An introduction to Reinforcement Learning

21st of June 2022

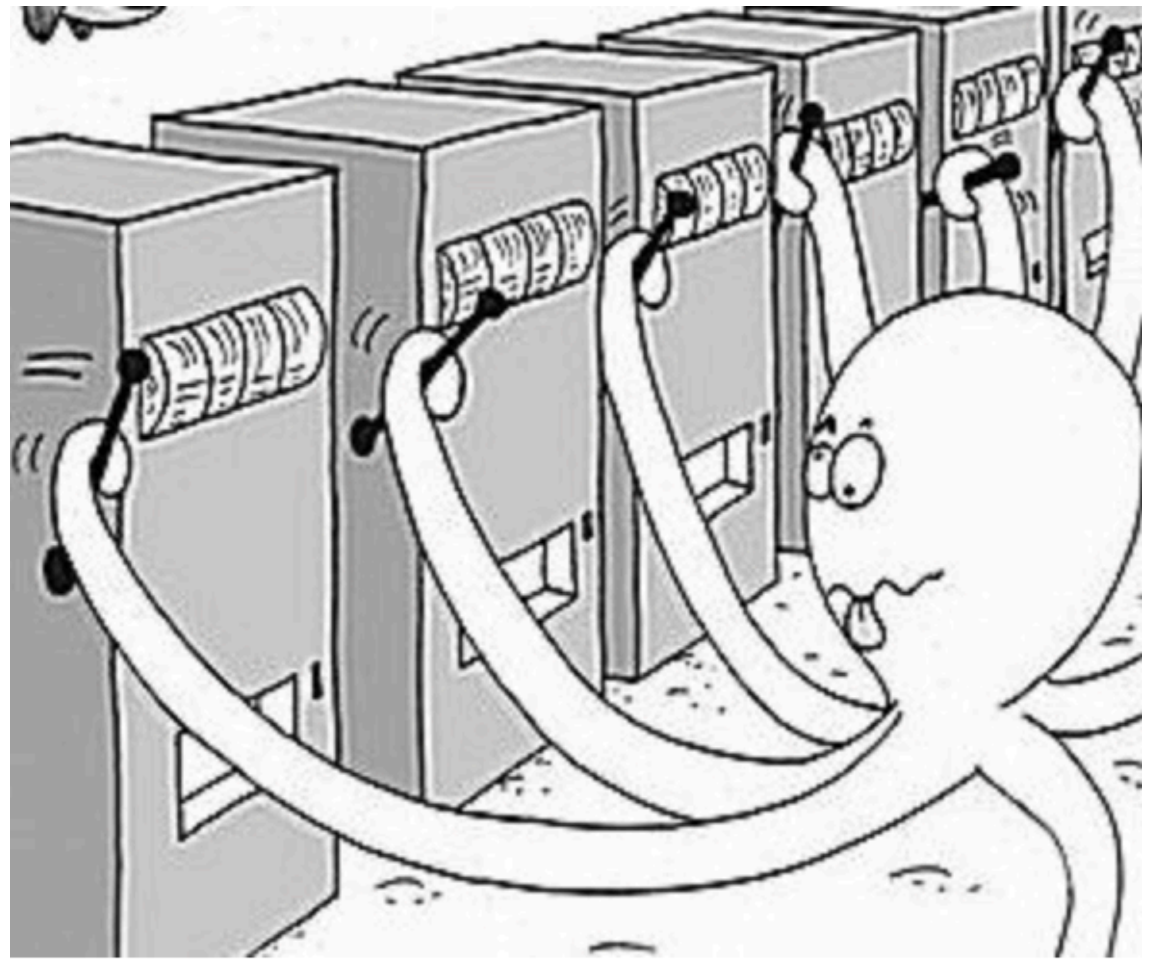
Multi-armed bandits

Greedy action selection:

$$P(a_t = a) = \begin{cases} 1 & \text{if } a_t = \text{argmax}_a \; V_t(a) \\ 0 & \text{otherwise} \end{cases}$$

Epsilon-greedy action selection:

$$P(a_t = a) = \begin{cases} 1 - \epsilon & \text{if } a_t = \text{argmax}_a \; V_t(a) \\ \epsilon / N & \text{otherwise} \end{cases}$$



Softmax action selection:

$$P(a_t = a) = \frac{e^{V_t(a) \cdot \beta}}{\sum_{i=1}^{N} e^{V_t(a_i) \cdot \beta}}$$

Action is governed by a **policy**:

$$\pi(a, s) = P(a_t = a \,|\, s_t = s)$$

Upper-confidence-bound (UCB) action selection:

$$P(a_t = a) = \text{argmax}_a [V_t(a) + c \cdot \sqrt{\frac{\ln t}{N_t(a)}}]$$

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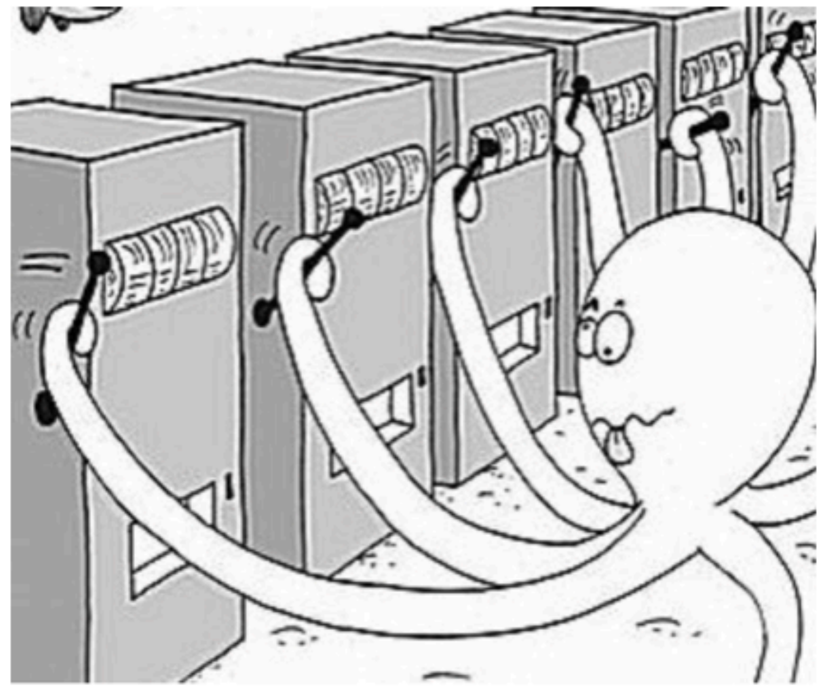

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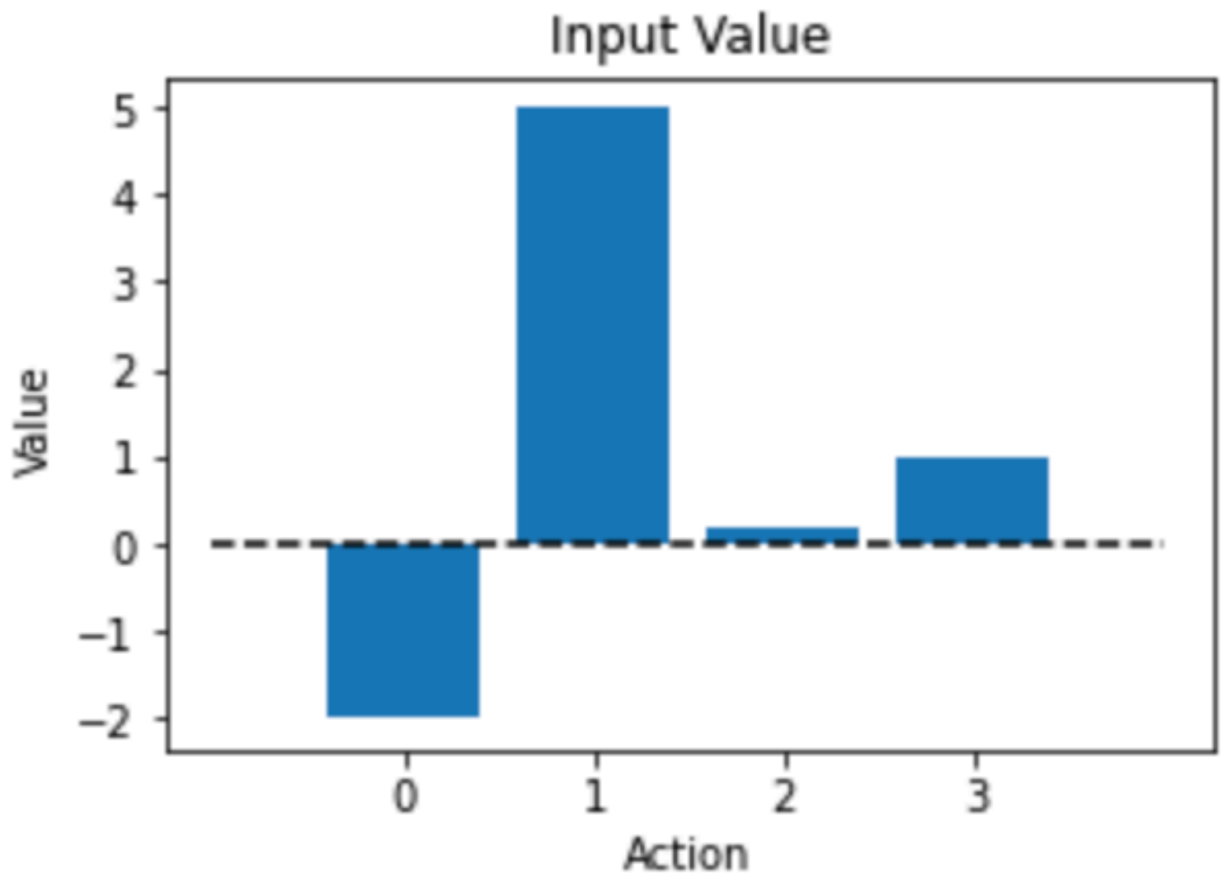

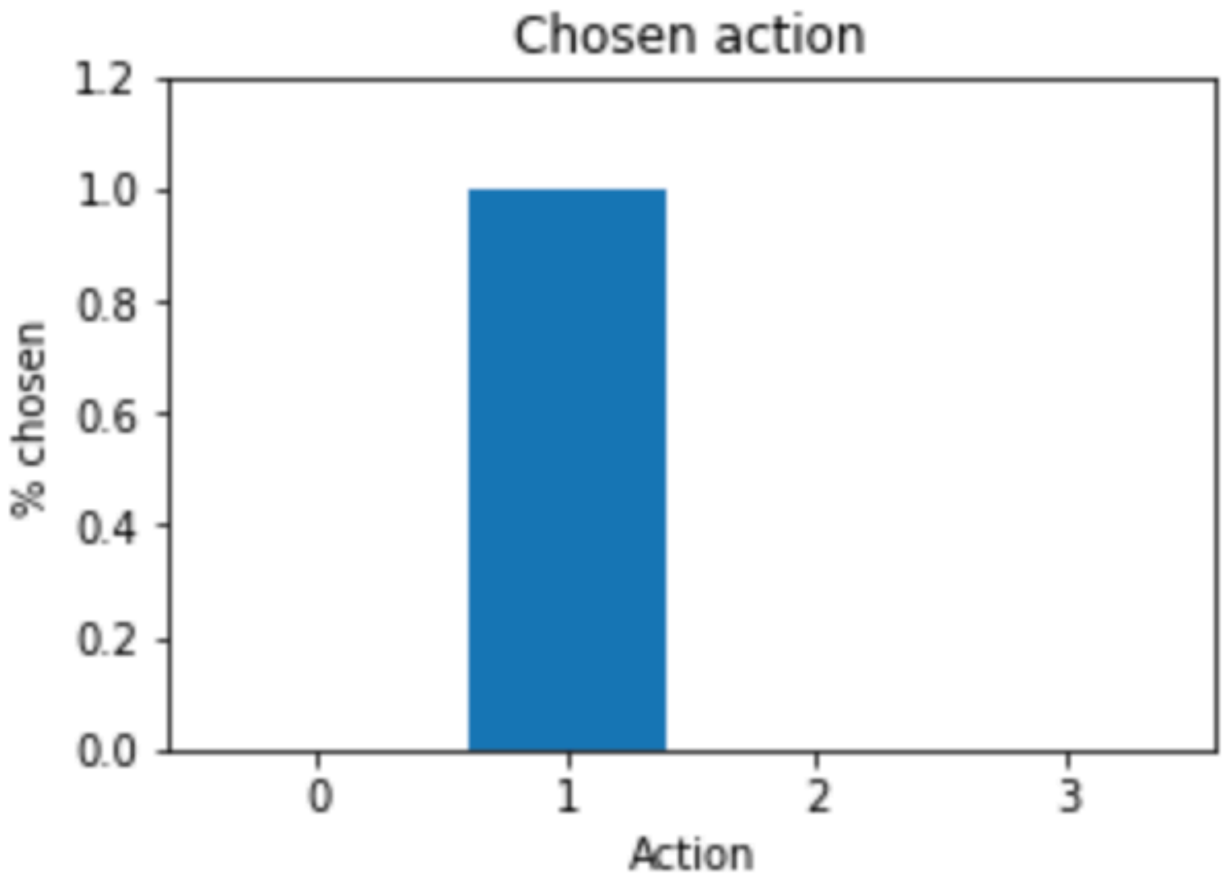

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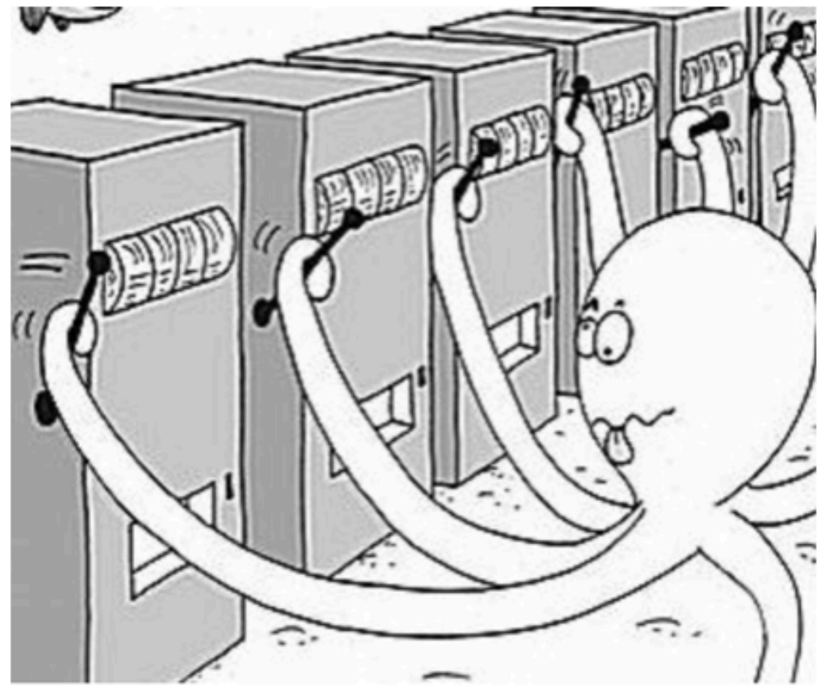

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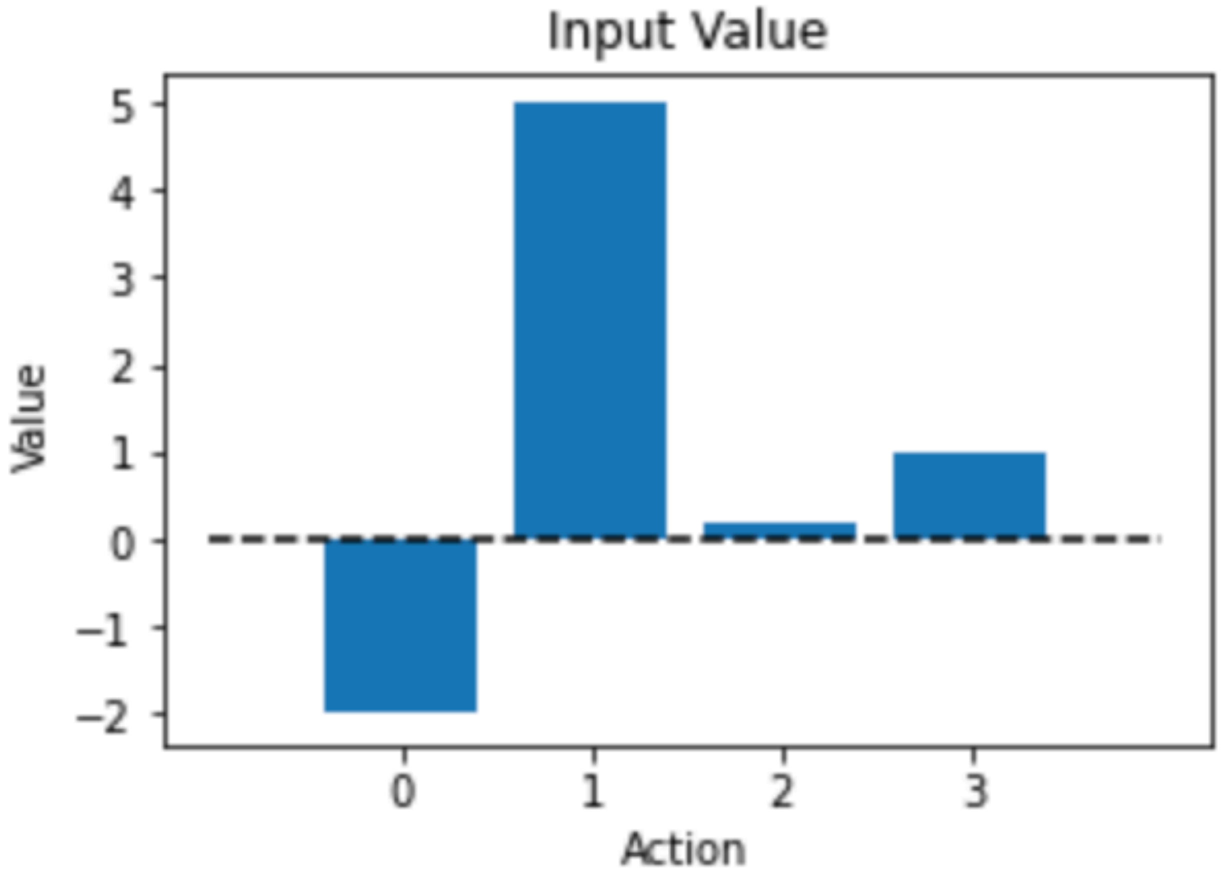

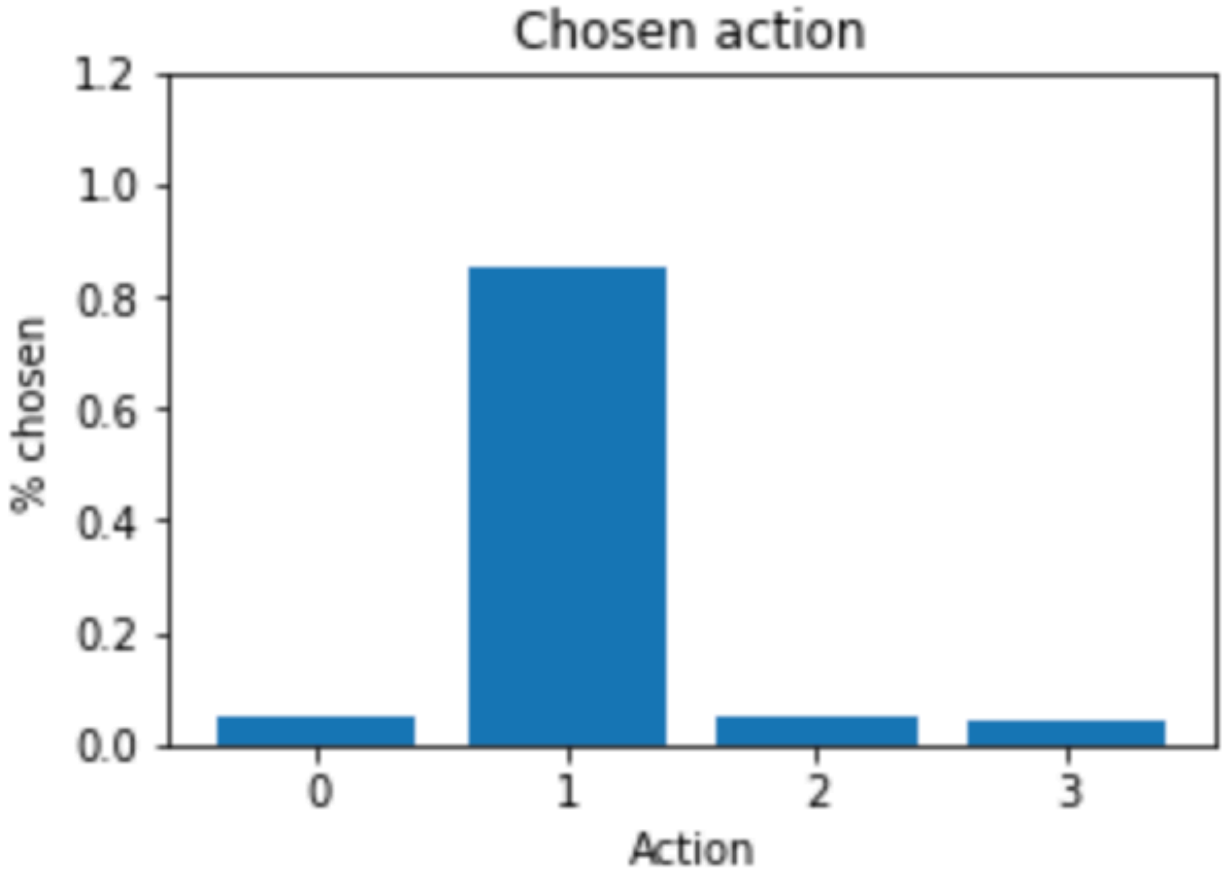

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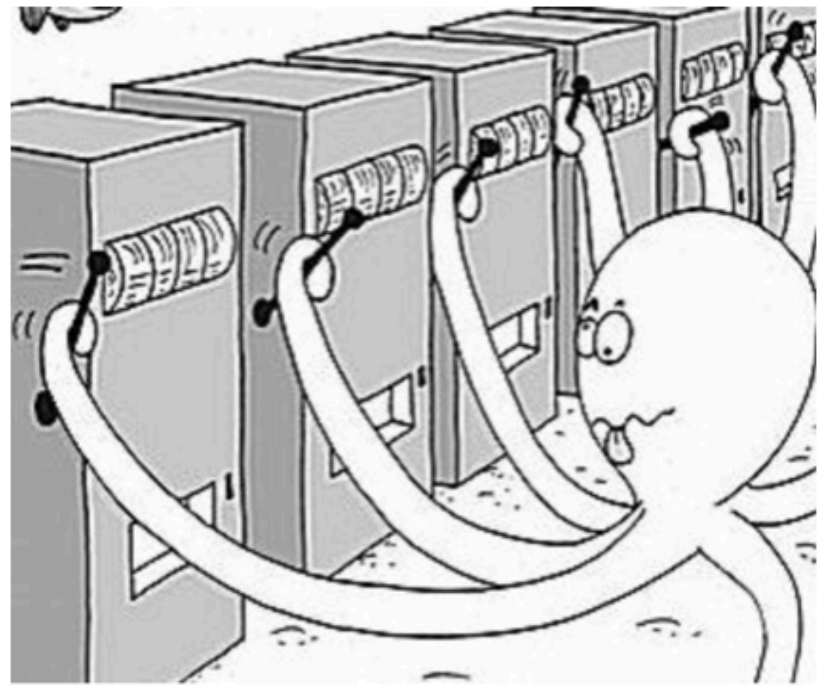

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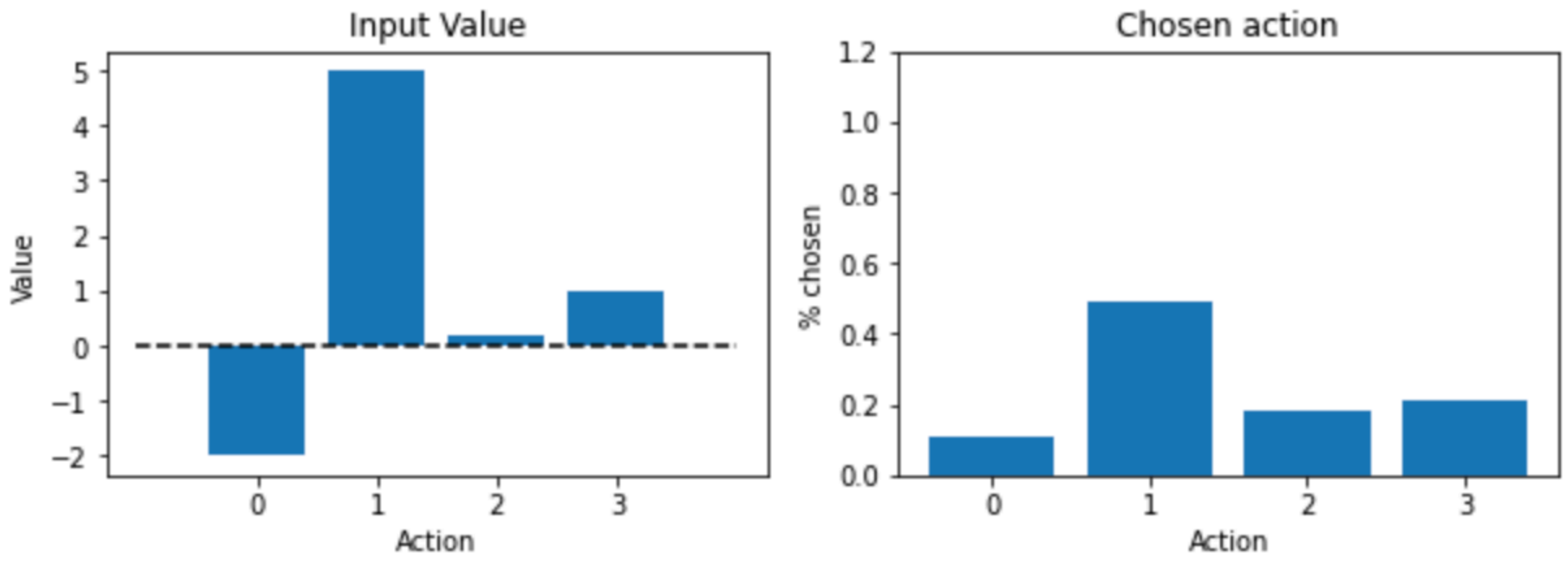

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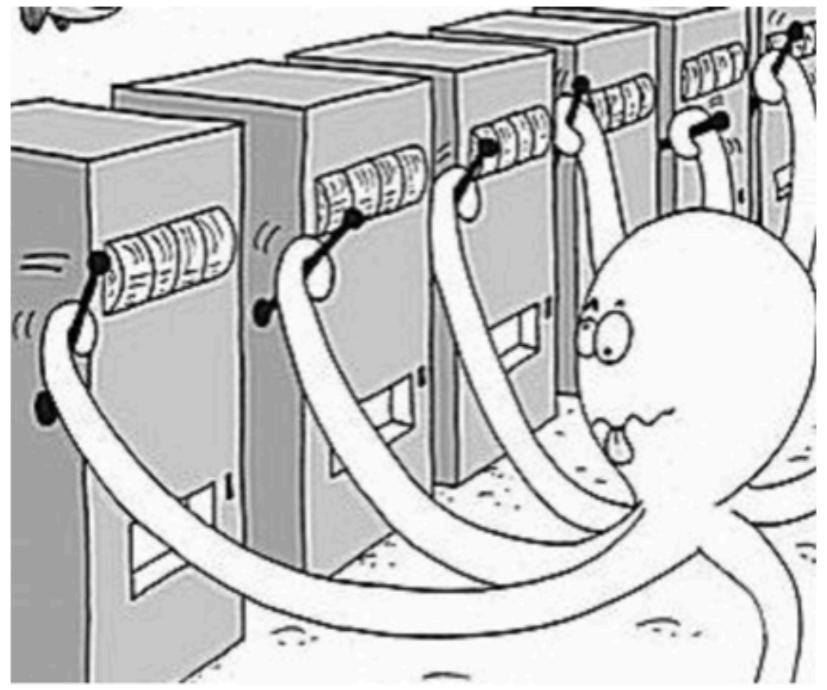

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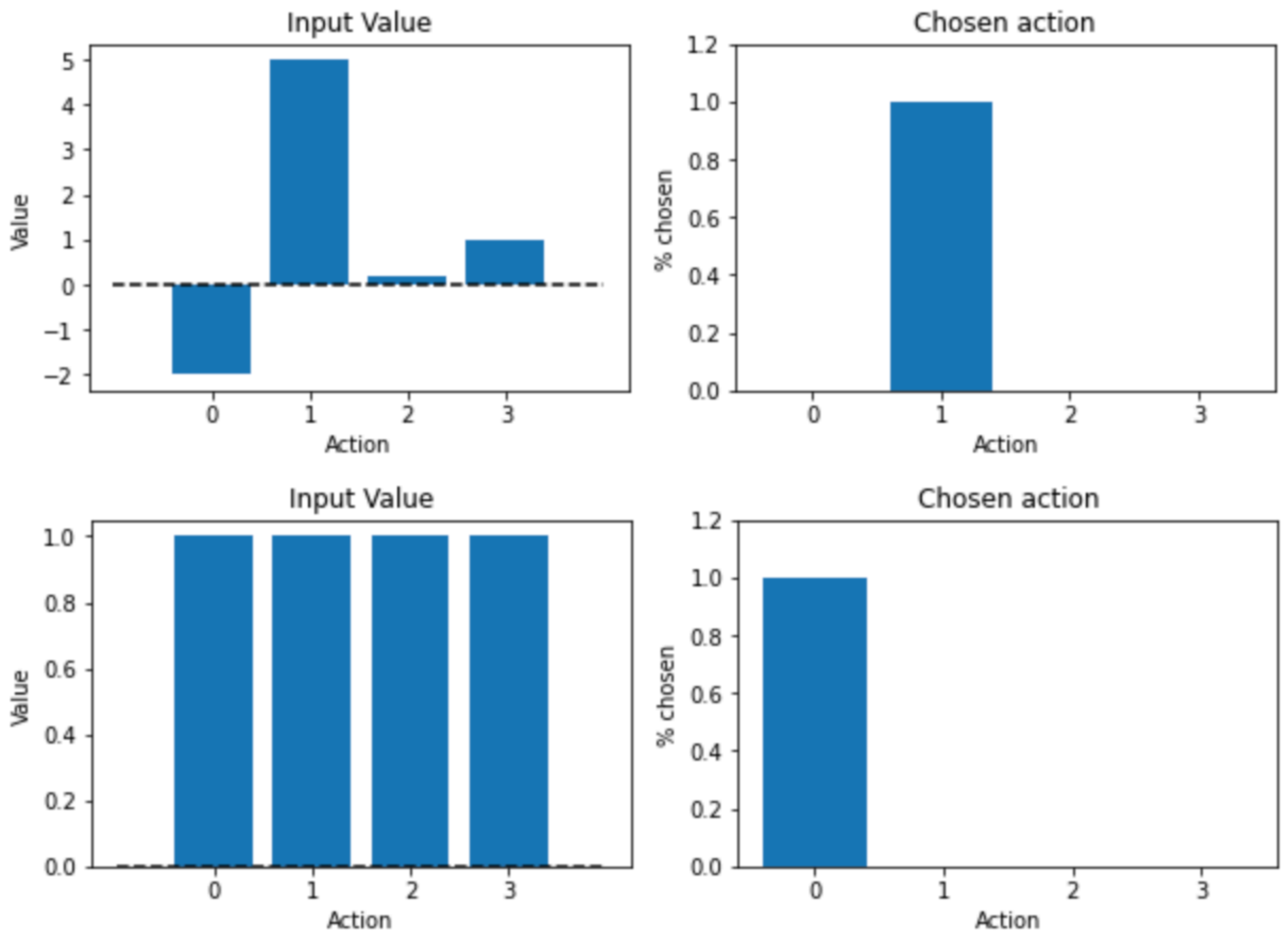

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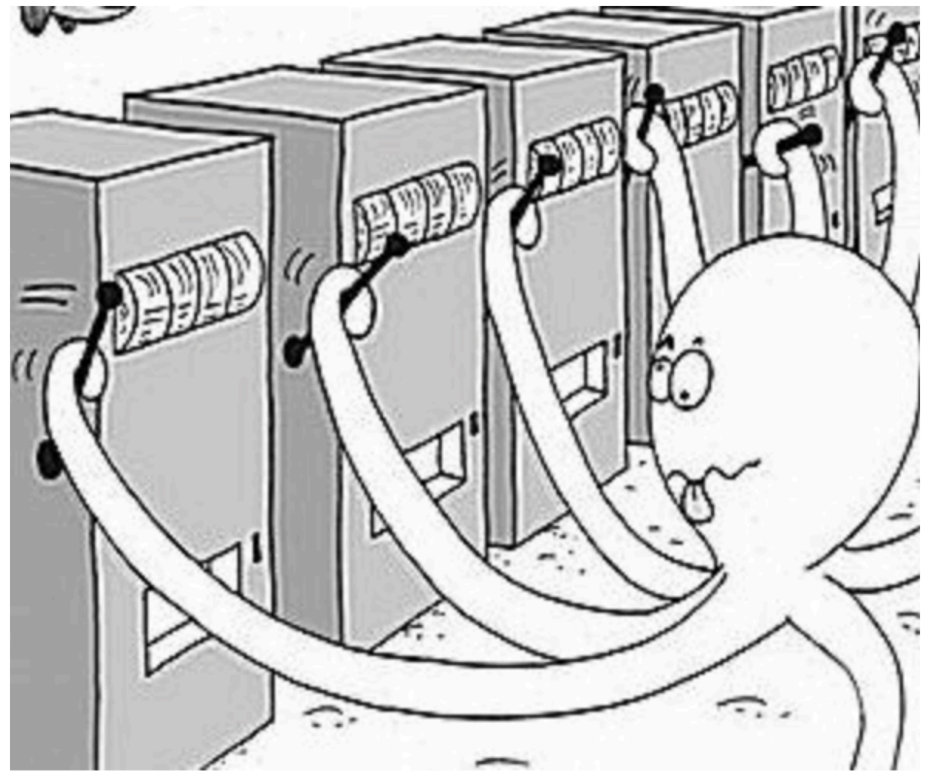

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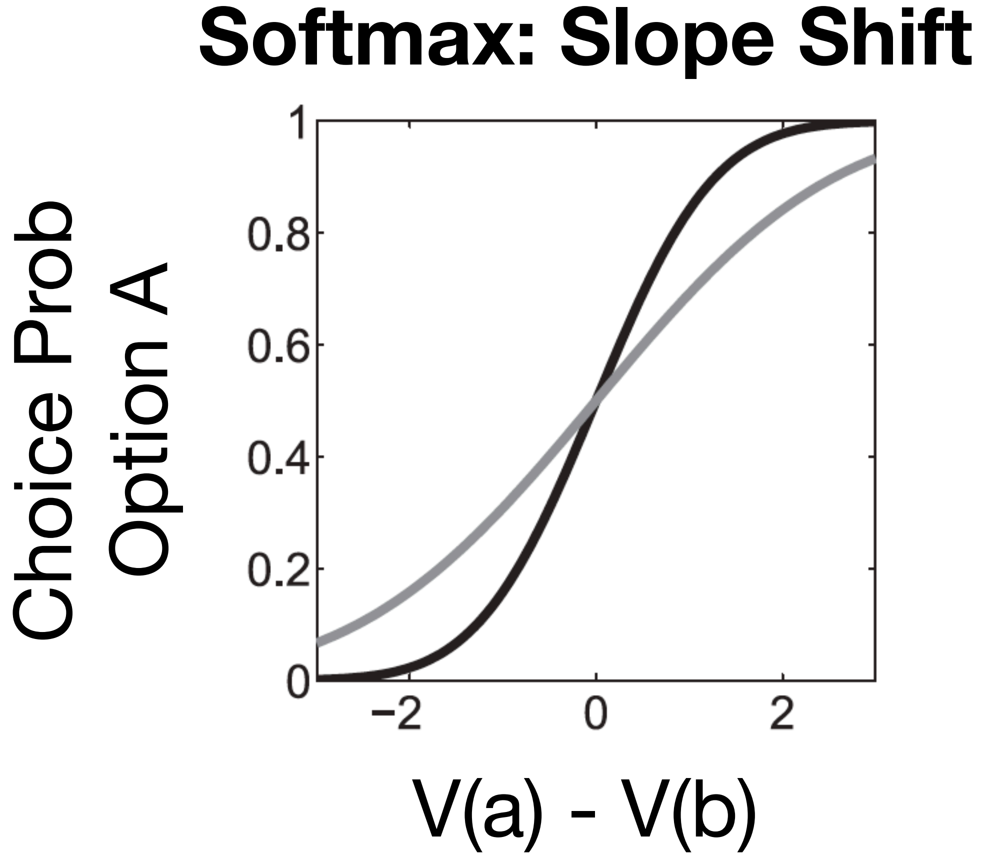
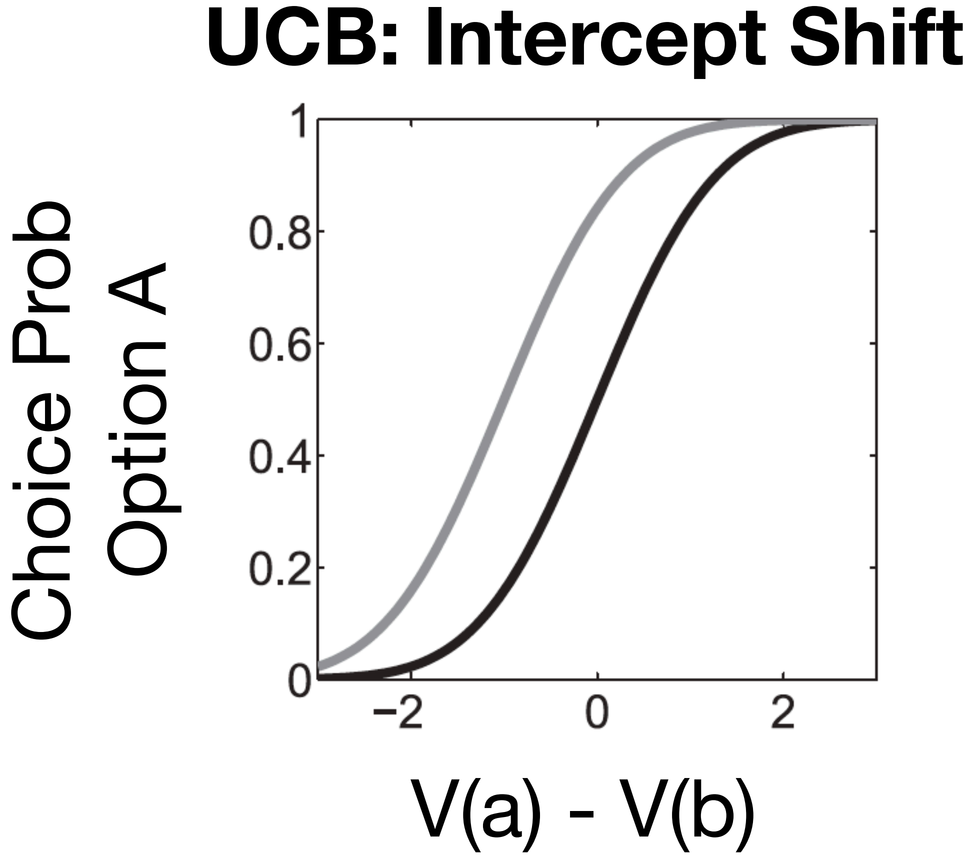
There's an interesting distinction between **random** and **goal-directed** exploration

Note:





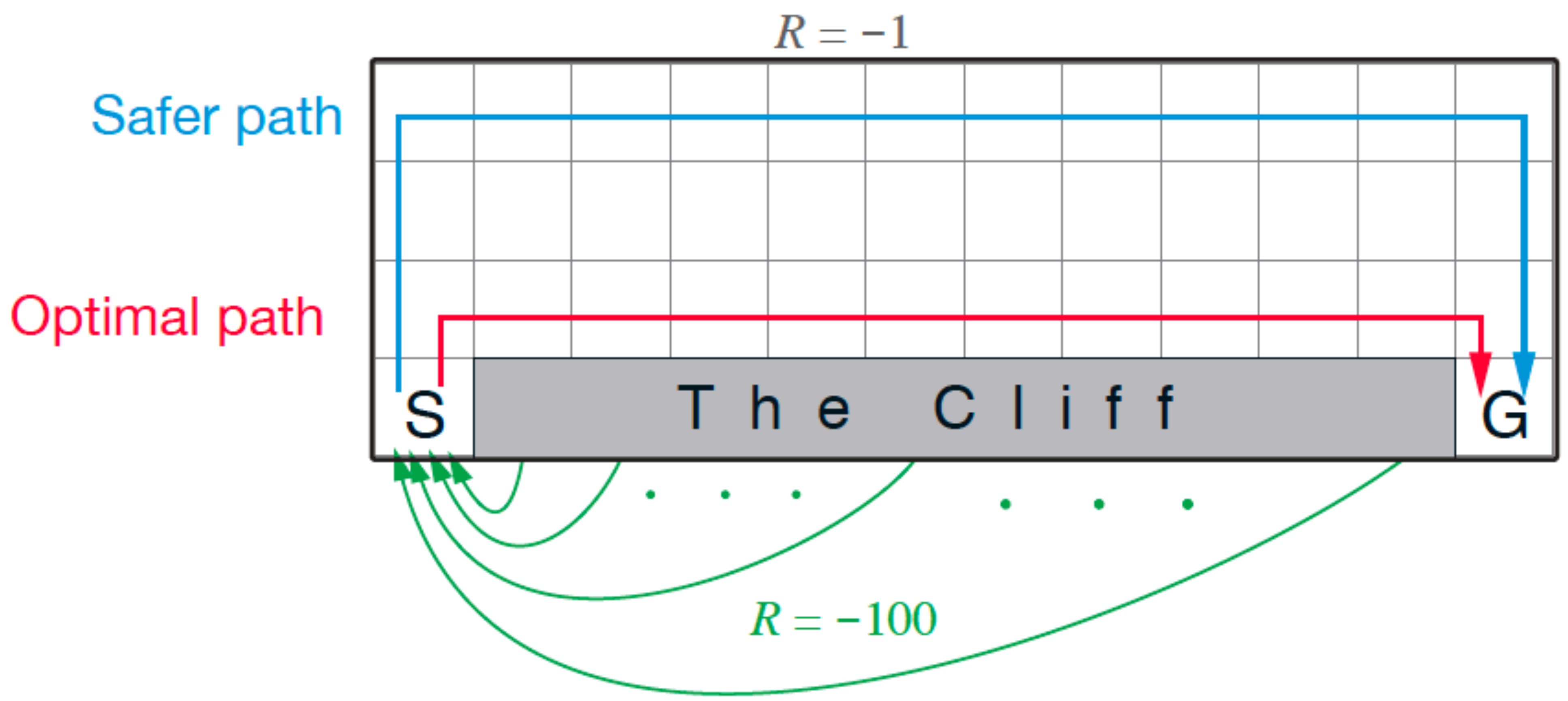
Gershman, Cognition 2017

Limitation of multi-armed bandit problems

Your current action does not influence what happens next!!

How can we solve sequential problems?

The textbook problem:
'Cliff-World'



The rules:

- Agent has to move from start (S) to goal (G)
- Reaching the goal results in a positive reward of +10
- Falling off the cliff results in a negative reward of -100
- Any other state results in a negative reward of -1

What's the problem the agent has to solve here??

Note the subtle introduction of the concept of '**transition probabilities**' here
- implicit, later: explicit

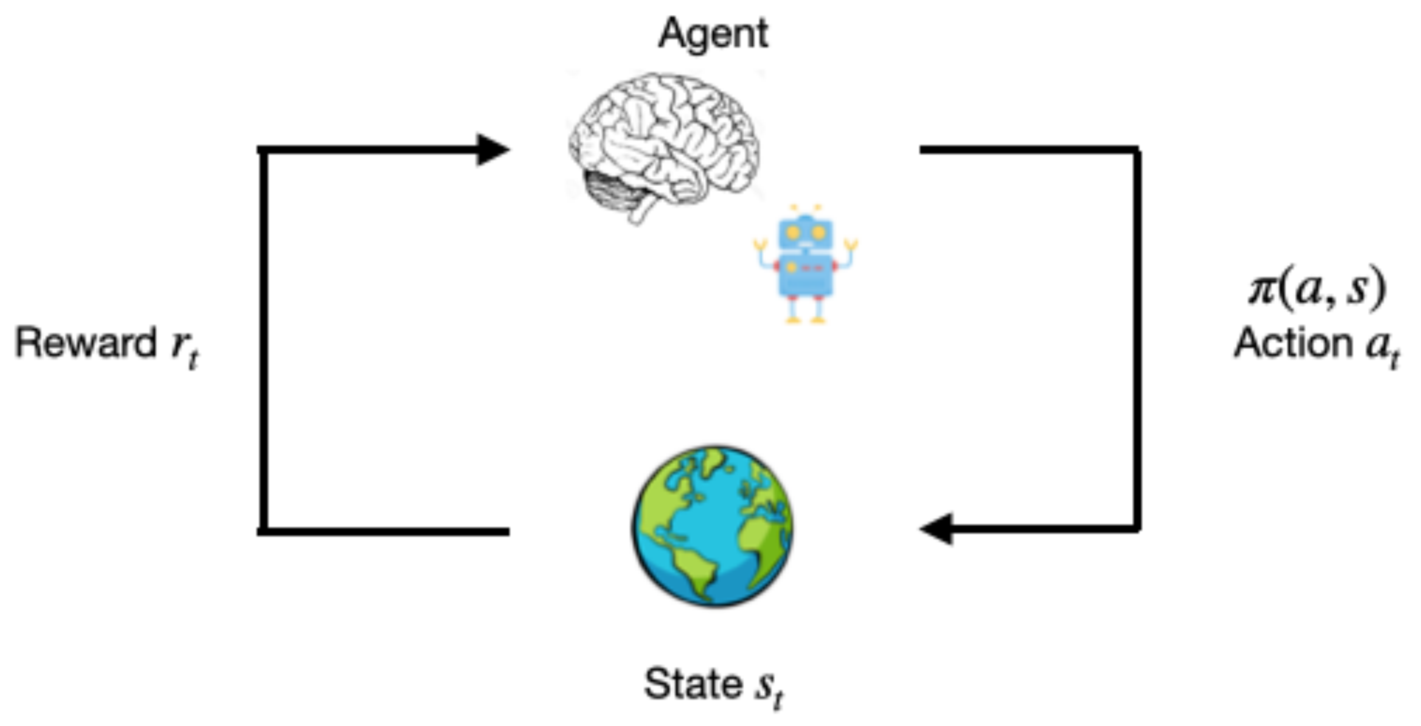
From classical to instrumental learning

TD Learning:

Prediction error

$$V(s_t) \leftarrow V(s_t) + \alpha \cdot (r + \gamma \cdot V(s_{t+1}) - V(s_t))$$

Learning rate   Discount rate



Q-Learning:

Prediction error

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \cdot (r + \gamma \cdot max_a Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t))$$

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What's the difference between $V(s_t)$ and $Q(s_t, a_t)$?

What's is $max_a Q(s_t, a_t)$ doing?

Note that this is just an update rule - doesn't tell us how to select an action!