 + 1$ najdi $g^{-1}(\langle 0, 2\rangle)$

$y = x^3 + 1$

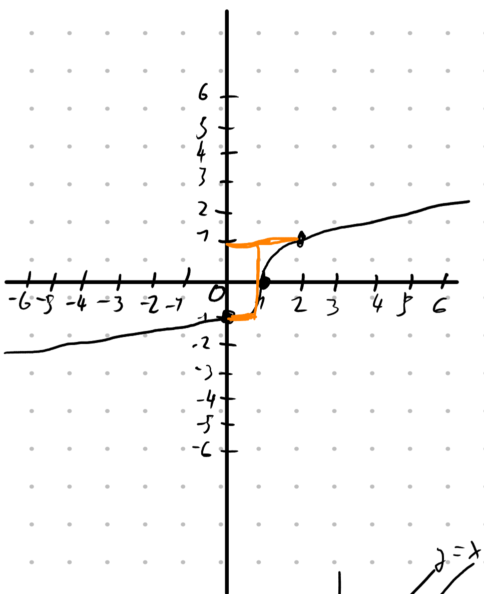
$x = y^3 + 1 \quad | -1$

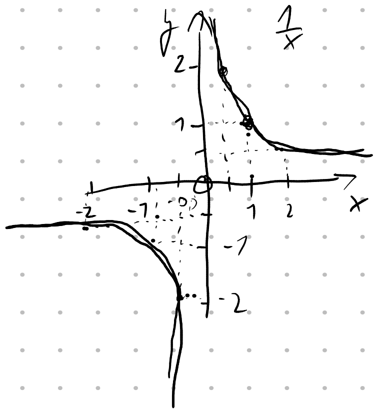
$x - 1 = y^3 \quad | \sqrt[3]{\ }$

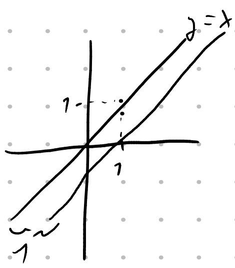
$\sqrt[3]{x-1} = y$

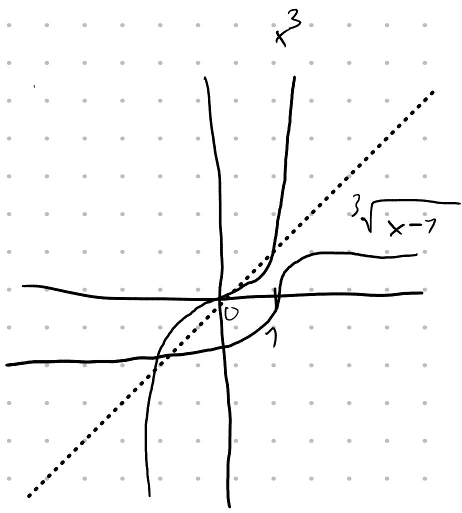
$\boxed{g^{-1}(x) = \sqrt[3]{x-1}}$

$\boxed{g^{-1}(\langle 0, 2\rangle) = \langle -1, 1\rangle}$









1) $f(x) = \dfrac{\sqrt{4x+3+x^2}}{x-2}$

$x-2 \neq 0 \;\backslash +2$

$\boxed{x \neq 2}$

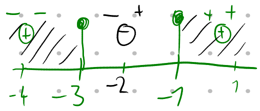
$4x+3+x^2 \geq 0$

$x^2+4x+3 \geq 0$

$(x+1)\cdot(x+3) \geq 0$

$x+1 = 0 \;\backslash -1$ $\quad$ $x+3 = 0 \;\backslash -3$

$\boxed{x = -1}$ $\quad$ $\boxed{x = -3}$



$\boxed{x \in \langle -\infty, -3\rangle \cup \langle -1, +\infty\rangle}$

$\boxed{D(f) = (\langle -\infty; -3\rangle \cup \langle -1; \infty\rangle) \setminus \{2\}}$

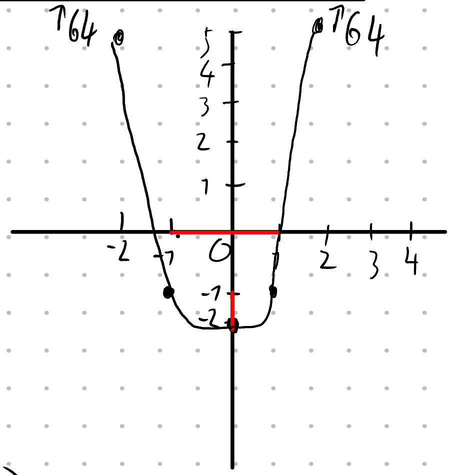
2) $g(x) = x^6 - 2$ najdi $g(\langle -1, 1\rangle)$

$y = x^6 - 2$

$x = y^6 - 2 \;\backslash +2$

$x+2 = y^6 \;\backslash \sqrt[6]{\;}$

$\boxed{\sqrt[6]{x+2} = y}$



$\boxed{g(\langle -1, 1\rangle) = \langle -2, -1\rangle}$

1) $f(x)=\sqrt{\frac{2}{x-2}-1}$

$D(f)=?$ $\qquad x-2\neq 0$

$\qquad\qquad\quad \boxed{x\neq 2}$

$\frac{2}{x-2}-1\geq 0$

$\frac{2}{x-2}-\frac{x-2}{x-2}\geq 0$

$\frac{2-x+2}{x-2}\geq 0$

$\frac{-x+4}{x-2}\geq 0$

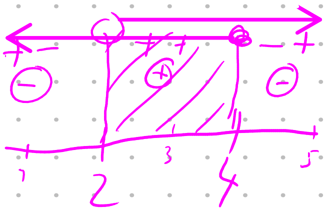
$-x+4\geq 0 \quad \backslash -4$

$-x\geq -4 \quad \backslash\cdot(-1)$

$x\leq 4$

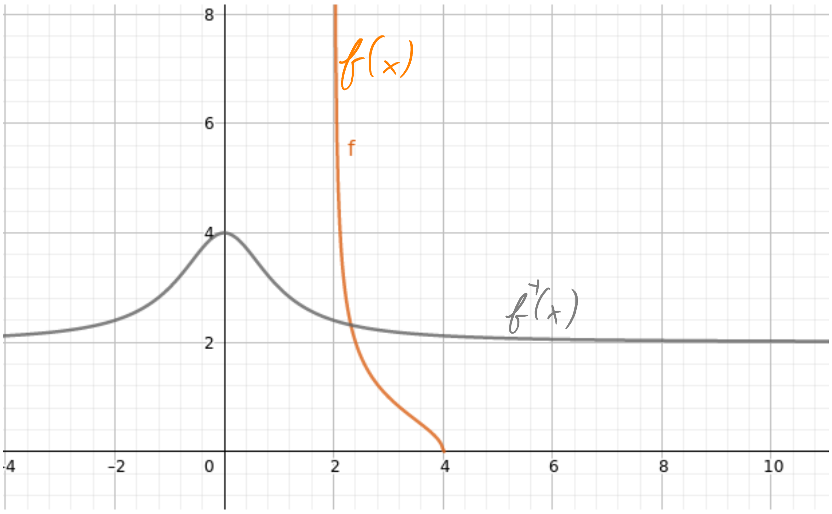
$x-2\geq 0 \quad \backslash +2$

$x\geq 2$



$\boxed{x\in(2,4\rangle}$

$\boxed{D(f)=(2,4\rangle}$



$f((2,+\infty))$

$\lim_{x\to 2^+}\sqrt{\frac{2}{x-2}-1}\to\sqrt{+\infty}\to+\infty$

$f(4)=0$

$\boxed{H(f)=\langle 0,+\infty)}$

---

2) $y=\sqrt{\frac{2}{x-2}-1}$

$x=\sqrt{\frac{2}{y-2}-1} \quad \backslash ()^2$

$x^2=\frac{2}{y-2}-1 \quad \backslash +1$

$x^2+1=\frac{2}{y-2} \quad \backslash\cdot(y-2)$

$(x^2+1)\cdot(y-2)=2$

$x^2y-2x^2+y-2=2 \quad \backslash +2 \backslash +2x^2$

$x^2y+y=2x^2+4$

$y(x^2+1)=2x^2+4 \quad \backslash\frac{1}{(x^2+1)}$

$\boxed{y=\frac{2x^2+4}{x^2+1}}$

$\boxed{f^{-1}(x)=\frac{2x^2+4}{x^2+1}}$

$D(f^{-1})=?$

$x^2+1\neq 0 \quad \backslash -1$

$x^2\neq -1$

obecně platí

$\boxed{x\in\mathbb{R}}$

1) $f(x)=\sqrt{\frac{12}{x+2}-3}$

$D(f)=?$ $\qquad x+2\neq 0 \mid -2$

$\boxed{x\neq -2}$

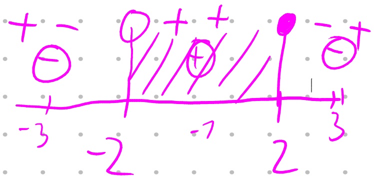
$\frac{12}{x+2}-3\geq 0$

$\frac{12-3\cdot(x+2)}{x+2}\geq 0$

$\frac{12-(3x+6)}{x+2}\geq 0$

$\frac{12-3x-6}{x+2}\geq 0$

$\frac{6-3x}{x+2}\geq 0$



$+6-3x=0 \mid -6$

$-3x=-6 \mid \cdot\frac{1}{-3}$

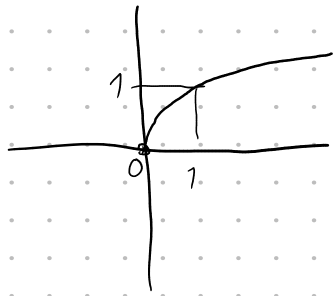
$\boxed{x=+2}$

$x+2=0 \mid -2$

$\boxed{x=-2}$

$\boxed{x\in(-2,2\rangle}$

$\boxed{D(f)=(-2,2\rangle}$



$H(f)=?$

$\lim_{x\to -2^+} f(x)=\sqrt{\frac{12}{0.000\ldots001}-3}=\sqrt{+\infty}=+\infty$

-1.99999

$\boxed{H(f)=\langle 0;+\infty)}$

f^{-1} $\qquad y=\sqrt{\frac{12}{x+2}-3}$ $\qquad \boxed{f^{-1}(x)=\frac{6-2x^2}{x^2+3}}$

$x=\sqrt{\frac{12}{y+2}-3} \quad \backslash\, ()^2$

$x^2=\frac{12}{y+2}-3 \quad \backslash +3$

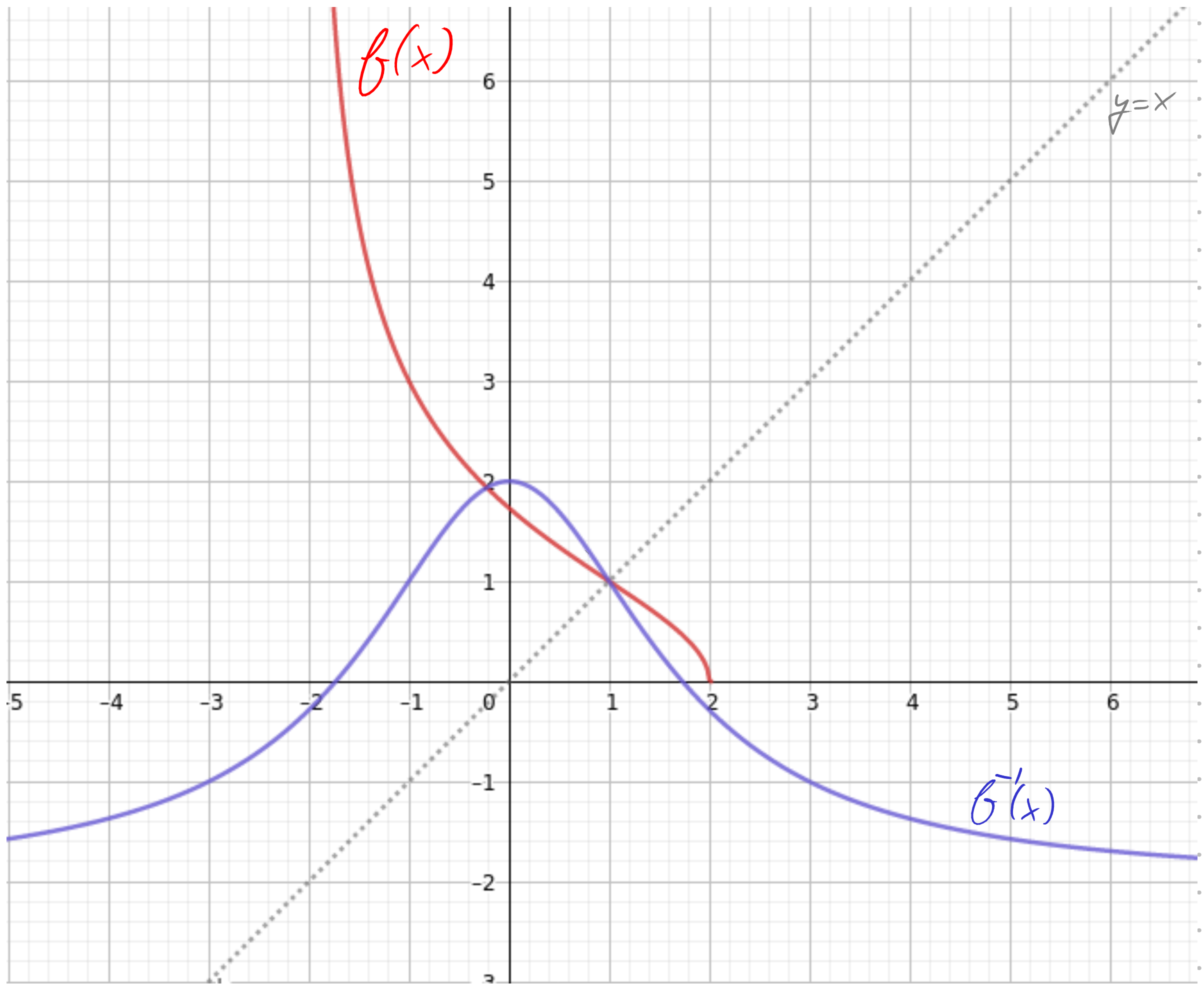
$x^2+3=\frac{12}{y+2} \quad \backslash \cdot(y+2)$

$(x^2+3)\cdot(y+2)=12$

$x^2y+2x^2+3y+6=12 \quad \backslash -6 \,\backslash -2x^2$

$y(x^2+3)=6-2x^2 \quad \backslash \cdot\frac{1}{(x^2+3)}$

$\boxed{y=\frac{6-2x^2}{x^2+3}}$



f(x)
y=x
f⁻¹(x)

Náročnější úlohy

$D(f)=D(g)=\mathbb{R}$

$H(f)=H(g)=\mathbb{R}$

5. Dokažte, že platí: Nechť f a g jsou rostoucí funkce na $\mathbb{R}$. Potom $f \circ g$ je rostoucí na $\mathbb{R}$.

def. rostoucí fce f: $\forall x_1, x_2 \in D(f);\ x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

$$\Big(\forall x, y \in D(f);\ x < y \Rightarrow f(x) < f(y)$$

$$\wedge$$

$$\forall x, y \in D(g);\ x < y \Rightarrow g(x) < g(y) \Big)$$

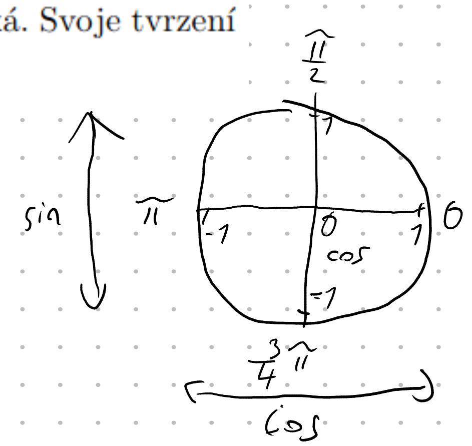
$$\Rightarrow$$

$$\forall x, y \in D(f);\ x < y \Rightarrow f(x) < f(y) \Rightarrow g(f(x)) < g(f(y)) \ \blacksquare$$

xschul06

10. Zjistěte, jestli je funkce $f(x) = 4\sin^4 x + 4\cos^4 x$ periodická. Svoje tvrzení zdůvodněte.

$$f(x) = 4\sin^4(x) + 4\cos^4(x)$$
$$f(x) = 4\cdot(\sin^4(x) + \cos^4(x))$$
$$f(x) = 4\cdot(g(x)) \; ; \; g(x) = \sin^4(x) + \cos^4(x)$$



- Pokud je fce f(x) T-periodická => f(A*x+B) je (T/|A|)-periodická (škálování periody)
- Pokud je fce f(x) T-periodická => A*f(x) je T-periodická (škálování hodnoty neponičí periodicitu)
- Pokud je fce g periodická => kompozice f(g(x)) je periodická pro každou fci f
- Pokud fce f je T-periodická a fce g je S-periodická => f+g, f-g, f*g, f/g jsou R-periodické, kde R je nejmeenší společný násobek S a T

nejdříve určíme periodu g(x):

$\cos(x)$, $\sin(x)$ 2π-periodické

$$g(x) = \sin^4(x) + \cos^4(x)$$

$$\sin^4(x) = \underbrace{\sin(x)}_{2\pi}\cdot\underbrace{\sin(x)}_{2\pi}\cdot\sin(x)\cdot\sin(x)$$

→ obdobně u $\cos^4(x)$

$\boxed{\cos^4(x) \text{ je } 2\pi\text{-periodická}}$

$nsn(2\pi, 2\pi) = 2\pi$

$nsn(nsn(2\pi, 2\pi)) = 2\pi$

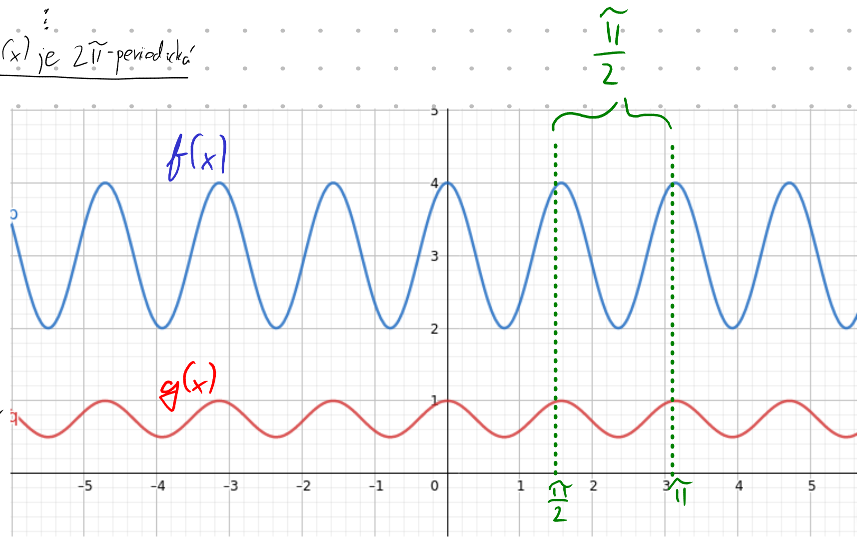
$nsn(-\|-) = 2\pi$

$\boxed{\sin^4(x) \text{ je } 2\pi\text{-periodická}}$

nsn = nejmenší společný násobek

2π-periodická + 2π-periodická
$\sin^4(x) + \cos^4(x)$
je $nsn(2\pi, 2\pi)$-periodická

=> $g(x)$ je 2π-periodická



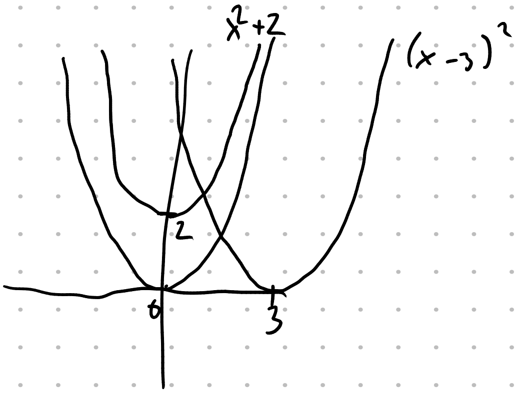
$$\boxed{\text{periodicita}(f) = \frac{\text{periodicita}(g)}{|4|} = \frac{2\pi}{4} = \frac{\pi}{2}} \blacksquare$$

Roland Schulz
(xschul06)

axiomy periodicity převzaty z https://math.fel.cvut.cz/mt/txtb/3/txe3bb3d.htm

3. cviko

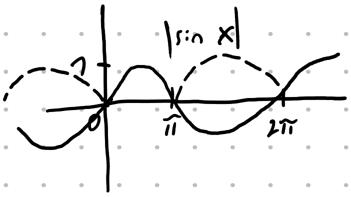
$y = x^2$

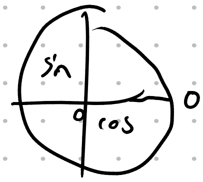


$\xleftarrow{-1}$

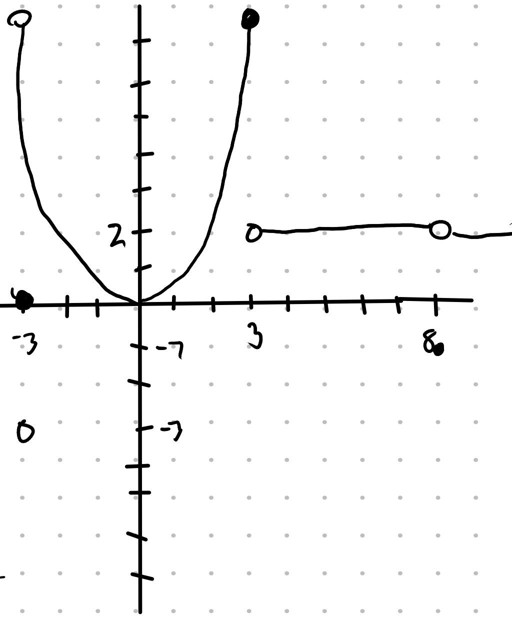
$\uparrow +2$

$y = \sin x$

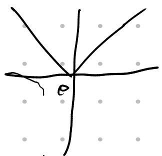




$$f(x) = \begin{cases} (-\infty, -3) \longrightarrow x \\ \{-3\} \longrightarrow 0 \\ (-3, 3\rangle \longrightarrow x^2 \\ (3, 8) \cup (8, +\infty) \longrightarrow 2 \\ \{8\} \longrightarrow -1 \end{cases}$$



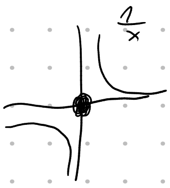
1) • zdola omezená na $D(f)$ $\quad f(x) = |x|$



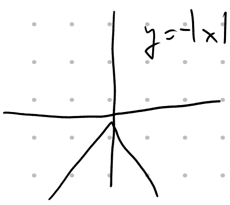
• není zdola omezená na $D(f)$ $\quad f(x) = x$

• omezená na $\langle -1, 1\rangle$ $\quad f(x) = -x^2 + 1$

• není omezená na $\langle -1, 1\rangle$, ale je definovaná

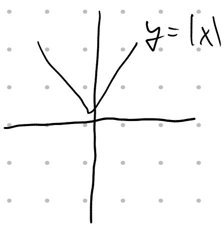


$$f(x) \begin{cases} \frac{1}{x} & \mathbb{R} \setminus \{0\} \\ 0 & x = 0 \end{cases}$$



• rostoucí na $\langle -1, 0\rangle$
klesající na $\langle 0, 1\rangle$

• nerostoucí na $\langle -1, 0\rangle$
neklesající na $\langle 0, 1\rangle$



Limity

- $\lim\limits_{x\to0}\frac{6x^2-1}{x^3-4x-5} = \frac{-1}{-5} = \frac{1}{5}$

- $\lim\limits_{x\to2}\frac{x^2-5x+6}{x^2-12x+20} = \frac{4-10+6}{4-24+20} = \frac{0}{0}$ nevíme

$\lim\limits_{x\to2}\frac{(x-2)(x-3)}{(x-2)(x-10)} = \lim\limits_{x\to2}\frac{x-3}{x-10} = \frac{2-3}{2-10} = \frac{-1}{-8} = \frac{1}{8}$

- $\lim\limits_{x\to\infty}\frac{x^4+x^3}{x^4-2x^3} = \lim\limits_{x\to\infty}\frac{x^3(x+1)}{x^3(x-2)} = \lim\limits_{x\to\infty}\frac{x+1}{x-2} = \lim\limits_{x\to\infty}\frac{x(1+\frac{1}{x})}{x(1-\frac{2}{x})} = \frac{1}{1} = 1$

$(A+B)(A-B) = A^2-B^2$ $\qquad A^2 - AB - AB - B^2 = A^2 - B^2$

- $\lim\limits_{x\to3}\frac{\sqrt{x^2+7}-4}{x^2-5x+6} = \lim\limits_{x\to3}\frac{\sqrt{x^2+7}-4}{x^2-5x+6}\cdot\frac{\sqrt{x^2+7}+4}{\sqrt{x^2+7}+4} = \lim\limits_{x\to3}\frac{x^2+7-16}{(x^2-5x+6)(\sqrt{x^2+7}+4)} =$

$= \lim\limits_{x\to3}\frac{x^2-9}{(x^2-5x+6)(\sqrt{x^2+7}+4)} = \lim\limits_{x\to3}\frac{(x-3)(x+3)}{(x-2)(x-3)(\sqrt{x^2+7}+4)} = \frac{6}{1\cdot(\sqrt{9+7}+4)} = \frac{6}{8} = \frac{3}{4}$

finty:

- $(A+B)(A-B) = A^2-B^2$ ✓
- rozklad na součin ✓
- vytknutí x ✓

- $\lim\limits_{x\to-\infty}\frac{6x^2-1}{x^3-4x-5} = \lim\limits_{x\to-\infty}\frac{x^3(\frac{6}{x}-\frac{1}{x})}{x^3(1-\frac{4}{x^2}-\frac{5}{x^3})} = \frac{0}{1} = 0$

$\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

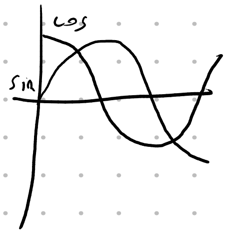
finta:

$\lim\limits_{x\to0}\frac{\sin x}{x} = 1$

- $\lim\limits_{x\to0}\frac{\operatorname{tg}5x}{\operatorname{tg}6x} = \lim\limits_{x\to0}\frac{\frac{\sin 5x}{\cos 5x}}{\frac{\sin 6x}{\cos 6x}} = \lim\limits_{x\to0}\frac{\sin 5x\cdot\cos 6x}{\sin 6x\cdot\cos 5x} = \lim\limits_{x\to0}\frac{\frac{\sin 5x}{5x}\cdot 5\cancel{x}\cdot\cos 6x}{\frac{\sin 6x}{6x}\cdot 6\cancel{x}\cdot\cos 5x} = \frac{1\cdot5\cdot1}{1\cdot6\cdot1} = \frac{5}{6}$

$\operatorname{tg}x = \frac{\sin x}{\cos x}$ $\qquad \lim\limits_{x\to0}\frac{\sin x}{x} = 1$

$$\sin^2 x + \cos^2 x = 1$$



- $\lim\limits_{x\to 0} \dfrac{\cos x - \cos^3 x}{x^2} = \lim\limits_{x\to 0} \dfrac{\cos x(\overbrace{1-\cos^2 x}^{\sin^2 x})}{x^2} = \lim\limits_{x\to 0} \dfrac{\overset{1}{\cos x}\cdot \overset{1}{\sin^2 x}}{x^2} = 1$

- $\lim\limits_{x\to 0} \dfrac{\sin x}{x^3} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{x \cdot x^2} = \lim\limits_{x\to 0} \dfrac{1}{x^2} = \dfrac{1}{0} = +\infty$

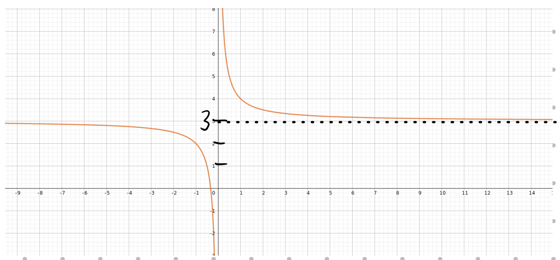
- $\lim\limits_{x\to \infty} \sin x$ neexistuje

---

4. cviko

$\frac{4}{4n} = \frac{1}{n}$

- $\lim\limits_{n\to\infty} \left(3 + \dfrac{4}{4n}\right) = \lim\limits_{n\to\infty}\left(3+\dfrac{1}{n}\right) = 3+0 = 3$



- $\lim\limits_{n\to\infty} (\sqrt[n]{4} - 16) = 4^{\frac{1}{n}} - 16 = 1 - 16 = -15$

?

- $\lim\limits_{n\to\infty} \dfrac{7n+5}{4-2n} = \lim\limits_{n\to\infty} \dfrac{\cancel{n}(7+\frac{5}{n})}{\cancel{n}(\frac{4}{n}-2)} = \lim\limits_{n\to\infty} \dfrac{7+\frac{5}{n}}{\frac{4}{n}-2} = \lim\limits_{n\to\infty} \dfrac{7+0}{0-2} = \dfrac{7}{-2} = -3{,}5$?

- $\lim\limits_{n\to\infty} \left(3 + \dfrac{2}{4n}\right)^3 = \lim\limits_{n\to\infty} (3+0)^3 = 27$

- $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^2+1}}{n+1} = \lim\limits_{n\to\infty} \dfrac{\sqrt{n^2(1+\frac{1}{n^2})}}{n(1+\frac{1}{n})} = \lim\limits_{n\to\infty} \dfrac{\cancel{n}\cdot\sqrt{1+\frac{1}{n^2}}}{\cancel{n}(1+\frac{1}{n})} = \dfrac{1}{1} = 1$

$$\lim_{x\to1}\left(\frac{1}{x^2-1}-\frac{2}{x^4-1}\right) = \lim_{x\to1}\frac{x^2+1-2}{(x^2-1)(x^2+1)} = \lim_{x\to1}\frac{\cancel{x^2-1}^{\,1}}{\cancel{(x^2-1)}(x^2+1)} = \frac{1}{2}$$

$$(x^4-1) = (x^2-1)\cdot(x^2+1)$$
$$(A^2-B^2) = (A-B)\cdot(A+B)$$

$$\lim_{x\to-2}\frac{\sqrt{x+6}-2}{x+2} = \lim_{x\to-2}\frac{\overbrace{\sqrt{x+6}}^{A}-\overbrace{2}^{B}}{x+2}\cdot\frac{\overbrace{\sqrt{x+6}}^{A}+\overbrace{2}^{B}}{\sqrt{x+6}+2} = \lim_{x\to-2}\frac{\cancel{x+6-2^2}^{\,1}}{\cancel{(x+2)}(\sqrt{x+6}+2)} =$$

$x+2$

$$= \lim_{x\to-2}\frac{1}{\sqrt{x+6}+2} = \frac{1}{\sqrt{-2+6}+2} = \frac{1}{4}$$

$\sqrt{4}=2$

$$\lim_{n\to\infty}\left(1+\frac{1}{n+5}\right)^{n+5} = \left|\begin{array}{l} t=n+5 \\ n\to\infty \\ \Downarrow \\ t\to\infty \end{array}\right| \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t} = e$$

definice e

Substituce!

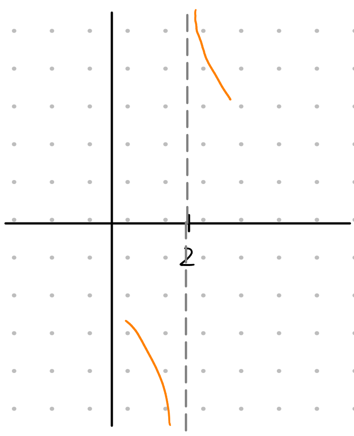
$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\overbrace{\frac{1}{n}}^{\to0}} = 1$$

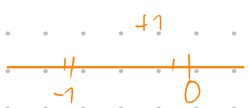
$$\lim_{x\to2}\frac{x^2-2}{x^2-3x+2} \overset{\frac{2}{0}}{=} \text{neexistuje}$$

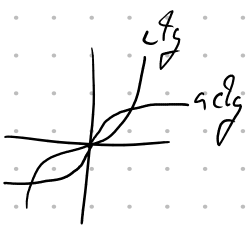
$$+\quad \lim_{x\to2^+}\frac{x^2-2}{\underbrace{(x-1)(x-2)}_{1\cdot(+0)}} \overset{\frac{2}{+0}}{=} +\infty$$

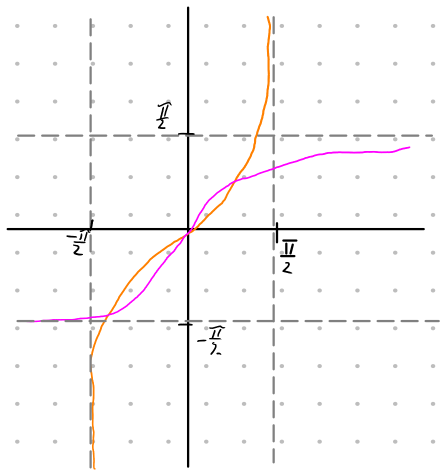
$$-\quad \lim_{x\to2^-}\frac{x^2-2}{\underbrace{(x-1)(x-2)}_{1\cdot(-0)}} \overset{\frac{2}{-0}}{=} -\infty$$



$$\lim_{x\to-1^-}\operatorname{arctg}\frac{1}{1+x} \overset{\frac{1}{0^-}}{=} \operatorname{arctg}-\infty = -\frac{\pi}{2}$$







- $\lim \frac{3n-2}{1-3n} = \frac{(1-3n)-1-2}{1-3n} = \frac{(1-3n)-3}{1-3n} = 1-\frac{3}{1-3n}$

chci do tvaru $\left(1-\frac{1}{n}\right)^n$

Asymptoty

$y = ax+b$

$a = \lim\limits_{x\to\pm\infty} \frac{f(x)}{x}$

$b = \lim\limits_{x\to\pm\infty} \left(f(x)-ax\right)$

Bez směrnice ABS

$\lim\limits_{x\to a^-} f(x) = \pm\infty$

$\lim\limits_{x\to a^+} f(x) = \pm\infty$

se směrnicí ASS

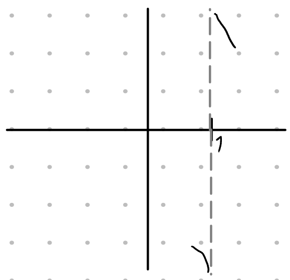
$y = ax \cdot b$

$\lim\limits_{x\to\pm\infty} \left[f(x)-(ax-b)\right] = 0$

$a = \lim\limits_{x\to\pm\infty} \frac{f(x)}{x}$

$b = \lim\limits_{x\to\pm\infty} \left(f(x)-ax\right)$

- $f_1(x) = \frac{x}{x-1}$



ABS:

$a=1$: $\lim\limits_{x\to 1^-} \frac{x}{x-1} \overset{\frac{1}{-0}}{=} -\infty$

$\lim\limits_{x\to 1^+} \frac{x}{x-1} = \overset{\frac{1}{+0}}{+\infty}$

ASS:

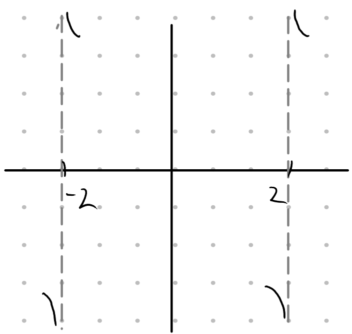
$a = \lim\limits_{x\to\infty} \frac{\frac{x}{x-1}}{x} \quad \lim\limits_{x\to\infty} \frac{\cancel{x}}{x-1} \cdot \frac{1}{\cancel{x}} = \lim\limits_{x\to\infty} \frac{1}{x-1} \overset{\frac{1}{\infty}}{=} 0$

dosadit $\lim\limits_{x\to\infty} \frac{1}{x-1} = 0$

$b = \lim\limits_{x\to\infty} \left(\frac{x}{x-1} - 0\cdot x\right) = \lim\limits_{x\to\infty} \left(\frac{x-0x\cdot(x-1)}{x-1}\right) = \lim\limits_{x\to\infty} \frac{x-0\cdot x^2-0\cdot x}{x-1} = \lim\limits_{x\to\infty} \frac{x^2\cdot\left(\frac{1}{x}-0+\frac{0}{x}\right)}{x^2\cdot\left(\frac{1}{x}-\frac{1}{x^2}\right)} = 1$

$y = 1$

- $f_2(x) = \frac{x^3+2}{x^2+4} = \frac{x^3+2}{(x-2)(x+2)}$



ABS:

$a=2$

$\lim\limits_{x\to 2^-} \frac{x^3+2}{\underset{(-0)\ (\cdot 4)}{(x-2)(x+2)}} \overset{\frac{10}{-0}}{=} -\infty$

$\lim\limits_{x\to 2^+} \frac{x^3+2}{\underset{(+0)\cdot(4)}{(x-2)(x+2)}} \overset{\frac{10}{+0}}{=} +\infty$

$a=-2$

$\lim\limits_{x\to -2^-} \frac{x^3+2}{\underset{(-4)\cdot(-0)}{(x-2)(x+2)}} \overset{\frac{-6}{+0}}{=} -\infty$

$\lim\limits_{x\to -2^+} \frac{x^3+2}{\underset{(-4)\cdot(+0)}{(x-2)(x+2)}} \overset{\frac{-6}{-0}}{=} +\infty$

$f_2(x)$ ASS:

$$a = \lim_{x \to \infty} \frac{\frac{x^3+2}{(x-2)(x+2)}}{x} = \lim_{x \to \infty} \frac{x^3+2}{x(x^2-4)} = \lim_{x \to \infty} \frac{x^3+2}{x^3-4x} = \lim_{x \to \infty} \frac{x^3(1+\frac{2}{x^3})}{x^3(1-\frac{4}{x^2})} = 1$$

$$b = \lim_{x \to \infty} \left(\frac{x^3+2}{x^2-4} - x \right) = \lim_{x \to \infty} \left(\frac{x^3+2 \overbrace{- x(x^2-4)}^{-x^3+4x}}{x^2-4} \right) = \lim_{x \to \infty} \frac{2+4x}{x^2-4} = \lim_{x \to \infty} \frac{\overbrace{x^2\left(\frac{2}{x^2}+\frac{4}{x}\right)}^{0+0}}{\underbrace{x^2\left(1-\frac{4}{x^2}\right)}_{1-0}} = \frac{0}{1} = 0$$

$y = x$

$$f_3\left(\frac{1}{x+1} + \frac{1}{x} + \frac{1}{x-1}\right) = \frac{\overbrace{x(x-1)}^{x^2-x} + \overbrace{(x+1)(x-1)}^{x^2-1} + \overbrace{x(x+1)}^{x^2+x}}{\underbrace{x \cdot (x+1) \cdot (x-1)}_{x \cdot x^2-1}} = \frac{3x^2-1}{\underbrace{x \cdot (x^2-1)}_{x^3-x}} = x^3\left(\frac{3}{x} - \frac{1}{x^3}\right)$$

ABS

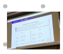
1, -1, 0

ASS

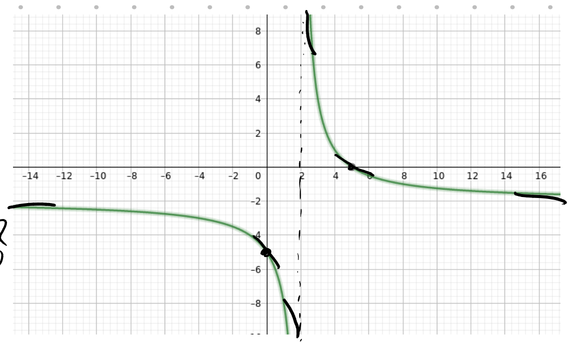
$y=0$

$a=0$

$b=0$



$\textcircled{A}$



1) $\lim\limits_{x\to\pm\infty} f(x)$ + v bodeh nespojitosti + graf

$f(x) = \dfrac{2x-10}{2-x}$ $\boxed{x \neq 2}$ $D_f = \mathbb{R} \setminus \{2\}$

$$\lim_{x\to+\infty} \frac{2x-10}{2-x} = \lim_{x\to+\infty} \frac{x\left(2-\frac{10}{x}\right)}{x\left(\frac{2}{x}-1\right)} = \lim_{x\to\infty} \frac{2-\overbrace{\frac{10}{x}}^{\to 0}}{\underbrace{\frac{2}{x}}_{0}-1} = \frac{2}{-1} = \underline{\underline{-2}}$$

$$\lim_{x\to-\infty} \frac{2x-10}{2-x} = \lim_{x\to-\infty} \frac{x\left(2-\frac{10}{x}\right)}{x\left(\frac{2}{x}-1\right)} = \lim_{x\to-\infty} = \frac{2-\overbrace{\frac{10}{x}}^{\to -0}}{\underbrace{\frac{2}{x}}_{-0}-1} = \frac{2}{-1} = -2$$

v bodeh nespojitosti (stari)

A.B.S:

$$\bullet \lim_{x\to 2^-} \frac{2x-10}{2-x} = \lim_{x\to 2^-} \frac{x\left(2-\frac{10}{x}\right)}{x\left(\frac{2}{x}-1\right)} = \lim_{x\to 2^-} \frac{2-\frac{10}{x}}{\frac{2}{x}-1} = \frac{2-\overbrace{\frac{10}{1,9999\ldots}}^{\to 5^+}}{\frac{2}{1,9999}-1} = \frac{2-5^+}{1^+-1} = \frac{-3^+}{0^+} = \underline{-\infty}$$

$$= \frac{3,9999\ldots-10}{2-1,99999\ldots} = \frac{-6^+}{0^+} = \underline{-\infty}$$

$-$ 2 $+$

Cviko – Derivace

$x^n = n \cdot x^{n-1}$

$x^1 = 1 \cdot x^0 = 1$

$c' = 0$

$(e^x) = e^x$

- $y = \pi x^3 - 7x$

  $y' = \underline{\pi 3x^2 - 7}$

$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

- $y = (x^3 + 8) \cdot (x - 2)$

  $y = x^4 - 2x^3 + 8x - 16$

  $y' = 4x^3 - 2 \cdot (3x^2) + 8$

  $= \underline{4x^3 - 6x^2 + 8}$ $\blacksquare$

  $y = (x^3 + 8) \cdot (x - 2)$

  $y' = (3x^2) \cdot (x - 2) + (x^3 + 8)(1)$

  $y' = 3x^3 - 6x^2 + x^3 + 8 = \underline{4x^3 - 6x^2 + 8}$ $\blacksquare$

- $y = e^x \cdot (x^2 - 1)$

  $y' = e^x \cdot (x^2 - 1) + e^x \cdot (2x) = \underline{e^x(x^2 - 1 + 2x)}$ $\blacksquare$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

- $y = \dfrac{x + 5}{x^2}$

  $$y' = \frac{1 \cdot (x^2) - (x + 5) \cdot (2x)}{(x^2)^2} = \frac{\overbrace{x^2 - 2x^2}^{-x^2} - 10x}{x^4} =$$

  $$= \frac{\cancel{x}(-x - 10)}{\cancel{x}(x^3)} = \frac{-x - 10}{x^3} \blacksquare$$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

umět zpaměti

$f(g(x)) = f'(x) \cdot g'(x)$

---

vnější složka

- $y = \sqrt{\sin x} = (\sin x)^{\frac{1}{2}}$

  vnitřní složka

  $$y' = \frac{1}{2}(\sin x)^{-\frac{1}{2}} \cdot \cos x = \frac{\cos x}{2(\sin x)^{\frac{1}{2}}}$$

- $y=\frac{1}{2}(x-\sin x\cdot\cos x)$

$$y'=\frac{1}{2}\cdot\left(1-\left(\overset{f'}{\cos x}\cdot\overset{g}{\cos x}+\overset{f}{\sin x}\cdot\overset{g'}{(-\sin x)}\right)\right)=\frac{1}{2}\left(1-\left(\cos^2 x-\sin^2 x\right)\right)=$$

$$=\frac{1}{2}\left(\overbrace{1-\cos^2 x}^{\sin^2 x}+\sin^2 x\right)=\frac{1}{2}\left(\sin^2 x+\sin^2 x\right)=\frac{1}{2}\left(2\sin^2 x\right)=\underline{\sin^2 x}\ \blacksquare$$

- $y=\ln\frac{5+4x}{3+7x}$

$$(\ln x)'=\frac{1}{x}$$

$$y'=\frac{1}{\frac{5+4x}{3+7x}}\cdot\frac{4(3+7x)-(5+4x)7}{(3+7x)^2}=\frac{3+7x}{5+4x}\cdot\frac{12+28x-35-28x}{(3+7x)\cdot(3+7x)}=\frac{-23}{(5+4x)\cdot(3+7x)}\ \blacksquare$$

- $y=x^{\sin x}=e^{\sin x\cdot\ln x}$

$$f(x)^{g(x)}=e^{\ln\left(f(x)^{g(x)}\right)}=e^{g(x)\cdot\ln f(x)}$$

$$y'=e^{\overset{f}{\sin x}\cdot\overset{g}{\ln x}}\cdot\left(\overset{f'}{\cos x}\cdot\overset{g}{\ln x}+\overset{f}{(\sin x)}\cdot\overset{g'}{\frac{1}{x}}\right)$$

$\boxed{\text{zeptat se}}$

- $y=x^{\ln x}=e^{\ln(x^{\ln x})}=e^{(\ln x)^2}$

$$y'=e^{(\ln x)^2}\cdot 2(\ln x)\frac{1}{x}=\frac{2\ln x\cdot e^{(\ln x)^2}}{x}\ \blacksquare$$

Tečny

cze-cs.techambition.com /demo/ /lesson/ 17/steps

- $y = \frac{3x-4}{2x-3}$ $\quad A = [2, ?]$ (x_0, y_0)

1) $y_0 = \frac{3 \cdot 2 - 4}{2 \cdot 2 - 3} = \frac{2}{1} = 2$ $\qquad A = [2,2]$

$x_0 = 2$

TEČNA

$$y - y_0 = f'(x_0) \cdot (x - x_0) \quad \Big| \quad y = kx + q$$

směrnicový tvar

$$f'(x) = \frac{3 \cdot (2x-3) - (3x-4) \cdot 2}{(2x-3)^2} = \frac{6x - 9 - 6x + 8}{(2x-3)^2} = \frac{-1}{(2x-3)^2} \to \text{v každém bodě klesá}$$

2) $f'(x_0) = f'(2) = -1$

$y - 2 = -1 \cdot (x - 2)$

$y - 2 = -x + 2 \quad \backslash +2$

$\underline{y = -x + 4}$ ■

kolmá ke směrnici

normála:

$$y - y_0 = -\frac{1}{f'(x_0)} (x - x_0)$$

$y - 2 = -\frac{1}{-1}(x - 2)$

$y - 2 = x - 2 \quad \backslash +2$

$\underline{y = x}$ ■

- $y = x^3 - 3x$; tečny // s osou x

$f'(x_0) = 0$

$f'(x) = 3x^2 - 3$

$3x_0^2 - 3 = 0 \quad \backslash +3$

$3x_0^2 = 3 \quad \backslash \frac{\cdot}{3}$

$x_0^2 = 1$

$x_0 \in \{-1, 1\}$

$x_0 = -1 \Rightarrow y_0 = -1 + 3 = 2 \quad A_2 = [-1, 2]$

$x_0 = 1 \Rightarrow y_0 = 1 - 3 = -2 \quad A_1 = [1, 2]$

Tečny

$y - y_0 = f'(x_0) \cdot (x - x_0)$, $f'(x_0) \to 0$

A_1: $y - (-2) = 0 \quad \backslash -2$

$\underline{y = -2}$

A_2: $y - (2) = 0 \quad \backslash +2$

$\underline{y = 2}$

Normála:

$x = 1$

$x = -1$

- $y = \ln(x+1)$ tečna a norm $A = [0, ?]$

$A = [0, 0]$

tečna:

$y = x$

odpověď

normála:

$y = -x$

- $y = \ln x$ tečna // s přímkou $p: 2x - y - 3 = 0$

$p: 2x - y - 3 = 0 \mid +y$

$\boxed{y = 2x - 3}$

nejsou splněné

$f'(x) = \frac{1}{x}$

$f'(x_0) = \frac{1}{x_0} = 2 \nearrow x_0 = \frac{1}{2}$

$\frac{1}{x_0} = 2x_0 - 3 \quad | \cdot x_0$

$1 = 2x_0^2 - 3x_0 \quad | -1$

$2x_0^2 - 3x_0 - 1 = y_0$

$x_0 = \frac{1}{2}$

$y_0 = \ln \frac{1}{2}$

tečna: $y = 2x - 1 + \ln \frac{1}{2}$

normála: $y = -\frac{1}{2}x + \frac{1}{4} + \ln x$

- $y = \sqrt[3]{x^3 - 1}$ teč. norm v $A = [1, ?]$

$A = [1, 0]$

Tečna: $x = 1$

Normála: $y = 0$

Příprava na test

Derivace:

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$(\ln x)' = \frac{1}{x}$

vnořené

$f(g(x))' = f'(x) \cdot g'(x)$

$f(x)^{g(x)} = e^{\ln(f(x)^{g(x)})} = e^{g(x) \cdot \ln f(x)}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$x^n = n \cdot x^{n-1}$ $\quad c' = 0$

$x^1 = 1 \cdot x^0 = 1$ $\quad (e^x) = e^x$

L'Hospital

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

pro: $a \in \mathbb{R} \cup \pm\infty$

- $f(x) = \sqrt{x \cdot \sqrt{x \cdot \sqrt{x}}} = \left(x \cdot \left(x \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= \sqrt{x^{1+\frac{1}{2}+\frac{1}{4}}} = \sqrt{x^{\frac{7}{4}}} = \left(x^{\frac{7}{4}}\right)^{\frac{1}{2}}$

$= \underline{x^{\frac{7}{8}}}$

$f(x)' = \frac{7}{8} x^{-\frac{1}{8}} =$ $\qquad \frac{7}{8} - \frac{8}{8} = -\frac{1}{8}$

$= \frac{7}{8 \cdot x^{\frac{1}{8}}} = \underline{\frac{7}{8 \cdot \sqrt[8]{x}}}$ ■

- $f(x) = (x^3 + 8) \cdot (x - 2) = x^4 - 2x^3 + 8x - 16$

$f(x)' = 4x^3 - 2 \cdot 3x^2 + 8 = \underline{4x^3 - 6x^2 + 8}$ ■

- $f(x) = \frac{(x^3+8) \cdot (x-2)}{(x^2+1) \cdot (x^3-1)} = \frac{x^4 - 2x^3 + 8x - 16}{x^5 - x^2 + x^3 - 1}$

$$f(x)' = \frac{(4x^3 - 2 \cdot 3x^2 + 8) \cdot (x^5 - x^2 + x^3 - 1) - (x^4 - 2x^3 + 8x - 16) \cdot (5x^4 - 2x + 3x^2)}{((x^2+1)(x^3-1))^2} =$$

$$= \frac{(3x \cdot (x-2) + (x^3+8) \cdot 1) \cdot (x^2+1) \cdot (x^3-1) - (x^3+8) \cdot (x-2) \cdot (2x \cdot (x^3-1) + (x^2+1) \cdot 3x^2)}{((x^2+1)(x^3-1))^2} =$$

$$= \frac{(3x^2 - 6x + x^3 + 8) \cdot (x^2+1) \cdot (x^3-1) - (x^3+8) \cdot (x-2) \cdot (2x^4 - 2x + 3x^4 + 3x^2)}{((x^2+1)(x^3-1))^2} = \underline{\text{jebu}}$$

- $f(x) = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} = \left(\frac{1 - x^{\frac{1}{2}}}{1 + x^{\frac{1}{2}}}\right)^{\frac{1}{2}} = \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}$

$$f(x)' = \frac{\frac{1}{2} \cdot (1-\sqrt{x})^{-\frac{1}{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}} \cdot \sqrt{1+\sqrt{x}} - \sqrt{1-\sqrt{x}} \cdot \frac{1}{2}(1+\sqrt{x})^{-\frac{1}{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}}}{1+\sqrt{x}} = \frac{-\frac{\sqrt{1+\sqrt{x}}}{4 \cdot \sqrt{1-\sqrt{x}} \cdot \sqrt{x}} - \frac{\sqrt{1-\sqrt{x}}}{4 \cdot \sqrt{1+\sqrt{x}} \cdot \sqrt{x}}}{1+\sqrt{x}}$$ ■

$$f(x) = \ln\left(x + \sqrt{1+x^2}\right)$$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b \cdot c}$$

$$f'(x) = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(x + \sqrt{1+x^2}\right)' =$$

$$= \frac{1 + \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x}{x+\sqrt{1+x^2}} = \frac{\frac{2x}{2\sqrt{1+x^2}} + 1}{x+\sqrt{1+x^2}} = \frac{\frac{x}{\sqrt{1+x^2}} + 1}{x+\sqrt{1+x^2}} = \frac{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}$$

$$= \frac{\sqrt{1+x^2}+x}{\sqrt{x^2+1} \cdot \left(\sqrt{x^2+1}+x\right)} = \frac{1}{\sqrt{x^2+1}} \quad \blacksquare$$

Další cviko

L'Hopital

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

$\frac{0}{0} \vee \frac{\pm\infty}{\pm\infty}$

$$\lim_{x\to -3} \frac{x^2-9}{x^2+4x+3} \overset{\frac{0}{0}\ LHP}{=} \lim_{x\to -3} \frac{2x}{2x+4} = \lim_{x\to -3} \frac{x}{x+2} = \frac{-3}{-1} = 3$$

$$\lim_{x\to 1}\left(\frac{1}{x-1} - \frac{1}{\ln x}\right) \overset{\infty-\infty}{=} \frac{\ln x - (x-1)}{x-1 \cdot \ln x} \overset{\frac{0}{0}}{=} \lim_{x\to 1} \frac{\ln x - x + 1}{x-1 \cdot \ln x} \overset{LHP}{=} \lim_{x\to 1} \frac{\frac{1}{x} - 1}{1\cdot \ln x + (x-1)\cdot \frac{1}{x}} =$$

$$\lim_{x\to 1} \frac{\frac{1}{x}(1-x)}{\frac{1}{x}(x\cdot \ln x + x - 1)} \overset{\frac{0}{0}}{=} \lim_{x\to 1} \frac{-1}{1 \ln x + x \cdot \frac{1}{x} + 1} = \lim_{x\to 1} \frac{-1}{\ln x + 2} = \frac{-1}{2}$$

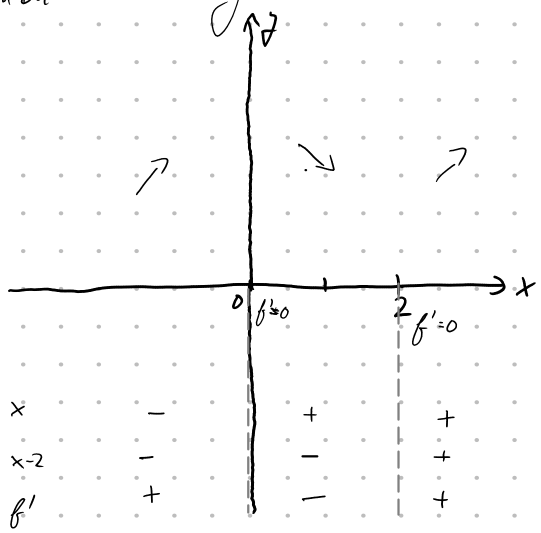
Extrémy fce

Lokální extrémy

$f(x) = (x-2)^2 \cdot (x+1) = x^3 - 3x^2 + 4$; najděte lok. extrémy

$D(f) = \mathbb{R}$

$f'(x) = 3x^2 - 3\cdot 2x = 3x(x-2)$



x	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
f'	$+$	$-$	$+$

$f(0)=4$ $\quad f(2)=0$

L. maximum 4 v 0 $\quad f(0)=4$

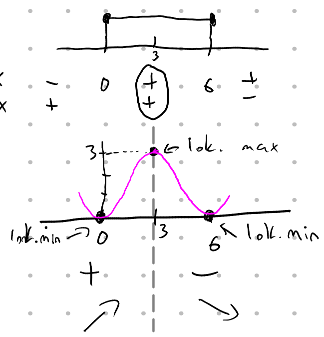
L. minimum 0 v 2 $\quad f(2)=0$

$f(x) = \sqrt{6x - x^2} = (6x - x^2)^{\frac{1}{2}}$

$6x - x^2 \geq 0$

$x(6-x) \geq 0$

		0	3	6	
x	$-$		$+$		$+$
$6-x$	$+$		$+$		$-$



$f(0)=0 \quad f(3)=3 \quad f(6)=0$

$f'(x) = \frac{1}{2}(6x - x^2)^{-\frac{1}{2}} \cdot (6-2x) = \frac{2(3-x)}{2\cdot(6x-x^2)^{\frac{1}{2}}}$

≥ 0

$6x - x^2 \neq 0$

Nemá Derivaci $\begin{cases} x=0 \\ x=6 \end{cases}$

1. Derivace
2. $f'(x) = 0$; $x \in \{??\}$

$f'(x) = 0$ pro $3 - x = 0$

$x = 3$

- $f(x) = \overset{f}{x^2}\cdot\overset{g}{(x-6)}$

$$f'(x) = \overset{f'}{2x}\cdot\overset{g}{(x-6)} \overset{+}{+} \overset{f}{x^2} \overset{\cdot g'}{-1} = \underline{2}x^2 - 12x + \underline{x^2} - 1 = \underline{3x^2 - 12x - 1}$$

Lok max $f(0) = 0$

lok min $f(4) = -32$

- $f(x) = \dfrac{x}{\ln x}$

lok min $f(e) = e$

- $f(x) = x^3 - 2|x|$

$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

pro $x>0$

lok max $f(0) = 0$

lok min $f\left(\frac{\sqrt{6}}{3}\right) = \frac{4\sqrt{6}}{9}$

$f'(x) = 3x^2 - 2 =$ (A^2 B^2)

$= 3\left(x^2 - \frac{2}{3}\right) =$

$= 3\left(x - \sqrt{\frac{2}{3}}\right)\left(x + \sqrt{\frac{2}{3}}\right) =$

$= 3\left(x - \frac{\sqrt{6}}{3}\right)\left(x + \frac{\sqrt{6}}{3}\right)$

$-$ $+$

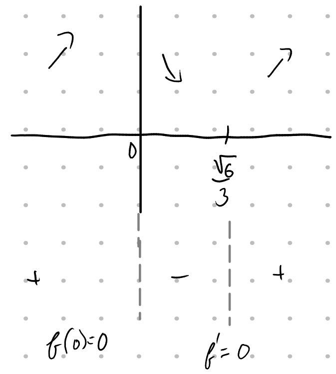
$+$ $+$

$$f(x) = \begin{cases} 3x^2 + 2 & x < 0 \\ \text{neex} & x = 0 \\ 3x^2 - 2 & x > 0 \end{cases}$$

$f(x) = x^3 - 2(-x) = x^3 + 2x$

$\boxed{= \frac{-\sqrt{6}\cdot 4}{9}}$

$f(x) = x^3 - 2x$



?

$f\left(\frac{\sqrt{6}}{3}\right) = \left(\frac{\sqrt{6}}{2}\right)^3 - 2\cdot\frac{\sqrt{6}}{3} =$

$= \frac{\sqrt{6}}{3}\left(\frac{6}{9} - 2\right) =$

$= \frac{\sqrt{6}}{3} \cdot \frac{6-18}{9} = \frac{\sqrt{6}}{3} \cdot \frac{4}{3}$

$f(x) = x^5 - 5x^4 + 5x^3 + 1$ pro $x \in \langle -2, 1 \rangle$

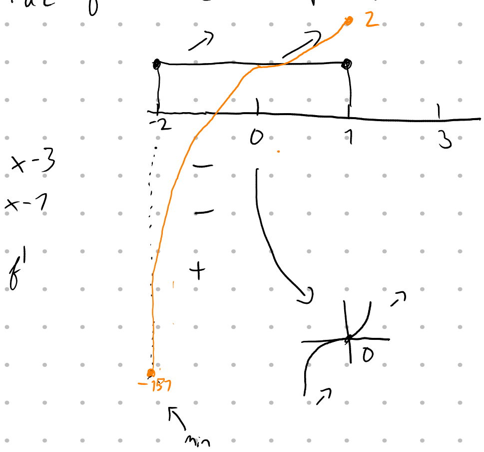
$f'(x) = 5x^4 - 5\cdot 4x^3 + 5\cdot 3x^2 =$

$= 5x^2(x^2 - 4x + 3)$

$= 5x^2(x-3)(x-1)$

$\Rightarrow 0$

Kde f' kladné a záporné?

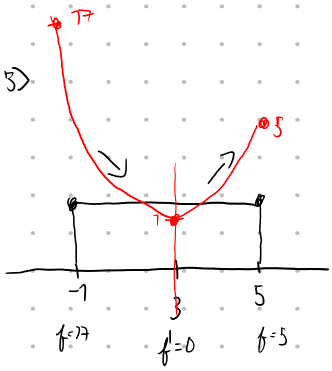


$f(-2) = \dots = 32 - 80 - 40 + 1 = -151$

$f(1) = 1 - 5 + 5 + 1 = 2$

$f(x) = x^2 - 6x + 10 \quad x \in \langle -1, 5 \rangle$

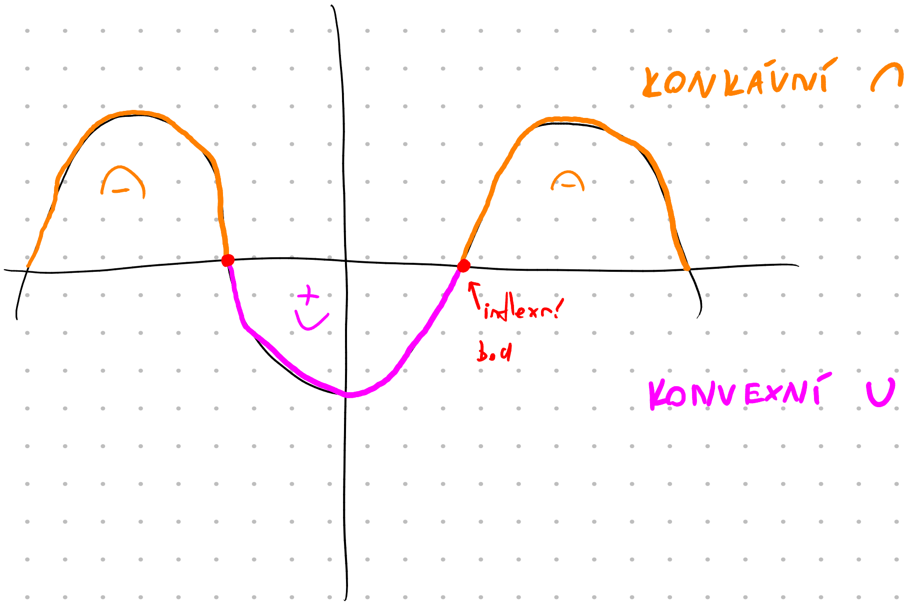
$f'(x) = 2x - 6 = 2\cdot(x-3)$



$f(3) = 9 - 18 + 10 = 1$ → minimum

$f(5) = 5$

$f(-1) = 17$ → maximum



platí:

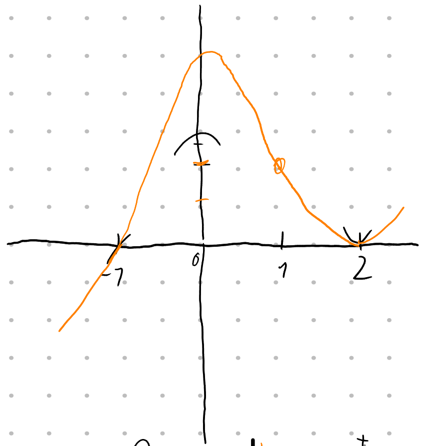
- konkávní → $f'' < 0$
- konvexní $f'' > 0$
- inflexní bod $f'' = 0$

↳ do konkávy kávu nenaleješ

$f(x) = (x-2)^2(x+1) = x^3-3x^2+4$

$f'(x) = 3x^2-6x = 3x(x-2)$

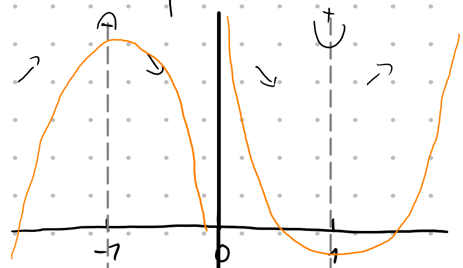
$f''(x) = 3\cdot 2x - 6 = 6x-6 = 6(x-1)$ →



$f(x) = \frac{x^2+1}{x}$ ⚠ $x \neq 0$

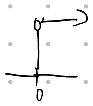
$f'(x) = \frac{2x\cdot x-(x^2+1)\cdot 1}{x^2} = \frac{x^2-1}{x^2} = \frac{(x-1)(x+1)}{x^2}$

$f''(x) = \frac{2x\cdot x^2-(x^2-1)2x}{(x^2)^2} = \frac{2x(x^2-x^2+1)}{x^4} = \frac{2x}{x^4} = \frac{2}{x^3}$



$x-1$	−	−	−	+
$x+1$	−	+	+	+
f'	+ ↗	− ↘	− ↘	+ ↗
f''	−	−	+	+

$x \in \mathbb{R}^+$



$f(x) = x + \frac{1}{x}$ co nejmenší

$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2} = \frac{(x+1)(x-1)}{x^2}$

$x-1$	−	+
$x+1$	+	+
f'	−	+

$f(1) = 1 + \frac{1}{1} = 2$ → minimum

$$\lim_{x \to 0} \frac{\sin x}{x} = 0$$

$$(A+B)(A-B) = A^2 - B^2$$

$$\sin^2 x + \cos^2 x = 1$$

$$tg\,x \quad \frac{\sin x}{\cos x}$$

$$\lim_{x \to \infty} \left(1 - \frac{1}{x}\right)^x = e$$

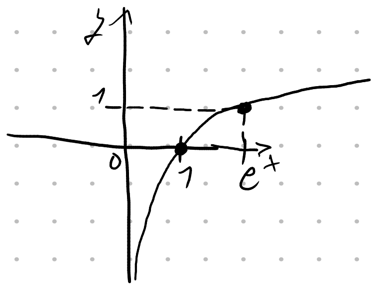
$$x^2 + (a+b)x + (a \cdot b)$$

$$\downarrow$$

$$(x+a) \cdot (x+b)$$

z přípravy na test dávaný zpátky

$$\lim_{x\to 2^+} \frac{2x-10}{2-x} = \frac{4^+-10}{2-2^+} = \frac{-6^+}{0^-} = \boxed{+\infty}$$



2) $g(x) = \ln f(x)$

$$g(x) = \ln \frac{2x-10}{2-x}$$

$$\lim_{x\to \pm\infty} g(x)$$

Rule or special case	Formula
Product	$\ln(xy) = \ln(x) + \ln(y)$
Quotient	$\ln(x/y) = \ln(x) - \ln(y)$
Log of power	$\ln(x^y) = y\ln(x)$
Log of e	$\ln(e) = 1$
Log of one	$\ln(1) = 0$
Log reciprocal	$\ln(1/x) = -\ln(x)$

$4^+-10 \to = -6^+$ $\quad$ $2-2^+ \to = 0^-$

$$\lim_{x\to 2^+} \ln\frac{2x-10}{2-x} = \lim_{x\to 2^+} (\ln 2x-10) - (\ln 2-x) = \lim_{x\to 2^+} \ln -6^+ - \ln 0^- = \ln\frac{-6^+}{0^-} =$$
$$= \ln \infty = \infty$$

$\to 10^- - 10 = 0^-$

$$\lim_{x\to 5^-} \ln\frac{2x-10}{2-x} = \lim_{x\to 5^-} \ln\frac{0^-}{-3^+} = \lim_{x\to 5^-} \ln 0 = \boxed{-\infty}$$

$2-x \to -3^+$ $\quad$ $\to 0$

$x \neq 2$

$$\frac{2x-10}{2-x} > 0$$

$2x-10 \neq 0 \quad | +10$

$2x \neq 10 \quad | \cdot \frac{1}{2}$

$\boxed{x \neq 5}$

$- + \quad + - \quad$ 2 $\quad$ 5

Test limity

zeptal se

$$\lim_{x\to -3} \frac{x^2-9}{x^2+4x+3} = \frac{(x+3)(x-3)}{(x+1)(x+3)} = \frac{x-3}{x+1} = \underline{0}$$ [redacted]

$\frac{0}{-2}$

$$\frac{2}{(8)\cdot(\sqrt{25}+5)} = \frac{2}{8\cdot 10}$$

$$\lim_{x\to 1} \frac{(x+1)}{(x+7)(\sqrt{x^2+24}+5)} = \frac{2}{80}$$ [redacted]