Theory of narrow-band mode-locking

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Abstract

- It should be emphasized that the mode locking of lasers can be thought as two types of balance: The balance between pulse broadening and pulse shortening; the balance between the phase shift by GVD and one by nonlinearity (self-phase modulation).
- If the former balance is not obtained, the mode locking pulses will not occur; if only the former balance is obtained, the mode-locking pulses can occur but will be unstable; if both balances are obtained, the stable mode locking pulse will occur.

Abstract

- Basically, Stark effect makes multiple longitudinal modes oscillating simultaneously become possible.
- What' s more, gain-line splitting (Stark splitting) induced by an intra-cavity laser field can form a dip or dips in gain-line shape, with which a balance between pulse broadening and pulse shortening can be achieved. That is to say, the first balance is obtained and the gain medium with such a gain-line shape can produce short pulses. [1986, J. J. Sanchez-Mondragon]
- Since the Stark splitting is symmetric, there is no additional group-velocity dispersion (GVD). The SML pulses without dispersion compensation will occur in those lasers whose gain media have a dip in the gain-line shape, if the nonlinearity and GVD of the host materials can be neglected.

[1986, J. J. Sanchez-Mondragon]. "Pulse compression caused by a spectral hole in an inhomogeneously broadened line of an amplifier", Centro de Investigaciones en Optica, *A.* C., Apdo. *Postal* 948, Leon, Gto. *37000,* Mexico

Abstract

- Narrow gain width makes mode-locking easier (so it can start mode-locking by itself): stronger intra-cavity laser field owing to larger gain can generate frequency shift of the Stark splitting, which is beneficial to mode-locking.
- Since the gain width is small, the allowed oscillation frequencies are limited, which means less longitudinal laser modes would anticipate in mode-locking, leading to a possibly less stable mode-locking state with a larger pulse width in the time domain compared with that of large gain width laser gain material.

Abstract

- By increasing the laser cavity length, thus reducing the fundamental frequency of the cavity, more stable mode-locking lasers could be attained with shorter pulse width in the time domain.
- The above statement is true on condition that the pulses would be sustained in the extended long laser cavity. Fortunately, that is true for our Pr:YLF solid-state laser. The extended laser cavity is in the air, there should be no phase shift by GVD or by nonlinearity (self-phase modulation).

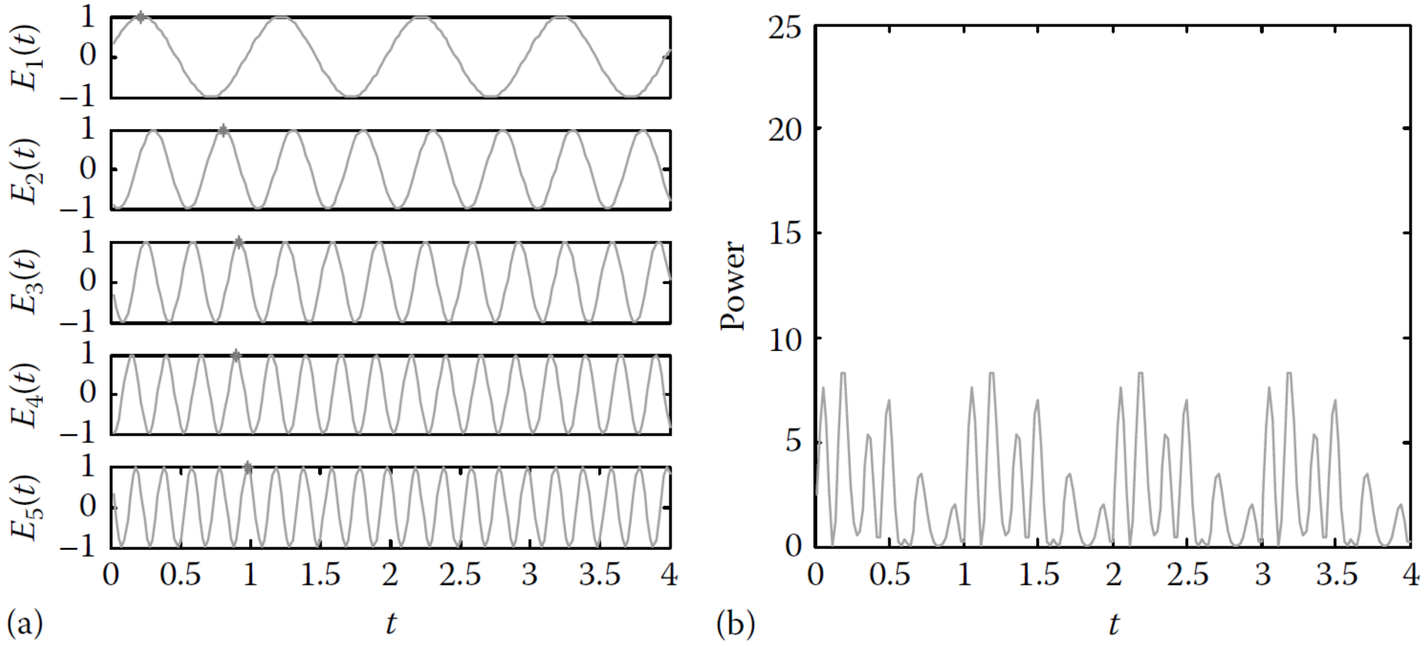
1. Mode-Locking Theory

a laser can oscillate at a number of resonant frequencies whose spacing is equal to the fundamental frequency ω_R of the cavity:

$$\omega_i - \omega_{i-1} = \frac{\pi c}{L} = \omega_R$$

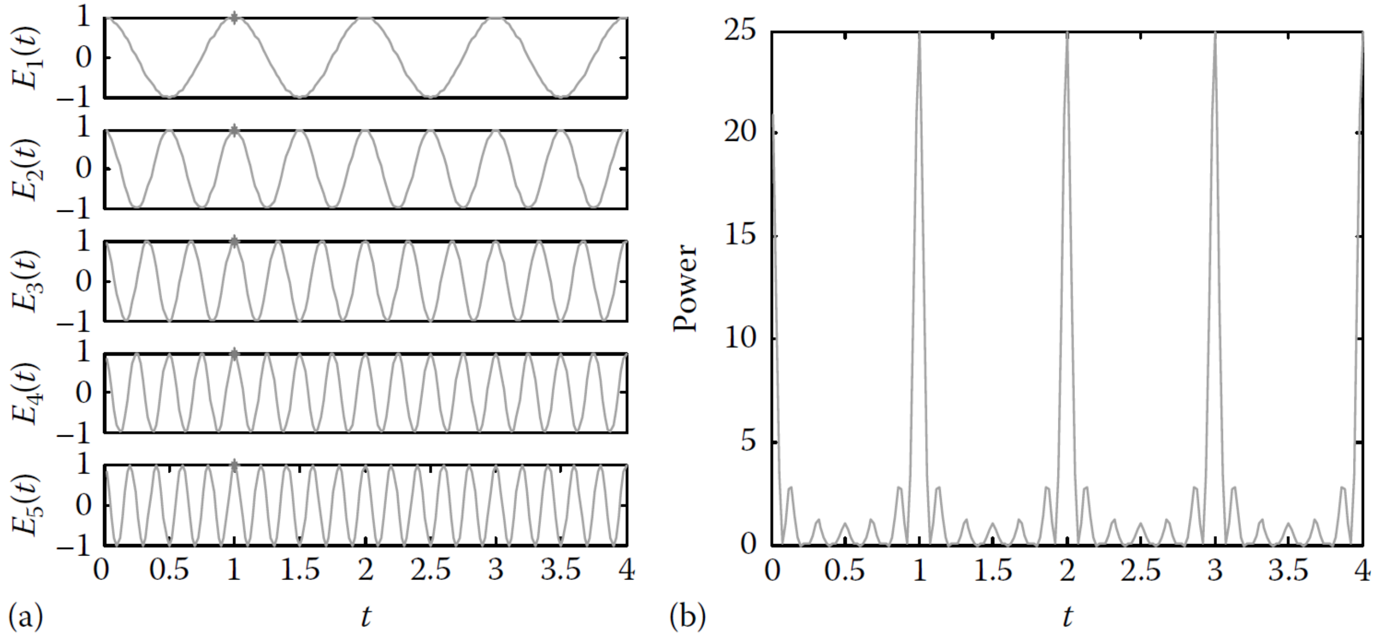
The output electric field of the generated lightwave in the temporal domain is the summation of all the oscillating modes given as

$$e(t) = \sum_n E_n e^{j\left[\left(\omega_0 + n\omega_R\right)t + \Phi_n\right]}$$



when the modes are forced to lock together, that means all the N modes are either in phase or different in a multiple number of 2π, E_n
and Φn will be constants. The simplest case is when E_n = 1, and Φ_n = 0

$$e(t) = \cos \omega_0 t \frac{\sin(N\omega_R t / 2)}{\sin(\omega_R t / 2)}$$



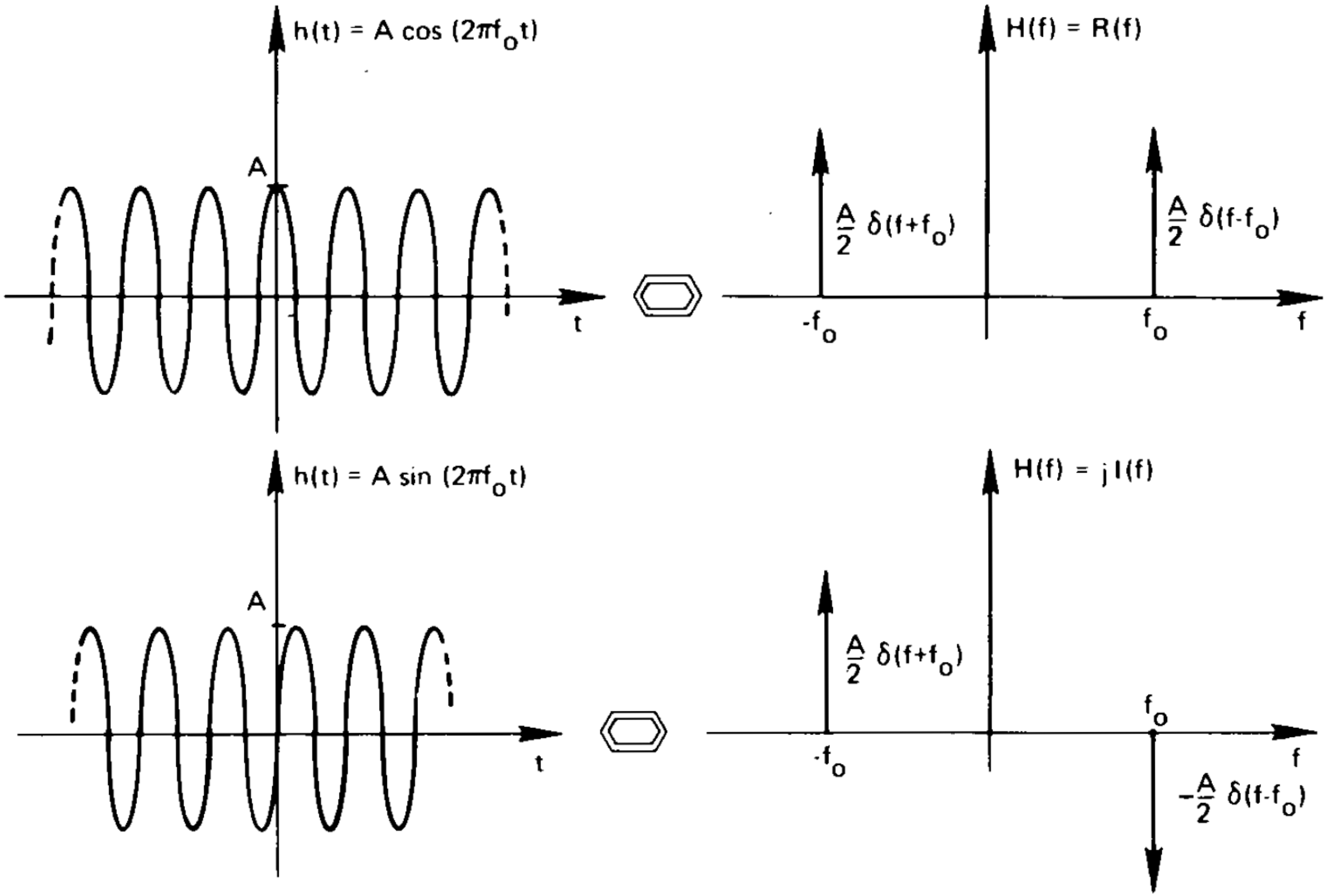
which is an oscillation at frequency ω_0 modulated with the *sinc* envelope function. The average power is thus given by

$$P(t) \propto \frac{\sin^2(N\omega_R t / 2)}{\sin^2(\omega_R t / 2)}$$

The periodic train of pulses that have the following properties

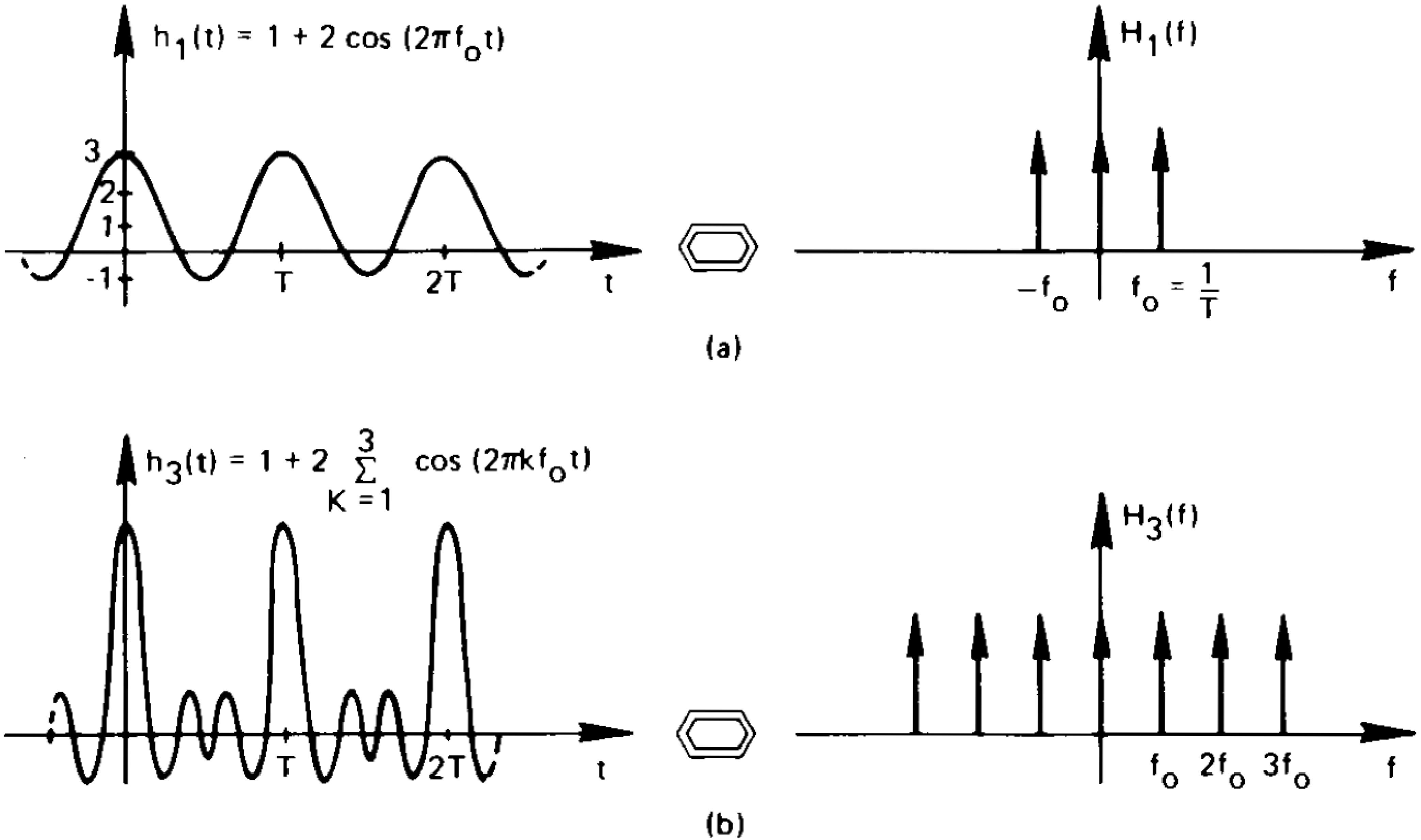
(1)the pulse period is $T = 2\pi/\omega_R$;
(2)the peak power is N times the average power;
(3)the peak field amplitude is N times the amplitude of a single mode;
(4)the pulse width, defined as the time from the peak to the first zero, is $\tau = T/N$, which shortens as N increases.

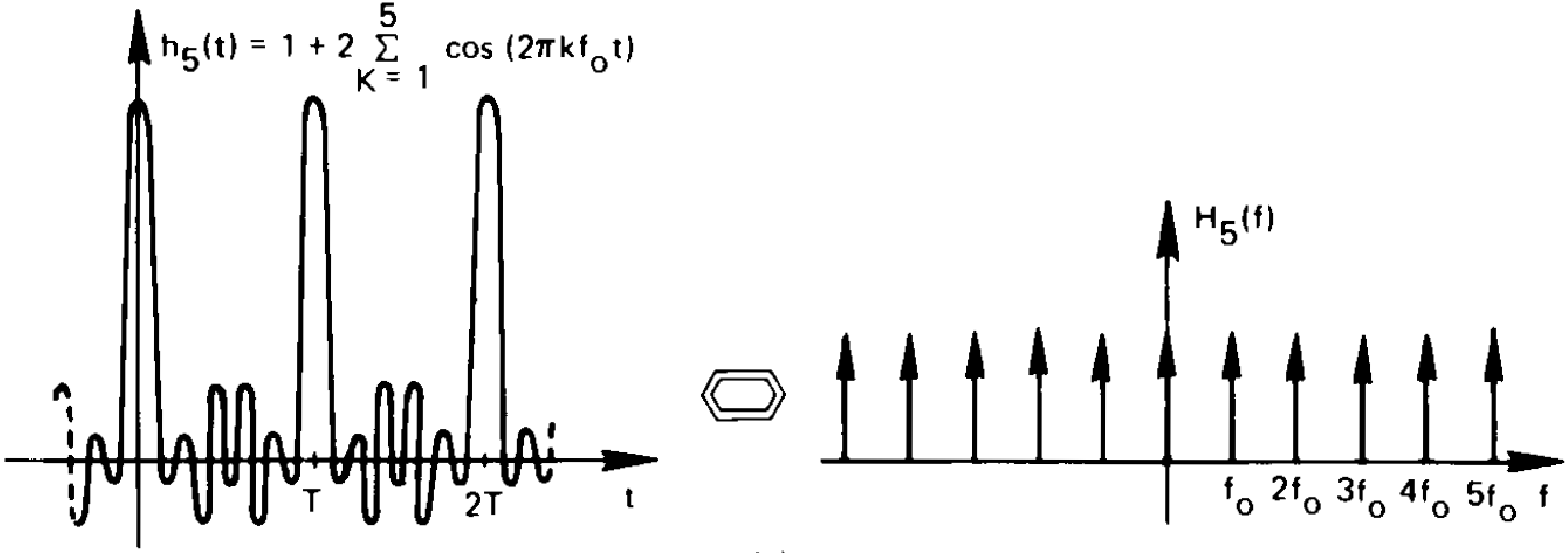
Appendix: Fourier Transform



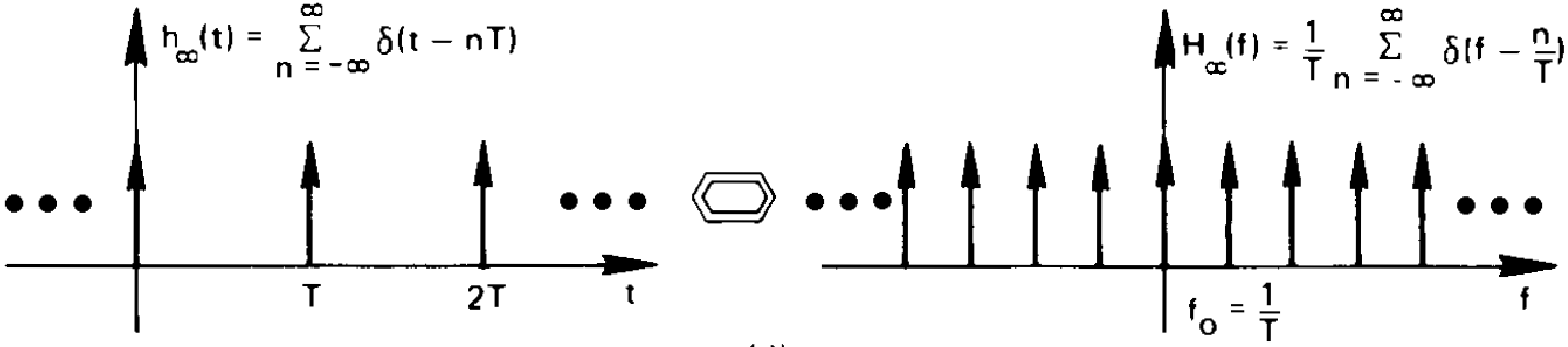
Note that the Fourier transform is imaginary

Graphical development of the Fourier transform of a sequence of equal distant impulse function

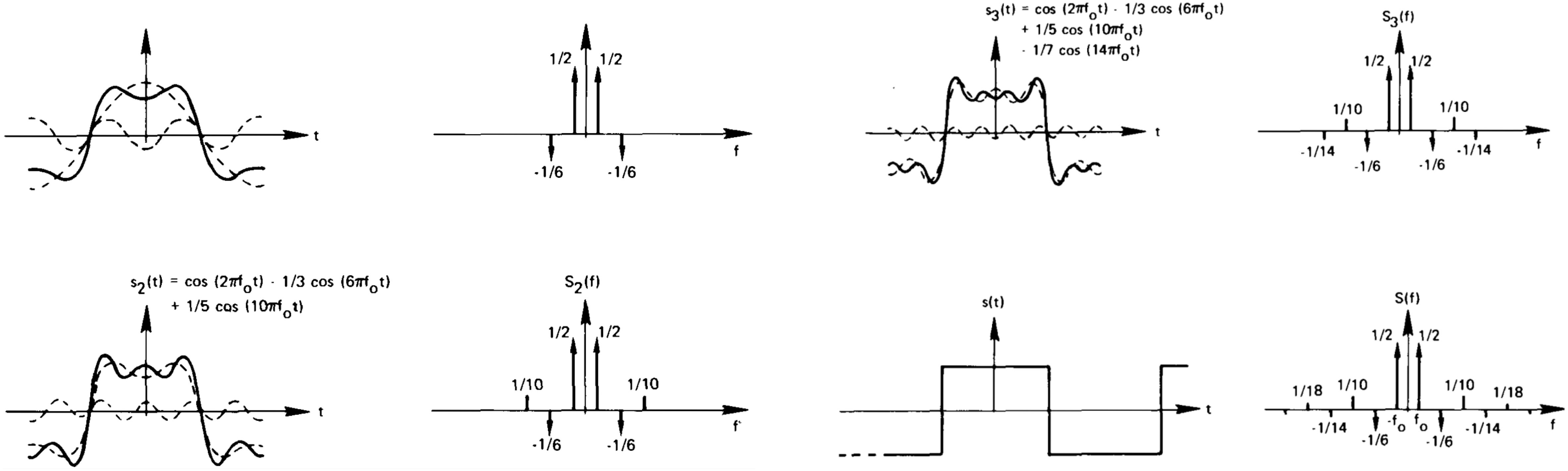



$h_5(t) = 1 + 2 \sum_{K=1}^{5} \cos(2\pi k f_o t)$
T
2T
t
$H_5(f)$
f_o $2f_o$ $3f_o$ $4f_o$ $5f_o$ f

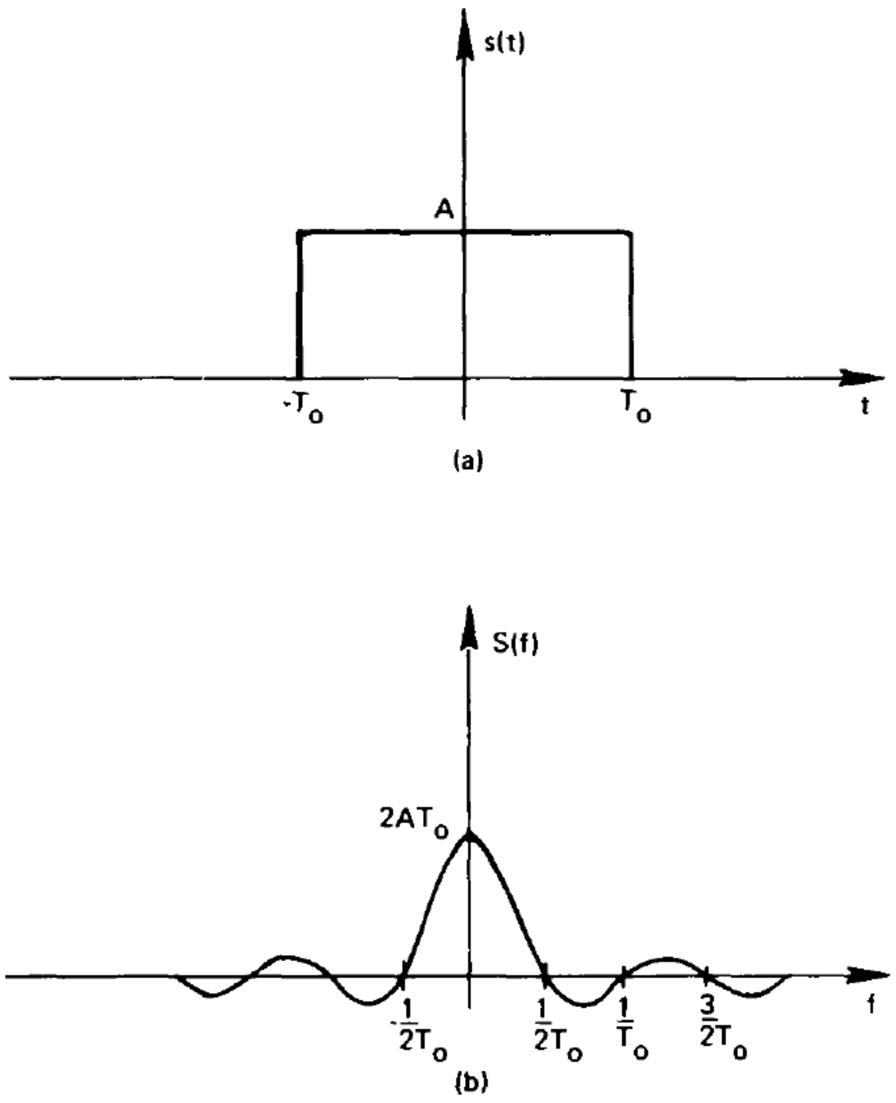
(c)


$h_\infty(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$
T
2T
t
$H_\infty(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T})$
$f_o = \frac{1}{T}$
f

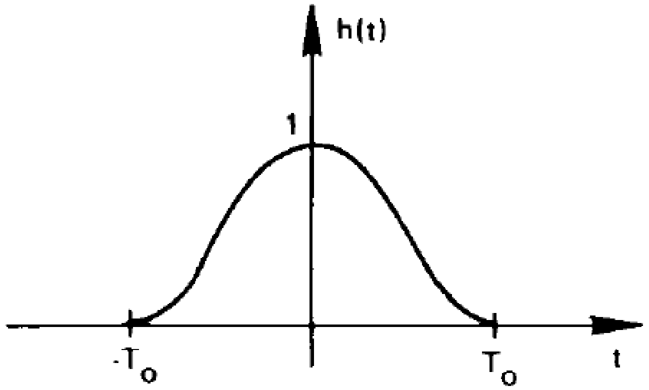
(d)



Fourier transform of a square wave function (Fourier series)



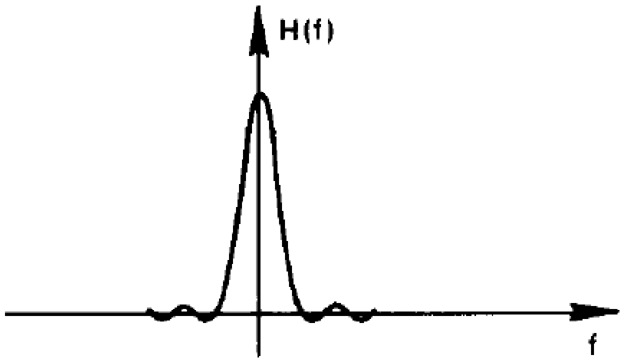
Fourier transform of a pulse waveform

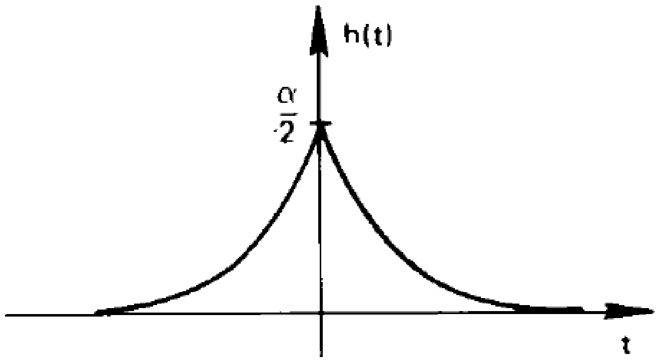


$$h(t)=\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi t}{T_0}\right) \qquad |t|\leq T_0$$
$$= 0 \qquad |t|>T_0$$

$$\Longleftrightarrow$$

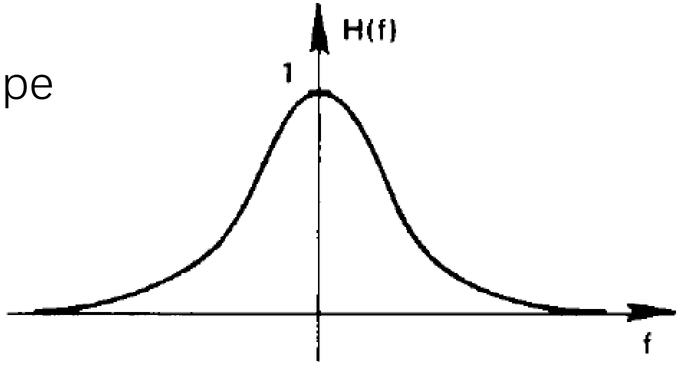
$$H(f)=\frac{1}{2}Q(f)+\frac{1}{4}\left[Q\left(f+\frac{1}{2T_0}\right)+Q\left(f-\frac{1}{2T_0}\right)\right]$$
$$Q(f)=\frac{\sin(2\pi T_0 f)}{\pi f}$$

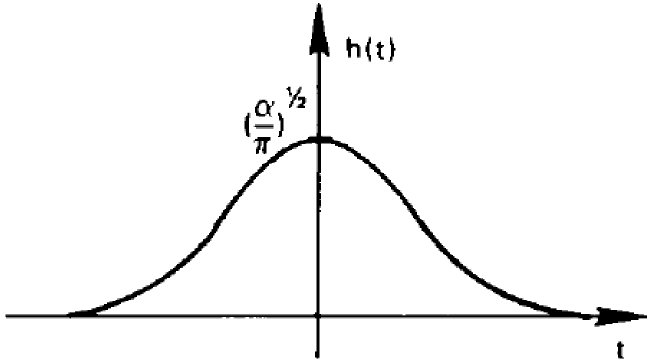




Lorentzian line shape

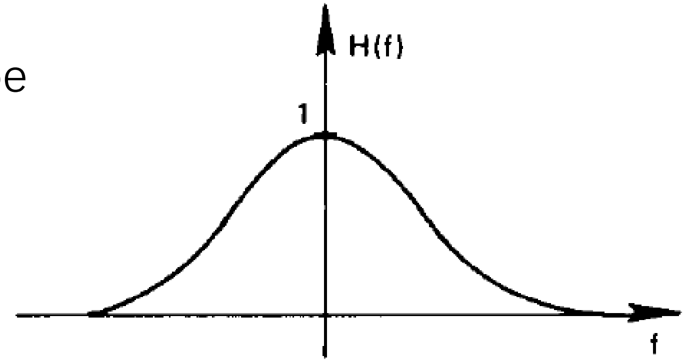
$$h(t)=\frac{1}{2}\alpha\exp(-\alpha|t|) \quad \Longleftrightarrow \quad H(f)=\frac{\alpha^2}{\alpha^2+4\pi^2 f^2}$$





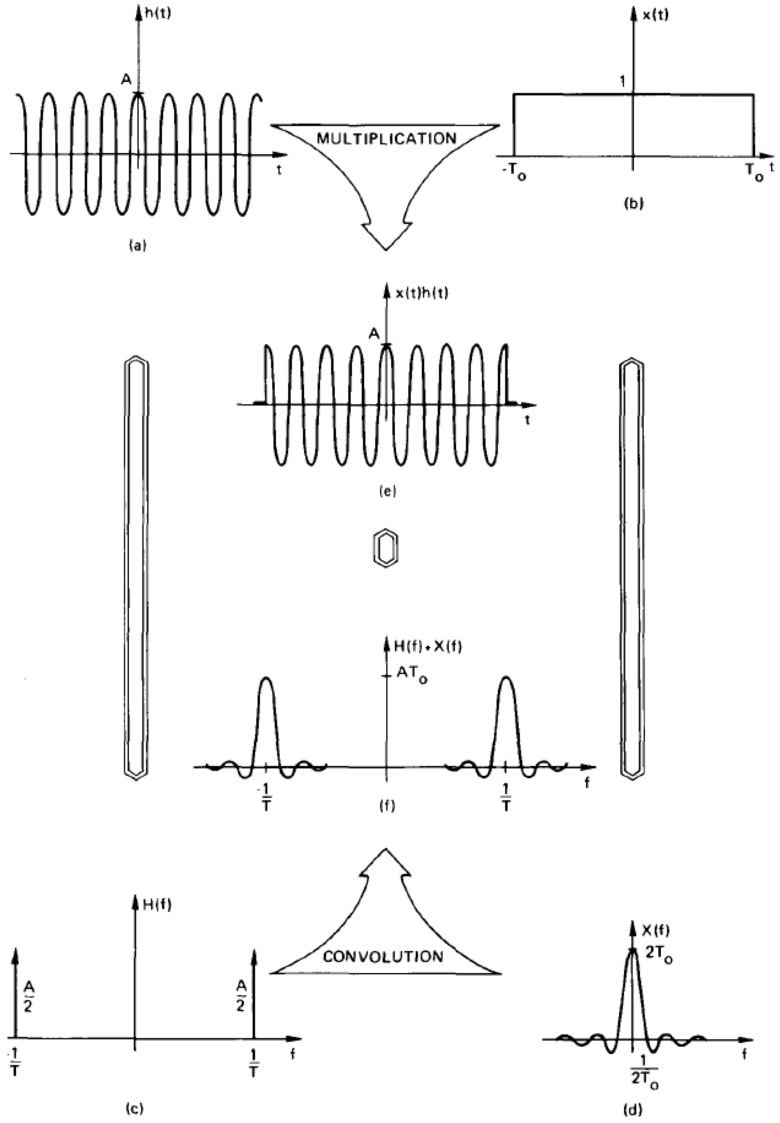
Gaussian line shape

$$h(t)=\left(\frac{\alpha}{\pi}\right)^{1/2}\exp(-\alpha t^2) \quad \Longleftrightarrow \quad H(f)=\exp\left(\frac{-\pi^2 f^2}{\alpha}\right)$$

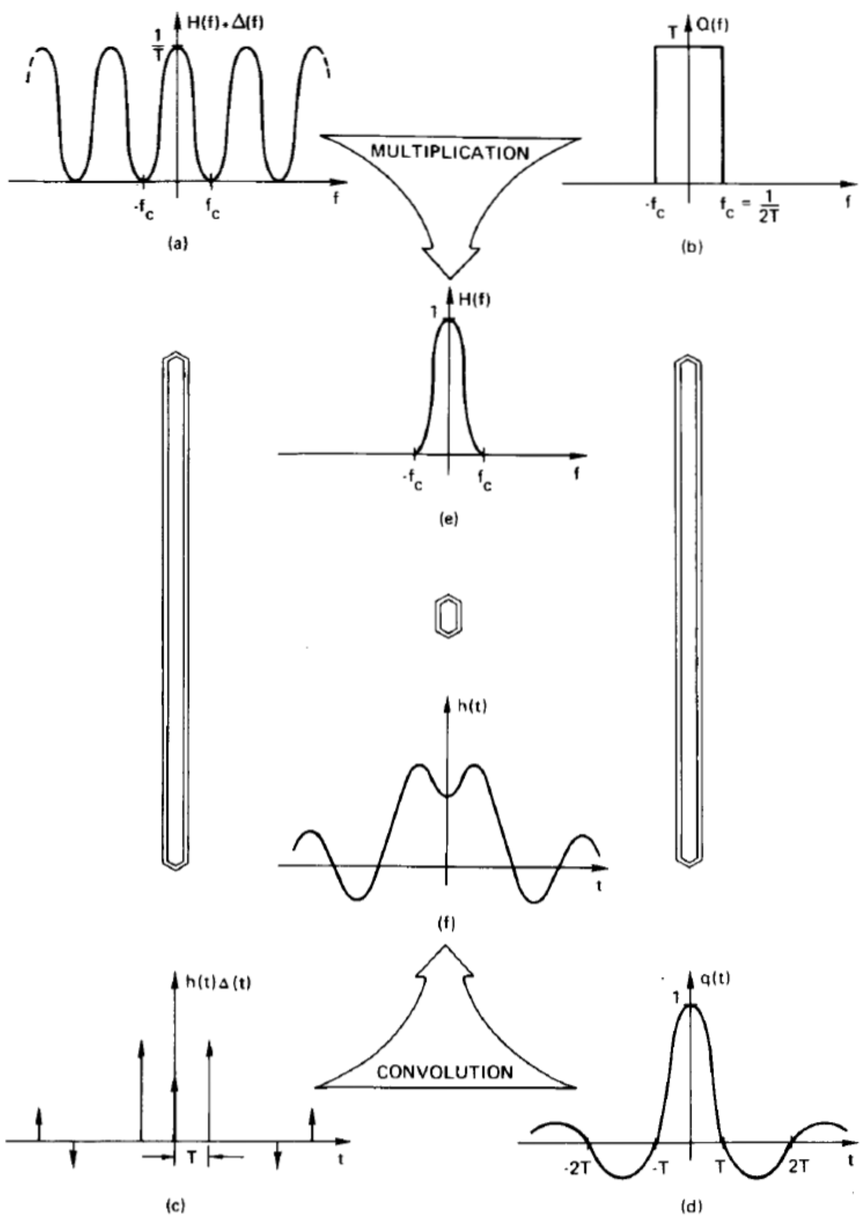


Frequency Convolution Theorem

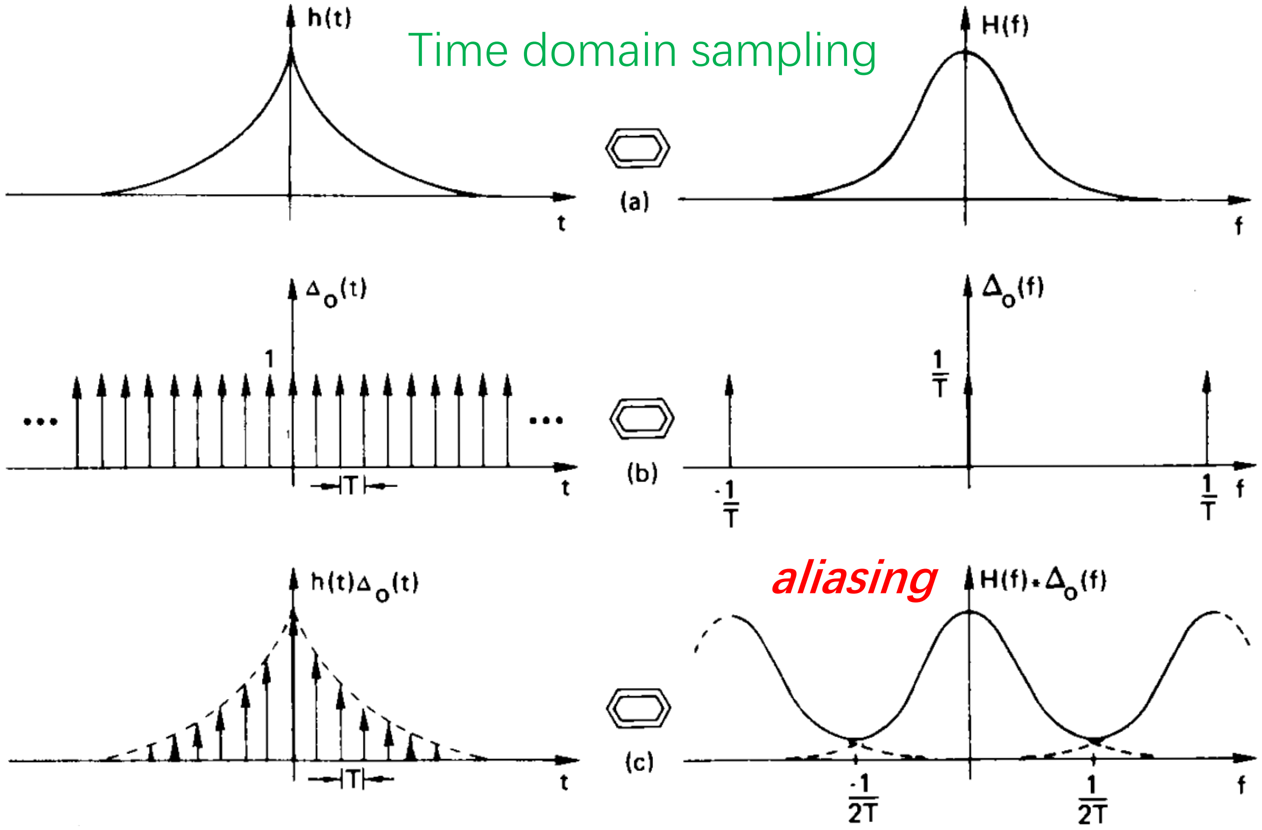
$$h(t)x(t) \Longleftrightarrow H(f) * X(f)$$

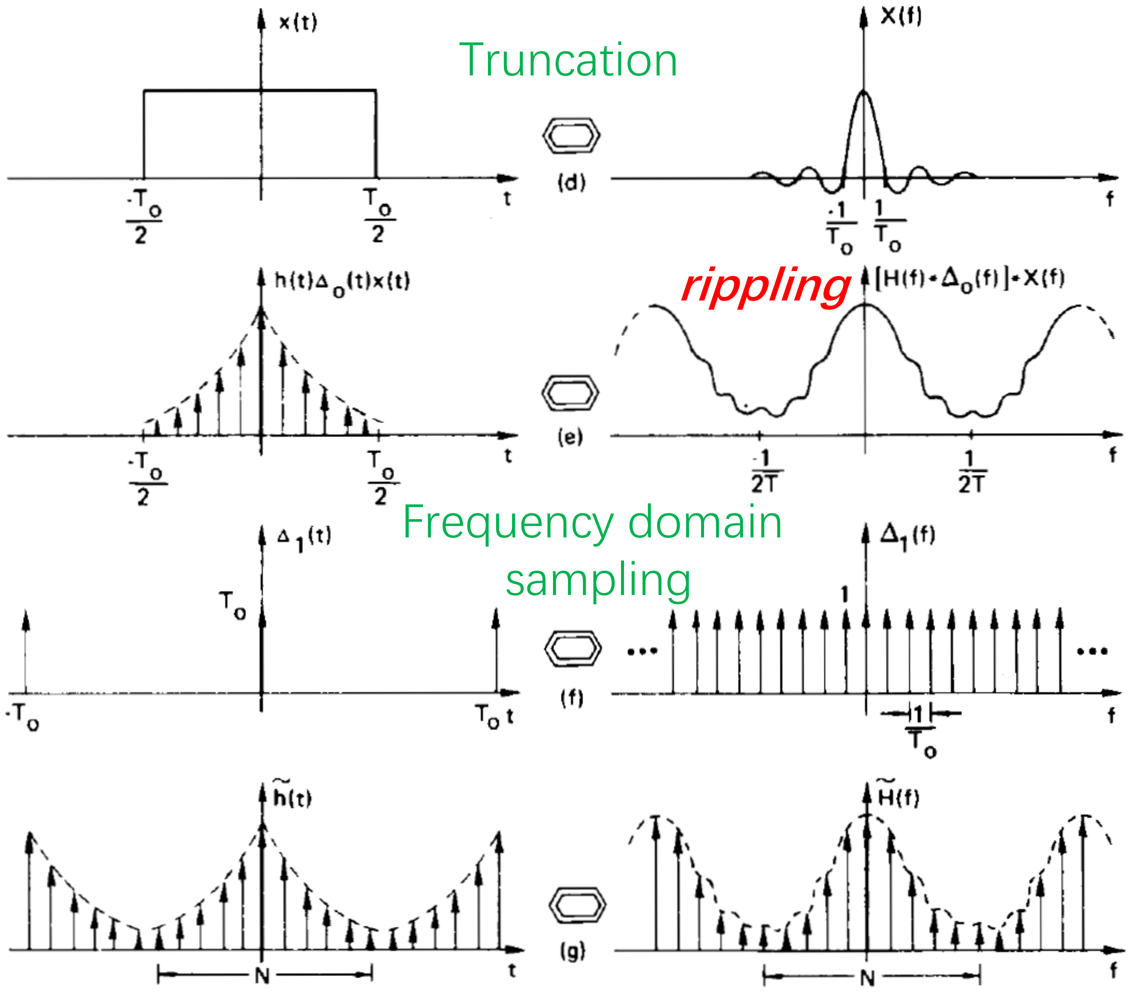


Graphical derivation of the sampling theorem



5. The Discrete Fourier Transform





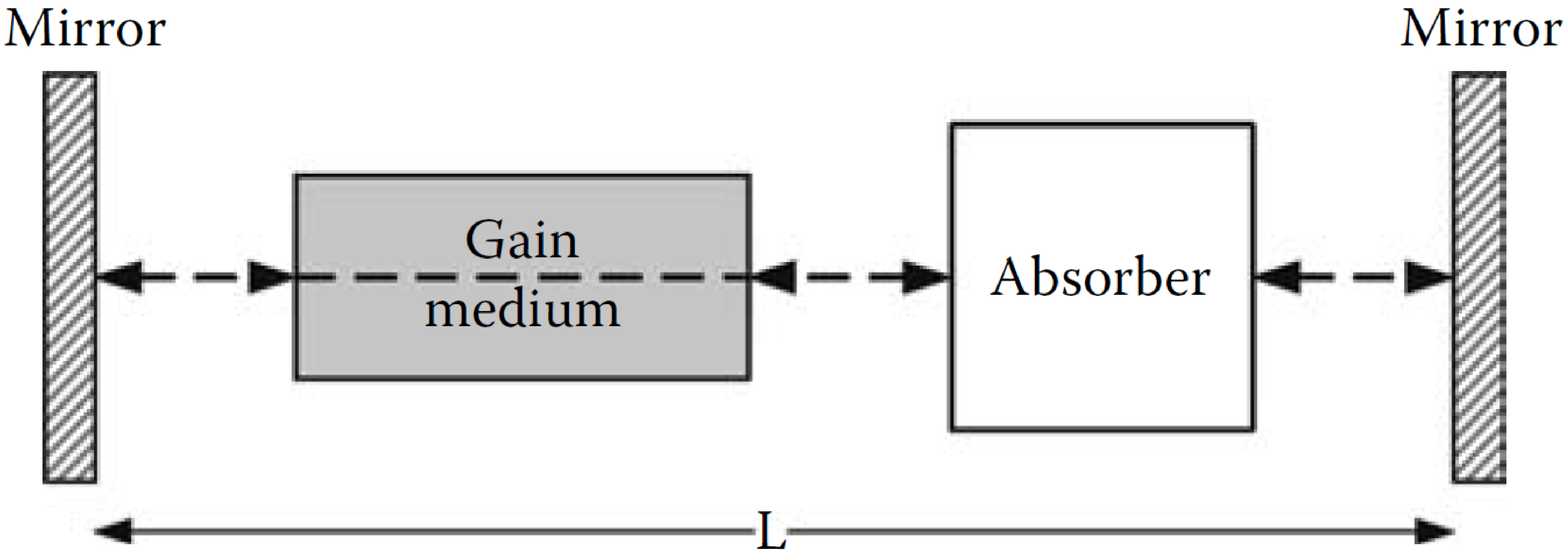
Note that sampling in the time/frequency domain resulted in a periodic function of frequency/time;
N time samples and N frequency values represent one period of the time and frequency

2. Mode-Locking Techniques

2.1 Passive Mode Locking

2.1.1 Using saturable absorber

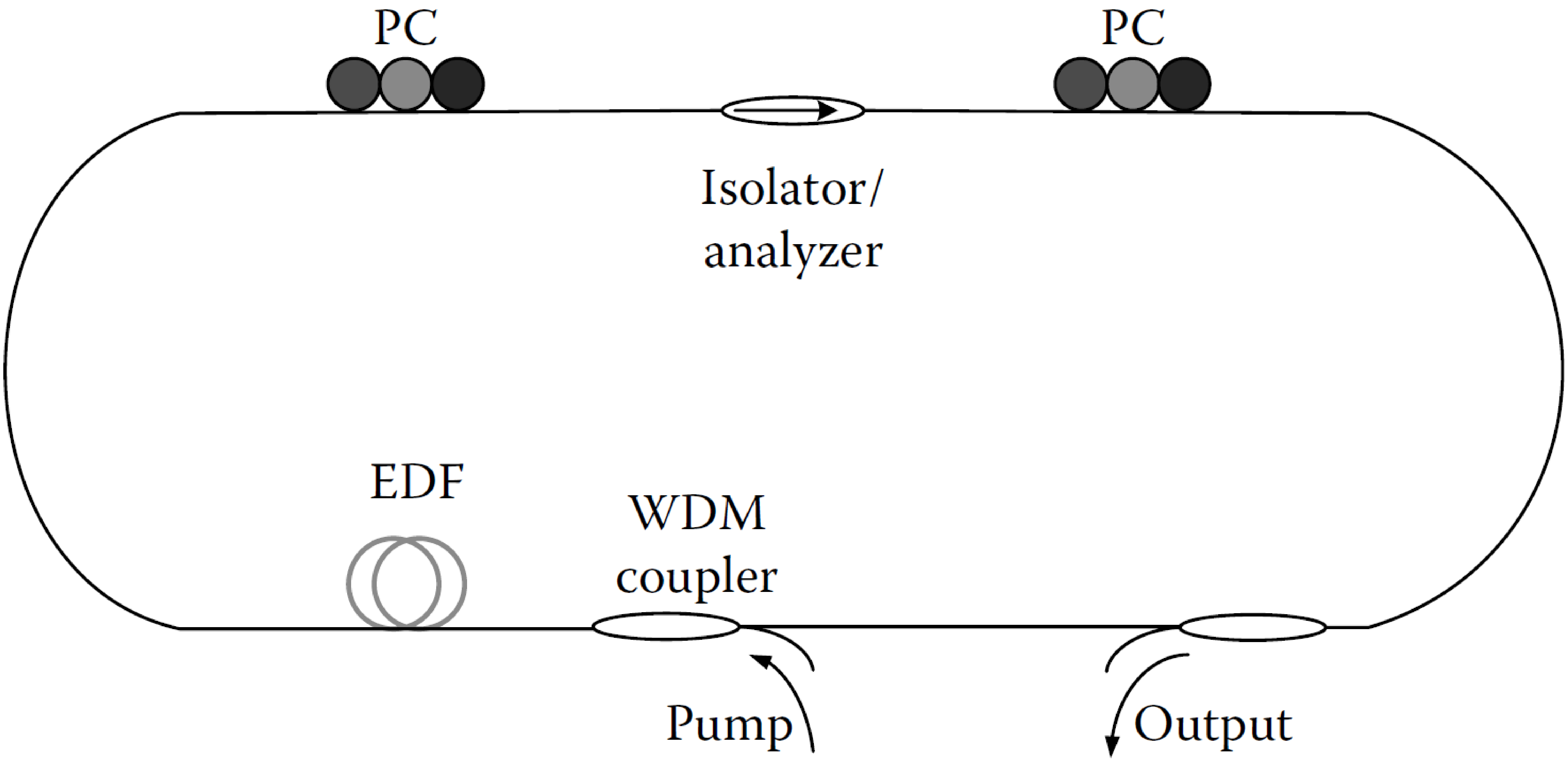
Passive mode locking technique refers to those locking without any external radio frequency (RF) signal for seeding. The pulses are formed passively through the internal structure of the laser that gives more advantages (less loss, high gain) to signal if it travels in the pulse form rather than in a CW. The simplest method is to insert a saturable absorber into the cavity. The saturable absorber is a nonlinear optical component whose absorption coefficient decreases when the optical intensity increases.
Thus, the pulse train with high peak intensity would pass through the absorber with much less loss as compared to that in a CW laser with several modes as the energy available is concentrated in the periodic pulses, thus a stable pulse train can be formed in the cavity.



Schematic of a passive MLL integrated with a saturable absorber in the cavity

2.1.2 Using polarization rotation technique

Instead of using the saturable absorber, a nonlinear polarization rotation (NPR) technique can be used for passive mode locking, as shown in the Figure below. In this system, a nonlinear phase shift is imposed on the signal with high peak power and thus rotates its polarization state to align with the analyzer axis. The signal thus passes through the analyzer with minimum loss. On the other hand, the CW signal with a low average power experiences zero nonlinear phase shift and thus its polarization state is not rotated to align with the analyzer axis. It is then blocked by the analyzer. Therefore, pulses with high peak power are formed in the cavity instead of the CW with a low average power.



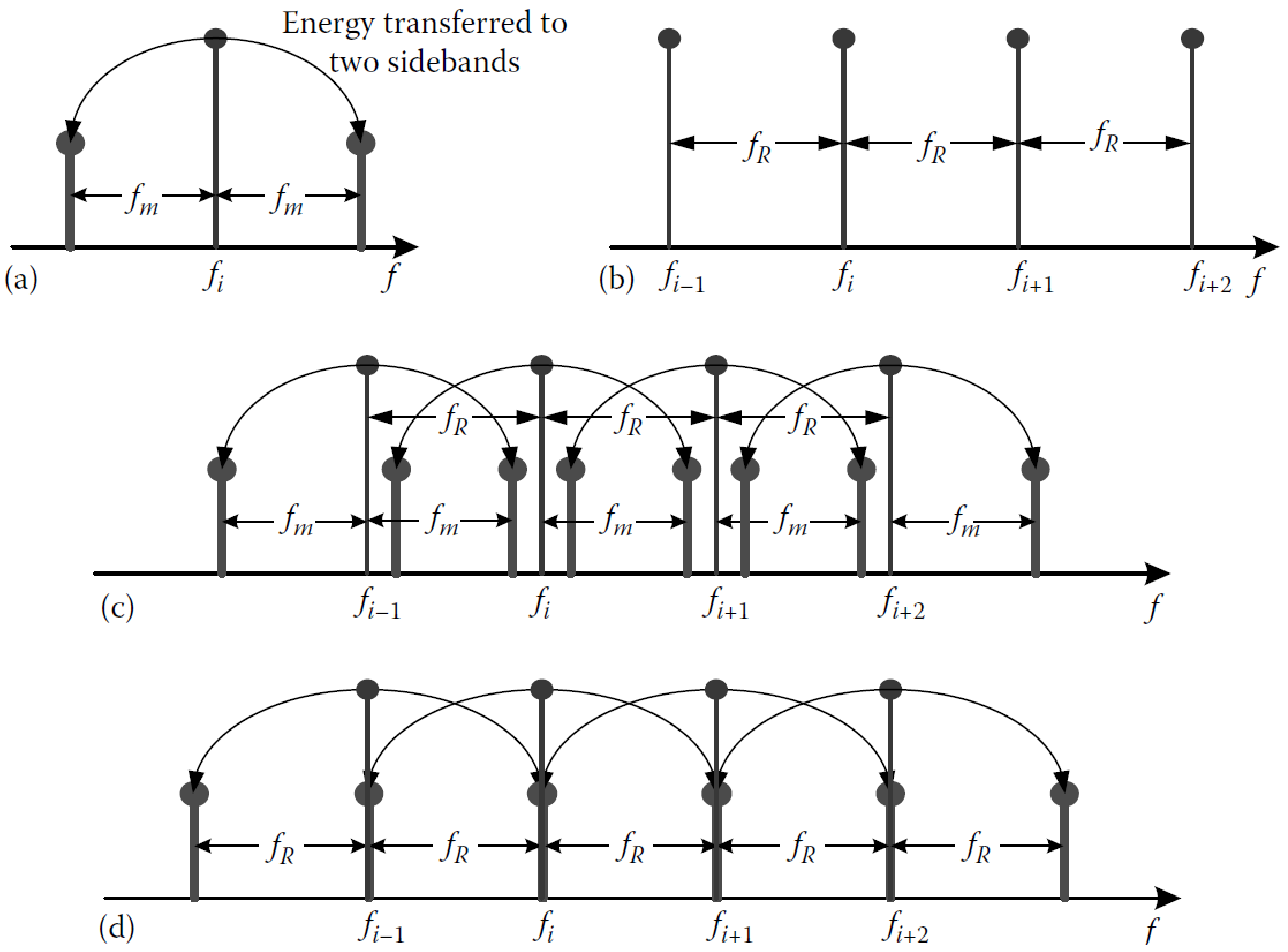
Schematic of an NPR passive MLFL

2.1.3 Advantages and Disadvantages of Passive MML

A passive mode-locked laser (MLL) has the potential to produce a very short laser pulse, down to femtoseconds. The shortest pulse generated in the communication wavelength window to date is 5 fs?
Moreover, passive mode-locking is a self-locking phenomenon, it means that the pulse is formed without any external modulating signal.
However, the spacing between two pulses is varied from pulse to pulse since there is no control mechanism to force the pulses to be equally spaced. This is the disadvantage that makes passive MLLs not suitable for high-speed optical transmission systems, in which precise timing from pulse to pulse is required.

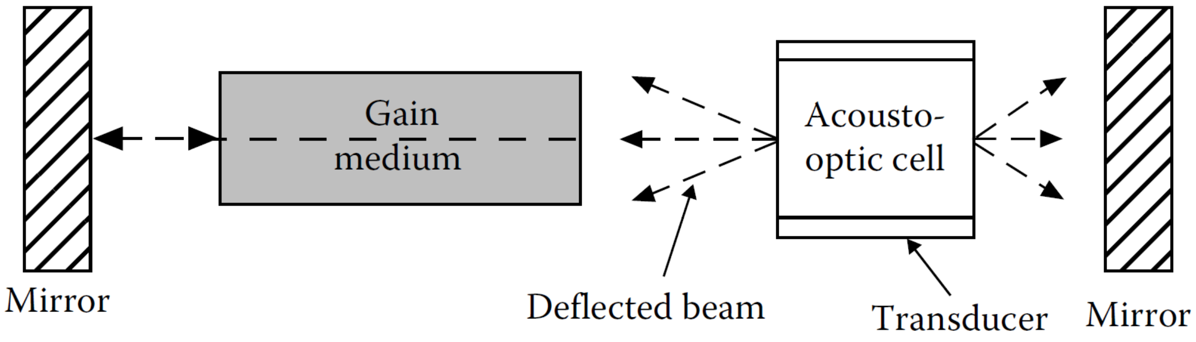
2.2.1 Active Mode Locking by Amplitude Modulation

In actively MLLs, mode locking can be induced either by amplitude modulation (AM), phase modulation (PM), or frequency modulation (FM). When amplitude is modulated by an external RF signal with a frequency of f_m, the oscillating mode at the optical frequency f_0 shades its energy to the two side bands located at f_0+f_m and f_0-f_m, as shown in the Figure below. If the modulating frequency f_m is chosen so that $f_m=f_R$, the side bands are coincident with the adjacent modes of the laser and hence the energy from ith mode is transferred to its adjacent $(i+1)$th and $(i-1)$th modes. In other words, the energies of the modes in the laser are transferred from one to the other. This causes the phase of the modes locked together and hence a mode-locked pulse train is formed.



Energy transferred to two sidebands
f_m
f_m
(a)
f_i
f
f_R
f_R
f_R
(b)
f_{i-1}
f_i
f_{i+1}
f_{i+2}
f
f_R
f_R
f_R
f_m
f_m
f_m
f_m
f_m
(c)
f_{i-1}
f_i
f_{i+1}
f_{i+2}
f
f_R
f_R
f_R
f_R
f_R
(d)
f_{i-1}
f_i
f_{i+1}
f_{i+2}
f

In actively MLLs, mode locking is induced by modulating the gain or loss of the cavity with an external signal at the fundamental frequency f_R. A common method of active mode locking is inserting an acousto-optic modulator (AOM) into the cavity, as shown in the Figure below. This is the first method used in the experiment for the observation of mode-locked pulses in a helium-neon laser. The AOM is used to deflect light energy from the cavity by varying the refractive index of the medium in a standing wave profile. Thus, the undeflected beam is amplitude modulated.



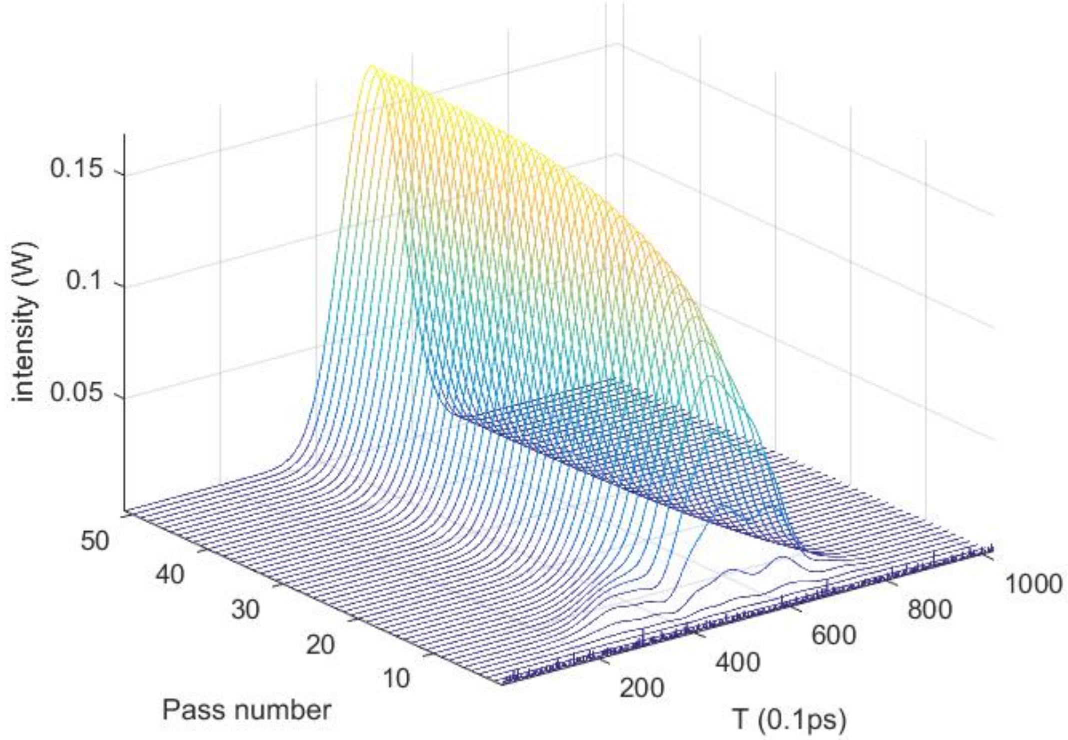
AOM is widely used in several MLLs as it offers a very low insertion loss. However, the operating frequency of the AOM is limited and far below the pulse rate required for modern optical communication, typically, from a few hundred megahertz to a few gigahertz depending on the acoustic velocity of the acousto-optic material.

Due to the limited bandwidth of AOMs, they are normally replaced by $LiNbO_3$ (铌酸锂) intensity or phase modulators whose bandwidth may reach 40 GHz or higher in high repetition rate actively MLLs.

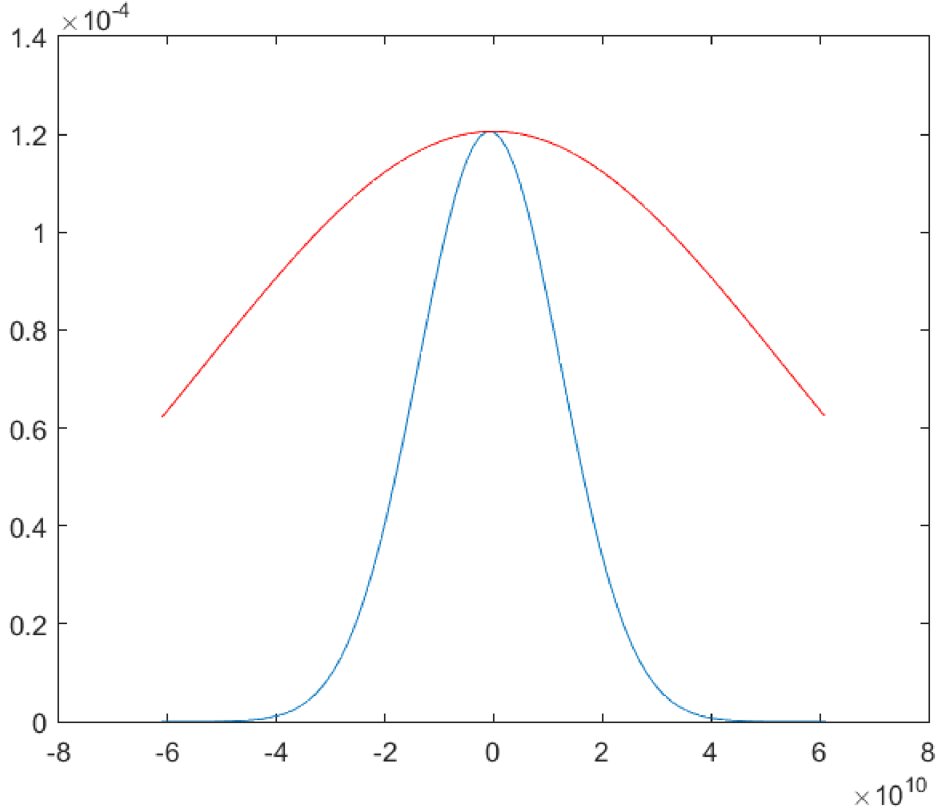
In addition, all-fiber cavity is also used in those lasers to take advantage of fiber lasers such as low threshold, single mode, and high beam quality. Moreover, high optical intensity in a small cross section of the fiber enables one to obtain nonlinear effects for short pulses with even a low pump power.

Another method of mode locking is to modulate the gain of the amplifying medium as in Refs. [?, ?]. In those experiments, the gain of the semiconductor amplifier is modulated by an external modulating signal. The lightwaves therefore experience a periodical gain and form the mode-locked pulses in the cavity. A semiconductor laser has the potential to generate very short pulses due to its wide spectral gain region.

2.2.2 Stimulation of Active Mode Locking by Amplitude Modulation

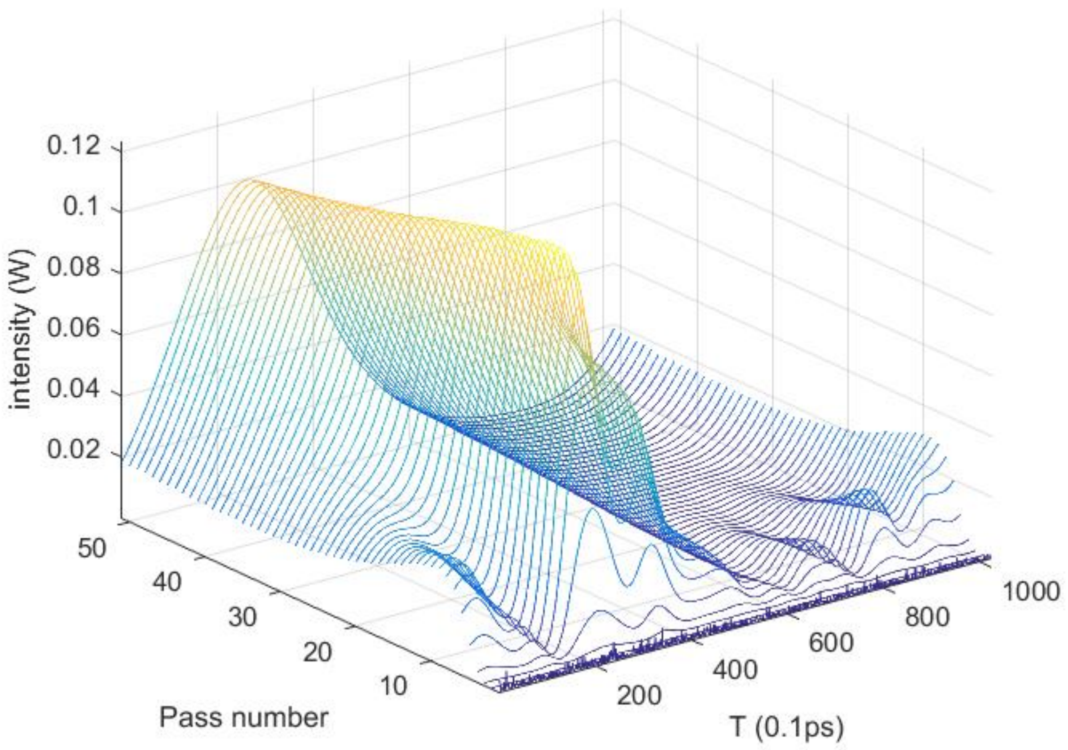


Pulse generation

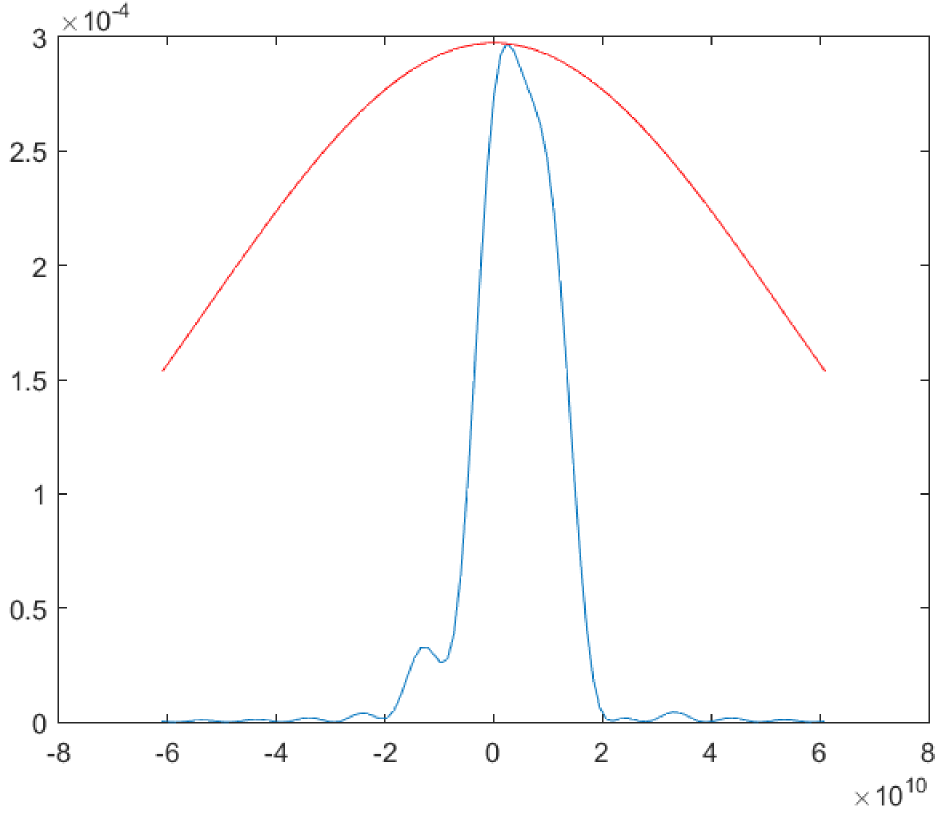


Spectrum

Without AM, mode locking will not be stable



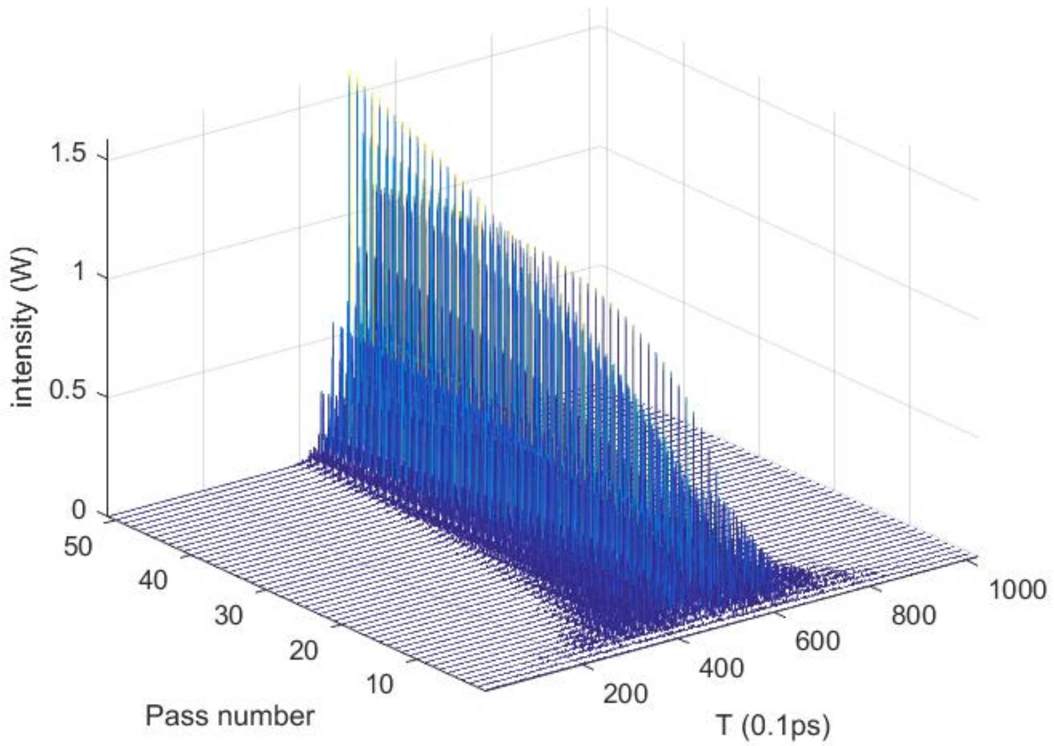
Pulse generation without AM



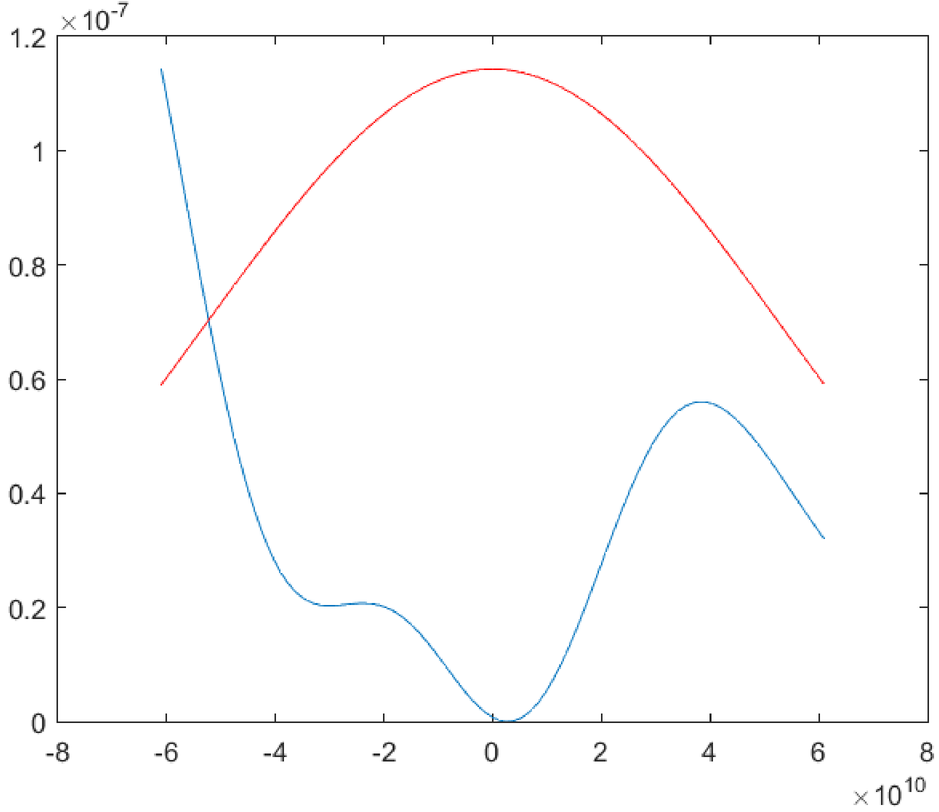
Spectrum without AM

In a mode-locked fiber ring laser (MLFRL),

Without frequency filter, high frequency noise would deform the laser pulses



Pulse generation without frequency filter



Spectrum without frequency filter

2.3 The mechanism of Kerr-lens mode-locking is supposed to be soft-aperture effect

$Nd:YVO_4$[2008, Y. F. Chen], $Nd:GdVO_4$[2009, Y. F. Chen], $Ti:Al_2O_3$ mode-locking lasers[1999]

[2008, Y. F. Chen]. National Chiao Tung University, Hsinchu, Taiwan, "Compact efficient multi-GHz Kerr-lens modelocked diode-pumped $Nd:YVO_4$ laser", OE
[2009, Y. F. Chen], "Picosecond optical vortex converted from multigigahertz self-mode-locked high-order Hermite–Gaussian Nd:GdVO4 lasers", OL
[1999]. U. Morgner, F.X. Kartner, S.H. Cho, Y. Chen, H.A. Haus, J.G. Fujimoto, E.P. Ippen, V. Scheuer, G. Angelow, and T. Tschudi, Sub-two-cycle pulses from a Kerr-lens mode-locked Ti: Sapphire laser, Opt. Lett., 24, 411–413, 1999.

2.4 For the laser materials that is not a self-focusing medium, a Hard-aperture is used to control transverse modes

- In the experiments of CuBr vapor lasers, highly stable SML pulses have been obtained by using an aperture to control transverse modes. [1992, J. Geng]

[1992, J. Geng]. J. Geng, G. Zhang, X. Song, and F. Lin, “Highly stable self-mode locking and the longitudinal mode structure in CuBr laser” Appl. Phys. Lett. 60, 2969(1992).

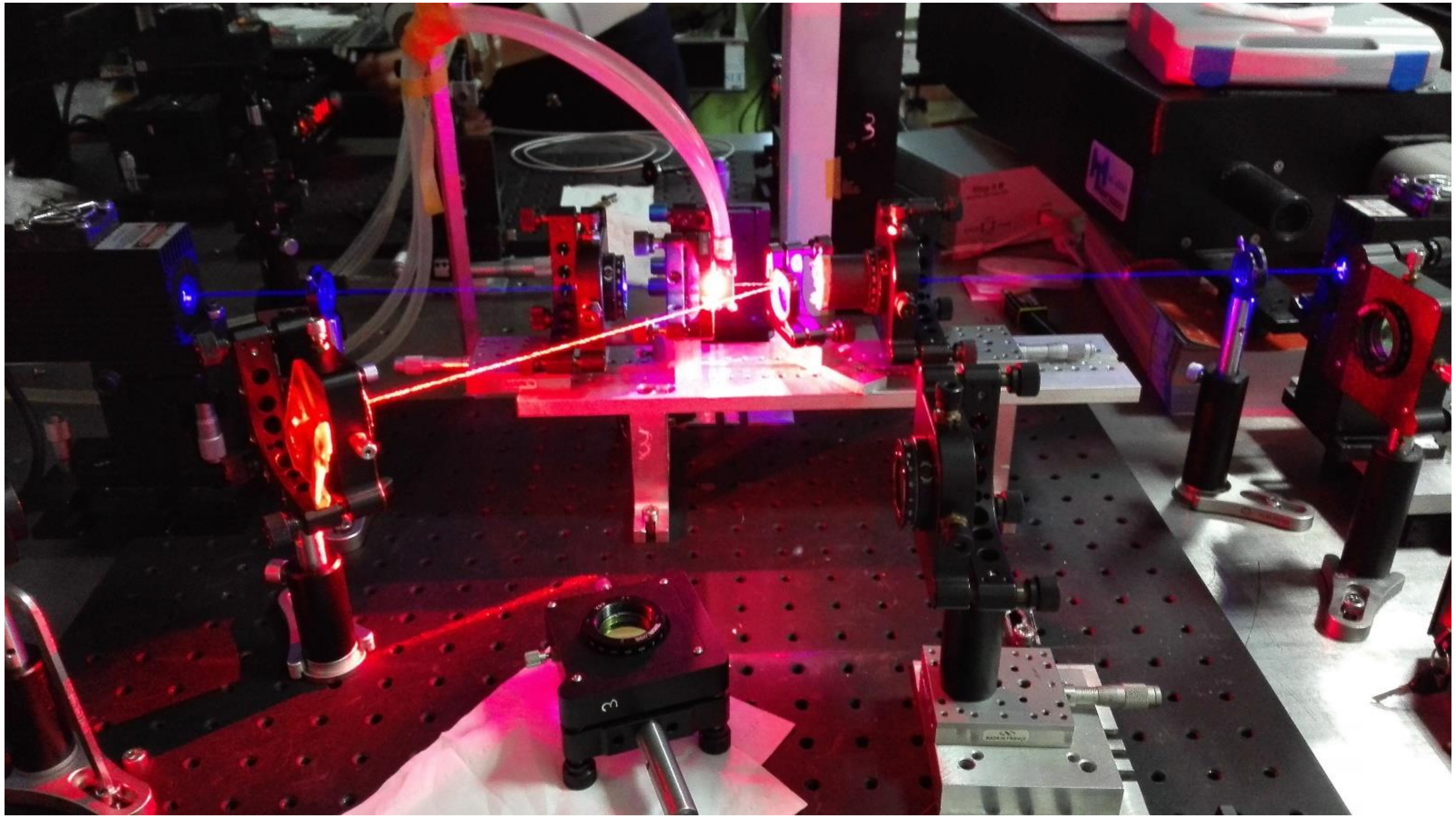
2.5 Narrow-band width self-mode-locking lasers

Gas lasers generally has narrow band width of the gain

- a copper vapor laser not only has the largest power density (about 10^5 W/cm^2), but also has the largest gain linewidth (about 10^{10} Hz). If the electric field is induced by the intra-cavity laser field. The power density must be about 10^5-10^6 W/cm^2 to satisfy the SML criterion. In fact, it is difficult to reach such a power density by an intra-cavity laser field. Therefore, the SML is unstable.
- For a He-Ne laser, its gain linewidth is the narrowest and its power density is the lowest in three kinds of lasers. The analysis shows that its SML is usually unstable again.
- For a CuBr laser, its linewidth is narrower than a copper vapor laser since its operating temperature is lower, then its SML needs lower power density; although its linewidth is larger than a He-Ne laser, its power density has much larger. Therefore, its SML is the easiest and stablest in the three kinds of lasers.

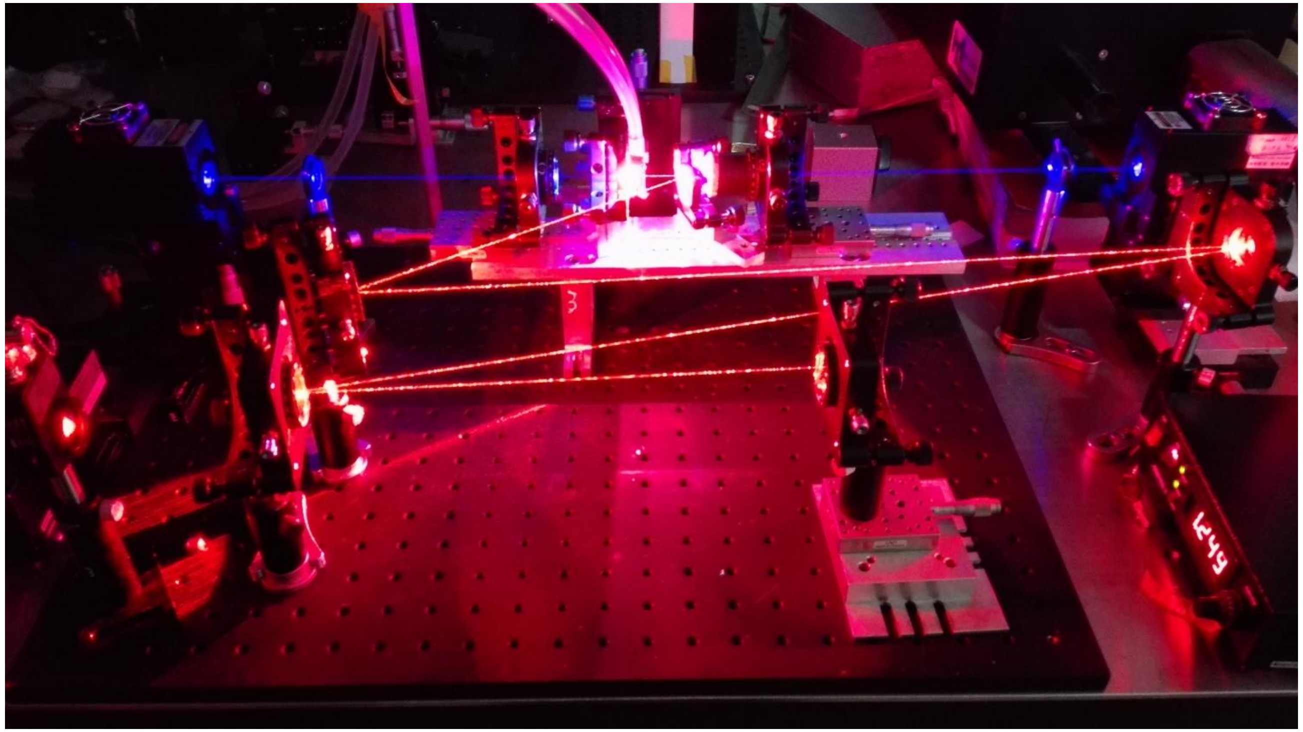
Self-Mode-Locking

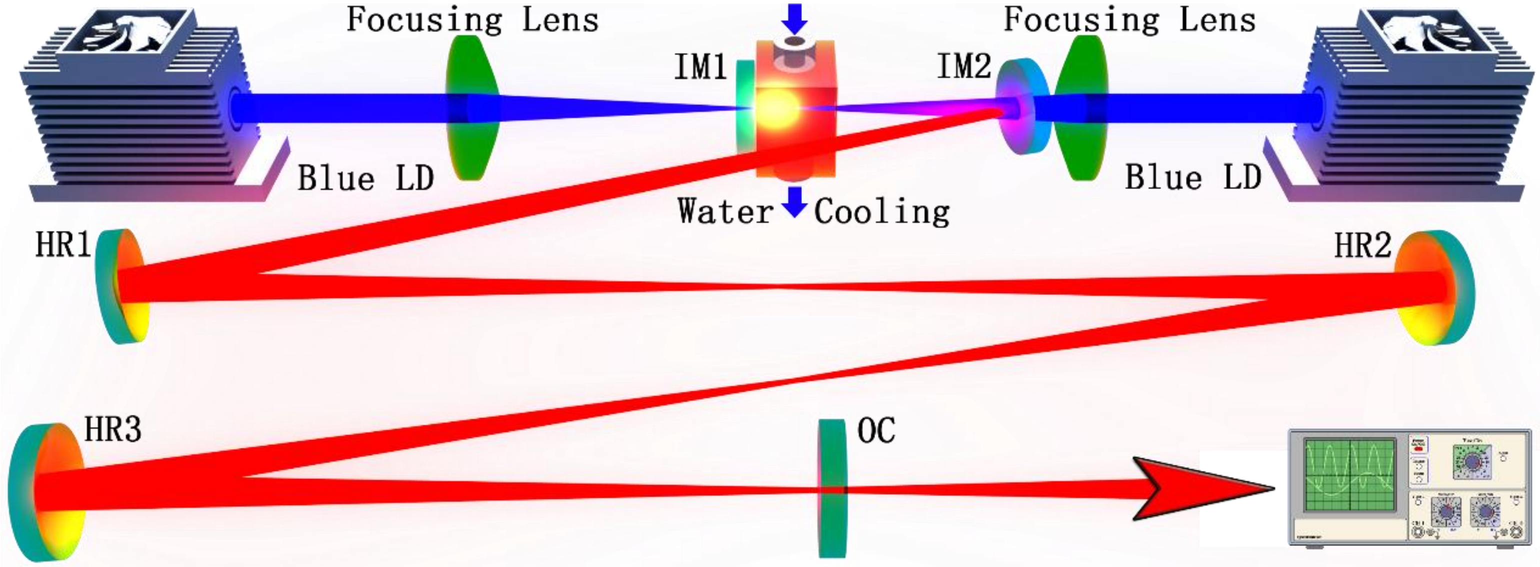
0.3 m cavity

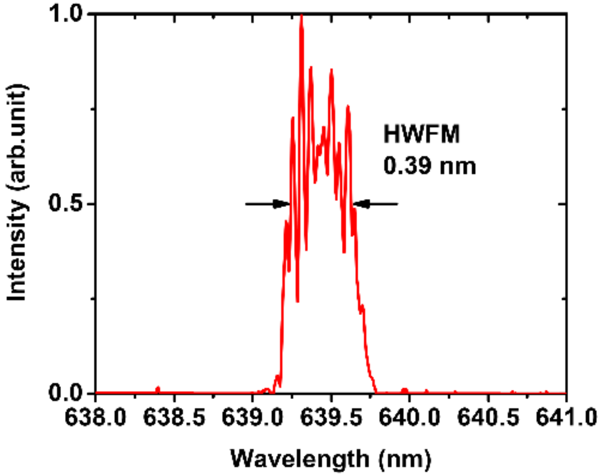


Kerr mode-locking

1.8 m cavity





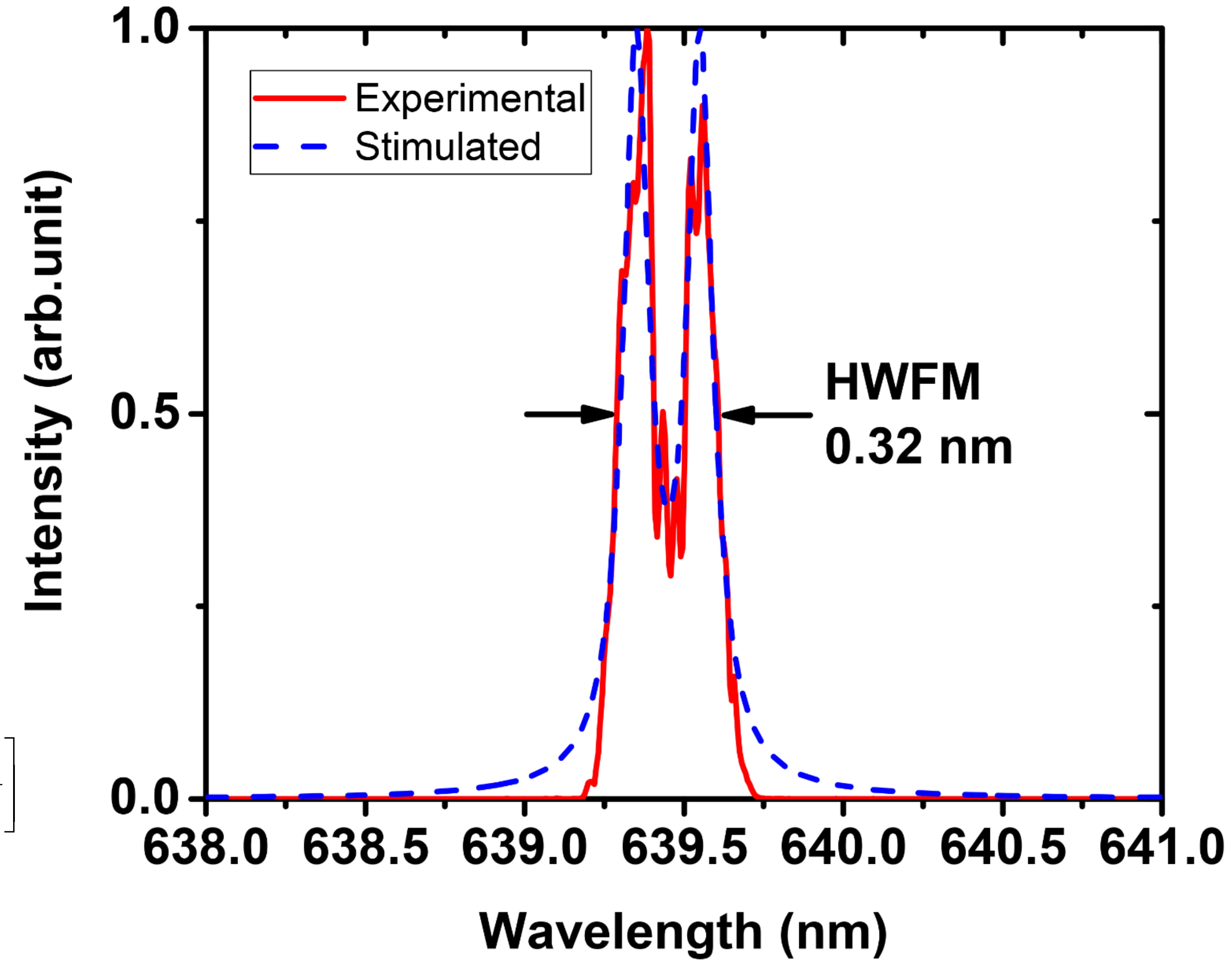


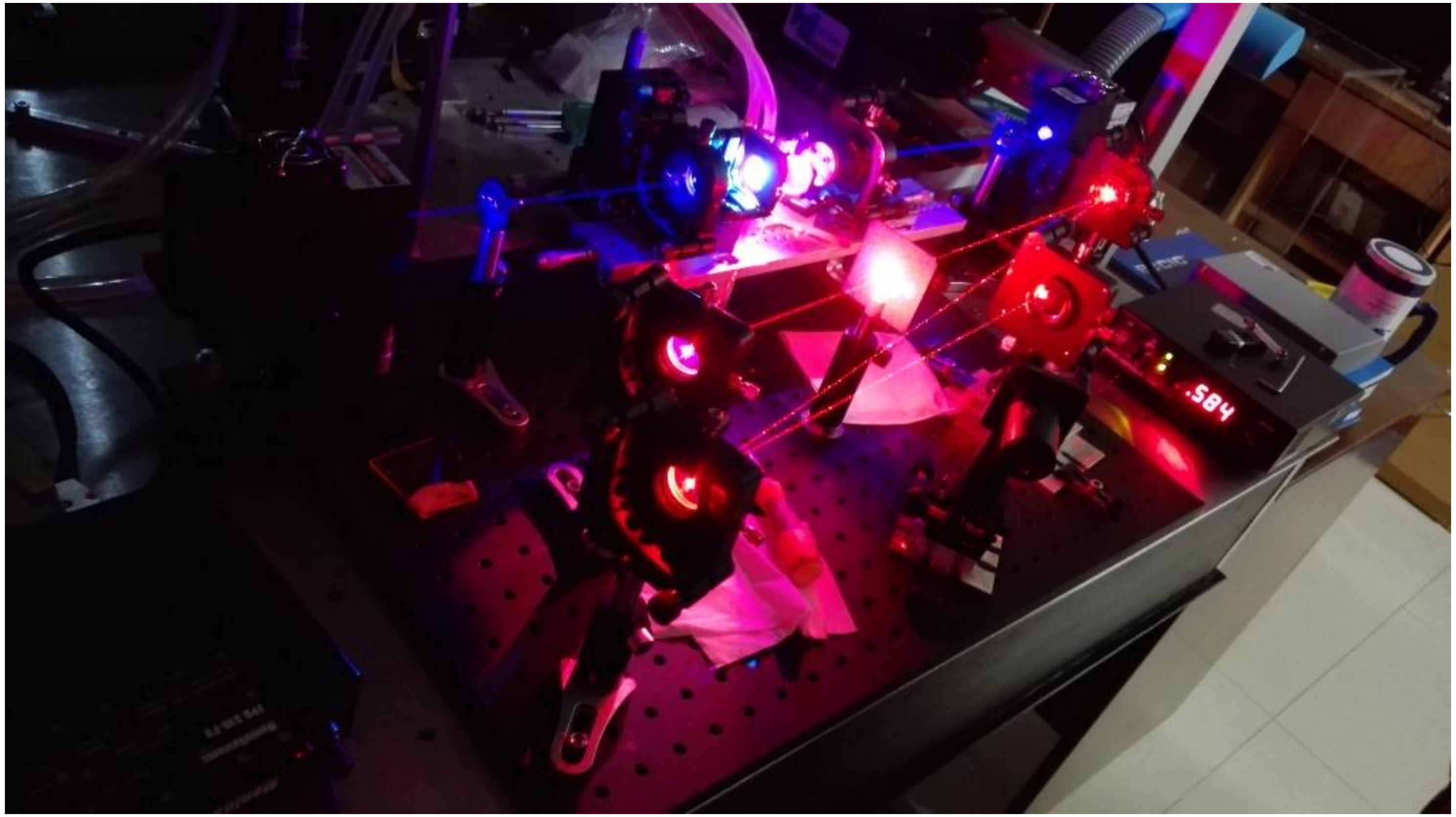
Lorentzian line shape

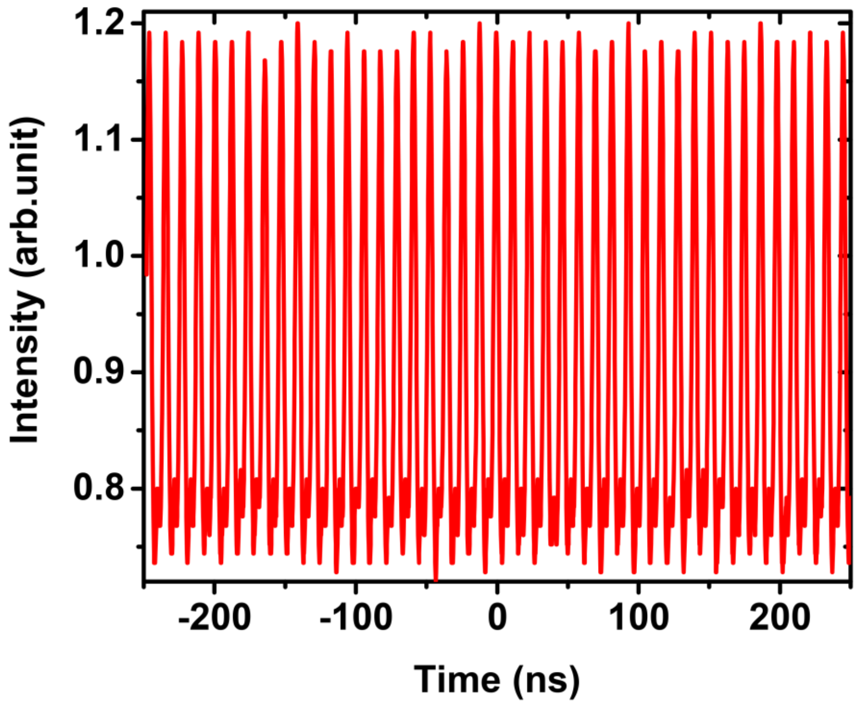
$$g(\omega) = g(\omega_0) \cdot \frac{\Delta\omega_g/(2\pi)}{(\Delta\omega_g/2)^2 + (\omega - \omega_0)^2}$$

$$g(\omega) = \frac{1}{2} g(\omega_0) \cdot \left[\frac{\Delta\omega_g/(2\pi)}{(\Delta\omega_g/2)^2 + (\omega - \omega_0 + \Delta\omega_s)^2} + \frac{\Delta\omega_g/(2\pi)}{(\Delta\omega_g/2)^2 + (\omega - \omega_0 - \Delta\omega_s)^2} \right]$$

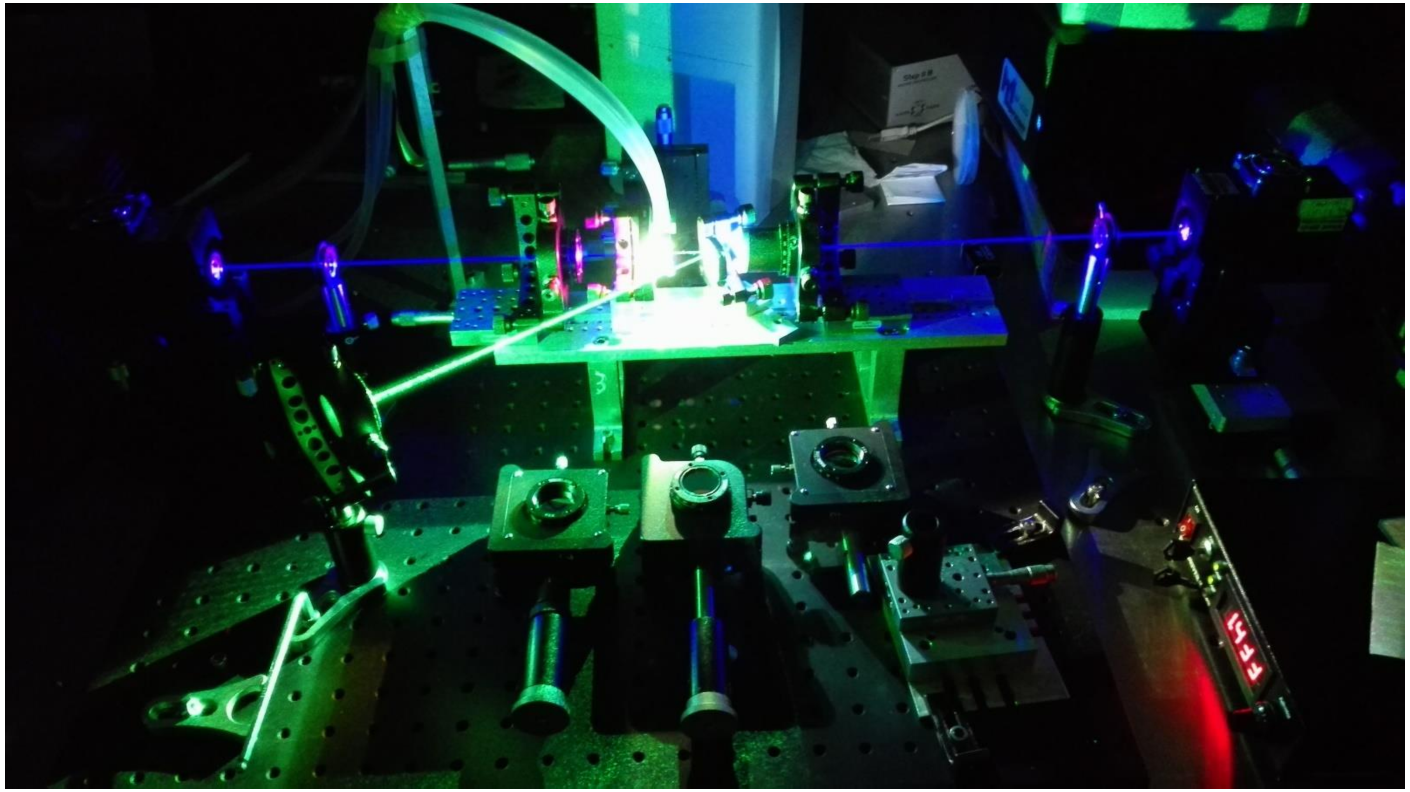
Line shape with the frequency shift of the Stark splitting from the unperturbed frequency induced by the intra-cavity laser field

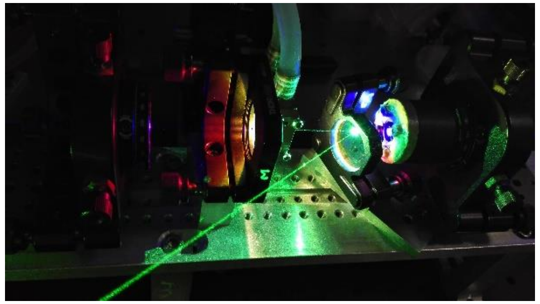


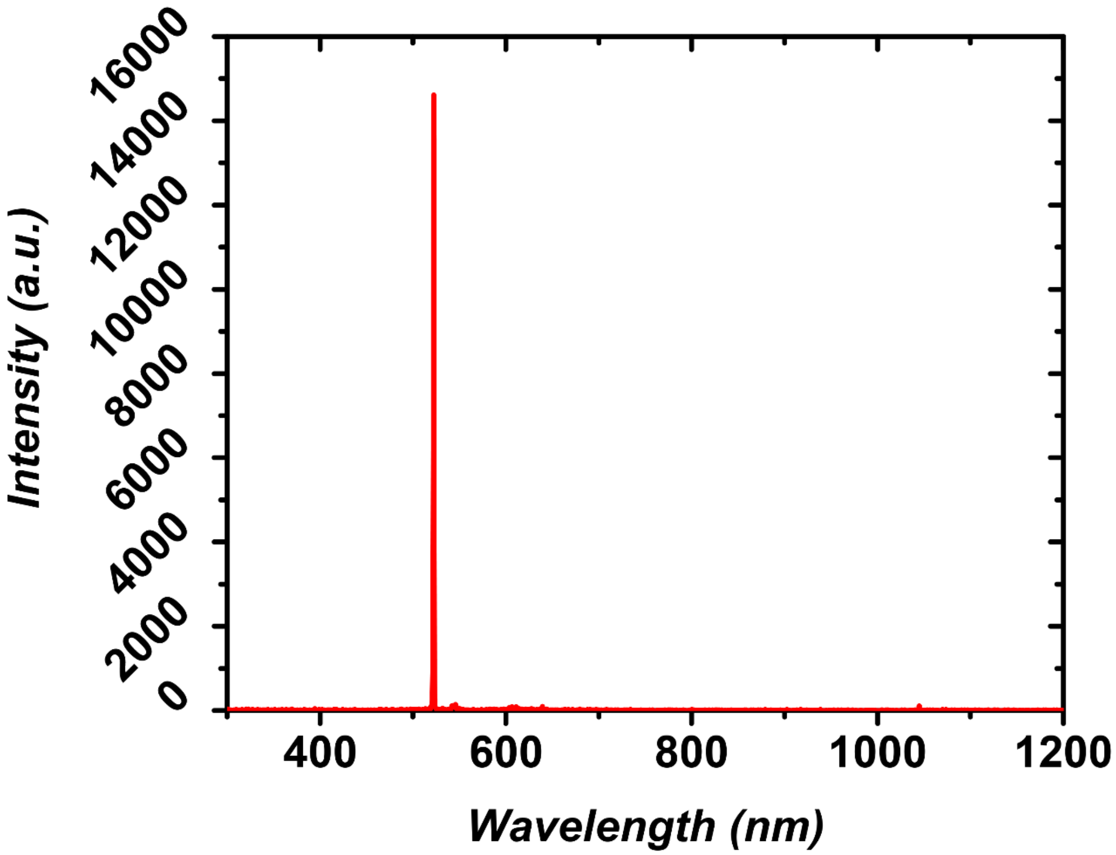

.584

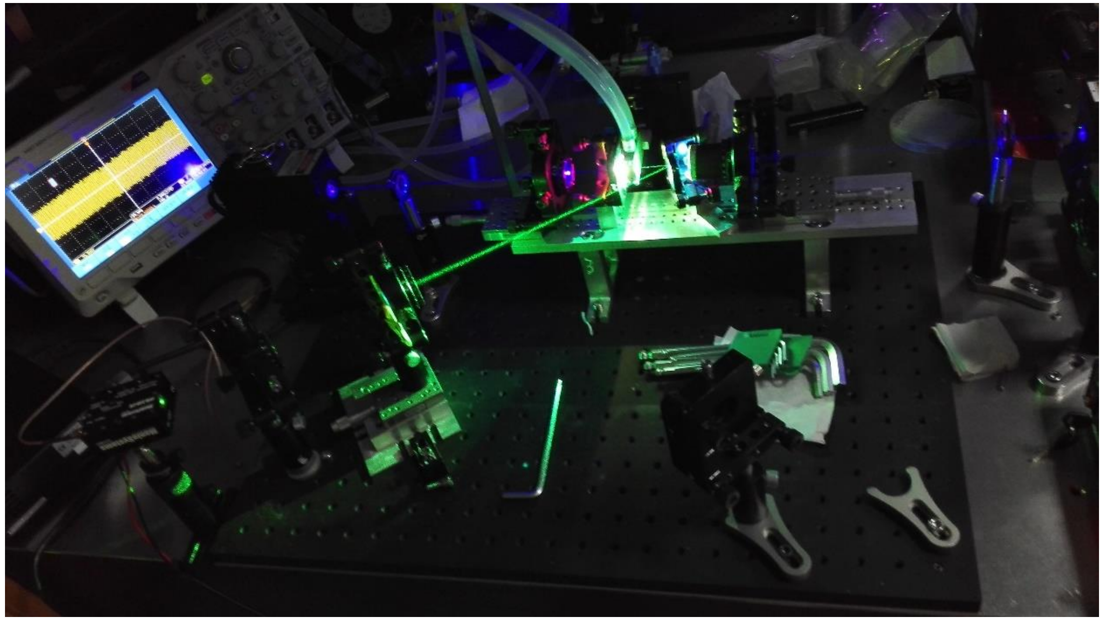

1.2
1.1
1.0
0.9
0.8
Intensity (arb.unit)
-200
-100
0
100
200
Time (ns)

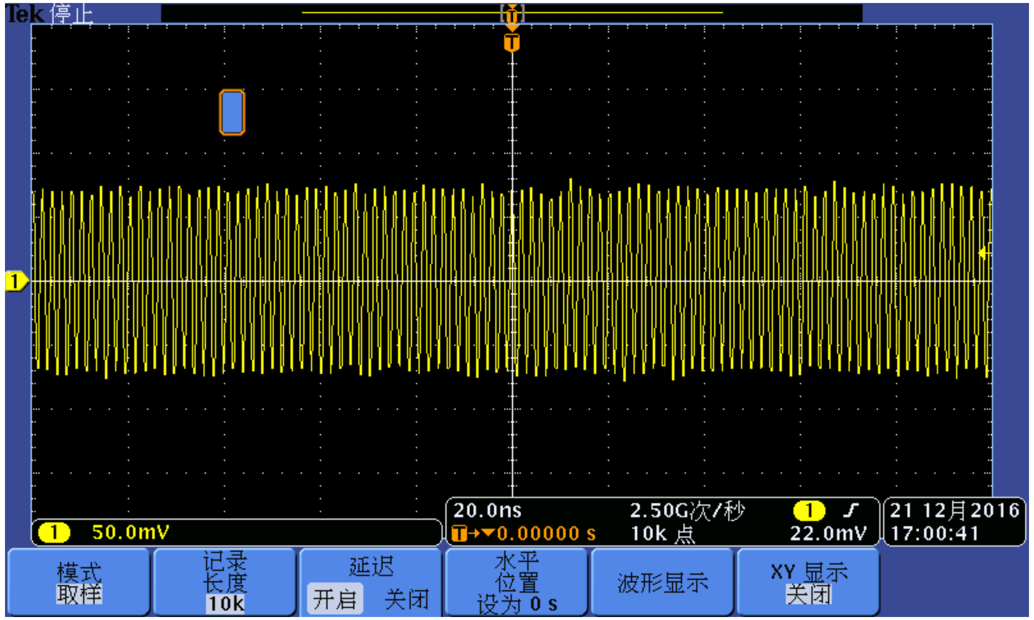
Green Self-mode-locking

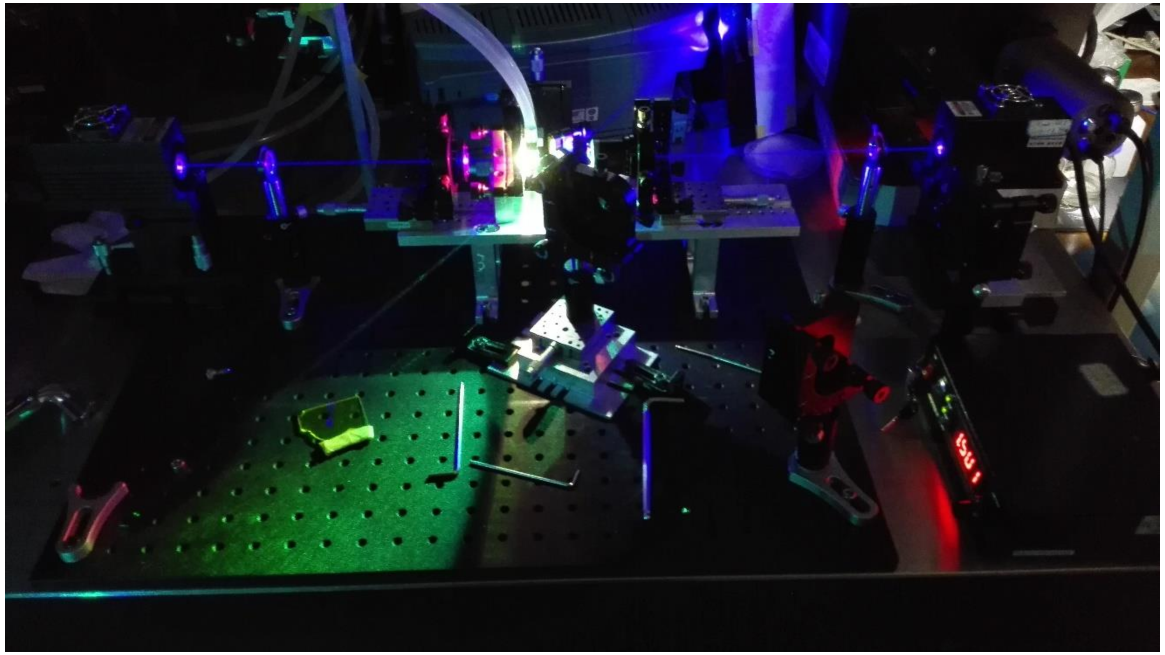


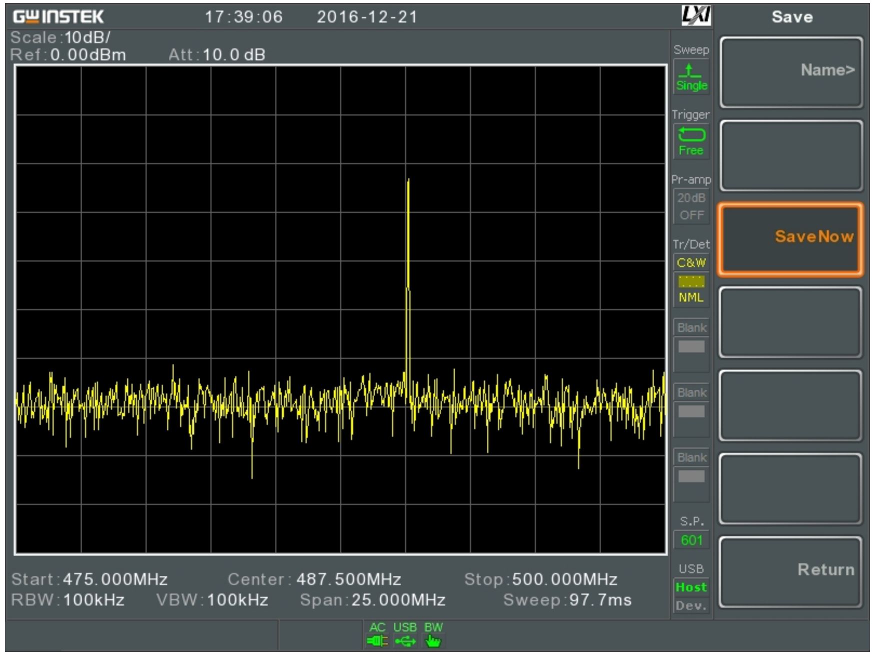


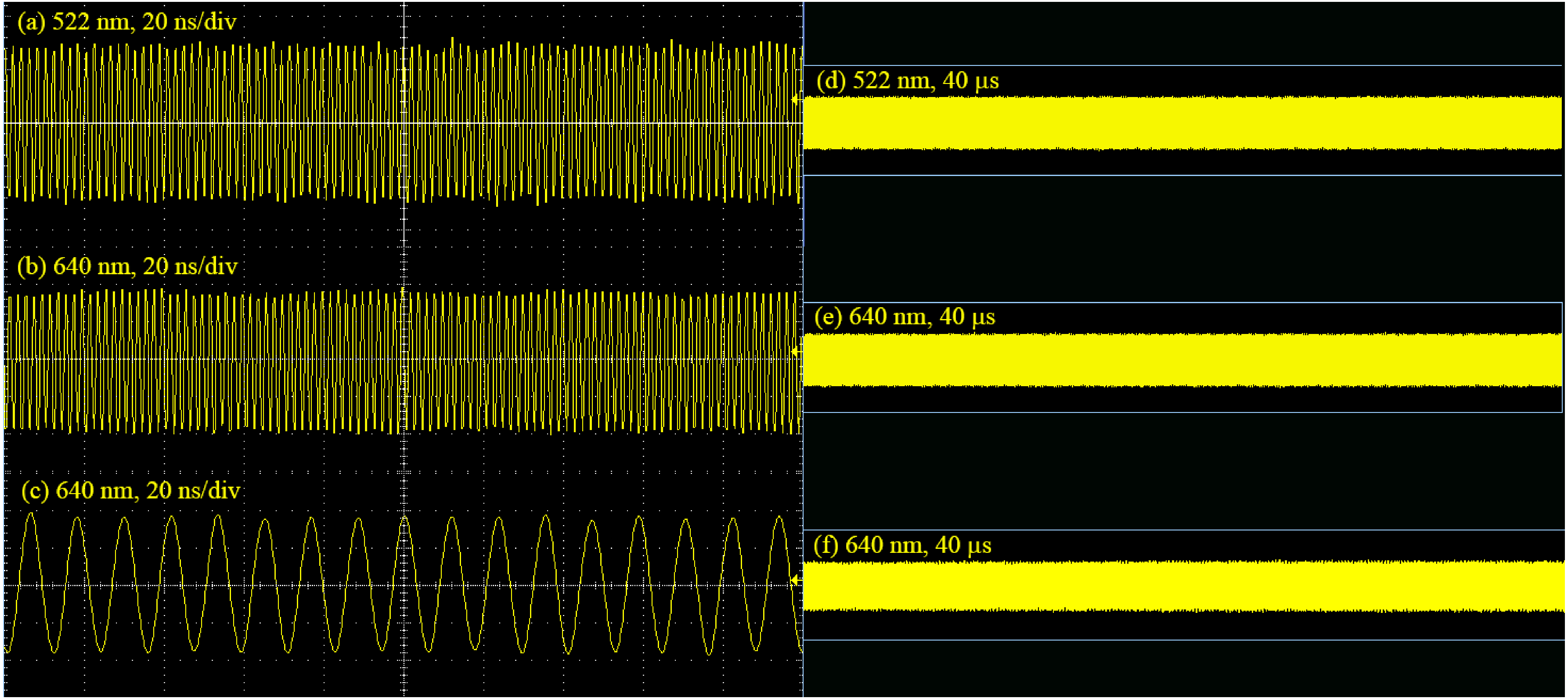


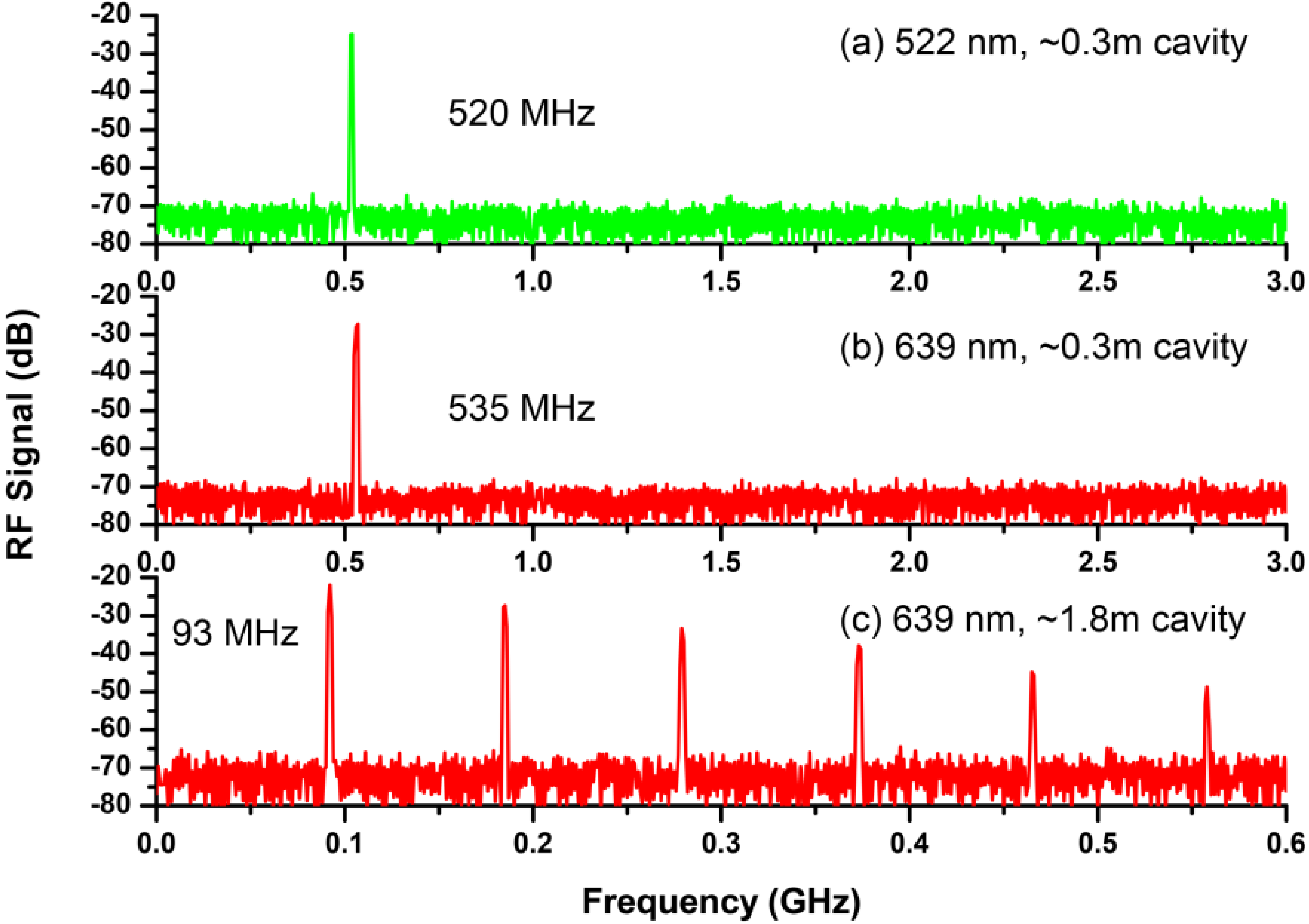



Tek 停止
20.0ns
2.50G次/秒
10k 点
22.0mV
21 12月 2016
17:00:41
50.0mV
0.00000 s
模式
取样
记录长度
10k
延迟
开启 关闭
水平位置
设为 0 s
波形显示
XY 显示
关闭




GWINSTEK
17:39:06
2016-12-21
Scale:10dB/
Ref:0.00dBm
Att:10.0 dB
Save
Name>
Save Now
Return
Sweep
Single
Trigger
Free
Pr-amp
20dB
OFF
Tr/Det
C&W
NML
Blank
Blank
Blank
S.P.
601
USB
Host
Dev.
Start:475.000MHz
Center:487.500MHz
Stop:500.000MHz
RBW:100kHz
VBW:100kHz
Span:25.000MHz
Sweep:97.7ms


(a) 522 nm, 20 ns/div
(b) 640 nm, 20 ns/div
(c) 640 nm, 20 ns/div
(d) 522 nm, 40 µs
(e) 640 nm, 40 µs
(f) 640 nm, 40 µs



(a) 522 nm, ~0.3m cavity
520 MHz
(b) 639 nm, ~0.3m cavity
535 MHz
(c) 639 nm, ~1.8m cavity
93 MHz
RF Signal (dB)
Frequency (GHz)
-20
-30
-40
-50
-60
-70
-80
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.1
0.2
0.3
0.4
0.5
0.6


谢谢！