Self-starting condition for additive-pulse mode-locked lasers

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We analyze self-starting in additive-pulse mode-locked lasers and find that dynamic gain saturation can play a determining role. A simple condition for self-starting is presented for comparison with experiments on different laser systems.

Interest in ultrashort-pulse generation with coupled-cavity lasers was stimulated several years ago by Mollenauer and Stolen's description of the soliton laser.[1] Subsequent experiments[2,3] and numerical simulations[4] by others revealed that soliton behavior is not a necessary ingredient and that lasers coupled to optical fiber cavities with positive dispersion can also produce ultrashort pulses. An analytical description of the general pulse-shortening process, termed additive-pulse mode locking (APM), has since been presented[5,6] and has been used to describe the characteristics of several different coupled-cavity laser systems.[5,7,8] Recently it was discovered experimentally that the process is self-starting with Ti:Al_2O_3,[8] although with color-center lasers it has required synchronous pumping for initiation.[9] It is the purpose of this Letter to analyze the condition for self-starting and to explain the difference between these different experimental results. We find a simple condition for energy growth of an intensity fluctuation in these lasers.

Previous analysis of APM[5,6] has shown how coupling between a laser and an external nonlinear cavity can be modeled as an intensity-dependent reflectivity of the laser end mirror. Under proper adjustment this dynamic reflectivity produces pulse shortening with each reflection until a steady-state balance is achieved with pulse spreading owing to bandwidth limiting and dispersion. Particularly strong shortening occurs when the static phase offset between the two cavity lengths is chosen so that reflectivity varies linearly with fluctuations in the internal laser intensity. Under these conditions, the dynamic reflectivity may be represented as a dynamic change in round-trip gain equal to

$$\Delta g = \kappa s(t), \tag{1}$$

where $s(t)$ is a photon flux perturbation about the average flux S_0 and κ is the proportionality constant that, for the case of APM, depends on the nonlinearity, the loss, and the strength of coupling between cavities. We note that there is a functional equivalence between Eq. (1) and the dynamic gain change produced by a fast saturable absorber[10] inserted into the laser cavity. In both cases, shorter pulses experience increased net overall gain in the laser as well as differential gain between peak and wings.

This increased net gain for the shortest pulse (of a given energy) provides both the amplitude instability and the selection mechanism necessary for evolution to the mode-locked steady state. Under the assumption that the gain of the laser medium has such a long response time that it saturates only with average power, mode locking always self-starts from fluctuations even though the speed with which pulse shortening occurs depends on the duration of the initial fluctuation. This is the condition that has been previously assumed for passive mode locking with a fast saturable absorber.[10]

On the other hand, if dynamic gain saturation is taken into account, the start-up may not be so automatic. Assuming that the recovery time of the gain medium is much slower than the duration of the fluctuation, we find from straightforward rate-equation analysis that our perturbation changes the gain by an amount

$$\Delta g = -\sigma g \int_{-\infty}^{t} s(t)\mathrm{d}t, \tag{2}$$

where g is the saturated gain prior to the perturbation and σ is the emission cross section of the gain medium. We have also assumed that the fractional change in gain is small compared with one and that the intensity fluctuation has a duration that is less than the round-trip time. Integration is simply meant to begin before the fluctuation.

Figure 1 illustrates the dynamic relationship between the gain increase described by Eq. (1) and the saturation described by Eq. (2) for two different cases. The top plot defines the intensity fluctuation for the sequence. In the middle and bottom plots the solid curves and dashed curves indicate the gain changes given by Eqs. (1) and (2), respectively. In the middle plot there is no gain saturation ($\sigma = 0$) and the positive intensity fluctuation creates a window of increased gain for itself, indicated by the shaded region. In the bottom plot we see how an increase in σ results in a decrease in the region of self-induced gain.

We now test for the stability of a short (pulselike) temporal increase of intensity on a cw intensity. The net change in gain it produces is obtained by adding Eqs. (1) and (2). To first order, this then induces an additional intensity change by modulating S_0, the average intensity. We determine whether this induced change increases the energy of the original perturba-

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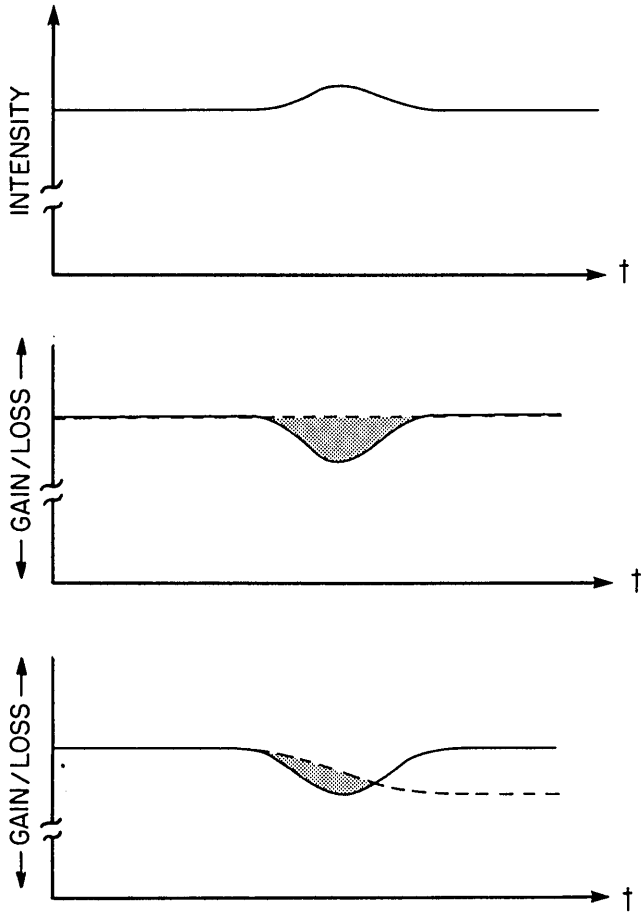

Fig. 1. Dynamic pulse gain achieved by APM without gain saturation ($\sigma = 0$; middle plot) and with gain saturation (bottom plot). The shaded areas indicate regions of net instantaneous gain resulting from the difference between the APM response (solid curves) and the response of the gain medium (dashed curves). The top plot shows the assumed intensity perturbation.

tion by calculating its overlap with $s(t)$. Thus, summing Eqs. (1) and (2), multiplying by $s(t)$, and integrating over the fluctuation, we find the condition for the perturbation to be increased,

$$\kappa \int_{-\infty}^{\infty} s^2(t)\mathrm{d}t - \sigma g \int_{-\infty}^{\infty} s(t) \int_{-\infty}^{t} s(t')\mathrm{d}t'\mathrm{d}t > 0, \qquad (3)$$

or, more simply,

$$\kappa/g > \sigma \left\{ \frac{\left[\int_{-\infty}^{\infty} s(t)\mathrm{d}t\right]^2}{\int_{-\infty}^{\infty} s^2(t)\mathrm{d}t} \right\}. \qquad (4)$$

It is assumed that $s(t)$ is well contained within a single round-trip period of the laser. Integration is meant only to encompass the fluctuation but is written from $-\infty$ to $+\infty$ for analytical convenience. An analogous derivation applies for a short temporal decrease of intensity. The changes given by Eqs. (1) and (2) both change sign. A net decrease in gain then enhances the temporal decrease and makes it unstable as well. It is this instability of the perturbations, either positive or negative, that leads to pulse formation.

Examination of the pulse-shape-dependent quantity in the braces of relation (4) yields

$$\frac{\left[\int_{-\infty}^{\infty} s(t)\mathrm{d}t\right]^2}{\int_{-\infty}^{\infty} s^2(t)\mathrm{d}t} \qquad (5)$$

where τ_p is the pulse width of $s(t)$ and β is a numerical factor that depends somewhat on pulse shape but is of order unity. For example, if τ_p is defined as a FWHM, $\beta = 0.85$ for a sech^2 shape and $\beta = 0.75$ for a Gaussian shape.

Finally from relations (1)–(5) we derive the simple start-up condition,

$$\kappa/g > \beta\sigma\tau_p, \qquad (6)$$

which reveals that the gain cross section of the laser medium is a key parameter for start-up. Clearly, additional active mode locking can help greatly by introducing a short pulse width τ_p. Purely passive start-up, however, depends on a randomly generated pulse (either from noise or mode beating), and the importance of σ is dominant. In this light it is clear why $Ti{:}Al_2O_3$, which has an emission cross section of 2.7×10^{-19} cm^2,[11] should self-start more easily than the KCl:Tl°(1) color-center laser, which has a cross section of 1.3×10^{-17} cm^2.[12]

The analogy of APM with mode locking by a fast saturable absorber is worth some additional comments. Although fast saturable absorber mode locking has been effective with some flash-lamp systems, it has not been realized in cw operation. This is due, in part, to the conflicting requirements of fast response, large absorption change, and low insertion loss. The present analysis shows that, in addition, the self-starting of a system containing a fast saturable absorber is not guaranteed. The APM system, because it contains more adjustable parameters than one has with a physical absorber, is more easily used to satisfy several requirements simultaneously. Start-up may be made easier by making the fiber longer or, more generally, by increasing the nonlinearity of the coupled cavity. Reflectivity of the coupling mirror and loss in the auxiliary cavity can also be optimized for a given loss of the laser cavity.

Since dynamic gain saturation is neither required nor desirable for APM, the scheme is well suited for use with laser media that have long upper-state lifetimes. With some such media, however, the nonlinearity responsible for APM may also produce Q switching. Successful design of an APM system therefore depends on a selection of parameters that favors self-starting in the sense discussed above but avoids a Q-switching instability.[13] In this regard, one difference between APM and the fast saturable absorber analogy should be noted. The coupled-cavity nonlinearity induces dynamic phase shifts as well as changes in the effective gain. On short pulses (fluctuations shorter than a round-trip period) these dynamic shifts produce a slight chirp. With slower intensity changes they may be viewed as mode-pulling, or frequency-shifting effects. These latter effects may therefore also play a role in the overall laser buildup process.

Finally, we note that if the nonlinearity in the auxiliary cavity is slow rather than fast, the change described by Eq. (1) follows the integral of the intensity rather than the intensity itself. Then the start-up condition becomes independent of the initial pulse width and analogous to the condition usually given for slow absorber passive mode locking: that the effective cross section of the saturable absorber be greater than that of the gain.[14,15] Gain saturation then helps and is required.

In summary, we have presented a condition for passive start-up of ultrashort-pulse generation in APM lasers. It supplements previous conditions given for pulse shortening in these systems and explains some reported experimental differences. It also supplements earlier research on fast saturable absorber mode locking by pointing out the influence of dynamic gain saturation.

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