3 Your Task

Evaluate each of the following lambda expressions, showing each step of the computation along the way. Note that some (but not all) of the following expressions may require α-conversion. You should not perform α-conversion unless you believe there is variable capture that warrants it. As an example computation:

$$(\lambda x.(\lambda y.xy)z)\,(\lambda x.xy) \xrightarrow{\alpha} (\lambda x.(\lambda a.xa)z)\,(\lambda x.xy) \xrightarrow{\beta} (\lambda a.(\lambda x.xy)a)z \xrightarrow{\beta} (\lambda x.xy)z \xrightarrow{\beta} zy$$

Problem 1. $(\lambda x.x)\ y$ = y

Problem 2. $(\lambda x.y)\ x$ = y

Problem 3. $(\lambda x.x\ y)\ (\lambda y.y\ z)$ = $(\lambda y.yz)y$ = yz

Problem 4. $(\lambda x.x\ y)\ (\lambda a.a\ b)\ p$ = $(\lambda a.ab)yp$ = ybp

Problem 5. $(\lambda x.x\ y)\ (\lambda a.b\ a)\ p$ = $(\lambda a.ba)yp$ = byp

Problem 6. $(\lambda x.(\lambda y.x\ y))\ y \overset{\alpha}{=} (\lambda x.(\lambda y.xy))a$ = $\lambda y.ay$ -3 should rename the bound y's not the free one

Problem 7. $(\lambda x.y\ x)\ y$ = yy

Problem 8. $(\lambda x.\lambda y.x\ y\ z)\ (\lambda x.x\ y)\ z$ = -3 need to alpha convert y
$(\lambda xy.xyz)(\lambda x.xy)z = (\lambda x.xy)zz$ = zyz

Problem 9. $(\lambda x.y\ x)\ x$ = yx

Problem 10. $(\lambda y.y\ x)\ (\lambda z.z\ y)$ = $(\lambda z.zy)x$ = xy