Mode-locked lasers with nonlinear external cavities

K. J. Blow and David Wood

British Telecom Research Laboratories, Martlesham Heath, Ipswich IP5 7RE, UK

Received August 31, 1987; accepted November 9, 1987

A nonlinear element, when introduced into an external cavity, is shown to improve the mode locking of lasers. Mode locking is achieved by inducing coupling between the cavity modes, thus permitting more efficient transmission of phase information. We begin by discussing a phenomenological laser model for homogeneously broadened systems. The model laser is coupled to an external cavity and contains a nonlinear element. The returning pulse from this external cavity is then mixed with a circulating pulse in the laser at the output mirror. We have considered two nonlinear elements, a saturable absorber and a saturable amplifier. Although these elements have quite different pulse-shaping effects, they both cause considerable improvement in mode-locked performance.

INTRODUCTION

The soliton laser[1] provides a striking example of the power of external cavities. The second cavity, containing a single-mode optical fiber, increases the mode-locked bandwidth of the *F*-center laser, thus permitting the production of much shorter pulses. The initial explanation of this effect was that the laser output stabilized at a point when the field in the optical fiber constituted a double soliton. The temporal width of this soliton was determined by the condition that the propagation distance in the optical fiber was equal to the length at which the soliton returned to its initial shape.[1] However, theoretical[2] and experimental[3] investigations have shown that the operating point is more complicated. Although the optical fiber field is near the double soliton, there is a significant contribution from the radiation (nonsoliton), and the pulse returned to the laser is generally shorter than the output pulse. We are thus led to speculate that the existence of solitons in the external cavity is not the crucial ingredient for successful operation.

In this paper we will study the effect that a simple nonlinear element in an external cavity has on the mode-locked performance of lasers. To see why nonlinearity, in general, should be important let us consider the process of mode locking. A mode-locked pulse train results when the longitudinal modes of the laser cavity have some coherent phase relationship with one another. The simplest condition is that all modes have the same phase. For this to occur there must be some communication among the modes. This mode coupling can be induced in a number of ways, such as loss modulation (as in electro-optic mode lockers) and by synchronous pumping (as in dye and *F*-center lasers). Alternatively, passive mode locking can be achieved in cw-pumped semiconductor lasers by using a saturable absorber in the cavity[4] to provide this communication. We will be considering the situation in which the laser is imperfectly mode locked by some means, such as loss modulation. Then it would be necessary to introduce a bandwidth-limiting element into the cavity to produce transform-limited pulses by preventing lasing in the unlocked modes. Although a nonlinear element can be used in the main cavity to enhance the mode locking, a nonlinear element will also raise the lasing threshold. If, however, the nonlinear element is placed in an external cavity, the mode coupling can be achieved without affecting the lasing properties. When the nonlinear external cavity is added, the number of locked modes can increase in a similar way, and the bandwidth filter can be used to select a desired pulse width.

In the following section we will describe the equations used to model the various elements of a coupled-cavity laser system. We consider a homogeneously broadened gain medium in the laser cavity together with saturable absorbers and saturable amplifiers in the external cavity. We then discuss the results obtained from these models and how the improvements in mode locking are achieved.

MODEL EQUATIONS

In this section we will describe the equations used to model the lasing media, the cavity coupling, and the external nonlinear elements. We begin with the laser model.

In our study of the soliton laser we used an inhomogeneously broadened laser model, the details of which can be found in Ref. 2. Each pulse was represented by its decomposition into the modes of the laser cavity, and the evolution was written as a mapping from one pulse to the next. We use the same approach here but with a model appropriate to a homogeneously broadened laser; however, most of the qualitative features described here do not depend critically on the choice of laser model.

The homogeneous model is based on the active mode-locking theory of Haus.[5] The evolution of the laser field A within the active medium is described by the equation

$$A^{n+1} = \frac{\alpha A'^{n}}{(1+I)(1+\beta^2\omega^2)} + \Delta\,\frac{\mathrm{d}^2 A'^{n}}{\mathrm{d}\omega^2} + N, \tag{1}$$

where $I = \int |A^2|^2 \mathrm{d}t$. The steady-state pulse width results from the competition between the compression induced by the mode locking, represented by the parameter Δ, and the finite bandwidth of the equivalent filter, β (i.e., the effective bandwidth of any intracavity filter and the gain bandwidth). The mode-locking term describes the effect of preferential amplification at the peak of the gain modulation. N is an additive noise source that serves to destabilize the weakly mode-locked states. In the steady state and in the absence

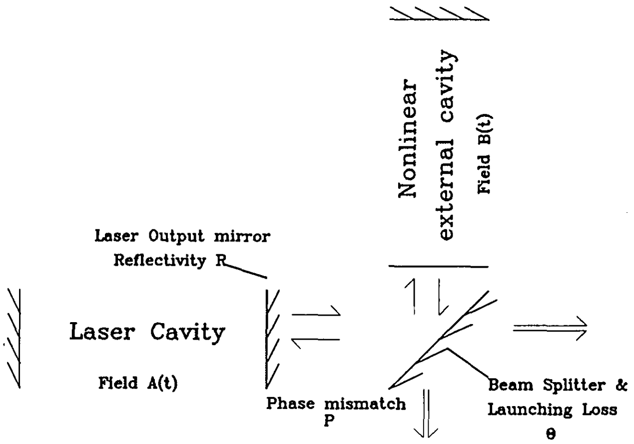

Fig. 1. The two coupled cavities.

of noise, this equation has bound-state solutions, with I as an eigenvalue. In the small β limit, Eq. (1) can be related to the quantum-mechanical harmonic oscillator whose solutions are the Gaussian–Hermite polynomials.[5] The pulse width is then given by $\Delta^{-1/4}(2\beta)^{1/2}$, showing the balance between mode-locking and filtering effects.

COUPLING BETWEEN THE CAVITIES

Figure 1 shows the arrangement of the two cavities. The coupling between the laser cavity field A and the external cavity field B is described by three parameters. The first parameter is the effect of the laser output mirror, which has a field reflectivity R. The second parameter is the combined loss incurred at the beam splitter and on launching into the external cavity, θ. The final parameter, P, is the optical phase mismatch between the two cavities. Our previous study showed that this parameter is important.[2] We assume that the two cavities have the same optical length, apart from subwavelength differences given by the parameter P. These effects lead to the following equations for the coupling of the two cavities:

$$A'(t) = RA - i\theta(1 - R^2)^{1/2}e^{iP\pi}B,$$
$$B'(t) = -i\theta(1 - R^2)^{1/2}A + \theta^2 Re^{iP\pi}B. \tag{2}$$

NONLINEAR EXTERNAL CAVITY

In this paper we use two simple models for the nonlinearity of the external cavity. These models are chosen to illustrate simple physics, and they avoid the computationally intensive integration of the nonlinear Schrödinger equation.[2] In each of these models we assume that the nonlinearity is effectively instantaneous.

The first model that we use is that of a saturable absorber described by

$$B^{n+1}(t) = \frac{|B'^n|^2}{1 + |B'^n|^2} B'^n(t), \tag{3}$$

which gives rise to pulse compression. The second model is that of a saturable amplifier described by

$$B^{n+1}(t) = \frac{1 + |B'^n|^2}{1 + 2|B'^n|^2} B'^n(t), \tag{4}$$

which gives rise to pulse broadening. The transmission functions are defined in time rather than in frequency space. This definition ensures that coupling will occur between the modes of different frequency in the fundamental laser cavity.

These models represent two extreme transmission characteristics. The differences will be reflected in different values of the cavity phase mismatch required to achieve mode locking. In the limit of small θ we can neglect the last term of Eqs. (2), which is $O(\theta^2)$. Thus the returning wave will be in phase with the cavity when $P = 1$ and out of phase when $P = 0$. The saturable absorber is optimum for $P = 1$, as this adds preferentially to the high-intensity parts of the laser cavity field. In contrast, the saturable amplifier is optimum for $P = 0$, as the external cavity field now subtracts preferentially from the low-intensity parts of the laser field. At larger values of θ the situation is more complex because ringing in the external cavity pulls the phase around, and the optimum values of P are shifted.

RESULTS

Equations (1)–(4) define our model, and we can now iterate them to obtain the steady-state solutions. In all our calculations the following points apply:

1. The pulse will be represented in a periodic region of length T, which is smaller than the round-trip time τ. The active mode-locking parameter is scaled so that the results do not depend on T. This scaling was achieved by equating the curvature at $t = 0$ of the real modulation function, $1 + \epsilon \cos(2\pi t/\tau)$, with the model version, $1 + \Delta \cos(2\pi t/T)$, thus yielding the relation $\Delta = \epsilon(T/\tau)^2$.
2. The cavity field is assumed to be symmetric under the transformation $t \rightarrow -t$, and the noise is represented by low-amplitude (symmetric) white noise.
3. The low signal gain α is set to $\alpha = 2$. The principal effect of this parameter is to set the scale of the fields, which is unimportant in the normalized equations used here.
4. The laser bandwidth β is set to $\beta = 0.316$. This parameter sets the time scale of the pulse relative to T.
5. The mirror reflectivity R is set to $R = 0.9$. Smaller values simply give longer transients in the laser evolution.

In Fig. 2 we show the evolution of the laser model in the limit of weak coupling to the external nonlinear medium, $\theta = 0.001$. The mode-locking parameter $\Delta = 0.005$ is chosen to be sufficiently small that the noise term makes a significant contribution to the evolution. The laser pulse settles into a relatively broad state with a rms pulse width of 1.4. The rms width is used at this point because the FWHM is ill defined during the early stages of the evolution. As Δ increases, the laser pulse width decreases, and its shape becomes more Gaussian.

In Fig. 3 we show the evolution of the rms pulse width; curve a corresponds to the parameters of Fig. 2. The FWHM of the final values is roughly 0.7 of the rms width. Curves b and c of Fig. 3 show the effect of the saturable absorber and the saturable amplifier, respectively, with $\theta = 0.8$ in each case. The values of P were chosen to be in the middle of the range over which mode locking was achieved. The external cavity has changed the situation dramatically.

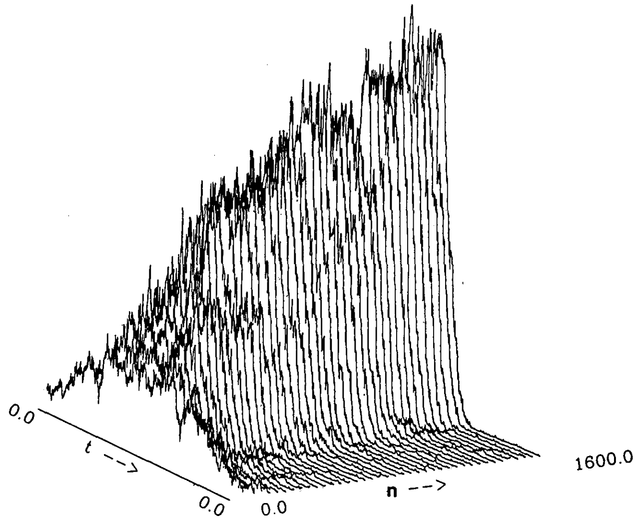

Fig. 2. Evolution of the laser with negligible coupling to the external cavity: $\beta = 0.005, \theta = 0.001$.

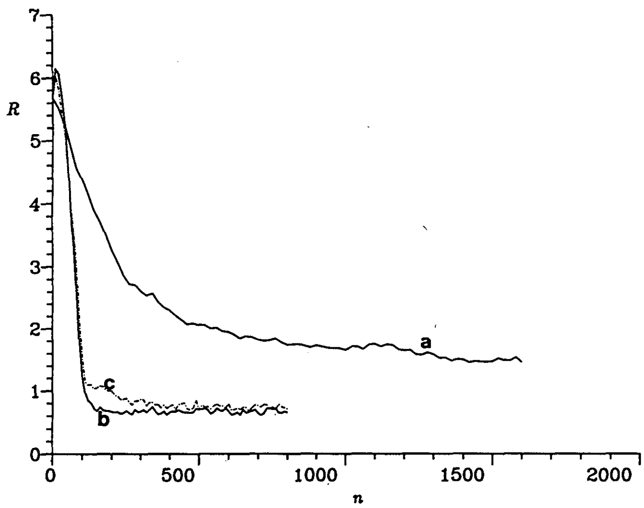

Fig. 3. Evolution of the rms pulse width R as a function of iteration number n: a, as in Fig. 1; b, with the saturable absorber, $\theta = 0.8$; c, with the saturable amplifier, $\theta = 0.8$.

The transient region has decreased, and the final state has a rms pulse width of 0.6, which is determined largely by the filter bandwidth. The mode locking of the laser is dominated by the external cavity. These results show little difference between the effects of pulse compression and pulse broadening in the external cavity. The important effect is the coupling of modes of different frequency in the fundamental cavity, although different values of P were used in each case.

When the active mode locking was small, $0 < \Delta < 0.01$, we found that the laser was passively mode locked by the external cavity for both models of nonlinearity and that the general properties of the output were unaffected by Δ. The only role of the active process, in this limit, is to define the position of the pulse to be at the peak of the gain modulation. When $\Delta = 0$, the model is translationally invariant in time. Consequently, the model is unaffected by a shift in the pulse position, which will perform a slow random walk owing to the effects of the additive noise. However, because of the

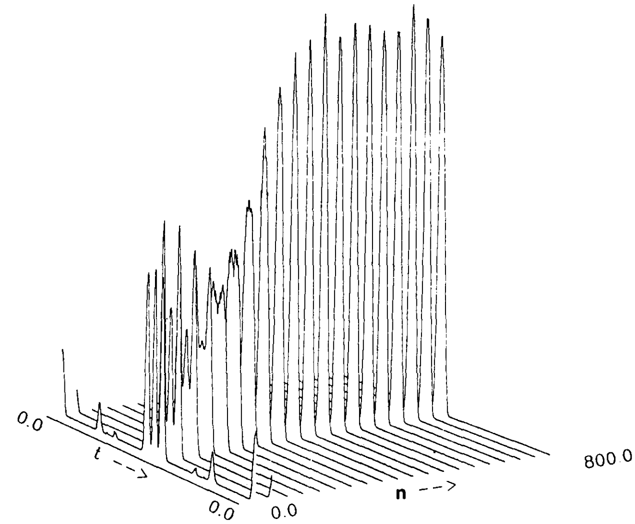

Fig. 4. Evolution of the laser coupled to a saturable amplifier in the external cavity: $\beta = 0.005, \theta = 0.8, P = 0$.

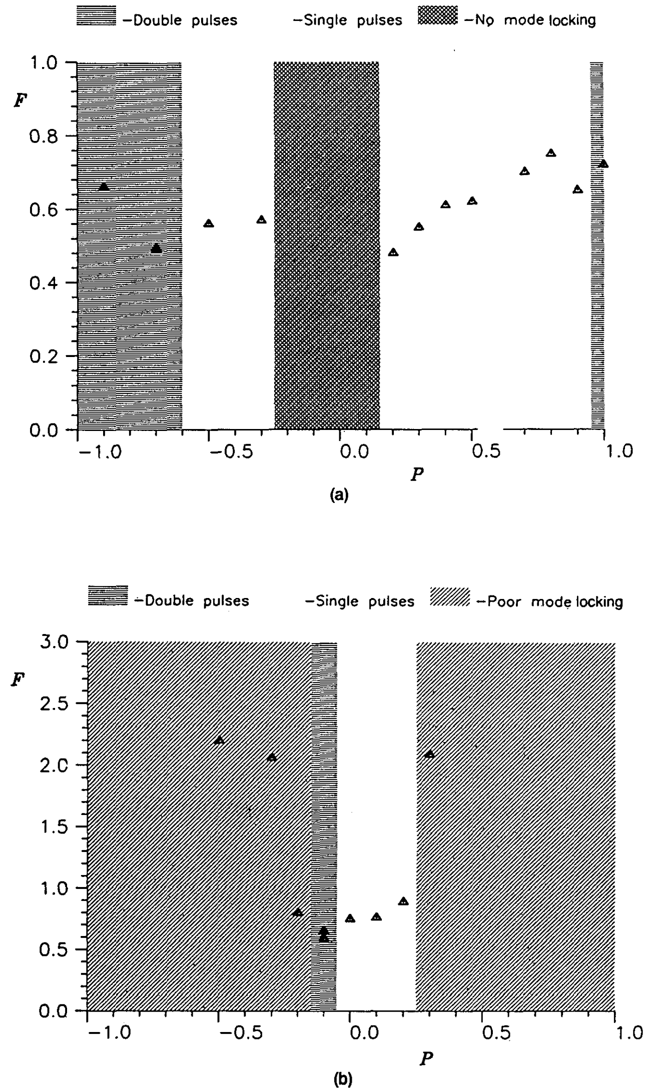

Fig. 5. Steady-state FWHM pulse width F as a function of the cavity mismatch P: (a) saturable absorber, (b) saturable amplifier.

symmetry in our numerical simulations, only two positions correspond to a single pulse: the center and the edge of the periodic region. The final position of the pulse, in this limit, is determined by the initial field, and the effects of noise and pulses were observed in both positions.

To demonstrate further the effectiveness of nonlinear external coupling we show, in Fig. 4, the pulse evolution corresponding to curve c in Fig. 3, i.e., with a saturable absorber. The steady-state pulses are not affected greatly by the noise because all the available energy has been compressed into the laser pulse so that it is now much more intense than the noise term.

We now turn our attention to the effects of the cavity phase-mismatch parameter P. In Figs. 5(a) and 5(b) we show the FWHM pulse width as a function of P for the saturable absorber and saturable amplifier in the external cavity, respectively. These results were taken after 400 iterations, when the evolution had reached the quasi-steady state. The results are periodic in P, which is defined over the range $-1 < P < 1$. The behavior of the saturable absorber and saturable amplifier are different, and we discuss the saturable absorber first. When $-0.25 < P < 0.2$, no mode locking is observed. The pulse width decreases as P approaches 0.2 from above, but the intensity of the pulse decreases, thus making the noise term more important. As P increases (from 0.2) the width increases until $P \sim 0.95$, at which point the laser starts to produce double pulses. In the double-pulsed region we found two distinct stable states with pulses of similar shape but separated by different distances. These states arise from a bifurcation in the single pulsed solution at about $P = 0.95$. The double pulsing continues until $P \sim -0.6$, at which point a further region of clean mode locking is found up to $P < -0.25$. The optimum mode locking, judged from the combination of pulse width and peak intensity, was found to occur at approximately $P = 0.5$.

The results for the saturable amplifier [Fig. 5(b)] show a much narrower region of mode-locked behavior. When $P < -0.15$ and $P > 0.25$, poor mode locking is observed. The pulses are extremely noisy and mostly have widths larger than those obtained with the laser model alone. Several data points were off the scale of Fig. 5(b). A region of good mode locking was found for $-0.05 < P < 0.25$. The pulse width decreases as P approaches -0.05 from above, at which point double pulsing is observed as before with its associated bistability. The optimum mode locking was found to be near $P = 0.1$, and the pulse quality around this value was as good as that obtained with the saturable absorber.

CONCLUSIONS

Our calculations show that a nonlinear external cavity can improve the mode-locked performance of lasers dramatically. This improvement is not limited to the case of a saturable absorber that injects shorter pulses, with a correspondingly larger bandwidth, back into the laser. Rather than being due to the injection of energy into modes far from the peak of the gain, the mechanism is one of inducing coupling between the longitudinal modes of the fundamental laser cavity. This mechanism permits more efficient communication of phase information to the edges of the laser gain bandwidth. To demonstrate this we used a saturable amplifier that returns pulses with a narrower bandwidth to the laser, and we obtained improved mode-locked performance.

Our results show the sensitivity of the mode-locked pulse train to the phase mismatch of the two cavities. Thus it will be necessary, in general, to use cavity stabilization schemes similar to those used in the soliton laser.[6]

When the active component of the mode locking is sufficiently small, the external cavity takes over, and the laser becomes passively mode locked. In this limit the position of the final pulse is determined by the initial field, as the model is now invariant under time translations. This transition to passive mode locking has been observed in the soliton laser,[6] where it was possible to turn off the YAG mode locker, as shown by our calculations.

The results presented here and those of our previous study of the soliton laser[2] indicate that shorter mode-locked pulses can be produced by coupling a laser to an external cavity containing any element that has a nonlinear response to the optical field. This conclusion is also supported by recent results[7] that are based on an integral equation formulation of the problem. We have achieved the same effects with homogeneously broadened laser models and inhomogeneously broadened laser models. Thus the interpretation, in terms of mode coupling induced by the nonlinearity in the external cavity, is a generic description of the mode-locking process.

ACKNOWLEDGMENT

The authors thank the director of British Telecom Research Laboratories for permission to publish.

REFERENCES

1. L. F. Mollenauer and R. H. Stolen, "The soliton laser," Opt. Lett. **9,** 13–15 (1984).
2. K. J. Blow and D. Wood, "Stability and compression of pulses in the soliton laser," IEEE J. Quantum Electron. **QE-22,** 1109 (1986).
3. L. F. Mollenauer, "Solitons in optical fibers and the soliton laser," Philos. Trans. R. Soc. London Ser. A **315,** 437–450 (1985).
4. Y. Silberberg, P. W. Smith, D. J. Eilenberger, D. A. B. Miller, A. C. Gossard, and W. Wiegmann, "Passive mode locking of a semiconductor diode laser," Opt. Lett. 11, 507–509 (1984).
5. H. A. Haus, "A theory of forced mode locking," IEEE J. Quantum Electron. **QE-11,** 323–330 (1975).
6. F. M. Mitschke and L. F. Mollenauer, "Stabilizing the soliton laser," IEEE J. Quantum Electron. **QE-22,** 2242–2250 (1986).
7. P. Belanger, "The soliton laser: a simplified model," J. Opt. Soc. Am. B 5 (to be published).