Self-starting issues of passive self-focusing mode locking

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An analysis of passive mode locking with the intracavity self-focusing effect is presented, and the self-starting issue is addressed. The analysis shows that the initial pulse-shortening force of this mechanism is too weak to start the mode locking from mode beating or noise and how it can be greatly enhanced with a dilute dye saturable absorber in the cavity. This analysis also shows how the pulse-width evolution is related to the pulse-shortening force in cw mode-locking lasers.

Since the first report of passive mode locking in a $Ti{:}Al_2O_3$ laser,[1] there has been great interest in passive mode locking of solid-state lasers in general.[2] For lasers with a large bandwidth, such as the $Ti{:}Al_2O_3$ laser, the interest has been on the generation of tunable femtosecond pulses, whereas for lasers that are not tunable but can be pumped by laser diodes, such as the Nd:YAG and Nd:YLF lasers, the potential in commercial applications is obvious.

In a cw passively mode-locked laser, it is essential to have a mechanism that produces a gain or loss modulation that favors the growth of the pulse peak. In the colliding-pulse mode-locking laser, a gain window that amplifies the pulse peak is produced by the combined dynamic saturation in the gain and absorber jets, whereas in the newly developed passive additive-pulse mode locking, the differential gain arises from the intracavity interferometer and the self-phase modulation. The interferometer translates the phase modulation into amplitude modulation, which favors the growth of the pulse peak.[3,4] Since the self-phase-modulation effect is nonresonant, tuning in frequency does not interfere with the mode-locking mechanism. This presents a great advantage over colliding-pulse mode locking. The disadvantage of passive additive-pulse mode locking is the requirement of matching the lengths of the two coupled cavities to a submicrometer accuracy. Such a stringent condition is hard to maintain in practice. To circumvent the stability requirement, single-cavity configurations of additive-pulse mode locking have been proposed.[3,4]

Recently, alternative ways of utilizing the Kerr nonlinearity for passive mode locking have been reported.[5–8] At least two groups have identified their mode-locking mechanisms to be the self-focusing effect.[6,7] Self-focusing affects the transverse modes in the cavity and produces power-dependent diffraction loss. However, unlike with additive-pulse mode locking, it is found that the self-focusing mode locking does not self-start. External perturbations must be applied to start the mode locking. On the other hand, self-starting passive mode locking with a saturable absorber in $Ti{:}Al_2O_3$ lasers has been reported,[8] without an explanation of the mode-locking mechanism. The purpose of this Letter is to clarify the self-starting issue of this new mode-locking mechanism.

In a low-loss, stable laser resonator, the q parameter of the fundamental Gaussian mode is determined by the following self-consistent equation:

$$q = \frac{Aq + B}{Cq + D}, \tag{1}$$

where A, B, C, and D are the matrix elements of the combined $ABCD$ matrix over a round trip. Modes with q different from the self-consistent q have larger loss and will be damped out in the long run.[9] In a pulsed laser with intracavity self-focusing effect, the q parameter is a function of the instantaneous power $I(t)$,

$$q = q[I(t)]. \tag{2}$$

In this case, it is possible to design the laser in such a way that it favors high power. Since the average power is limited by the rate equations, the only way for the laser to operate at high power is to become mode locked. To verify this idea, we have numerically analyzed the cavity modes and diffraction loss as a function of the instantaneous power for the $Ti{:}Al_2O_3$ laser shown in Fig. 1. In Fig. 1, self-focusing occurs at the $Ti{:}Al_2O_3$ gain medium ($n_2 = 0.7 \times 10^{-20}\ m^2/W$).[10] When the power is high, the gain medium acts as a distributed lens, and the beam diameter at the position of the iris is smaller than in the low-power case. The iris acts as a spatial filter that produces a loss bias against the low-power portion of the pulses. When the diameter of the iris is much larger than the laser wavelength, diffraction loss can be calculated by throwing away the portion of the Gaussian beam that is blocked by the iris.

Figure 2(a) shows the beam radius (defined by $1/e$ of the peak field strength) of the fundamental transverse mode for two intensities at the position of the iris. The solid line is for the cw low-power case, and the dashed lines are for a power of 1.0×10^5 W. The short dashed line is calculated with $d_1 = 4.96$ cm and $d_2 = 5.16$ cm, while the long dashed line is for $d_1 = 5.16$ cm and $d_2 = 4.96$ cm. The calculation

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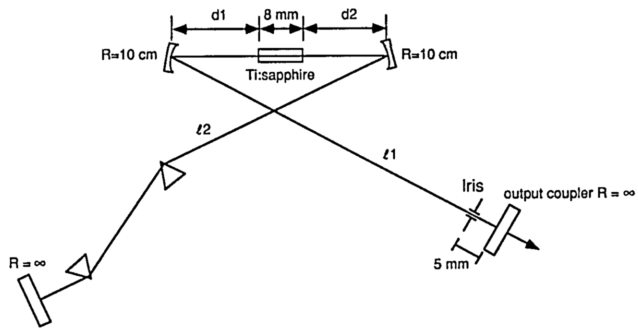

Fig. 1. Schematic diagram of a laser utilizing the intracavity self-focusing effect for cw mode locking. Self-focusing occurs at the gain medium. The arm lengths l_1 and l_2 are 75 and 50 cm, respectively.

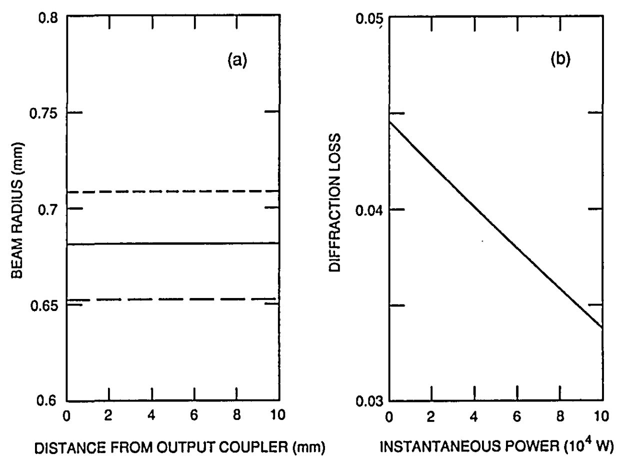

Fig. 2. (a) Solid line, beam radius around the iris shown in Fig. 1 for the cw low-power case; short dashed line, beam radius around the iris for a power of 1.0×10^5 W and $d_1 = 4.96$ cm and $d_2 = 5.16$ cm; long dashed line, beam radius around the iris for a power of 1.0×10^5 W and $d_1 = 5.16$ cm and $d_2 = 4.96$ cm. (b) Diffraction loss as a function of the instantaneous power. The diameter of the iris is chosen to be 1.7 mm.

was done numerically with the beam-propagation method, which has been widely used in waveguide calculations.[11] It is seen that for $d_1 = 5.16$ cm and $d_2 = 4.96$ cm, the beam radius is smaller for higher power. From the beam radius, diffraction loss as a function of the instantaneous power can be obtained. This figure also shows that d_1 and d_2 are sensitive design parameters. If they are not chosen correctly, the laser may favor cw instead of pulsed operation.

Figure 2(b) shows the diffraction loss as a function of the instantaneous power for the laser in Fig. 1. The slope of the diffraction loss in Fig. 2(b) is almost a constant. Therefore one may write the instantaneous gain of the laser as

$$g(t) = g_0 + cI(t), \tag{3}$$

with g_0 the cw gain and $c = 1.1 \times 10^{-7}$/W. Such a system is equivalent to a laser with a fast saturable absorber.[12] From this figure one can see that if the laser is already mode locked, the differential loss between the pulse peak and wings can be as high as 0.011 for 1-ps pulses. However, for an initial 10% fluctuation of the cw power ($\approx$10 W), the differential loss is only approximately 1.1×10^{-7}, which is far below the mode-locking threshold.[4,13,14] The calculation indicates that the mode locking will not self-start.

One may enhance the initial pulse-shortening force with a real dye saturable absorber, which need not be fast. This is because in the self-starting stage, where the pulse width is long, a slow dye saturable absorber is fast enough in this time scale. For a saturable absorber with finite relaxation time, the differential loss Δl is approximately

$$\Delta l(t) = -\frac{\sigma l}{\hbar\omega A_a}\int_{-\infty}^{t} I(t')\exp\left(-\frac{t-t'}{T_1}\right)\mathrm{d}t', \tag{4a}$$

where σ is the absorption cross section, T_1 is the relaxation time, l is the linear loss, and A_a is the beam cross section at the absorber. When T_1 is much smaller than the width of $I(t)$, Eq. (4a) can be reduced to

$$\Delta l(t) = -\frac{\sigma l T_1}{\hbar\omega A_a}I(t). \tag{4b}$$

Using HITCI ($\sigma = 10^{-15}$ cm^2, $T_1 = 1.2$ ns) as an example,[15] if the concentration is chosen to be 2×10^{-6} M, the thickness of the jet is 100 μm, and the beam diameter at the absorber is 80 μm, then the differential loss calculated from Eq. (4b) is approximately 1.2×10^{-3}. This number is not only much larger than the value obtained before without the absorber but is also larger than the initial differential gain for the self-starting additive-pulse mode-locking $Ti:Al_2O_3$ laser.[3,4] This indicates that with the addition of a saturable absorber, the laser will self-start. The self-starting scheme analyzed in this Letter can be applied equally well to the proposed single-cavity additive-pulse mode locking,[3,4] because of the similarity between these two mode-locking mechanisms.

In the rest of this Letter we clarify the role of differential gain in the pulse-width evolution of cw mode-locked lasers and estimate the time scale for the pulse to shorten to its steady state from an initial seed. From Eqs. (3) and (4a), the equation of motion for the power waveform is

$$I_{n+1}(t) = I_n(t)\left[g_n + \frac{\sigma l}{\hbar\omega A_a}\int_{-\infty}^{t} I(t') \times \exp\left(-\frac{t-t'}{T_1}\right)\mathrm{d}t' + cI_n(t)\right], \tag{5}$$

where g_n is the combined gain of the laser media and the absorber at the nth round trip. Because the emission cross section of the $Ti:Al_2O_3$ laser is small, we have ignored the dynamic saturation of the gain media. Because the bandwidth of the gain medium is much larger than the frequency span of the nanosecond to picosecond transient pulses, we have also ignored the bandwidth-limiting effect. For the convenience of analysis, we assume that $I_n(t)$ has the following form:

$$I_n(t) = \frac{E}{2\tau_n}\mathrm{sech}^2\left(\frac{t}{\tau_n}\right), \tag{6}$$

where E is the pulse energy and τ_n is the pulse width ($\approx$0.57 times the FWHM pulse width), defined by the integral

$$\tau_n^{\,2} = \frac{12}{\pi^2 E} \int t^2 I_n(t)\mathrm{d}t. \tag{7}$$

Assume that the laser is running at its steady-state power, so that the pulse energy does not change in the mode-locking process. Integrating both sides of Eq. (5), we have the following identity:

$$E = \int I_{n+1}(t)\mathrm{d}t = E\left[g_n + \frac{cE}{3\tau_n} + \frac{f(\tau_n)E\sigma l}{\hbar\omega A_a}\right], \tag{8}$$

$$f(\tau_n) = \frac{\dfrac{\tau_n}{T_1} + 4}{8\left(\dfrac{\tau_n}{T_1} + 2\right)^2}. \tag{9}$$

Because the linear loss from the absorber is very small ($\approx$0.012) owing to its low concentration, the opening up of the absorber has little effect on the pulse energy. Multiplying both sides of Eq. (5) by t^2/E and integrating, we obtain another identity of the pulse width,

$$\tau_{n+1}^{\,2} = g_n\tau_n^{\,2} + c_1 cE\tau_n + \frac{h(\tau_n)E\sigma l}{\hbar\omega A_a}\tau_n^{\,2}, \tag{10}$$

$$c_1 = \frac{3}{\pi^2}\int_{-\infty}^{\infty} x^2 \,\mathrm{sech}^4(x)\mathrm{d}x \approx 0.131,$$

$$h(\tau_n) = \frac{\left(\dfrac{\tau_n}{T_1}\right)^3 + 8\left(\dfrac{\tau_n}{T_1}\right)^2 + 28\left(\dfrac{\tau_n}{T_1}\right) + 64}{32\left(\dfrac{\tau_n}{T_1} + 2\right)^4}. \tag{11}$$

In evaluating the integrals of the last terms in the right-hand side of Eqs. (8) and (10), in order to obtain closed-form results, we have made the approximation of $\mathrm{sech}^2(x)$ by $\exp(-2x)$ for $x > 0$ and $\exp(2x)$ for $x < 0$. This approximation does not affect the bulk part of our estimation. Since τ_n changes only slightly after each round trip, Eqs. (8) and (10) can be replaced by a differential equation of $\tau(n) \equiv \tau_n$,

$$2\tau(n)\frac{\mathrm{d}\tau(n)}{\mathrm{d}n} = -\left(\frac{1}{3} - c_1\right)cE\tau_n - [f(\tau_n) - h(\tau_n)]\frac{E\sigma l}{\hbar\omega A_a}\tau_n^{\,2}. \tag{12}$$

It can be seen in Eq. (12) that for large τ_n (the initial stage), the pulse shortening is governed by the saturable absorber, while for small τ_n (the final stage), it is determined by the diffraction loss. Indeed, saturable absorber enhances the pulse shortening in the starting stage. Equation (12) shows how the pulse-width evolution is governed by the differential gain. From the solution of Eq. (12), the time for the pulses to shorten from 5 ns to 100 fs FWHM is estimated to be approximately 20,000 to 30,000 round trips. This prediction agrees well with experimental data.[16]

In our analysis, we have assumed that the dispersion effect is compensated by the prism pair and have ignored the self-phase-modulation effect. Since these two effects can work together to allow the propagation of an optical soliton, one might think that they could play a role in the pulse formation. Indeed, since the optical soliton is the steady-state solution of this mode-locking mechanism,[12] these two effects must affect the steady-state pulse shape. However, in the starting stage (pulse width from 10 ns to 10 ps), the pulse width is too long to feel the slightly negative dispersion that is designed to balance the self-phase modulation. This can be seen from the fact that it takes 1 km of optical fiber to broaden a 10-ps pulse to 14 ps near 830 nm. Without dispersion, self-phase modulation alone does not broaden or shorten the pulses. Thus we may conclude that in this particular system dispersion and self-phase modulation have little effect on the pulse-width evolution, except in the final stage of the mode-locking process.

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