WARWICK
THE UNIVERSITY OF WARWICK

The dynamics of money and wealth

Sam Forbes

Stefan Grosskinsky

IM903 – Complexity in the Social Sciences

March 21, 2019

Outline

1. Intro to the monetary system/Motivation

   [Bank of England Quarterly Bulletin 2014 Q1]

2. Role of the money supply in the economy

3. Inequality and wealth distribution

   [Yakovenko, Barkley Rosser: Colloquium: Statistical mechanics of money, wealth, and income, 2010]

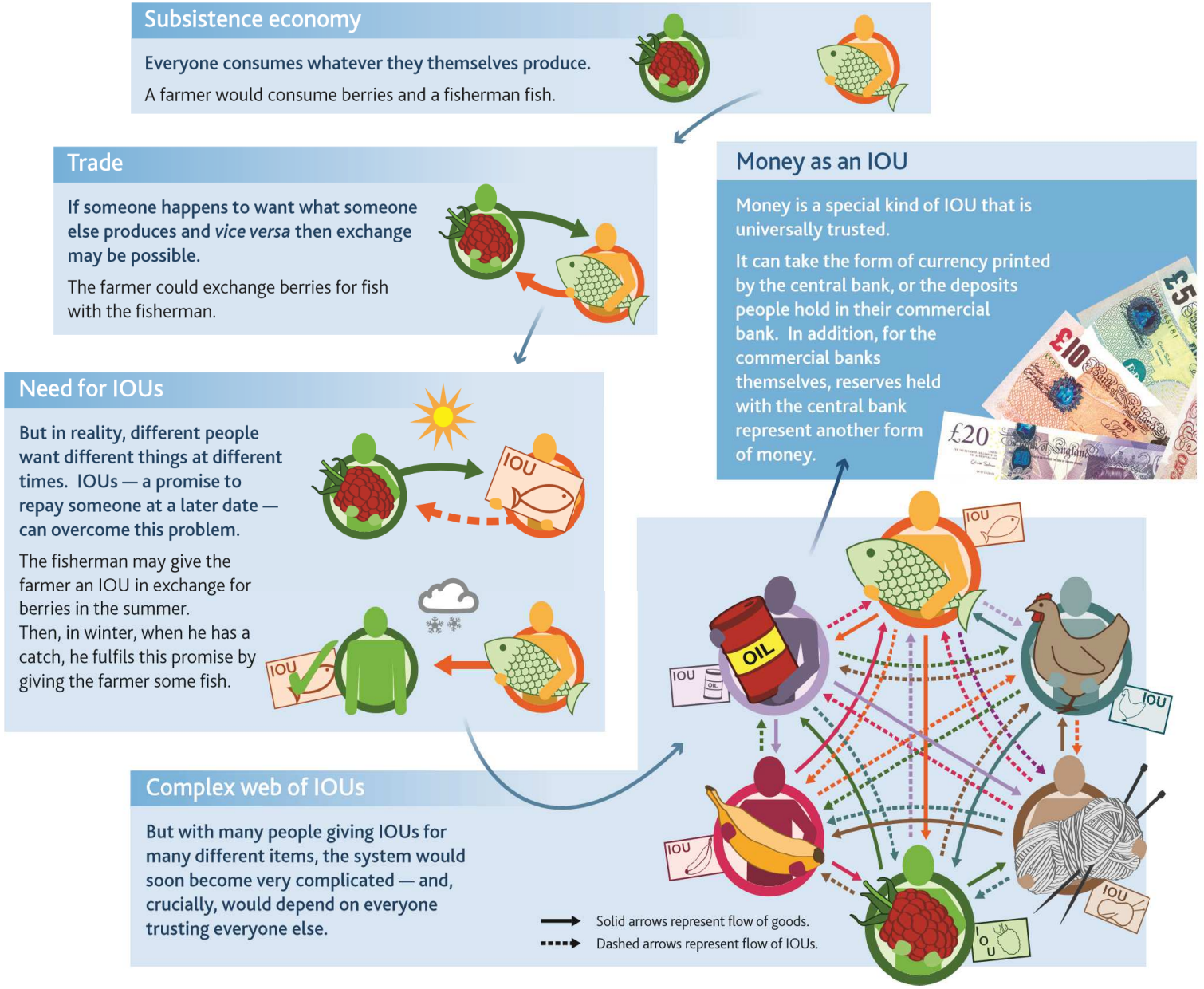
Money

- unit of account
- medium of exchange
- Money as a ‘store’ of value

Wealth

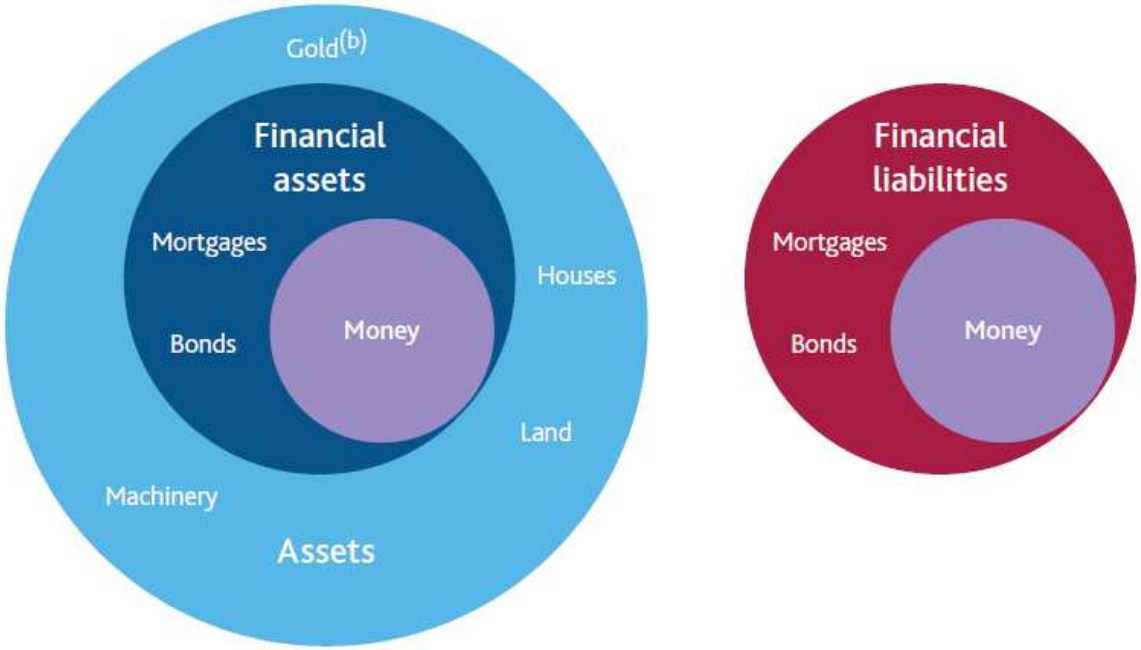
= Assets – Liabilities

- Measured in GBP
- Money is a **special** financial asset



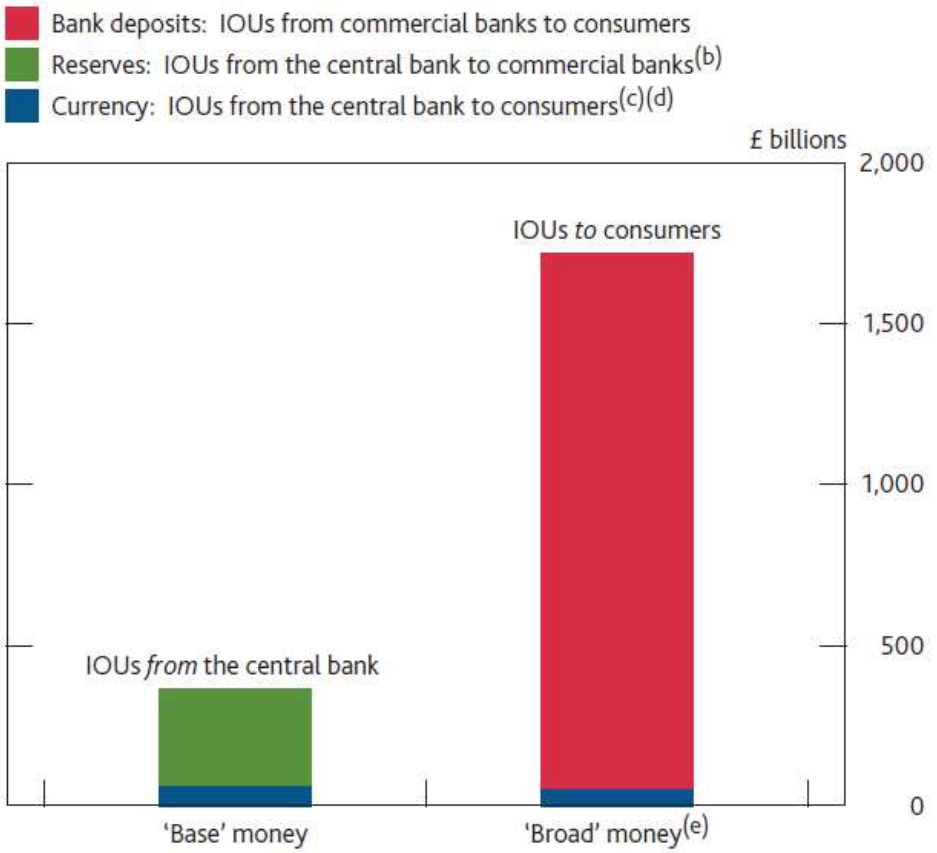
The monetary system

Figure 1 Money and other assets and liabilities(a)



(a) Figure is highly stylised for ease of exposition: the quantities of each asset/liability shown do not correspond to the actual quantities in the economy.
(b) By statistical convention, some holdings of gold (such as by the government) are classed as a financial asset rather than a non-financial asset in economic accounts.

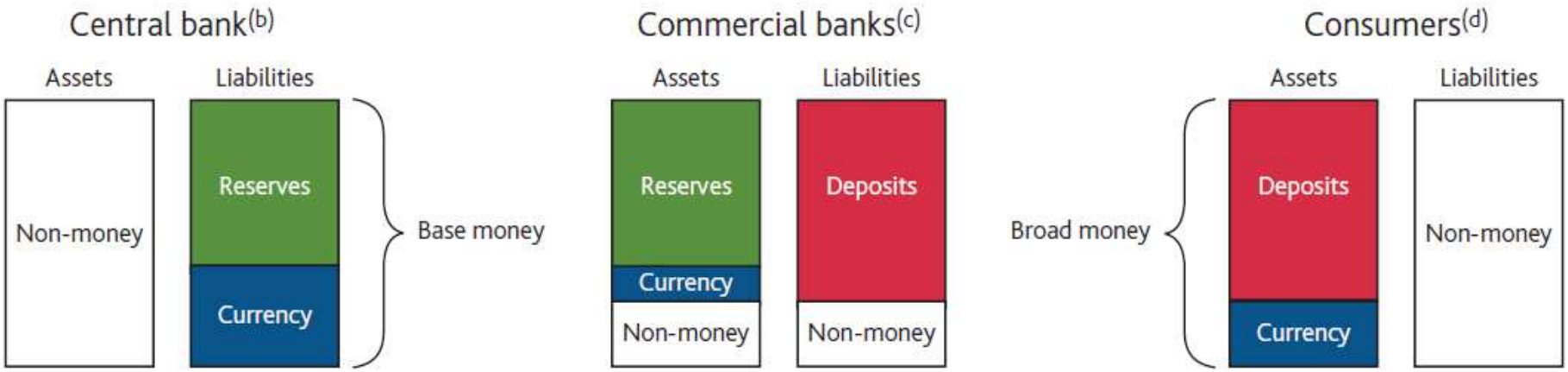
Chart 1 Amounts of money in circulation(a)



(a) All data are for December 2013.
(b) Reserves balances at the Bank of England held by banks and building societies, non seasonally adjusted. Data are measured as the monthly average of weekly data.
(c) Currency in base money includes notes and coin in circulation outside the Bank of England, including those in banks' and building societies' tills. Data are measured as the monthly average of weekly data.
(d) Currency in broad money includes only those notes and coins held by the non-bank private sector, measured as the month-end position.
(e) M4 excluding intermediate other financial corporations.

The monetary system

Figure 2 Stylised balance sheets of different types of money holders and issuers in the economy[a]



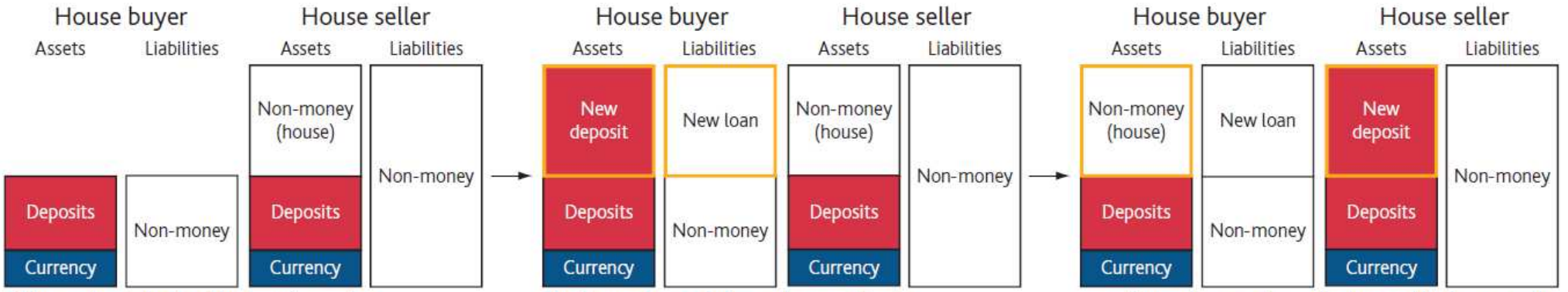
(a) Balance sheets are highly stylised for ease of exposition: the quantities of each type of money shown do not correspond to the quantities actually held on each sector's balance sheet.
(b) Central bank balance sheet only shows base money liabilities and matching assets. In practice the central bank holds other non-money liabilities. Its non-money assets are mostly made up of government debt. Although that government debt is held by the Bank of England's Asset Purchase Facility, so does not appear directly on the Bank of England's consolidated balance sheet.
(c) Commercial banks' non-money assets would include government debt and non-money liabilities would include long-term debt and equity.
(d) Consumers represent the private sector of households and companies. Balance sheet only shows broad money assets and corresponding liabilities. Consumers' non-money liabilities would include secured and unsecured loans.

- Money in the modern economy: an introduction, Bank of England Quarterly Bulletin 2014 Q1
- Money creation in the modern economy, Bank of England Quarterly Bulletin 2014 Q1

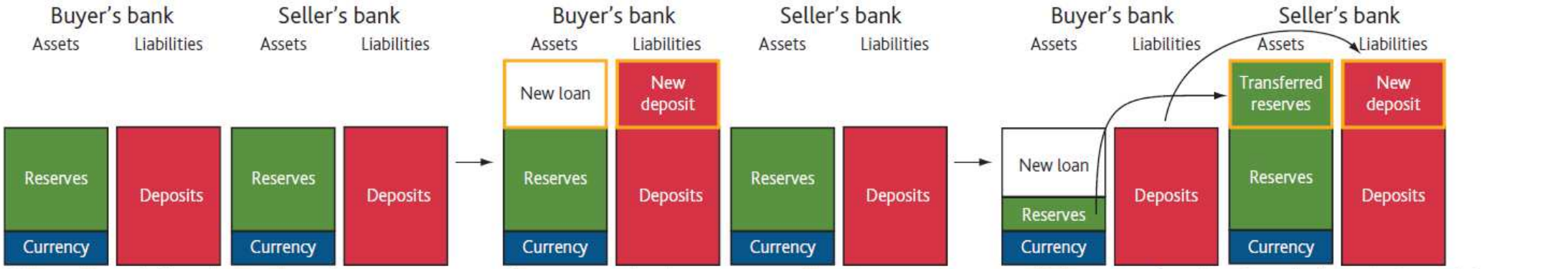
The monetary system

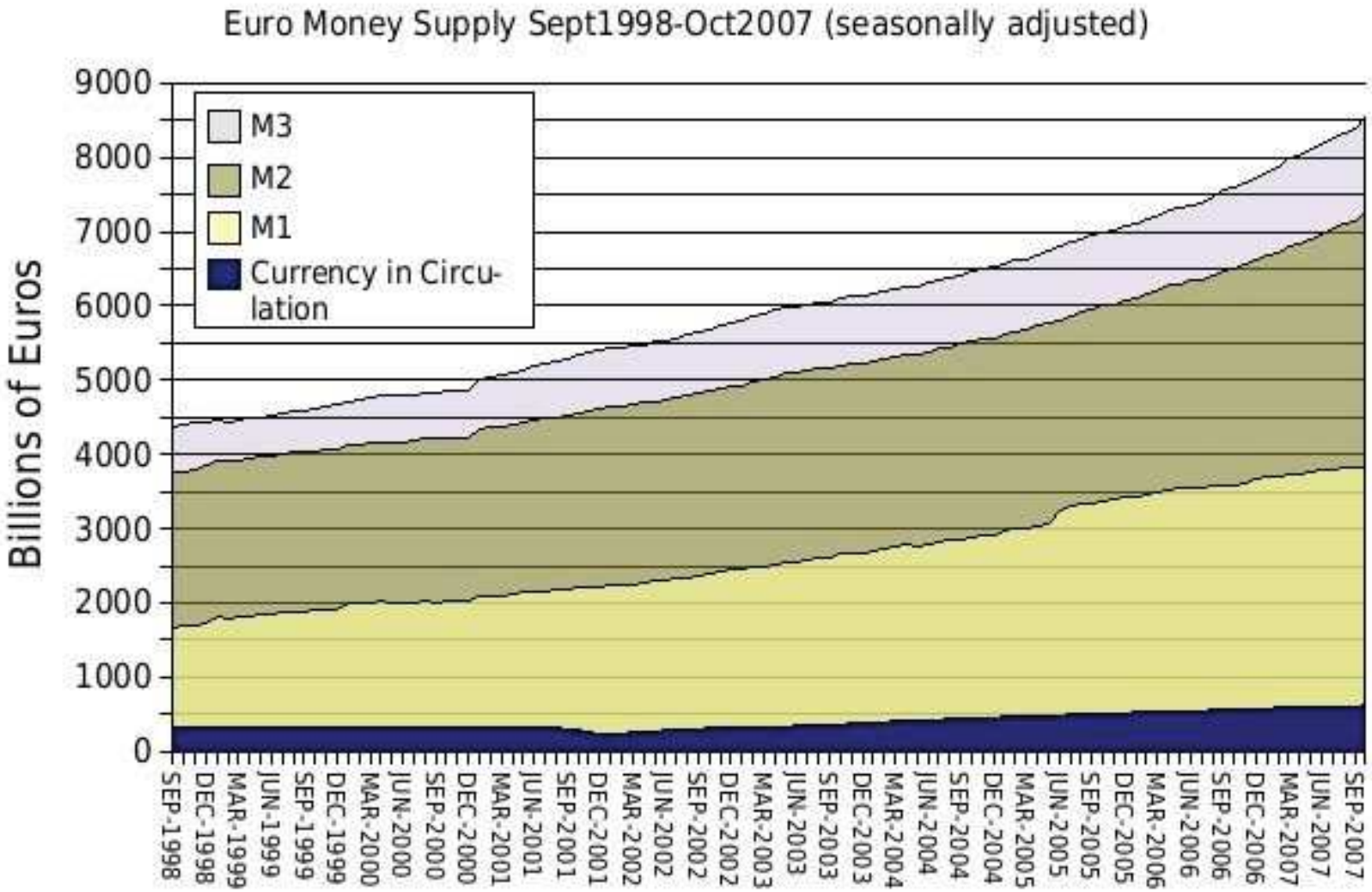
Figure 2 Money creation for an individual bank making an additional loan[(a)]

Changes to the balance sheets of the house buyer and seller



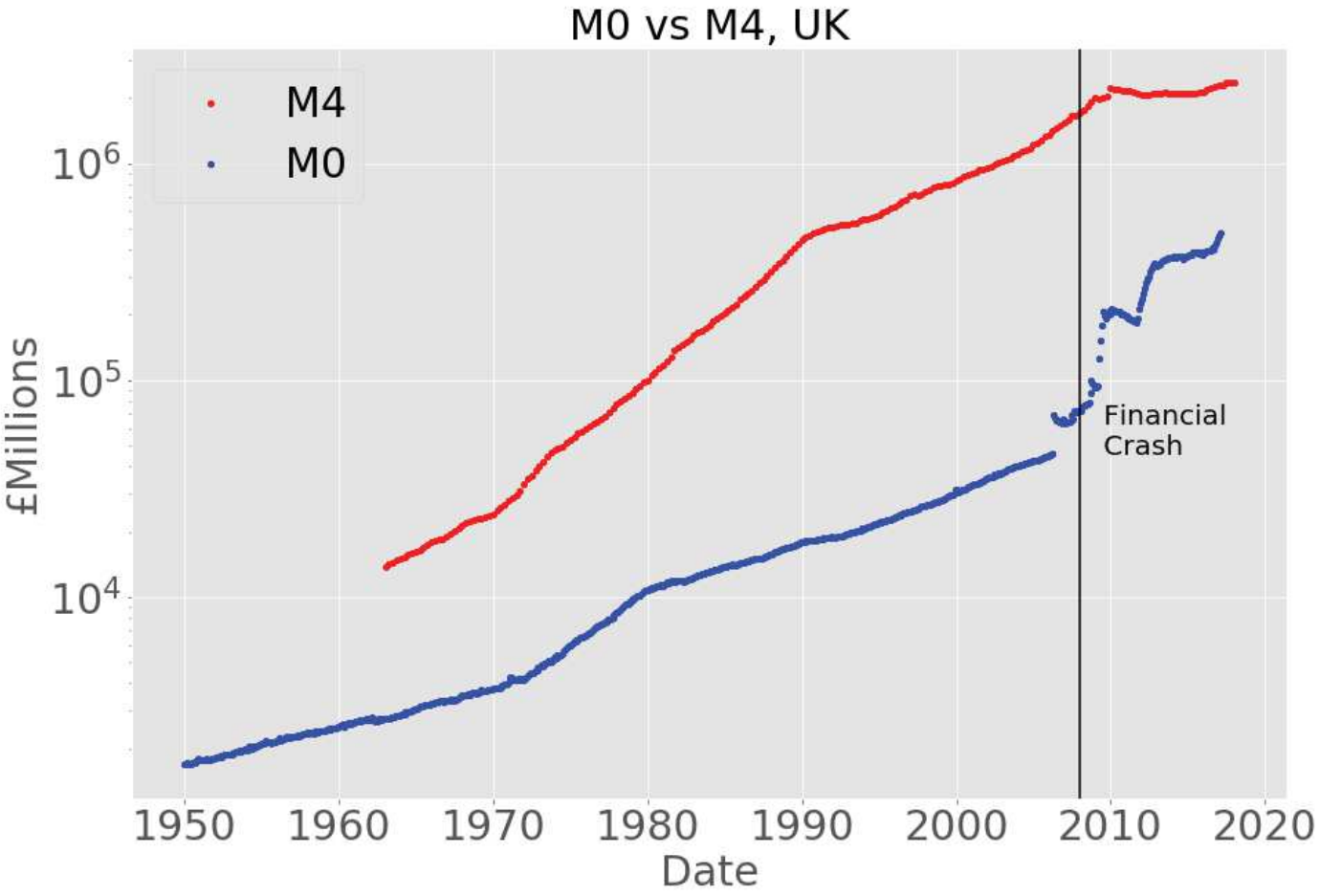
Changes to the balance sheets of the house buyer and seller's banks





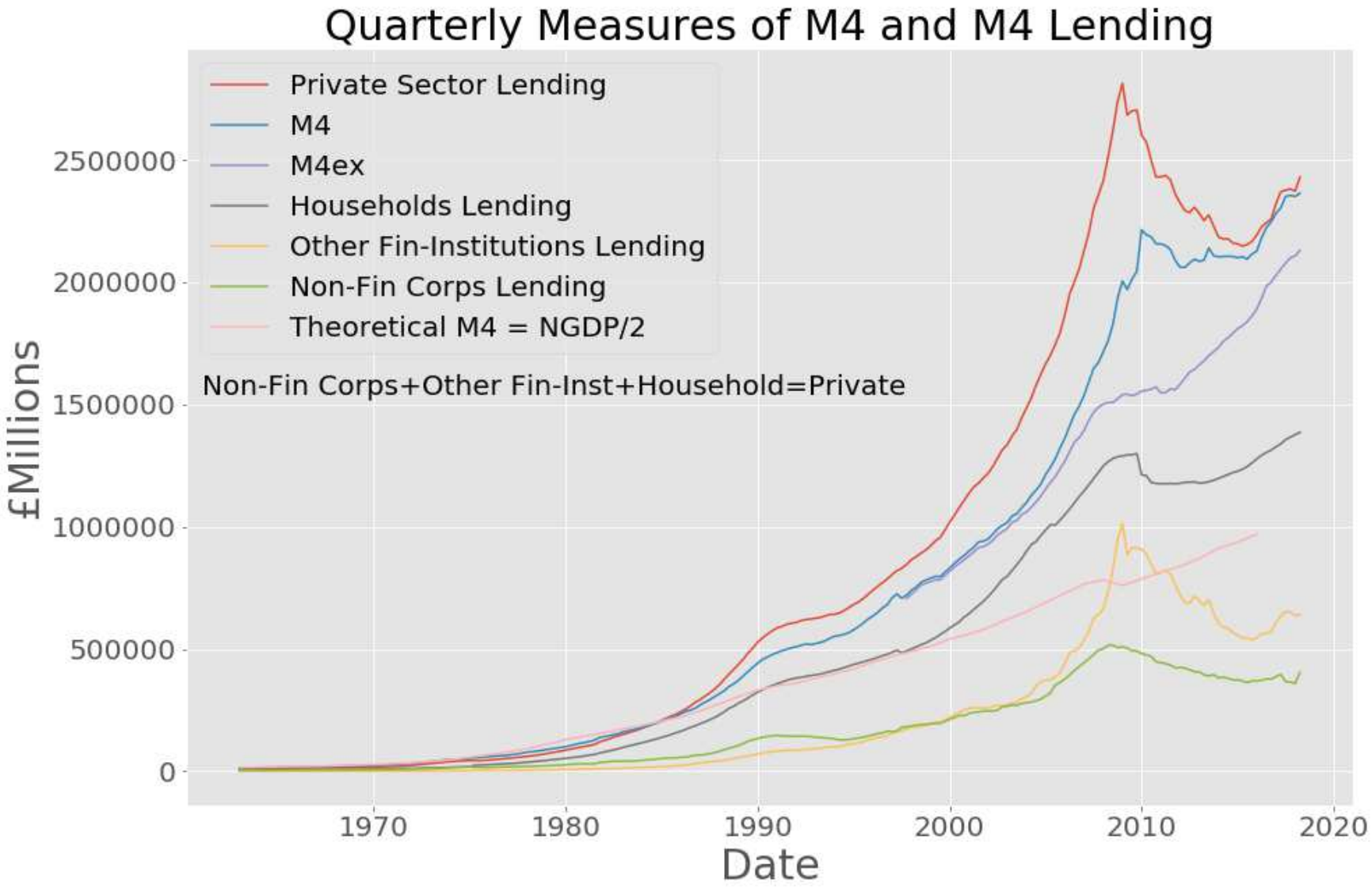
https://en.wikipedia.org/wiki/Money_supply

Money supply for UK



The Prime Minister just told a nurse who hasn't had a pay rise for eight years:

"There's no magic money tree."


Quarterly Measures of M4 and M4 Lending
Private Sector Lending
M4
M4ex
Households Lending
Other Fin-Institutions Lending
Non-Fin Corps Lending
Theoretical M4 = NGDP/2
Non-Fin Corps+Other Fin-Inst+Household=Private
£Millions
2500000
2000000
1500000
1000000
500000
0
1970
1980
1990
2000
2010
2020
Date

GDP

Definition (OECD): aggregate measure of production equal to the sum of the gross values added of all resident and institutional units engaged in production (plus any taxes, and minus any subsidies, on products not included in the value of their outputs)

Measurement: Production approach, Income approach, **Expenditure approach**

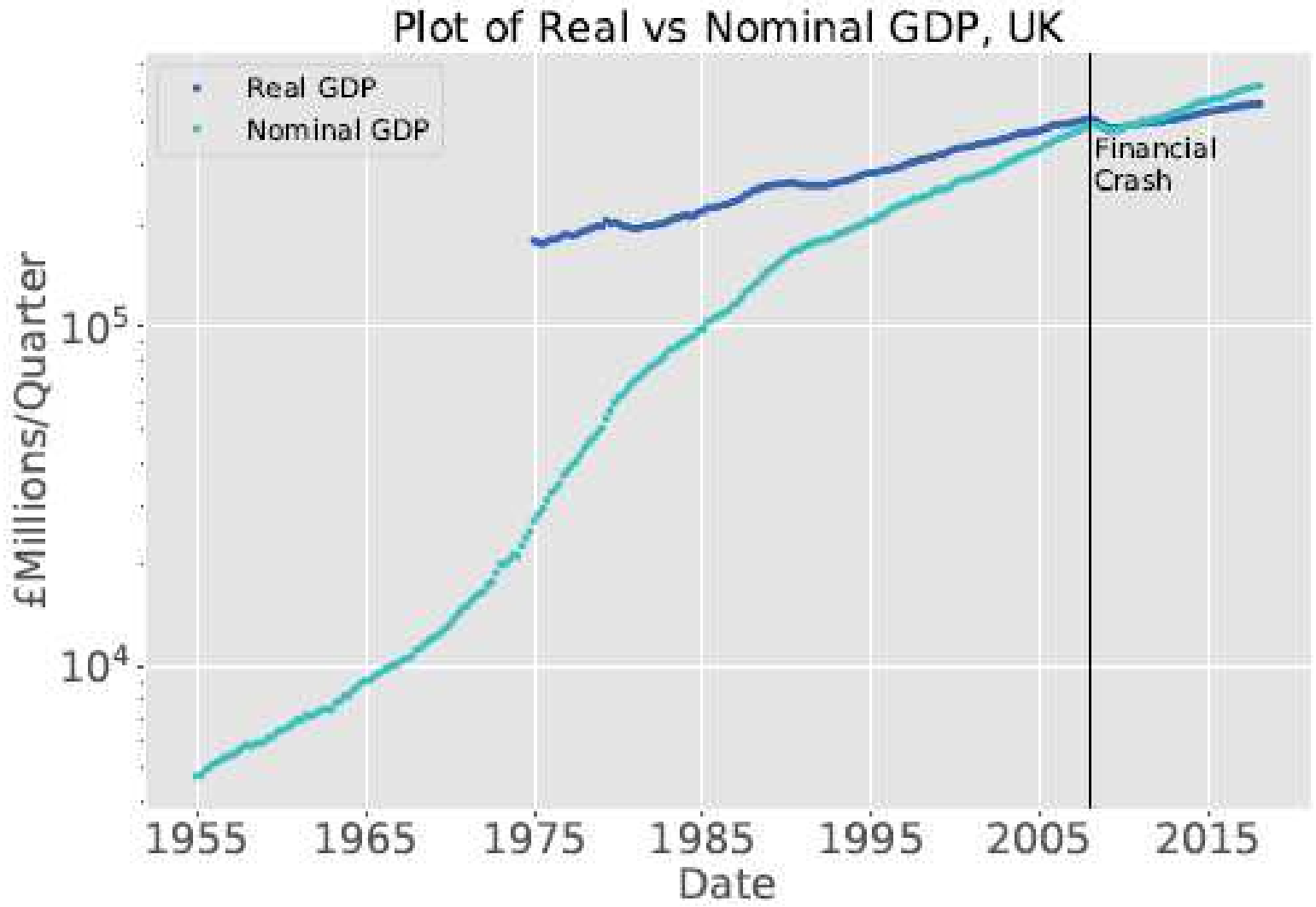
GDP = Consumption + Investment + Government spending + Exports - Imports

Unit: GBP/year or GBP/quarter

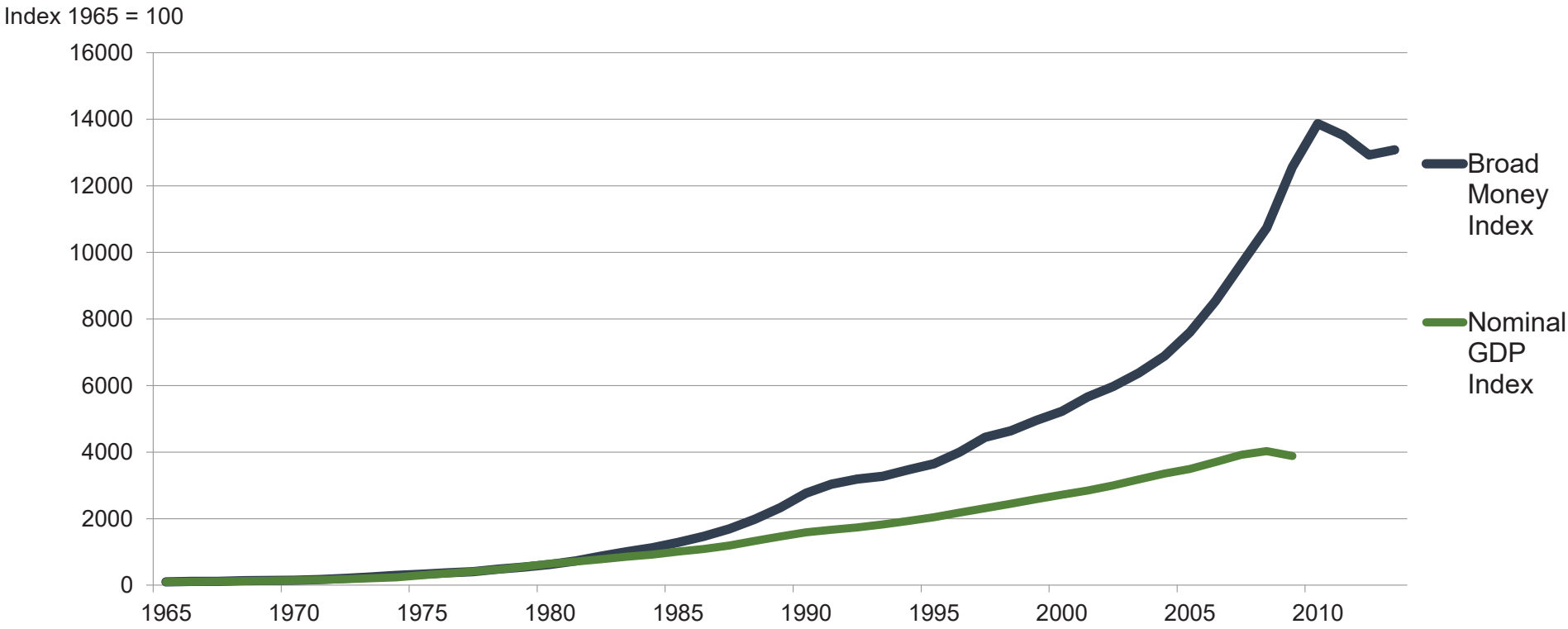
Nominal GDP – in current prices

Real GDP – corrected for inflation

https://www.ons.gov.uk/economy/grossdomesticproductgdp

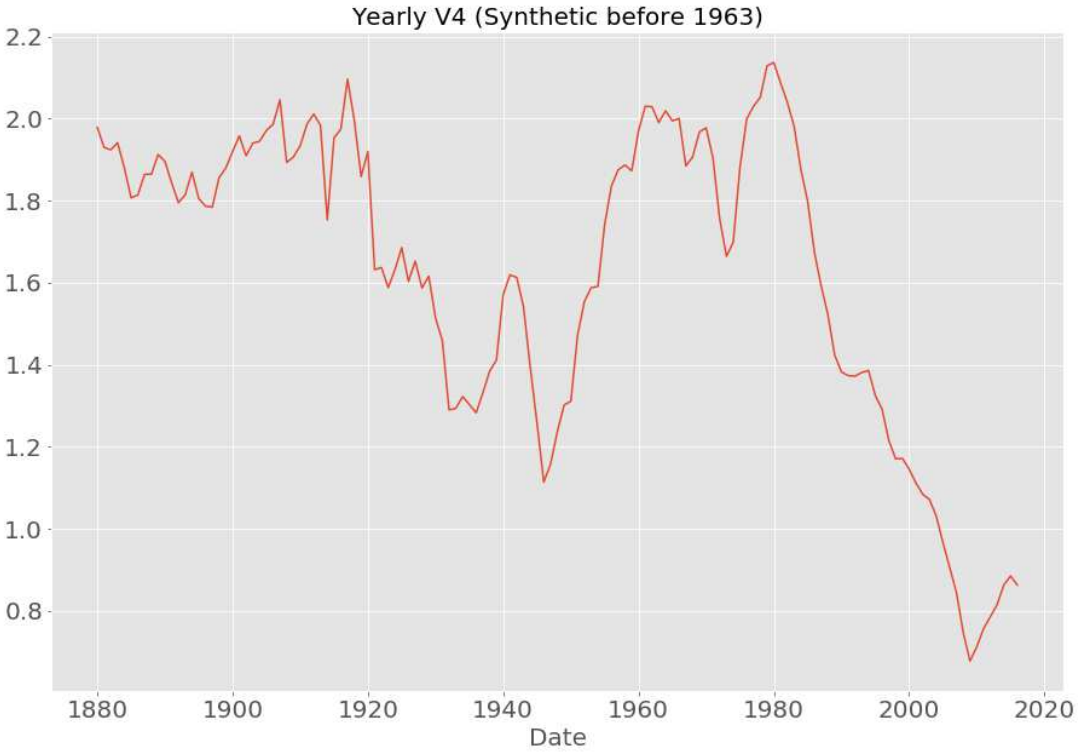

Plot of Real vs Nominal GDP, UK
Real GDP
Nominal GDP
Financial Crash
£Millions/Quarter
10^5
10^4
1955
1965
1975
1985
1995
2005
2015
Date

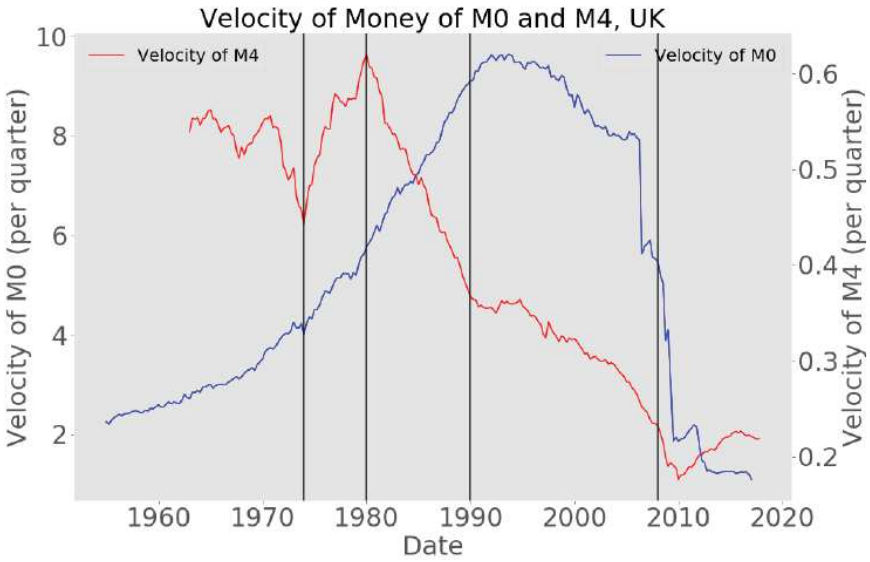
Money supply and GDP



Sources: Bank of England statistical database (series LPQAUYM)
Hills, S., Thomas, R., & Dimsdale, N. (2010). The UK recession in context — what do three centuries of data tell us? Bank of England Q4 Quarterly Bulletin

Velocity of money

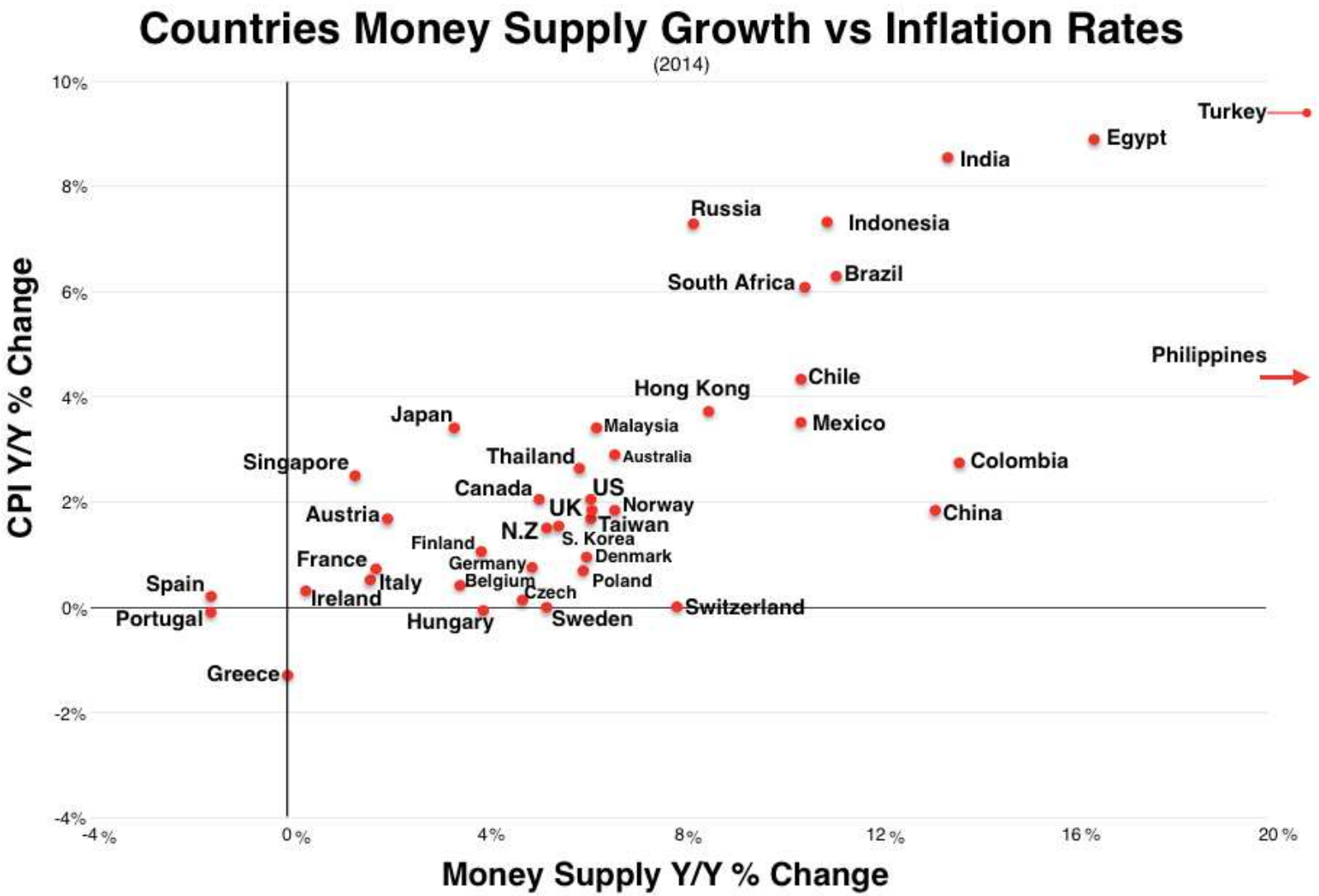




Velocity V = nominal GDP / Money supply

[V] = 1/year

Inflation in the UK



https://en.wikipedia.org/wiki/Money_supply

Austerity

The Prime Minister just told a nurse who hasn't had a pay rise for eight years:

"There's no magic money tree."

Summary of some facts

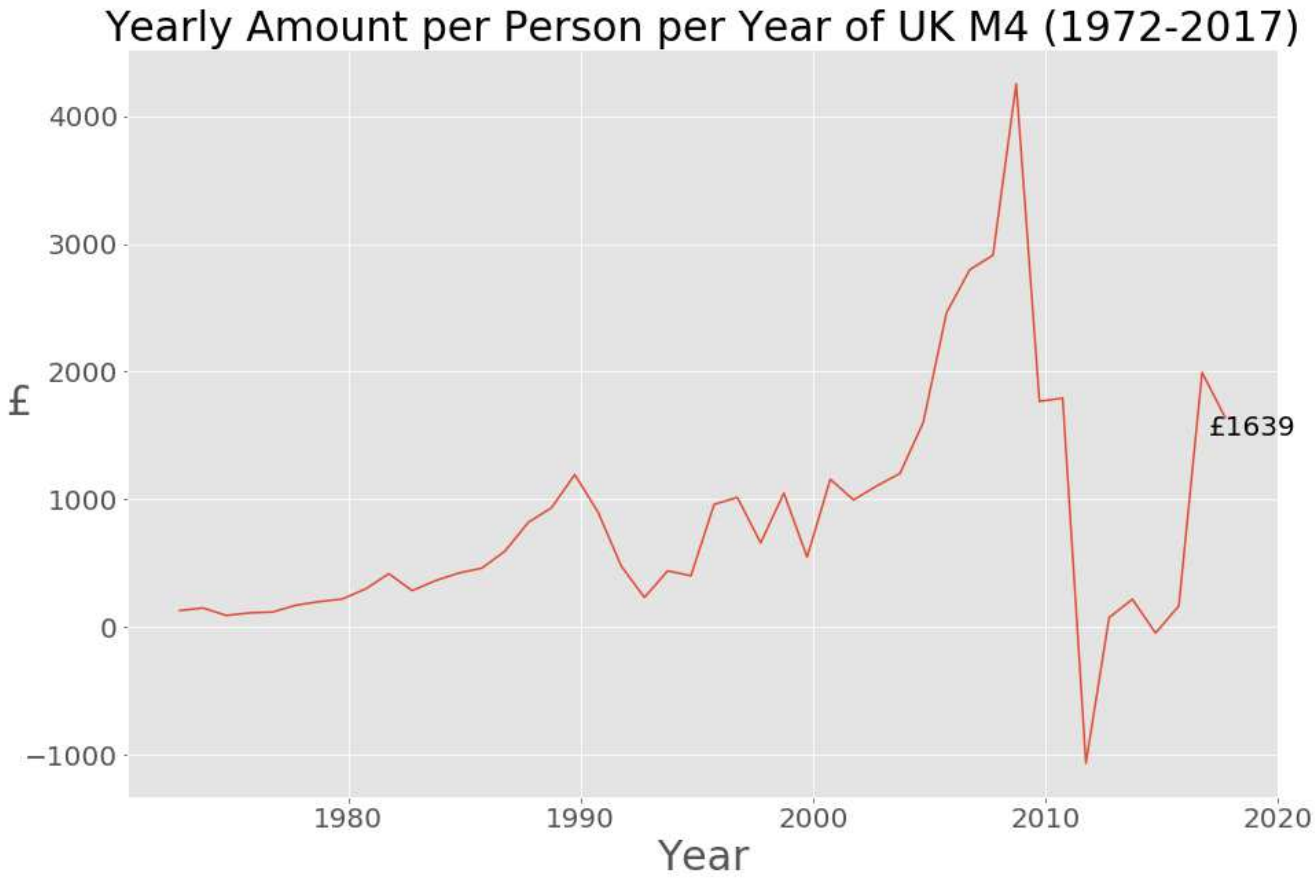
(from mid 2018)

27 M Households with about 2.45 people/household (66 M people)

M4 money supply	Debt	
total **£ 2.4 Trillion**	Household **£ 1.6 Trillion**	Public **£ 1.8 Trillion**
£ 88 K per household (M4ex £ 81K)	**£ 59 K** per household	(The Money Charity)
M4 growth: ~ **£ 100 Billion a year**	Public debt growth: **~ £ 100 Billion a year**	

GDP: **£ 2 Trillion/year**

Motivation to understand monetary dynamics

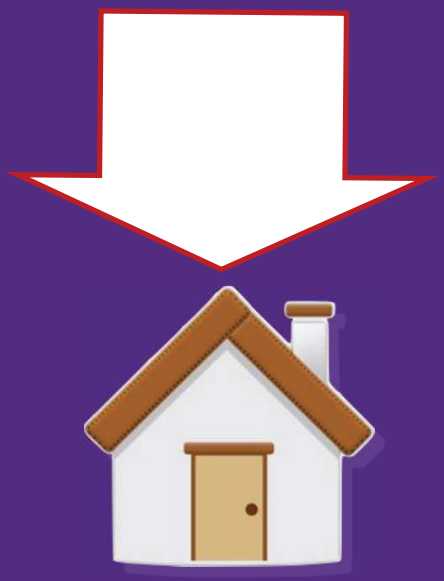


https://www.theguardian.com/society/2019/mar/18/universal-basic-income-could-be-covered-reversing-welfare-spending-cuts-plan-uk

WHERE DOES THE MONEY GO?*

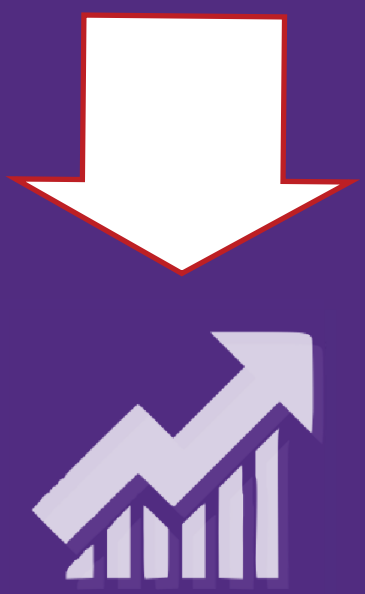
Stock of loan debt, Q3 2018
Source: Bank of England

50%



Mortgages

26%



Financial

17%

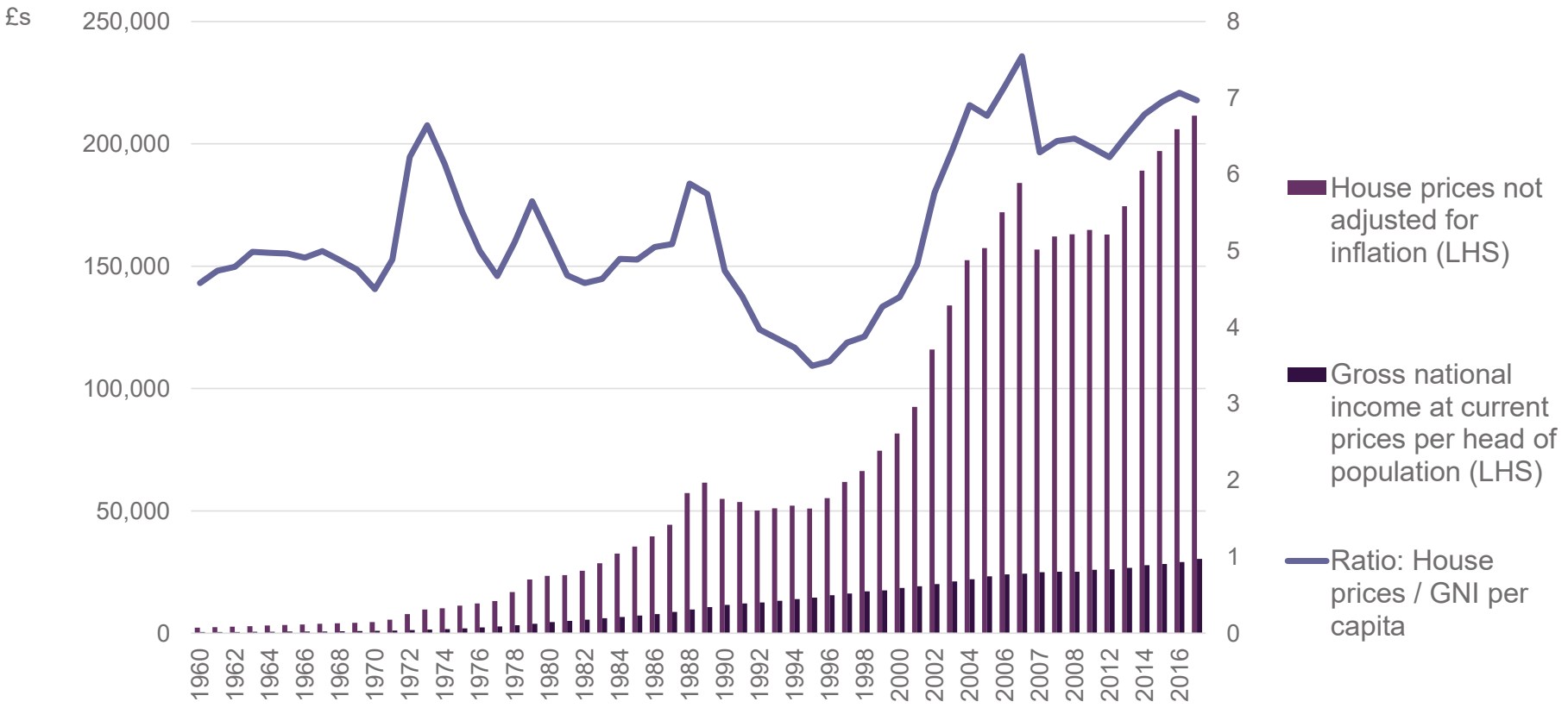
Non-financial business

5%

Credit cards and personal loans

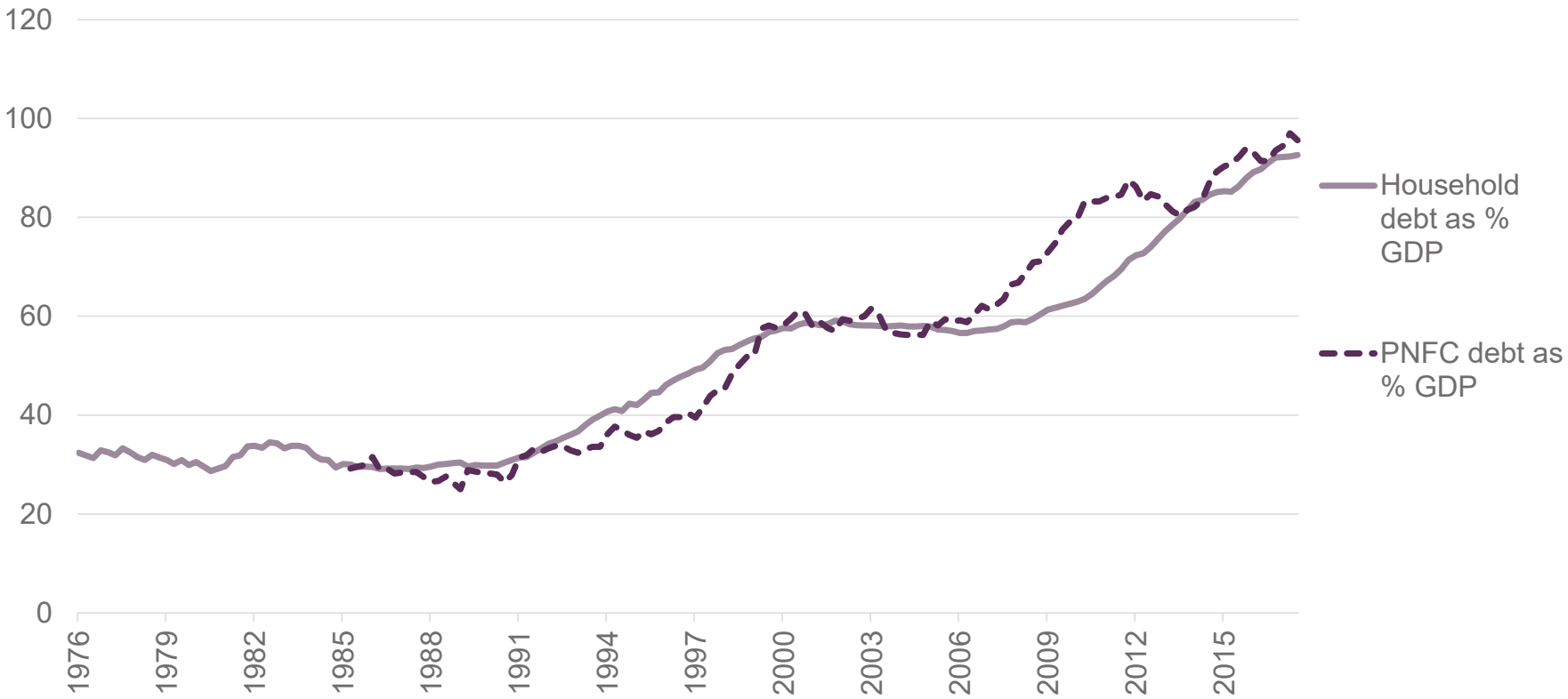
*government = 2%

Housing crisis



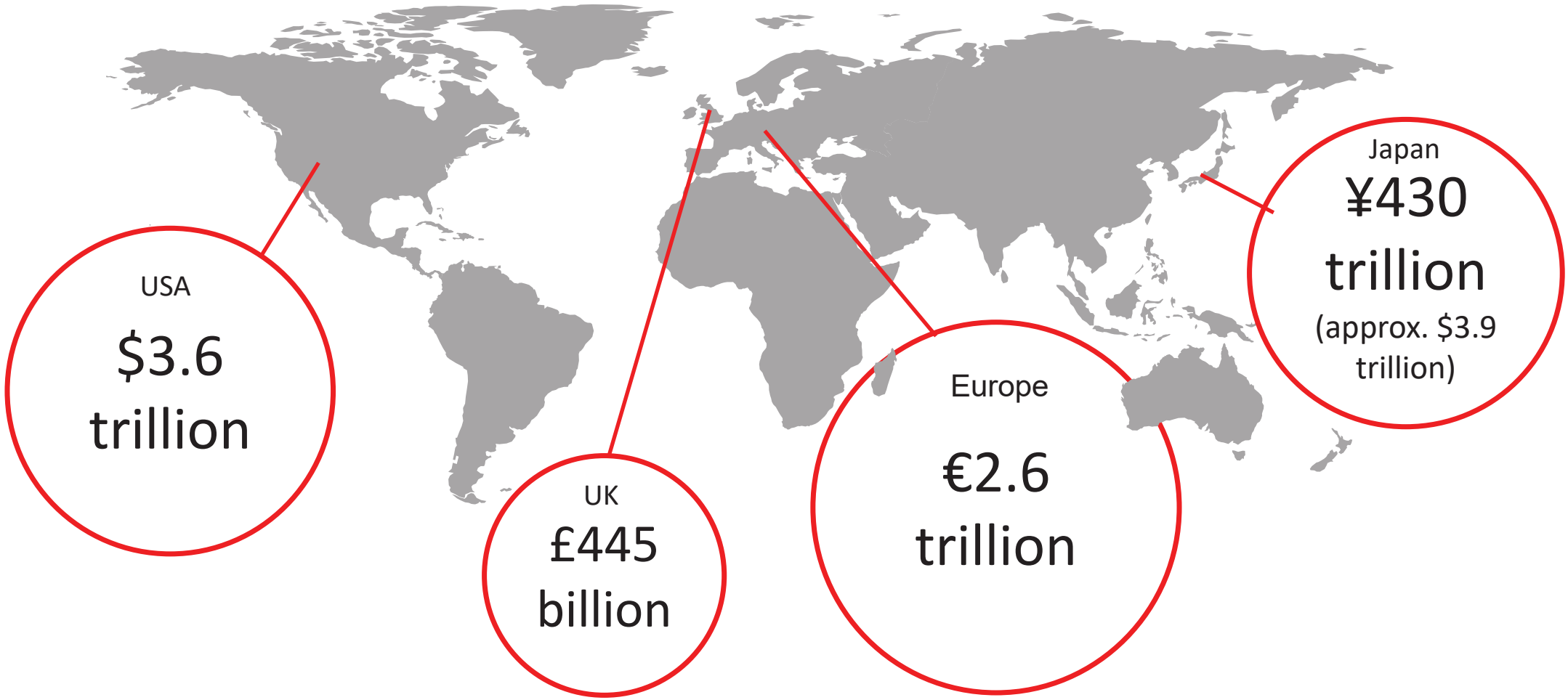
Sources: Nationwide UK House Price Index;
European Commission Economic and Financial Affairs, AMECO statistics

Over-indebtedness



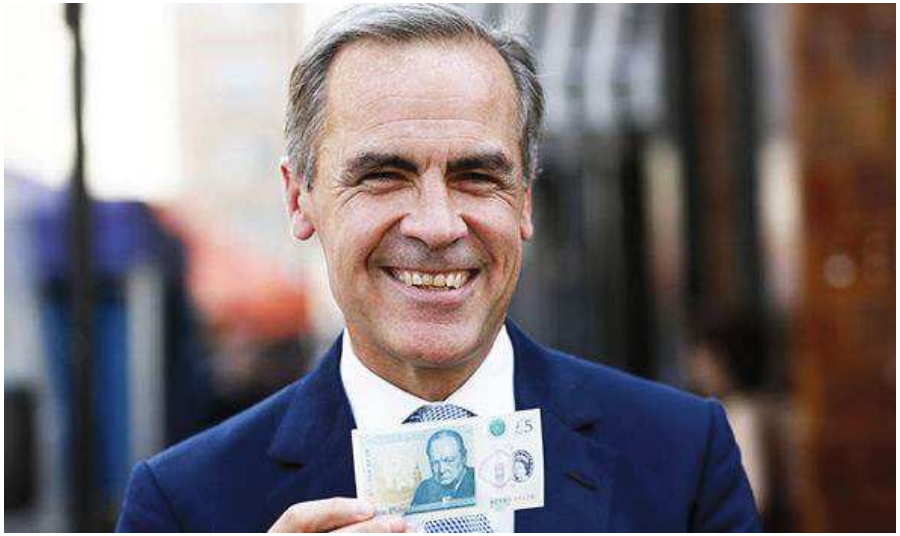
Source: Bank for International Settlements statistical warehouse, Tables F2, F3

Introducing Quantitative Easing (QE)



QE in a nutshell

- the Bank of England creates new money and uses it to buy financial assets from banks
- aim: 'to inject money into the economy in order to revive nominal spending' (Benford *et al.*, 2009)
  - 'bank lending channel'
  - 'wealth channel'
  - 'portfolio rebalancing channel'

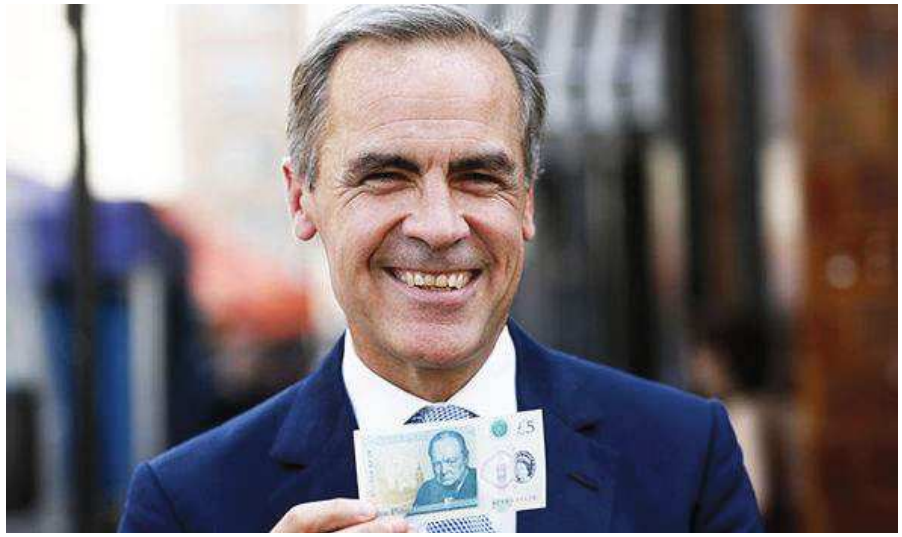


- not very effective stimulus: for every £1 of money created via QE, UK GDP increased by just 10p-15p (see Joyce *et al., 2011*)
- much newly created money either stays in banking sector, or inflates asset prices

Benford, J., Berry, S., Nikolov, K., Young, C., and Robson, M. (2009). 'Quantitative easing'. BoE Quarterly Bulletin, 49(2)

Joyce, M., Tong, M., & Woods, R. (2011). The United Kingdom's quantitative easing policy: design, operation and impact. BoE Quarterly Bulletin, 51(3)

worked for Goldman Sachs from 2002 until 2005



from 1990 to 2003

Wanted: new fiscal-monetary relationship

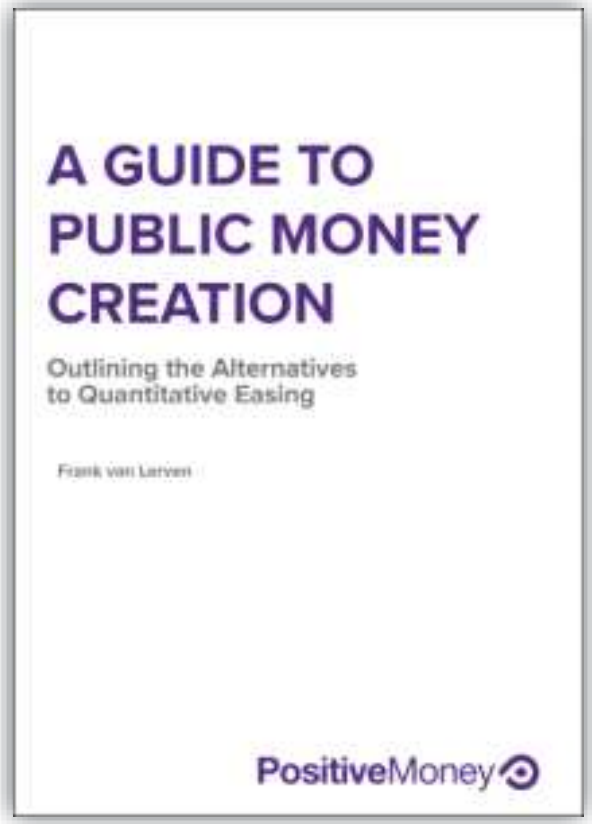
Bank of England

UK Government

NHS

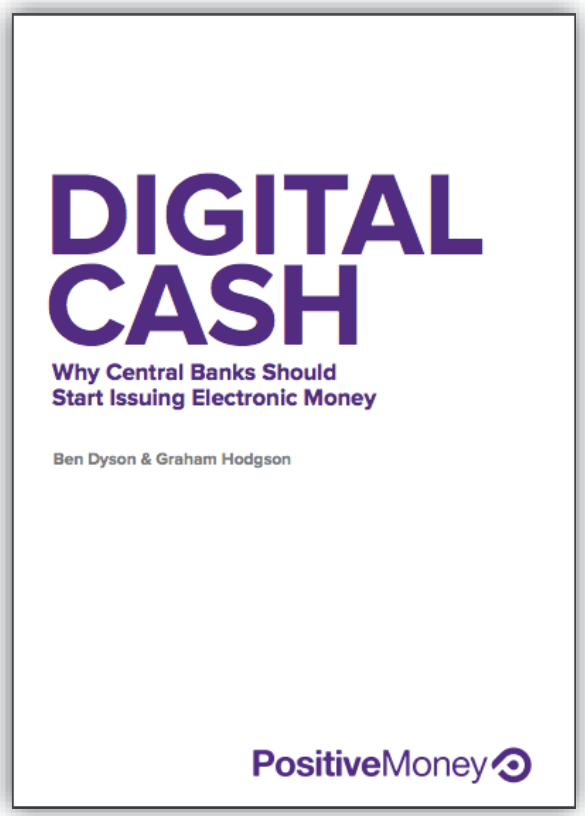
1. QE for People, or 'Monetary Financing'

- instead of going into financial markets, new money could be created to fund society's needs
- if £445bn of new money distributed evenly amongst the population, every UK citizen would receive £6,834
  - **'helicopter money'**
- alternatively, new money created by the Bank of England could go to schools, hospitals, construction, clean energy…
  - **'Green New Deal'**



2. Digital cash…

- electronic version of notes and coins
- design:
  - exists alongside physical cash
  - doesn't require an account at a high street bank
- advantages
  - promotes competition and innovation in current accounts
  - makes the payment system more resilient
  - removes a reason for "too big to fail"
  - expands the options for monetary policy
  - could promote financial inclusion



Outline

1. Intro to the monetary system/Motivation
2. Role of the money supply in the economy
3. Inequality and wealth distribution

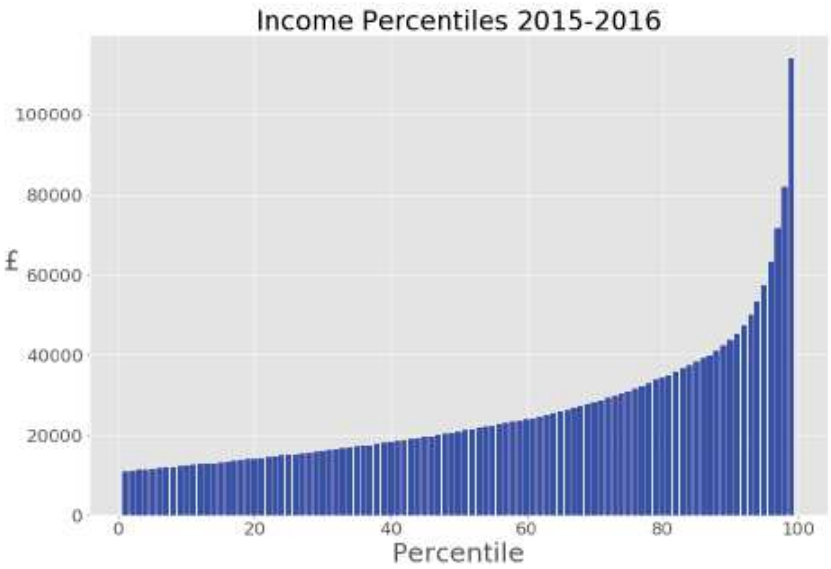
Wealth distribution

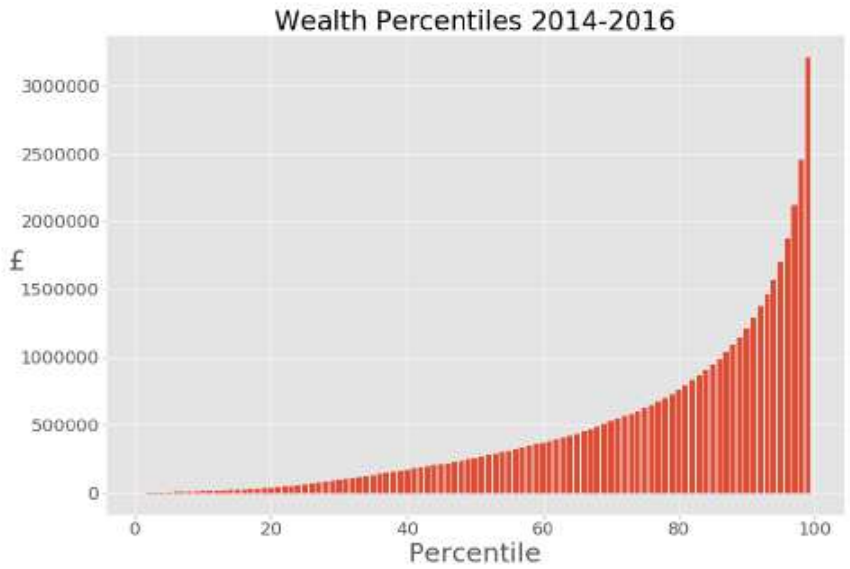
Figure 5: Breakdown of aggregate total wealth, by deciles and components^1^

Great Britain, July 2014 to June 2016

Source: Wealth and Assets Survey, Office for National Statistics

Wealth distribution

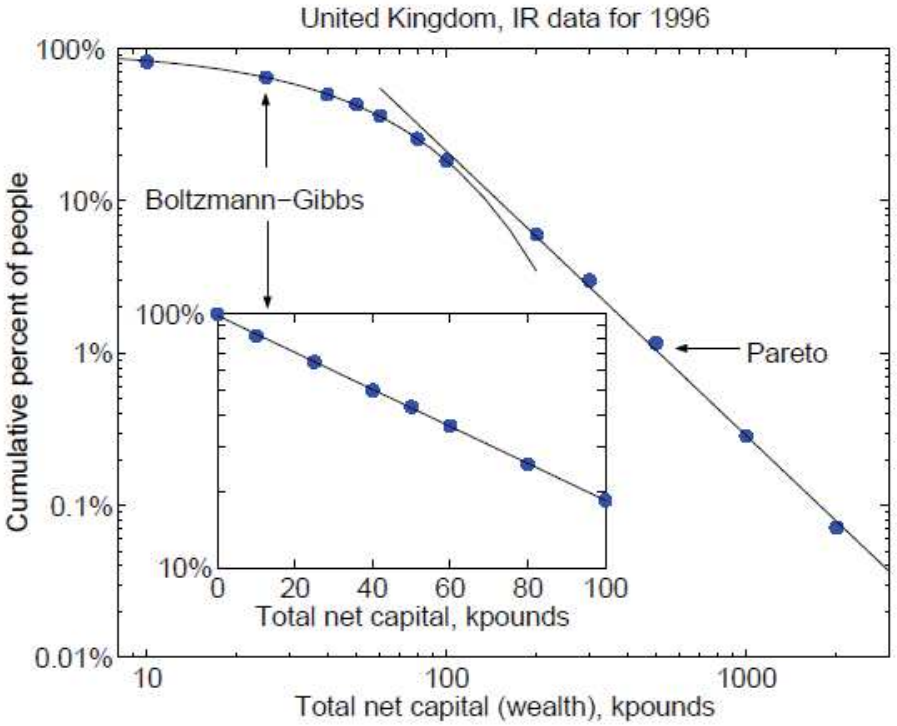


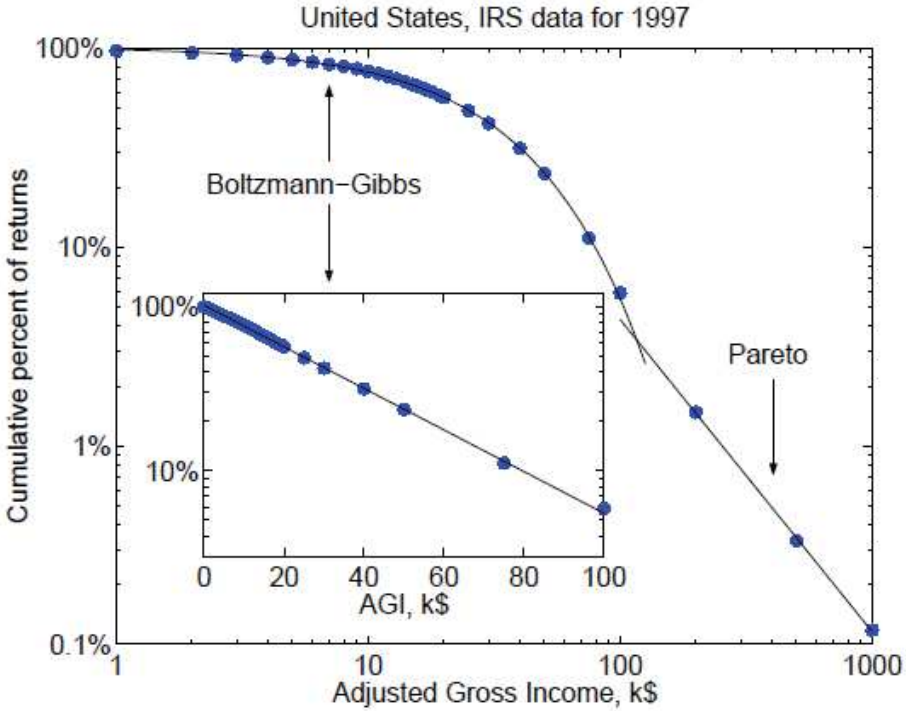


	Income (£)	Wealth (£)
Bottom 1%	10800	-5129
Top 1%	114000	3208546
Median	20800	259430
Mean	25679	458556

Source: Office for National Statistics

Wealth distribution

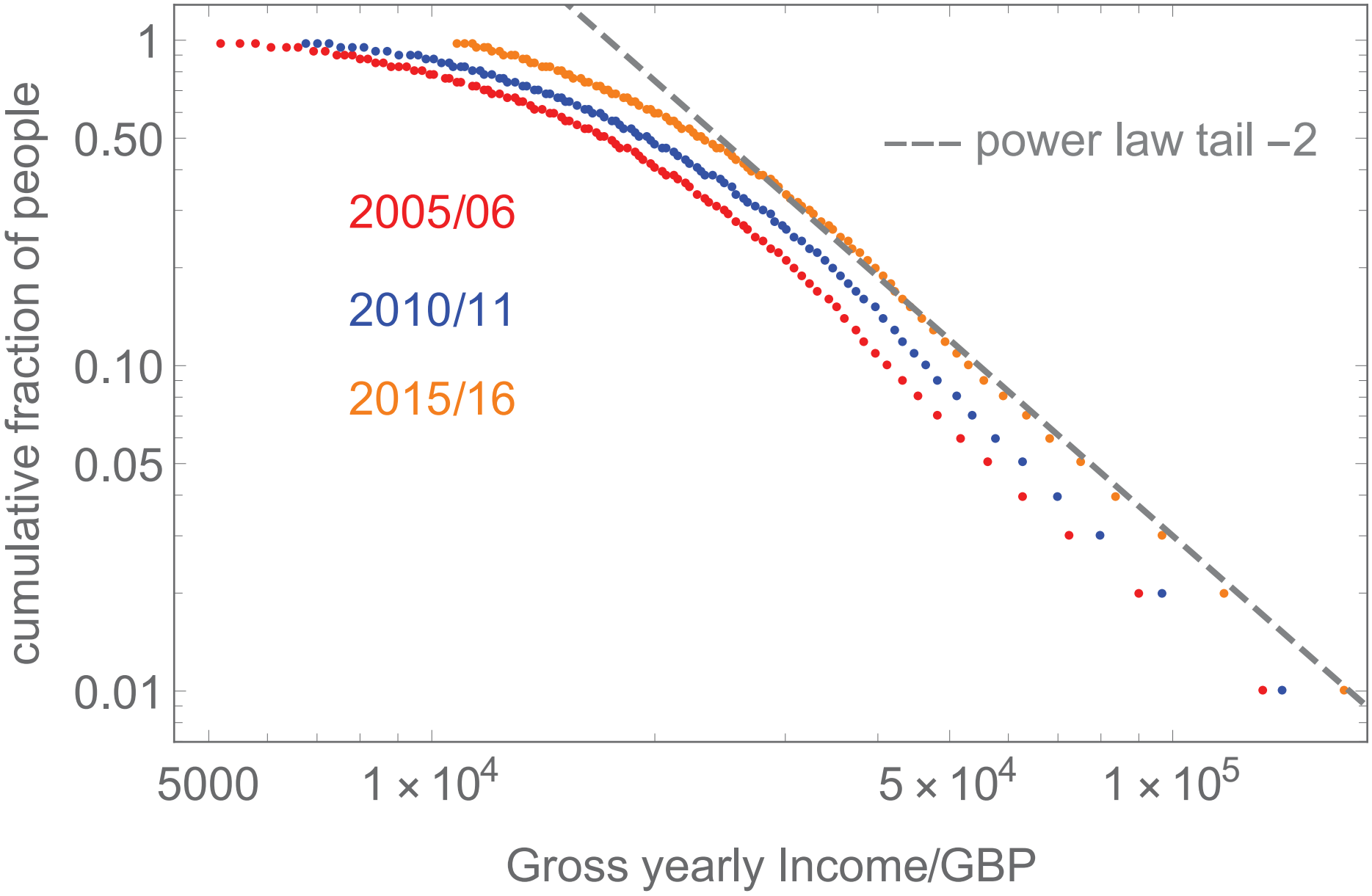




→ **Econophysics**

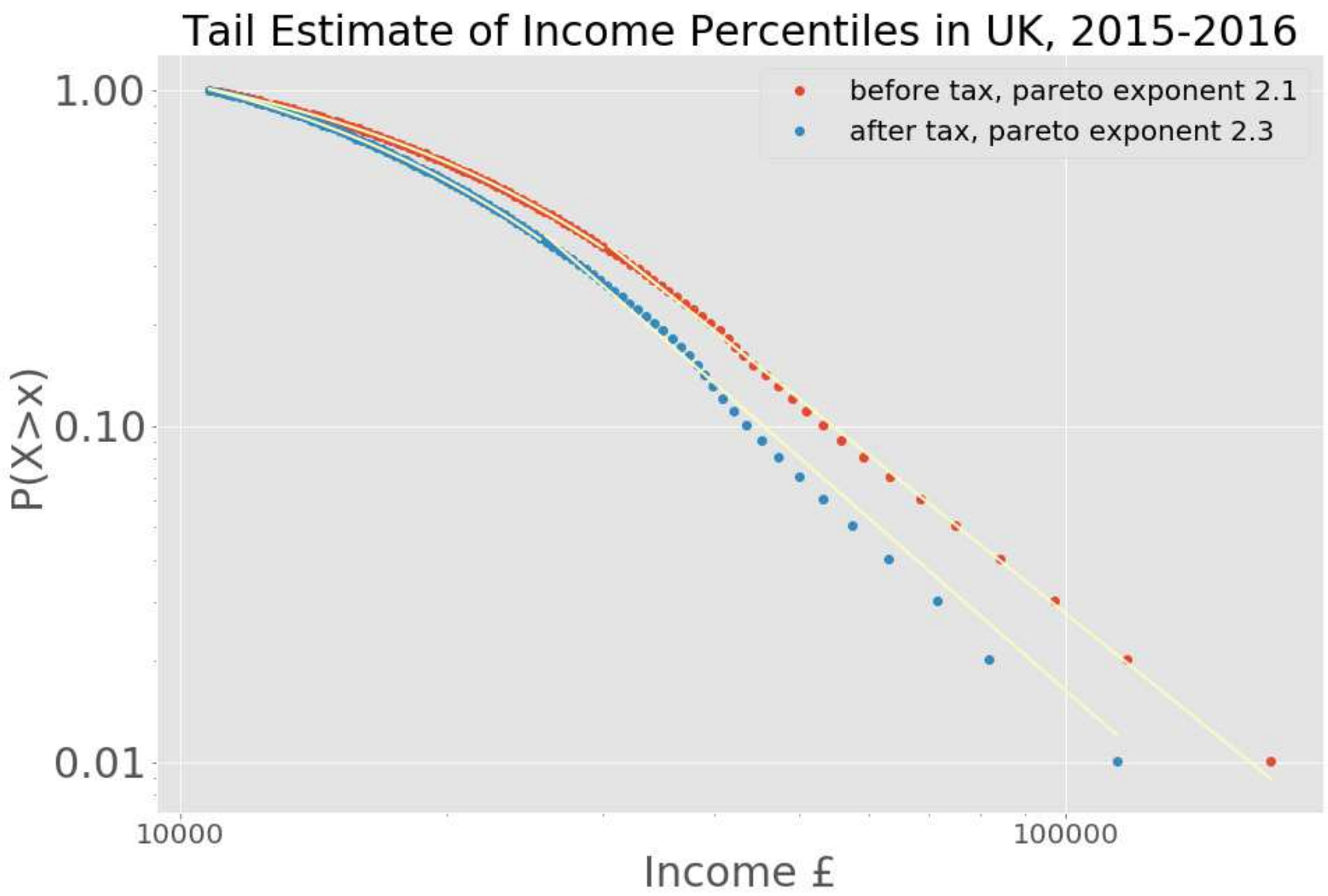
[Drăgulescu, Yakovenko (2000); Yakovenko, Barkley Rosser Jr (2009)]

Income distribution



[Office for National Statistics]

Income distribution



[Office for National Statistics]

Random exchange models

N agents with fixed amount of total wealth

states $w = (w_i : i = 1, \dots, N)$ with $w_i \geq 0$ wealth of agent i

additive exchange $w, w' \leftrightarrow w - \Delta, w' + \Delta$ with rate $r(\Delta) \geq 0$

- $\Delta > 0$ is fixed or i.i.d. random,

  Δ and the state space can be discrete or continuous.

- **Boltzmann-Gibbs/Exponential** distribution $\mathbb{P}_\lambda[W = w] = \prod_i \frac{e^{-\lambda w_i}}{Z(\lambda)}$,

  with $\lambda > 0$ satisfies **detailed balance**

$$e^{-\lambda w_i} e^{-\lambda w_j} r(\Delta) = e^{-\lambda(w_i - \Delta)} e^{-\lambda(w_j + \Delta)} r(\Delta) \ ,$$

  and is therefore **reversible** (and stationary) for the process

- dynamics **conserves total wealth**, i.e. $|w(t)| = \sum_i w_i(t) = |w(0)|$

  λ parametrizes average wealth per person ($= 1/\lambda$ for pure exp. case)

$$\mathbb{P}_M[W = w] = \mathbb{P}_\lambda\big[W = w \big| \, |W| = M\big] = \frac{1}{Z_{N,M}}$$

  is stationary for all $M \geq 0$ and **uniform!** on all allowed configurations

Random exchange models

multiplicative exchange

$$w, w' \to w - \Delta, w' + \Delta \quad \text{with rate} \quad r(\Delta, w) \geq 0$$

- $\Delta > 0$ is a fixed or random fraction of w
- The process is not reversible and the BG/exponential distribution is in general not stationary
- For $\Delta = \gamma w$ with fixed $\gamma \in (0, 1)$ the stationary distribution is similar to a **Gamma law** with density

$$f_{\alpha,\beta}(w) \sim x^{\alpha-1} e^{-x/\beta}$$

  where $\alpha = -\ln 2 / \ln(1 - \gamma)$ and average wealth is $\alpha\beta$
- in general **no heavy tails** also for different versions including saving propensities and more general rates $r(\Delta, w, w')$

Random exchange models

heterogeneous exchange

$$w_i, w_j \to w_i - \Delta, w_j + \Delta \quad \text{with rate} \quad r(\Delta, w_i, w_j, i, j) \geq 0$$

- several examples that lead to heavy-tailed stationary distributions
- simplest case: $r = r(\Delta)e^{\lambda_i \Delta}$

  $\mathbb{P}_\lambda[W = w] = \prod_i \frac{e^{-\lambda_i w_i}}{Z(\lambda_i)}$ satisfies detailed balance

$$e^{-\lambda_i w_i} e^{-\lambda_j w_j} r(\Delta) e^{\lambda_i \Delta} = e^{-\lambda_i (w_i - \Delta)} e^{-\lambda_j (w_j + \Delta)} r(\Delta) e^{\lambda_j \Delta} \ ,$$

- independent exp. r.v.s with disordered parameter can show **heavy tails**; distribution conditioned on total wealth may also exhibit **condensation**
- many different versions have been studied, see references in

  [Yakovenko, Barkley Rosser: Colloquium: Statistical mechanics of money, wealth, and income, 2010]

Growth models

N agents, wealth growing in discrete timesteps and increments

states $w(t) = (w_i(t) : i = 1, .., N)$ with $w_i(t) \geq 0$ wealth of agent i at time t

- models are in general not tending to stationary distributions!
- **additive growth** $w_i \to w_i + 1$ with prob. $1/n$

$$w_i(t) = w_i(0) + \mathrm{Bin}(t, 1/n) \approx \mathrm{Poi}(t/n) \quad \text{as } n \to \infty$$

- **linear reinforcement** $w_i \to w_i + 1$ with prob. $w_i / \sum_j w_j$

$$w_i(t) \approx \mathrm{Geo}(t/n) \quad \text{as } n \to \infty$$

- **heterogeneous reinforcement** $w_i \to w_i + 1$ with prob. $p_i w_i / \sum_j w_j$
  if p_i are i.i.d. uniform $[0, 1]$ see heave tails

Bouchaud, Mézard (2000)

- $w_i(t) \in \mathbb{R}$ wealth of agent i evolves in continuous time with SDE

$$\frac{d}{dt} w_i(t) = w_i(t)\big(\mu + \eta_i(t)\big) + \sum_{j \neq i} J_{ji} w_j(t) - \sum_{j \neq i} J_{ij} w_i(t)$$

  with i.i.d. **Gaussian white noise** $\langle \eta_i(t) \eta_j(s) \rangle = 2\sigma^2 \delta_{ij} \delta(t-s)$

- **mean field** $J_{ij} \equiv J$ implies $\langle w \rangle(t) = \langle w \rangle(0)\, e^{(\mu+\sigma^2)t}$ for average wealth, and for **relative wealth** $\tilde{w}_i = w_i / \langle w \rangle$ we get

$$\frac{d}{dt} \tilde{w}_i(t) = (\eta_i(t) - \sigma^2)\tilde{w}_i(t) + J\big(1 - \tilde{w}_i(t)\big)$$

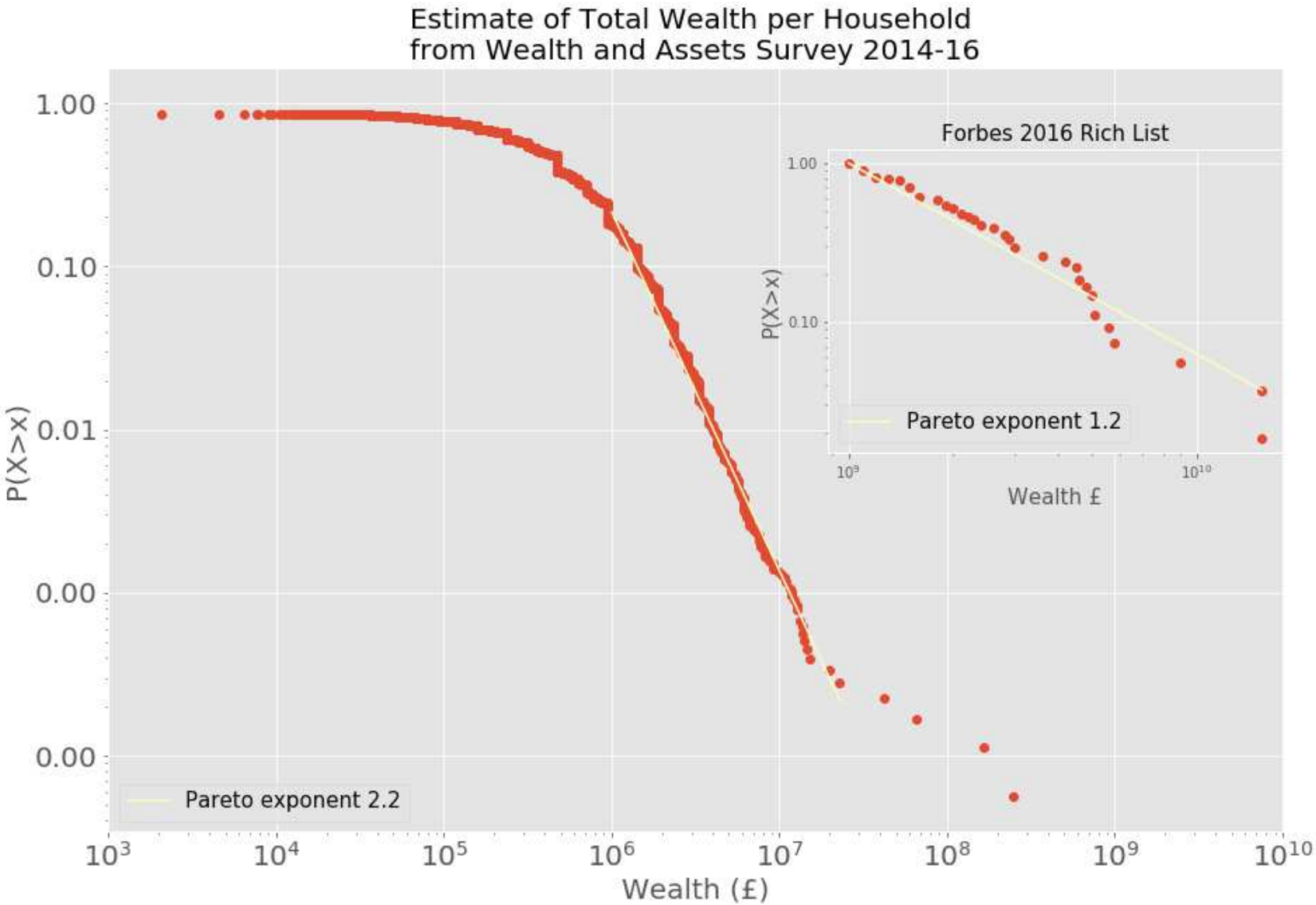
- The corresponding **Fokker-Planck equation** for PDF $f(\tilde{w}, t)$

$$\frac{\partial}{\partial t} f = \frac{\partial\big(J(\tilde{w}-1) + \sigma^2 \tilde{w}\big) f}{\partial \tilde{w}} + \sigma^2 \frac{\partial}{\partial \tilde{w}} \left(\tilde{w} \frac{\partial(\tilde{w} f)}{\partial \tilde{w}} \right)$$

  has **stationary distribution** $f^*(\tilde{w}) \propto e^{-J/(\sigma^2 \tilde{w})} \big/ \tilde{w}^{2+J/\sigma^2}$.

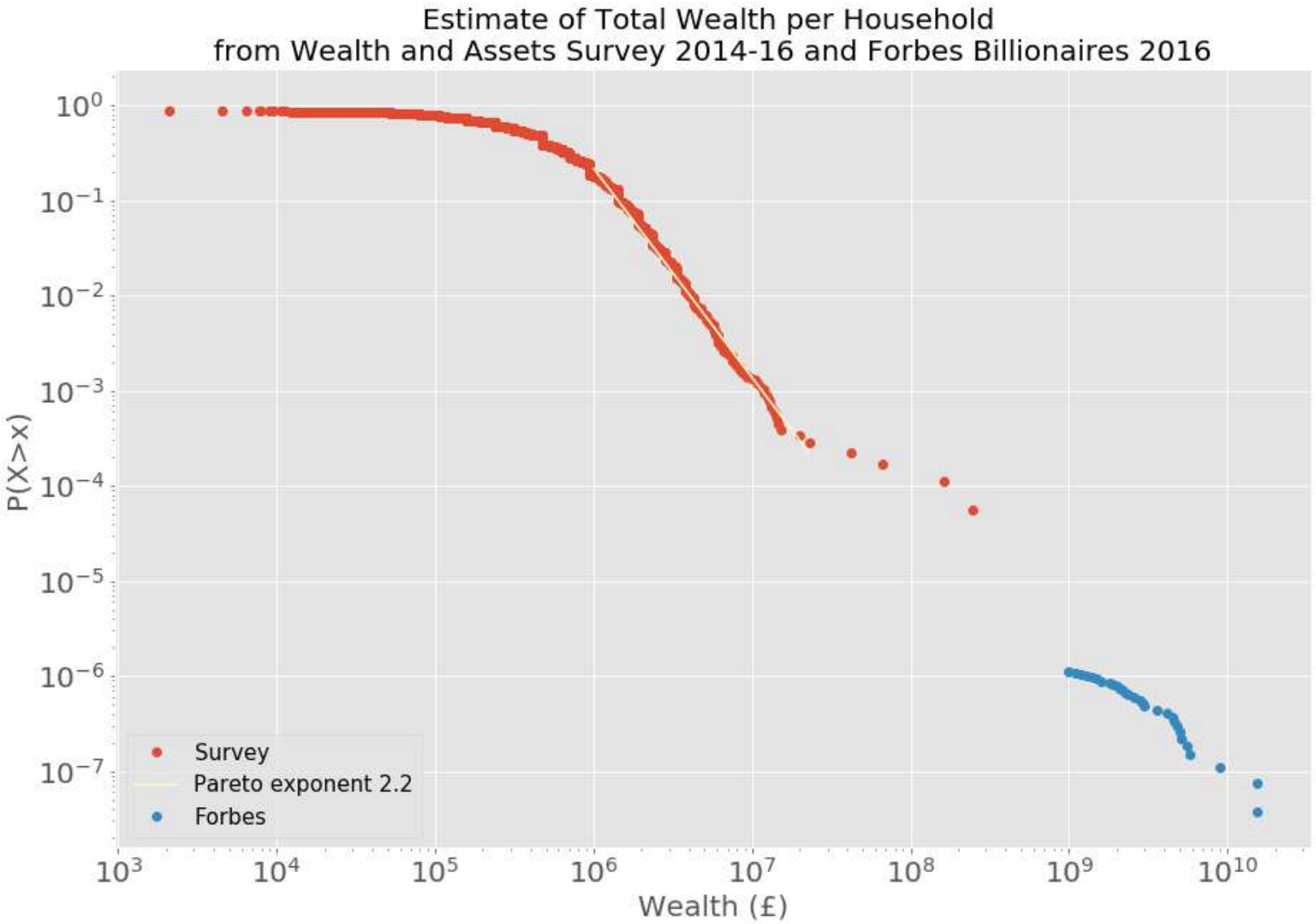
  with a **power law tail** with exponent $1 + J/\sigma^2$.

Wealth distribution data



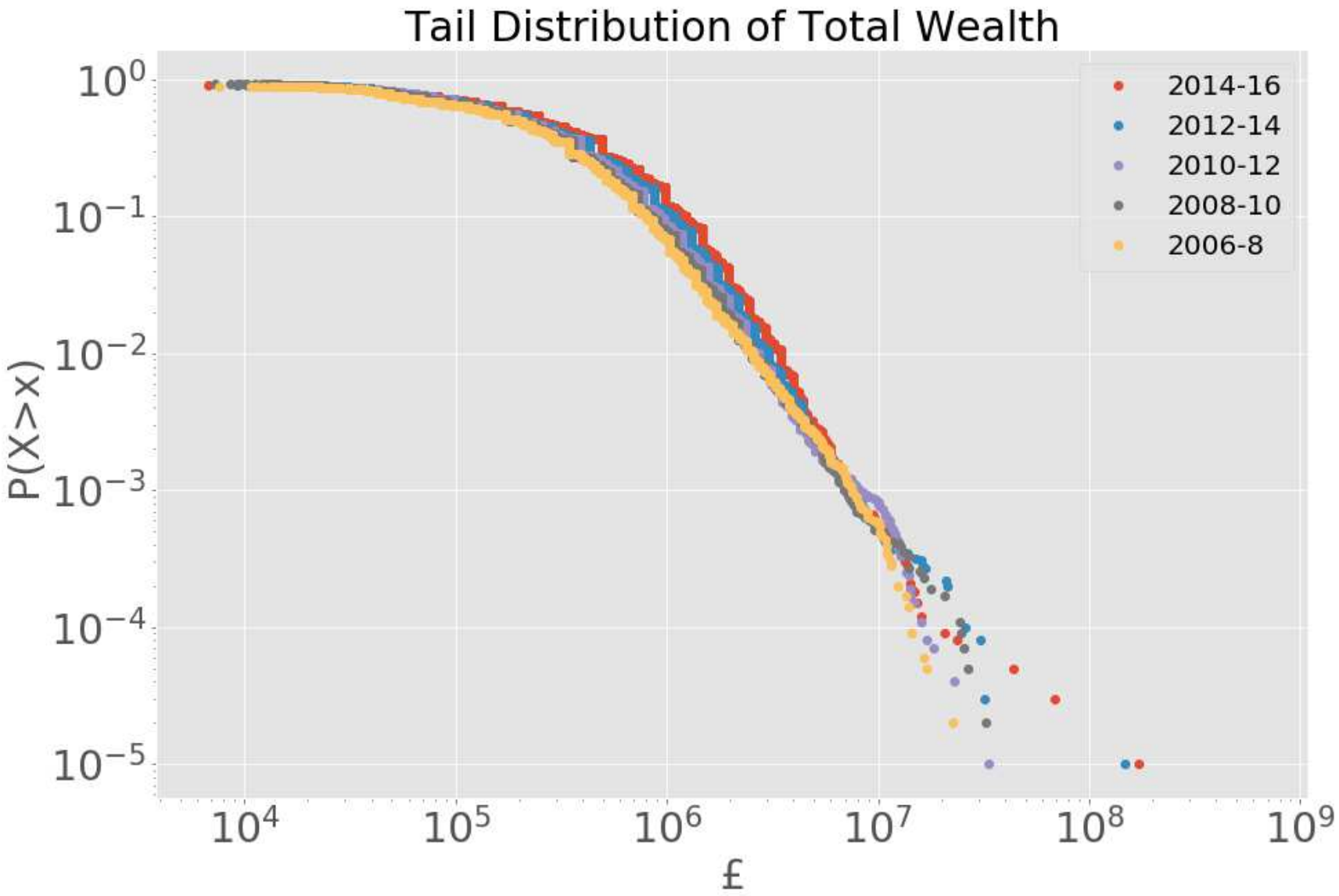
[Office for National Statistics, Wealth and Assets Survey]

Wealth distribution data



[Office for National Statistics, Wealth and Assets Survey]

Wealth distribution data



[Office for National Statistics, Wealth and Assets Survey]

Nonlinear reinforcement

[Vallejos, H. A., Nutaro, J. J., and Perumalla, K. S. (2018). J Econ Interact Coord, Springer.]

- N agents, initial wealth $w_i(0) > 0$ (integer)
- add **wealth packet** of size 1 in discrete timesteps

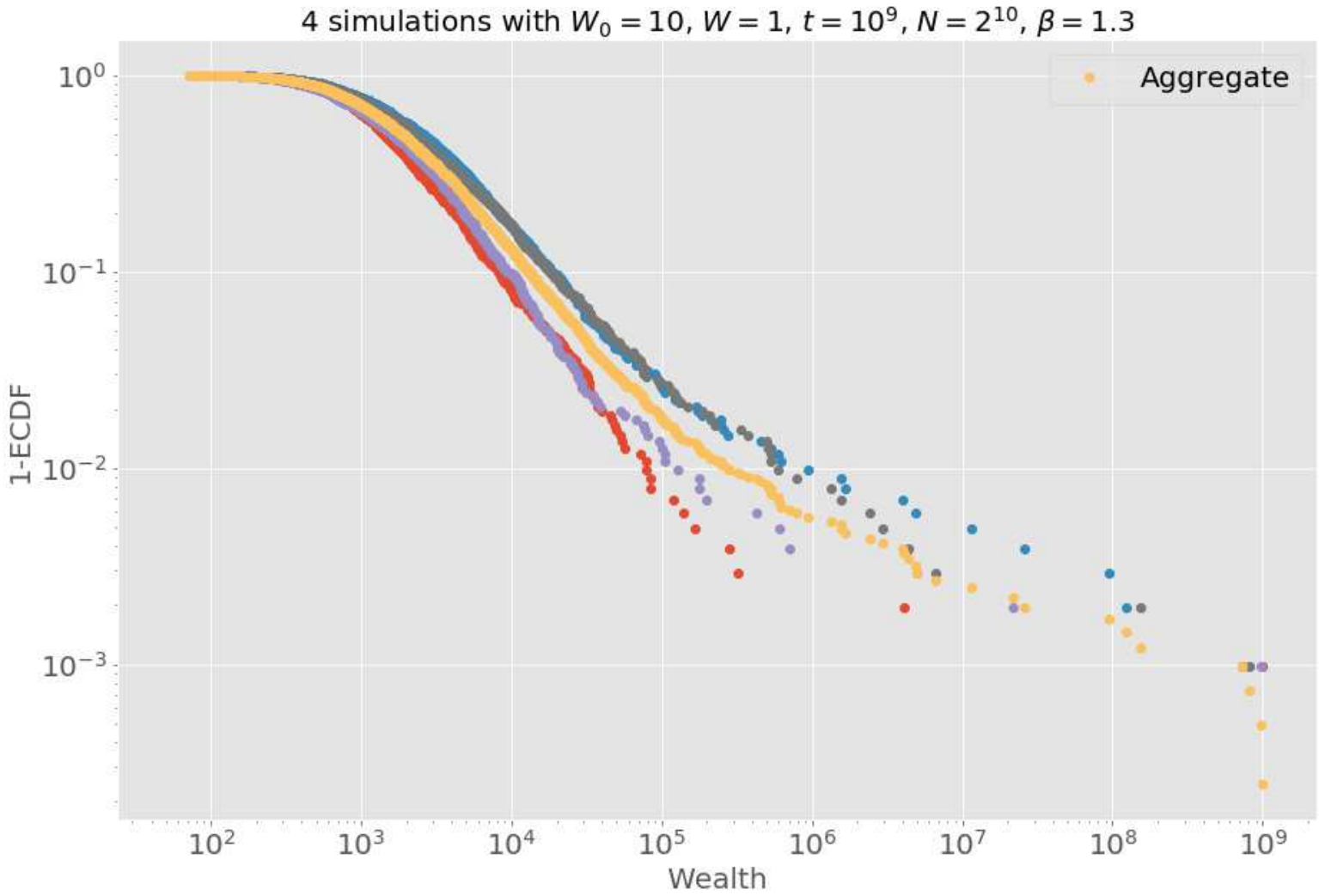
$$\text{to agent } i \text{ with prob.} \quad w_i(t)^\gamma \Big/ \sum_j w_j(t)^\gamma$$

- **known** (preferential attachment, Pólya urns): $M = \max_i w_i$, $S = \sum_i w_i$

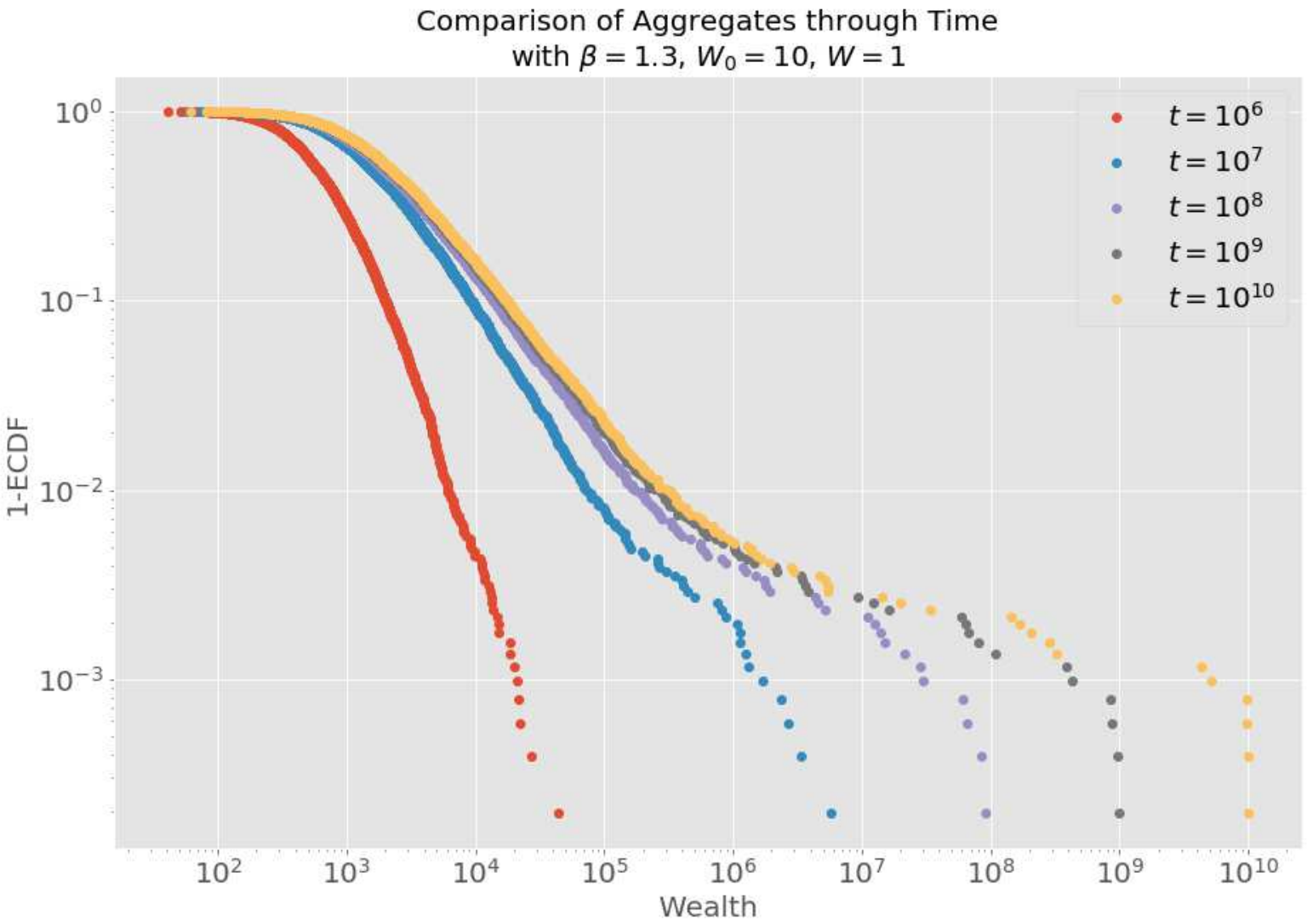
$$M(t)/S(t) \to 1 \quad \text{as } t \to \infty$$
$$S(t) - M(t) \to \Delta \quad \text{as } t \to \infty \quad \text{(stationary value)}$$
$$\Delta \text{ has power-law tail } (-\gamma \text{ if } N = 2)$$

- transient behaviour may be realistic
  or limiting behaviour excluding the richest person
  dependence on initial condition!

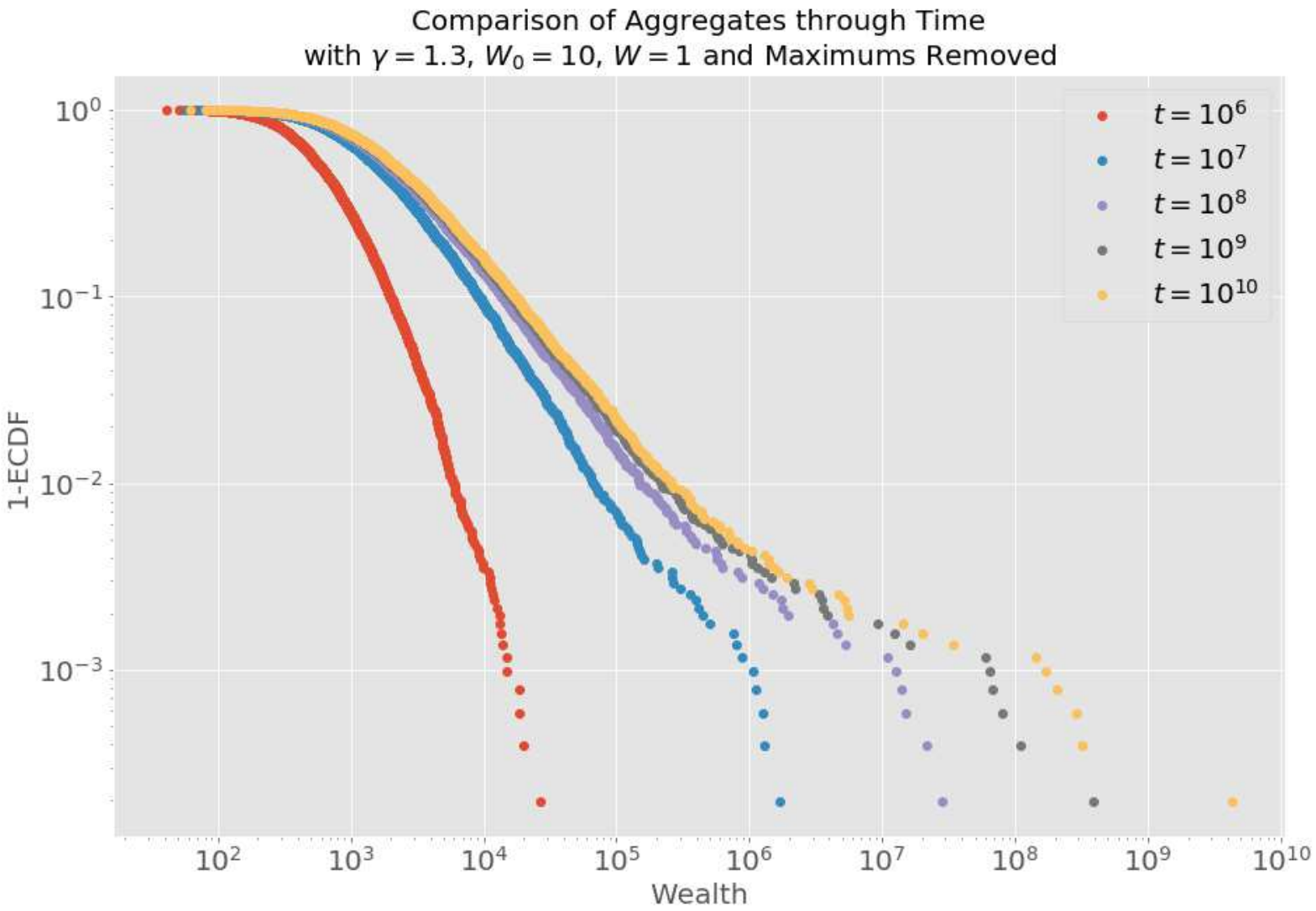
Pure growth model



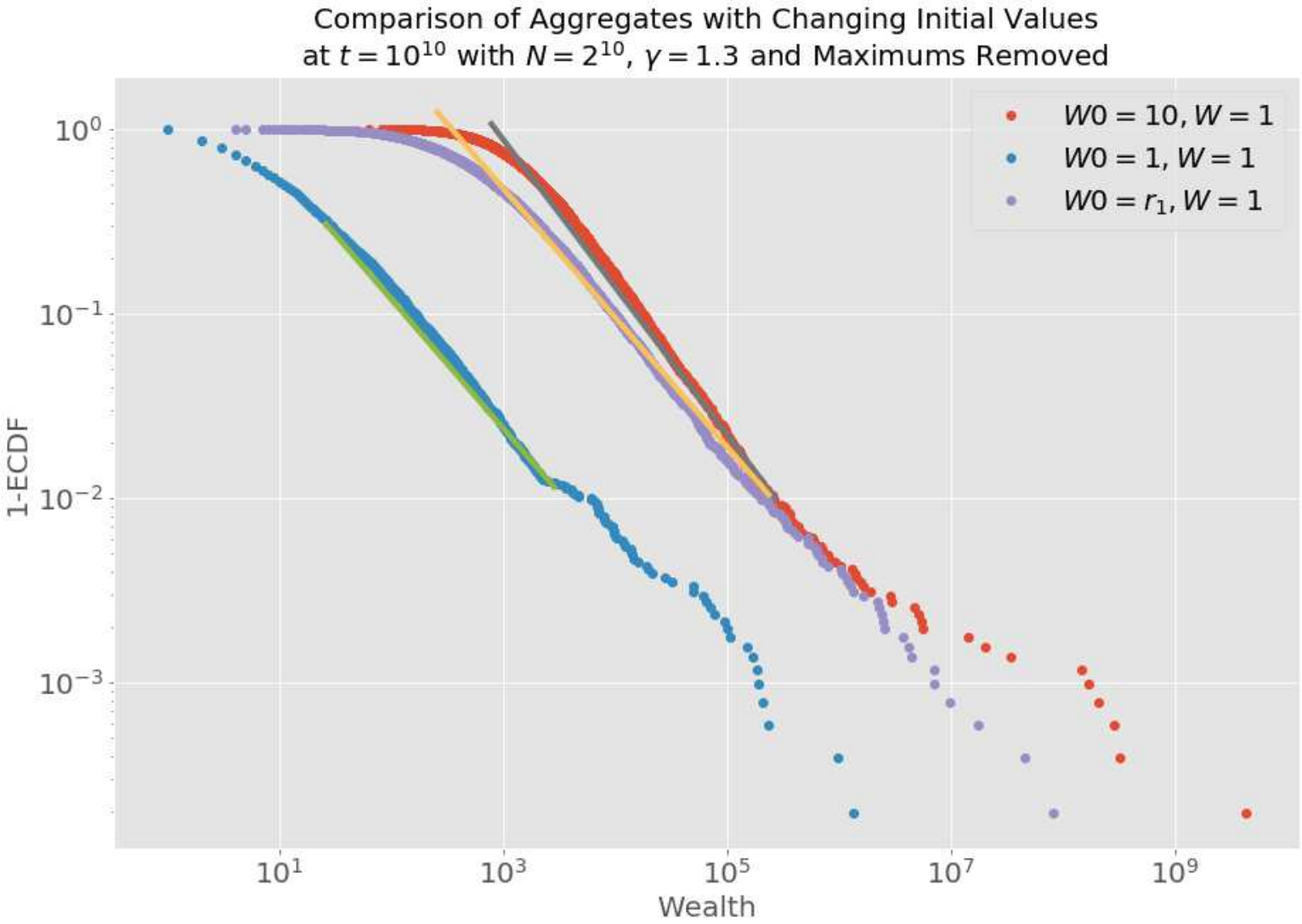
Pure growth model



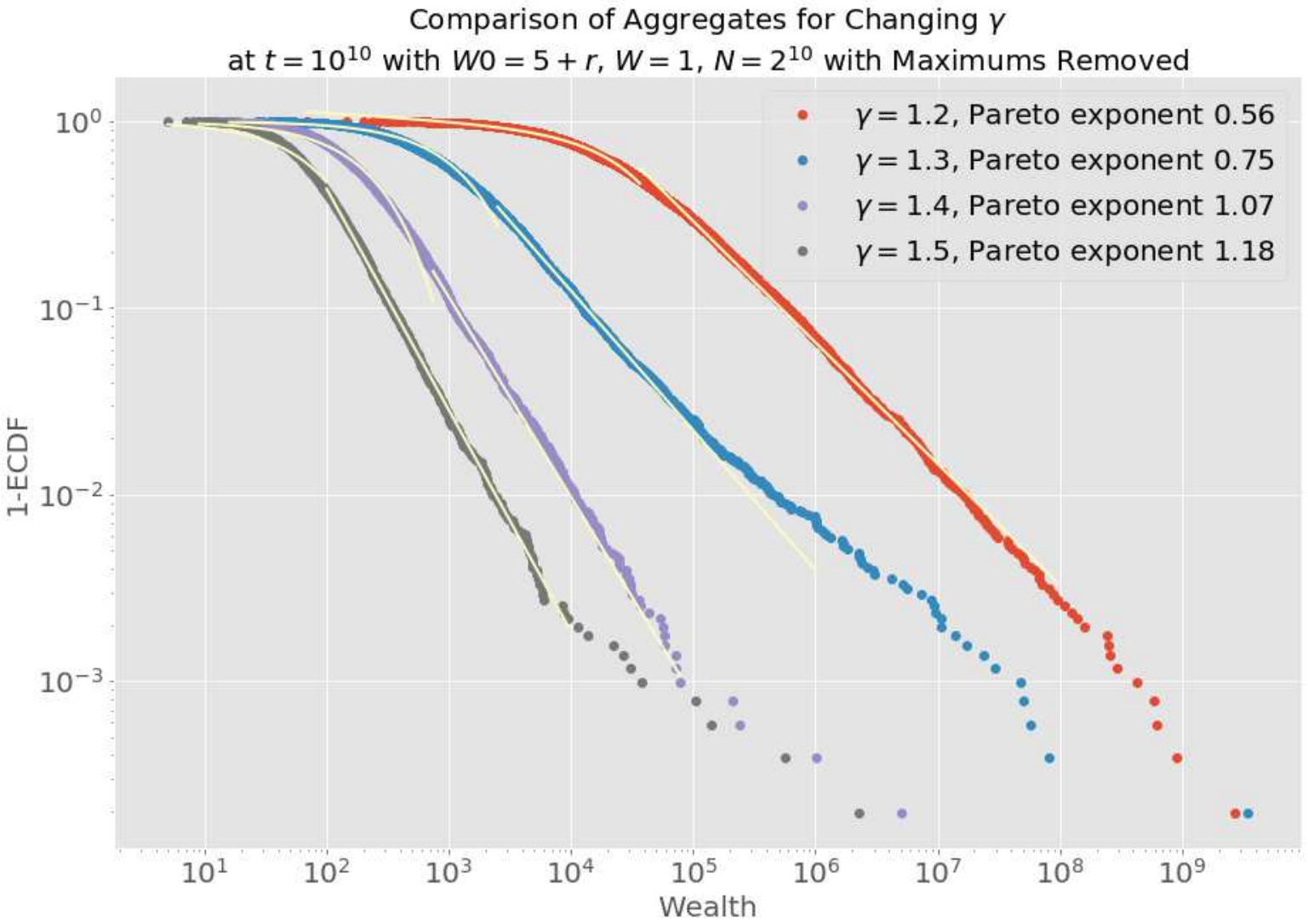
Pure growth model



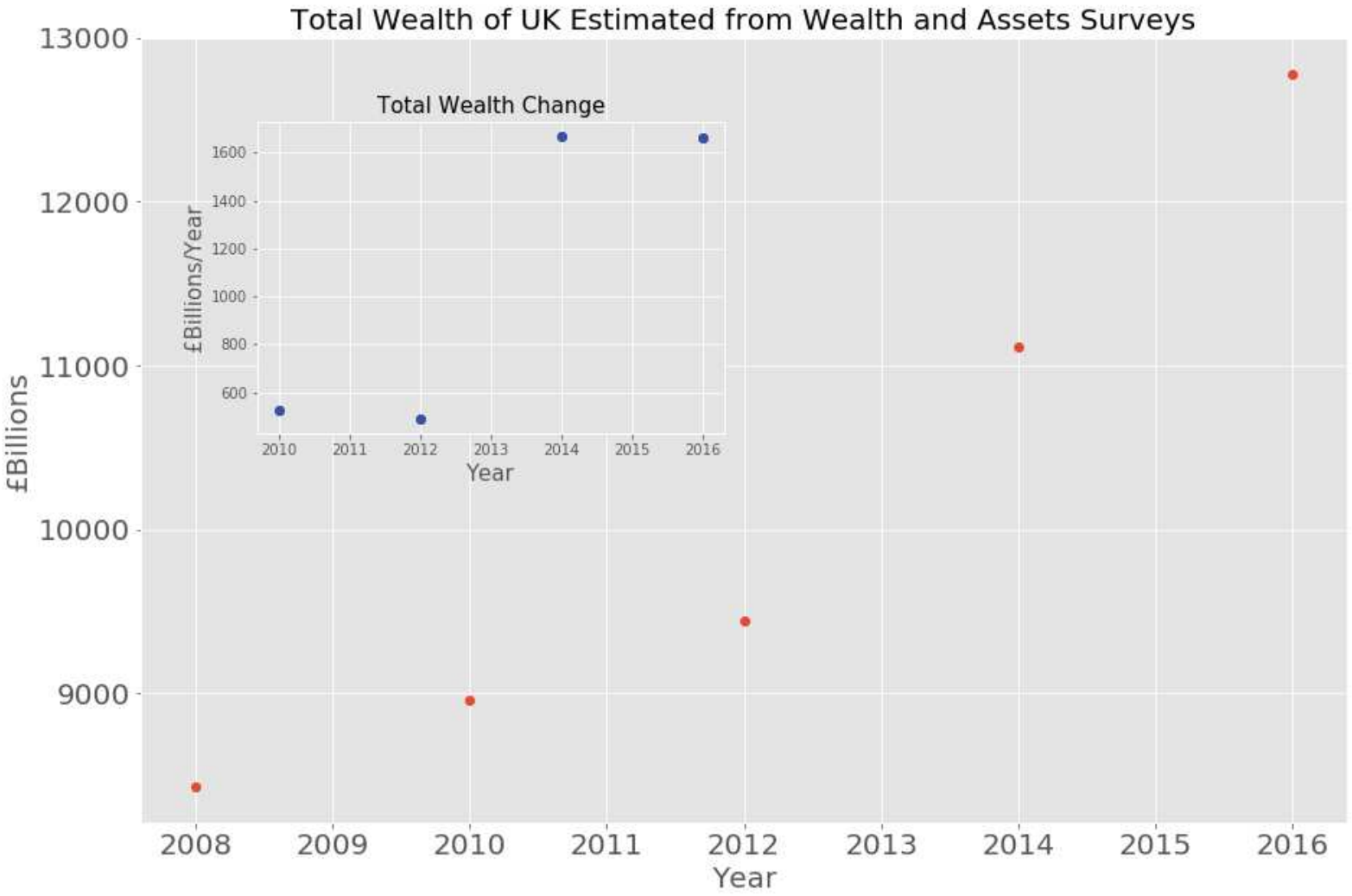
Pure growth model



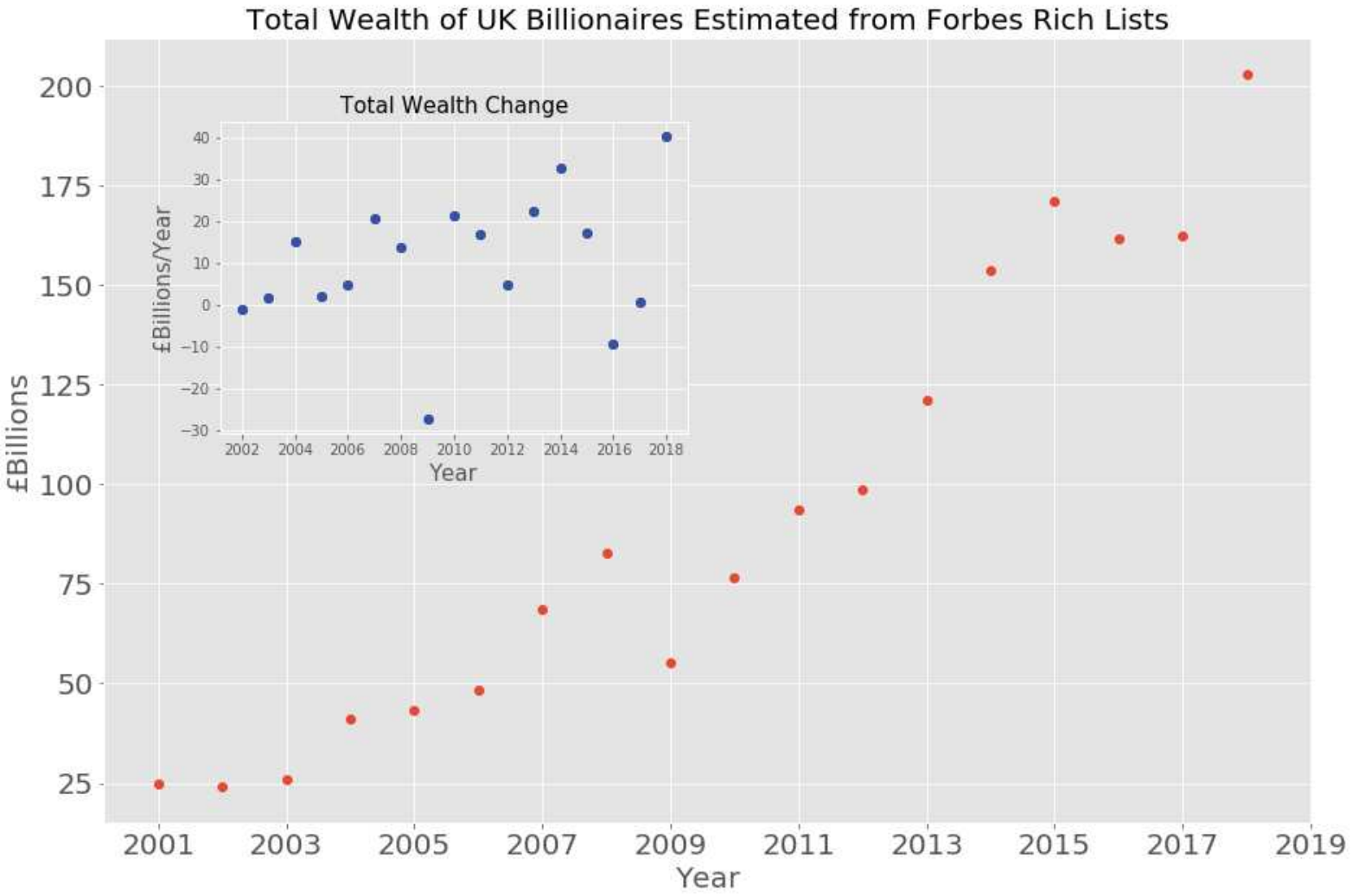
Pure growth model



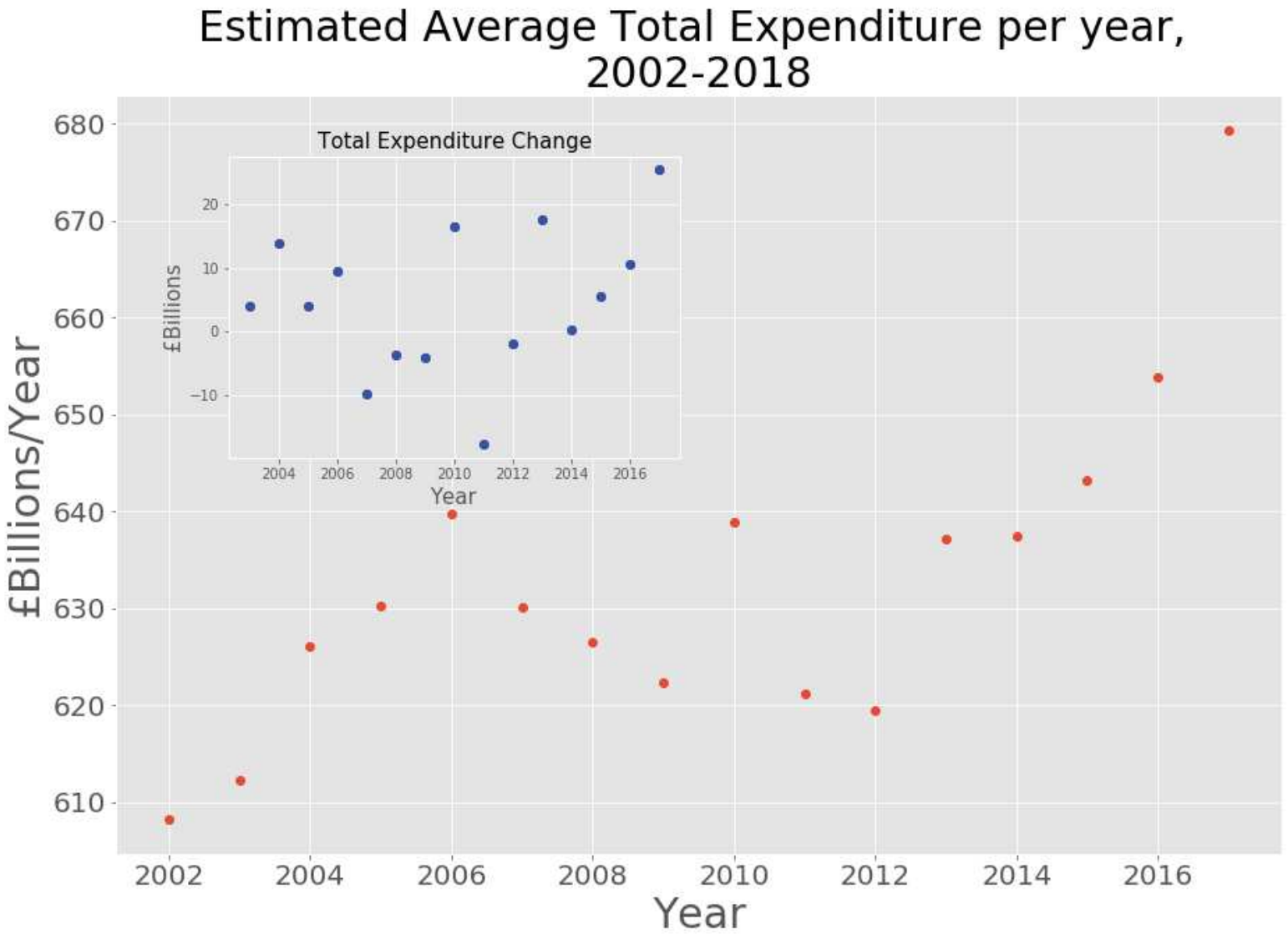
Growth model with redistribution



Growth model with redistribution



Growth model with redistribution



Growth model with redistribution