Structures for additive pulse mode locking

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The theory of additive pulse mode locking (APM) is developed in closed form under a linearization approximation. The pulse parameters are determined in terms of the gain, the gain dispersion, the group-velocity dispersion, and the self-phase modulation. Stability regimes are established. Various possible configurations of laser systems that can produce APM are presented. Some of them permit single-cavity realizations that may not require interferometric length stabilization circuits. In general, the results are applicable to a wide range of fast saturable absorber mode-locked systems.

1. INTRODUCTION

Additive pulse mode locking (APM) has been employed successfully for short-pulse production from several solid-state lasers. In APM, pulse shortening occurs by the coherent addition of self-phase-modulated pulses.[1,2] This pulse-shortening mechanism produces a mode locking similar to fast saturable-absorber mode locking. The saturable-absorber action is extremely fast, since it is based on the self-phase modulation (SPM) of the Kerr nonlinearity. In addition, the effective magnitude of the nonlinearity or the effective cross section of the saturable absorber can be designed explicitly by variation of passive cavity parameters. Thus APM can be applied to a wide range of lasers.

In this paper we expand APM theory to consider the case in which the laser contains SPM and dispersion in addition to APM pulse shaping. With negative dispersion, SPM can also produce a solitonlike pulse compression mechanism that significantly enhances the laser performance. We then extend APM theory to consider different laser geometries including the nonlinear Michelson and Mach–Zehnder interferometers as well as single-cavity designs using Kerr effect polarization rotation. This investigation provides the theoretical basis for the application of APM to a wide range of solid-state and fiber lasers. The results are, in fact, applicable to any fast saturable-absorberlike mode-locked system.

The precursor to the current APM laser was the soliton laser invented by Mollenauer and Stolen.[3] The soliton laser generated ultrashort pulses from a mode-locked color center laser by using soliton pulse shaping in an optical fiber contained in an external optical cavity. The soliton laser is also significant since it was one of the first demonstrations of all-optical feedback for the control of femtosecond laser operation. It was originally conjectured that the negative dispersion and the soliton pulse shaping were required for ultrashort-pulse generation in the soliton laser. However, recent investigations have shown that this technique can be generalized to a wide range of laser systems.

Computer simulations by Blow and Wood[4] and experiments by Blow and Nelson[5] and Kean *et al.*[6] demonstrated that short pulses could also be obtained in the positive-dispersion regime of the fiber. A number of subsequent investigations were performed with color-center lasers demonstrating dramatic pulse shortening from 20 ps to approximately 100 fs in NaCl (Ref. 7) as well as KCl (Ref. 1) systems. A unifying analytical theory for the mode locking, termed additive pulse mode locking, followed.[1,2] Self-starting APM was observed in Ti:Al_2O_3, permitting ultrashort-pulse generation without the need for external modulation,[8,9] and a theoretical criterion for achieving self-starting was developed.[10] Self-starting APM was then extended to other laser systems, including lamp-pumped and diode-pumped Nd:YAG, Nd:YLF, and Nd:glass.[11–14] These investigations demonstrate that APM can be applied to a wide range of solid-state lasers. Since APM provides fast saturable-absorberlike mode locking, it can be used to generate much shorter pulses than those previously possible. Pulses as short as 1.7 ps and 800 fs have been obtained in Nd:YAG and Nd:glass lasers, respectively.[11,14]

Recently, ultrashort-pulse generation in the Ti:Al_2O_3 laser has become a particularly active and important area of investigation. These experiments suggest that intracavity SPM and group-velocity dispersion (GVD) are important for determining ultrashort-pulse generation performance. Ti:Al_2O_3 is an excellent model laser system for the investigation of mode locking in solid-state lasers, since it has a broad gain bandwidth (tunable from 670 nm to greater than 1000 nm) and can produce high output and intracavity powers. With the use of APM, pulses as short as 700 fs have been generated directly; pulse durations as short as 200 fs were produced with the use of an external diffraction grating pair for chirp compensation.[8] Pulses as short as 200 fs have been produced with APM and intracavity dispersion compensation with a prism pair.[15] Active mode locking using acousto-optic modulation has generated pulses of 6 ps, with pulses of 1.3 ps possible when intracavity dispersion compensation is provided by a Gires–Tournois interferometer.[16] Passive mode locking using saturable-absorber dyes has resulted in pulses as short as 2.3 ps, and further shortening of the pulses was achieved with intracavity dispersion compensation.[17] Investigators have suggested that intracavity

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SPM and negative GVD produce a solitonlike pulse-shortening mechanism.[17]

Recently, Spence *et al.*[18] demonstrated mode locking in $Ti:Al_2O_3$, using a single laser cavity without a saturable absorber, an active modulator, or a nonlinear external cavity. Stable ultrashort pulses as short as 2 ps were generated when the laser was misaligned so that two modes were simultaneously oscillating. Pulses as short as 90 fs were generated with the use of intracavity dispersion compensation. This discovery suggests the importance of intracavity SPM and dispersion compensation.

Other researchers have investigated a variety of techniques for obtaining pulse shaping and mode locking, using SPM and Kerr effect nonlinearities. A nonlinear Michelson interferometer has been demonstrated for mode locking of the CO_2 laser.[19] Fiber loop mirrors using an antiresonant ring configuration have been investigated theoretically and experimentally.[20,21] Mode locking has been demonstrated with a novel figure-eight fiber laser.[22] Finally, the Kerr effect has been investigated theoretically and proposed for pulse shortening and mode locking.[23]

If sufficient intracavity peak intensities can be achieved, a wide range of mode-locking geometries, including nonlinear Michelson and Mach–Zehnder geometries as well as single-cavity geometries using Kerr effect polarization rotation, should be possible for use with bulk solid-state lasers. The use of single-cavity geometries is particularly attractive, since mode locking can be achieved without the need for interferometric cavity length stabilization.

In this paper we give a short review of the theory of APM with two cavities. We use a new ansatz to solve for the pulse parameters, one that is an exact solution to the problem at hand, rather than the assumed Gaussian pulse shape of the original APM paper. Whereas there is no qualitative change in the result, quantitatively there are some differences.

Next, we generalize the class of APM systems by including the nonlinear (Mach–Zehnder) interferometer as a means of synthesizing an artificial saturable absorber. The advantage of this scheme is that it can be accomplished without the use of two separate cavities that normally call for stabilization of the optical path difference in the two cavities. Of course, the path difference in the two arms of the interferometer still must be kept fixed. Sometimes this can be accomplished by a clever design of the system so that the path difference is automatically maintained constant.

From the generic interferometric system one may derive many equivalent systems, some realized with a single cavity, some with coupled cavities. We set up a general menu of APM systems that are all equivalent and characterizable by four parameters that explain the entire cw operation of the system. We concentrate on the case of a long relaxation time of the laser medium, so that the gain is approximately constant within the time interval of one single pulse. Our paper emphasizes an aspect of APM that may not have received sufficient attention before, namely, that the role of the artificial saturable absorber is not only to provide pulse shaping; this can also be done by SPM and negative dispersion (maybe produced artificially with a prism pair). A key part of the saturable absorber's role is to provide stabilization of the pulses against prepulsing and postpulsing that would arise if the gain level preceding or following the main pulse were above the loss level. We show further that, as in the case of active mode locking, in which the addition of a Kerr medium led to pulse shortening by a factor of 2 before the system was driven unstable,[24] the APM system can benefit from SPM by a factor of 2.75.

2. GENERAL EQUATION FOR A FAST SATURABLE ABSORBER AND ADDITIVE PULSE MODE LOCKING

It was pointed out before[1,2,8] that APM is similar to mode locking with a fast saturable absorber.[25] The SPM of the Kerr medium is transformed (at least in part) into an amplitude modulation that provides smaller loss for high intensities and larger loss for small intensities. In what follows we shall set up the master equation for fast saturable absorber mode locking, supplemented with the effects of GVD and SPM. We shall solve the equation and investigate its stability properties. Then we shall show that this equation describes the operation of APM with an auxiliary cavity. Finally, we shall see that a class of nonlinear interferometric systems obeys the same master equation. First, let us consider a simple ring resonator in which there are elements that produce linear loss and phase shift, dispersive gain, GVD, SPM, and saturable absorption (see Fig. 1).

Denote the complex amplitude of the temporal envelope of the electric field of frequency ω_0 by $a[=a(t)]$. The (small) linear loss and phase shift per pass produce a change in a, Δa, that is a complex multiple of a:

$$\Delta a = -(l + jx)a \tag{2.1}$$

(the power loss per pass is clearly $2l|a|^2$). If the gain per pass is small, its effect on the field amplitude is[25]

$$\Delta a = g\left(1 + \frac{1}{\Omega_g{}^2}\frac{d^2}{dt^2}\right)a, \tag{2.2}$$

where Ω_g is the bandwidth of the gain line approximated to be parabolic. The GVD produces the change

$$\Delta a = jD\frac{d^2}{dt^2}a, \tag{2.3}$$

where D describes the magnitude of the dispersion. If the

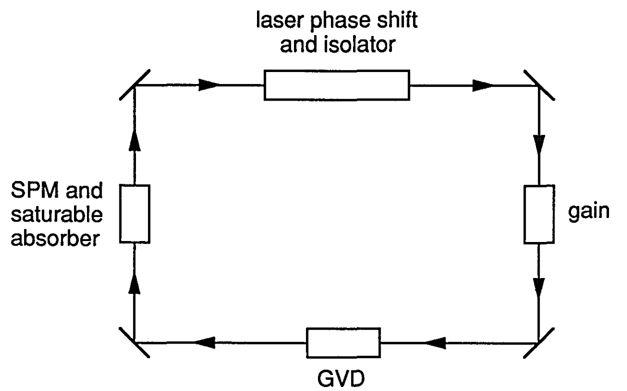


Fig. 1. Schematic of ring cavity with gain, gain dispersion, SPM, GVD, fast saturable absorption, and linear loss and phase shift.

second derivative of the propagation constant of a fiber of length d is k'', then its dispersion parameter D is

$$D = \frac{1}{2}k''d\,.$$

SPM causes a phase shift that is proportional to $|a|^2$:

$$\Delta a = -j\delta|a|^2 a \tag{2.4}$$

(e.g., if a fiber of length d has a nonlinear index n_2, and $|a|^2$ is normalized to give the power, then

$$\delta = \frac{\omega_0}{c}\frac{n_2 d}{\mathcal{A}_{\text{eff}}},$$

where $\mathcal{A}_{\text{eff}}$ is the effective cross section of the mode). The saturable-absorber action is described by

$$\Delta a = \gamma|a|^2 a, \tag{2.5}$$

where γ is inversely proportional to the saturation intensity; γ must be positive so that the loss is reduced with increasing intensity.

In addition, there may be a small carrier frequency shift away from exact alignment with one of the Fabry–Perot resonances of the linear resonator by itself. Such a carrier frequency shift $\Delta\omega_0$ expresses itself in a phase shift per pass $\psi = (\Delta\omega_0/c)L_{\text{eff}}$, where L_{eff} is the effective optical length of the resonator. The associated change Δa is

$$\Delta a = \exp(-j\psi)a - a = -j\psi a. \tag{2.6}$$

We assume that all effects per pass are small and, therefore, additive. In the steady state all changes must add to zero. This fact leads to the master equation:

$$\left[-j\psi - (l + jx) + g\left(1 + \frac{1}{\Omega_g^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right) + jD\frac{\mathrm{d}^2}{\mathrm{d}t^2} + (\gamma - j\delta)|a|^2\right]a = 0. \tag{2.7}$$

It turns out, as we shall show in Sections 4 and 5, that all APM systems to be discussed henceforward obey the same master equation following linearization of the Kerr action. Before we discuss them, we shall look carefully at the solution of Eqs. (2.7) and its stability.

3. SOLUTION OF THE MASTER EQUATION

There is an exact solution of Eq. (2.9), as previously recognized by Martinez *et al.*[26] We introduce the ansatz

$$a = A\,\text{sech}(t/\tau)\exp[j\beta ln\ \text{sech}(t/\tau)] \tag{3.1}$$

into Eq. (2.9). Note that the pulse is described by three parameters: the pulse width τ, the chirp parameter β, and the pulse amplitude A. Thus the ansatz (3.1) permits chirped-pulse solutions. The second derivative of the ansatz is

$$\tau^2\frac{\mathrm{d}^2}{\mathrm{d}t^2}a = \left[-(2 + 3j\beta - \beta^2)\text{sech}^2\left(\frac{t}{\tau}\right) + (1 + j\beta)^2\right]a. \tag{3.2}$$

The master equation reduces to an equation involving a constant multiplier of the ansatz and a sech^2 multiplier. Setting them equal to zero individually, we obtain two equations:

$$-j\psi + g - l - jx + \frac{(1 + j\beta)^2}{\tau^2}\left(\frac{g}{\Omega_g^2} + jD\right) = 0, \tag{3.3}$$

$$\frac{1}{\tau^2}\left(\frac{g}{\Omega_g^2} + jD\right)(2 + 3j\beta - \beta^2) = (\gamma - j\delta)A^2. \tag{3.4}$$

These are two complex equations that can be solved for four unknowns. The first two unknowns are the pulse width and the chirp parameter, which can be found from Eq. (3.4) for a known amplitude A of the pulse. Equation (3.3) serves to determine the unknown phase shift ψ and the gain g. Now, the gain itself is a function of the energy of the pulse if the gain-relaxation time is long compared with one round-trip time. To lowest order, one may determine the gain g by requiring it to balance the loss ($g = l$), ignoring at first the changes of gain that are due to the finite bandwidth of the pulse. Suppose that the gain saturation is adequately described by the formula

$$g = \frac{g_0}{(1 + 2A^2\tau/P_s T_R)} = \frac{g_0}{1 + W/P_s T_R}, \tag{3.5}$$

where $W = 2A^2\tau$ is equal to the energy of the pulse, P_s is an effective saturation power, and T_R is the round-trip time. From Eq. (3.5) one may determine a value of $2A^2\tau$, which can then be introduced into Eq. (3.4) to find the pulse width and the chirp parameter.

When the energy W is assumed to be determined by the requirement $g = l$, it is appropriate to introduce the normalized pulse width

$$\tau_n = (W\Omega_g^2/2g)\tau \tag{3.6}$$

(note that τ_n is not dimensionless) and the normalized dispersion parameter

$$D_n = (\Omega_g^2/g)D. \tag{3.7}$$

With these normalizations we may write

$$(1/\tau_n)(1 + jD_n)(2 + 3j\beta - \beta^2) = \gamma - j\delta. \tag{3.8}$$

This equation gives only one solution for the chirp parameter β and the pulse width τ_n. We find for the chirp parameter, equating real and imaginary parts,

$$\frac{3\beta}{2 - \beta^2} = \frac{\delta + \gamma D_n}{\delta D_n - \gamma} \equiv \frac{1}{\chi}. \tag{3.9}$$

Solving for β, one finds that

$$\beta = -\frac{3}{2}\chi \pm \left[\left(\frac{3}{2}\chi\right)^2 + 2\right]^{1/2}. \tag{3.10a}$$

Only one of the two signs is acceptable because both the real and imaginary parts of Eq. (3.4) must balance with the proper sign (see Table 1). Having determined the chirp parameter β, one may calculate the normalized pulse width from Eq. (3.8):

$$\tau_n = \frac{2 - 3\beta D_n - \beta^2}{\gamma} = \frac{-2D_n - 3\beta + D_n\beta^2}{\delta}. \tag{3.10b}$$

Table 1. Signs of β and Signs of Square Root

		$\delta+\gamma D_n$ +	$\delta+\gamma D_n$ −
$\delta D_n-\gamma$	+	$\beta^2>2$; $\chi>0$; $\beta<0$; −	$\beta^2>2$; $\chi<0$; $\beta>0$; +
	−	$\beta^2<2$; $\chi<0$; $\beta<0$; −	$\beta^2<2$; $\chi>0$; $\beta>0$; +

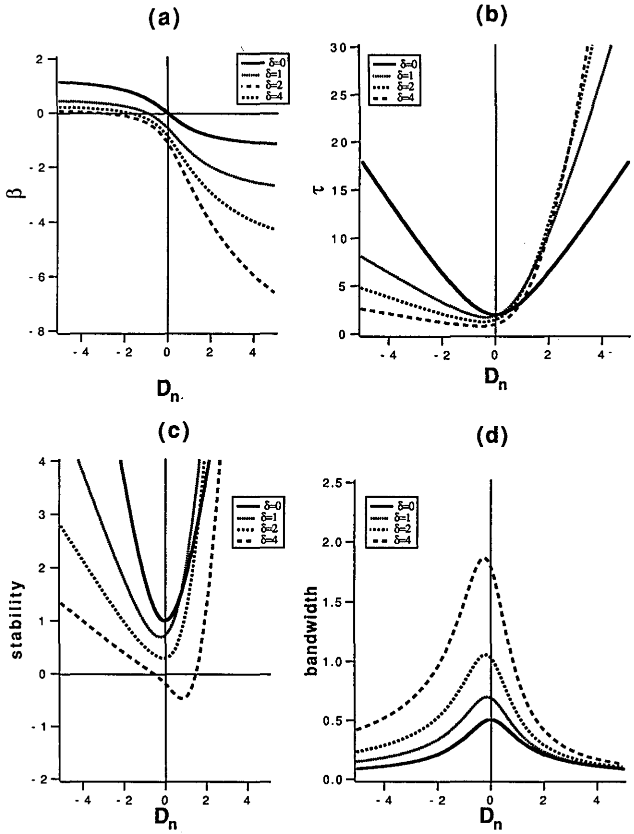


Fig. 2. Pulse parameters versus normalized dispersion D_n for $\gamma = 1$ and different SPM parameters δ: (a) the chirp parameter β, (b) the inverse normalized pulse width τ, (c) the bandwidth $(1 + \beta^2)^{1/2}/\tau$, (d) the stability criterion.

The normalized bandwidth is given by the pulse duration and the chirp:

$$w_n = (1 + \beta^2)^{1/2}(1/\tau_n). \tag{3.10c}$$

Figure 2 shows the normalized pulse width τ_n, the chirp parameter β, and the bandwidth w_n as functions of the normalized dispersion for $\gamma = 1$ and different values of the SPM parameter δ. Figure 3 shows similar families of curves for a constant SPM parameter δ and different values of the saturable absorption parameter γ. Two distinct regimes of short-pulse operation can be observed corresponding to positive and negative GVD, respectively.

For positive GVD the chirp parameter β can become quite large. This is an interesting operating regime worth consideration in detail. The equation for the pulse parameters, [Eq. (3.4)] gives a contribution to the pulse width from gain dispersion that changes sign, an indication that the gain dispersion, instead of lengthening the pulse, shortens it. At first sight this may seem paradoxical. However, if a pulse is chirped, then frequency filtering can, in fact, shorten it if it shaves off the high- and low-frequency wings. The pulse is kept in balance by a lengthening that is due to positive GVD. Of course, the gain dispersion also narrows the pulse spectrum. This spectral narrowing is compensated for by SPM. The pulse duration is strongly influenced by the saturable absorption or the APM parameter γ (Fig. 3).

For negative GVD, short-pulse durations can be generated that are relatively chirp free. In this case SPM and negative GVD produce the pulse compression or solitonlike pulse-shaping mechanism.[17] Pulse shaping in colliding-pulse mode-locked (CPM) ring dye laser and in Ti:Al_2O_3 lasers with negative intracavity GVD has previously been attributed to this effect.[17,27,28] Increasing the magnitude of the SPM parameter δ (Fig. 2) has a strong effect on the pulse duration in the negative GVD regime. In contrast, increasing the saturable absorption or the APM parameter γ has a much weaker effect (Fig. 3).

Appreciable pulse shortening seems feasible with the introduction of additional SPM. However, one must note that the entire range of values of pulse widths is not acceptable, because not all the pulse widths are stable. Stability prevails only if the gain before and after the pulse is less than the loss. If this is not the case, noise perturbations preceding or following the pulse will have a chance to grow and to destroy the pulse. The stability criterion follows from the real part of Eq. (3.3) because it gives the gain when a pulse is present ($1/\tau^2 \neq 0$) and when

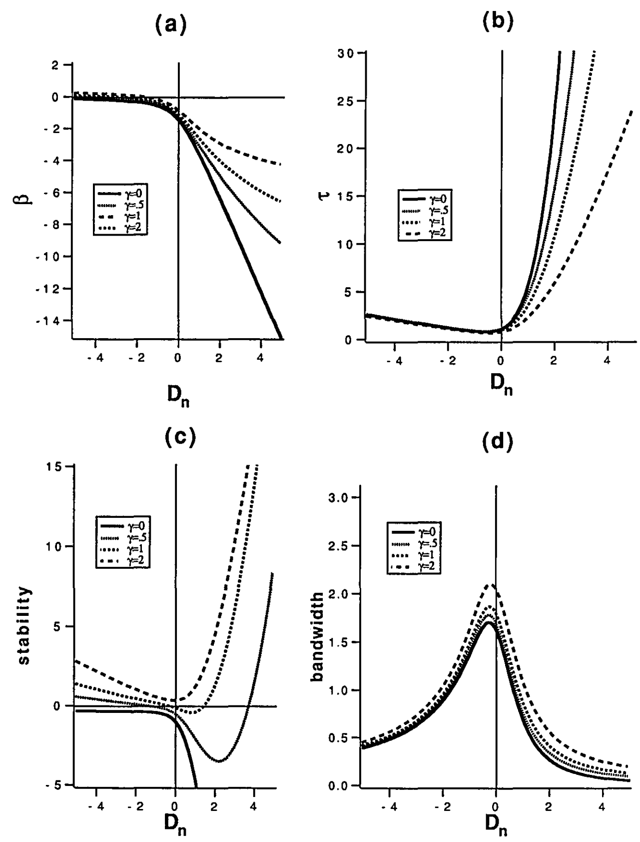


Fig. 3. Pulse parameters versus normalized dispersion D_n for $\delta = 4$ and different saturable absorption parameters γ: (a) the chirp parameter β, (b) the normalized pulse width τ_n, (c) the bandwidth $(1 + \beta^2)^{1/2}/\tau$, (d) the stability criterion.

a pulse is absent ($1/\tau^2 = 0$). From the real part of Eq. (3.3) we have

$$g - l + \frac{1-\beta^2}{\tau^2}\frac{g}{\Omega_g^2} - \frac{2\beta D}{\tau^2} = 0. \tag{3.11}$$

Denote the value of gain for cw excitation by g_{cw}. Then $g_{cw} = l$, and

$$g - g_{cw} = -(1-\beta^2)\frac{g}{\Omega_g^2\tau^2} + \frac{2\beta D}{\tau^2} < 0. \tag{3.12}$$

This is the stability criterion. At first sight it is rather odd. It involves the gain dispersion and the GVD. GVD should not really enter into an equation that balances gain against loss. For this reason it is worth investigating the stability criterion further and looking at the contribution to the power gain-and-loss balance of each term in the master equation. The procedure is to multiply Eq. (2.7) by a^* and integrate over all time and then to add the complex conjugate. The resulting equation gives the generation and the absorption of power directly:

$$\int_{-\infty}^{\infty} a^*\left[g\left(1 + \frac{1}{\Omega_g^2}\frac{d^2}{d\tau^2}\right) - l + \gamma|a|^2\right]a\, dt + \text{c.c.} = 0. \tag{3.13}$$

Integration by parts gives

$$2(g-l)\int_{-\infty}^{\infty}|a|^2\, dt - \frac{2g}{\Omega_g^2}\int_{-\infty}^{\infty}\left|\frac{da}{dt}\right|^2 dt + 2\gamma\int |a|^4\, dt = 0. \tag{3.14}$$

In the above expression the GVD term does not appear at all. Now one can identify clearly the role of each term. The first term is the net power generated if the gain exceeds the loss. Of course, stable operation requires that the loss level be above the gain level. The second term gives a negative contribution. If the radiation is not cw, it is of finite bandwidth, and the effective gain that it experiences is lower than that of cw radiation. The two negative contributions are balanced by the APM (equivalent saturable absorber) action. Were it not for this term, the gain level would have to be above the loss level, and the pulses would be unstable. When the sech solution is introduced into Eq. (3.14), one obtains

$$\left[2(g-l) - \frac{2}{3}\frac{g}{\Omega_g^2\tau^2}(1+\beta^2) + \frac{4}{3}\gamma A^2\right]\int |a|^2\, dt = 0. \tag{3.15}$$

When the A^2 term is eliminated by means of Eq. (3.4), one obtains Eq. (3.12). Now, however, the role of the stabilization by APM is clear. The mysterious appearance of the GVD term in the stability condition is thus explained by two steps: (a) the application of power balance and (b) the substitution of the APM contribution in terms of the pulse parameters. The stability condition is evaluated in Figs. 2(d) and 3(d). Only the range of values for which the stability parameter

$$(1-\beta^2) - 2\beta D_n > 0 \tag{3.16}$$

are acceptable as stable.

4. COUPLED-CAVITY ADDITIVE PULSE MODE LOCKING

Figure 4(a) shows schematically an APM system with two ring cavities. The coupler is characterized by an (amplitude) coupling r, and each ring cavity contains gain and Kerr media for the sake of generality. We focus on the usual case in which the intensity in the auxiliary cavity (2) is low compared with that in the main cavity (1) and the properties of the auxiliary cavity can be adjusted without an appreciable effect on the lasing condition in the gain cavity. This is, in fact, one of the main advantages of APM systems: they permit adjustment of the parameters of the artificial saturable absorber simulated by the auxiliary cavity without affecting the lasing conditions strongly and directly. Figure 4(b) shows the standing-wave cavity version.

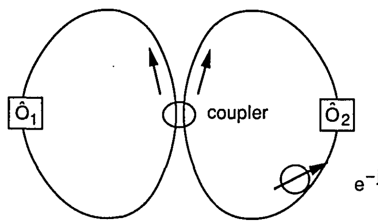
The incident and transmitted waves at the coupler (mirror) obey the scattering equations[1]

$$b_1 = ra_1 + (1-r^2)^{1/2}a_2, \tag{4.1}$$

$$b_2 = (1-r^2)^{1/2}a_1 - ra_2. \tag{4.2}$$

The Kerr media (1 and 2) produce the time-dependent nonlinear phase shifts Φ_1 and Φ_2 with GVD parameters D_1 and D_2, respectively. Here we define Φ_1 and Φ_2 as the full nonlinear phase swings that are due to the Kerr effect. Strictly, the Kerr media need not be solely responsible for the GVD; other components in the system may contribute. We shall assume that all components have a small effect on the pulse passing through them. Their operators commute, since they can be expanded to first order:

$$\hat{O}_i = \exp(-j\Phi_i)\left(1 + g_i + g_i\frac{1}{\omega_{g_i}^2}\frac{d^2}{dt^2} + jD_i\frac{d^2}{dt^2}\right)$$
$$\simeq 1 - j\Phi_i + g_i\left(1 + \frac{1}{\omega_{g_i}^2}\frac{d^2}{dt^2}\right) + jD_i\frac{d^2}{dt^2} \equiv 1 + \hat{o}_i, \tag{4.3}$$



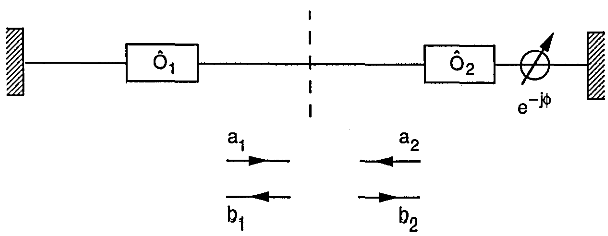


Fig. 4. Schematic of the two-cavity APM system.

where g_i is the net gain, i.e., it includes the linear loss. The loss in the auxiliary cavity is represented by L. The phase shift is porportional to the intensity:

$$\Phi_i = \kappa_i |a_1|^2, \quad i = 1, 2, \tag{4.4}$$

where κ_i is proportional to the length of the Kerr medium (fiber) and the Kerr coefficient and is inversely proportional to the cross-sectional area of the optical beam (mode). Note that we use the intensity a_1 in the definition of κ_i. When the Kerr phase shift produced in resonator 2 is considered, a proper scaling factor is contained in κ_2 to account for the ratio of powers in the two cavities. In the weak-perturbation limit assumed for our analysis the order of the different components can be interchanged without change of the operation. In the sequel all operators describe effects following one round trip.

The wave b_2 experiences a phase delay ϕ and a loss characterized by the factor $L(<1)$ before it returns as the incident wave a_2. If L is small, the reflection coefficient operator seen on side 1 of the mirror is found to be[1]

$$\Gamma = b_1/a_1 = r + L(1 - r^2)\exp(-j\phi)\hat{O}_2. \tag{4.5}$$

The wave a_1 is the transformed wave b_1, transformed by the operator $\hat{O}_1$, where $\hat{O}_1$ includes the action of gain, gain dispersion, and loss, and GVD in the gain cavity:

$$a_1 = \hat{O}_1 b_1. \tag{4.6}$$

The closure condition requires that the wave a_1 reproduce itself within a phase shift that may be associated with an induced shift of the carrier frequency:

$$\exp(-j\theta)\hat{O}_1\Gamma a_1 = a_1. \tag{4.7}$$

The phase shift θ must be handled with care. The logical step is to expand the system operation from cw operation and treat the pulse operation as a perturbation of the cw operation. In this way one determines the gain for cw operation first and then ascertains the gain required for pulsed operation. If the cw gain is smaller than the pulsed gain, pulsed operation is stable.

Cw operation corresponds to setting $\hat{O}_1 = 1 + g_1$. This is because the derivatives in the operators produce no effect and the Kerr phase shift is negligible, since the peak intensity of cw operation is much smaller than that of pulsed operation. The phase $\theta = \theta_0 + \psi$ can be written as a contribution from cw operation, θ_0, and a contribution from pulsed operation, ψ. In cw operation the frequency adjusts itself so that there is no net phase shift following one round trip, or

$$\exp(-j\theta_0)(1 + g_1)\Gamma_0 a_1 = a_1, \tag{4.8}$$

where Γ_0 is the reflection coefficient for cw operation:

$$\Gamma_0 = r + L(1 - r^2)\exp(-j\phi)(1 + g_2). \tag{4.9}$$

This closure condition shows that $\exp(-j\theta_0)\Gamma_0$ must be of zero phase. To first order, this gives the following relation for θ_0:

$$\sin\theta_0 \simeq \theta_0 = -L\frac{1 - r^2}{r}\sin\phi. \tag{4.10}$$

Physically, this means that the frequency of the cw oscillation adjusts itself to account for the phase of Γ_0 produced by the auxiliary cavity.

Introducing relation (4.10) into Eq. (4.8) and expanding all terms to first order in ψ and the operators $\hat{o}_i$, one obtains the master equation (2.7) with the following identifications:

$$\frac{g}{{\Omega_g}^2} = r\frac{g_1}{{\omega_{g1}}^2} + L(1 - r^2)\left(\frac{g_2}{{\omega_{g2}}^2}\cos\phi + D_2\sin\phi\right), \tag{4.11}$$

$$D = rD_1 + L(1 - r^2)(D_2\cos\phi - D_1\sin\phi), \tag{4.12}$$

$$\gamma = L(1 - r^2)(\kappa_1 - \kappa_2)\sin\phi, \tag{4.13}$$

$$\delta = \kappa_1 + L(1 - r^2)\kappa_2\cos\phi, \tag{4.14}$$

$$l = 1 - \Gamma_0, \tag{4.15}$$

$$x = 0, \tag{4.16}$$

$$g = rg_1 + L(1 - r^2)g_2\cos\phi. \tag{4.17}$$

Here the product $\theta_0\hat{o}_1$ has been retained because it is of the same order as $L(1 - r^2)\hat{o}_2$. Note that an empty auxiliary cavity can provide stabilization if the main cavity contains a Kerr medium.

Γ_0 has been replaced by unity where it multiplies a first-order quantity. The loss in the auxiliary cavity is included in L. If $\kappa_1 = 0$, note that use of a medium with a positive Kerr coefficient (i.e., κ_2) requires a negative bias phase ϕ. This phase is maintained by a feedback circuit. A positive D_2 may produce, according to relation (4.10), a negative gain dispersion (higher gain off line center). This would destabilize the carrier frequency.

5. GENERALIZED INTERFEROMETRIC SYSTEM AND ITS VARIANTS

Figure 5(a) shows a ring cavity that contains a nonlinear interferometer. The interferometer has Kerr and gain media in both arms and also in the return path. We describe the media by operators (4.3).

We shall assume that the interferometer disturbs the system only slightly; i.e., its loss is small. This is the case when either ϕ is small or $1 - r^2$ is small. The former is the case of the Sagnac ring reflector, which is not discussed here. We shall focus on the case of small $1 - r^2$ and possibly large ϕ.

The power splits in the interferometer as r^2 and $1 - r^2$; a relative phase shift in arm (2) is $\exp(-j\phi)$, and the operators $\hat{O}_1$ and $\hat{O}_2$ operate in each arm; finally, the operator $\hat{O}_3$ operates in the common ouput arm. Following one round trip the amplitude a must be reproduced in the steady state, except for the phase shift $\theta = \theta_0 + \psi$:

$$\exp(-j\theta)[r^2\hat{O}_1 + (1 - r^2)\exp(-j\phi)\hat{O}_2]\hat{O}_3 a = a. \tag{5.1}$$

Again, we expand from the cw steady state. The frequency of the steady state adjusts itself so that the net phase shift is zero; i.e.,

$$\sin\theta_0 \simeq \theta_0 = -\frac{1 - r^2}{r^2}\sin\phi. \tag{5.2}$$

When Eq. (5.1) is expanded to first order in ψ, $1 - r^2$, and the operators $\hat{o}_i$, one obtains a master equation of the form

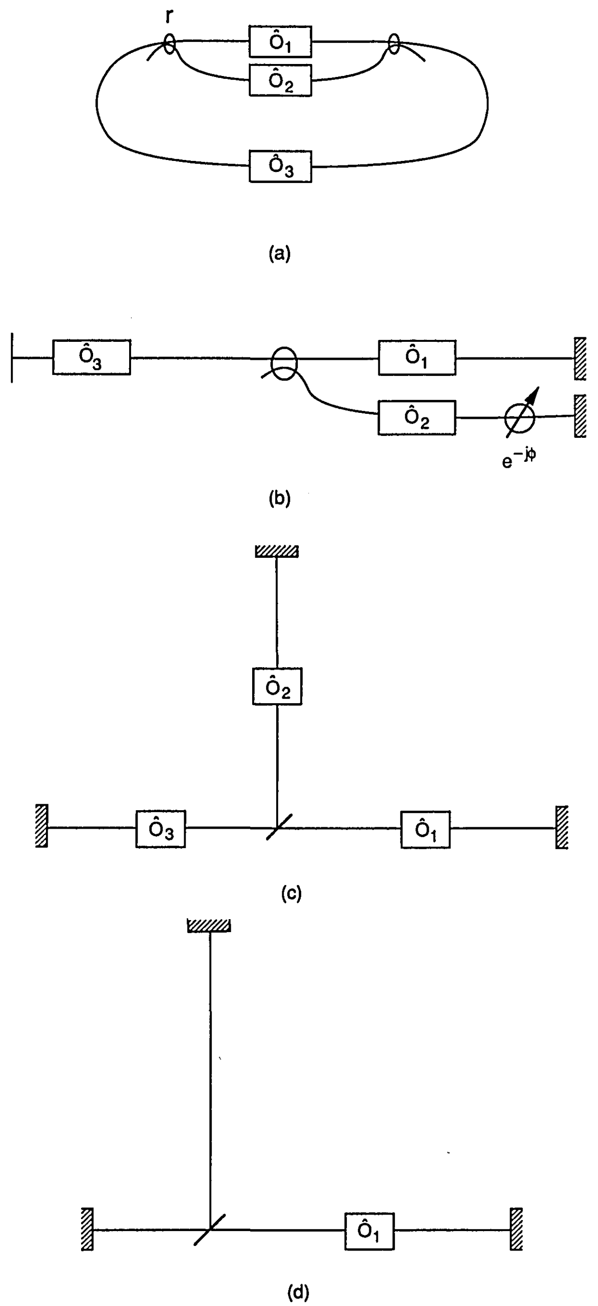


Fig. 5. Interferometric APM systems. (a) Ring cavity with an interferometer. (b) Standing-wave cavity—half of the cavity of Fig. 4(a). (c) Two-cavity system with the properties of system (b); APM with an auxiliary cavity and a beam splitter. (d) Special case of (c) involving a linear cavity [simplified version of Fig. 4(c)].

of Eq. (2.7) with the identifications

$$\frac{g}{\Omega_g^2} = r^2\frac{g_1}{\omega_{g1}^2} + (1 - r^2)\frac{g_2}{\omega_{g2}^2}\cos\phi + \frac{g_3}{\omega_{g3}^2} - (1 - r^2)(D_1 - D_2)\sin\phi, \tag{5.3}$$

$$D = r^2D_1 + (1 - r^2)D_2 + D_3 - (1 - r^2) \times \left(\frac{g_1}{\omega_{g1}^2} - \frac{g_2}{\omega_{g2}^2}\sin\phi\right), \tag{5.4}$$

$$\gamma = (\kappa_2 - \kappa_1)(1 - r^2)\sin\phi, \tag{5.5}$$

$$\delta = r^2\kappa_1 + (1 - r^2)\kappa_2\cos\phi + \kappa_3, \tag{5.6}$$

$$l = (1 - r^2)(1 - \cos\phi), \tag{5.7}$$

$$x = (\sin\phi)(1 - r^2), \tag{5.8}$$

$$g = r^2g_1 + (1 - r^2)g_2\cos\phi + g_3. \tag{5.9}$$

The product $\theta_0 r^2\hat{o}_1$ has been retained because it is of the same order as $(1 - r^2)\hat{o}_2$.

Clearly, adjustments of the system parameters must be made so that the coefficients are of the proper sign and the stability criterion of Figs. 2 and 3 is obeyed; g/Ω_g^2 and γ must be positive. If δ turns out to be negative, the plots of Figs. 2 and 3 still apply with the signs of D_n and β reversed.

Figures 5(b)–5(d) show other variations of the ring resonator structure of Fig. 5(a). Clearly, the system equations as applied to a short circulating pulse will not change if the Kerr and gain media in the system of Fig. 5(a) are doubled in number and then the system is split with perfectly reflecting mirrors at its symmetry plane. The resulting system is shown in Fig. 5(b). The ring resonator and the standing-wave resonator of Fig. 5(b) are described by the same form of the equation as long as the countertraveling parts of the pulse do not meet in the nonlinear elements. This can be arranged when the pulses are short. However, the self-starting condition may be different in ring resonators and standing-wave resonators.

A coupler can be replaced by a beam splitter. When this is done, the system of Fig. 5(b) changes into that of Fig. 5(c), which is a form of the Michelson interferometer cavity studied in Ref. 19. The system of Fig. 5(d) deserves more attention. A pulse traveling from the left-hand mirror to the right-hand mirror splits into two pulses of the same shape because it encounters no nonlinearities or dispersion. The pulse entering the gain and Kerr media returns to the beam splitter. There it meets the returning pulse from the vertical arm and interferes with it at the linear mirror, producing an outgoing pulse of the same shape as the starting pulse, in the steady state. Hence the pulse returning from the nonlinear medium must be unchanged in shape from the pulse that entered it. The interference at the mirror produces no pulse-shaping action in the steady state. However, the auxiliary cavity can still stabilize, as Eq. (5.5) shows, since $\gamma \neq 0$ when $\kappa \neq 0$.

The interferometric APM systems of Fig. 5 can be adapted so as to obviate active stabilization. An example is a nonlinear fiber ring reflector that interferes with the two countertraveling pulses that pass through the same fiber counterclockwise and are insensitive to index fluctuations that are slow compared with the transit time. Two polarizations in a single fiber can also serve as the two excitations of an equivalent fiber interferometer that is also extremely stable and does not need active feedback stabilization.[29]

6. DISCUSSION

The theory that we have developed describes the mode-locking performance of laser systems. Stability, pulse duration, chirp, and bandwidth are determined as functions of saturable absorption (APM action), SPM, and dispersion. For an understanding of the pulse-shaping mechanisms and their dependence on laser parameters, it is helpful to consider special cases of the solutions described in Section 3.

First, suppose that all parameters of the system are fixed except the GVD. This is a realistic situation because prism pairs [30] are routinely used to adjust the intracavity GVD in short-pulse lasers. Intracavity GVD has been demonstrated to optimize pulse performance in the CPM dye laser as well as in the Ti:Al_2O_3 laser.[15-18,27] As shown in Figs. 2 and 3, for large negative values of GVD the laser is stable and generates short pulses with relatively little chirp. Decreasing the amount of negative GVD, approaching zero GVD, will decrease the pulse duration, with the shortest pulses being generated at the point where the laser becomes unstable. This instability occurs because the pulses become so short that their effective gain becomes less than that for cw. The size and the existence of this unstable region are functions of the intracavity SPM and APM action (saturable absorption). The predicted laser behavior for negative GVD corresponds closely to the dispersion tuning behavior observed in the CPM laser.

If the GVD is increased to positive values, the laser will again become stable for pulse generation. However, the pulses generated for positive GVD will be long and highly chirped. The pulse duration and chirp increase as the positive GVD is increased. The pulse-shortening mechanism for positive GVD differs significantly from the pulse compression or solitonlike mechanism that occurs for negative GVD. For positive GVD, pulse shortening requires the pulse to be highly chirped and shortened by bandwidth limiting. Since the pulses are approximately linearly chirped, one can achieve shorter-pulse durations by compensating for this chirp external to the laser. If this is done, the pulse duration may be reduced to the inverse of the pulse bandwidth. However, since the pulse bandwidths are less for positive GVD than for negative GVD, the minimum pulse duration performance for a given laser requires negative intracavity GVD. The predicted behavior for positive GVD corresponds closely with experimental observations in the APM Ti:Al_2O_3 lasers, where chirped pulses are generated if intracavity prisms are not used.

In order to optimize laser performance, we now consider the effects of varying intracavity SPM and APM (saturable-absorber) action as shown in Figs. 2 and 3. The greater is the SPM (the parameter δ), the more negative GVD is needed to keep the pulse stable for a given saturable-absorber action γ. The pulse durations become shorter than those for zero SPM. However, for a given saturable absorption (γ), increasing SPM (δ) without limit (and the negative GVD with it) does not produce arbitrarily short pulses. Our numerical studies indicate that the pulse duration may be reduced by only a factor of approximately 2.75 from the value that would be achieved with saturable absorption alone. That this factor of 2.75 is truly an upper limit has also been supported by analytical derivation.[31] Thus, if the saturable absorption or the APM action is weak, introduction of SPM and GVD does not improve the pulse width by more than approximately a factor of 2.75. This behavior is not unlike that observed with active mode locking,[23] except that a larger pulse-width reduction (a factor of 2.75 rather than of 2) is achievable in the present case.

It should be emphasized, however, that the effects of GVD are extremely important in achieving short pulse durations, since there can be an order-of-magnitude difference in the pulse durations that are generated for negative and positive GVD. If zero or negative GVD is used, pulse durations that correspond to a significant fraction of the gain bandwidth can be achieved with relatively small amounts of saturable absorption. Consider for simplicity zero GVD and SPM. From Eq. (3.4) one finds that

$$g/\Omega_g{}^2\tau^2 = \gamma A^2/2\,.$$

Now assume that the saturable absorber or the APM action γA^2 is equal to 1%. Further, assume that the power gain per pass is 5%. Solving for the pulse width, we find that

$$\tau = (1/\Omega_g)\sqrt{1/5}\,.$$

Hence pulses comparable with the inverse gain linewidth can be achieved with only 1% APM action.

The nonlinear phase shifts necessary to realize such APM are easily achieved at low powers in optical fiber systems. Even in nonfiber systems significant nonlinearity is produced by the focusing of intracavity power in the gain medium itself. Pulses of 100-fs duration in an intracavity average power of 5 W have peak powers of 0.5 MW. If one assumes a nonlinear index comparable with that of fused silica, the phase shift produced by SPM on such a 0.5-MW pulse, freely focused in a medium longer than the confocal parameter, is of the order of 1 rad. This can be the basis for single-cavity APM. The calculation also indicates how strong the underlying SPM can be and why its compensation is so important in some systems.

7. SUMMARY

The theoretical results described above predict that extremely short pulse durations can be achieved with only modest saturable absorption. With a proper balance of self-phase modulation (SPM) and negative group-velocity dispersion (GVD) one can obtain an additional shortening, relative to that of the zero-dispersion case, by a factor of as much as 2.75. On the other hand, with improperly balanced SPM and GVD one gets pulse broadening and/or instability.

Although our theory has been formulated to describe systems that achieve fast saturable absorption by using additive pulse mode locking (APM), it can be applied to describe the operation of any laser that uses a fast saturable-absorberlike nonlinearity. In addition to multiple, coupled-cavity arrangements, single-cavity systems may operate with APM behavior when two different spatial modes or two different polarization modes oscillate simultaneously. Dynamic self-focusing is yet another mechanism for introducing saturable-absorberlike behavior into a single cavity.[32] In all these cases one can take advantage of a fast index nonlinearity to produce an effective fast saturable absorber. And one does so without the severe insertion loss penalty usually associated with a real absorber.

Thus the key ingredients in ultrashort-pulse lasers are intracavity SPM, APM or a saturable absorber, and intracavity GVD compensation. This paper provides a theoretical basis for using them to optimize system design. It should be possible to develop a versatile ultrashort-pulse generation technology with solid-state and fiber lasers to achieve pulse durations that utilize an appreciable fraction of their gain bandwidths.

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