Self-starting of passively mode-locked lasers

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The self-starting condition of passively mode-locked lasers, including those locked by additive-pulse mode locking, is reexamined. A quantitative model is developed for a threshold intensity for self-starting based on the hypothesis that spurious reflections have to be overcome by coherent injection signals of sufficient amplitude.

Saturable-absorber mode locking and additive-pulse mode locking (APM) are practically important mechanisms for the generation of short optical pulses.[1-11] If these mode-locking processes are self-starting, one avoids the need for other starting mechanisms, such as an active mode locker or synchronous pumping of the gain.

In another paper we investigated the necessary condition for self-starting.[12] It applies both to saturable-absorber action and APM action. According to this condition, the gain cross section has to be small enough compared with the effective saturable-absorber cross section that a temporary peak in the laser radiation experience net gain. Experimentally, it is often found that saturable-absorber cross sections substantially larger than those predicted are required and also that a threshold power of the cw radiation in the laser has to be exceeded. The original paper did not contain such a criterion.

Later, Krausz *et al.*[13] came up with a criterion that contained a threshold power. Briefly, the theory postulated a coherence time for the initial pulse formed at random (for example, by mode beating of two cavity modes). Significant shortening of this pulse must then occur in a time short compared with the coherence time. No model for predicting this coherence time was presented.

In this Letter we propose a specific mechanism that opposes self-starting, give ways of estimating its magnitude, and arrive at an expression for a threshold intensity in terms of measurable parameters.

Every laser resonator contains spurious reflectors that create a multiple Fabry–Perot mode structure of unevenly spaced resonance frequencies. An initial pulse passing through a saturable absorber (or experiencing APM action) is modulated at a frequency equal to $2\pi/T_R$, where T_R is the round-trip time. The initial pulse will continue to grow and get shorter if, and only if, the injection signals produced by this modulation are large enough to lock the unevenly spaced resonator modes to the evenly spaced injection signals.

There is an equivalent interpretation of uneven resonator mode spacings that lends itself more readily to this analysis. A spurious reflection of a mode in a resonator changes the frequency of the mode by producing an injection signal. This injection signal must be overcome by the mode-locking injection signal if a pulse is to evolve from a beat among the many modes. This is the model used in the analysis.

Consider the system of Fig. 1. At a position inside the cavity, there is a reflection with the scattering matrix

$$S = \begin{bmatrix} r & \sqrt{1-r^2} \\ \sqrt{1-r^2} & -r \end{bmatrix}. \tag{1}$$

The incident waves are related to the reflected waves by

$$a_i = -b_i \exp(-j\phi_i), \qquad i = 1, 2. \tag{2}$$

Using the scattering matrix and Eq. (2), one finds the determinantal equation:

$$\exp[j(\phi_1 + \phi_2)] + r[\exp(j\phi_2) - \exp(j\phi_1)] - 1 = 0. \tag{3}$$

If one assumes that $r \ll 1$, one may make approximations and ignore terms of higher order in r. For $r = 0$, one finds that

$$\phi_1 + \phi_2 = 2\pi = 2\frac{\omega_0}{c}(l_1 + l_2). \tag{4}$$

One may set $\omega = \omega_0 + \Delta\omega$ and evaluate $\Delta\omega$ to first order in r:

$$\frac{\Delta\omega}{c}(l_1 + l_2) = r \sin\phi, \tag{5}$$

where

$$\phi = \frac{\omega_0}{c}(l_2 - l_1).$$

One finds that the frequency shift is proportional to r and that it depends sinusoidally on the phase characteristic of the position of the scatterer. Clearly, the sinusoidal dependence had to be expected by virtue of the fact that the scatterer is in a standing-wave pattern of the field.

We can use the above result to estimate the magnitude of the injection signal. Indeed, the equation of the nth mode by itself is

$$\frac{da_n}{dt} = j\omega_n a_n. \tag{6}$$

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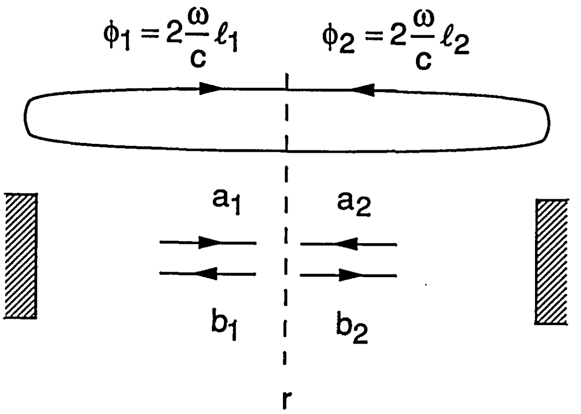

Fig. 1. Schematic of a resonator with reflection.

We normalize a_n so that $|a_n|^2$ is equal to the power traveling in one direction in mode n. In the presence of an injection signal that is due to a superposition of scatterers one has

$$\frac{da_n}{dt} = j\omega_n a_n + j \sum_i r_i(c/l) \sin \phi_i a_n, \tag{7}$$

where $l = l_1 + l_2$. When the modulation that is due to the saturable absorber is weak, as it is at the incipience of mode locking, only the first sidebands are of importance. These sidebands can be estimated by assuming the presence of two neighboring modes n and $n + 1$ of equal amplitude. The injection signals that are due to the random scatterers must be overcome by the injection signal produced by the saturable-absorber (or by APM) action. If the saturable-absorber loss is characterized by $\kappa|a(t)|^2$, the injection signal in the time domain is

$$\Delta a(t) = j\kappa|a(t)|^2 a(t). \tag{8}$$

In the frequency domain, as an injection signal in the equation of motion of the nth mode, this becomes

$$\frac{da_n}{dt} = j\omega_n a_n + j \sum_i r_i(c/l)\sin \phi_i a_n + js, \tag{9}$$

where

$$s = \frac{\kappa}{T_R}(a_{n+1}{}^* a_n)a_{n+1}.$$

The factor $1/T_R = c/2l$ normalizes the injection to the round-trip time of the cavity. If we assume that the two modes share the combined average power traveling in one direction in the resonator P, then

$$\kappa\frac{P}{2} > 2|\sum r_i \sin \phi_i|. \tag{10}$$

Thus we arrive at the result that a minimum power is required for mode locking in the presence of reflections. If more than two modes are present initially, this threshold can be lower. For N equal-amplitude modes ($N \gg 1$) the minimum average power is given by

$$\kappa\frac{(N-1)}{2}P > 2\left|\sum r_i \sin \phi_i\right|. \tag{11}$$

The factor $N - 1$ can be understood by noting that the superposition over all the modes involves a double sum giving a factor of the order of $N(N - 1)$. The power in each mode is proportional to $1/N$; hence the factor of $N - 1$. In the time domain, inequality (11) implies a trade-off between the threshold power and the duration of the shortest intensity fluctuation initially available, $\tau_f \simeq T_R/(N - 1)$:

$$\frac{\kappa P}{2}\frac{T_R}{\tau_f} > 2\left|\sum r_i \sin \phi_i\right|. \tag{12}$$

Turning the problem around, we can estimate the magnitude of the required scattering suppression by noting that peak modulations $\kappa|a(t)|^2$ of the order of unity are achieved in APM lasers when full mode locking has set in and a short pulse of duration τ_p has been produced. Since the average power is approximately independent of pulse duration, in lasers with long gain recovery times this implies an injection level into a_n when only two modes are present of

$$\kappa|a_{n+1}|^2 \simeq \frac{\tau_p}{T_R}. \tag{13}$$

We thus require for self-starting that

$$\left|\sum r_i \sin \phi_i\right| < \frac{\tau_p}{T_R}. \tag{14}$$

This is indeed a severe constraint on the magnitude of the scattering, since (τ_p/T_R) can be of the order of 10^{-5}. It is, most likely, an overly conservative estimate. If we assume that the pulse builds up from the beating of many modes, i.e., from a shorter pulse, then the injection signal from the saturable absorber is larger. Yet, in broad outline, the above argument points up the major importance of eliminating stray Fabry–Perot effects in the resonator. In our experiments we do observe that minor changes of the reflectivity of the fiber end in the auxiliary cavity of an APM system affect self-starting.[9] An apparent threshold for the initiation of mode locking has also been documented.[7]

The explanation of the self-starting threshold seems to give plausible explanations of some other experimental observations as well. It is common practice to start APM systems that do not self-start by moving the mirror of an auxiliary Fabry–Perot or Michelson interferometer. This motion of the mirror is likely to compensate temporarily for the interferometric effect of the internal reflection. Within this time the pulses are given a chance to grow so short that they become self-sustaining. Also, APM systems left alone are known to self-start suddenly. Fortuitous displacements of the scattering points may lead to temporary partial cancellation of the injection signals and thus provide temporarily the condition for self-starting.

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