Power Electronics

Exercise: Pulse Width Modulation Methods

2013

1 Theory

1.1 Introduction

As known in the last exercise, with fundamental frequency clocking (FFC) it is possible to generate three-phase voltages for motor drive. Although the frequency can be changed, it is not able to generate difference voltage magnitudes and forms. Pulse width modulation (PWM) method is the solution of these problems. The basic idea is described in Figure 1 and Figure 2.
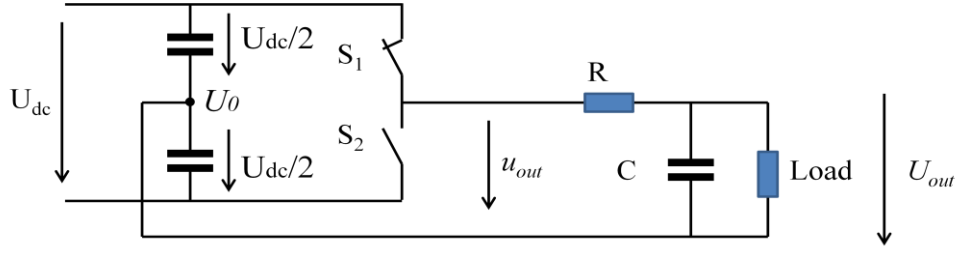

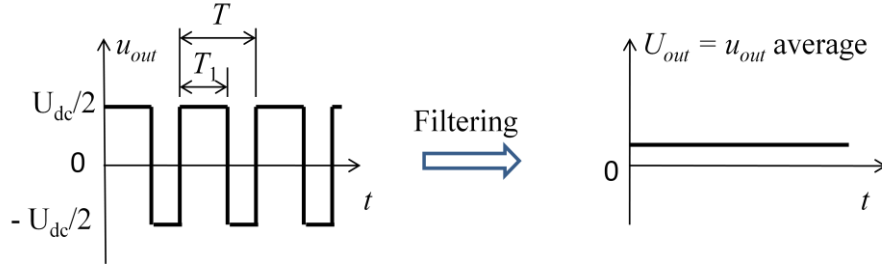Figure 1. A conceptual circuit for realizing PWM

Figure 2. Voltages in a PWM circuit

The input voltage is DC and has a constant value U_{dc}. Switches S_1 and S_2 are turned on and off periodically with cycle T. During time T_1 switch S_1 conducts (ON) and S_2 isolates (OFF). In the rest of the period $(T-T_1)$, the switch status is reversed. With this process we will get a voltage u_{out} as shown in the left part of Figure 2. The average (DC component) of the output voltage u_{out} would be a constant voltage U_{out} with a value that is different than U_{dc}. The value U_{out} is

$$U_{out} = (\frac{T_1}{T} - \frac{1}{2}) \times U_{dc}, \tag{1}$$

where (T_1/T) is duty cycle, which is usually represented with a percentage value. The output voltage is directly dependent on the duty cycle. By continuously changing the duty cycle, the output voltage is changed continuously, too.

1.2 SPWM

Based on the PWM concept, if the duty cycle is changed sinusoidally, a sinusoidal voltage will be generated at the output. The question then becomes how to change the duty cycle with a sinusoidal rule. The following figure illustrates one method, which is named Sub-oscillation PWM (SPWM).
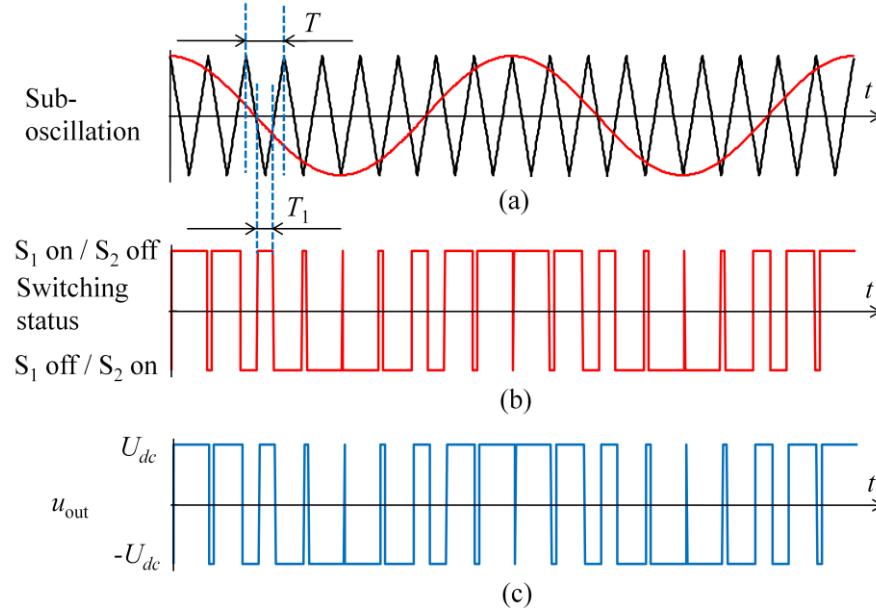

Figure 3. Concept of SPWM

Referring to Figure 3, it is expected to generate a sinusoidal voltage as shown by the red curve in Figure 3a. This signal is named reference signal. In parallel, we take a triangle signal with a much higher frequency than that of the reference signal. The triangular signal is called carrier signal. Its frequency is the PWM frequency. These two signals are compared. At the time when the reference signal is larger than the triangle signal, the upper switch is turned on and the lower switch is off; otherwise, the upper switch is off and the lower switch is on, as shown in Figure 3c.

In the above figure the generated voltage approximates the sinusoidal values very roughly. This lies in the low frequency of the triangular signal. If this signal is much higher than that of the reference signal, the low frequency components of the PWM output can be very close to the waveform of the reference signal. In the practice, the PWM frequency is usually above 8 kHz. For a 50 Hz driving voltage for a motor, the generate PWM voltages can approximate sinusoidal values very well.

Based on this concept, we could get the following circuit for generating three-phase voltages. It is actually the inverter circuit as was studied for the FFC.
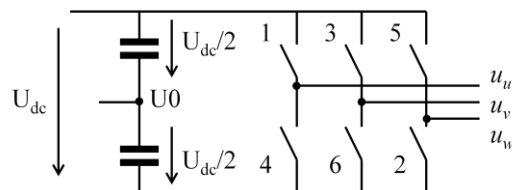

Figure 4. Structure of a three-phase inverter

1.3   Modulation index

In the concept shown in Figure 3, it is able to adjust the output voltage by adjusting the magnitude of the reference signal. If the maximum value of the reference is equal to the maximum value of the triangle signal, the SPWM method will generate the highest available voltage. To measure the ability of a PWM method to deliver AC power, the term Modulation Index is defined:

$$m = \frac{U_{\text{PWM}}}{U_{\text{FFC}}}, \tag{2}$$

where U_{PWM} is the magnitude of the first harmonic of the voltage generated by a given PWM method, and U_{FFC} is the magnitude of the first harmonic of the voltage generated by FFC method.

It is known that the first harmonic of the FFC voltage is the first term of its Fourier deconstruction. This is shown in the following figure.
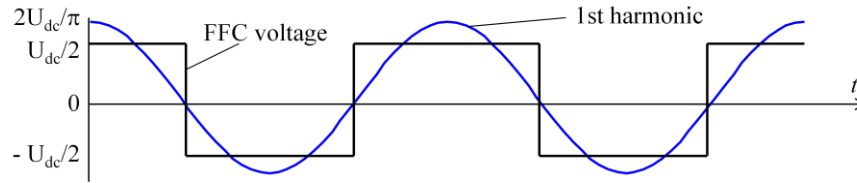

Figure 5. FFC voltage and ist first harmonic

According to the Fourier transformation, the maximum value of the first harmonic of a square wave is $(4/\pi)$ times of the maximum value of the square wave signal. If the DC bus voltage is U_{dc}, the maximum FFC voltage compared to U_0 is $U_{\text{dc}}/2$. Thus the maximum value of its first harmonic is $(2U_{\text{dc}}/\pi)$.

The maximum output line voltage of SPWM method is equal to half of the DC bus voltage, namely $U_{\text{dc}}/2$. Therefore, the maximal modulation index of SPWM is

$$m_{\text{SPWM}} = \frac{U_{dc}/2}{2U_{dc}/\pi} = \frac{\pi}{4} \approx 0.785. \tag{3}$$

Using other PWM methods, it is possible to have a higher modulation index, meaning those methods can output more power in the first harmonic.

1.4   Other PWM methods

Besides SPWM, other PWM methods are proposed for generating three-phase voltages to deliver more power, including third-harmonic PWM, sixty-degree PWM and space vector PWM. All these methods are aimed at making better use of the DC bus voltage and thus increasing modulation index. These methods are shortly mentioned here without detailed analysis.

1.4.1 Third-harmonic PWM

In this method, the reference signal is not a pure sinusoidal wave, but the sum of the fundamental and the third harmonic. It is shown with curve $f(\omega t)$ in the following figure. Same as the SPWM method, this signal will be compared with the triangle signal to generate PWM signals.
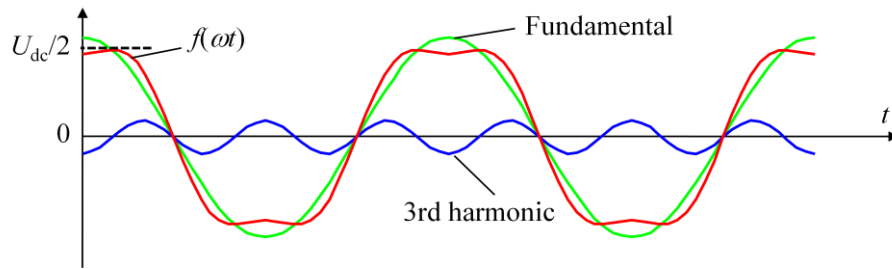
Figure 6. Reference signal of third-harmonic PWM

This concept is derived from the nature of three-phase motors that the third harmonic will be filtered out in the windings. And thus only the fundamental part will remain.

The modulation index of this method reaches 1.

1.4.2 Sixty-degree PWM

The sixty-degree PWM is an extension of third-harmonic PWM. It is based on the nature of the motor that not only third harmonic, but also all none-even triple harmonics are filtered out by the windings. Adding all these harmonics with the fundamental together, a function with flat segments are obtained as shown in the following figure. The period of the flat part covers 60° signal phase.

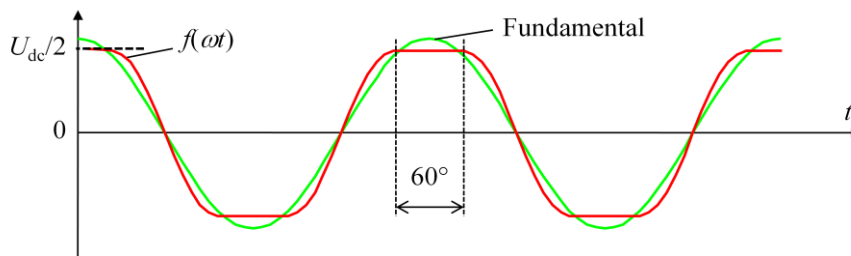The modulation index of this method can reach 1, too.

Figure 7. Concept of sixty-degree PWM

1.4.3 Space vector PWM

Space vector PWM is implemented in a different manner as SPWM. It is derived based on the space vector concept. This method will be discussion in a later exercise.

1.5　Dead time

It is known that the two power switches of a bridge arm in an inverter should not be turned on simultaneously. In order to absolutely avoid any interlace of the "on" state during the switching moment, a short "dead time" is inserted between the positive pulses of the two switches in one bridge arm.
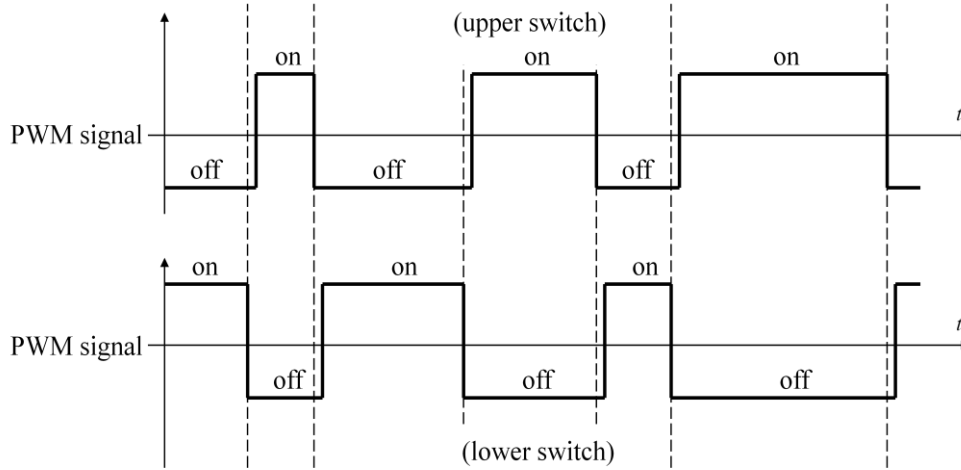

Figure 8. Dead time inserted between every two positive pulses

The length of the dead time is determined by the characteristic of the switching components.

Obviously, the insertion of the dead time in every PWM cycle distorts the output voltages. In accurate motor control, this negative effect will be compensated by prolong some pulses.

1.6　Synchronized PWM

In PWM, the reference signal and the triangular signal could be synchronized or asynchronized. Their difference is shown with the following figure.
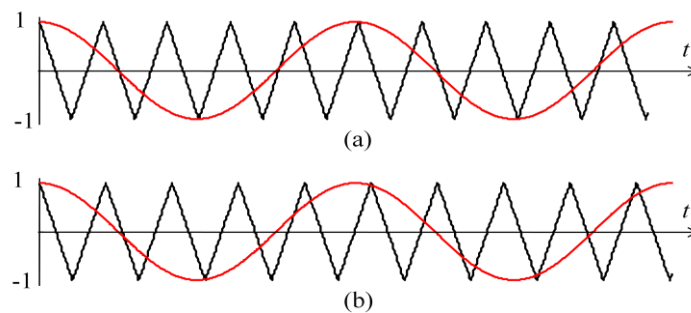

Figure 9. Synchronized (a) and asynchronized (b) PWM

In synchronized PWM (Figure 9a), the frequency of the triangle signal is an integral multiple of that of the reference signal. Therefore, the generated PWM signal is identical in every cycle of a reference signal with constant frequency. This ensures a stable voltage output and is important in high power applications, where low PWM frequencies are used.

On the contrary, aynchronized PWM (Figure 9b) doesn't ensure the frequency relationship between both signals. This method is simple but causes different voltage forms in difference cycles. However, if the

triangle frequency is much higher than the reference frequency, this influence is negligible. This is usually the case in the normal application for middle and low power applications.

1.7   Symmetric PWM

The symmetric and asymmetric PWM methods are explained with the following figure:
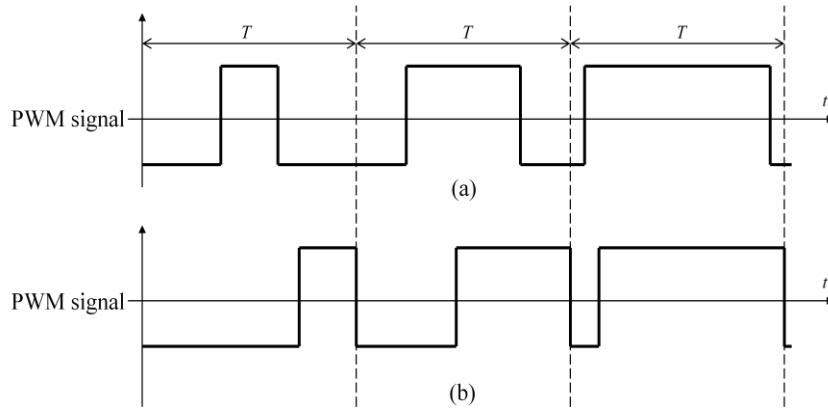

Figure 10. Symmetric (a) and asymmetric (b) PWM

In symmetric PWM, the positive (or negative) pulse of every PWM cycle is located in the middle of the cycle period, while in the asymmetric PWM, the pulses are usually aligned to the start or the end of the PWM cycle.

Practically, asymmetric methods are easier to be realised, but symmetric methods evoke fewer harmonic interferences. Therefore, symmetric PWM should be used if possible.

2 Exercises

2.1 Exercise 1

2.1.1 Problem

It is expected to generate a sinusoidal voltage using SPWM method. The circuit and the signals are given in Figure 11. The maximal value of the sinusoidal reference signal is half of that of the carrier signal.

(1) Please draw the waveform of the output voltage u_{out} and mark the voltage values in the diagram.
(2) Please calculate the value of modulation index for this SPWM application.
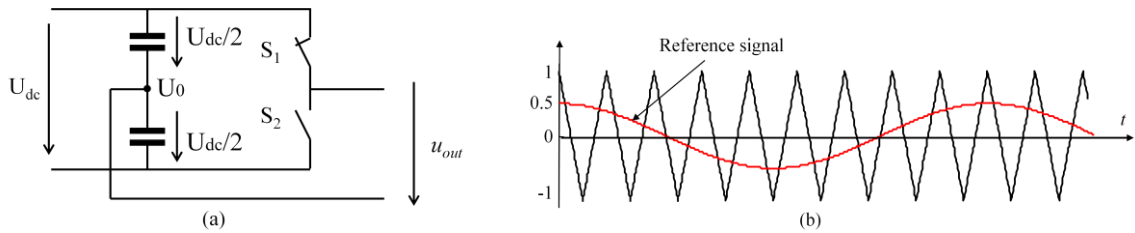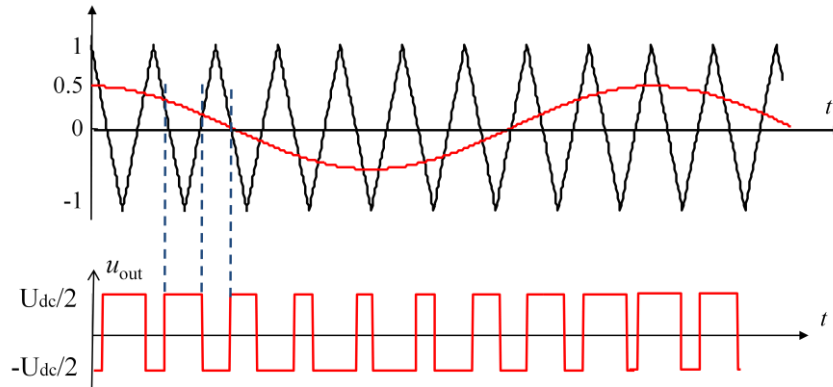

Figure 11. (a) PWM circuit, (b) Carrier and reference signals for SPWM

2.1.2 Solution to question 1

2.1.3 Solution to question 2

Based on the theory above it is known that the maximal value of the first harmonic of the FFC voltage generated with the circuit in this exercise is $\frac{4}{\pi} \cdot \frac{U_{dc}}{2}$. The maximal value of the sinusoidal voltage generated in this exercise is $\frac{U_{dc}}{4}$. Thus the modulation index is

$$m = \frac{U_{dc}/4}{2U_{dc/\pi}} = \frac{\pi}{8} \approx 0.39$$

2.2 Exercise 2

2.2.1 Problem

For a three-phase motor, before starting up, all windings should get a zero voltage. Supposing SPWM method is to be implemented in the power inverter, please draw the PWM signals for the three bridge arms of the inverter. The carrier signal is given in Figure 12. Dead time is not considered.

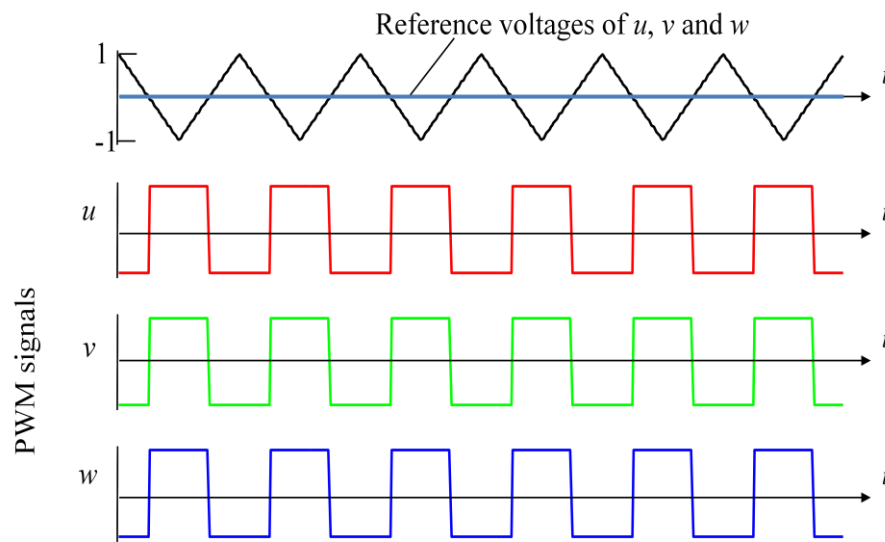Figure 12. The carrier signal for SPWM

2.2.2 Solution

Figure 13. PWM signals for zero voltages on the motor

It should be noted that the duty cycles for the three bridge arms are 50% (not 0%) in order to generate zero voltages for the motor.

2.3 Exercise 3

2.3.1 Problem

In an SPWM implementation that is realised with the circuit shown in Figure 14, it is expected to generate a three-phase sinusoidal voltage for the pure inductive load.
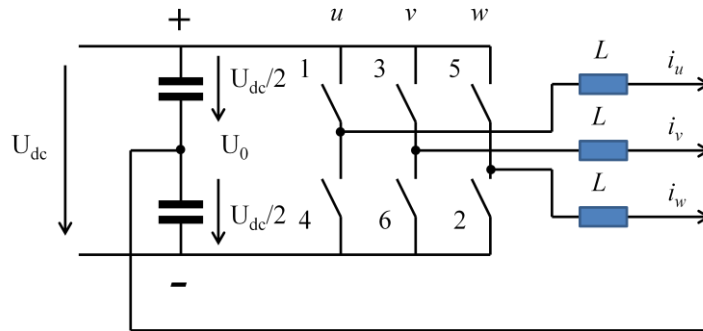

Figure 14. Structure of a three-phase inverter with inductive load

Taking potential U_0 as the reference zero voltage, the maximal value of the output voltage should be $U_{dc}/2$. The carrier and reference signals are given in Figure 15.
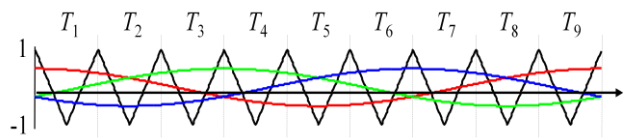

Figure 15. Structure of a three-phase inverter

Please

1. draw the PWM signals for the three voltages;
2. draw the curves of voltages;
3. draw the voltage space vector and describe its trajectory;
4. calculate the currents and draw the current space vector;
5. draw the current space vector again if the PWM frequency is 81 times of the reference three-phase signals.
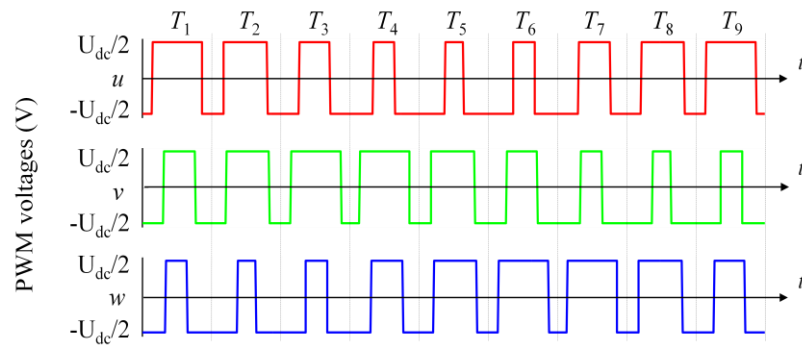
Note: For question 4 and 5 compute program (e.g. Matlab) is suggested to be used.
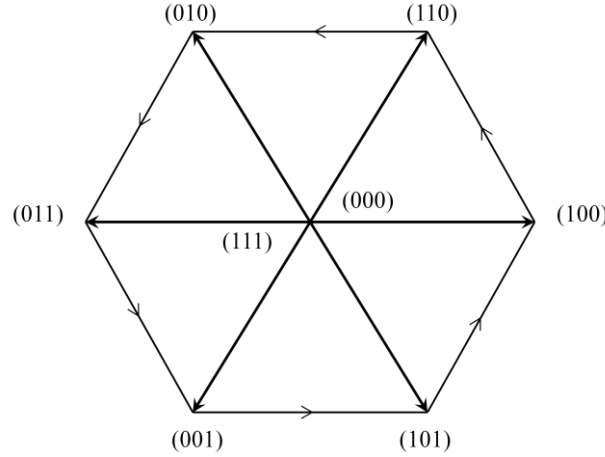
2.3.2 Solution to question 1
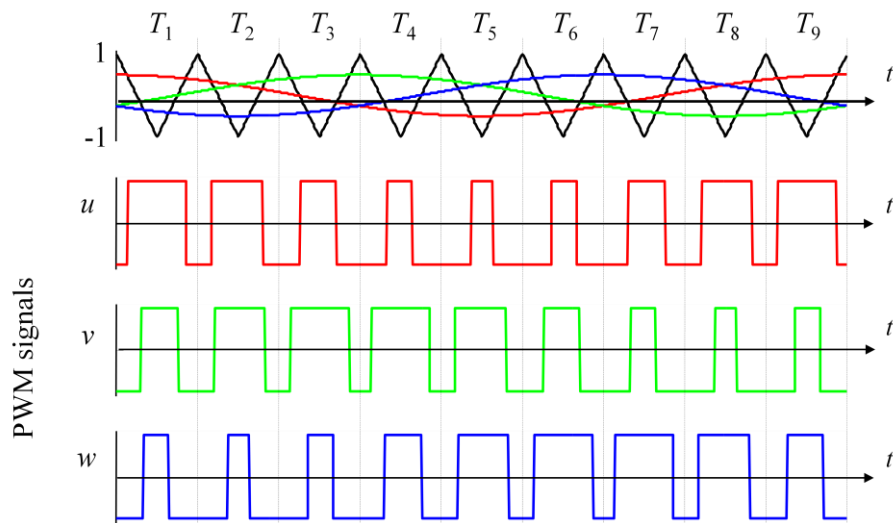
2.3.3 Solution to question 2

The voltage curves have the same form as the PWM signals.

2.3.4 Solution to question 3

The voltage space vectors are drawn based on the result of question 1, where high level is defined as "1" and low level is defined as "0". The vectors are shown in the following figure. All eight vectors are present.


To analyse the trajectory, let's check the vector sequence starting from $t = 0$. It is 000, 100, 110, 111, 110, 100, 000, 100, 110, 111, 110, 010, 000, 010, 110, 111, 110, 010, 000, ...

It is seen that the two zero vectors (000 and 111) are also present. Their function is to generate the zero voltage and thus reduce the average voltage magnitude.

It is also seen that the voltage space vector doesn't change continuously, same as the case in FFC. However, if the frequency of the triangle signal is high enough, the vector jumps very fast among the two adjacent none-zero vectors and the zero vectors. The average value will then be approximately continuous. This is further illustrated in the following figure.
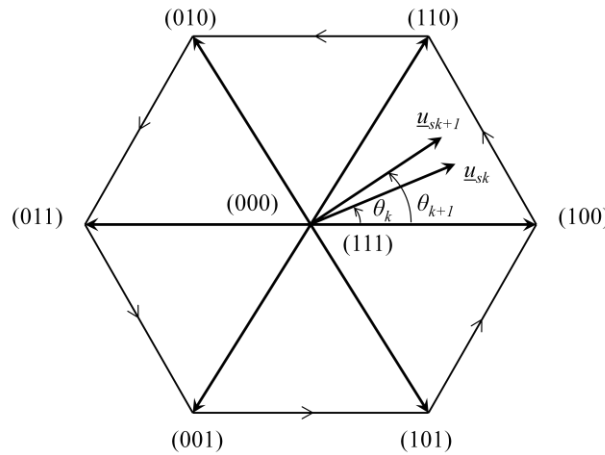
In the figure, $\underline{u}_{sk}$ and $\underline{u}_{sk+1}$ represent the voltage space vectors at PWM cycle T_k and T_{k+1}, respectively. From cycle k to $(k+1)$, the space vector changes only a small step. This approximates a continuous movement in a larger time scale.
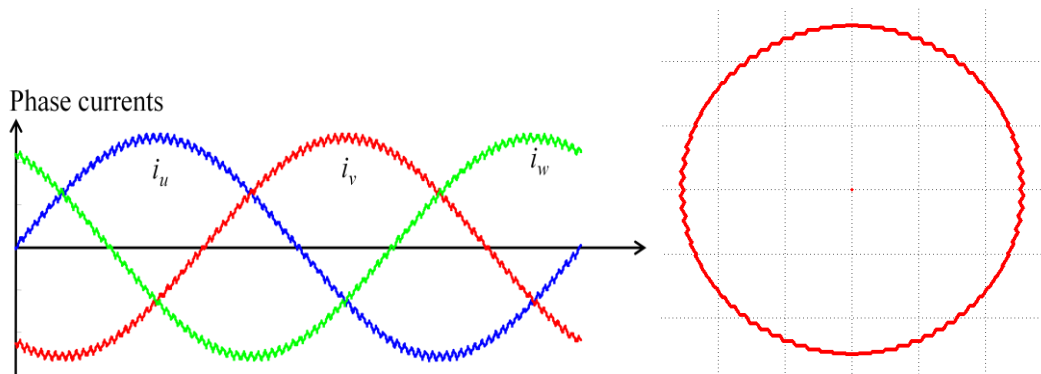
2.3.5 Solution to question 4

Phase currents and space vector trajectory (in red). The green circle represents the space vector trajectory of the ideal three-phase current.
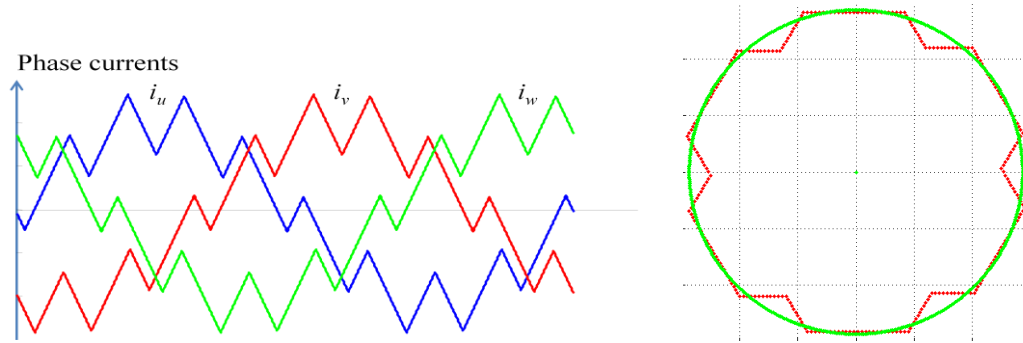
2.3.6 Solution to question 5

Phase current and space vector trajectory if PWM frequency is 81 times of the reference signal frequency.

3   Appendix

```matlab
% matlab code for exercise 2
% Directly copy the codes to MATLAB commend window to run the code.
% Remove "%" before the two lines "%tr = [tr, tr, tr];" for question 5.
% Dr. Michael Gao. TUM, EI, EAL, 2011


% init
clear;


% triangular signal
m = 0.5;      % max voltage
n = 20;       % number of data points in one triangle cycle
fg = 0;       % counting number of figures
trn = [(1:-1/n:-1)];          % rising edge of a triangle signal
trp = [(-1+1/n:1/n:1-1/n)]; % falling edge of a triangle signal
tr0 = [(1:-1/n:-1),(-1+1/n:1/n:1-1/n)]; % one triangle signal
tr = tr0;    % one triangle signal
% duplication of the triangle signal
% un-comment the lines to get higher PWM frequency
tr = [tr, tr, tr];
tr = [tr, tr, tr];
%tr = [tr, tr, tr];
%tr = [tr, tr, tr];


% sinusoidal reference signal
wt = 2*pi* (1:length(tr))/length(tr);
u_u0 = m*cos(wt);
u_v0 = m*cos(wt-pi*2/3);
u_w0 = m*cos(wt+pi*2/3);


% pwm signals and phase voltages
u_u1 = (u_u0>tr)-0.5;
u_v1 = (u_v0>tr)-0.5;
u_w1 = (u_w0>tr)-0.5;


% phase currents
i_u0 = cumsum(u_u1); i_u = i_u0 - mean(i_u0);
i_v0 = cumsum(u_v1); i_v = i_v0 - mean(i_v0);
i_w0 = cumsum(u_w1); i_w = i_w0 - mean(i_w0);


%----------------------------------------------
% plot
% triangle, reference signal and PWM signals
fg=fg+1;
figure(fg);clf;
subplot(4,1,1);
plot(wt,tr, 'k'); hold on;
plot(wt,u_u0,'r');
plot(wt,u_v0,'g');
plot(wt,u_w0,'b');
axis([0,wt(end),-1.1, 1.1]);
ylabel('duty cycle');
title('triangle and reference voltage signals');


% PWM signals and phase voltages
subplot(4,1,2);plot(wt,u_u1,'r'); axis([0,wt(end),-1.1, 1.1]);
ylabel('u');
subplot(4,1,3);plot(wt,u_v1,'g'); axis([0,wt(end),-1.1, 1.1]);
ylabel('v');
```

```matlab
subplot(4,1,4);plot(wt,u_w1,'b'); axis([0,wt(end),-1.1, 1.1]);
ylabel('w');

% phase current
fg=fg+1;
figure(fg);clf;
hold on;
title('phase current');
plot(wt,i_u); plot(wt,i_v,'r');plot(wt,i_w,'g'); grid on;


% ----------------------------------------
% space vector of voltage
ua = 2/3*(u_u1 - 0.5*u_v1 - 0.5*u_w1);
ub = 2/3*(sqrt(3)/2*u_v1 - sqrt(3)/2*u_w1);
fg=fg+1;
figure(fg);clf;
hold on;
title('space vector of voltage');
for k = 1:length(ua)
    plot([0,ua(k)], [0,ub(k)],'.-');
    %drawnow
end


% ----------------------------------------
% space vector of phase currents
ia = 2/3*(i_u - 0.5*i_v - 0.5*i_w);
ib = 2/3*(sqrt(3)/2*i_v - sqrt(3)/2*i_w);
fg=fg+1;
figure(fg);clf;
hold on;
title('space voltage trajectory of phase current');
for k = 1:length(ia)
    plot([0,ia(k)], [0,ib(k)],'r.');
    %drawnow
end
grid on
```

4 References

Valentine, Richard (1998). Motor control electronics handbook. New York: McGraw-Hill (pages from 246)