Model-based RL

Systems Computational Neuroscience
16th of November 2022

Philipp Schwartenbeck

Cognitive map seminar attendants: Sorry for the repetition..

Marrian Learning to Predict & Act

Ethology/computation

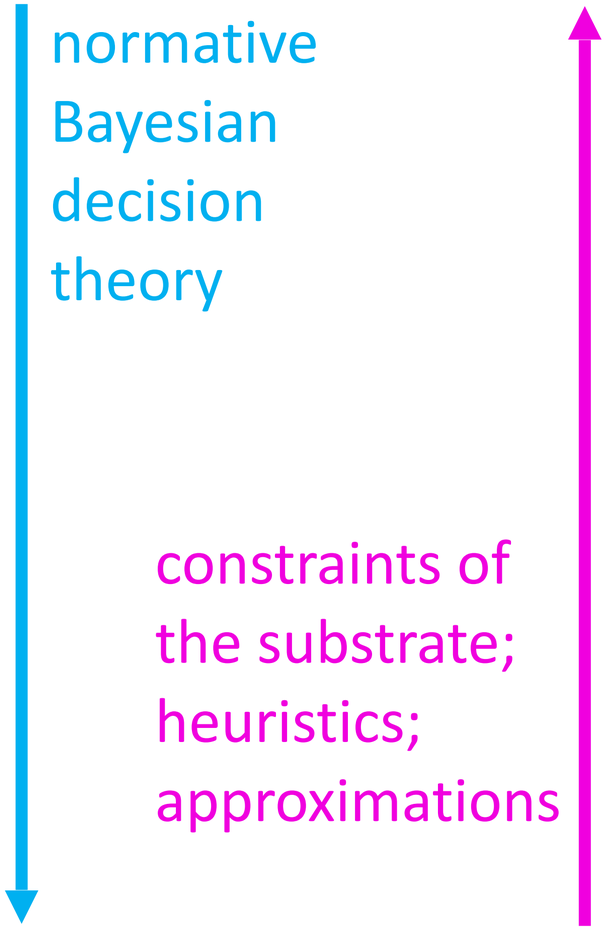
- Goal: maximise future pleasure; minimise future pain
- Logic: optimal control theory

Psychology/algorithm

- Classical/instrumental conditioning
- Learning from rewards and punishments
- **Model based RL for learning and planning**

Neuroscience/implementation

- Prefrontal cortex; hippocampus; entorhinal ctx; striatum
- Neuromodulation



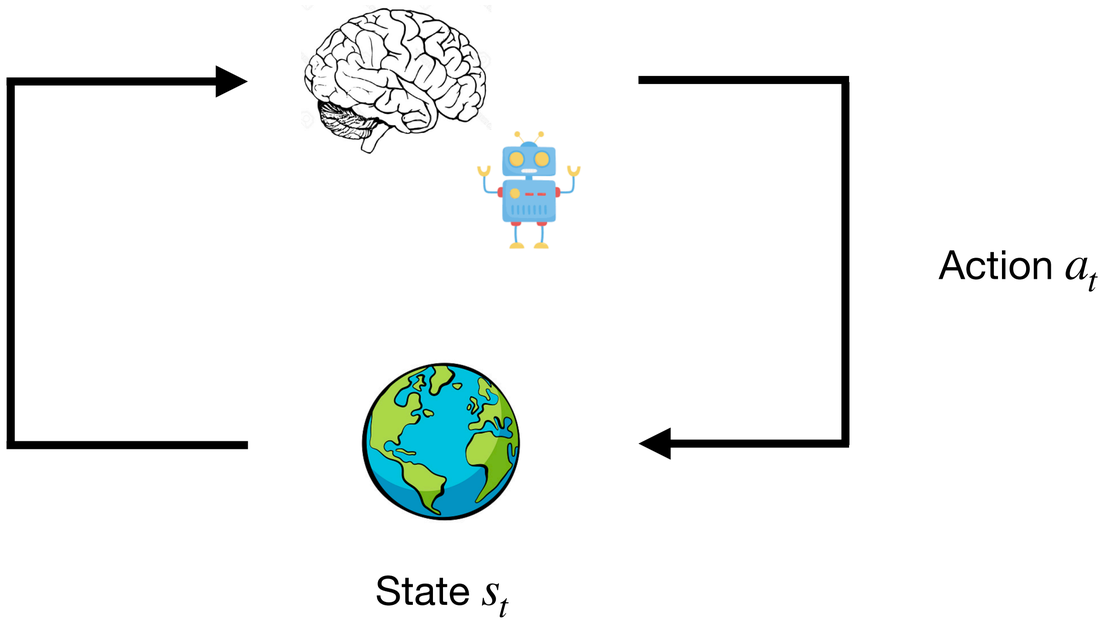
Basic setup of RL?

Based on a reward signal, agents learn **values of actions/states:**

$$V_\pi(s) = \mathbb{E}_\pi[R \,|\, s_0 = s]$$

Action is governed by a **policy**:

$$\pi(a, s) = P(a_t = a \,|\, s_t = s)$$



Agents can learn a **model of the environment** to make smarter decisions, e.g.:

$$P(s_{t+1} = s, r_{t+1} = r \,|\, s_t = s, a_t = a)$$

Overview

1. What happened previously..
    - Classical conditioning
    - Instrumental conditioning

2. Model-based reinforcement learning
    - De-valuation
    - Model-based RL for learning
    - Model-based RL for planning and action selection
    - Model-based RL and cognitive maps

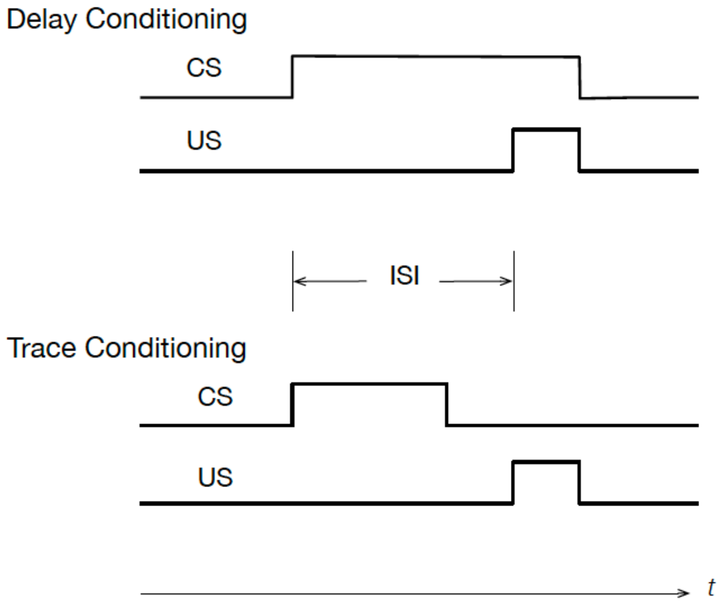
Slides will be on slack, all materials here: https://github.com/schwartenbeckph/Systems-Computational-Neuroscience

1. What happened previously

1.1 Classical Conditioning

Classical Conditioning

- Two learning algorithms you should know about:
  - **Rescorla-Wagner (RW-)Learning**
    - Learn stimulus-outcome associations
  - **Temporal Difference (TD-)Learning**
    - Learn stimulus-outcome associations *across* time



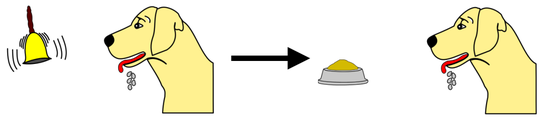
Basics of Learning: Rescorla-Wagner Learning

Learn associative strength between a CS and US

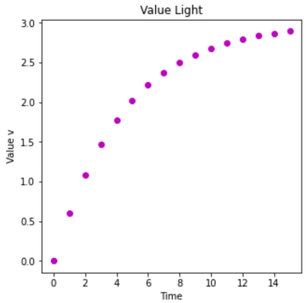
Prediction error

$$V(s) \leftarrow V(s) + \alpha \cdot (r - V(s))$$

Learning rate



$$V(\text{💡}) \leftarrow V(\text{💡}) + \alpha \cdot (\text{DDD} - V(\text{💡}))$$

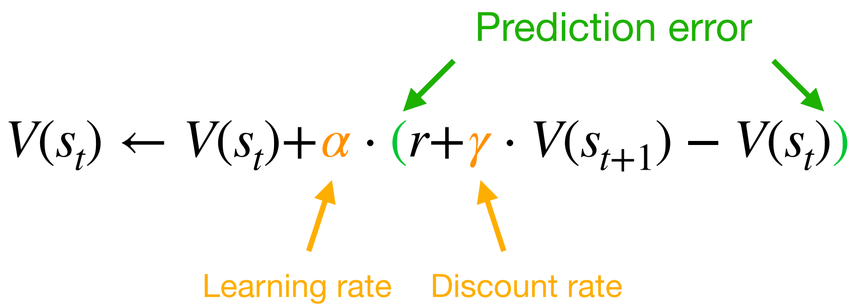


Link to code here

Temporal Difference Learning

- “If one had to identify one idea as **central** and **novel** to reinforcement learning, it would undoubtedly be temporal-difference (TD) learning.”

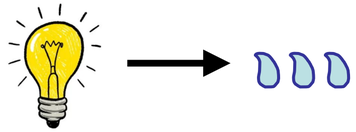
- Update based on other learned estimates, without waiting for final outcome (**bootstrap**)
  - Learn “a guess from a guess”



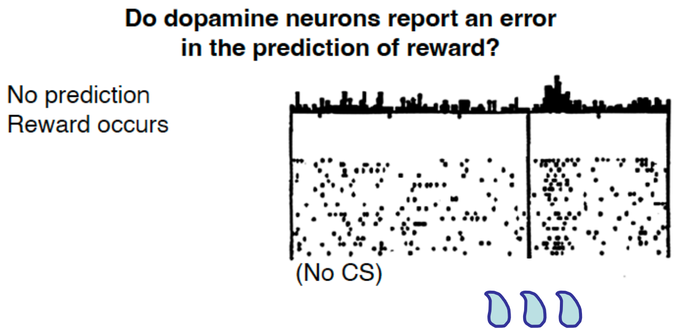
Can RL tell us anything about the brain?

$$V(s_t) \leftarrow V(s_t) + \alpha \cdot (r + \gamma \cdot V(s_{t+1}) - V(s_t))$$

Yes, quite a lot - it looks like **dopamine** (DA) is a key neuromodulator for (TD) reward learning

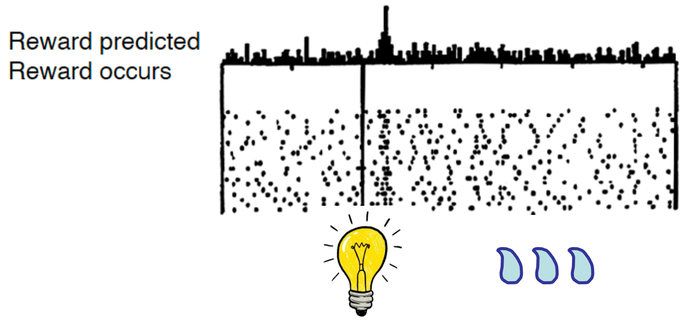


Dopamine neurons signal immediate reward

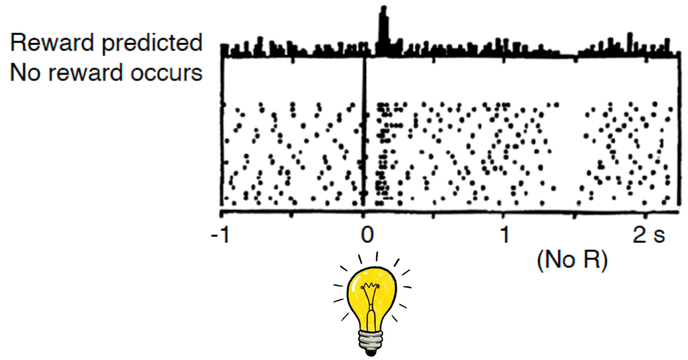


BUT: after training…

- DA signal reward prediction
- But not correctly predicted reward!

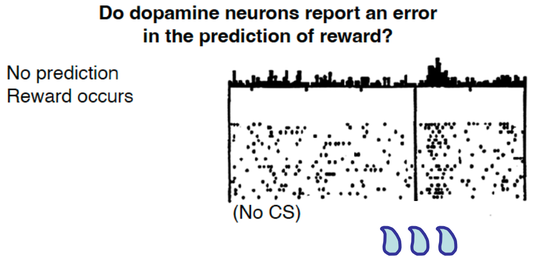


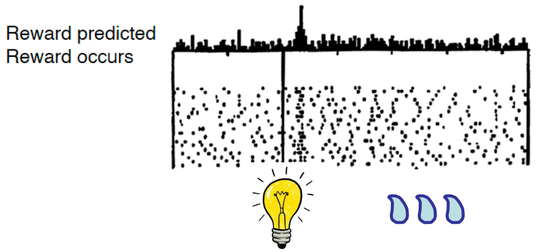
AND: it signals the unexpected omission of a reward!

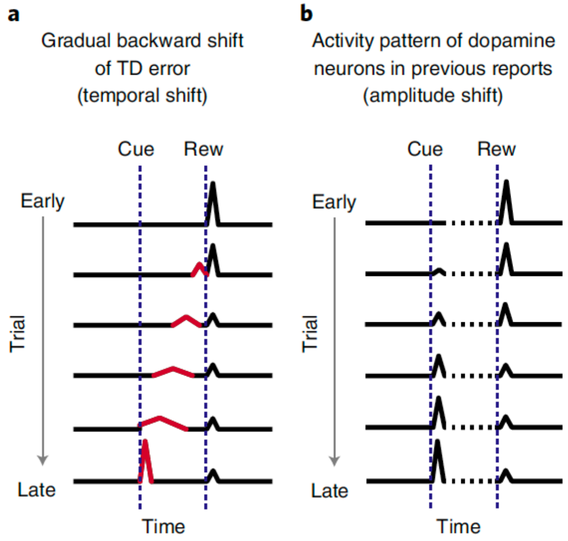


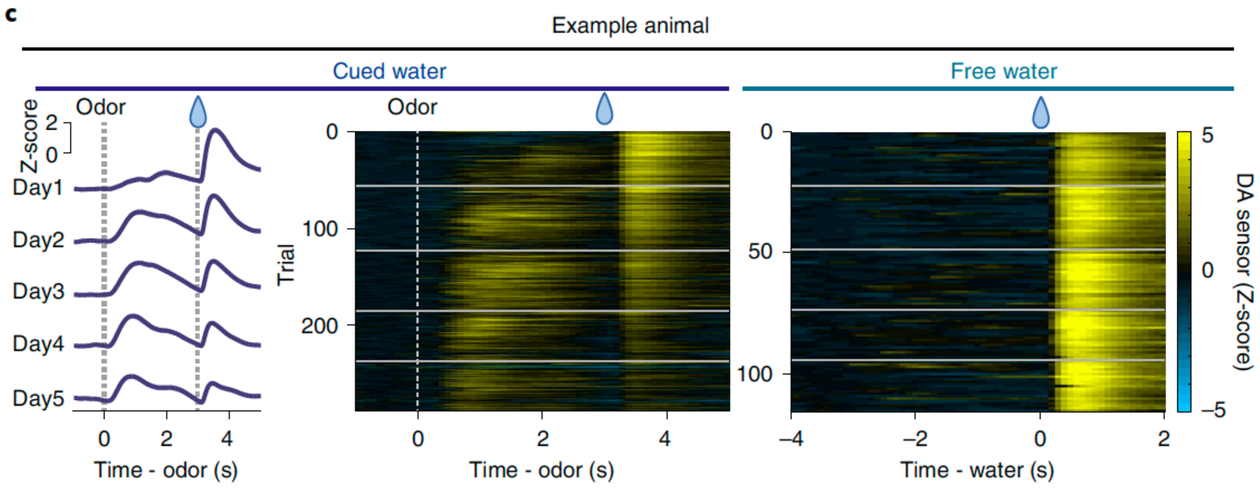
Schultz, Dayan & Montague (Science, 1997)

More TD-learning







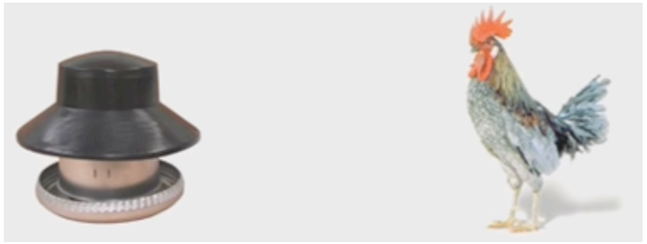


Amo, …, Watabe-Uchida, Nature Neuroscience 2022

1. What happened previously

1.2 Instrumental Conditioning

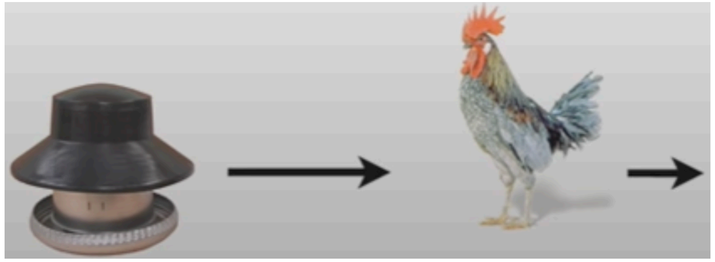
Evolutionary Programming



Chicken feeder

Approach: move away

Avoid: move closer

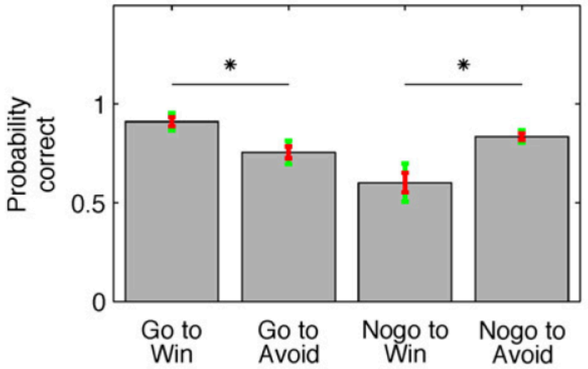


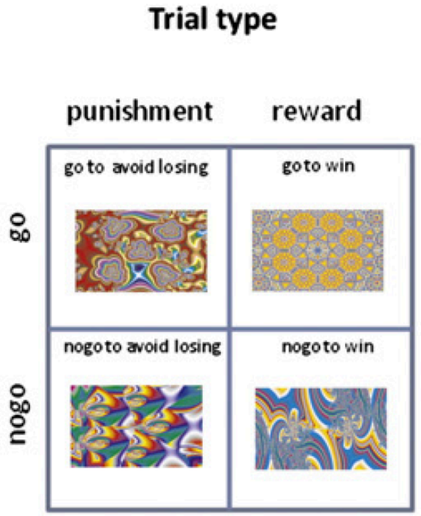
Chicken **never** learn this..

Hershberger, Animal Learning & Behavior, 1986

Evolutionary Programming

We see a strong signature of this in human behaviour:

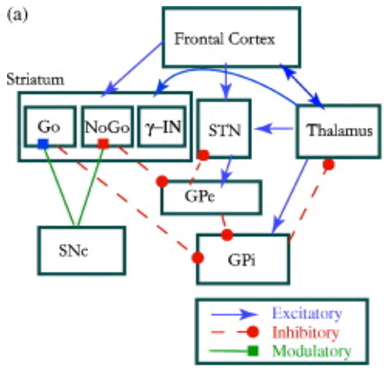


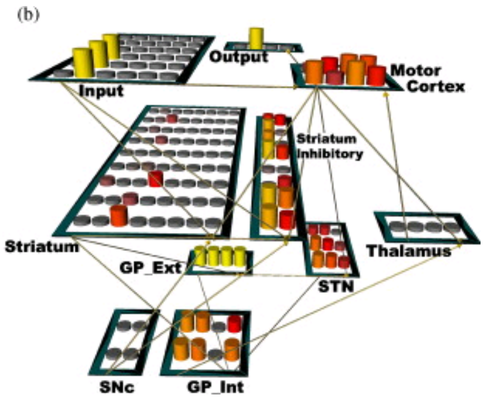


Guitart-Masip et al. 2011, 2012

And have biological process models for these effects

- **Direct**: D1: GO; learn from DA increase
- **Indirect**: D2: noGO; learn from DA decrease
- **Hyperdirect** (STN) delay actions given strongly attractive choices





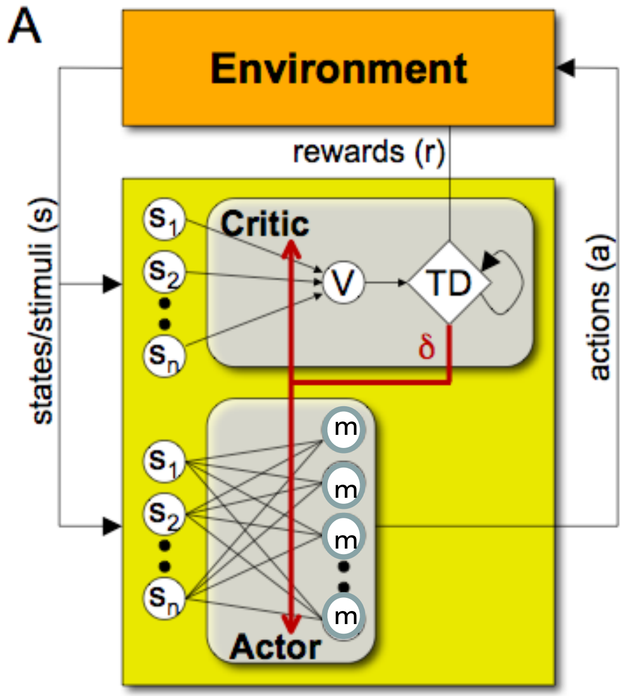
Frank & O'Reilly, Behavioural Neuroscience 2006

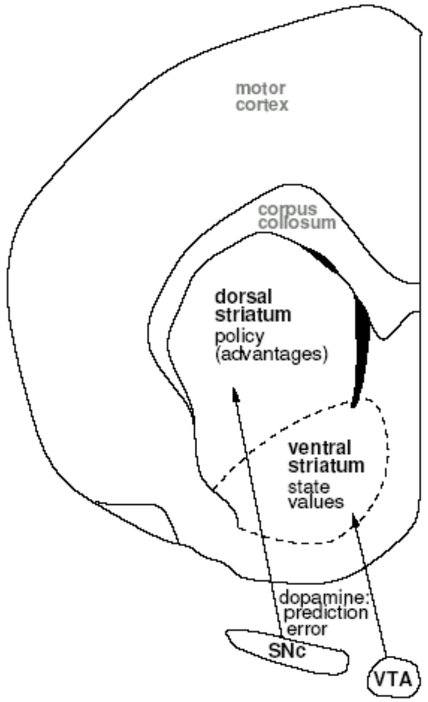
Actor/Critic

Learn values and policies iteratively based on the *same* teaching signal $\delta_t = r + \gamma \cdot V(s_{t+1}) - V(s_t)$

$$V(s_t) \leftarrow V(s_t) + \alpha_v \cdot \delta_t$$

$$\pi(s,a) \leftarrow \pi(s,a) + \alpha_\pi \cdot \delta_t$$





Dopamine signals to both **motivational** & **motor** regions

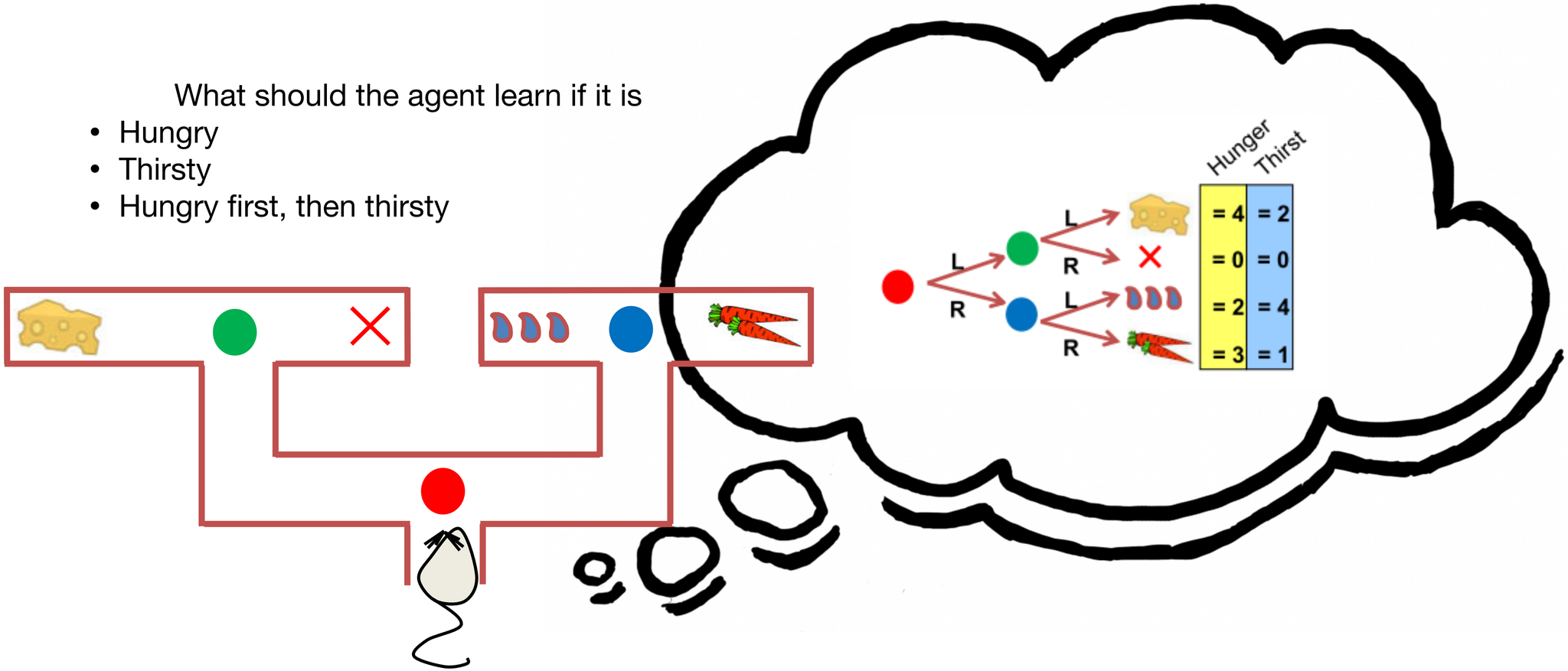
Suggestion: training both **values** & **policies**

Model-based Reinforcement Learning

What if we had a model to plan ahead?

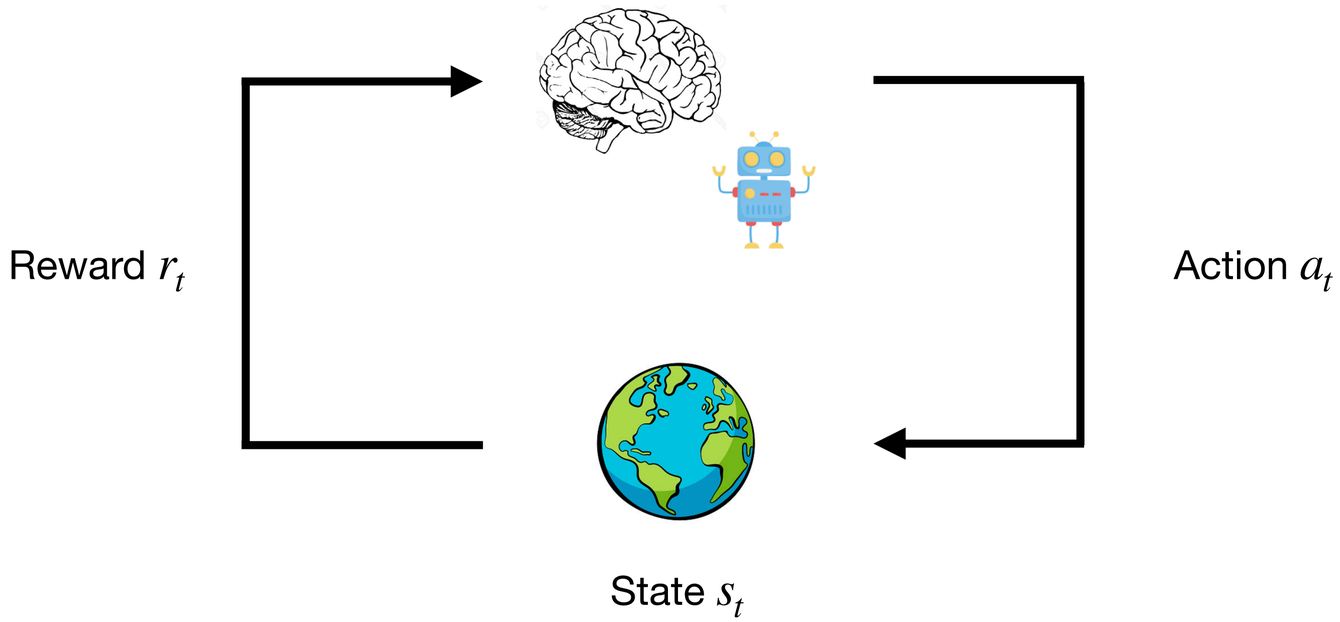
What should the agent learn if it is

- Hungry
- Thirsty
- Hungry first, then thirsty



2. Model-based RL

Basic setup: how do agents learn to act?



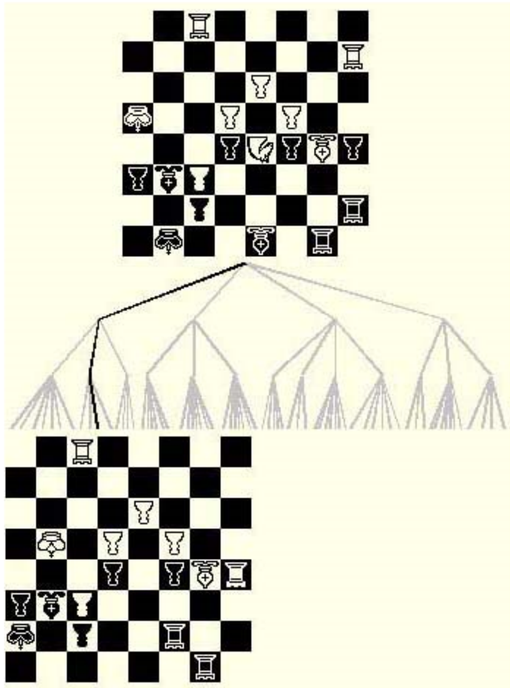
Agents can learn a **model of the environment** to make smarter decisions, e.g.:

$$P(s_{t+1} = s, r_{t+1} = r \,|\, s_t = s, a_t = a)$$

Different Decision-Makers

(At least) 3 ways to solve this:

1. Tree search
2. Position evaluation
3. Situation memory



Multiple Systems in RL

Model-based RL

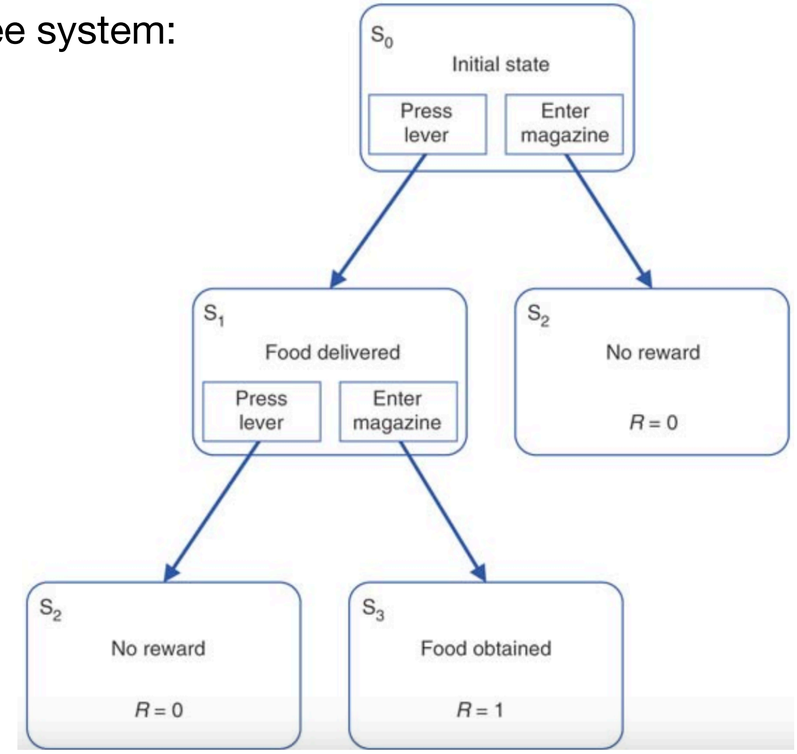
- Build a forward model of the task and outcomes
- Search in the forward model
- Optimal use of information, but computationally ruinous

- **Cached-based (model-free) RL**
  - Learn values, which summarise future worth
  - Computationally trivial but bootstrap-based - statistically inefficient

- Learn both – select according to **uncertainty**
  - Pro: **Robust** and **adaptive**
  - Con: **Competition** (and harder reverse engineering)

Multiple Systems in RL

Tree system:

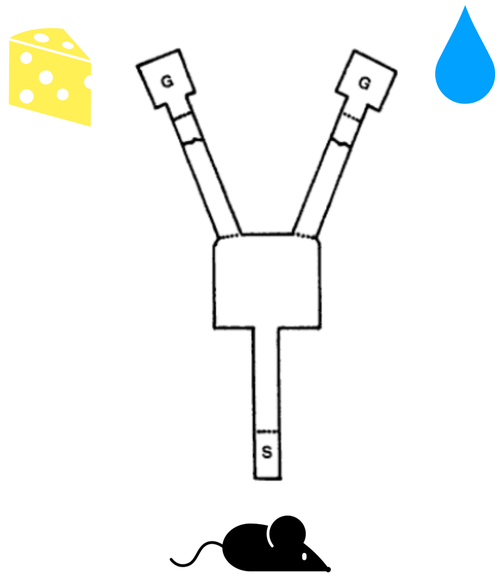


Cache system:

De-valuation

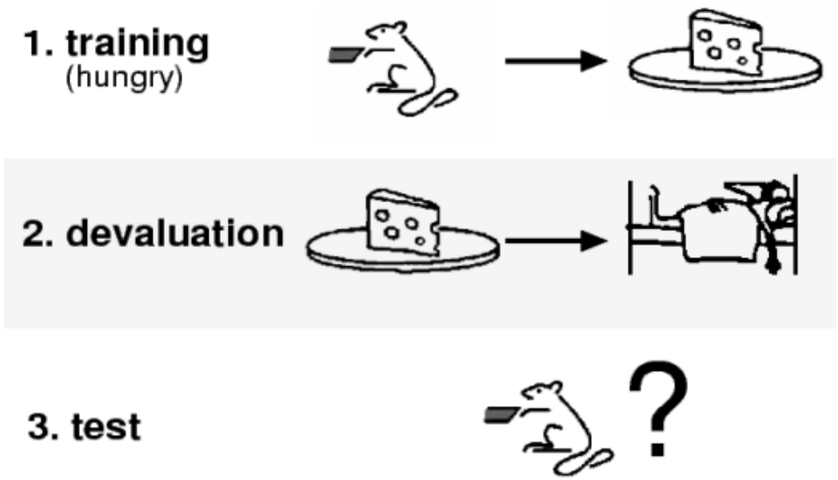
Outcome devaluation (*revaluation*): gold-standard test for forward model predicting outcomes of actions

Animal is trained to perform two different actions, with a different reward:



Akam, Costa, & Dayan, PLOS CB 2015

One reward is then **devalued**, for example by satiation.

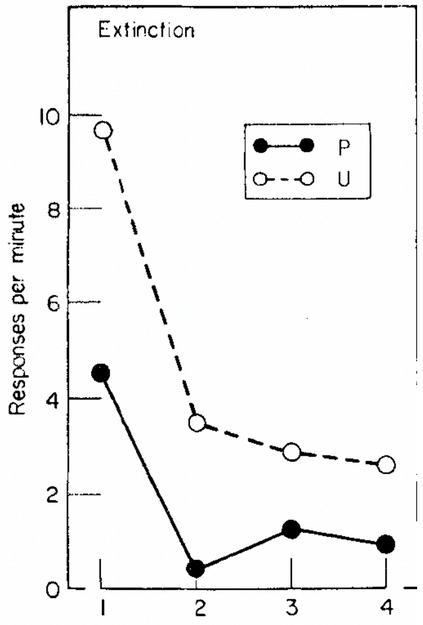


Drummond & Niv, Current Biology 2020

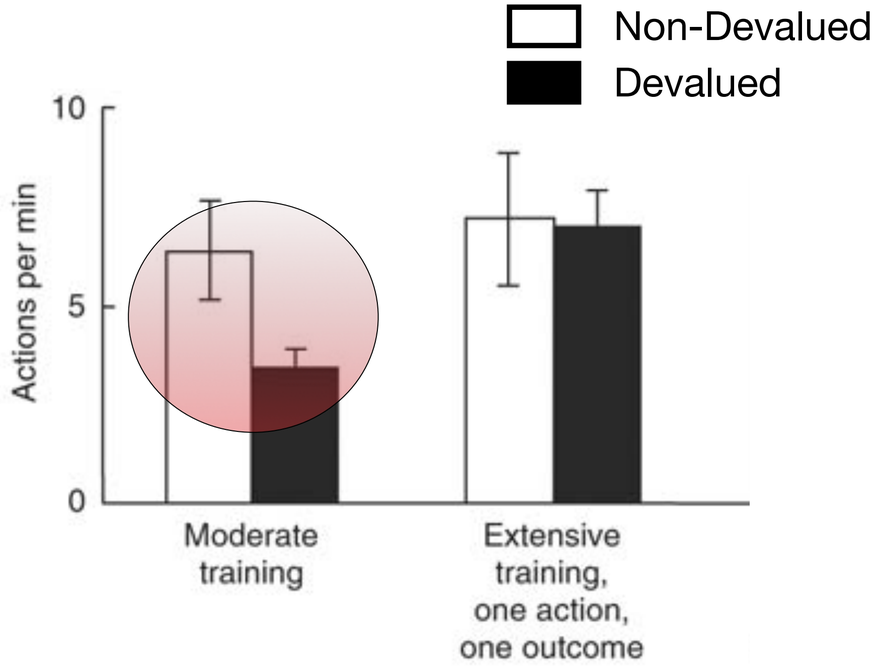
De-valuation

Outcome devaluation (*revaluation*): gold-standard test for forward model predicting outcomes of actions

Impact of this devaluation is tested in ‘extinction’, without providing outcomes.



Adams & Dickinson, Quarterly Journal of Experimental Psychology, 1981
Colwill & Rescorla, Journal of Experimental Psychology, 1985



Holland, Journal of Experimental Psychology, 2004
Dickinson & Balleine, Animal, Learning & Behaviour, 1994

Break?

What is the model in model-based RL?

$$P(s', r \mid s, a) = P(s_{t+1} = s', r_{t+1} = r \mid s_t = s, a_t = a)$$

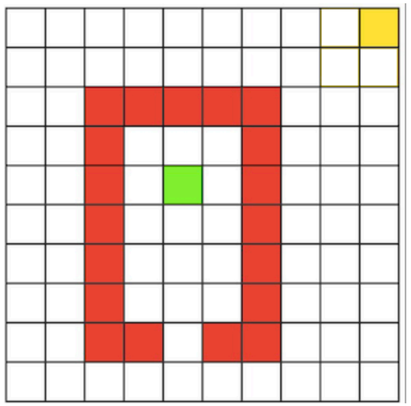
How can we make use of such models of the world?

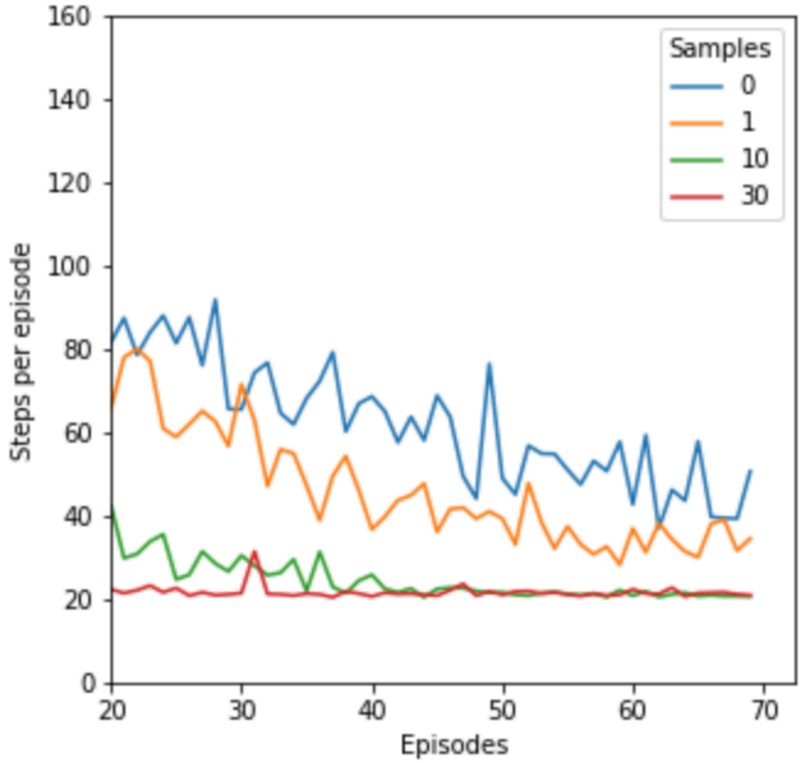
Learning

- Key idea: store experiences in world model $P(s', r \mid s, a)$
- Sample from this model to generate extra learning data
- This is called **DYNA-Q…**

DYNA-Q

Sample from world model $P(s', r \mid s, a)$ to generate additional learning data





$$P(s', r|s, a) = P(s_{t+1} = s', r_{t+1} = r|s_t = s, a_t = a)$$

$$Model(S, A) \leftarrow R, S'$$

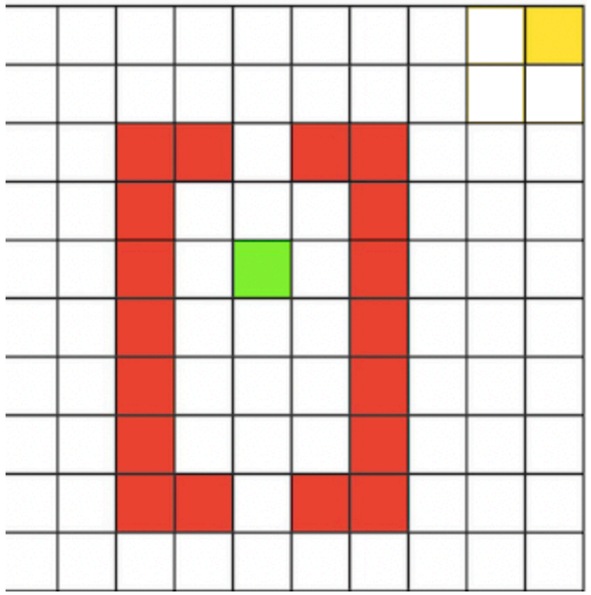
And during breaks ('offline rest'), they can sample from this experience and learn from these samples:

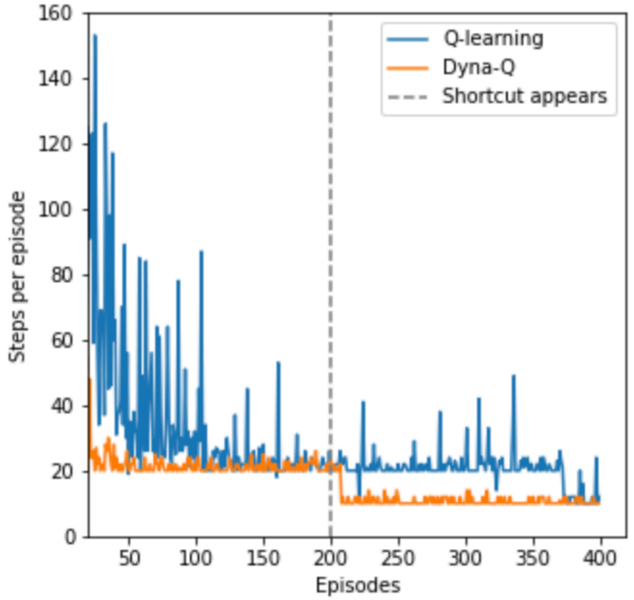
$S \leftarrow$ previously observed state
$A \leftarrow$ action previously taken in S
$R, S' \leftarrow Model(S, A)$
$Q(S, A) \leftarrow Q(S, A) + \alpha \left[R + \gamma \max_a Q(S', A) - Q(S, A)\right]$

Link to code here

DYNA-Q

This also helps with detecting shortcuts:



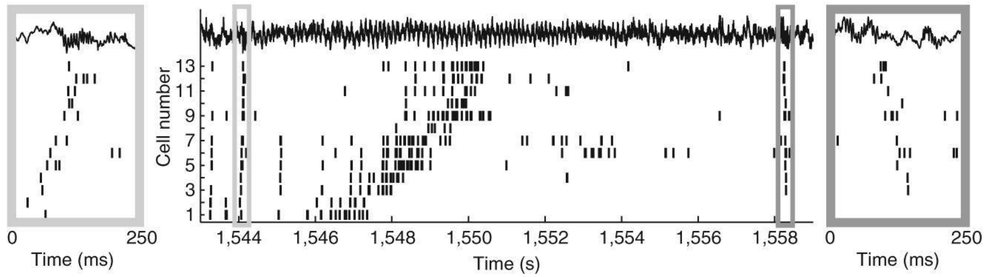


DYNA-Q - Replay as a candidate neural mechanism

DYNA-Q looks a lot like replay.

Replay as a computational mechanism in PFC and hippocampal formation

- i.e. fast reactivation of external states



Diba & Buzsaki (2007) Nature Neuroscience

Implicated in

- Learning from the *past* (credit assignment, Ambrose et al. (2016) Neuron)
- Planning *future* trajectories (Pfeiffer & Foster (2013) Nature)

What is the model in model-based RL?

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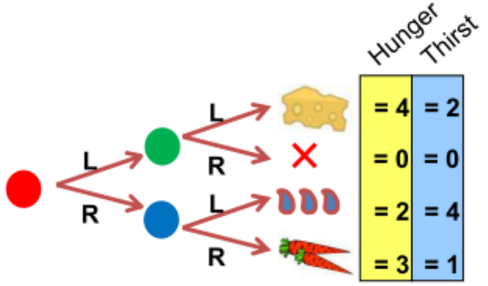
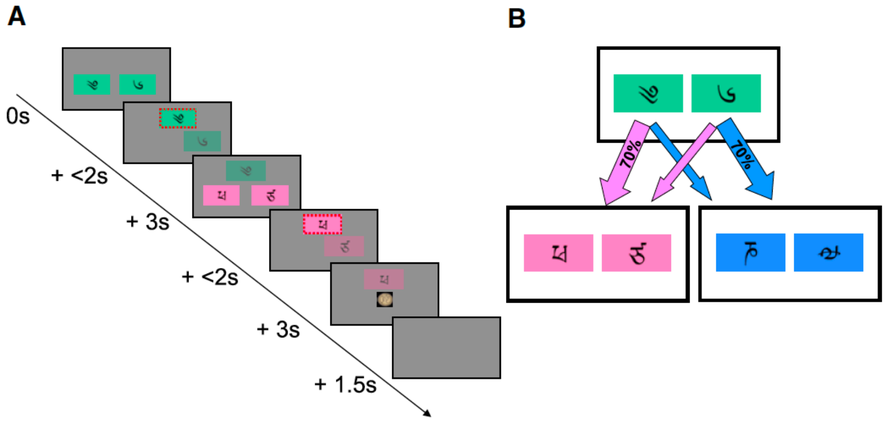
How can we make use of such models of the world?

Learning

- Key idea: sample from $P(s', r \mid s, a)$

Planning and **action selection**

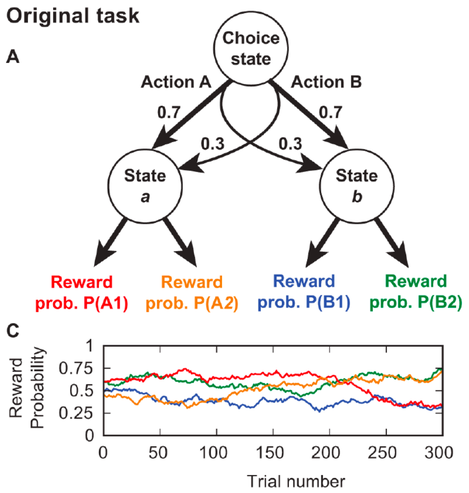




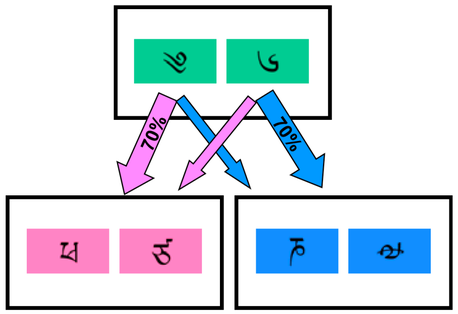
Daw, …, Dayan, & Dolan, Neuron, 2011

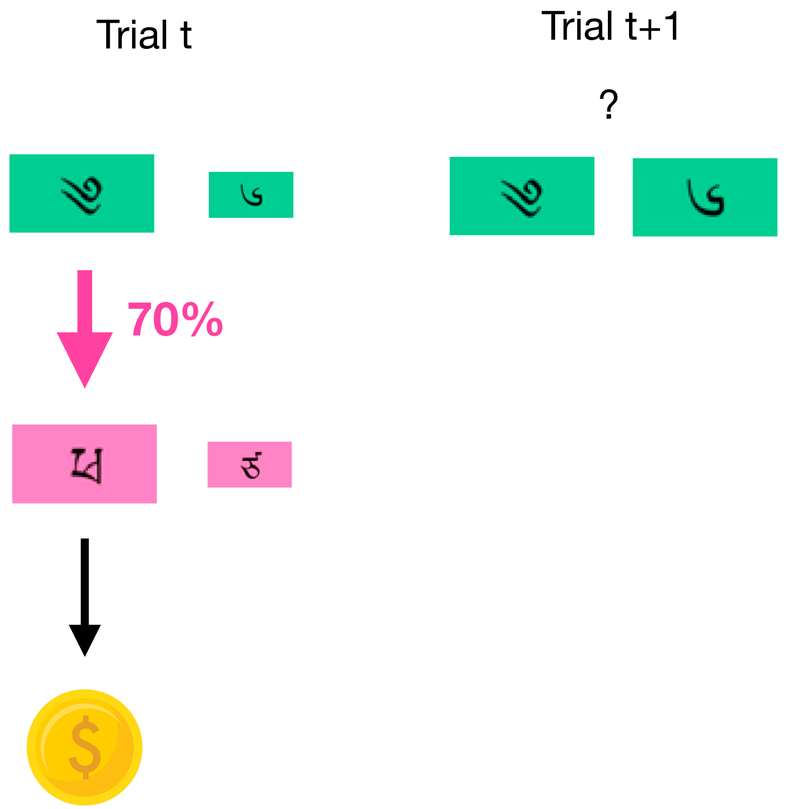
'Two-step task'

Two-step task: one of the most iconic RL tasks



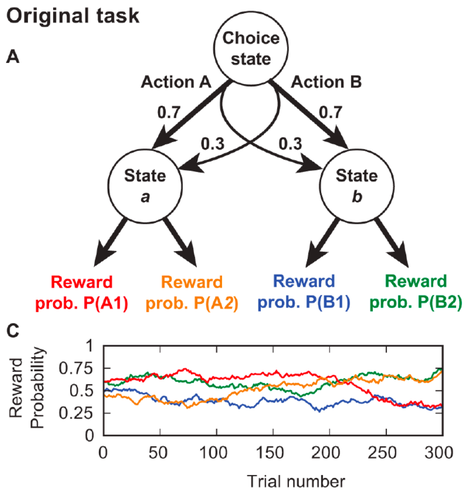
Akam, Costa, Dayan,
PLOS Computational Biology, 2015



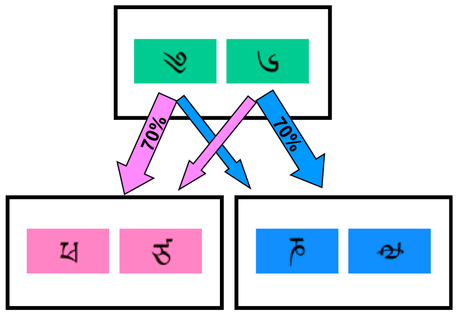


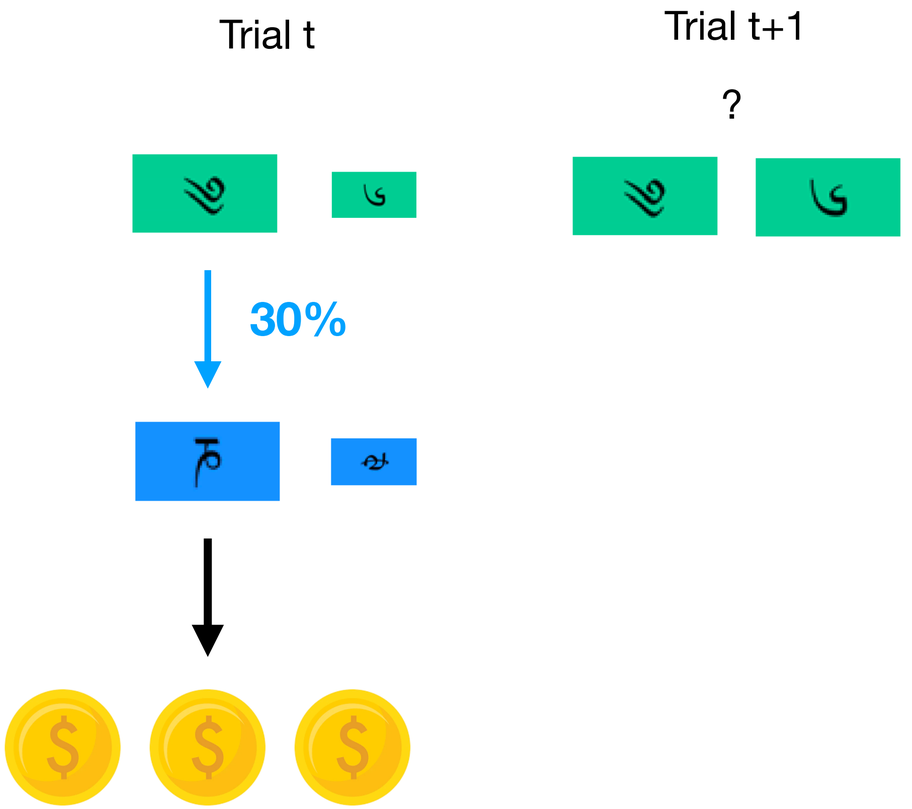
Which green option should the agent choose again at trial t+1?

Two-step task: one of the most iconic RL tasks



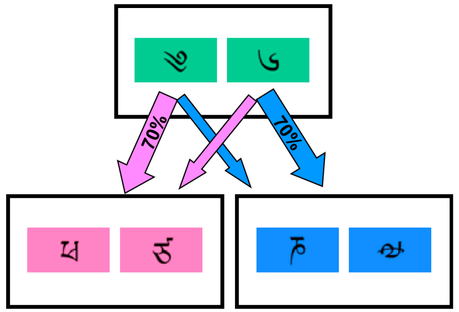
Akam, Costa, Dayan,
PLOS Computational Biology, 2015





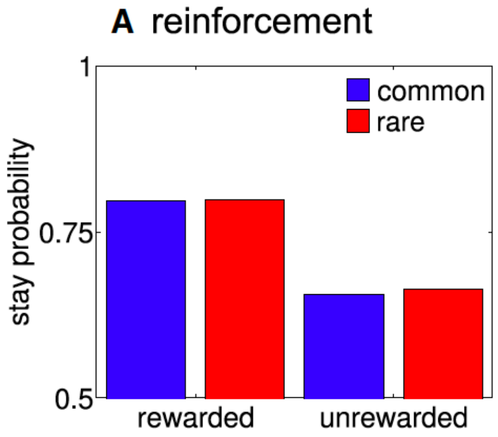
Which green option should the agent choose again at trial t+1?

Two-step task: one of the most iconic RL tasks

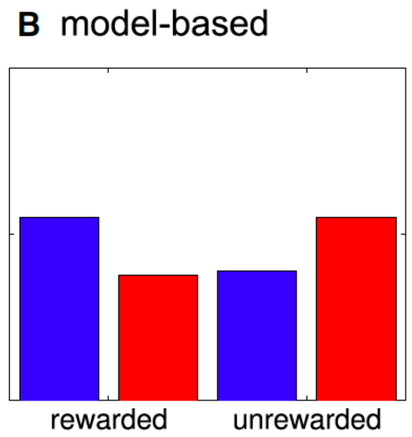


Trial t+1

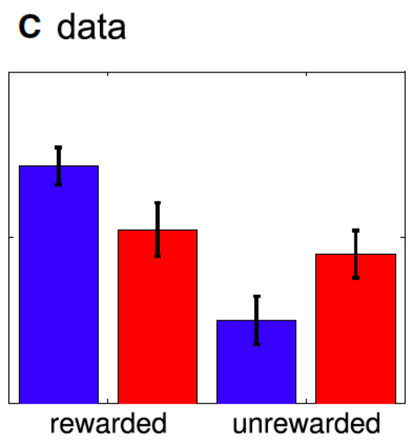
?



Model-free RL agent: repeat what is rewarding



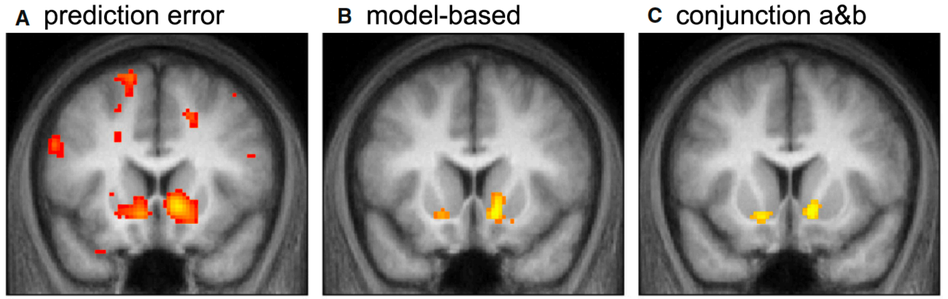
Model-based RL agent: repeat what is rewarding, but be clever

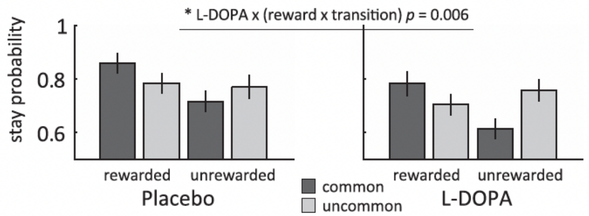


Real data: a mix of both

Two-step task: one of the most iconic RL tasks

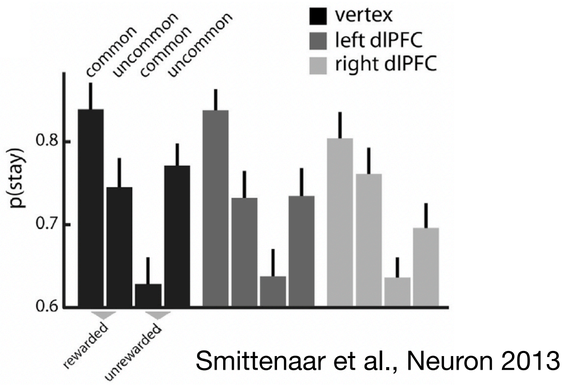
Model-free and model-based prediction errors in ventral striatum





Wunderlich et al., Neuron 2012

Increasing **dopamine**-levels makes participants more model-based



Smittenaar et al., Neuron 2013

Disrupting **dlPFC** leads to more model-free behaviour

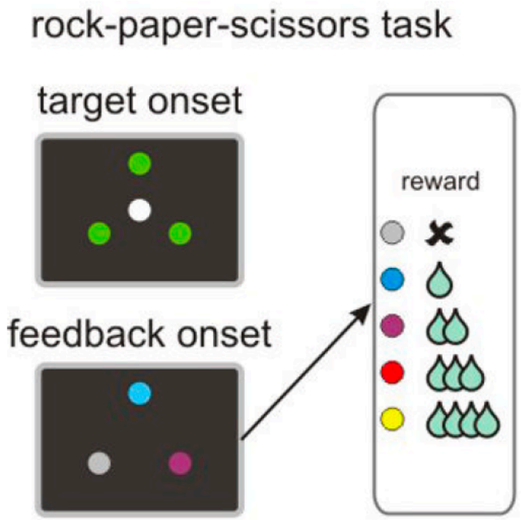
Model-based control…

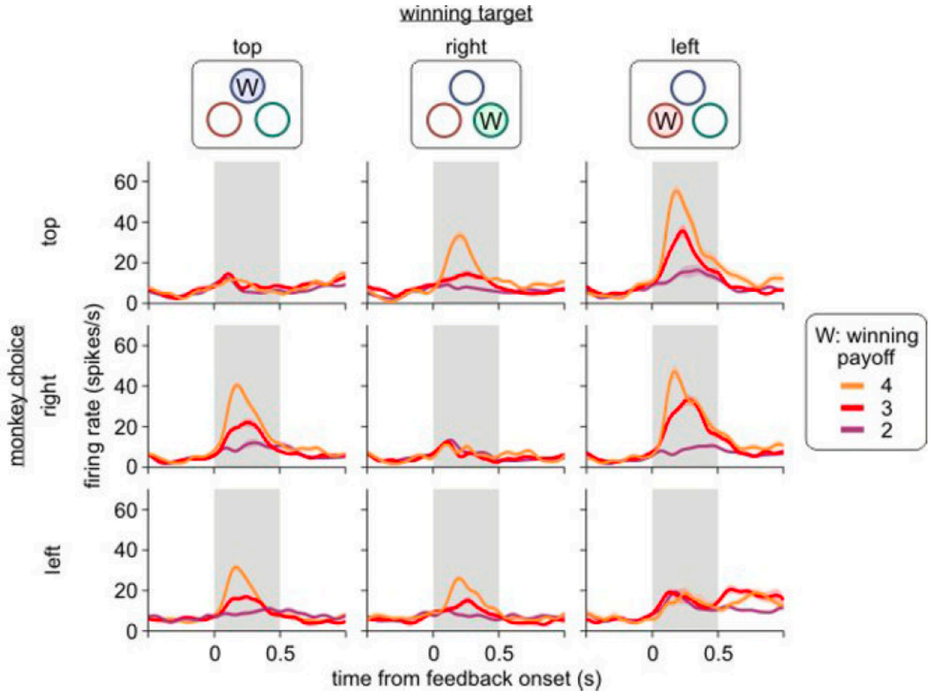
- Depends on **cognitive control** (Otto et al., Journ Cogn Neuroscience 2014)
- Depends on the **dorsal hippocampus** (Miller et al., Nat Neurosci 2017)
- May sometimes be **model-free control in disguise** (Akam et al., PLOS CB 2015)
- May reflect how well people **understand the task** (Silva & Hare, NHB 2020)

Model-based reasoning: counterfactuals

Some neurons in orbitofrontal cortex encode hypothetical outcomes:

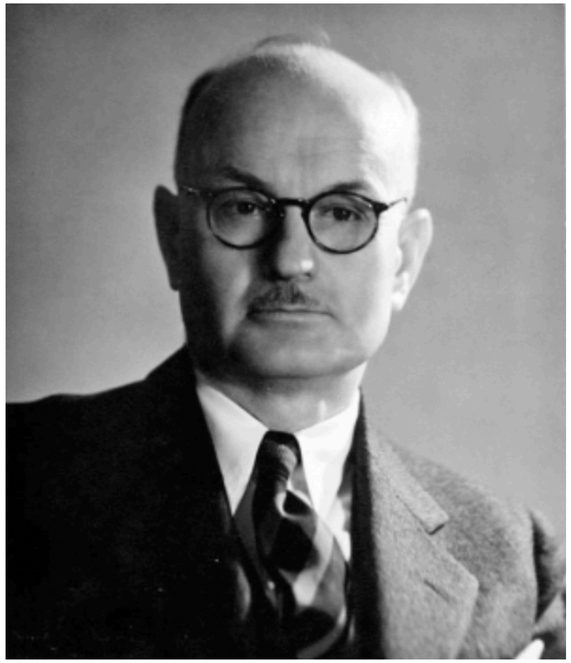
- Fire only if an *unchosen* option was rewarded

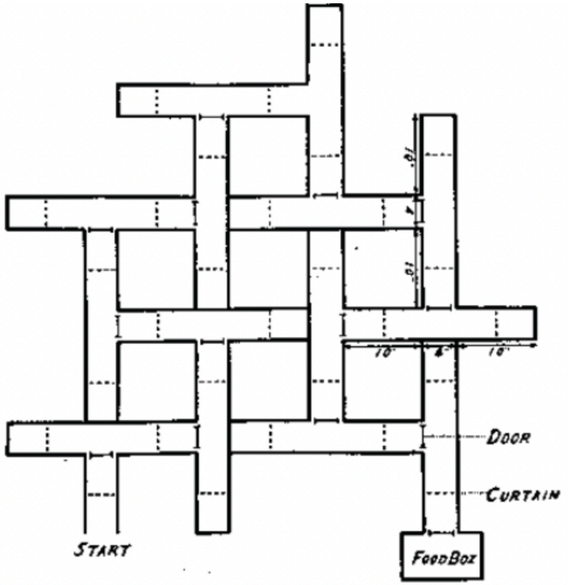




Abe & Lee, Neuron 2011
Lee et al., Annu Rev Neurosci. 2012

Cognitive maps for model-based RL?





Plan of maze
14-Unit T-Alley Maze

FIG. 1

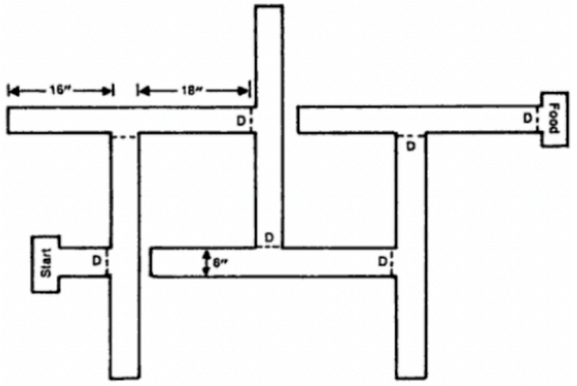
(From M. H. Elliott, The effect of change of reward on the maze performance of rats. *Univ. Calif. Publ. Psychol.*, 1928, **4**, p. 20.)

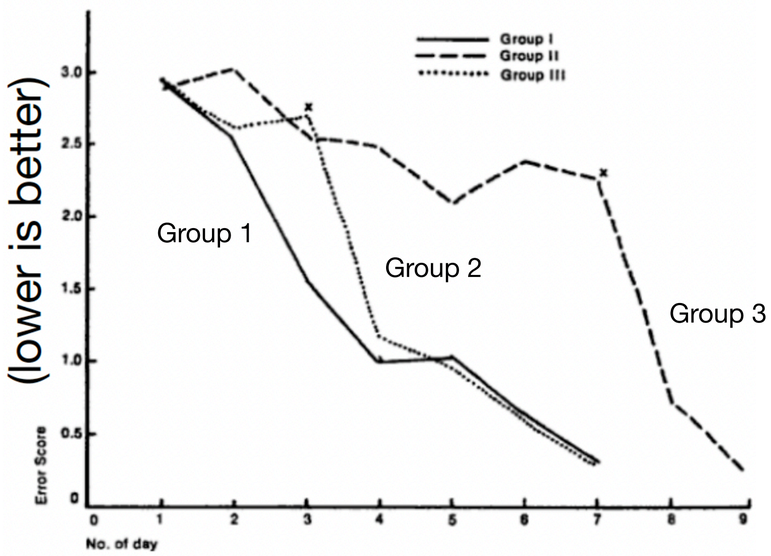
Tolman, Psychological Review, 1948

Cognitive maps for model-based RL?

Latent learning in RL: three groups of animals

- Group 1: always rewarded
- Group 2: reward added early
- Group 2: reward added late



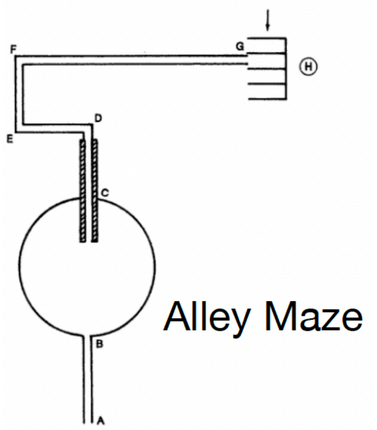


Key insight: whenever reward is added performance ‘catches up’ rapidly

- As if there is latent structural learning that can be used

Blodget, Univ. Calif. Publ. Psych., 1929

Cognitive maps for model-based RL?

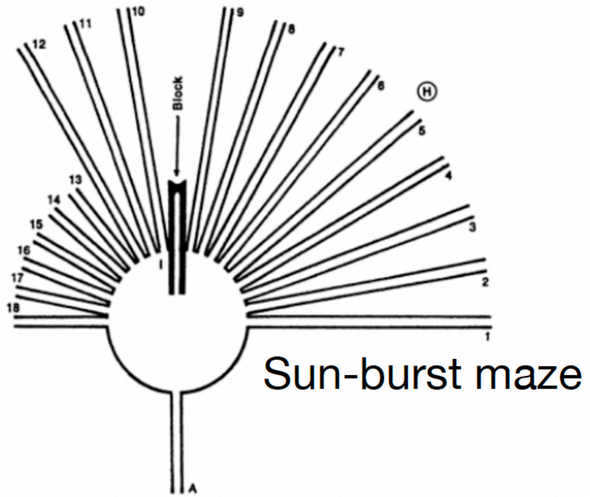


Alley Maze

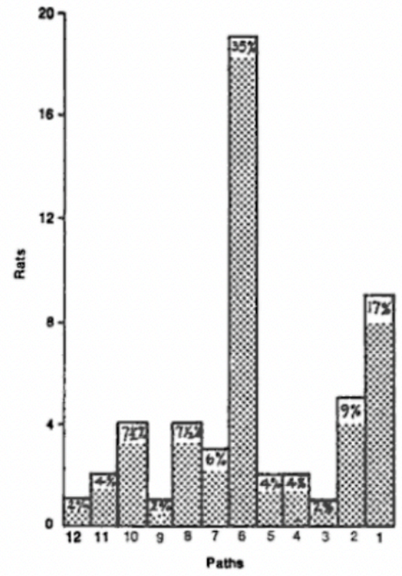
Rats are presented with a particular path to a goal G

When finding the original path blocked, rats choose the direct path to the goal in a ‘sun-burst maze’

Key insight: rats have a map of 2D space, and use it for model-based RL



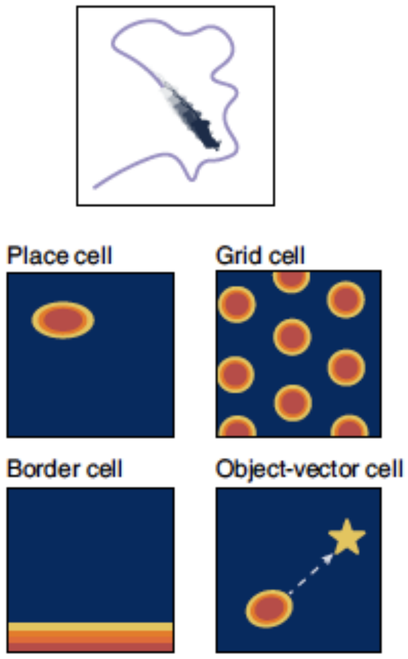
Sun-burst maze

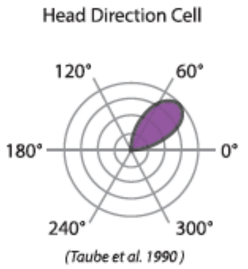


Tolman et al., Journ. Exp. Psych., 1946

Cognitive maps for model-based RL?

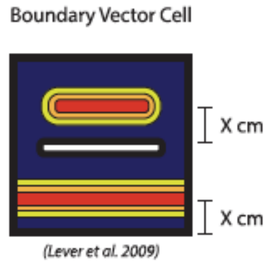
We have gained much insight into cognitive maps used in goal-directed navigation



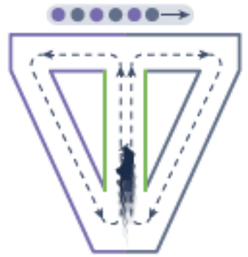


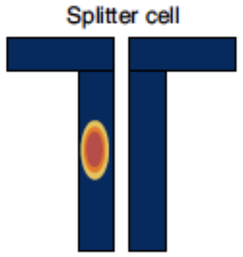
(Taube et al. 1990)

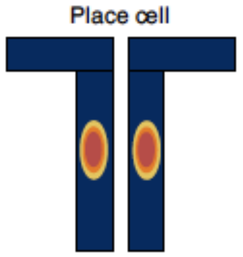
(Gauthier & Tank 2018)



(Lever et al. 2009)







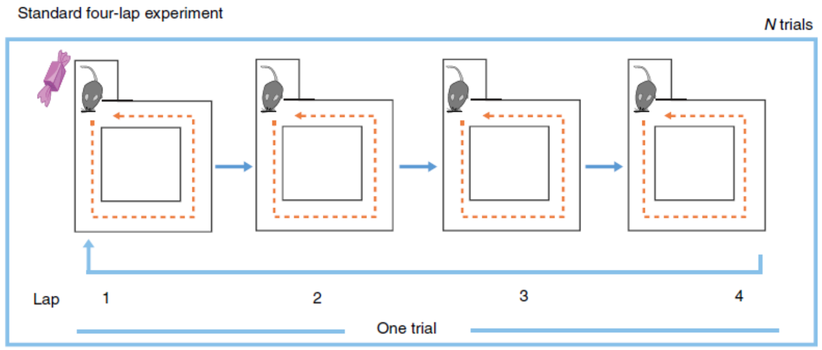
Is this a **basis set** over world structures?

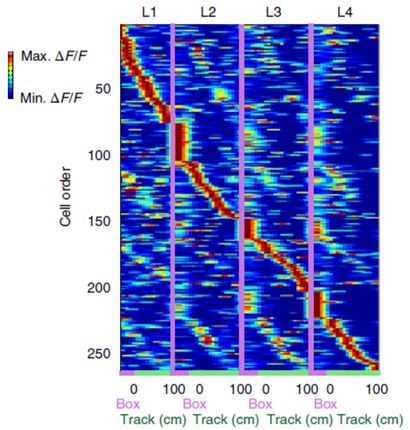
Whittington et al. (2022). How to build a cognitive map. Nature Neuroscience

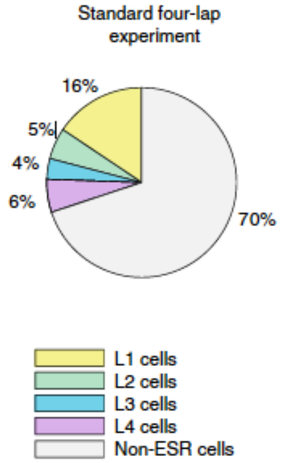
Behrens et al. (2018). What is a cognitive map? Organizing knowledge for flexible behavior. Neuron

A cognitive map for model-based RL - beyond space

Place cells for lap counting - relevant for reward:





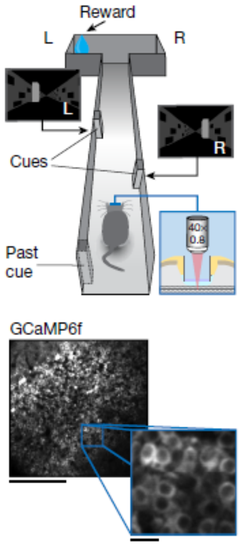


Important: cognitive maps might encode **task space**, rather than just physical space!

Sun et al. (2020). Hippocampal neurons represent events as transferable units of experience. Nature neuroscience

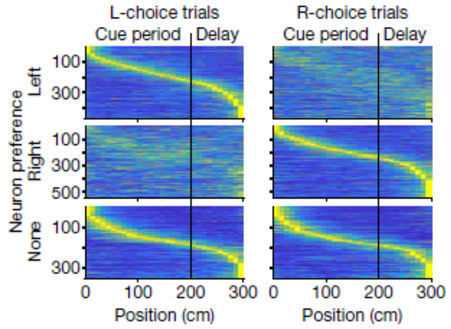
A cognitive map for model-based RL - beyond space

Place cells for evidence accumulation:

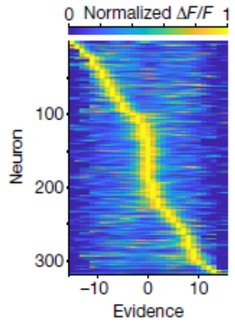


Cells in hippocampus (CA1) code for

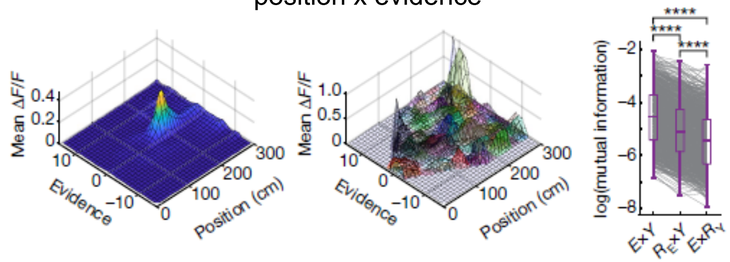
Position and subsequent choice



Evidence



Neural code looks like a conjunction position x evidence



Important: cognitive maps might encode **task space**, rather than just physical space!

Nieh et al. (2021). Geometry of abstract learned knowledge in the hippocampus. Nature

Summary

Model-free association learning powerful framework

- Remarkable success in explaining behaviour and neural phenomena
- Ressource-efficient but inflexible

Model-based RL: high flexibility but also costly

- Gold-standard probe: **de-valuation**
- Helps with **learning** and **planning/action selection**
- **Cognitive maps** as neural candidate for task model representation

Homework

Choose one of the following:

1. Reproduce a basic **DYNA-Q agent** in a maze task (with or without a shortcut)
   - https://github.com/schwartenbeckph/Systems-Computational-Neuroscience/blob/main/DYNA_Q_SysCompNeuro.ipynb

2. Reproduce the **two step task** and the **stay probabilities** for a model-free, model-based and mixed agent
   - https://github.com/schwartenbeckph/Systems-Computational-Neuroscience/blob/main/TwoStep_SysCompNeuro.ipynb

Please let me know if you are unfamiliar with Python notebooks!

Thank you