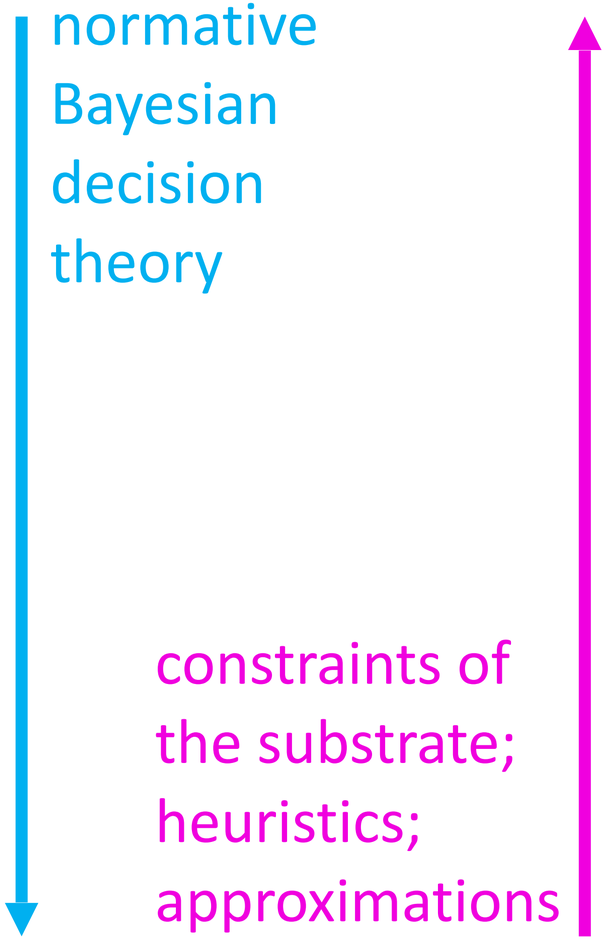
Model-based RL

Systems Computational Neuroscience
16th of November 2022

Marrian Learning to Predict & Act

- Ethology/computation
  - Goal: maximise future pleasure; minimise future pain
  - Logic: optimal control theory
- Psychology/algorithm
  - Classical/instrumental conditioning
  - Learning from rewards and punishments
  - **Model based RL for learning and planning**
- Neuroscience/implementation
  - Prefrontal cortex; hippocampus; entorhinal ctx; striatum
  - Neuromodulation



Motivated Choice

s

States

- Ignorant: have to represent and infer
  - Combine priors & likelihoods

a

Actions

- Affordances, go/no-go, X vs. Y

r

$r(s, a)$

Utilities

- Reproduction/homeostasis
- State-dependent

$Q(s, a)$

$\pi(s, a)$

Choice

- Action values and policy
- State-dependent action maximizing expected long-run reward

Overview

1. What happened previously..
    - Classical conditioning
    - Instrumental conditioning

2. Model-based reinforcement learning
    - De-valuation
    - Model-based RL for learning
    - Model-based RL for planning and action selection
    - Model-based RL and cognitive maps

1. What happened previously

What is reinforcement learning (RL)?

- RL is a **computational approach** to learning from **interactions** with the **environment**
  - Trial-and-error
  - Delayed reward

- Considers whole problem of **goal-directed** agent interacting with an **uncertain** environment

- RL agents
  - Have explicit goals
  - Sense aspects of their environments
  - Choose actions to influence their environments

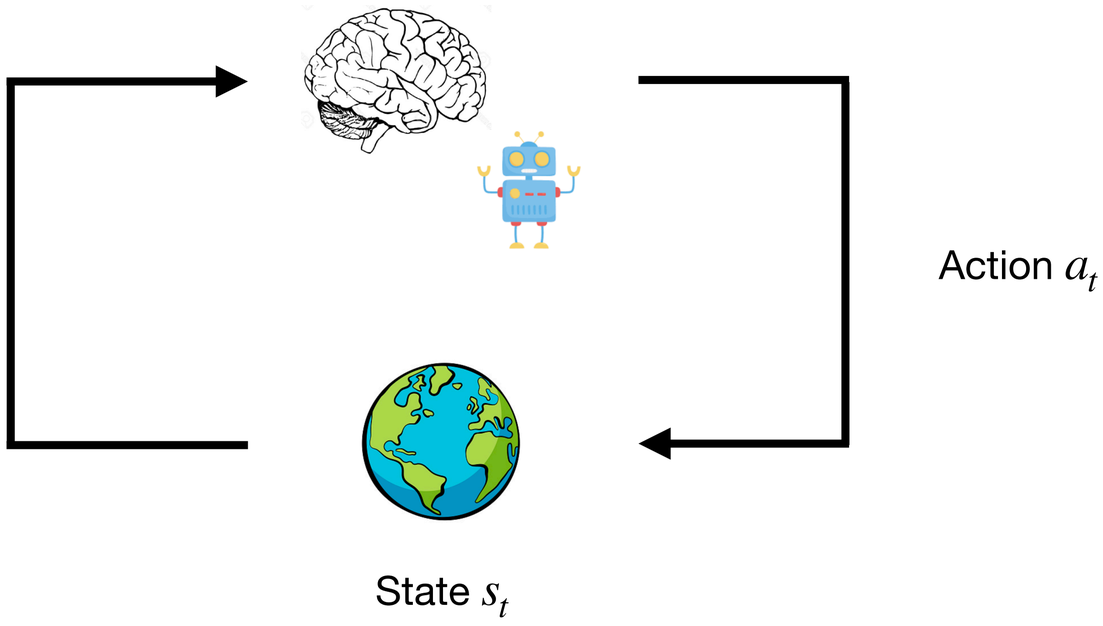
Basic setup of RL?

Based on a reward signal, agents learn **values of actions/states:**

$$V_\pi(s) = \mathbb{E}_\pi[R \,|\, s_0 = s]$$

Action is governed by a **policy**:

$$\pi(a, s) = P(a_t = a \,|\, s_t = s)$$



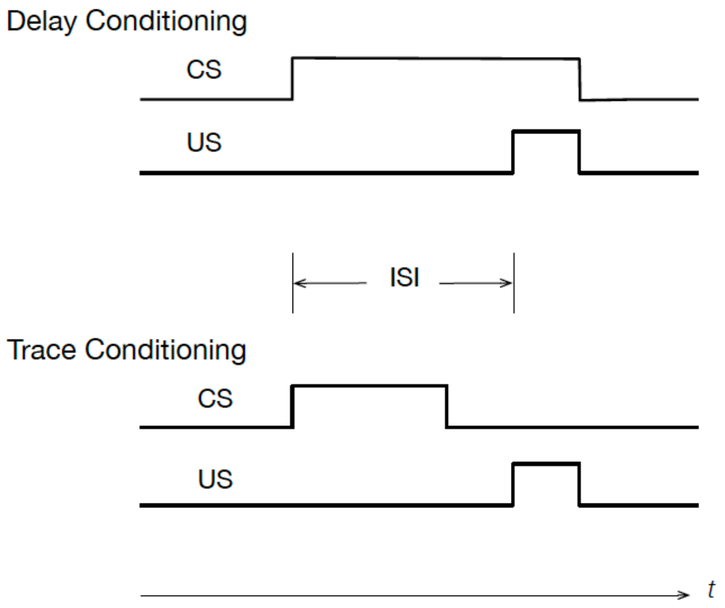
Agents can learn a **model of the environment** to make smarter decisions, e.g.:

$$P(s_{t+1} = s, r_{t+1} = r \,|\, s_t = s, a_t = a)$$

1.1 Classical Conditioning

Classical Conditioning

- Two learning algorithms you should know about:
  - **Rescorla-Wagner (RW-)Learning**
    - Learn stimulus-outcome associations
  - **Temporal Difference (TD-)Learning**
    - Learn stimulus-outcome associations *across* time



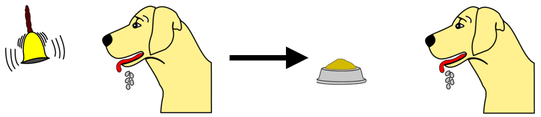
Basics of Learning: Rescorla-Wagner Learning

Learn associative strength between a CS and US

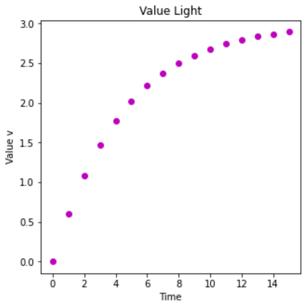
$$V(s) \leftarrow V(s) + \alpha \cdot (r - V(s))$$

Prediction error

Learning rate

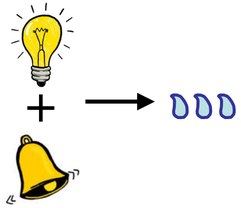


$$V(\text{💡}) \leftarrow V(\text{💡}) + \alpha \cdot (\text{DDD} - V(\text{💡}))$$



Link to code here

Introducing a second CS can lead to **blocking**:

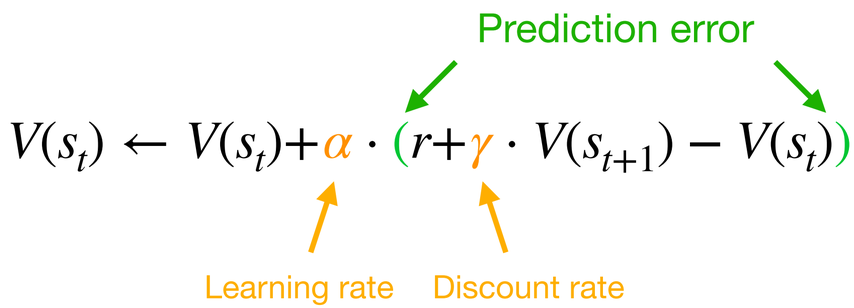


$$[V(\text{💡}) + V(\text{🔔})] \leftarrow [V(\text{💡}) + V(\text{🔔})] + \alpha \cdot (\text{DDD} - [V(\text{💡}) + V(\text{🔔})])$$

Temporal Difference Learning

- “If one had to identify one idea as **central** and **novel** to reinforcement learning, it would undoubtedly be temporal-difference (TD) learning.”

- Update based on other learned estimates, without waiting for final outcome (**bootstrap**)
  - Learn “a guess from a guess”

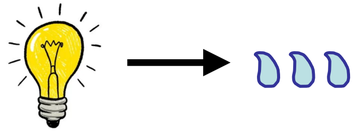


- Operates in ‘real-time’
  - t labels time steps within trials
  - Think of time between t and $t+1$ as a small time interval (e.g. 1ms)

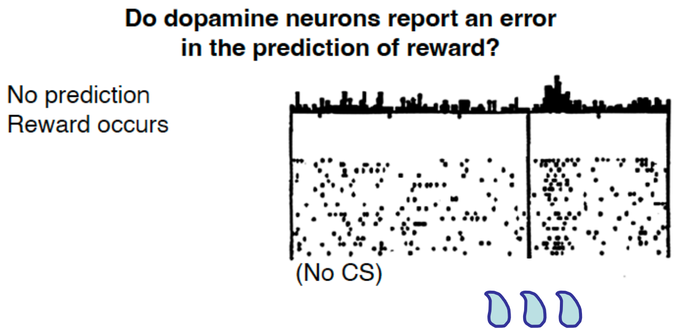
Can RL tell us anything about the brain?

$$V(s_t) \leftarrow V(s_t) + \alpha \cdot (r + \gamma \cdot V(s_{t+1}) - V(s_t))$$

- Yes, quite a lot.
- Particularly, it looks like dopamine (DA) is a key neurotransmitter for (TD) reward learning

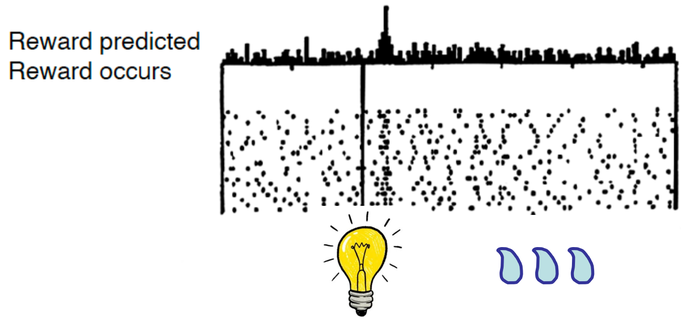


Dopamine neurons signal immediate reward

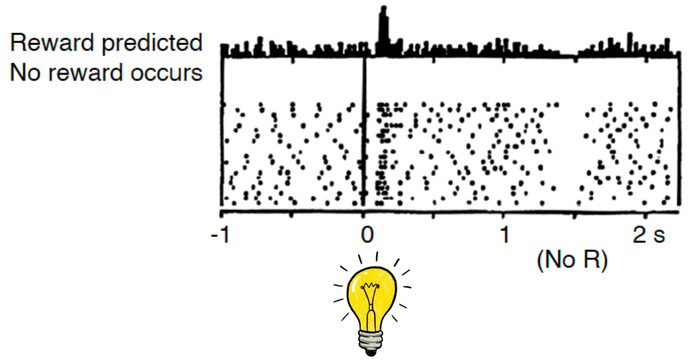


BUT: after training…

- DA signal reward prediction
- But not correctly predicted reward!



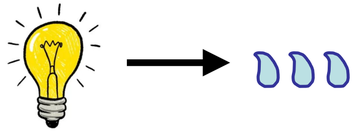
AND: it signals the unexpected omission of a reward!

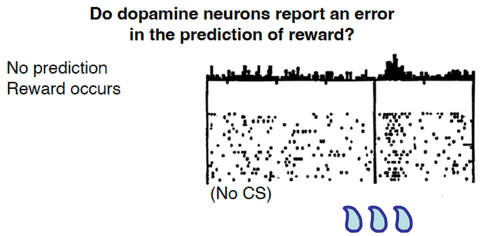


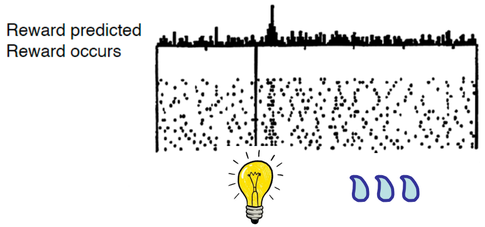
Schultz, Dayan & Montague (Science, 1997)

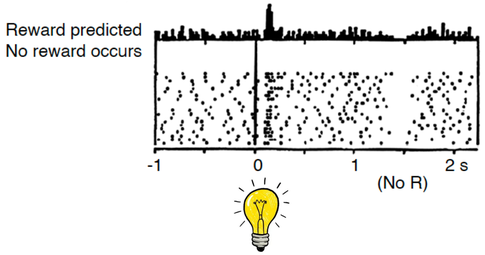
Temporal Difference Learning

$$V(s_t) \leftarrow V(s_t) + \alpha \cdot (r + \gamma \cdot V(s_{t+1}) - V(s_t))$$

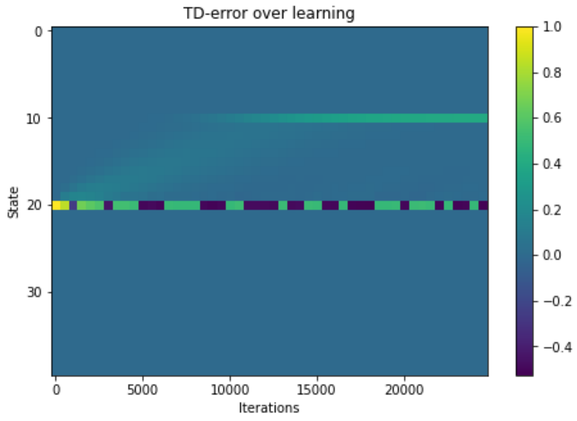




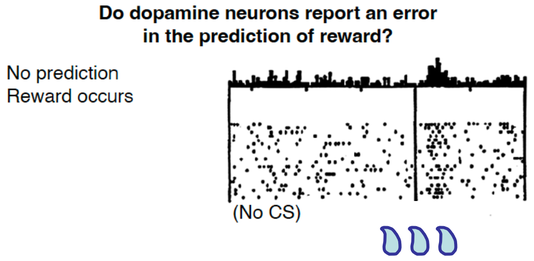


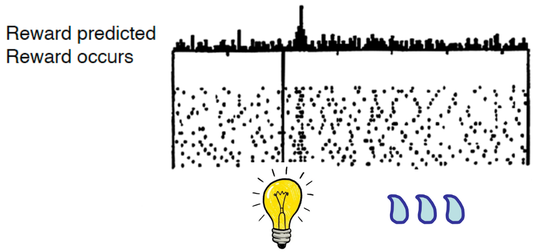


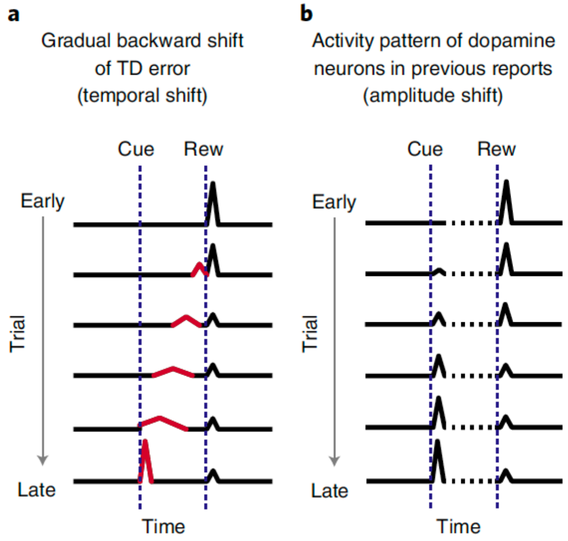
We can simulate this (link to code here):

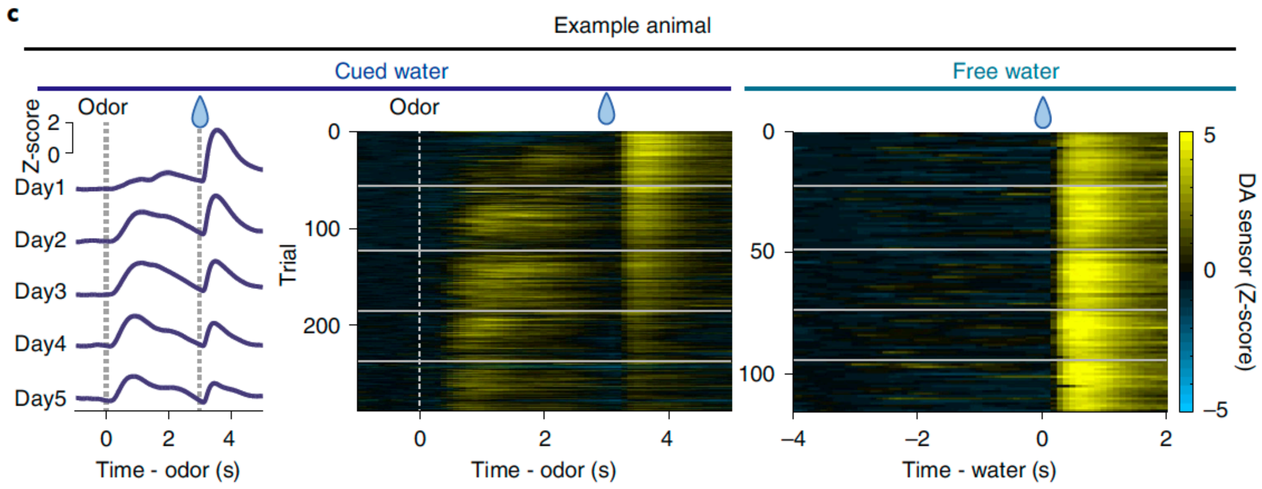


More TD-learning





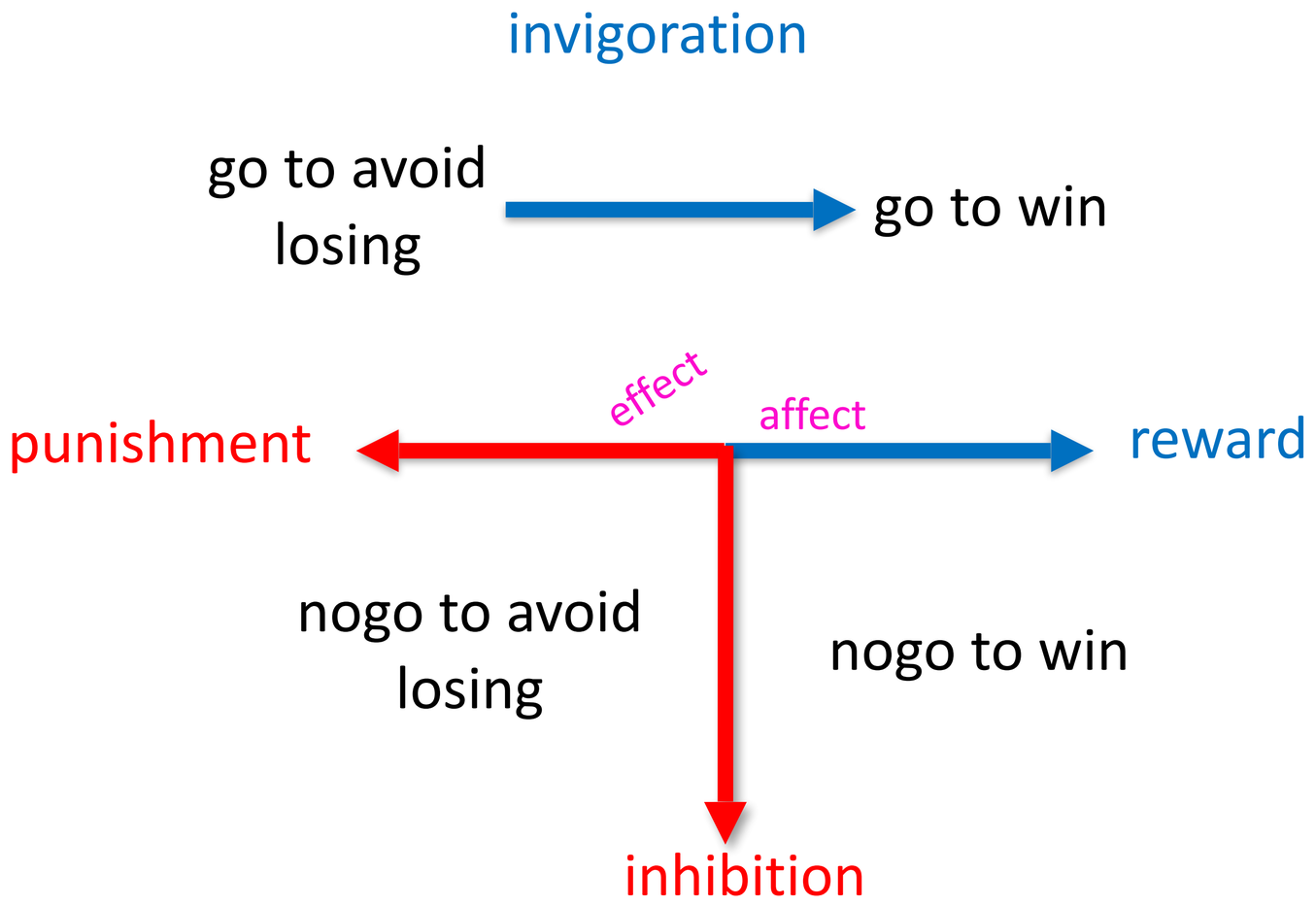




Amo, …, Watabe-Uchida, Nature Neuroscience 2022

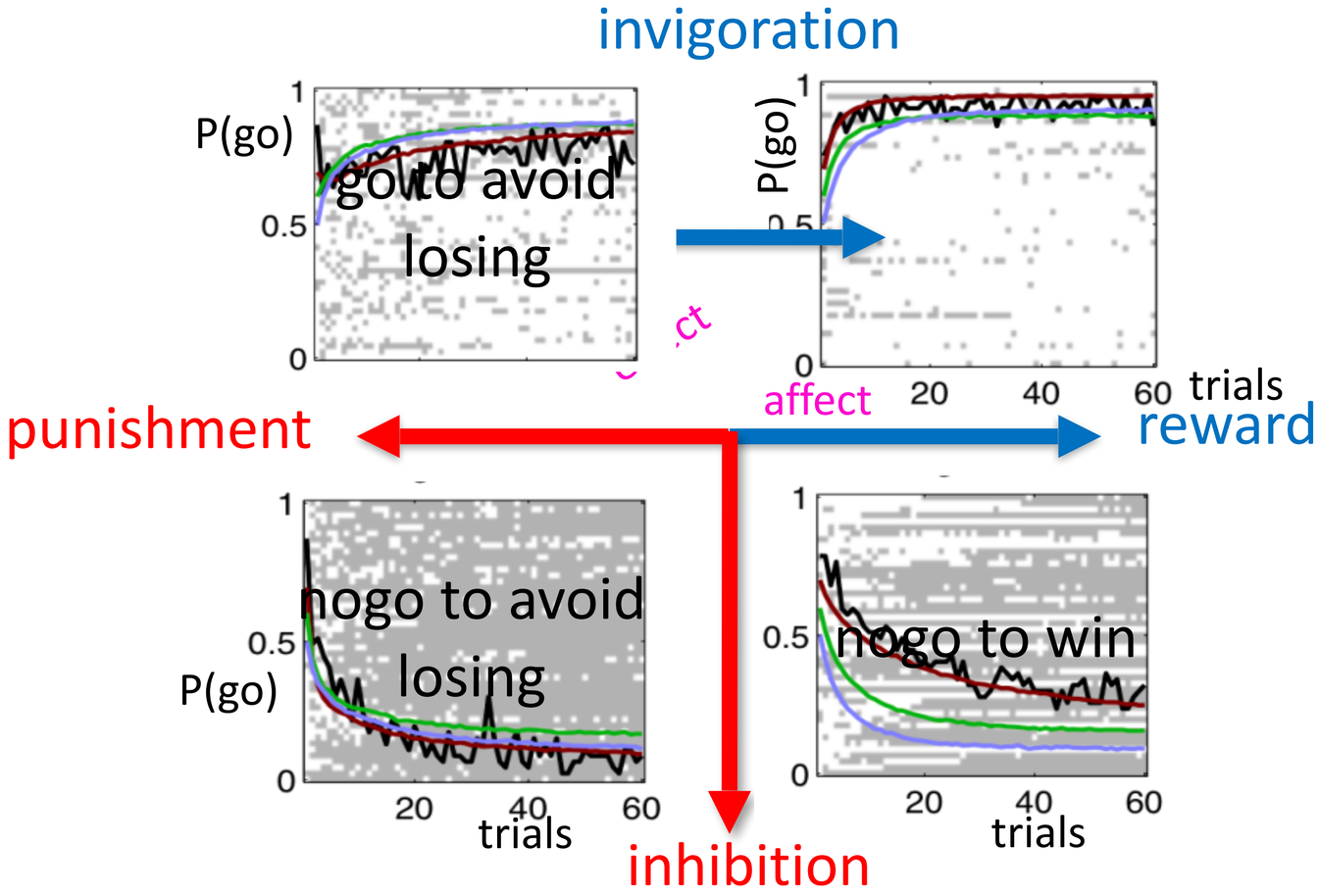
1.2 Instrumental Conditioning

Evolutionary Programming



Crockett et al, 2009; Boureau & Dayan, 2011

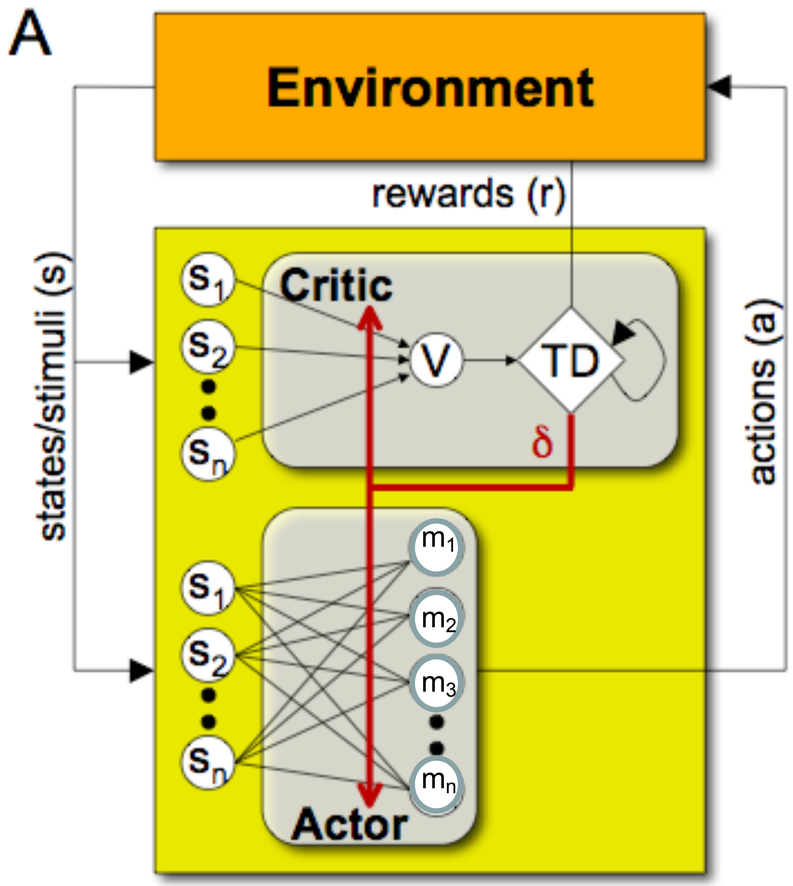
Evolutionary Programming

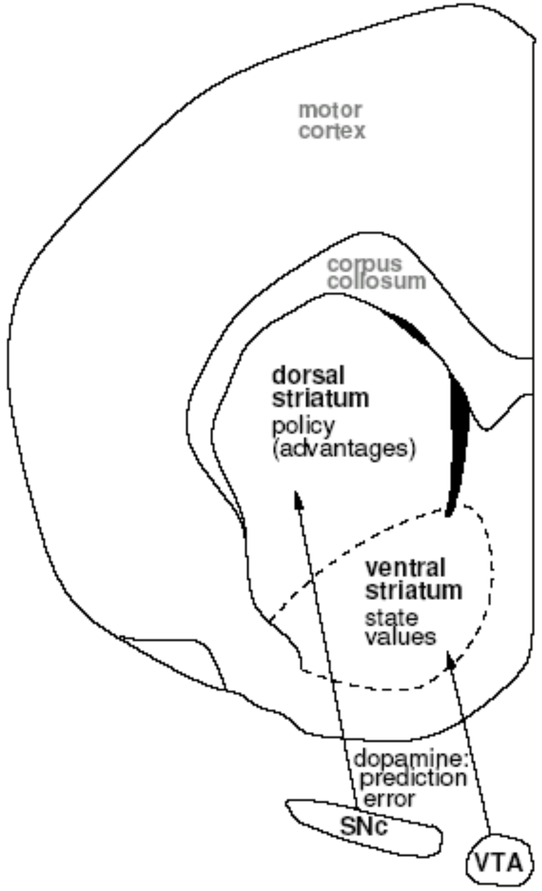


- **data**
- (delta rule)
- bias
  - go+b
- tremble
- Pavlovian
  - go+$\omega V(s_t)$

Crockett et al, 2009; Guitart-Masip et al, 2011; 2012

actor/critic



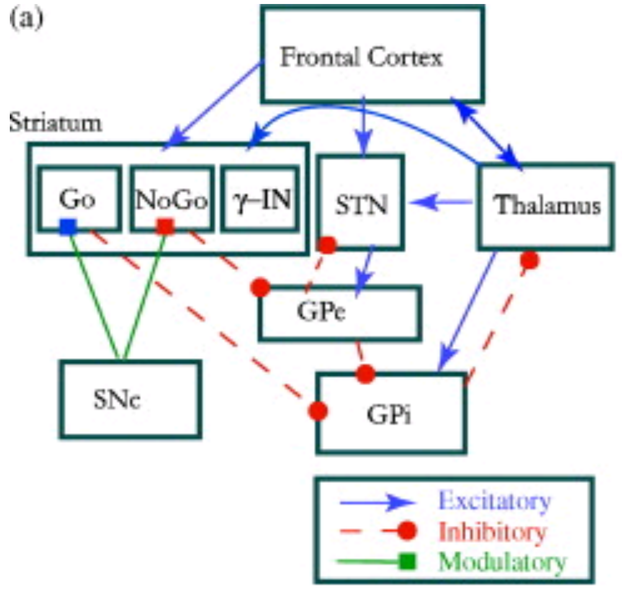


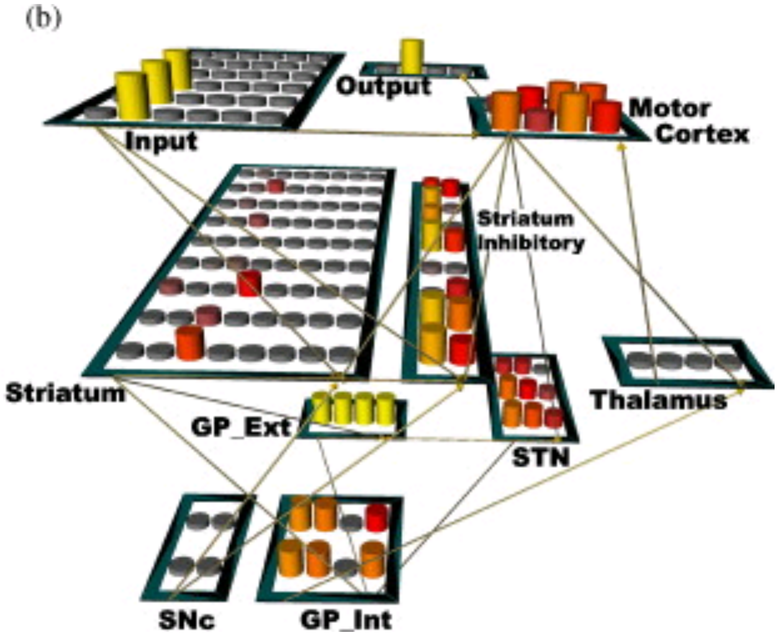
Dopamine signals to both **motivational** & **motor**

- Striatum appear, surprisingly the same

Suggestion: training both **values** & **policies**

Direct/Indirect Pathways



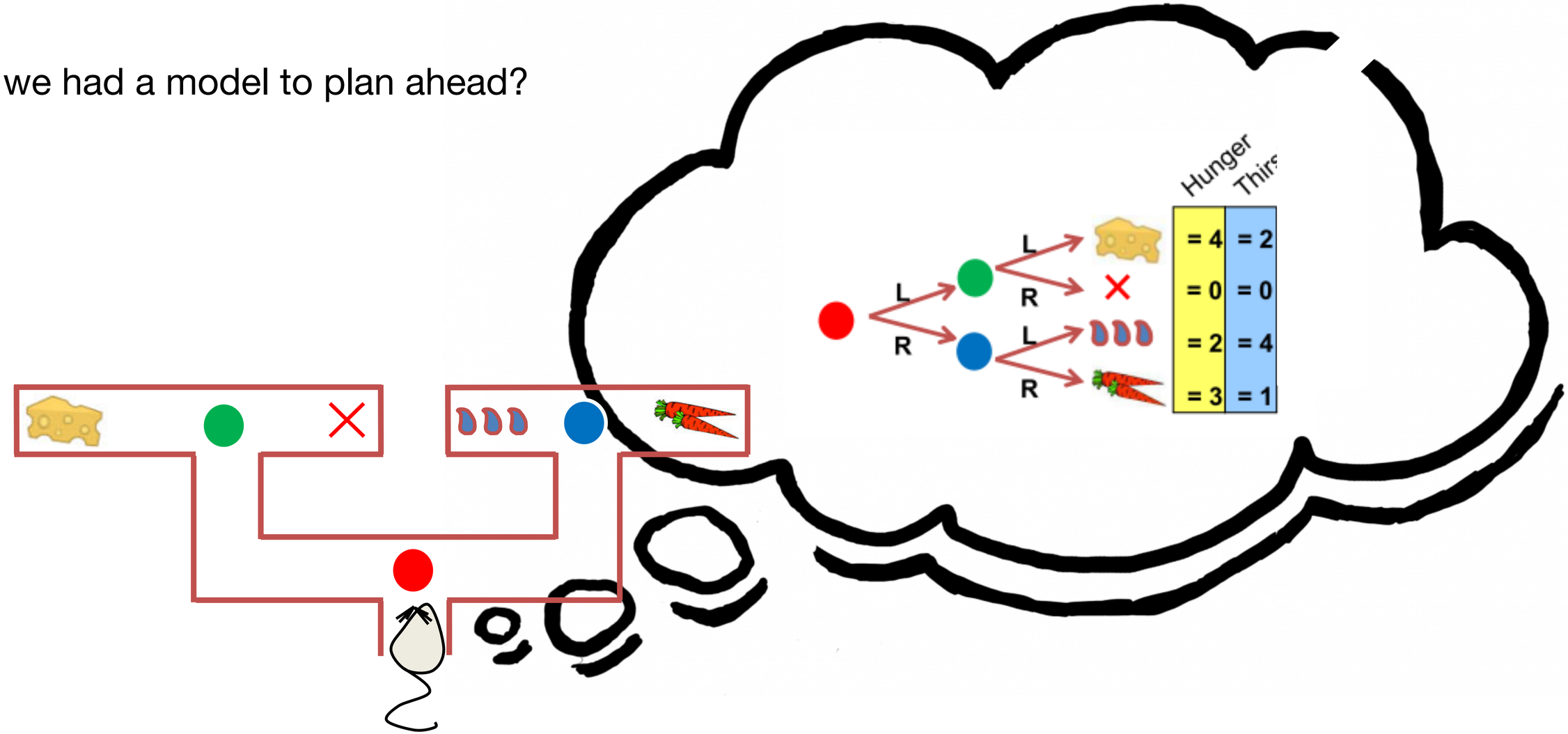


Michael Frank

- Direct: D1: GO; learn from DA increase
- Indirect: D2: noGO; learn from DA decrease
- Hyperdirect (STN) delay actions given strongly attractive choices

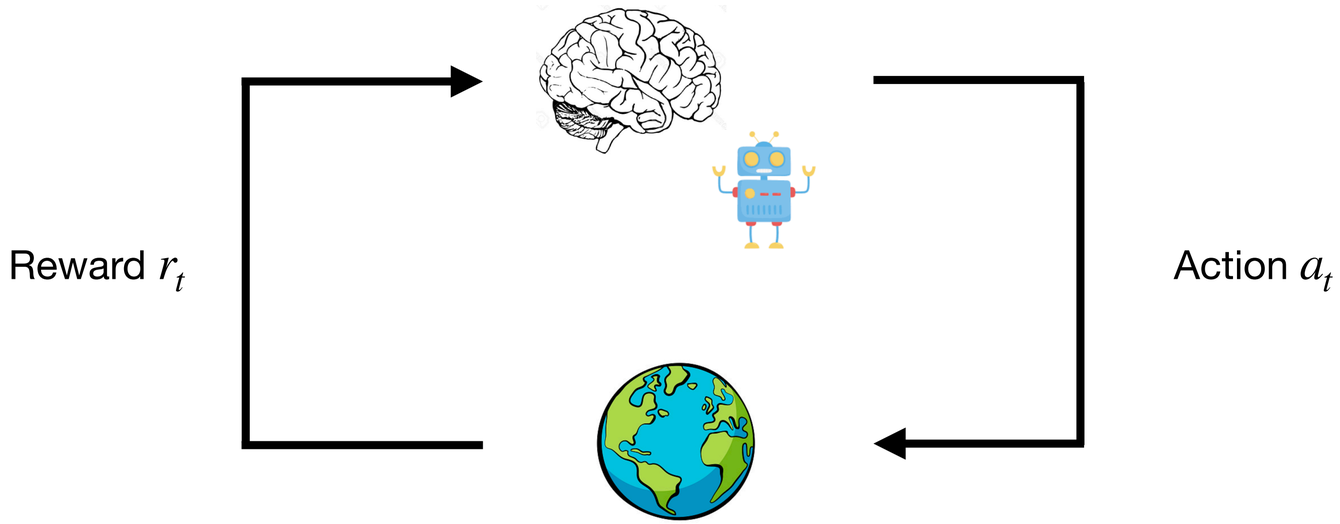
Model-based Reinforcement Learning

What if we had a model to plan ahead?



2. Model-based RL

Basic setup: how do agents learn to act?



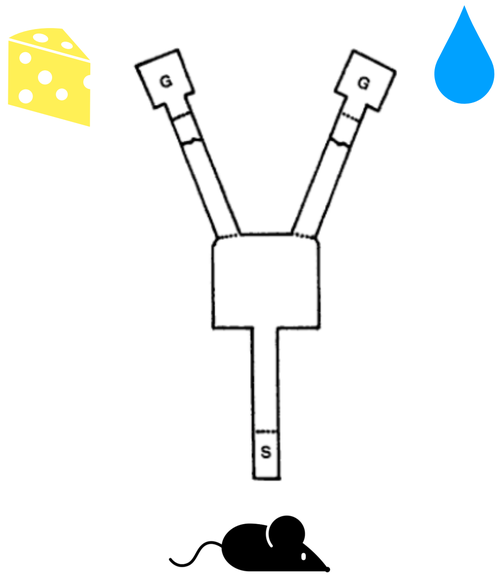
Agents can learn a **model of the environment** to make smarter decisions, e.g.:

$$P(s_{t+1} = s, r_{t+1} = r \,|\, s_t = s, a_t = a)$$

De-valuation

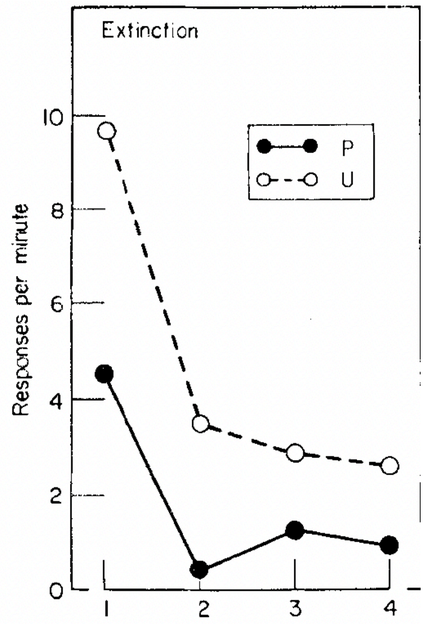
Outcome devaluation (*revaluation*): gold-standard test for forward model predicting outcomes of actions

Animal is trained to perform two different actions, with a different reward:



One reward is then devalued, for example by satiation.

Impact of this devaluation is tested in ‘extinction’, without providing outcomes.



Adams & Dickinson, Quarterly Journal of Experimental Psychology, 1981
Colwill & Rescorla, Journal of Experimental Psychology, 1985
Akam, Costa, & Dayan, PLOS CB 2015

What is the model in model-based RL?

$$P(s', r \mid s, a) = P(s_{t+1} = s', r_{t+1} = r \mid s_t = s, a_t = a)$$

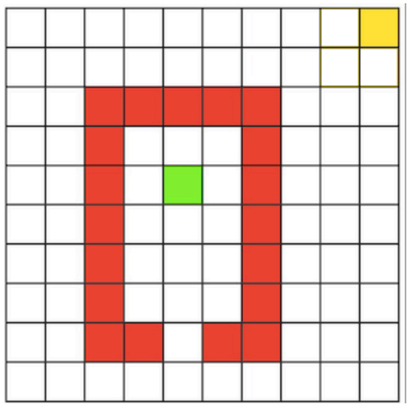
How can we make use of such models of the world?

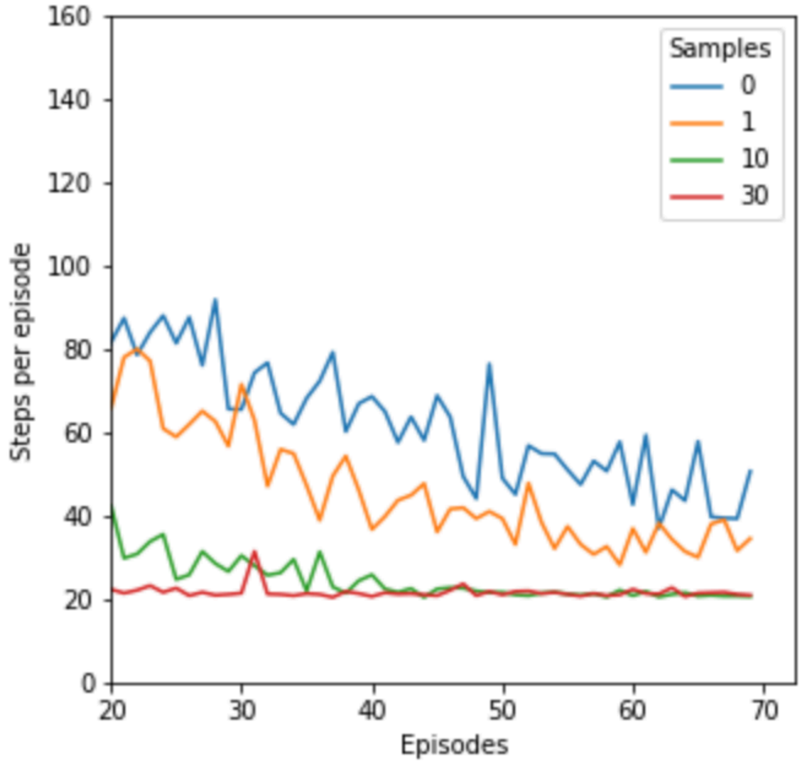
Learning

- Key idea: store experiences in world model $P(s', r \mid s, a)$
- Sample from this model to generate extra learning data
- This is called **DYNA-Q…**

DYNA-Q

Sample from world model $P(s', r \mid s, a)$ to generate additional learning data





$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha\Big(r_t + \gamma \max_a Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)\Big)$$

$$P(s', r|s, a) = P(s_{t+1} = s', r_{t+1} = r|s_t = s, a_t = a)$$

$$Model(S, A) \leftarrow R, S'$$

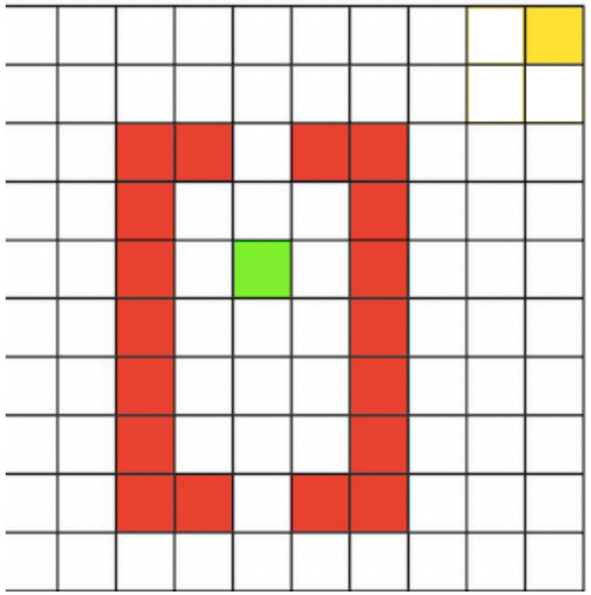
And during breaks ('offline rest'), they can sample from this experience and learn from these samples:

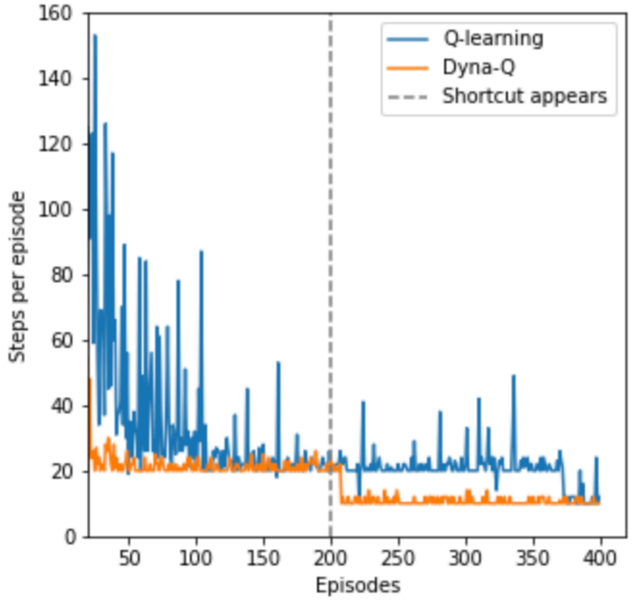
$S \leftarrow$ previously observed state
$A \leftarrow$ action previously taken in S
$R, S' \leftarrow Model(S, A)$
$Q(S, A) \leftarrow Q(S, A) + \alpha \left[R + \gamma \max_a Q(S', A) - Q(S, A)\right]$

Link to code here

DYNA-Q

This also helps with detecting shortcuts:



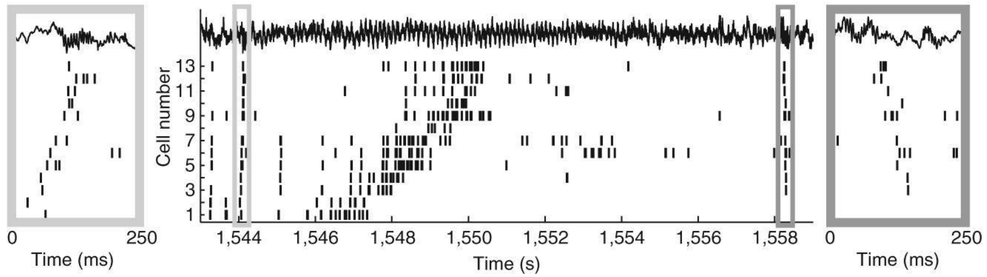


DYNA-Q - Replay as a candidate neural mechanism

DYNA-Q looks a lot like replay.

Replay as a computational mechanism in PFC and hippocampal formation

- i.e. fast reactivation of external states



Diba & Buzsaki (2007) Nature Neuroscience

Implicated in

- Learning from the *past* (credit assignment, Ambrose et al. (2016) Neuron)
- Planning *future* trajectories (Pfeiffer & Foster (2013) Nature)

MDPs basis for model-based RL

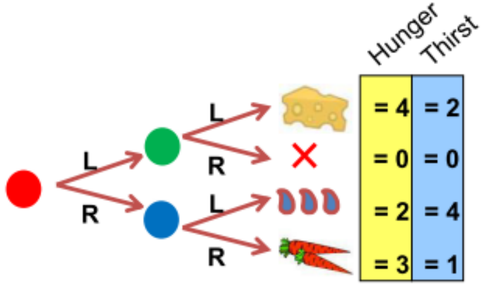
$$P(s', r \mid s, a) = P(s_{t+1} = s', r_{t+1} = r \mid s_t = s, a_t = a)$$

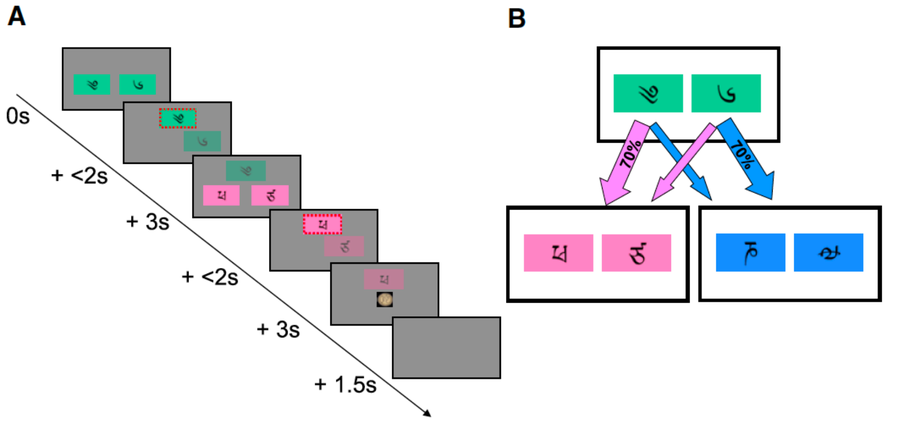
How can we make use of such models of the world?

Learning

- Key idea: sample from $P(s', r \mid s, a)$

Planning and **action selection**

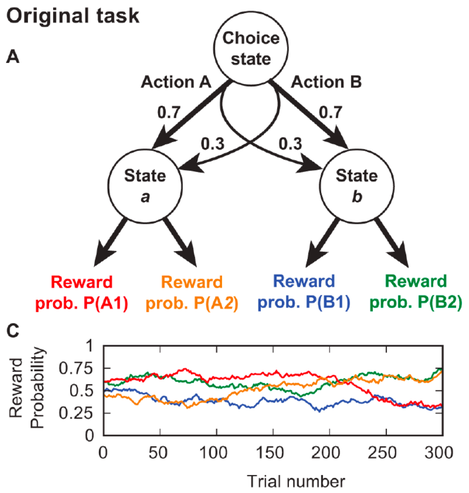




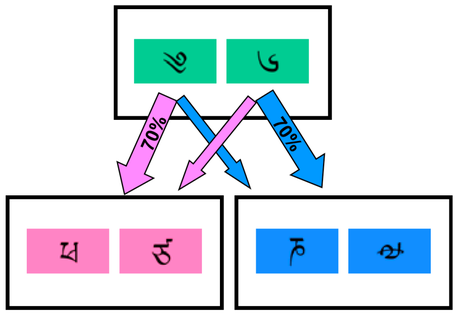
Daw, …, Dayan, & Dolan, Neuron, 2011

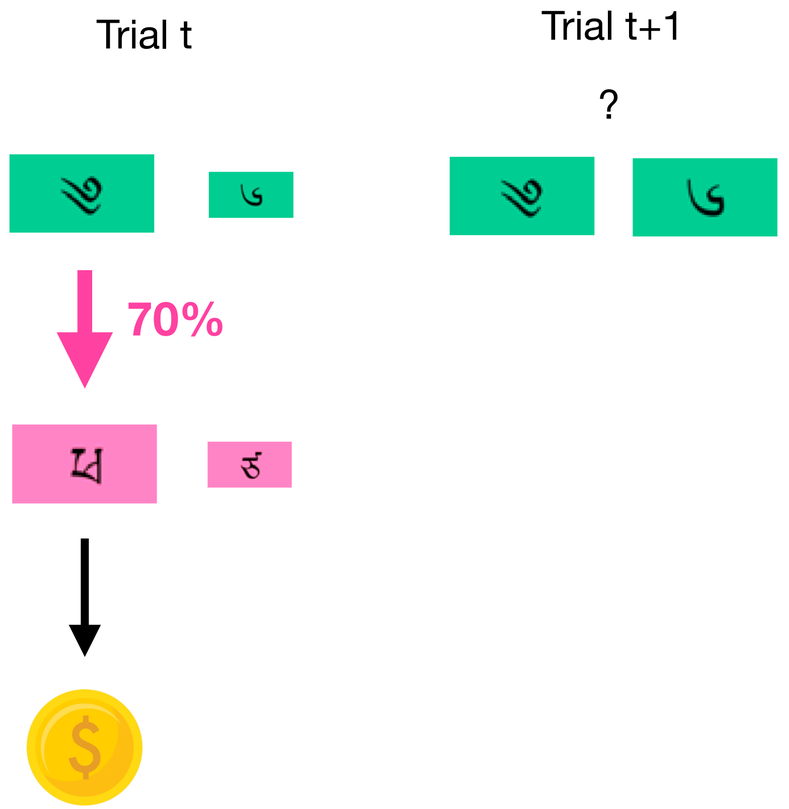
'Two-step task'

Two-step task: one of the most iconic RL tasks



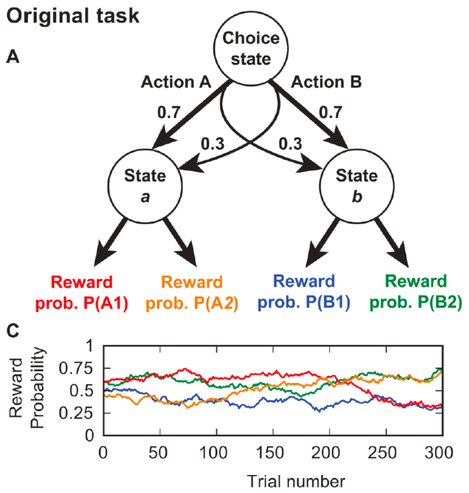
Akam, Costa, Dayan,
PLOS Computational Biology, 2015



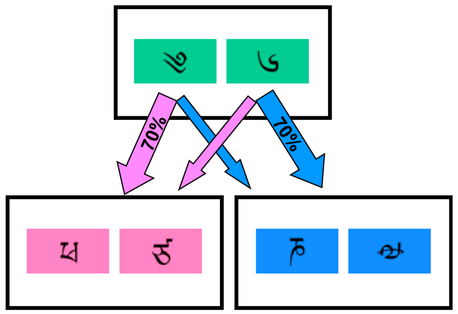


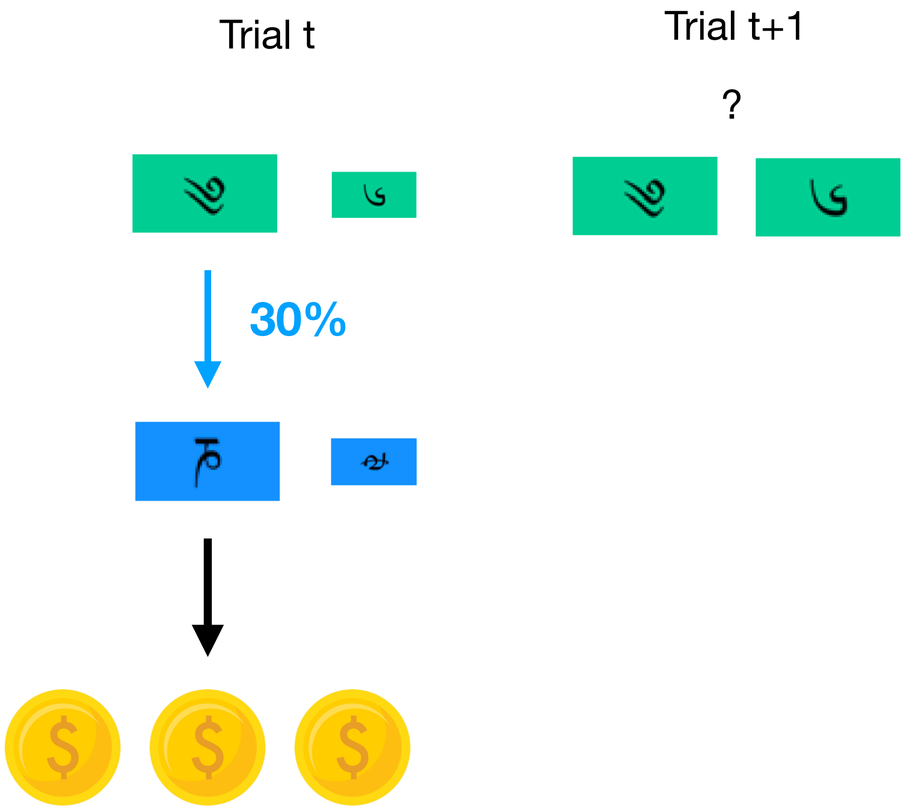
Which green option should the agent choose again at trial t+1?

Two-step task: one of the most iconic RL tasks



Akam, Costa, Dayan,
PLOS Computational Biology, 2015





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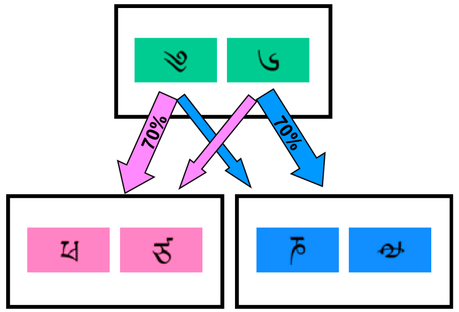
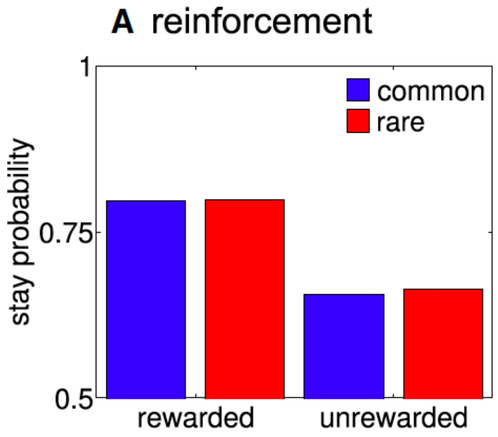
Two-step task: one of the most iconic RL tasks

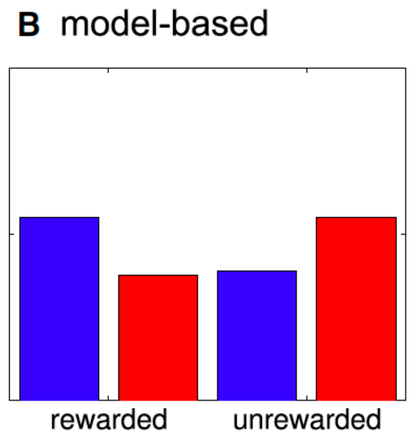


Trial t+1

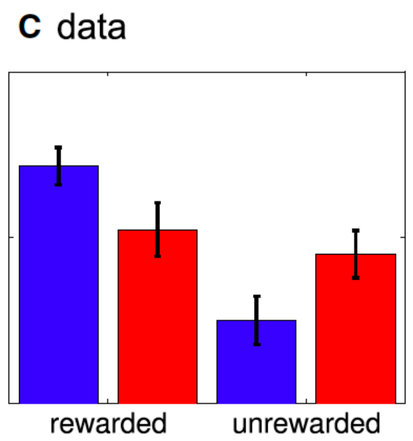
?



Model-free RL agent: repeat what is rewarding



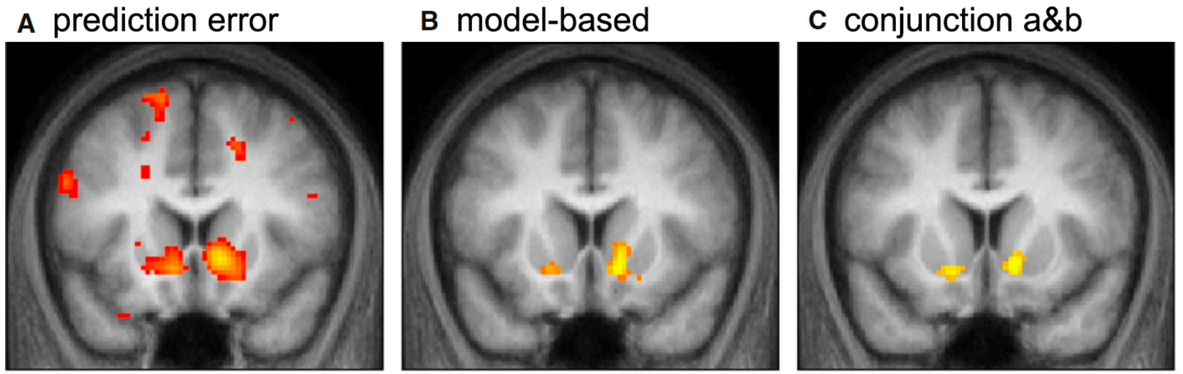
Model-based RL agent: repeat what is rewarding, but be clever



Really data: a mix of both

Two-step task: one of the most iconic RL tasks

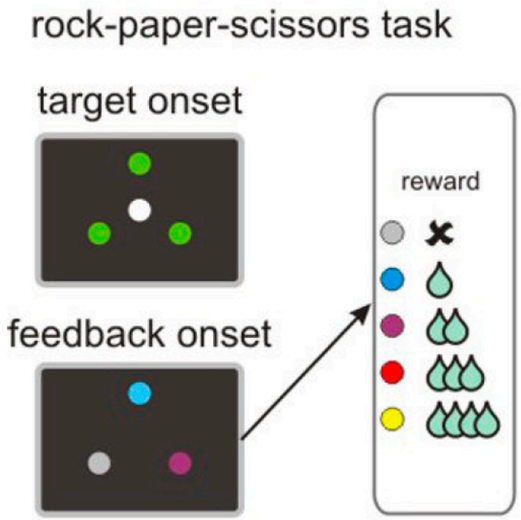
Model-free and model-based prediction errors in ventral striatum

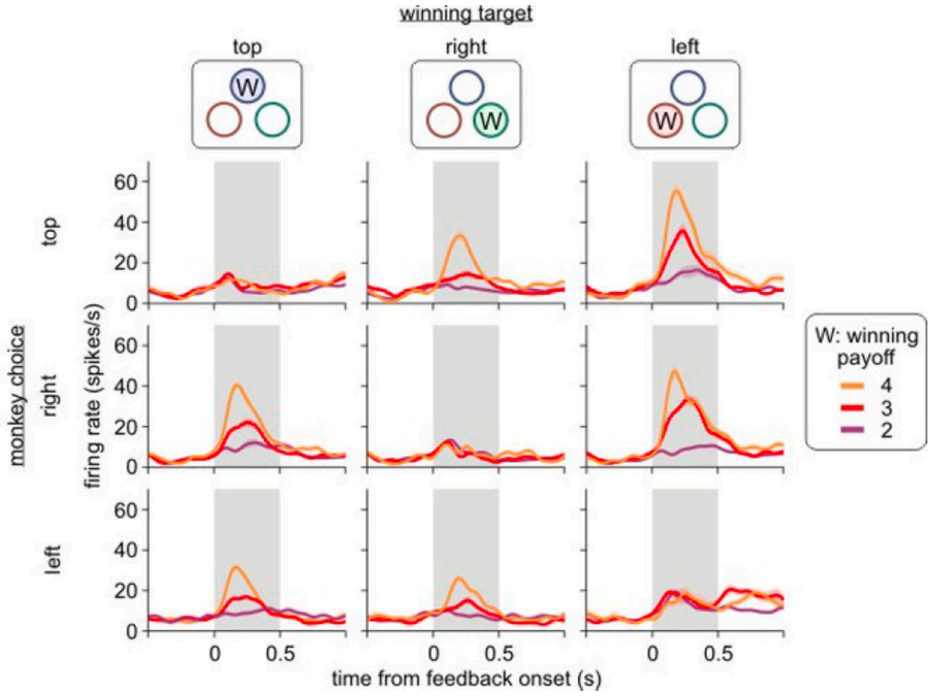


Model-based reasoning: counterfactuals

Some neurons in orbitofrontal cortex encode hypothetical outcomes:

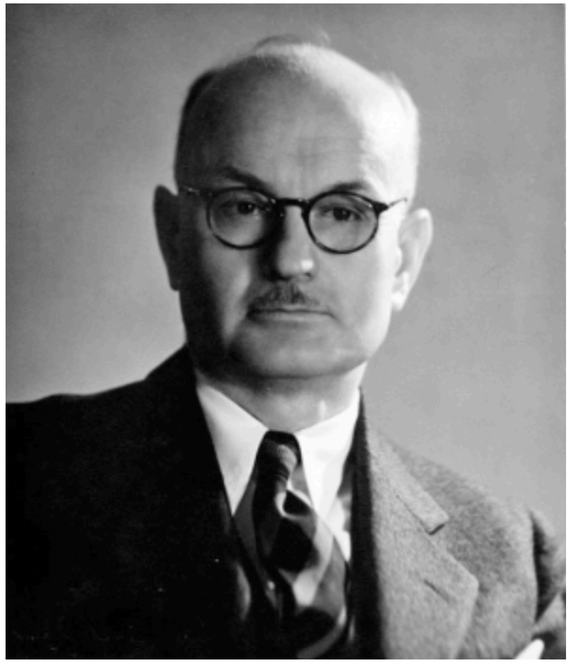
- Fire only if an *unchosen* option was rewarded

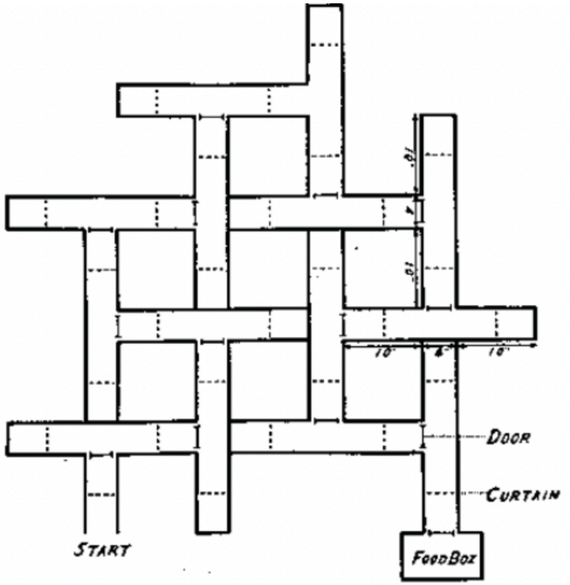




Abe & Lee, Neuron 2011
Lee et al., Annu Rev Neurosci. 2012

Cognitive maps for model-based RL?





Plan of maze
14-Unit T-Alley Maze

FIG. 1

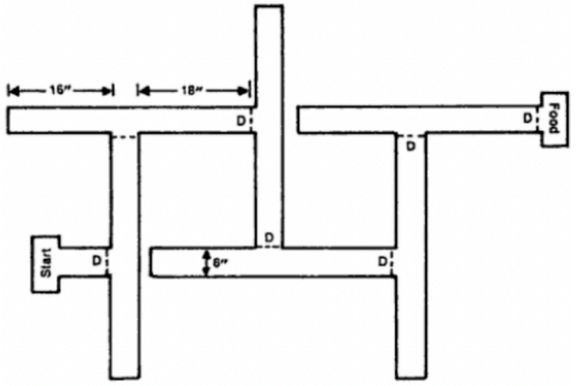
(From M. H. Elliott, The effect of change of reward on the maze performance of rats. *Univ. Calif. Publ. Psychol.*, 1928, **4**, p. 20.)

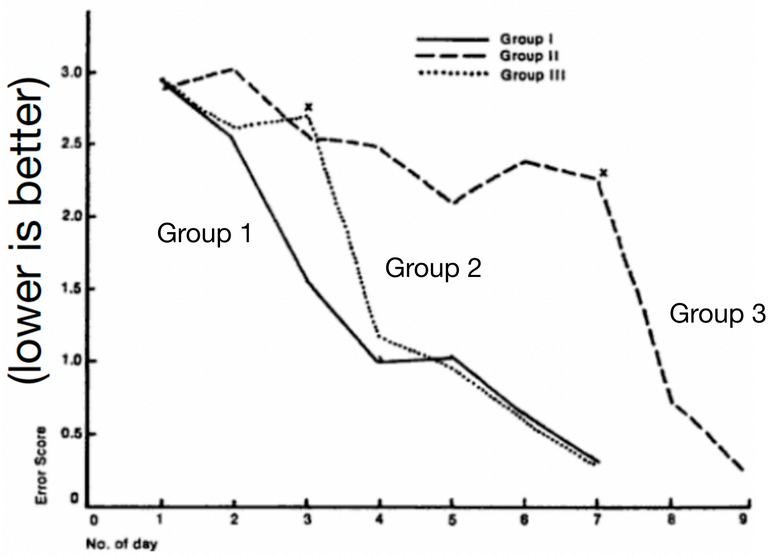
Tolman, Psychological Review, 1948

Cognitive maps for model-based RL?

Latent learning in RL: three groups of animals

- Group 1: always rewarded
- Group 2: reward added early
- Group 2: reward added late



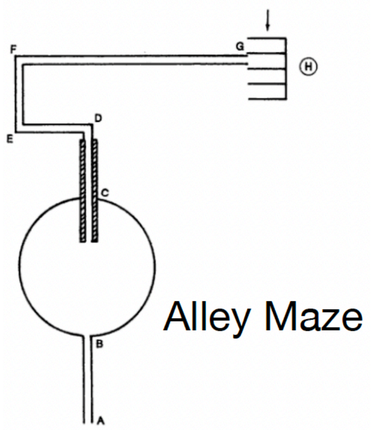


Key insight: whenever reward is added performance ‘catches up’ rapidly

- As if there is latent structural learning that can be used

Blodget, Univ. Calif. Publ. Psych., 1929

Cognitive maps for model-based RL?

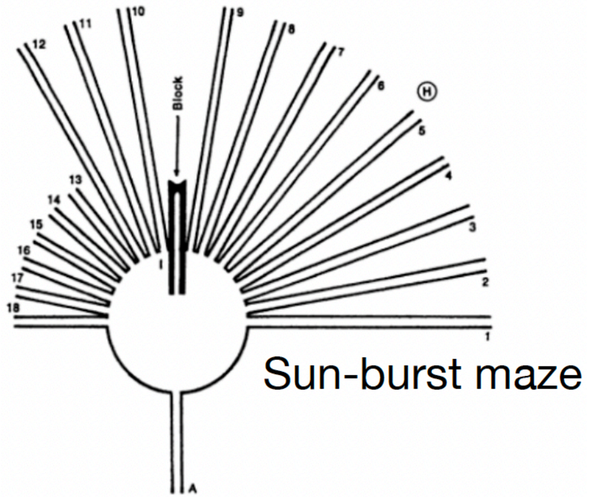


Alley Maze

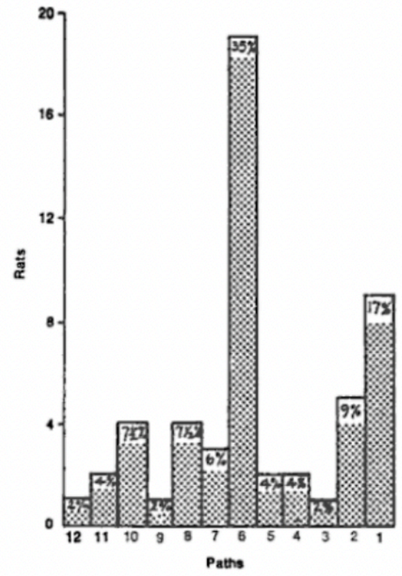
Rats are presented with a particular path to a goal G

When finding the original path blocked, rats choose the direct path to the goal in a ‘sun-burst maze’

Key insight: rats have a map of 2D space, and use it for model-based RL



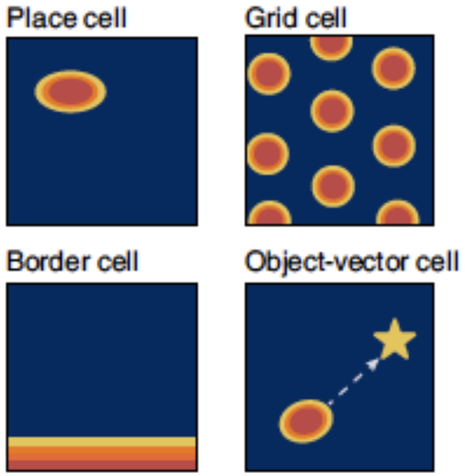
Sun-burst maze

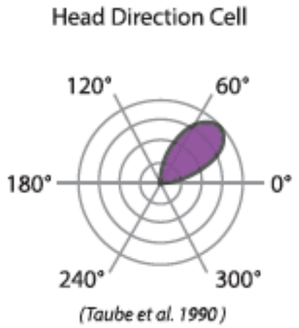


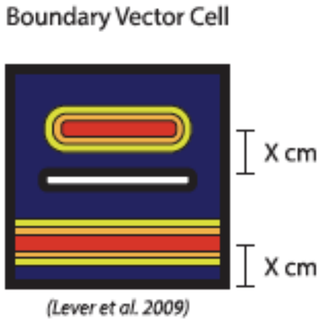
Tolman et al., Journ. Exp. Psych., 1946

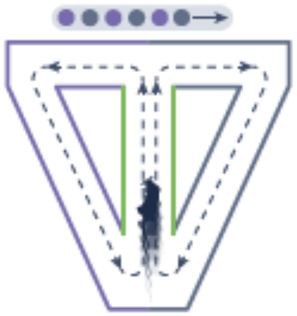
Cognitive maps for model-based RL?

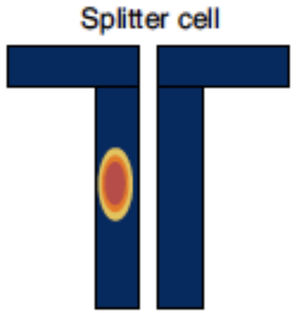
We have gained much insight into cognitive maps used in goal-directed navigation

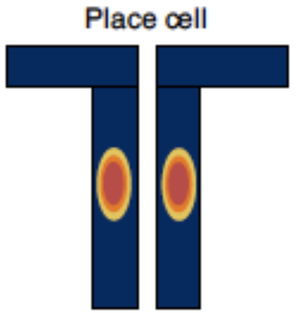












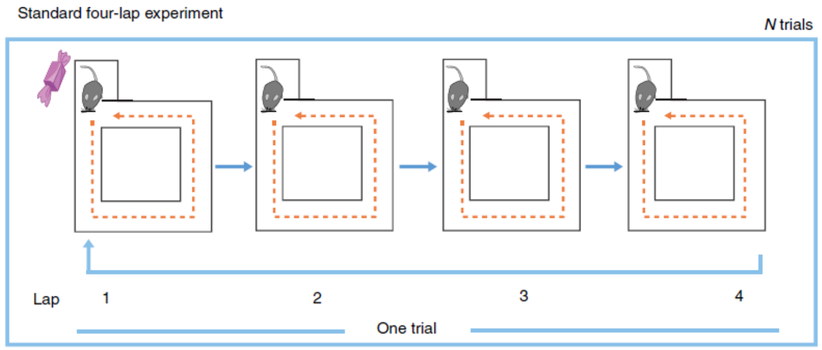
Is this a **basis set** over world structures?

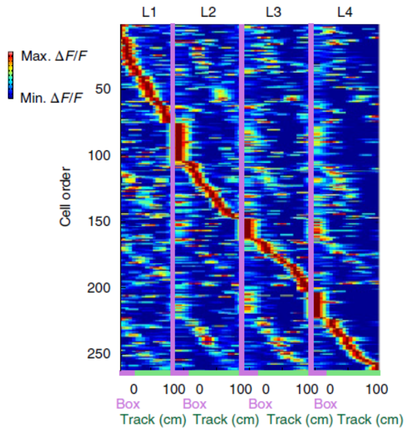
Whittington et al. (2022). How to build a cognitive map. Nature Neuroscience

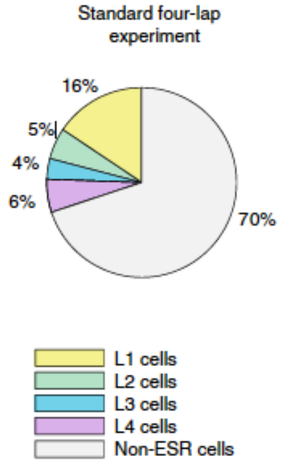
Behrens et al. (2018). What is a cognitive map? Organizing knowledge for flexible behavior. Neuron

A cognitive map for model-based RL - beyond space

Place cells for lap counting - relevant for reward:





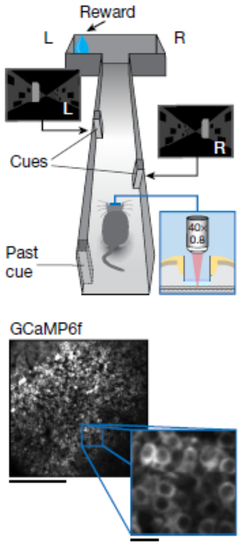


Important: cognitive maps might encode **task space**, rather than just physical space!

Sun et al. (2020). Hippocampal neurons represent events as transferable units of experience. Nature neuroscience

A cognitive map for model-based RL - beyond space

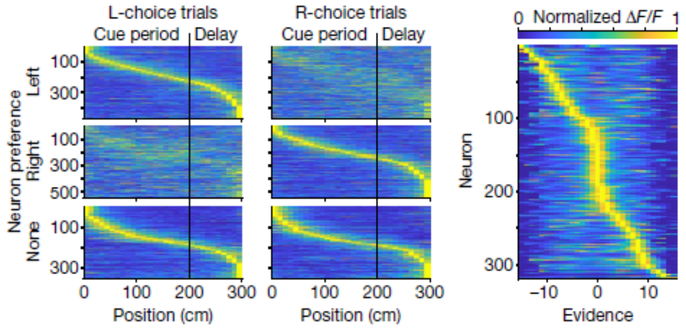
Place cells for evidence accumulation:



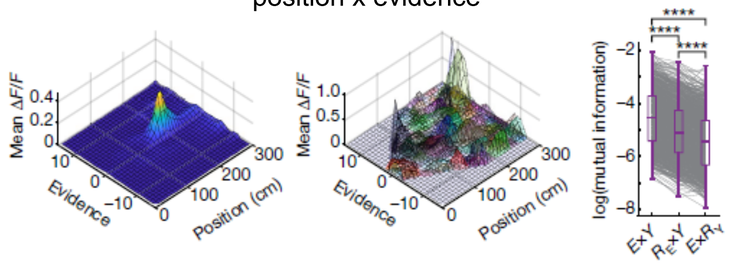
Cells in hippocampus (CA1) code for

Position and subsequent choice

Evidence



Neural code looks like a conjunction position x evidence



Important: cognitive maps might encode **task space**, rather than just physical space!

Nieh et al. (2021). Geometry of abstract learned knowledge in the hippocampus. Nature

Summary

Model-free association learning powerful framework for explaining biological behaviour

- Remarkable success in explaining neural phenomena
- Ressource-efficient but inflexible

Model-based RL: high flexibility but also costly

- Gold-standard probe: **de-valuation**
- Helps with **learning** and **planning/action selection**
- **Cognitive maps** as neural candidate for task model representation

Homework

Choose one of the following:

1. Reproduce a basic **DYNA-Q agent** in a maze task (with or without a shortcut)

   - Link

2. Reproduce the two step task and the stay probabilities for a model-free, model-based and mixed agent

   - Link

Thank you