           The Host-Density Ratio for Address Assignment Efficiency:
                        An update on the H ratio

Status of this Memo

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Abstract

   This document provides an update on the "H ratio" defined in RFC
   1715. It defines a new ratio which the authors claim is easier to
   understand.

1. Evaluating the efficiency of address allocation

   A naive observer might assume that the number of addressable objects
   in an addressing plan is a linear function of the size of the
   address. If this were true, a telephone numbering plan based on 10
   digits would be able to number 10 billion telephones, and the IPv4 32
   bit addresses would be adequate for numbering 4 billion computers
   (using the American English definition of a billion, i.e. one
   thousand millions.) We all know that this is not correct: the 10
   digit plan is stressed today, and it handles only a few hundred
   million telephones in North America; the Internet registries have
   started to implement increasingly restrictive allocation policies
   when there were only a few tens of million computers on the Internet.

   Addressing plans are typically organized as a hierarchy: in
   telephony, the first digits will designate a region, the next digits
   will designate an exchange, and the last digits will designate a
   subscriber within this exchange; in computer networks, the most
   significant bits will designate an address range allocated to a
   network provider, the next bits will designate the network of an
   organization served by that provider, and then the subnet to which
   the individual computers are connected. At each level of the

hierarchy, one has to provide some margins: one has to allocate more
digits to the region code than the current number of regions would
necessitate, and more bits in a subnet than strictly required by the
number of computers. The number of elements in any given level of
the hierarchy will change over time, due to growth and mobility.
If the current allocation is exceeded, one has to engage in
renumbering, which is painful and expensive. In short, trying to
squeeze too many objects into a hierarchical address space increases
the level of pain endured by operators and subscribers.

Back in 1993, when we were debating the revision of the Internet
Protocol, we wondered what the acceptable ratio of utilization was of
a given addressing plan. Coming out with such a ratio was useful to
assess how many computers could be connected to the Internet with the
current 32-bit addresses, as well as to decide the size of the next
generation addresses. The second point is now decided, with 128-bits
addresses for IPv6, but the first question is still relevant:
knowing the capacity of the current address plan will help us predict
the date at which this capacity will be exceeded.

Participants in the IPNG debates initially measured the efficiency of
address allocation by simply dividing the number of allocated
addresses by the size of the address space. This is a simple
measure, but it is largely dependent on the size of the address
space. Loss of efficiency at each level of a hierarchical plan has a
multiplicative effect; for example, 50% efficiency at each stage of a
three level hierarchy results in a overall efficiency of 12.5%. If
we want a "pain level indicator", we have to use a ratio that takes
into account these multiplicative effects.

The "H-Ratio" defined in RFC 1715 proposed to measure the efficiency
of address allocation as the ratio of the base 10 logarithm of the
number of allocated addresses to the size of the address in bits.
This provides an address size independent ratio, but the definition
of the H ratio results in values in the range of 0.0 to 0.30103, with
typical values ranging from 0.20 to 0.28. Experience has shown that
these numbers are difficult to explain to others; it would be easier
to say that "your address bits are used to 83% of their H-Density",
and then explain what the H-Density is, than to say "you are hitting
a H ratio of 0.25" and then explain what exactly the range is.

This memo introduces the Host Density ratio or "HD-Ratio", a proposed
replacement for the H-Ratio defined in RFC 1715. The HD values range
from 0 to 1, and are generally expressed as percentage points; the
authors believe that this new formulation is easier to understand and
more expressive than the H-Ratio.

2. Definition of the HD-ratio

   When considering an addressing plan to allocate objects, the host
   density ratio HD is defined as follow:

$$HD = \frac{\log(\text{number of allocated objects})}{\log(\text{maximum number of allocatable objects})}$$

   This ratio is defined for any number of allocatable objects greater
   than 1 and any number of allocated objects greater or equal than 1
   and less than or equal the maximum number of allocatable objects.
   The ratio is usually presented as a percentage, e.g. 70%. It varies
   between 0 (0%), when there is just one allocation, and 1 (100%), when
   there is one object allocated to each available address. Note that
   for the calculation of the HD-ratio, one can use any base for the
   logarithm as long as it is the same for both the numerator and the
   denominator.

   The HD-ratio can, in most cases, be derived from the H ratio by the
   formula:

$$HD = \frac{H}{\log 10(2)}$$

3. Using the HD-ratio as an indicator of the pain level

   In order to assess whether the H-Ratio was a good predictor of the
   "pain level" caused by a specific efficiency, RFC1715 used several
   examples of networks that had reached their capacity limit. These
   could be for example telephone networks at the point when they
   decided to add digits to their numbering plans, or computer networks
   at the point when their addressing capabilities were perceived as
   stretched beyond practical limits. The idea behind these examples is
   that network managers would delay renumbering or changing the network
   protocol until it became just too painful; the ratio just before the
   change is thus a good predictor of what can be achieved in practice.
   The examples were the following:

   * Adding one digit to all French telephone numbers, moving from 8
   digits to 9, when the number of phones reached a threshold of 1.0
   E+7.

$$HD(FrenchTelephone8digit) = \frac{\log(1.0E+7)}{\log(1.0E+8)} = 0.8750 = 87.5\%$$

$$HD(FrenchTelephone9digit) = \frac{\log(1.0E+7)}{\log(1.0E+9)} = 0.7778 = 77.8\%$$

* Expanding the number of areas in the US telephone system, making
the phone number effectively 10 digits long instead of "9.2" (the
second digit of area codes used to be limited to 0 or 1) for about
1.0 E+8 subscribers.

$$HD(USTelephone9.2digit) = \frac{\log(1.0E+8)}{\log(9.5E+9)} = 0.8696 = 87.0\ \%$$

$$HD(USTelephone10digit) = \frac{\log(1.0E+8)}{\log(1E+10)} = 0.8000 = 80.0\ \%$$

* The globally-connected physics/space science DECnet (Phase IV)
stopped growing at about 15K nodes (i.e. new nodes were hidden) in a
16 bit address space.

$$HD(DecNET\ IV) = \frac{\log(15000)}{\log(2^{16})} = 0.8670 = 86.7\ \%$$

From those examples, we can note that these addressing systems
reached their limits for very close values of the HD-ratio. We can
use the same examples to confirm that the definition of the HD-ratio
as a quotient of logarithms results in better prediction than the
direct quotient of allocated objects over size of the address space.
In our three examples, the direct quotients were 10%, 3.2% and 22.8%,
three very different numbers that don't lead to any obvious
generalization. The examples suggest an HD-ratio value on the order
of 85% and above correspond to a high pain level, at which operators
are ready to make drastic decisions.

We can also examine our examples and hypothesize that the operators
who renumbered their networks tried to reach, after the renumbering,
a pain level that was easily supported. The HD-ratio of the French
or US network immediately after renumbering was 78% and 80%,
respectively. This suggests that values of 80% or less corresponds
to comfortable trade-offs between pain and efficiency.

4. Using the HD-ratio to evaluate the capacity of addressing plans

   Directly using the HD-ratio makes it easy to evaluate the density of
   allocated objects. Evaluating how well an addressing plan will scale
   requires the reverse calculation. We have seen in section 3.1 that
   an HD-ratio lower than 80% is manageable, and that HD-ratios higher
   than 87% are hard to sustain. This should enable us to compute the
   acceptable and "practical maximum" number of objects that can be
   allocated given a specific address size, using the formula:

   number allocatable of objects
                = exp(HD x log(maximum number allocatable of objects))
                = (maximum number allocatable of objects)^HD

   The following table provides example values for a 9-digit telephone
   plan, a 10-digit telephone plan, and the 32-bit IPv4 Internet:

	Reasonable HD=80%	Painful HD=85%	Very Painful HD=86%	Practical Maximum HD=87%
9-digits plan	16 M	45 M	55 M	68 M
10-digits plan	100 M	316 M	400 M	500 M
32-bits addresses	51 M	154 M	192 M	240 M

   Note: 1M = 1,000,000

   Indeed, the practical maximum depends on the level of pain that the
   users and providers are willing to accept. We may very well end up
   with more than 154M allocated IPv4 addresses in the next years, if we
   are willing to accept the pain.

5. Security considerations

   This document has no security implications.

6. IANA Considerations

   This memo does not request any IANA action.

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   during the elaboration of the HD formula.

9. References

   [RFC1715] Huitema, C., "The H Ratio for Address Assignment
             Efficiency", RFC 1715, November 1994.

   [IANAV4] INTERNET PROTOCOL V4 ADDRESS SPACE, maintained by the IANA,
             http://www.iana.org/assignments/ipv4-address-space

   [DMNSRV] Internet Domain Survey, Internet Software Consortium,
             http://www.isc.org/ds/

   [NETSZR] Netsizer, Telcordia Technologies, http://www.netsizer.com/

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Acknowledgement