Mode locking of $Ti{:}Al_2O_3$ lasers and self-focusing: a Gaussian approximation

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We present an *ABCD* matrix model showing that self-focusing in the laser rod leads to modifications of the Gaussian beam parameters in cw-pumped $Ti{:}Al_2O_3$ lasers. Stabilization of self-mode-locking should result from these beam perturbations. Experimental measurements of beam modifications supporting this model are presented. The role of gain guiding is studied, and the limitations of the model are discussed.

Recently Spence *et al.*[1] reported the generation of 60-fs pulses using a cw-pumped $Ti{:}Al_2O_3$ laser. This laser was referred to as self-mode-locked since no passive or active element had to be introduced in the cavity in order to produce short pulses. Different groups[2–4] have reproduced this laser and have obtained similar results.

Self-mode-locking relying on pure self-phase-modulation was used previously to produce picosecond pulses in Nd:YAG lasers.[5,6] Unfortunately, this effect cannot be used alone since it does not provide any discrimination between the pulse and the noise. To use self-mode-locking in a cw-pumped laser one has to introduce an intracavity element that has an intensity-dependent transmission. This can be obtained by using absorption saturation in dyes or solids (usual passive mode locking) or various other nonlinear effects. Interferometrical addition of pulses coming from two coupled cavities,[7–9] intensity-dependent rotation of the polarization,[10–13] and spatial soliton propagation[14] have also been proposed and used with several lasers. We want to show in this Letter that transverse (i.e., spatial) effects due to self-focusing can be used for the same purpose and that such effects can explain the results obtained in $Ti{:}Al_2O_3$ lasers.[1–4,15] The model presented here relies on a Gaussian beam approximation. It cannot include effects such as diffraction by hard apertures or the time dependence of the focusing. Consequently this model merely demonstrates the fact that spatial nonlinear effects creates favorable conditions for stable mode locking. The analysis has the advantage of being simple and clearly shows that self-focusing in the laser rod changes the spatial properties of the laser beam. We also studied the effect of gain guiding and showed that this effect plays a nonnegligible role that cannot be entirely accounted for in our model.[15]

The basic principle used to obtain an intensity dependent transmission in the cavity can be understood by considering a beam incident upon a nonlinear medium followed by an aperture. Because of self-focusing, the nonlinear medium acts as an intensity-dependent lens. The transmission through the aperture will then be higher for a high-power beam than for a low-power beam. A laser cavity can include different kinds of apertures: hard apertures put in the cavity on purpose or soft apertures provided, for example, by the spatial profile of the gain in the cavity. In the following calculations we consider a z cavity similar to those used in Refs. 1–4 (Fig. 1). In principle it can be applied to any other cavity. The nonlinear medium and the gain medium can be the same optical element, or one can use two separate elements placed at different positions in the cavity.[14] The beam produced by $Ti{:}Al_2O_3$ lasers is almost perfectly Gaussian[2] and is considered as near TEM_{00} in this Letter. The size of the TEM_{00} beam in the cavity (hereafter designated as the laser beam as opposed to the pump beam) can be calculated by using the *ABCD* matrices method. When the intensity in the cavity increases, self-focusing due to the optical Kerr effect in the $Ti{:}Al_2O_3$ rod can no longer be neglected. To a first-order approximation, self-focusing of a Gaussian beam can be described as a self-induced quadratic index gradient. This approximation is valid on the axis but may fail in the edges of the beam. Like many other solid-state lasers, cw mode-locked $Ti{:}Al_2O_3$ lasers are pumped by another laser (an argon-ion laser in our case). The Gaussian nature of the pump beam induces a transverse gain profile that is not flat. In the approximation of low gain, the gain profile is Gaussian and can be approximated on axis by a parabola. Once again this approximation fails on the edges of the beam. Gain and self-focusing in a laser rod can be described by a unique complex propagation constant k,

$$k = k_0 - 1/2\, k_2 r^2. \qquad (1)$$

The real part of k_2 corresponds to self-focusing (we used a nonlinear index of $Ti{:}Al_2O_3$ equal to $3 \times$

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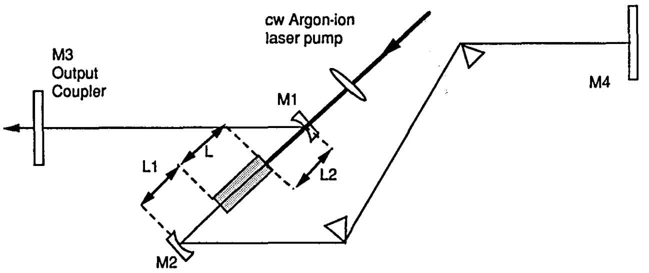

Fig. 1. Schematic of the cavity used in the calculations. The distances are $L1 = L2 = 73$ mm, $l = 20$ mm, and M1M3 = M2M4 = 750 mm. The radius of curvature of mirrors M1 and M2 is 150 mm.

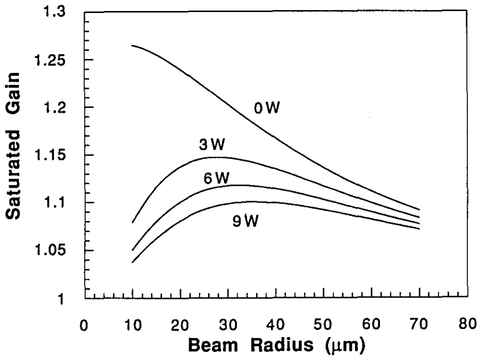

Fig. 2. Saturated gain averaged over the whole beam versus the beam radius for different average powers. The pump beam radius is 50 μm, and the small-signal gain is 0.25. The saturation intensity of Ti:sapphire is 330 kW/cm^2.

10^{16} cm^2/W), and the imaginary part corresponds to gain effects. Assumption of a quadratic radial dependence of k, although it overestimates real effects, makes it easy to write an *ABCD* matrix for a rod of length L,[15,16]

$$\begin{bmatrix} \cos \gamma L & 1/\gamma \sin \gamma L \\ -\gamma \sin \gamma L & \cos \gamma L \end{bmatrix}, \tag{2}$$

where $\gamma = \sqrt{k_2}/k_0$. As self-focusing depends on the beam size, which in turn depends on self-focusing, it is difficult to obtain a closed solution for the beam size in the cavity. Therefore we used an iterative numerical procedure. The calculations begin by computing the complex beam parameter with the assumption of a constant beam size in the rod in order to compute the self-focusing effect. We then segmented the crystal into 100 layers and calculated the *ABCD* matrix for each layer using the local beam parameters of the previous round trip. The beam is propagated back and forth in the cavity until a stable state is reached.

To begin, we only considered the effect of self-focusing (no gain effect). We used symmetrical cavities, but similar results can be obtained with different types of cavity.[4] We found that when the peak power increases, the beam radius at $1/e^2$ calculated at the rod center decreases almost linearly with a slope of approximately -1.5×10^{-5} μm/W. The very small value of the slope means that in general these lasers will not be self-starting. For typical cw self-mode-locked Ti:Al$_2$O$_3$ laser characteristics[2] (100 fs, 80 MHz, 3-W intracavity average power) the beam radius can be reduced by as much as 10%. The size of the beam at any point in the cavity is then intensity dependent. Placing an aperture in the cavity will introduce losses that are intensity dependent. In order to increase the mode-locking strength, the beam size on the aperture has to be smaller at high peak power than at low power. Even without any hard aperture in the cavity an increase in the net cavity gain with peak power can be obtained in this model with the inclusion of the gain spatial profile. The highest gain is obtained at the center of the pump beam, where the pump intensity is the greatest. Reducing the size of the intracavity beam in the gain region increases the gain seen by the laser since most of the energy is then concentrated at the center of the pump beam. In practice, the laser beam saturates the gain, and reducing the beam size does not always increase the gain. The evolution of the gain as a function of the beam radius for a pump waist radius of 50 μm and intracavity average powers ranging from 0 to 9 W is plotted in Fig. 2. This calculation neglects beam divergence but shows that if the initial beam size is large enough, reducing the beam size should increase the gain.

This first approach is, unfortunately, oversimplified, and as we are looking at relatively small changes in the beam characteristics there is no reason to neglect important effects such as gain guiding. The Ti:Al$_2$O$_3$ rod is pumped by a Gaussian beam that induces a higher gain on axis than on the edges. This effect is similar to that obtained with a variable-reflectivity mirror and has been extensively studied in gas lasers.[16–18] Figure 3 shows an example of the low-power beam evolution in the cavity when the gain guiding is included. Notice that even though this is a linear cavity, the beam size depends on the direction of propagation in the cavity. The effect of gain guiding was confirmed experimentally by recording the shape of the beam in the laser cavity described in Ref. 2 and similar to that presented in Fig. 1. Figure 4 gives the horizontal profile of the beam in the cavity, recorded at 20 cm to the right of mirror M2 under non-mode-

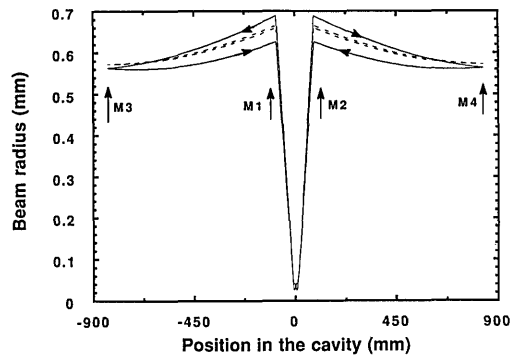

Fig. 3. Evolution of the laser beam radius in the cavity for a pump beam radius of 100 μm. The arrows give the direction of propagation. The solid curve corresponds to a low-peak-power beam ($P = 0$ W), and the dotted curve corresponds to a high-peak-power beam ($P = 150$ kW).

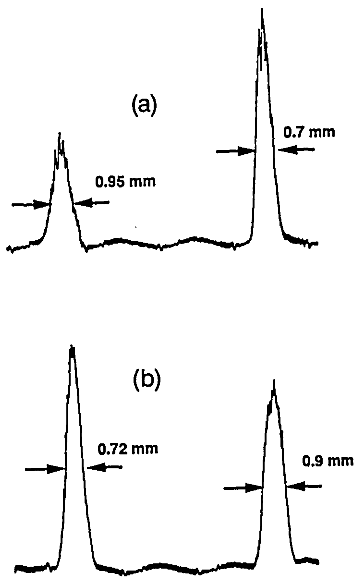

Fig. 4. Horizontal profiles of the beams propagating in our self-mode-locked $Ti:Al_2O_3$ laser in (a) non-mode-locked operation and (b) mode-locked operation. The profiles have been recorded at the first prism position (see Fig. 1). The right trace corresponds to the beam coming from the crystal, and the left trace corresponds to the beam coming back from mirror M4.

locked [Fig. 4(a)] and mode-locked [Fig. 4(b)] operation. Both mode-locked and non-mode-locked beams are modified by gain guiding. The fact that the experimental beam diameters do not correspond to those calculated in Fig. 3 comes from the different cavities used in the simulation and in the experiment and from the spatial evolution of the gain saturation, which cannot be perfectly taken into account in the *ABCD* matrix method.

The evolution of the beam size in the cavity at high peak power (150 kW) is also depicted in Fig. 3. Self-focusing still modifies the beam size. Nevertheless, from the simulations we have conducted, and in the frame of the approximations used in the *ABCD* matrix method, it appears that gain guiding tends to reduce the effect of self-focusing. The differential gain obtained because of the size reduction in the crystal is not sufficient by itself to sustain the mode locking. A hard aperture is needed to obtain stable mode locking. In previous experimental research we have already noticed this effect without understanding the reason. Note that the effect of gain guiding allows one to put an aperture almost anywhere in the cavity. In our laser, we use the apex of a prism located at 70 cm from the rear mirror to aperture the beam. The prism refracts only a part of the intracavity beam, which provides two extra output beams. The average power in these beams compared with that of the beam coming from the output coupler gives the losses introduced by aperturing the beam with the prism. Cw losses were approximately 2%, ten times higher than the mode-locked losses.

In summary we have presented a simplified explanation for the observation of stable self-mode-locking in $Ti:Al_2O_3$ lasers. Although we observed under certain circumstances self-starting generation of 100-fs pulses, self-mode-locking in these lasers is generally not self-starting but is self-sustaining. We think that self-focusing in the $Ti:Al_2O_3$ crystal can decrease the losses in the cavity, preventing the laser from switching back to cw operation. Cw and mode-locked regimes are the only two stable states of these lasers. Cw operation corresponds to high spatial losses but low spectral losses (i.e., narrow spectrum), while the mode-locked regime corresponds to low spatial losses and high spectral losses. Gain guiding limits the range over which the laser will mode lock and the number of laser media in which we can expect to see stable self-mode-locking. It is noticeable that a 100-MHz train of 100-fs pulses with a peak power sufficient to create self-focusing ($P_{peak} \approx 300$ kW $\approx$ self-focusing critical power) in a typical cavity leads to an average intensity ($\sim$300 kW/cm^2) close to the $Ti:Al_2O_3$ saturation intensity, which corresponds to a relatively flat gain profile. In any case, the differential gain obtained by considering only the soft aperture provided by the gain is too small to explain the mode locking, and one has to consider putting apertures in the cavity as experimentally confirmed by our measurements.

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