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Reed-Solomon Forward Error Correction (FEC) Schemes

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Abstract

This document describes a Fully-Specified Forward Error Correction (FEC) Scheme for the Reed-Solomon FEC codes over GF(2^^m), where m is in {2..16}, and its application to the reliable delivery of data objects on the packet erasure channel (i.e., a communication path where packets are either received without any corruption or discarded during transmission). This document also describes a Fully-Specified FEC Scheme for the special case of Reed-Solomon codes over GF(2^^8) when there is no encoding symbol group. Finally, in the context of the Under-Specified Small Block Systematic FEC Scheme (FEC Encoding ID 129), this document assigns an FEC Instance ID to the special case of Reed-Solomon codes over GF(2^^8).

Reed-Solomon codes belong to the class of Maximum Distance Separable (MDS) codes, i.e., they enable a receiver to recover the k source symbols from any set of k received symbols. The schemes described here are compatible with the implementation from Luigi Rizzo.

Table of Contents

1. Introduction

The use of Forward Error Correction (FEC) codes is a classical solution to improve the reliability of multicast and broadcast transmissions. The [RFC5052] document describes a general framework to use FEC in Content Delivery Protocols (CDPs). The companion document [RFC3453] describes some applications of FEC codes for content delivery.

Recent FEC schemes like [RFC5053] and [RFC5170] proposed erasure codes based on sparse graphs/matrices. These codes are efficient in terms of processing but not optimal in terms of correction capabilities when dealing with "small" objects.

The FEC schemes described in this document belongs to the class of Maximum Distance Separable codes that are optimal in terms of erasure correction capability. In others words, it enables a receiver to recover the k source symbols from any set of exactly k encoding symbols. They are also systematic codes, which means that the k source symbols are part of the encoding symbols. Even if the encoding/decoding complexity is larger than that of [RFC5053] or [RFC5170], this family of codes is very useful.

Many applications dealing with content transmission or content storage already rely on packet-based Reed-Solomon codes. In particular, many of them use the Reed-Solomon codec of Luigi Rizzo [RS-codec] [Rizzo97]. The goal of the present document is to specify an implementation of Reed-Solomon codes that is compatible with this codec.

The present document:

- o introduces the Fully-Specified FEC Scheme with FEC Encoding ID 2, which specifies the use of Reed-Solomon codes over GF(2^^m), where m is in {2..16},

- o introduces the Fully-Specified FEC Scheme with FEC Encoding ID 5, which focuses on the special case of Reed-Solomon codes over GF(2^^8) and no encoding symbol group (i.e., exactly one symbol per packet), and

- o in the context of the Under-Specified Small Block Systematic FEC Scheme (FEC Encoding ID 129) [RFC5445], assigns the FEC Instance ID 0 to the special case of Reed-Solomon codes over GF(2^^8) and no encoding symbol group.

For a definition of the terms Fully-Specified and Under-Specified FEC Schemes, see [RFC5052], Section 4.

2. Terminology

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in RFC 2119 [RFC2119].

3. Definitions Notations and Abbreviations

3.1. Definitions

This document uses the same terms and definitions as those specified in [RFC5052]. Additionally, it uses the following definitions:

Source symbol: unit of data used during the encoding process.

Encoding symbol: unit of data generated by the encoding process.

Repair symbol: encoding symbol that is not a source symbol.

Code rate: the k/n ratio, i.e., the ratio between the number of source symbols and the number of encoding symbols. By definition, the code rate is such that: 0 < code rate <= 1. A code rate close to 1 indicates that a small number of repair symbols have been produced during the encoding process.

Systematic code: FEC code in which the source symbols are part of the encoding symbols.

Source block: a block of k source symbols that are considered together for the encoding.

Encoding Symbol Group: a group of encoding symbols that are sent together within the same packet, and whose relationships to the source block can be derived from a single Encoding Symbol ID.

Source Packet: a data packet containing only source symbols.

Repair Packet: a data packet containing only repair symbols.

Packet Erasure Channel: a communication path where packets are either dropped (e.g., by a congested router, or because the number of transmission errors exceeds the correction capabilities of the physical layer codes) or received. When a packet is received, it is assumed that this packet is not corrupted.

3.2. Notations

This document uses the following notations:

L	the object transfer length in bytes.
k	the number of source symbols in a source block.
n_r	the number of repair symbols generated for a source block.
n	the encoding block length, i.e., the number of encoding symbols generated for a source block. Therefore: n = k + n_r.
max_n	the maximum number of encoding symbols generated for any source block.
B	the maximum source block length in symbols, i.e., the maximum number of source symbols per source block.
N	the number of source blocks into which the object shall be partitioned.
E	the encoding symbol length in bytes.
S	the symbol size in units of m-bit elements. When m = 8, then S and E are equal.
m	the length of the elements in the finite field, in bits. In this document, m belongs to {2..16}.
q	the number of elements in the finite field. We have: q = 2^^m in this specification.
G	the number of encoding symbols per group, i.e., the number of symbols sent in the same packet.
GM	the Generator Matrix of a Reed-Solomon code.
CR	the "code rate", i.e., the k/n ratio.
a^^b	a raised to the power b.
a^^-1	the inverse of a.
I_k	the k*k identity matrix.

3.3. Abbreviations

This document uses the following abbreviations:

ESI	Encoding Symbol ID.
FEC OTI	FEC Object Transmission Information.
RS	Reed-Solomon.
MDS	Maximum Distance Separable code.
GF(q)	a finite field (also known as Galois Field) with q elements. We assume that q = 2^^m in this document.

4. Formats and Codes with FEC Encoding ID 2

This section introduces the formats and codes associated with the Fully-Specified FEC Scheme with FEC Encoding ID 2, which specifies the use of Reed-Solomon codes over GF(2^^m).

4.1. FEC Payload ID

The FEC Payload ID is composed of the Source Block Number and the Encoding Symbol ID. The lengths of these two fields depend on the parameter m (which is transmitted in the FEC OTI) as follows:

- o The Source Block Number (field of size 32-m bits) identifies from which source block of the object the encoding symbol(s) in the payload are generated. There is a maximum of 2^^(32-m) blocks per object.

- o The Encoding Symbol ID (field of size m bits) identifies which specific encoding symbol(s) generated from the source block are carried in the packet payload. There is a maximum of 2^^m encoding symbols per block. The first k values (0 to k - 1) identify source symbols, the remaining n-k values identify repair symbols.

There MUST be exactly one FEC Payload ID per source or repair packet. In case of an Encoding Symbol Group, when multiple encoding symbols are sent in the same packet, the FEC Payload ID refers to the first symbol of the packet. The other symbols can be deduced from the ESI of the first symbol by incrementing sequentially the ESI.

```
    0                   1                   2                   3
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |     Source Block Number (32-8=24 bits)        | Enc. Symb. ID |
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 1: FEC Payload ID Encoding Format for m = 8 (Default)

```
    0                   1                   2                   3
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   | Src Block Nb (32-16=16 bits)  |  Enc. Symbol ID (m=16 bits)   |
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 2: FEC Payload ID Encoding Format for m = 16

The formats of the FEC Payload ID for m = 8 and m = 16 are illustrated in Figure 1 and Figure 2, respectively.

4.2. FEC Object Transmission Information

4.2.1. Mandatory Elements

- FEC Encoding ID: the Fully-Specified FEC Scheme described in this section uses FEC Encoding ID 2.

4.2.2. Common Elements

The following elements MUST be defined with the present FEC scheme.

- Transfer-Length (L): a non-negative integer indicating the length of the object in bytes. There are some restrictions on the maximum Transfer-Length that can be supported:

  max_transfer_length = 2^^(32-m) * B * E

  For instance, for m = 8, for B = 2^^8 - 1 (because the codec operates on a finite field with 2^^8 elements), and if E = 1024 bytes, then the maximum transfer length is approximately equal to 2^^42 bytes (i.e., 4 terabytes). Similarly, for m = 16, for B = 2^^16 - 1, and if E = 1024 bytes, then the maximum transfer length is also approximately equal to 2^^42 bytes. For larger objects, another FEC scheme, with a larger Source Block Number field in the FEC Payload ID, could be defined. Another solution consists in fragmenting large objects into smaller objects, each of them complying with the above limits.

- Encoding-Symbol-Length (E): a non-negative integer indicating the length of each encoding symbol in bytes.
- Maximum-Source-Block-Length (B): a non-negative integer indicating the maximum number of source symbols in a source block.
- Max-Number-of-Encoding-Symbols (max_n): a non-negative integer indicating the maximum number of encoding symbols generated for any source block.

Section 6 explains how to derive the values of each of these elements.

4.2.3. Scheme-Specific Elements

The following element MUST be defined with the present FEC scheme. It contains two distinct pieces of information:

- G: a non-negative integer indicating the number of encoding symbols per group used for the object. The default value is 1, meaning that each packet contains exactly one symbol. When no G parameter is communicated to the decoder, then the latter MUST assume that G = 1.
- m: The m parameter is the length of the finite field elements, in bits. It also characterizes the number of elements in the finite field: q = 2^^m elements. The default value is m = 8. When no finite field size parameter is communicated to the decoder, then the latter MUST assume that m = 8.

4.2.4. Encoding Format

This section shows the two possible encoding formats of the above FEC OTI. The present document does not specify when one encoding format or the other should be used.

4.2.4.1. Using the General EXT_FTI Format

The FEC OTI binary format is the following, when the EXT_FTI mechanism is used (e.g., within the ALC [ALC] or NORM [NORM] protocols).

```
     0                   1                   2                   3
     0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
    |   HET = 64    |    HEL = 4    |                               |
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+                               +
    |                      Transfer Length (L)                      |
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
    |       m       |       G       |   Encoding Symbol Length (E)  |
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
    |  Max Source Block Length (B)  |  Max Nb Enc. Symbols (max_n)  |
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 3: EXT_FTI Header Format

4.2.4.2. Using the FDT Instance (FLUTE specific)

When it is desired that the FEC OTI be carried in the FDT (File Delivery Table) Instance of a FLUTE session [FLUTE], the following XML attributes must be described for the associated object:

- o FEC-OTI-FEC-Encoding-ID
- o FEC-OTI-Transfer-Length (L)
- o FEC-OTI-Encoding-Symbol-Length (E)
- o FEC-OTI-Maximum-Source-Block-Length (B)
- o FEC-OTI-Max-Number-of-Encoding-Symbols (max_n)
- o FEC-OTI-Scheme-Specific-Info

The FEC-OTI-Scheme-Specific-Info contains the string resulting from the Base64 encoding (in the XML Schema xs:base64Binary sense) of the following value:

```
     0                   1
     0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
    |       m       |       G       |
    +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 4: FEC OTI Scheme Specific Information To Be Included in the FDT Instance

When no m parameter is to be carried in the FEC OTI, the m field is set to 0 (which is not a valid seed value). Otherwise, the m field contains a valid value as explained in Section 4.2.3. Similarly, when no G parameter is to be carried in the FEC OTI, the G field is set to 0 (which is not a valid seed value). Otherwise, the G field contains a valid value as explained in Section 4.2.3. When neither m nor G are to be carried in the FEC OTI, then the sender simply omits the FEC-OTI-Scheme-Specific-Info attribute.

During Base64 encoding, the 2 bytes of the FEC OTI Scheme-Specific Information are transformed into a string of 4 printable characters (in the 64-character alphabet) that is added to the FEC-OTI-Scheme-Specific-Info attribute.

5. Formats and Codes with FEC Encoding ID 5

This section introduces the formats and codes associated with the Fully-Specified FEC Scheme with FEC Encoding ID 5, which focuses on the special case of Reed-Solomon codes over GF(2^^8) and no encoding symbol group.

5.1. FEC Payload ID

The FEC Payload ID is composed of the Source Block Number and the Encoding Symbol ID:

o The Source Block Number (24-bit field) identifies from which source block of the object the encoding symbol in the payload is generated. There is a maximum of 2^^24 blocks per object.

o The Encoding Symbol ID (8-bit field) identifies which specific encoding symbol generated from the source block is carried in the packet payload. There is a maximum of 2^^8 encoding symbols per block. The first k values (0 to k - 1) identify source symbols; the remaining n-k values identify repair symbols.

There MUST be exactly one FEC Payload ID per source or repair packet. This FEC Payload ID refers to the one and only symbol of the packet.

```
 0                   1                   2                   3
 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
|        Source Block Number (24 bits)          | Enc. Symb. ID |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 5: FEC Payload ID Encoding Format with FEC Encoding ID 5

5.2. FEC Object Transmission Information

5.2.1. Mandatory Elements

- FEC Encoding ID: the Fully-Specified FEC Scheme described in this section uses FEC Encoding ID 5.

5.2.2. Common Elements

The Common elements are the same as those specified in Section 4.2.2 when m = 8 and G = 1.

5.2.3. Scheme-Specific Elements

No Scheme-Specific elements are defined by this FEC scheme.

5.2.4. Encoding Format

This section shows the two possible encoding formats of the above FEC OTI. The present document does not specify when one encoding format or the other should be used.

5.2.4.1. Using the General EXT_FTI Format

The FEC OTI binary format is the following, when the EXT_FTI mechanism is used (e.g., within the ALC [ALC] or NORM [NORM] protocols).

```
 0                   1                   2                   3
 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
|   HET = 64    |    HEL = 3    |                               |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+                               +
|                      Transfer Length (L)                      |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
|   Encoding Symbol Length (E)  | MaxBlkLen (B) |     max_n     |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 6: EXT_FTI Header Format with FEC Encoding ID 5

5.2.4.2. Using the FDT Instance (FLUTE specific)

When it is desired that the FEC OTI be carried in the FDT Instance of a FLUTE session [FLUTE], the following XML attributes must be described for the associated object:

- FEC-OTI-FEC-Encoding-ID

- FEC-OTI-Transfer-Length (L)
- FEC-OTI-Encoding-Symbol-Length (E)
- FEC-OTI-Maximum-Source-Block-Length (B)
- FEC-OTI-Max-Number-of-Encoding-Symbols (max_n)

6. Procedures with FEC Encoding IDs 2 and 5

This section defines procedures that are common to FEC Encoding IDs 2 and 5. In case of FEC Encoding ID 5, m = 8 and G = 1. The block partitioning algorithm that is defined in Section 9.1 of [RFC5052] MUST be used with FEC Encoding IDs 2 and 5.

6.1. Determining the Maximum Source Block Length (B)

The finite field size parameter, m, defines the number of non-zero elements in this field, which is equal to: q - 1 = 2^^m - 1. Note that q - 1 is also the theoretical maximum number of encoding symbols that can be produced for a source block. For instance, when m = 8 (default) there is a maximum of 2^^8 - 1 = 255 encoding symbols.

Given the target FEC code rate (e.g., provided by the user when starting a FLUTE sending application), the sender calculates:

```
max1_B = floor((2^^m - 1) * CR)
```

This max1_B value leaves enough room for the sender to produce the desired number of parity symbols.

Additionally, a codec MAY impose other limitations on the maximum block size. Yet it is not expected that such limits exist when using the default m = 8 value. This decision MUST be clarified at implementation time, when the target use case is known. This results in a max2_B limitation.

Then, B is given by:

```
B = min(max1_B, max2_B)
```

Note that this calculation is only required at the coder, since the B parameter is communicated to the decoder through the FEC OTI.

6.2. Determining the Number of Encoding Symbols of a Block

The following algorithm, also called "n-algorithm", explains how to determine the maximum number of encoding symbols generated for any source block (max_n) and the number of encoding symbols for a given block (n) as a function of the target code rate.

AT A SENDER:

Input:

B: Maximum source block length, for any source block. Section 6.1 explains how to determine its value.

k: Current source block length. This parameter is given by the block partitioning algorithm.

CR: FEC code rate, which is given by the user (e.g., when starting a FLUTE sending application). It is expressed as a floating point value.

Output:

max_n: Maximum number of encoding symbols generated for any source block.

n: Number of encoding symbols generated for this source block.

Algorithm:

```
max_n = ceil(B / CR);

if (max_n > 2^^m - 1), then return an error ("invalid code rate");

n = floor(k * max_n / B);
```

AT A RECEIVER:

Input:

B: Extracted from the received FEC OTI.

max_n: Extracted from the received FEC OTI.

k: Given by the block partitioning algorithm.

Output:

```
n
```

Algorithm:

```
n = floor(k * max_n / B);
```

It is RECOMMENDED that the "n-algorithm" be used by a sender, but other algorithms remain possible to determine max_n and/or n.

At a receiver, the max_n value is extracted from the received FEC OTI. Since the Reed-Solomon decoder does not need to know the actual n value, using the receiver part of the "n-algorithm" is not necessary from a decoding point of view.

However, a receiver may want to have an estimate of n for other reasons (e.g., for memory management purposes). In that case, a receiver knows that the number of encoding symbols of a block cannot exceed max_n. Additionally, if a receiver believes that a sender uses the "n-algorithm", this receiver MAY use the receiver part of the "n-algorithm" to get a better estimate of n. When this is the case, a receiver MUST be prepared to handle symbols with an Encoding Symbol ID superior or equal to the computed n value (e.g., it can choose to simply drop them).

7. Small Block Systematic FEC Scheme (FEC Encoding ID 129) and Reed-Solomon Codes over GF(2^^8)

In the context of the Under-Specified Small Block Systematic FEC Scheme (FEC Encoding ID 129) [RFC5445], this document assigns the FEC Instance ID 0 to the special case of Reed-Solomon codes over GF(2^^8) and no encoding symbol group.

The FEC Instance ID 0 uses the Formats and Codes specified in [RFC5445].

The FEC scheme with FEC Instance ID 0 MAY use the block partitioning algorithm defined in Section 9.1 of [RFC5052] to partition the object into source blocks. This FEC scheme MAY also use another algorithm. For instance, the CDP sender may change the length of each source block dynamically, depending on some external criteria (e.g., to adjust the FEC coding rate to the current loss rate experienced by NORM receivers) and inform the CDP receivers of the current block length by means of the EXT_FTI mechanism. This choice is out of the scope of the current document.

8. Reed-Solomon Codes Specification for the Erasure Channel

Reed-Solomon (RS) codes are linear block codes. They also belong to the class of MDS codes. A [n,k]-RS code encodes a sequence of k source elements defined over a finite field GF(q) into a sequence of n encoding elements, where n is upper bounded by q - 1. The implementation described in this document is based on a generator matrix built from a Vandermonde matrix put into systematic form.

Sections 8.1 to 8.3 specify the [n,k]-RS codes when applied to m-bit elements, and Section 8.4 specifies the use of [n,k]-RS codes when applied to symbols composed of several m-bit elements. The use described in Section 8.4 is the crux of this specification.

A reader who wants to understand the underlying theory is invited to refer to references [Rizzo97] and [MWS77].

8.1. Finite Field

A finite field GF(q) is defined as a finite set of q elements that has a structure of field. It contains necessarily q = p^^m elements, where p is a prime number. With packet erasure channels, p is always set to 2. The elements of the field GF(2^^m) can be represented by polynomials with binary coefficients (i.e., over GF(2)) of degree lower or equal to m-1. The polynomials can be associated with binary vectors of length m. For example, the vector (11001) represents the polynomial 1 + x + x^^4. This representation is often called polynomial representation. The addition between two elements is defined as the addition of binary polynomials in GF(2) and the multiplication is the multiplication modulo a given irreducible polynomial over GF(2) of degree m. Note that all the roots of this polynomial are in GF(2^^m) but not in GF(2).

The chosen polynomial representation of the finite field GF(2^^m) is completely characterized by the irreducible polynomial. The following polynomials are chosen to represent the field GF(2^^m), for m varying from 2 to 16:

m = 2, "111" (1+x+x^^2)

m = 3, "1101", (1+x+x^^3)

m = 4, "11001", (1+x+x^^4)

m = 5, "101001", (1+x^^2+x^^5)

m = 6, "1100001", (1+x+x^^6)

m = 7, "10010001", (1+x^^3+x^^7)

m = 8, "101110001", (1+x^^2+x^^3+x^^4+x^^8)

m = 9, "1000100001", (1+x^^4+x^^9)

m = 10, "10010000001", (1+x^^3+x^^10)

m = 11, "101000000001", (1+x^^2+x^^11)

m = 12, "1100101000001", (1+x+x^^4+x^^6+x^^12)

m = 13, "11011000000001", (1+x+x^^3+x^^4+x^^13)

m = 14, "110000100010001", (1+x+x^^6+x^^10+x^^14)

m = 15, "1100000000000001", (1+x+x^^15)

m = 16, "11010000000010001", (1+x+x^^3+x^^12+x^^16)

In order to facilitate the implementation, these polynomials are also primitive. This means that any element of GF(2^^m) can be expressed as a power of a given root of this polynomial. These polynomials are also chosen so that they contain the minimum number of monomials.

8.2. Reed-Solomon Encoding Algorithm

8.2.1. Encoding Principles

Let s = (s_0, ..., s_{k-1}) be a source vector of k elements over GF(2^^m). Let e = (e_0, ..., e_{n-1}) be the corresponding encoding vector of n elements over GF(2^^m). Being a linear code, encoding is performed by multiplying the source vector by a generator matrix, GM, of k rows and n columns over GF(2^^m). Thus:

e = s * GM.

The definition of the generator matrix completely characterizes the RS code.

Let us consider that n = 2^^m - 1 and that 0 < k <= n. Let us denote by alpha the root of the primitive polynomial of degree m chosen in the list of Section 8.1 for the corresponding value of m. Let us consider a Vandermonde matrix of k rows and n columns, denoted by V_{k,n}, and built as follows: the {i, j} entry of V_{k,n} is v_{i,j} = alpha^^(i*j), where 0 <= i <= k - 1 and 0 <= j <= n - 1. This matrix generates a MDS code. However, this MDS code is not systematic, which is a problem for many networking applications. To obtain a systematic matrix (and code), the simplest solution consists in considering the matrix V_{k,k} formed by the first k columns of V_{k,n}, then to invert it and to multiply this inverse by V_{k,n}. Clearly, the product V_{k,k}^^-1 * V_{k,n} contains the identity matrix I_k on its first k columns, meaning that the first k encoding elements are equal to source elements. Besides, the associated code keeps the MDS property.

Therefore, the generator matrix of the code considered in this document is:

```
GM = (V_{k,k}^^-1) * V_{k,n}
```

Note that, in practice, the [n,k]-RS code can be shortened to a [n',k]-RS code, where k <= n' < n, by considering the sub-matrix formed by the n' first columns of GM.

8.2.2. Encoding Complexity

Encoding can be performed by first pre-computing GM and by multiplying the source vector (k elements) by GM (k rows and n columns). The complexity of the pre-computation of the generator matrix can be estimated as the complexity of the multiplication of the inverse of a Vandermonde matrix by n-k vectors (i.e., the last n-k columns of V_{k,n}). Since the complexity of the inverse of a k*k-Vandermonde matrix by a vector is O(k * (log(k))^^2), the generator matrix can be computed in 0((n-k)* k * (log(k))^^2)) operations. When the generator matrix is pre-computed, the encoding needs k operations per repair element (vector-matrix multiplication).

Encoding can also be performed by first computing the product s * V_{k,k}^^-1 and then by multiplying the result with V_{k,n}. The multiplication by the inverse of a square Vandermonde matrix is known as the interpolation problem and its complexity is O(k * (log(k))^^2). The multiplication by a Vandermonde matrix, known as the multipoint evaluation problem, requires O((n-k) * log(k)) by using Fast Fourier Transform, as explained in [GO94]. The total complexity of this encoding algorithm is then O((k/(n-k)) * (log(k))^^2 + log(k)) operations per repair element.

8.3. Reed-Solomon Decoding Algorithm

8.3.1. Decoding Principles

The Reed-Solomon decoding algorithm for the erasure channel allows the recovery of the k source elements from any set of k received elements. It is based on the fundamental property of the generator matrix, which is such that any k*k-submatrix is invertible (see

[MWS77]). The first step of the decoding consists in extracting the k*k submatrix of the generator matrix obtained by considering the columns corresponding to the received elements. Indeed, since any encoding element is obtained by multiplying the source vector by one column of the generator matrix, the received vector of k encoding elements can be considered as the result of the multiplication of the source vector by a k*k submatrix of the generator matrix. Since this submatrix is invertible, the second step of the algorithm is to invert this matrix and to multiply the received vector by the obtained matrix to recover the source vector.

8.3.2. Decoding Complexity

The decoding algorithm described previously includes the matrix inversion and the vector-matrix multiplication. With the classical Gauss-Jordan algorithm, the matrix inversion requires O(k^^3) operations and the vector-matrix multiplication is performed in O(k^^2) operations.

This complexity can be improved by considering that the received submatrix of GM is the product between the inverse of a Vandermonde matrix (V_(k,k)^^-1) and another Vandermonde matrix (denoted by V', which is a submatrix of V_(k,n)). The decoding can be done by multiplying the received vector by V'^^-1 (interpolation problem with complexity O(k * (log(k))^^2)) then by V_{k,k} (multipoint evaluation with complexity O(k * log(k))). The global decoding complexity is then O((log(k))^^2) operations per source element.

8.4. Implementation for the Packet Erasure Channel

In a packet erasure channel, each packet (including its symbol(s), since packets contain G >= 1 symbols) is either correctly received or erased. The location of the erased symbols in the sequence of symbols MUST be known. The following specification describes the use of Reed-Solomon codes for generating redundant symbols from the k source symbols and for recovering the source symbols from any set of k received symbols.

The k source symbols of a source block are assumed to be composed of S m-bit elements. Each m-bit element corresponds to an element of the finite field GF(2^^m) through the polynomial representation (Section 8.1). If some of the source symbols contain less than S elements, they MUST be virtually padded with zero elements (this can be the case for the last symbol of the last block of the object). However, this padding does not need to be actually sent with the data to the receivers.

The encoding process produces n encoding symbols of size S m-bit elements, of which k are source symbols (this is a systematic code) and n-k are repair symbols (Figure 7). The m-bit elements of the repair symbols are calculated using the corresponding m-bit elements of the source symbol set. A logical u-th source vector, comprised of the u-th elements from the set of source symbols, is used to calculate a u-th encoding vector. This u-th encoding vector then provides the u-th elements for the set encoding symbols calculated for the block. As a systematic code, the first k encoding symbols are the same as the k source symbols, and the last n-k repair symbols are the result of the Reed-Solomon encoding.

```
          Input:  k source symbols

    0             u                                S-1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |             |X|                                 | source symbol 0
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |             |X|                                 | source symbol 1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
                . . .
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |             |X|                                 | source symbol k-1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+

                  *

        +--------------------+
        |  generator matrix  |
        |         GM         |
        |       (k x n)      |
        +--------------------+

                  |
                  V

        Output: n encoding symbols (source + repair)

    0             u                                S-1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |             |X|                                 | enc. symbol 0
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |             |X|                                 | enc. symbol 1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
                . . .
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
   |             |Y|                                 | enc. symbol n-1
   +-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

Figure 7: Packet Encoding Scheme

An asset of this scheme is that the loss of some encoding symbols produces the same erasure pattern for each of the S encoding vectors. It follows that the matrix inversion must be done only once and will be used by all the S encoding vectors. For large S, this matrix inversion cost becomes negligible in front of the S vector-matrix multiplications.

Another asset is that the n-k repair symbols can be produced on demand. For instance, a sender can start by producing a limited number of repair symbols and later on, depending on the observed erasures on the channel, decide to produce additional repair symbols, up to the n-k upper limit. Indeed, to produce the repair symbol e_j, where k <= j < n, it is sufficient to multiply the S source vectors with column j of GM.

9. Security Considerations

9.1. Problem Statement

A content delivery system is potentially subject to many attacks: some of them target the network (e.g., to compromise the routing infrastructure, by compromising the congestion control component), others target the Content Delivery Protocol (CDP) (e.g., to compromise its normal behavior), and finally some attacks target the content itself. Since this document focuses on a FEC building block independently of any particular CDP (even if ALC and NORM are two natural candidates), this section only discusses the additional threats that an arbitrary CDP may be exposed to when using this building block.

More specifically, several kinds of attacks exist:

- o those that are meant to give access to confidential content (e.g., in case of non-free content),
- o those that try to corrupt the object being transmitted (e.g., to inject malicious code within an object or to prevent a receiver from using an object),
- o and those that try to compromise the receiver's behavior (e.g., by making the decoding of an object computationally expensive).

These attacks can be launched either against the data flow itself (e.g., by sending forged symbols) or against the FEC parameters that are sent either in-band (e.g., in an EXT_FTI or FDT Instance) or out-of-band (e.g., in a session description).

9.2. Attacks against the Data Flow

First of all, let us consider the attacks against the data flow.

9.2.1. Access to Confidential Objects

Access control to the object being transmitted is typically provided by means of encryption. This encryption can be done over the whole object (e.g., by the content provider, before the FEC encoding process), or be done on a packet per-packet basis (e.g., when IPsec Encapsulating Security Payload (ESP) is used [RFC4303]). If access control is a concern, it is RECOMMENDED that one of these solutions be used. Even if we mention these attacks here, they are not related nor facilitated by the use of FEC.

9.2.2. Content Corruption

Protection against corruptions (e.g., after sending forged packets) is achieved by means of a content integrity verification/sender authentication scheme. This service can be provided at the object level, but in that case a receiver has no way to identify which symbol(s) are corrupted if the object is detected as corrupted. This service can also be provided at the packet level. In this case, after removing all forged packets, the object may be recovered sometimes. Several techniques can provide this source authentication/content integrity service:

o At the object level, the object MAY be digitally signed (with public key cryptography), for instance by using RSASSA-PKCS1-v1_5 [RFC3447]. This signature enables a receiver to check the object integrity, once the object has been fully decoded. Even if digital signatures are computationally expensive, this calculation occurs only once per object, which is usually acceptable.

o At the packet level, each packet can be digitally signed. A major limitation is the high computational and transmission overheads that this solution requires (unless Elliptic Curve Cryptography (ECC) is used). To avoid this problem, the signature may span a set of symbols (instead of a single one) in order to amortize the signature calculation. But if a single symbol is missing, the integrity of the whole set cannot be checked.

o At the packet level, a Group Message Authentication Code (MAC) [RFC2104] scheme can be used; for instance, by using HMAC-SHA-256 with a secret key shared by all the group members (i.e., the sender(s) and receivers). Thanks to the secret key, this technique creates a cryptographically secured digest of a packet that is sent along with the packet. The Group MAC scheme does not create prohibitive processing load nor transmission overhead, but it has a major limitation: it only provides a group authentication/integrity service since all group members share the same secret group key, which means that each member can send a forged packet. It is therefore restricted to situations where group members are fully trusted (or in association with another technique as a pre-check).

- At the packet level, TESLA [RFC4082] is a very attractive and efficient solution that is robust to losses, provides a true authentication/integrity service, and does not create any prohibitive processing load or transmission overhead. Yet checking a packet requires a small delay (a second or more) after its reception.

Techniques relying on public key cryptography (digital signatures and TESLA during the bootstrap process, when used) require that public keys be securely associated to the entities. This can be achieved by a Public Key Infrastructure (PKI), or by a PGP Web of Trust, or by pre-distributing the public keys of each group member.

Techniques relying on symmetric key cryptography (group MAC) require that a secret key be shared by all group members. This can be achieved by means of a group key management protocol, or simply by pre-distributing the secret key (but this manual solution has many limitations).

It is up to the developer and deployer, who know the security requirements and features of the target application area, to define which solution is the most appropriate. Nonetheless, in case there is any concern of the threat of object corruption, it is RECOMMENDED that at least one of these techniques be used.

9.3. Attacks against the FEC Parameters

Let us now consider attacks against the FEC parameters (or FEC OTI). The FEC OTI can either be sent in-band (i.e., in an EXT_FTI or in an FDT Instance containing FEC OTI for the object) or out-of-band (e.g., in a session description). Attacks on these FEC parameters can prevent the decoding of the associated object: for instance, modifying the B parameter will lead to a different block partitioning at a receiver thereby compromising decoding; or setting the m parameter to 16 instead of 8 with FEC Encoding ID 2 will increase the processing load while compromising decoding.

It is therefore RECOMMENDED that security measures be taken to guarantee the FEC OTI integrity. To that purpose, the packets carrying the FEC parameters sent in-band in an EXT_FTI header extension SHOULD be protected by one of the per-packet techniques described above: digital signature, group MAC, or TESLA. When FEC OTI is contained in an FDT Instance, this FDT Instance object SHOULD be protected, for instance, by digitally signing it with XML digital signatures [RFC3275]. Finally, when FEC OTI is sent out-of-band (e.g., in a session description), this FEC OTI SHOULD be protected, for instance, by digitally signing the object that includes this FEC OTI.

The same considerations concerning the key management aspects apply here also.

10. IANA Considerations

Values of FEC Encoding IDs and FEC Instance IDs are subject to IANA registration. For general guidelines on IANA considerations as they apply to this document, see [RFC5052].

This document assigns the Fully-Specified FEC Encoding ID 2 under the "ietf:rmt:fec:encoding" name-space to "Reed-Solomon Codes over GF(2^^m)".

This document assigns the Fully-Specified FEC Encoding ID 5 under the "ietf:rmt:fec:encoding" name-space to "Reed-Solomon Codes over GF(2^^8)".

This document assigns the FEC Instance ID 0 scoped by the Under-Specified FEC Encoding ID 129 to "Reed-Solomon Codes over GF(2^^8)". More specifically, under the "ietf:rmt:fec:encoding:instance" sub-name-space that is scoped by the "ietf:rmt:fec:encoding" called "Small Block Systematic FEC Codes", this document assigns FEC Instance ID 0 to "Reed-Solomon Codes over GF(2^^8)".

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