



in resonators with cylindrical symmetry. The first-order loss variation caused by an aperture is proportional to the small-signal relative spot size variation, defined as<sup>9,13</sup>

$$\delta = \left( \frac{1}{w} \frac{dw}{dp} \right)_{p=0}, \quad (1)$$

where  $w$  is the spot size at the aperture plane and  $p$  is the power normalized to the critical power for self-focusing.<sup>12</sup> If  $\delta$  is negative and sufficiently large, an aperture gives rise to KLM because the losses decrease as the power increases. In particular, at the plane of end mirror  $M_1$ ,  $\delta$  can be expressed as

$$\delta_1 = - \frac{\alpha_1 + \alpha_2 S}{2(\alpha_1 + \alpha_2 + 2\alpha_1 \alpha_2 S)}, \quad (2)$$

with

$$\begin{aligned} \alpha_i &= B_{ri}/d_e + A_{ri} \quad (i = 1, 2), \\ S &= A_0 D_0 + B_0 C_0, \end{aligned} \quad (3)$$

where  $B_{ri}$  and  $A_{ri}$  are the matrix elements defined in Fig. 1(b),  $d_e$  is the equivalent length of the Ti:sapphire rod, and  $A_0$ ,  $B_0$ ,  $C_0$ , and  $D_0$  are the elements of the one-way matrix relevant to propagation from mirror  $M_1$  to  $M_2$ . Note that the resonator is optically stable if  $-1 < S < 1$ . As a first approximation, Eq. (2) can be applied to the two equivalent one-dimensional resonators separately. One therefore obtains a value,  $\delta_{1\xi}$ , for the tangential plane and a different one,  $\delta_{1\eta}$ , for the sagittal plane. Since, however, Eq. (2) was derived for resonators with cylindrical symmetry and does not strictly hold for astigmatic beams, this approximation was compared with more-accurate iterative numerical calculations. The results, in agreement with those of Ref. 11, have shown that for most of the resonators commonly used the effect of astigmatism is to increase  $\delta_{1\xi}$  and reduce  $\delta_{1\eta}$  with respect to the predictions of Eq. (2). Our approach therefore also describes reasonably well, apart from a scale factor, the mode behavior also in astigmatic resonators. It should be noted that Eq. (2) allows one to obtain in a simple way most of the results previously reported in the literature and often obtained with more-complicated calculations.<sup>5-11</sup>

To design and optimize a resonator that best exploits the nonlinearity for KLM, one can start from Eq. (2) and find the resonator parameters that maximize  $|\delta|$ . The most critical parameters turn out to be the distance between the folding mirrors,  $z$ , which essentially determines the optical stability, and the distance between mirror  $M_3$  and the closer face of the Ti:sapphire rod,  $x$ , which controls the focusing condition without affecting the resonator stability. Therefore a practical way to optimize the resonator is to plot  $\delta$  as a function of  $x$  and  $z$  and find the regions in which the best trade-off between large values of  $|\delta|$  and acceptable tolerances can be achieved. To demonstrate experimentally the validity of this procedure we applied it to various resonators of the type shown in Fig. 1(a), each characterized by different distances  $L_1$  and  $L_2$  and operating in both the HMS

and LMS regions. For each resonator we systematically changed the value of  $\delta$  by varying both the position of the Ti:sapphire rod and the folding distance while checking whether KLM could be achieved.

For the experiments we set up an argon-laser-pumped Ti:sapphire laser in the configuration of Fig. 1(a). The 20-mm-long Ti:sapphire rod and the folding mirrors were mounted on precision translators. A pair of SF10 prisms placed 35 cm apart compensates the dispersion. Slit  $S_1$  was placed close to mirror  $M_1$  and was used to sustain KLM, whereas slit  $S_2$  was used for tuning and to avoid wavelength instabilities. In the cw regime, with 5% coupling, the typical output power was 850 mW at 4.5 W of pump power. KLM was initiated with the help of piezoactuator that vibrates mirror  $M_1$  at 300 Hz with an amplitude of  $<1.5 \mu\text{m}$ . The position of the folding mirrors was directly read on the micrometers of the mirror translation stages. To obtain the mirror distance,  $z$ , we initially performed a calibration procedure. The resonator was aligned to work in the HMS stability region, and the distance  $z$  was progressively reduced until laser action stopped. We assumed that in this condition the mirrors were in the confocal arrangement corresponding to the stability limit. The folding distances of the other three stability limits read on the micrometers corresponded to the theoretical values within 0.2 mm. The zero reference of the micrometer of the rod translation stage was calibrated with the help of a precision ruler placed between mirror  $M_3$  and the closer rod face. For each resonator and for each value of  $x$  and  $z$ , first the laser was aligned for maximum output power; then slit  $S_1$ , oriented to cut the beam in the  $\xi$  direction, was progressively closed until pure KLM operation, without any cw component in the spectrum, was achieved. By varying the distance of the pump focusing mirror, we adjusted the pump beam waist position in such a way that KLM could not be obtained without slit  $S_1$  as a result of gain-guiding effects in the active material. The pulse duration ranged between 50 and 60 fs FWHM with a spectrum of  $\sim 15$  nm FWHM centered at 800 nm.

We first considered an asymmetric resonator, with  $L_1 = 500$  mm and  $L_2 = 1100$  mm, operating in either the HMS or the LMS region. Slit  $S_1$  was inserted into the short arm of the cavity because our calculations, in agreement with Refs. 8 and 11, showed that, as a function of  $x$  and  $z$ , broader regions with negative  $\delta_{1\xi}$  could be achieved. Figure 2 shows the contour curves of  $\delta_{1\xi}$  as a function of  $x$  and  $z$ . On the same plot we also show, with filled circles, the points for which KLM could be experimentally obtained. The results, in good agreement with the theoretical model, demonstrate that KLM could be achieved only for  $\delta_{1\xi} < 0$ . Qualitatively, for  $|\delta_{1\xi}| > 0.5$ , KLM required only slight cavity perturbations to be initiated and remained stable for hours; on the other hand, for lower  $|\delta_{1\xi}|$ , KLM required careful adjustment of the aperture size and was sensitive to perturbations. The same experiments were performed with a nearly symmetric resonator, with  $L_1 = 700$  and  $L_2 = 900$ : Fig. 3 shows that similar results in agreement with the theory have also been obtained. Because of the

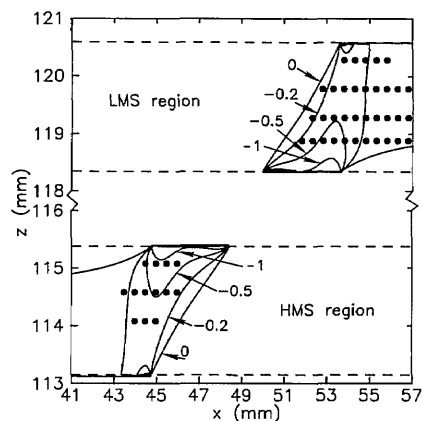


Fig. 2. Contour curves of the small-signal relative spot size variation  $\delta_{1\xi}$  (parameter of the curves) in the tangential plane of the resonator with  $L_1 = 500$  mm and  $L_2 = 1100$  mm. Only curves of negative  $\delta_{1\xi}$  are shown. The filled circles mark the points where KLM has been experimentally achieved. The dashed horizontal lines are the stability limits.

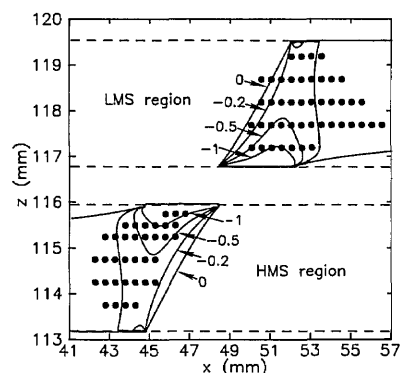


Fig. 3. Same as in Fig. 2 for the resonator with  $L_1 = 700$  mm and  $L_2 = 900$  mm.

astigmatism effects, KLM could not be achieved with the aperture oriented to cut the beam in the  $\eta$  directions. In the HMS region both resonators were, as expected, sensitive to misalignment, and KLM was in general more difficult to achieve and less stable. For this reason there are fewer experimental data points in Figs. 2 and 3 in the HMS regions than in the LMS regions. Although, as already mentioned in the literature,<sup>7-13</sup> large nonlinear loss modulations can be achieved by bringing the resonator close to the stability limits, where  $S = \pm 1$ , approaching the stability limit too closely degrades the laser performance. Our results indicate that the best trade-off in terms of output power, stability, and tolerance to the exact positions of the components is achieved by operating the laser in the LMS region at a distance of  $\Delta z = 0.5$ – $1$  mm from the lower stability limit.

As Figs. 2 and 3 show, both asymmetric and nearly symmetric cavities are suitable for KLM, provided that the rod position  $x$  and the folding distance  $z$  are properly adjusted. However, these figures also show that the region in the  $xz$  plane for which high values of  $|\delta|$  can be achieved progressively broadens as the resonator approaches the symmetric configuration, in contradiction to Ref. 11. As an example, for  $\delta_1 = -1$  to be reached, the maximum allowed distance from

the stability limit is 0.22 mm for the asymmetric cavity of Fig. 2 and 0.44 mm for the cavity of Fig. 3, which is closer to a symmetric one. The highest tolerance is reached with a perfectly symmetric cavity ( $L_1 = L_2$ ). Moreover, in a symmetric resonator the HMS and the LMS regions become joined<sup>14</sup> and result in a single stability region in terms of  $z$ . In the center of this wider region the resonator becomes equivalent to a confocal one with a well-behaved transverse mode and no power reduction even if the stability parameter  $S$  is  $-1$ . Therefore, if the rod is suitably positioned and the distance  $z$  is set close to the center of the stability region,  $\delta_{1\xi}$  can even diverge to  $\pm\infty$  without sacrificing the laser stability. Experimentally, with a symmetric cavity ( $L_1 = L_2 = 850$  mm), in a range of  $\pm 0.6$  mm around  $z = 116$  mm (corresponding to the center of the stability region) KLM did not require external perturbations to start but was completely self-starting. This regime was stable for hours and easily reproducible since only minor alignments were required from day to day. The amplitude fluctuations on a millisecond time scale were  $<1\%$ . The perfect symmetry of the resonator was not necessary for self-starting: in fact, we could change  $L_1$  by as much as 50 mm and still keep the laser in the self-starting regime. Also, the tolerance on the exact position of the rod was as much as 2 mm. On the basis of these results and of the above considerations we attributed the self-starting behavior to the very high value of  $\delta_1$  achieved with these resonators, but further studies of the phenomenon are in progress.

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