

3 Your Task

Evaluate each of the following lambda expressions, showing each step of the computation along the way. Note that some (but not all) of the following expressions may require α -conversion. You should not perform α -conversion unless you believe there is variable capture that warrants it. As an example computation:

$$(\lambda x. (\lambda y. xy)z) (\lambda x. xy) \xrightarrow{\alpha} (\lambda x. (\lambda a. xa)z) (\lambda x. xy) \xrightarrow{\beta} (\lambda a. (\lambda x. xy)a)z \xrightarrow{\beta} (\lambda x. xy)z \xrightarrow{\beta} zy$$

Problem 1. $(\lambda x. x) y = y$

Problem 2. $(\lambda x. y) x = y$

Problem 3. $(\lambda x. x y) (\lambda y. y z) = (\lambda y. y z) y = yz$

Problem 4. $(\lambda x. x y) (\lambda a. a b) p = (\lambda a. ab) y p = ybp$

Problem 5. $(\lambda x. x y) (\lambda a. ba) p = (\lambda a. ba) y p = byp$

Problem 6. $(\lambda x. (\lambda y. x y)) y \stackrel{\alpha}{=} (\lambda x. (\lambda y. xy)) a = \lambda y. ay$

Problem 7. $(\lambda x. y x) y = y y$

Problem 8. $(\lambda x. \lambda y. x y z) (\lambda x. x y) z = (\lambda x y. xy z) (\lambda x. xy) z = (\lambda x. xy) z z = z y z$

Problem 9. $(\lambda x. y x) x = y x$

Problem 10. $(\lambda y. y x) (\lambda z. z y) = (\lambda z. zy) x = xy$