

$$01. \sum_{k=1}^n (c \cdot a_k + b_k) = c \cdot \sum_{k=1}^n a_k + \sum_{k=1}^n b_k \rightarrow \sum_{k=1}^n \Theta[f(k)] = \Theta\left[\sum_{k=1}^n f(k)\right]$$

$$02. \sum_{k=1}^n k = \frac{n(n+1)}{2} \quad 03. \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \quad 04. \sum_{k=1}^n k^3 = \left[\sum_{k=1}^n k\right]^2 = \frac{n^2(n+1)^2}{4}$$

$$05. \sum_{k=1}^n k^4 = \frac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30} \quad 06. \sum_{k=1}^n k^p \approx \Theta(N^{p+1})$$

$$07. \sum_{k=0}^n x^k = \frac{x^{n+1} - 1}{x - 1} \quad (\text{PG finita, razão } x \neq 1) \quad 08. \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad (\text{PG infinita, razão } |x| < 1)$$

$$09. \sum_{k=1}^n \frac{1}{k} = \ln(n) + O(1) \quad 10. \sum_{k=1}^{\infty} k \cdot x^k = \frac{x}{(1-x)^2} \quad \text{para } |x| < 1$$

$$11. \sum_{k=1}^n k \cdot x^k = \frac{x - (n+1)x^{n+1} + n \cdot x^{n+2}}{(1-x)^2} \quad \text{para } x \neq 1$$

$$12. \sum_{k=1}^n k \cdot 2^k = (n-1) \cdot 2^{n+1} + 2 \quad 13. \sum_{k=1}^{n-1} \frac{1}{k(k+1)} = \sum_{k=1}^{n-1} \frac{1}{k} - \sum_{k=1}^{n-1} \frac{1}{k+1}$$

$$14. a) \sum_{k=1}^n \binom{n}{k} = 2^n \quad b) \sum_{k=1}^n \binom{k}{m} = \binom{n+1}{m+1} \quad c) \max_{1 \leq k \leq n} \left\{ \binom{n}{k} \right\} = \binom{n}{\lceil n/2 \rceil}$$

$$15. n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} - \frac{571}{2488320n^4} + \dots\right) \quad (\text{Stirling series})$$

$$16. S(n) = c \cdot S(n-1) + g(n) \rightarrow S(n) = c^{n-1} S(1) + \sum_{i=2}^n c^{n-i} g(i) \quad \leftarrow \text{RECORRÊNCIAS}$$

$$17. T(n) = a \cdot T(n/b) + f(n) \rightarrow T(n) = \Theta\left(n^{\log_b a}\right) + \sum_{i=0}^{h-1} a^i f\left(\frac{n}{b^i}\right) \quad \text{para } h = \log_b n$$

Master theorem If $T(n) = aT(\lceil n/b \rceil) + O(n^d)$ for some constants $a > 0$, $b > 1$, and $d \geq 0$, then: ➡

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$$T(n) = \begin{cases} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a \end{cases}$$

18. Equação homogênea característica:

$$a_0 T(n) + a_1 T(n-1) + a_2 T(n-2) + \dots + a_k T(n-k) = 0$$

Raízes distintas: $r_1 \neq r_2 \neq r_3 \dots \neq r_p \rightarrow T(n) = c_1 r_1^N + c_2 r_2^N \dots c_p r_p^N$

Raízes múltiplas: $r_1 = r_2 = r_3 \dots = r_p \rightarrow T(n) = c_1 r_1^N + c_2 N r_1^N \dots c_p N^{p-1} r_p^N$

Raízes Complexas Distintas: $r = a \pm bi \rightarrow T(n) = c \cdot p^N$ onde $p = \sqrt{a^2 + b^2}$

Raízes complexas múltiplas: $T(n) = c_1 p^N + c_2 N p^N$ onde $p = \sqrt{a^2 + b^2}$