Monochromatic light diffraction propagation via angular spectrum method

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Abstract

The study implements the angular spectrum method of propagation to simulate diffraction patterns of different input monochromatic wavefront. The solutions were calculated via a Fast Fourier Transform algorithm facilitated by the *diffractsim* Python library. Input monochromatic waves are composed by a choice of different aperture and phase masks. By varying the wavelength and screen distance as parameters, the study qualitatively explored their effects to the formed diffraction patterns, the phase profile, and the intensity profile at the observation plane z distance away from the input plane.

Keywords: wave diffraction, angular spectrum method, FFT

1 Introduction

There are different ways to calculate propagation of coherent optical fields. A common method is to use Huygen's principle. This is done so by considering each point in the field of the initial plane to act as a point source and emit a spherical wave. In the observation plane, the contributions are then added up from all the point sources to calculate the resulting wave. This principle gives us. the *Fresnel propagation* method. In the far field limit is the distance to the observation plane is increased to infinity, Fresnel propagation reduces to the *Fraunhofer propagation*. Such propagation reduces the phase factor to a constant and the propagated field is simply a fourier transform of the initial field. [1]

In this exploration, we use another method of propagation: the *angular spectrum propagation* method. Such propagation is done by decomposing the field present in the initial plane into a sum of 2D plane waves, propagating each plane waves to the plane of observation, and summing up over the resulting plane waves. Given a complex field u at the initial $z = 0$ plane expressed as

$$u(x', y', 0; t) = U(x', y', 0)e^{-i\omega t} \tag{1}$$

and defining the *angular spectrum* of $U(x, y, z)$ to be its 2D Fourier transform

$$A(k_x, k_y; z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(x, y, z)e^{-(k_x x+k_y y)i}\mathrm{d}x\mathrm{d}y \tag{2}$$

and, inversely,

$$U(x, y, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(k_x, k_y; z)e^{(k_x x+k_y y)y}\mathrm{d}k_x dk_y \tag{3}$$

which satisfies the Helmholtz equation with solution in $A(k_x, k_y; z)$ given by

$$A(k_x, k_y; z) = A(k_x, k_y; 0)e^{-ik_z z} \qquad k_z = \sqrt{k^2 - k_x^2 + k_y^2} \tag{4}$$

The disturbance at $z = -L$ can be written as

$$U(x, y, -L) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(k_x, k_y; 0)e^{k_z L}e^{(k_z x+k_y y)y}\mathrm{d}k_x dk_y \tag{5}$$

To represent the shape of the aperture, we can define the amplitude transmittance function $t_A(x', y')$ as a binary variable $t_A \in \{0, 1\}$ denoting block or pass states, respectively. That is,

$$U(x', y', 0) = U_0(x', y', 0)t_A(x, y) \tag{6}$$

In theory, we simply need to solve the propagated complex field (Eq. (5)) via the initial angular spectrum (Eq. (2)) which can be found from the Fourier transform of U coupled with its corresponding amplitude transmittance function (Eq. (6)). For simulation purposes, we resort to numerical discretization schemes involving Fast Fourier Transform algorithms to solve these equations.

2 Methodology

The paper aims to investigate the variations in the diffraction pattern, its phase portrait, and its intensity portrait as we vary the input intensity, input phase, wavelength of the wave, and the screen distance from the initial plane to the observation plane. For computation purposes, the *diffractsim* Python library (link at the appendix) contains the necessary algorithm to calculate the system of equations via the Fast Fourier Transform algorithm which builds upon the *numpy* and *scipy* Python scientific packages.

We use a set of five different images for different purposes: as phase masks or as intensity masks/apertures. The first part of the simulation fixes the phase mask of the input plane to be a single image of choice while varying the aperture, screen distance, and wavelength. Likewise, the second half of he simulation fixes the aperture of the input plane to be a a single image of choice while also varying the aperture, screen distance, and wavelength.

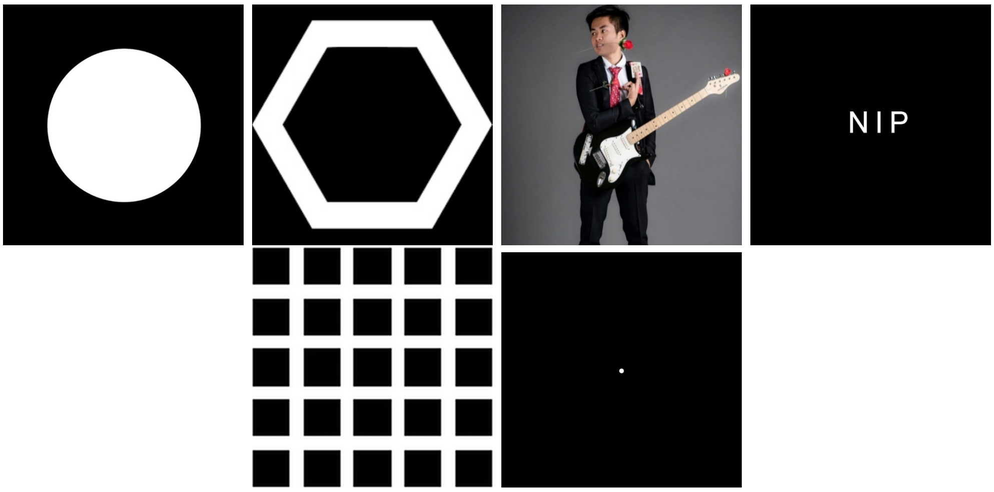

Figure 1: From left to right, then top to bottom: Images are referred to as Image A, Image B, Image C, Image D, Image E, and Image F, respectively

The output images consist (i) diffraction pattern with color corresponding to the diffracted wavelength, (ii) phase map of the resulting wave at the observation plane, and (iii) intensity map of the resulting wave at the the observation plane coupled with a intensity plot profile of the $y = 0$ slice. The latter two is color-mapped appropriate with a corresponding color bar scale to denote either the phase (running from $-\pi$ to π) or the square root of the intensity.

3 Simulation Images

This section simulates the propagation of wave using Image B, Image F, and Image E, respectively in the following sections, as aperture while fixing Image C as the phase mask. The screen distance levels used are (i) 0 cm, (ii) 35 cm, (iii) 70 cm and the wavelength levels used are (i) 400 nm, (ii) 500 nm, and (iii) 600 nm, respectively in the following sections. For the fixed setup, we use a constant 532.8 nm wavelength.

3.1 Amplitude 1

3.1.1 Varying Screen Distance

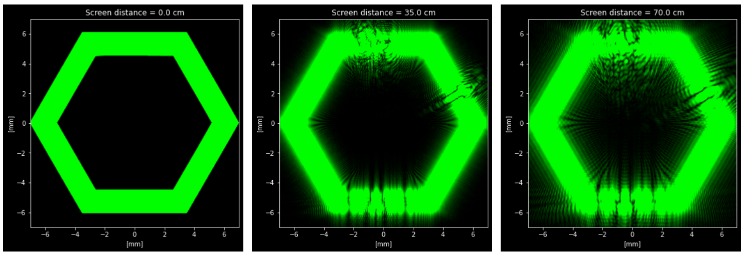

Figure 2: Simulation shows diffraction pattern using Image B as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

Observe that the effects of the phase mask shifts the initial phase of the incoming wave such that as the screen distance increases, there exists an additional diffracting behvaior along the perimeter of the phase mask.

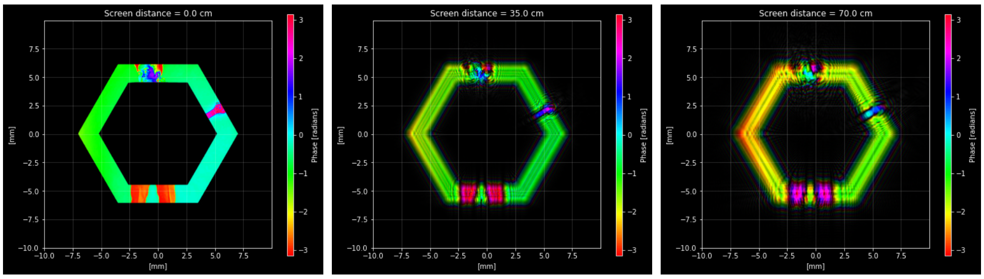

Figure 3: Simulation shows phase profile using Image B as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

From the phase profiles of the hexagon, a uniform oscillating phase shift can be seen along the area not affected by the phase masks. The area affected by the phase mask were also phase-shifted by a uniform value along with a rippling behavior.

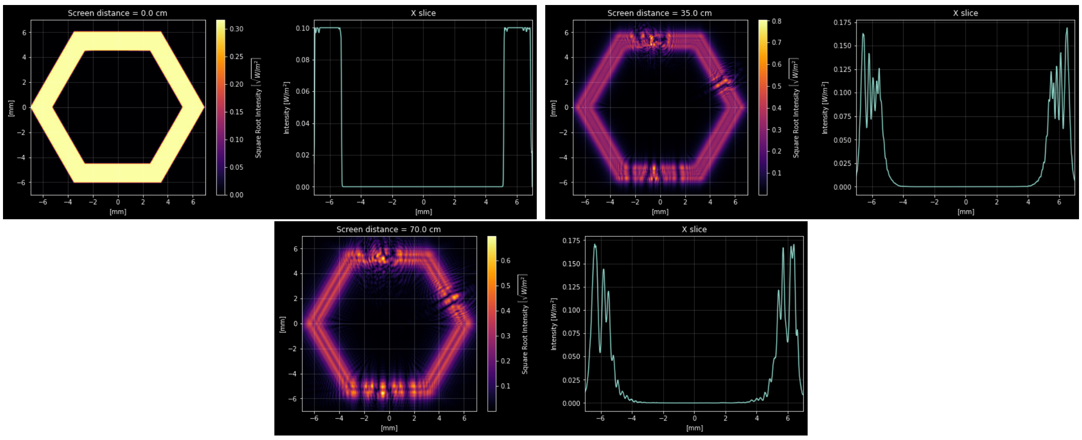

Figure 4: Simulation shows intensity profile using Image B as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

The phase mask also causes a rippling effect in the intensity domain. The intensity profile at the $y = 0$ slice shows that diffraction causes the constant intensity of the hexagon to decrease and spread out into a rippled wave behavior - characteristic of the wavy diffraction pattern.

3.1.2 Varying Wavelength

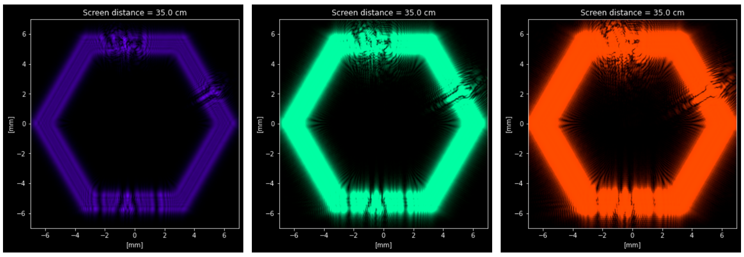

Figure 5: Simulation shows diffraction pattern using Image B as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

Simulation shows that increasing the wavelength increases the amount of diffractive ripple effects that can be observed on the observation plane.

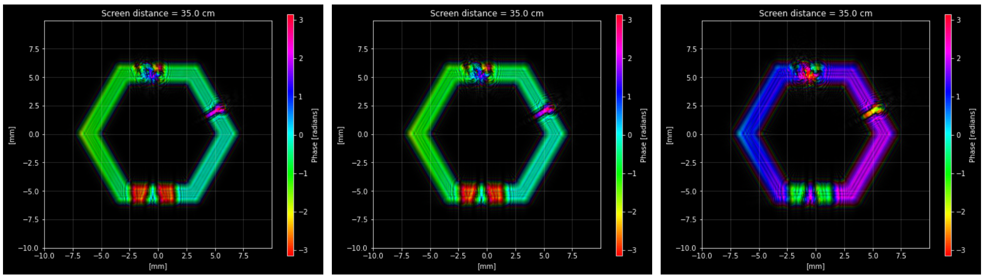

Figure 6: Simulation shows phase profile using Image B as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

It can also be observed that the wavelength of light propagated also dictates how much the phase generally shifts as the wave is diffracted.

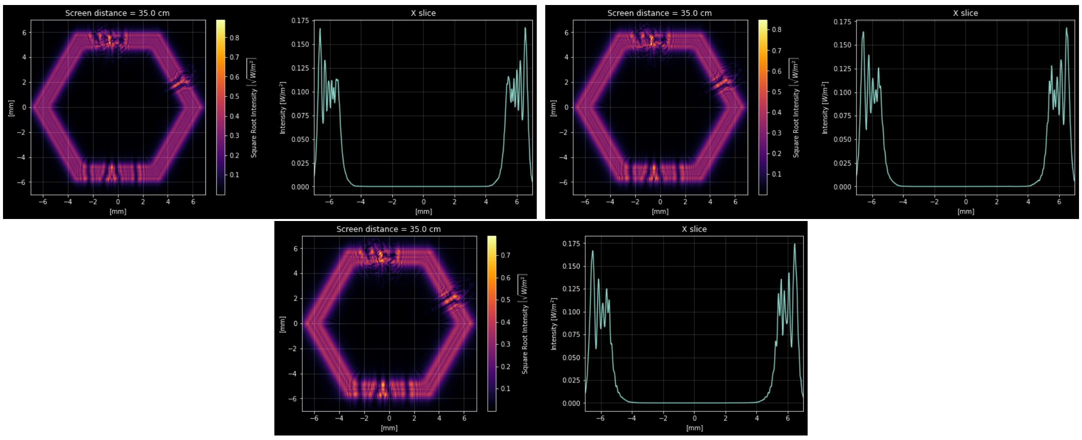

Figure 7: Simulation shows intensity profile using Image B as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

This simulation shows no general observable effects of varying monochromatic light wavelength to the intensity profile of the diffracted wave. Although not immediately noticeable, some diffractive behavior can be shown as the sinusoidal shape of the intensity plot is altered.

3.2 Amplitude 2

Throughout the rest of the simulation, we merely replicate the variation of parameters using different image samples.

3.2.1 Varying Screen Distance

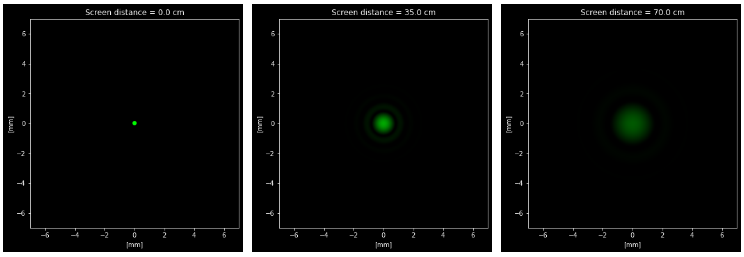

Figure 8: Simulation shows diffraction pattern using Image F as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

Diffraction causes the point sources to diffuse out in a spherical fashion.

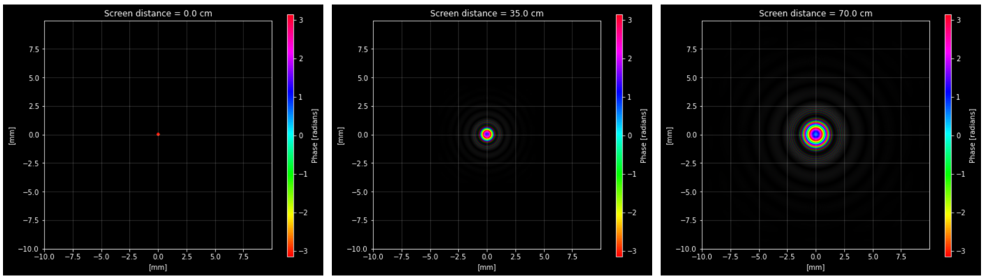

Figure 9: Simulation shows phase profile using Image F as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

Analysis of the phase profile shows a characteristic spherical wavefront of periodic phase behavior. Increasing the screen distance introduces more "rings" in the phase portrait signifying greater variations in the phase radially.

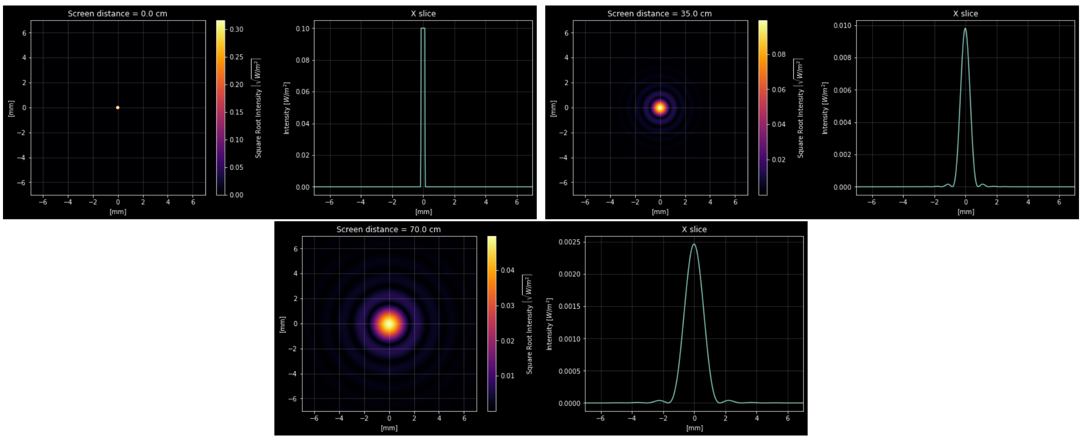

Figure 10: Simulation shows intensity profile using Image F as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

Both the 2D and 1D slice of the intensity profile and the phase profile show the characteristic diffraction images of a point source: the *Airy disk* diffraction pattern.

3.2.2 Varying Wavelength

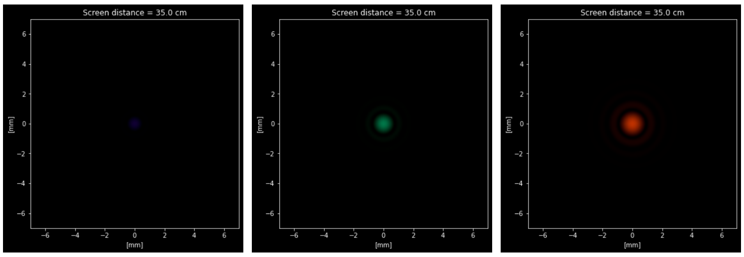

Figure 11: Simulation shows diffraction pattern using Image F as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

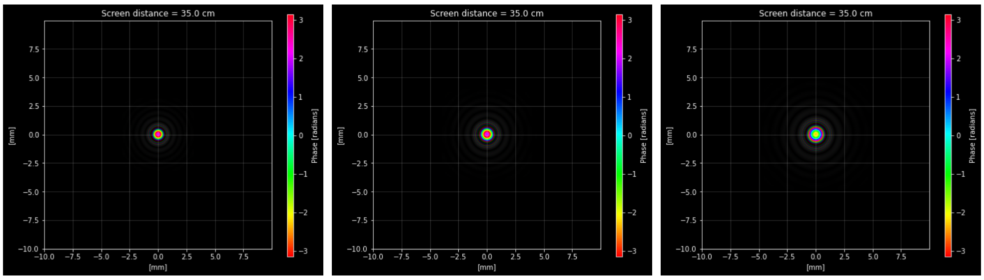

Figure 12: Simulation shows phase profile using Image F as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

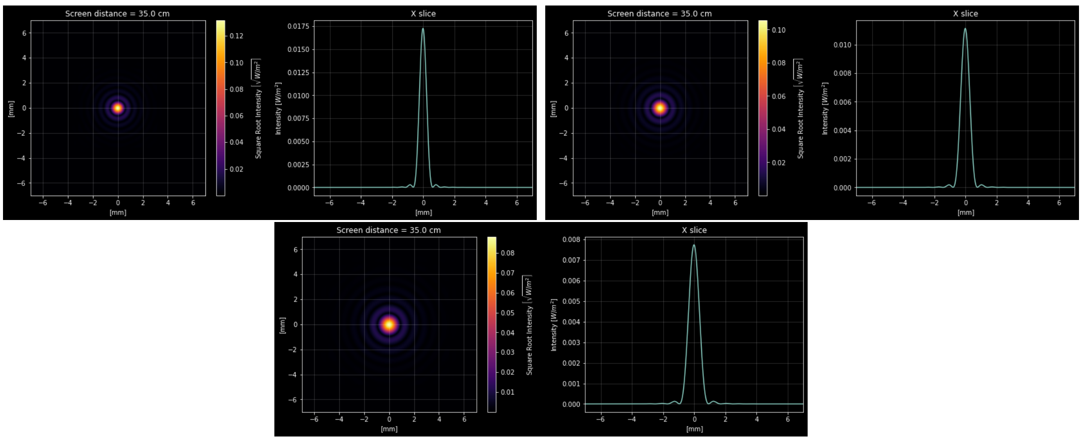

Figure 13: Simulation shows intensity profile using Image F as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

Increasing the wavelength increases the prominence of diffraction in the formed image, shifts the phase portrait of the observed image, and introduces diffractive behavior to the intensity by lowering the peak of the $y = 0$ slice profile and making the shape wider.

At this point, we have already covered the main effects on the output (pattern, phase, intensity) from the input (screen distance and wavelength). The rest of the section shows additional resources exhibiting the behavior of the previously discussed observations.

3.3 Amplitude 3

This section simulates the propagation of wave using Image B, Image F, and Image E, respectively in the following sections, as aperture while fixing Image C as the phase mask. The screen distance levels used are (i) 0 cm, (ii) 35 cm, (iii) 70 cm.

3.3.1 Varying Screen Distance

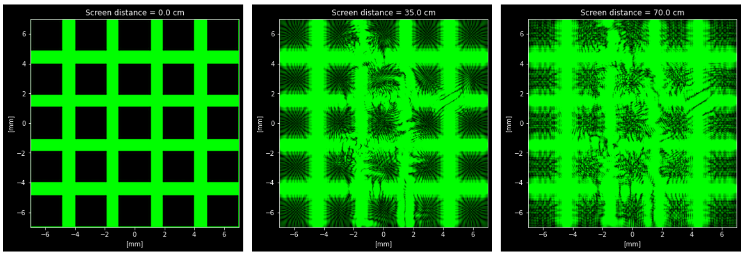

Figure 14: Simulation shows diffraction pattern using Image E as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

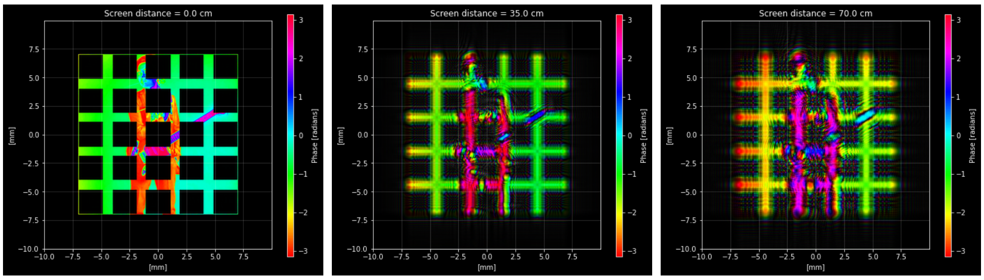

Figure 15: Simulation shows phase profile using Image E as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

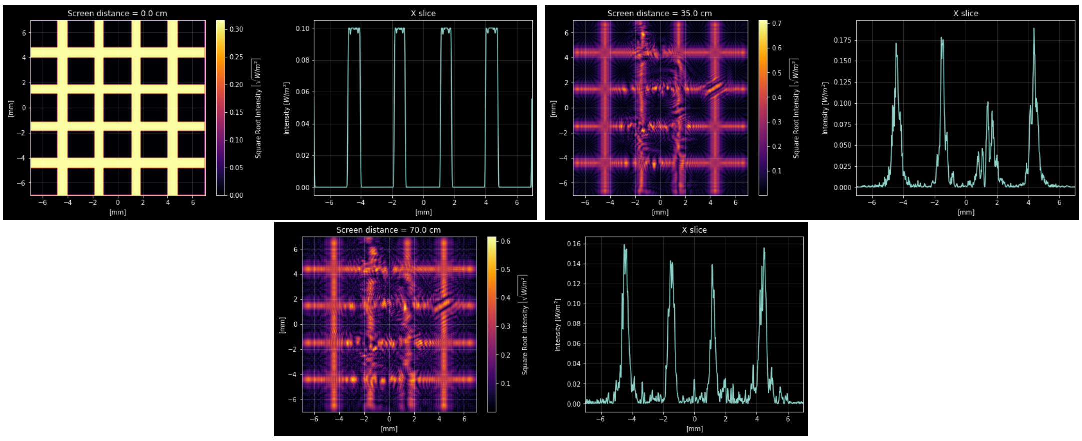

Figure 16: Simulation shows intensity profile using Image E as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

3.3.2 Varying Wavelength

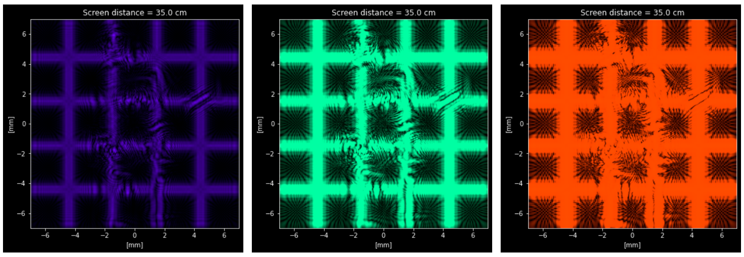

Figure 17: Simulation shows diffraction pattern using Image E as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

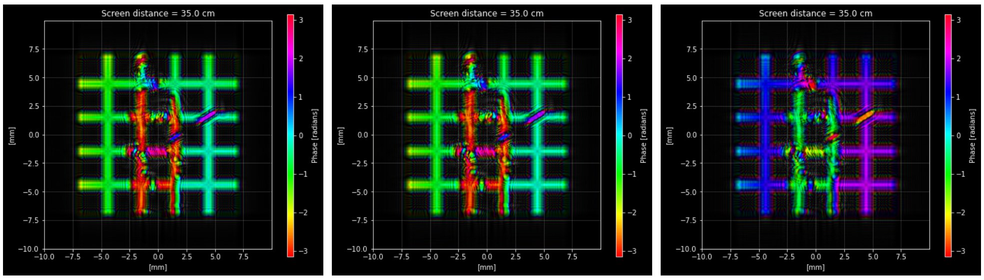

Figure 18: Simulation shows phase profile using Image E as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

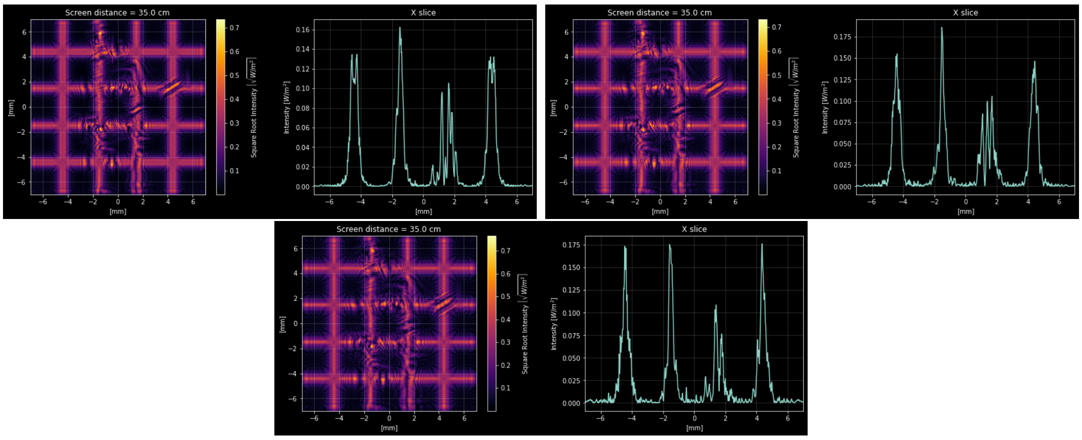

Figure 19: Simulation shows intensity profile using Image E as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

This section simulates the propagation of wave using Image C, Image E, and Image D, respectively in the following sections, as phase masks while fixing Image A as the aperture. The screen distance levels used are (i) 0 cm, (ii) 35 cm, (iii) 70 cm and the wavelength levels used are (i) 400 nm, (ii) 500 nm, and (iii) 600 nm, respectively in the following sections.

3.4 Phase 1

3.4.1 Varying Screen Distance

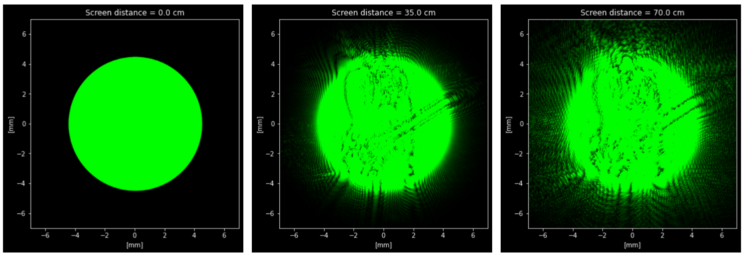

Figure 20: Simulation shows diffraction pattern using Image A as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

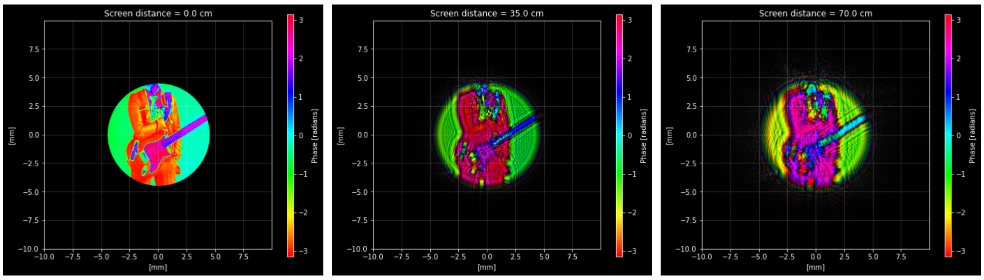

Figure 21: Simulation shows phase profile using Image A as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

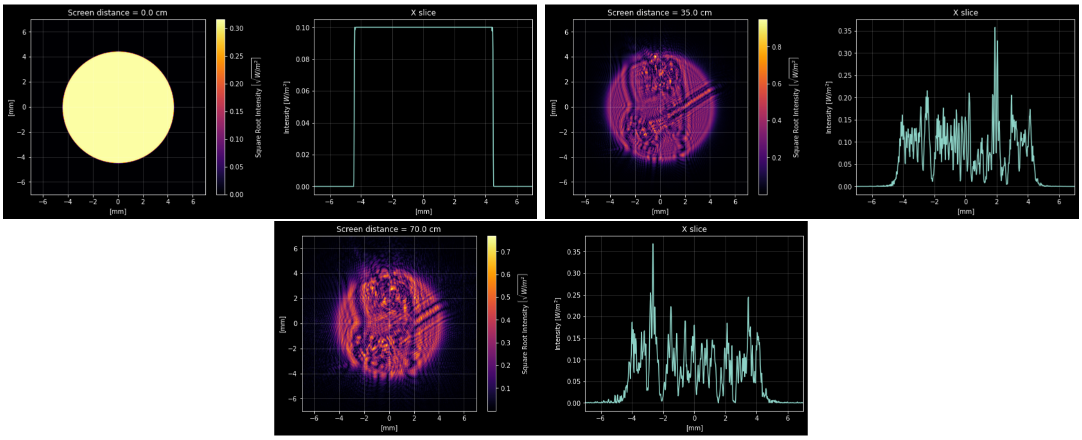

Figure 22: Simulation shows intensity profile using Image A as aperture and Image C as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

3.4.2 Varying Wavelength

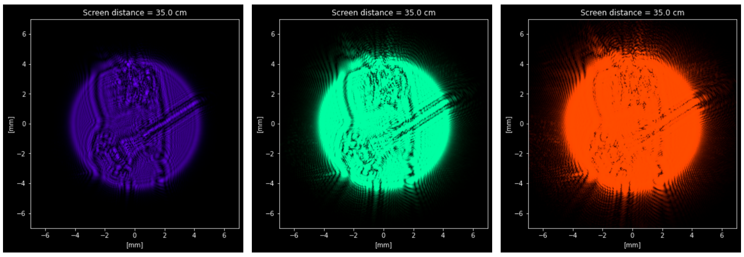

Figure 23: Simulation shows diffraction pattern using Image A as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

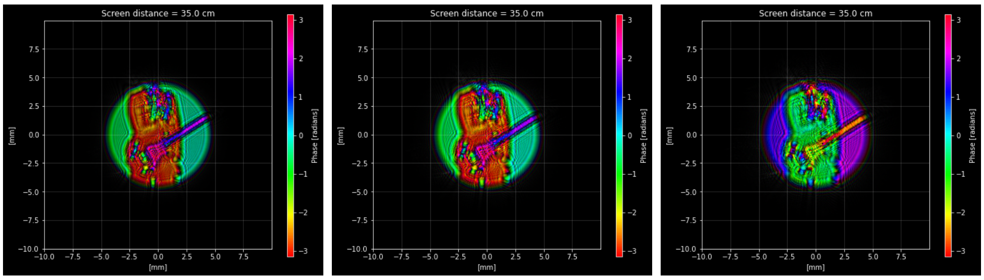

Figure 24: Simulation shows intensity profile using Image A as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

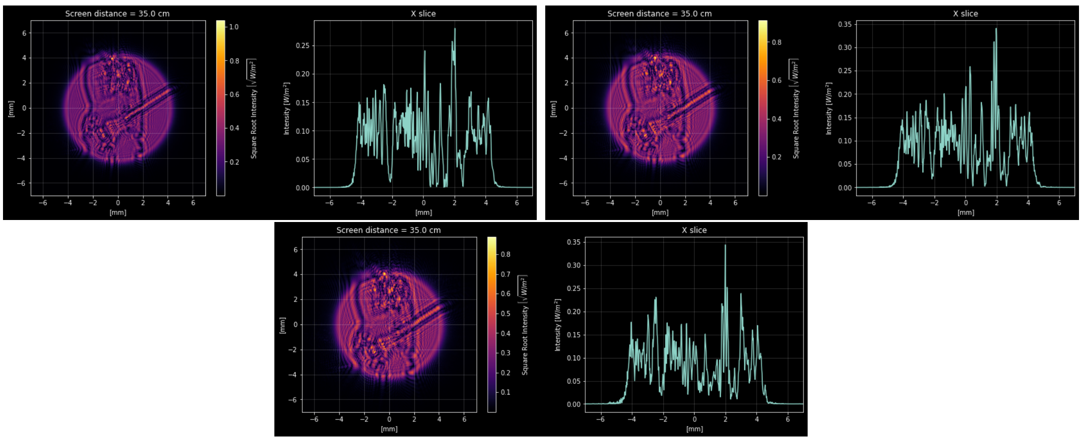

Figure 25: Simulation shows diffraction pattern using Image A as aperture and Image C as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

3.5 Phase 2

3.5.1 Varying Screen Distance

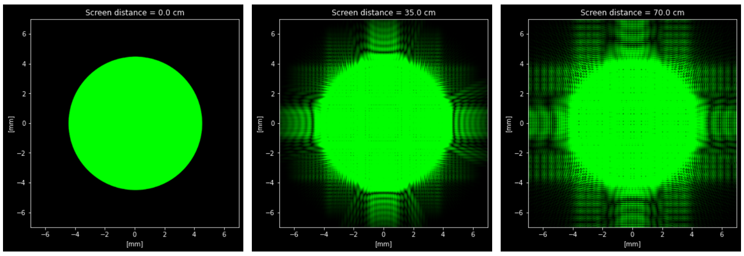

Figure 26: Simulation shows diffraction pattern using Image A as aperture and Image E as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

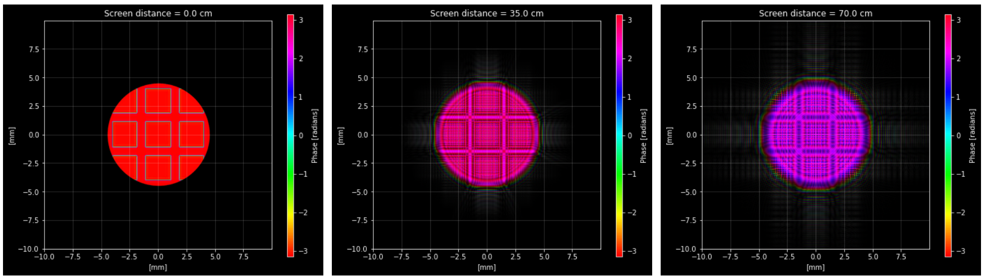

Figure 27: Simulation shows phase profile using Image A as aperture and Image E as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

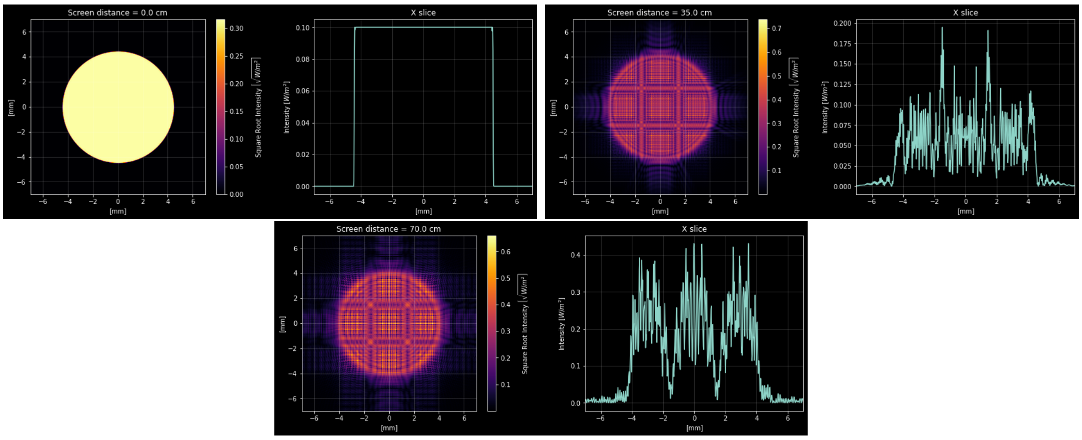

Figure 28: Simulation shows intensity profile using Image A as aperture and Image E as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

3.5.2 Varying Wavelength

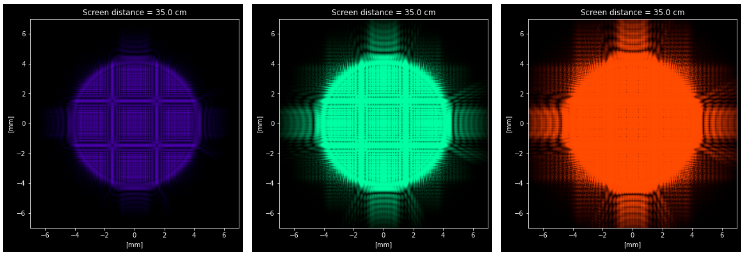

Figure 29: Simulation shows diffraction pattern using Image A as aperture and Image E as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

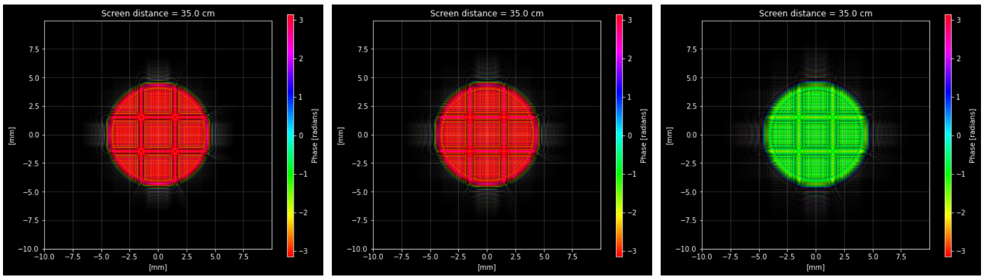

Figure 30: Simulation shows phase profile using Image A as aperture and Image E as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

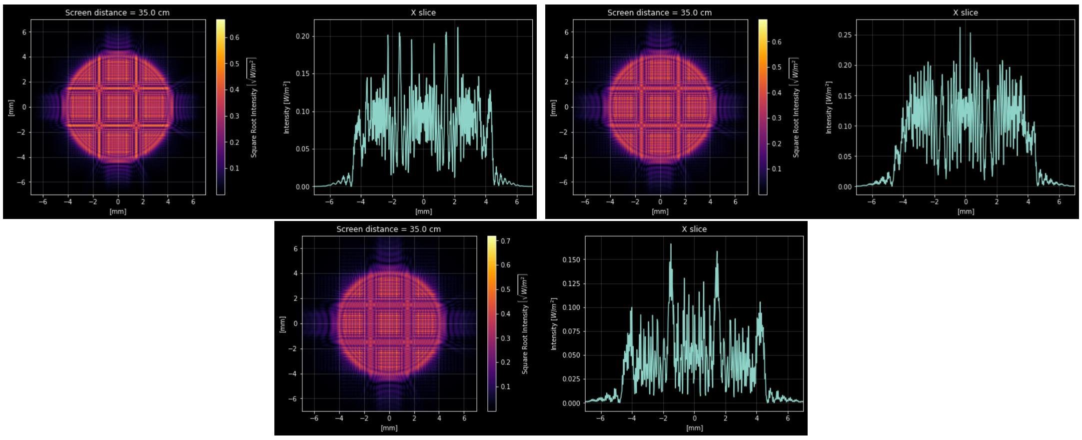

Figure 31: Simulation shows intensity profile using Image A as aperture and Image E as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

3.6 Phase 3

3.6.1 Varying Screen Distance

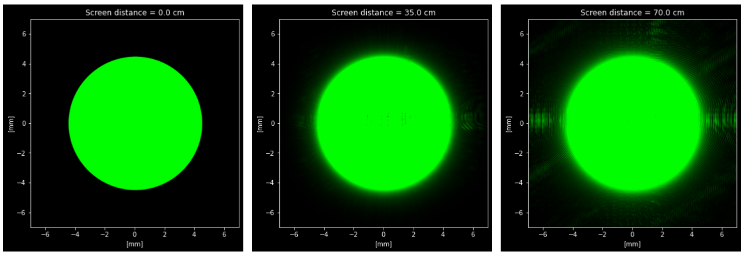

Figure 32: Simulation shows diffraction pattern using Image A as aperture and Image D as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

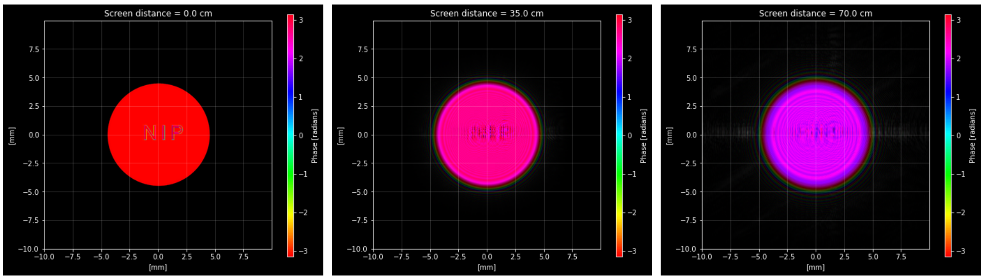

Figure 33: Simulation shows phase profile using Image A as aperture and Image D as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

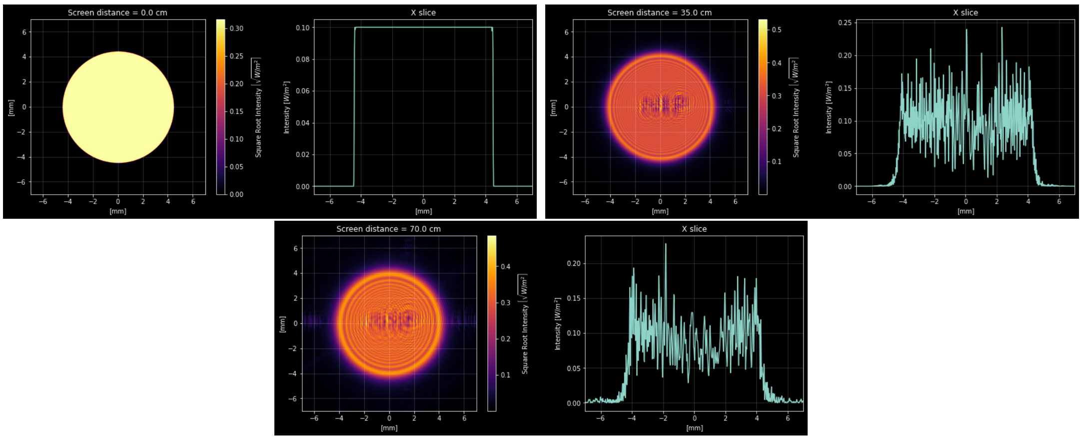

Figure 34: Simulation shows intensity profile using Image A as aperture and Image D as phase mask diffracted at 0 cm, 35 cm, 70 cm, from left to right.

3.6.2 Varying Wavelength

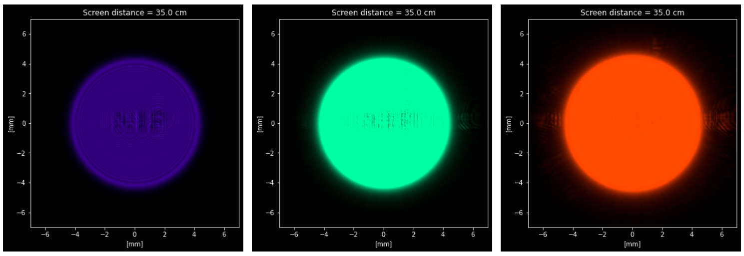

Figure 35: Simulation shows diffraction pattern using Image A as aperture and Image D as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

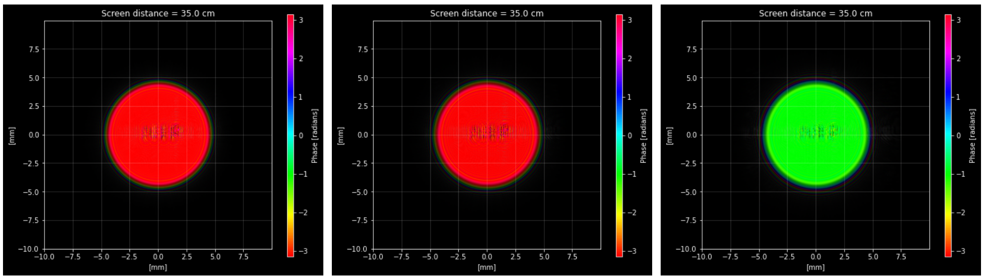

Figure 36: Simulation shows phase profile using Image A as aperture and Image D as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

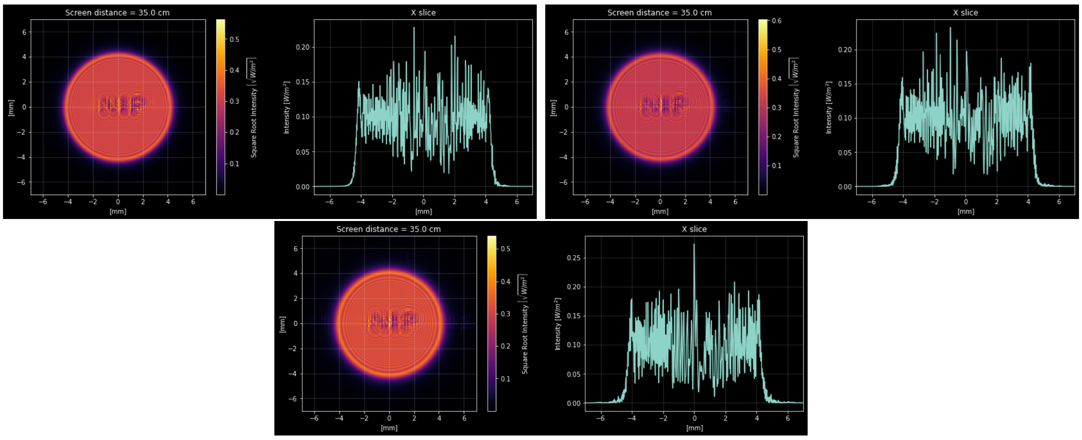

Figure 37: Simulation shows intensity profile using Image A as aperture and Image D as phase mask diffracted at wavelengths 400 nm, 500 nm, and 600 nm, from left to right.

4 Conclusions

Thee study utilizes a qualitative approach using the angular spectrum method solved via numerical approach to explore the effects of screen distance and wavelengths to diffraction effects on the diffraction pattern, phase profile, and intensity profile of the wavefront formed from the combination of different images serving as intensity and phase masks. In general, the following observations are common across all simulation samples. Screen distance generally increases the diffraction ripple effect on the formed image at the observation plane. The corresponding phase map increases in amount of variations spatially and the intensity profile becomes stouter - overall magnitude is decreased, the shape becomes wider and sinusoidal patterns emerge. Meanwhile, increasing the wavelength of the incoming monochromatic light increases the amount of diffraction ripple effects on the formed pattern made at a constant screen distance, produces an overall phase shift, and introducing an inconclusive modification on the intensity profile.

References

[1] J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill Companies, United States of America, 1988).

5 Appendix

All simulation image samples,code, project version histories, and other relevant files used can be found here: `https://github.com/schwarzschlyle/applied-optics`

The diffractsim Github repository where the numerical algorithms used in solving the equations of the angular spectrum method can be found here: `https://github.com/rafael-fuente/diffractsim`