# Is Timing Everything? Measurement Timing and the Ability to Accurately Model Longitudinal Data

by

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# ABSTRACT

IS TIMING EVERYTHING? MEASUREMENT TIMING AND THE ABILITY TO ACCURATELY MODEL LONGITUDINAL DATA


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The preface pretty much says it all. This is additional content. The preface pretty much says it all. This is additional content. The preface pretty much says it all. This is additional content. The preface pretty much says it all. This is additional content. The preface pretty much says it all. This is additional content.

# DEDICATION

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# ACKNOWLEDGEMENTS

I want to thank a few people.You can have a dedication here if you wish. You can have a dedication here if you wish.You can have a dedication here if you wish.You can have a dedication here if you wish.You can have a dedication here if you wish.You can have a dedication here if you wish.You can have a dedication here if you wish.

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and and analysis goals. For the design, I conducted a 3(time structuredness: time-structured data, time-unstructured data resulting from a fast response rate, time-unstructured data resulting from a slow response rate) x 4(number of measurements: 5, 7, 9, 11) x 6(sample size: 30, 50, 100, 200, 500, 1000) study. For the analysis, I examined whether the number of measurements and sample sizes needed to obtain high modelling accuracy (i.e., low bias, high precision) increased as time structuredness decreased.

## 4.1 Methods

### 4.1.1 Variables Used in Simulation Experiment

#### 4.1.1.1 Independent Variables

##### 4.1.1.1.1 Number of Measurements

For the number of measurements, I used the same values as in Experiment 1 of 5, 7, 9, and 11 measurements (see number of measurements for more discussion)).

##### 4.1.1.1.2 Sample Size

For sample size, I used the same values as in Experiment 2 of 30, 50, 100, 200, 500, and 1000 (see sample size for more discussion).

##### 4.1.1.1.3 Time Structuredness

*Time structuredness* describes the extent to which, at each time point, data are obtained at the exact same time point. The manipulation of time structuredness was adopted from the manipulation used in Coulombe et al. (2016) with a slight modification. In Coulombe et al. (2016), time-unstructured data were generated according to an exponential pattern such that most data were obtained at the beginning of the response window, with a smaller amount of data being obtained towards the end of the response window. Importantly, Coulombe et al. (2016) employed a non-continuous function for generating time-unstructured data: A binning method was employed such that 80% of the data were obtained within a time period equivalent to 12% (fast response rate) or 30% (slow response rate) of the entire response window. Using a response window length of 10 days with a fast response rate, the procedure employed by Coulombe et al. (2016) for generating time-unstructured data would have generated the following percentages of data in each of the four bins (note that, using the data generation procedure for Coulombe et al. (2016), the effective response window length for a fast response rate would be 4 days in the current example instead of 10 days):[1]

1) Bin 1: 60% of the data would be generated in the initial 10% length of the response window (0–0.40 day).
2) Bin 2: 20% of the data would be generated in the next 20% length of the response response window (0.40–1.20 days).
3) Bin 3: 10% of the data would be generated in the next 30% length of the response window (1.20–2.40 days).
4) Bin 4: the remaining 10% of the data would be generated in the remaining 40% length of the response window (2.40–4.00 days).

Note that, summing the data percentages and time durations from the first two bins yields an 80% cumulative response rate that is obtained in the initial 12% length of the full-length response window of 10 days (i.e., $(\frac{1.2}{10})100\% = 12\%$). Also note that, in Coulombe et al. (2016), a data point in each bin was randomly assigned a measurement time within the bin's time range. In the current example where the full-length response



[1] The data generation procedure in (ref:coulombe2016) for a fast response rate assumed that all of the data were collected within the initial 40% length of the nominal response window length (i.e., 4 days in the current example).

window had a length of 10 days, a data point obtained in the first bin would be randomly assigned a measurement time between 0–0.40. Although Coulombe et al. (2016) generated time-unstructured data to resemble data collection conditions—–response rates have been shown to follow an exponential pattern (Dillman et al. (2014); Pan (2010))—–the use of a pseudo-continuous binning function for generating time-unstructured data lacked ecological validity because response patterns are more likely to follow a continuous function. To improve on the time structuredness manipulation of Coulombe et al. (2016), I developed a more ecologically valid manipulation by using a continuous function. Specifically, I used the the exponential function shown below in Equation 4.1 to generate time-unstructured data:

$$y = M(1 - e^{-ax}), \tag{4.1}$$

where $x$ stores the time delay for a measurement at a particular time point, $y$ represents the cumulative response percentage achieved at a given $x$ time delay, $a$ sets the rate of growth of the cumulative response percentage over time, and $M$ sets the range of possible y values. Two important points need to be made with respect to the $M$ parameter (range of possible $y$ values) and the response window length used in the current simulations. First, because the range of possible values for the cumulative response percentage ($y$) is 0–1 (data can be collected from a 0% to a maximum of 100% of respondents; $\{y : 0 \leq y \leq 1\}$), the M parameter had a value of 1 (M = 1). Second, the response window length in the current simulations was 36 days, and so the range of possible time delay values was between 0–36 ($\{x : 0 \leq x \leq 36\}$).[2]

To replicate the time structuredness manipulation in Coulombe et al. (2016) using the continuous exponential function of Equation 4.1, the growth rate parameter ($a$) had to be calibrated to achieve a cumulative response rate of 80% after either 12% or 30% of the response window length of 36 days. The derivation below solves for $a$, with Equation 4.2 showing the equation for computing $a$.

$$
\begin{aligned}
y &= M(1 - e^{-ax}) \\
y &= M - Me^{-ax} \\
y &= 1 - e^{-ax} \\
e^{-ax} &= 1 - y \\
-ax\log(e) &= \log(1 - y) \\
a &= \frac{\log(1 - y)}{-x} \qquad (4.2)
\end{aligned}
$$

Because the target response rate was 80%, y took on a value of .80 (y = .80). Given that the response window length in the current simulations was 36 days, x took on a value of 4.32 (12% of 36) when time-unstructured data were defined by a fast response rate and 10.80 (30% of 36) when time-unstructured data were defined by a slow response rate. Using Equation 4.2 yielded the following growth rate parameter values for fast and slow

[2] A value of 36 days was used because the generation of time-unstructured data had to remain independent of the manipulation of measurement number (i.e., the response window lengths used in generating time-unstructured data could not vary with the number of measurements). To ensure the manipulations of measurement number and time structuredness remained independent, the reponse window length had to remain constant for all measurement number conditions with equal spacing. Looking at Table **??**, the longest possible response window that fit within all measurement number conditions with equal spacing was the interval length of the 11-measurement condition (i.e., 36 days).

response rates ($a_{fast}$, $a_{slow}$):

$$a_{fast} = \frac{\log(1 - .80)}{-4.32} = 0.37$$
$$a_{slow} = \frac{\log(1 - .80)}{-10.80} = 0.15$$

Therefore, to obtain 80% of the data with a fast response rate (i.e., in 4.32 days), the growth parameter ($a$) needed to have a value of 0.37 ($a_{fast} = 0.37$) and, to obtain 80% of the data with a slow response rate (i.e., in 10.80 days), the growth parameter ($a$) needed to have a value of 0.15 ($a_{slow} = 0.15$). Using the above growth rate values derived for the fast and slow response growth rate parameters ($a_{fast}$, $a_{slow}$), the following functions were generated for fast and slow response rates:

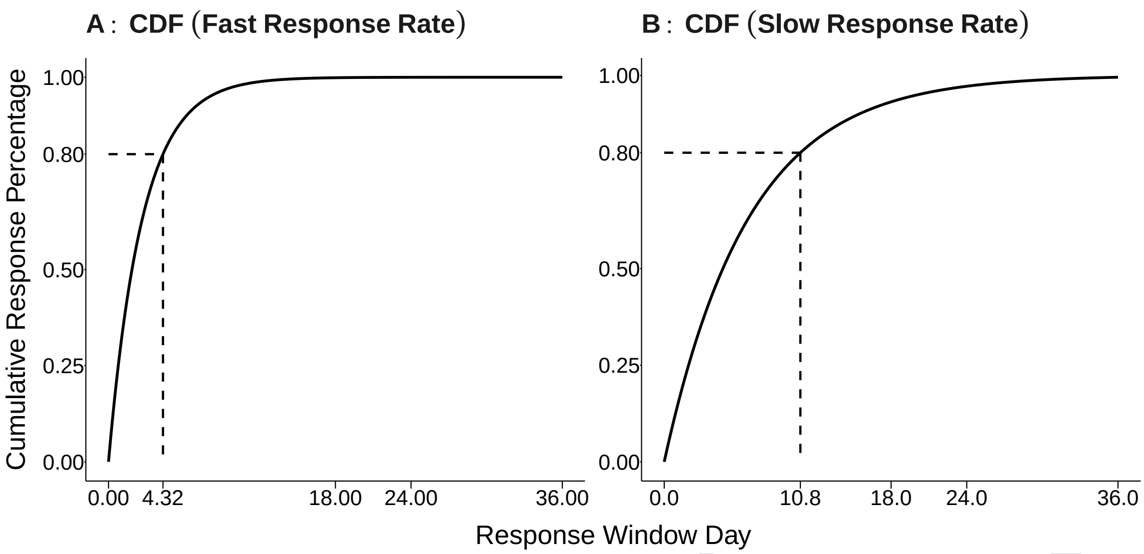
$$f_{fast}(x) = M(1 - e^{a_{fast}x}) = M(1 - e^{-0.37x}) \text{ and} \tag{4.3}$$

$$f_{slow}(x) = M(1 - e^{a_{slow}x}) = M(1 - e^{-0.15x}). \tag{4.4}$$

Using Equations 4.3–4.4, Figure 10 shows the resulting cumulative distribution functions (CDF) for time-unstructured data that show the cumulative response percentage as a function of time. Panel A shows the cumulative distribution function for a fast response rate (Equation 4.3), where an 80% response rate was obtained in 4.32 days. Panel B shows the cumulative distribution function for a slow response rate (Equation 4.4), where an 80% response rate was obtained in 10.80 days.

**Figure 4.1**

*Cumulative Distribution Functions (CDF) With Fast and Slow Response Rates*




*Note.* Panel A: Cumulative distribution function for a fast response rate (Equation 4.3), where an 80% response rate is obtained in 4.32 days. Panel B: Cumulative distribution function for a slow response rate (Equation 4.4), where an 80% response rate is obtained in 10.80 days.

#### 4.1.1.2 Constants

Because the nature of change not manipulated in Experiment 3, I set it to have a constant value across all cells. To keep the nature of change constant across all cells, I set the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$) to have a value of 180. Another variable set to a constant value across the cells was measurement spacing (equal spacing was used).

#### 4.1.1.3 Dependent Variables

##### 4.1.1.3.1 Convergence Success Rate

The proportion of iterations in a cell where models converged defined the **convergence success rate**.[3] Equation (4.5) below shows the calculation used to compute the

[3]Specifically, convergence was obtained if the convergence code returned by OpenMx was 0.

convergence success rate:

$$\text{Convergence success rate} = \frac{\text{Number of models that successfully converged in a cell}}{n}, \tag{4.5}$$

where $n$ represents the total number of models run in a cell.

#### 4.1.1.3.2 Bias

Bias was calculated to evaluate the accuracy with which each logistic function parameter was estimated. As shown below in Equation (4.6), *bias* was obtained by calculating the difference between the population value set for a parameter and the average estimated value in each cell.

$$\text{Bias} = \text{Population value for parameter} - \text{Average estimated value} \tag{4.6}$$

Bias was calculated for the fixed- and random-effect parameters of the baseline ($\theta_{fixed}$, $\theta_{random}$), maximal elevation ($\alpha_{fixed}$, $\alpha_{random}$), days-to-halfway elevation ($\beta_{fixed}$, $\beta_{random}$), and the halfway-triquarter delta parameters ($\gamma_{fixed}$, $\gamma_{random}$).

#### 4.1.1.3.3 Precision

In addition to computing bias, precision was calculated to evaluate the confidence with which each parameter was estimated in a given cell. *Precision* was obtained by computing the range of values covered by the middle 95% of values estimated for a logistic parameter in each cell. By using the middle 95% of estimated values, a plausible range of population estimates was obtained.

### 4.1.2 Overview of Data Generation

Data generation was computed the same way as in Experiment 1 (see data generation) with one addition to the procedure needed for time structuredness. The section that follows details how time structuredness was simulated.

#### 4.1.2.0.1 Simulation Procedure for Time Structuredness

To simulate time-unstructured data, response rates at each collection point followed an exponential pattern described by either a fast or slow response rate (for a review, see time structuredness). Importantly, data generated for each person at each time point had to be sampled according to a probability density function defined by either the fast or slow response rate cumulative distribution function. In the current context, a *probability density function* describes the probability of sampling any given time delay value $x$ where the range of time delay values is 0–36 ($\{x : 0 \leq x \leq 36\}$). To obtain the probability density functions for fast and slow response rates, the response rate function shown in Equation (**??**) was differentiated with respect to $x$ to obtain the function shown below in Equation 4.7[4]:

$$\begin{aligned} f' = \frac{\partial f(x)}{\partial x} &= \frac{\partial}{\partial x} M(1 - e^{-ax}). \\ &= M(e^{-ax}a) \end{aligned} \tag{4.7}$$

To compute the probability density function for the fast response rate cumulative distribution function, the growth rate parameter $a$ was set to 0.37 in Equation 4.7 to obtain

[4]Euler's notation for differentiation is used to represent derivatives. In words, $\frac{\partial f(x)}{\partial x}$ means that the derivative of the function $f(x)$ is taken with respect to $x$.

the following function in Equation 4.8:

$$f'_{fast}(x) = M(e^{-a_{fast}x}a_{fast}) = M(e^{-0.37x}0.37). \tag{4.8}$$

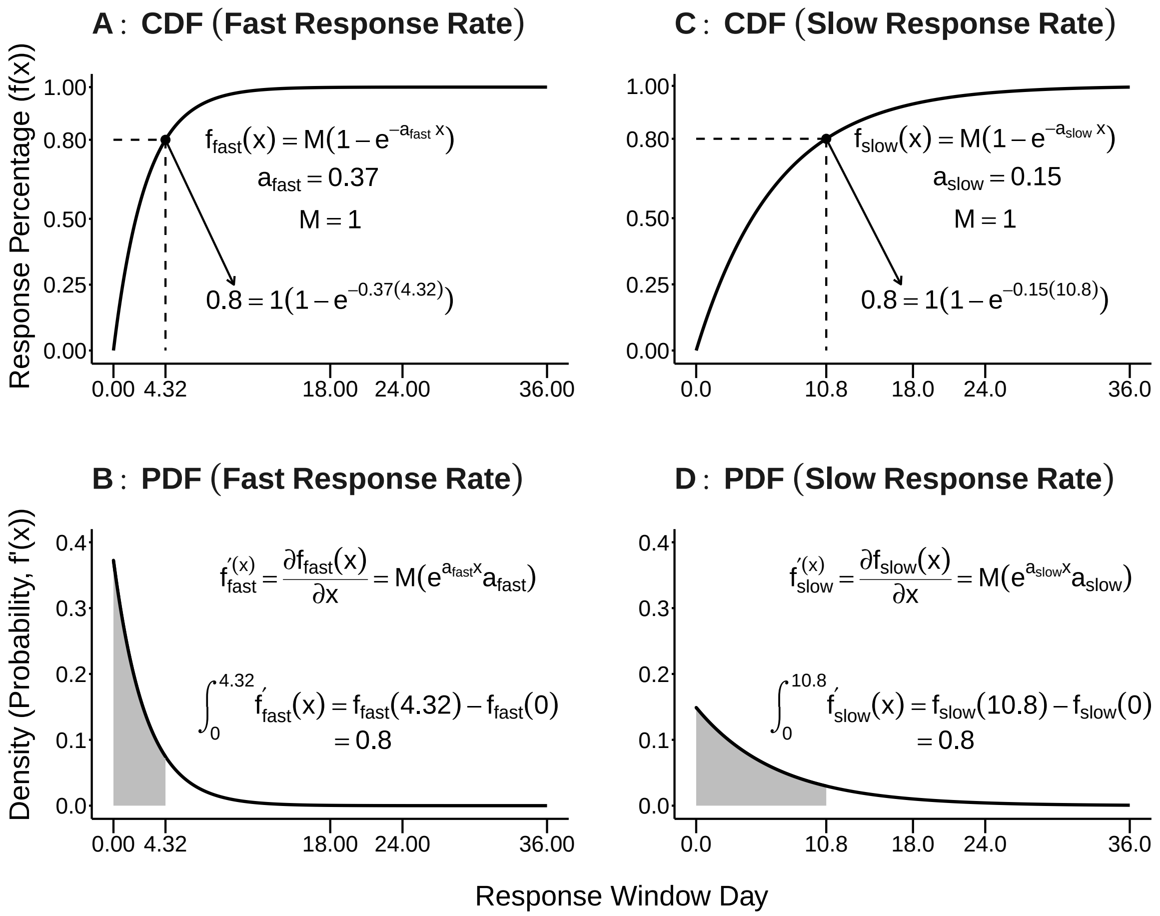
To compute the probability density function for the slow response rate cumulative distribution function, the growth rate parameter $a$ was set to 0.15 in Equation 4.7 to obtain the following function in Equation 4.9:

$$f'_{slow}(x) = M(e^{-0.15}a_{slow}) = M(e^{-0.15}0.15). \tag{4.9}$$

Figure 4.2 shows the fast and slow response cumulative distribution functions (CDF) and their corresponding probability density functions (PDF). Panel A shows the cumulative distribution function for the fast response rate (with a growth parameter value $a$ set to 0.37; see Equation 4.3) and Panel B shows the probability density function that results from computing the derivative of the fast response rate cumulative distribution function with respect to $x$ (see Equation 4.8). Panel C shows the cumulative distribution function for the slow response rate (with a growth parameter value $a$ set to 0.15; see Equation 4.4)) and Panel D shows the probability density function that results from computing the derivative of the slow response rate cumulative distribution function with respect to $x$ (see Equation 4.9 and section on time structuredness for more discussion). For the fast response rate functions, an 80% response rate is obtained after 4.32 days or, equivalently, 80% of the area underneath the probability density function is obtained at 4.32 days ($\int_0^{4.32} f'_{fast}(x) = 0.80$; the integral from 0 to 4.32 of the probability density function for a fast response rate $f'(x)_{fast}$ is 0.80). For the slow response rate functions, an 80% response rate is obtained after 10.80 days or, equivalently, 80% of the area underneath the probability density function is obtained at 10.80 days ($\int_0^{10.80} f'_{slow}(x) = 0.80$; the integral from 0 to 10.80 of the probability density function for a slow response rate $f'(x)_{slow}$ is 0.80).

**Figure 4.2**

*Cumulative Distribution Functions (CDF) and Probability Density Functions (PDF) for Fast and Slow Response Rates*




*Note.* Panel A: Cumulative distribution function for the fast response rate (with a growth parameter value $a$ set to 0.37; see Equation 4.3). Panel B: Probability density function that results from computing the derivative of the fast response rate cumulative distribution function with respect to $x$ (see Equation 4.8). Panel C: Cumulative distribution function for the slow response rate (with a growth parameter value $a$ set to 0.15; see Equation 4.4). Panel D: Probability density function that results from computing the derivative of the slow response rate cumulative distribution function with respect to $x$ (see Equation 4.9 and Time Structuredness for more discussion on time structuredness). For the fast response rate functions, an 80% response rate is obtained after 4.32 days or, equivalently, 80% of the area underneath the probability density function is obtained at 4.32 days ($\int_0^{4.32} f'_{fast}(x) = 0.80$). For the slow response rate functions, an 80% response rate is obtained after 10.80 days or, equivalently, 80% of the area underneath the probability density function is obtained at 10.80 days ($\int_0^{10.80} f'_{slow}(x) = 0.80$).

Having computed probability density functions for fast and slow response rates, time delays could be generated to create time-unstructured data. To generate time-unstructured data for a person at a given time point, a time delay was first generated by sampling values according to the probability density function defined by either a fast or slow response rate (Equations 4.8–4.9). The sampled time delay was then added to the value of the current measurement day, with the combined measurement day then being plugged into the logistic function (Equation **??**) along with a set of person-specific parameter values to generate an observed score at a given time point for a given person.

### 4.1.3 Modelling of Each Generated Data Set

Each generated data set was modelled using the structured latent growth curves outlined in Experiment 1 (see data modelling. For a detailed explanation of how the logistic function was fit into the structural equation modelling framework, see Technical Appendix B.

### 4.1.4 Analysis of Data Modelling Output and Accompanying Visualizations

Analysis and visualization was conducted as outlined in Experiment 1 (see analysis and visualization).

## 4.2 Results and Discussion

In the sections that follow, I organize the results by presenting them for each level of time structuredness (time-structured data, time-unstructured data resulting from a fast response rate, time-unstructured data resulting from a slow response rate). Importantly, only the results for the day-unit parameters will be presented (i.e., fixed- and random-effect days-to-halfway elevation and halfway-triquarter delta parameters [$\beta_{fixed}$, $\beta_{random}$, $\gamma_{fixed}$, $\gamma_{random}$, respectively]). The results for the likert-unit parameters (i.e., fixed- and random-effect baseline and maximal elevation parameters [$\theta_{fixed}$, $\theta_{random}$, $\alpha_{fixed}$, $\alpha_{random}$, respectively]) were largely trivial and so are presented in Appendix C).

For each level of time structuredness, I first provide a concise summary of the results and then provide a detailed report of the estimation accuracy of each day-unit parameter of the logistic function. Because the lengths of the detailed reports are considerable, I first provide concise summaries to establish a framework to interpret the detailed reports. The detailed report of each time structuredness level will summarize the results of each day-unit's bias/precision plot, report partial $\omega^2$ values, and then provide a qualitative summary.

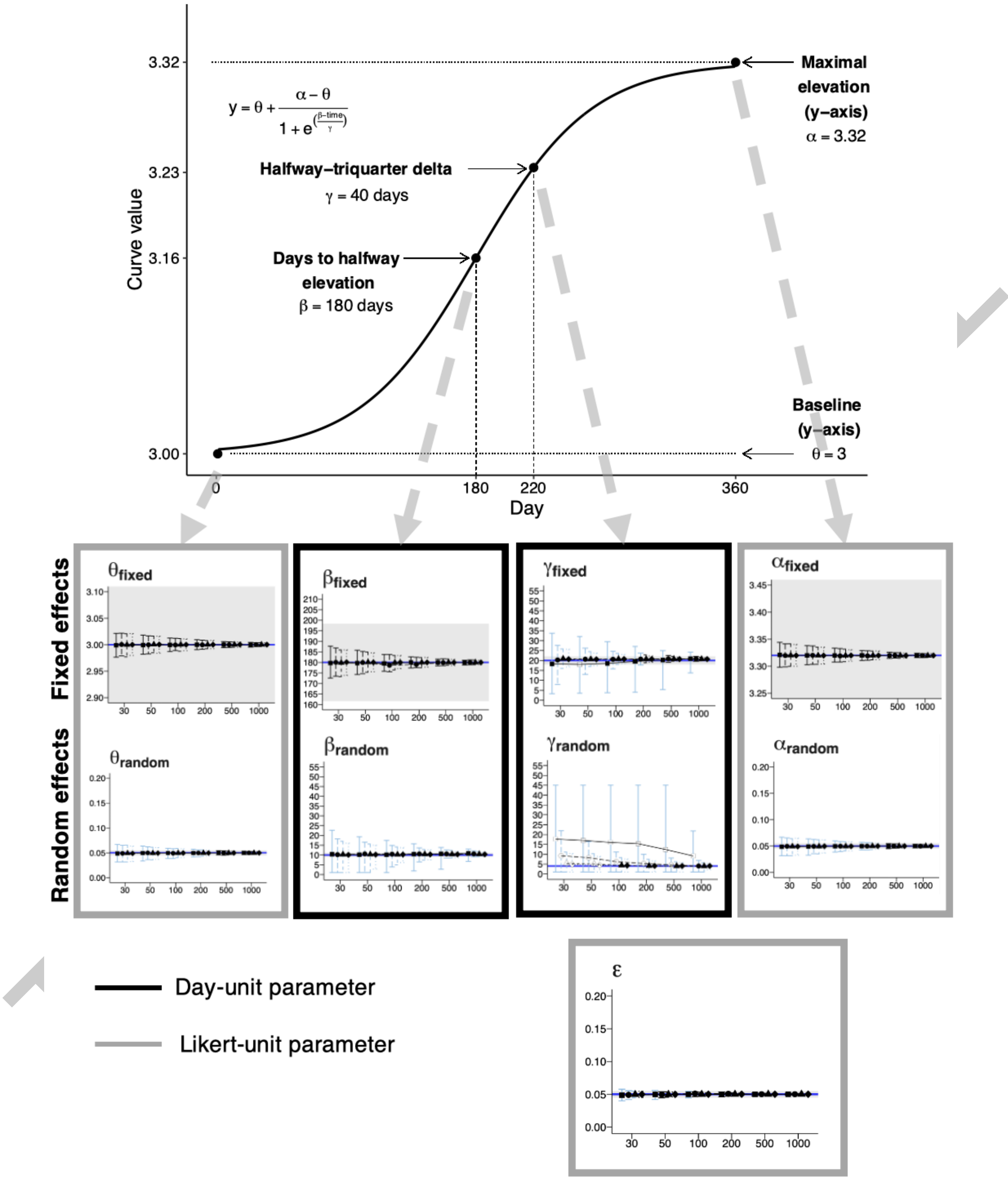
### 4.2.1 Framework for Interpreting Results

To conduct Experiment 3, the three variables of number of measurements (4 levels), sample size (6 levels), and time structuredness (3 levels) were manipulated, which yielded a total of 72 cells. Importantly, within each cell, bias and precision values were also computed for each of the nine parameters estimated by the structured latent growth curve models (for a review, see modelling of each generated data set). Thus, because the analysis of Experiment 3 computes values for many dependent variables, interpreting the results can become overwhelming. Therefore, I will provide a framework to help the reader efficiently navigate the results section.

Because I will present the results of Experiment 3 by each level of time structuredness, the framework I will describe in Figure **??** shows a template for the bias/precision plots that I will present for each level of time structuredness. The results presented for each time structuredness level contain a bias/precision plot for each of the nine estimated parameters. Each bias/precision plot shows the bias and precision for the estimation of one parameter across all measurement number and nature-of change levels. Within each bias/precision plot, dots indicate the average estimated value (which indicates bias bias) and error bars represent the middle 95% range of estimated values (which indicates precision). Bias/precision plots with black outlines show the results for day-unit parameters and plots with gray outlines show the results for Likert-unit parameters. Importantly, only the results for the day-unit parameters will be presented (i.e., fixed- and random-effect days-to-halfway elevation and halfway-triquarter delta parameters [$\beta_{fixed}$, $\beta_{random}$, $\gamma_{fixed}$, $\gamma_{random}$, respectively]). The results for the Likert-unit parameters (i.e., fixed- and random-effect baseline and maximal elevation parameters [$\theta_{fixed}$, $\theta_{random}$, $\alpha_{fixed}$, $\alpha_{random}$, respectively]) were largely trivial and so are presented in Appendix B. Therefore, the results of time structuredness level will only present the bias/precision plots for four parameters (i.e., the day-unit parameters).

**Figure 4.3**

*Set of Bias/Precision Plots Constructed for Each Spacing Schedule in Experiment 2*




*Note.* A parameter estimation plot is constructed for each parameter of the logistic function (see Equation ??). Note that each parameter of the logistic function is modelled as a fixed and random effect along with an error term ($\epsilon$; for a review, see Figure ??).

### 4.2.2 Pre-Processing of Data and Model Convergence

After collecting the output from the simulations, non-converged models (and their corresponding parameter estimates) were removed from subsequent analyses. Table **??** in Appendix B provides the convergence success rates for each cell in Experiment 3. Model convergence was almost always above 90% and convergence rates rates below 90% only occurred in two cells with five measurements.

### 4.2.3 Time-Structured Data

For time-structured data, Table 4.1 provides a concise summary of the results for the day-unit parameters (see Figure 4.4 for the corresponding parameter estimation plots). The sections that follow will present the results for each column of Table 4.1 and provide elaboration when necessary.

Before presenting the results for equal spacing, I provide a brief description of the concise summary table created for each spacing schedule and shown for equal spacing in Table 4.1. ext in the 'Unbiased' and 'Precise' columns indicates the measurement number-sample size pairings that, respectively, result in unbiased and precise estimation. Emboldened text in the 'Unbiased' and 'Qualitative Description' columns indicates the measurement number-sample size pairing needed to, respectively, obtain unbiased estimates and the greatest improvements in bias and precision across all day-unit parameters (acceptable precision not achieved in the estimation of all day-unit parameters with equal spacing). The 'Error Bar Length' column indicates the error bar length that results from using the lower-bounding measurement number-sample size pairing listed in the 'Qualitative Description' column.

**Table 4.1**

*Concise Summary of Results for Time-Structured Data in Experiment 3*

| | | | Description | |
|---|---|---|---|---|
| Parameter | Unbiased | Precise | Qualitative Description | Error Bar Length |
| $\beta_{fixed}$ (Figure 4.4A) | All cells | All cells | Unbiased and precise estimation in all cells | 15.13 |
| $\gamma_{fixed}$ (Figure 4.4B) | All cells | NM $\geq$ 9 with $N$ = 500 | Largest improvements in precision using **NM = 7 with $N \geq$ 200** or **NM = 9 with $N \leq$ 100** | 9.79 |
| $\beta_{random}$ (Figure 4.4C) | All cells | No cells | Largest improvements in precision with NM = 7 | 17.22 |
| $\gamma_{random}$ (Figure 4.4D) | **NM $\geq$ 9 with $N \geq$ 200** | No cells | Largest improvements in precision using **NM = 7 with $N \geq$ 200** or **NM = 9 with $N \leq$ 100** | 10.08 |

*Note.* Text in the 'Unbiased' and 'Precise' columns indicates the measurement number-sample size pairings that, respectively, result in unbiased and precise estimation. Emboldened text in the 'Unbiased' and 'Qualitative Description' columns indicates the number of measurements needed to, respectively, obtain unbiased estimates and the greatest improvements in bias and precision across all day-unit parameters (acceptable precision not achieved in the estimation of all day-unit parameters with equal spacing). 'Error Bar Length' column indicates the maximum error bar length that results from using the measurement number-sample size recommendation listed in the 'Qualitative Description' column. Parameter names and population values are as follows: $\beta_{fixed}$ = fixed-effect days-to-halfway elevation parameter = 180; $\gamma_{fixed}$ = fixed-effect halfway-triquarter delta parameter = 20; $\beta_{random}$ = random-effect days-to-halfway elevation parameter = 10; $\gamma_{random}$ = random-effect halfway-triquarter delta parameter = 4. NM = number of measurements.

#### 4.2.3.0.1 Bias

Before presenting the results for bias, I provide a description of the set of parameter estimation plots shown in Figure 4.4 and in the results sections for the other spacing schedules in Experiment 2. Figure 4.4 shows the parameter estimation plots for each day-unit parameter and Table ?? provides the partial $\omega^2$ values for each independent variable of each day-unit parameter. In Figure 4.4, blue horizontal lines indicate the population values for each parameter (with population values of $\beta_{fixed}$ = 180.00, $\beta_{random}$ = 10.00, $\gamma_{fixed}$ = 20.00, and $\gamma_{random}$ = 4.00). Gray bands indicate the ±10% margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Panels A–B show the parameter estimation plots for the fixed- and random-effect days-to-halfway elevation parameters ($\beta_{fixed}$ and $\beta_{random}$, respectively). Panels C–D show the parameter estimation plots for the fixed- and random-effect triquarter-halfway delta parameters ($\gamma_{fixed}$ and $\gamma_{random}$, respectively). Note that random-effect parameter units are in standard deviation units.

With respect to bias for time-structured data, estimates are biased (i.e., above the acceptable 10% cutoff) for each day-unit parameter in the following cells:

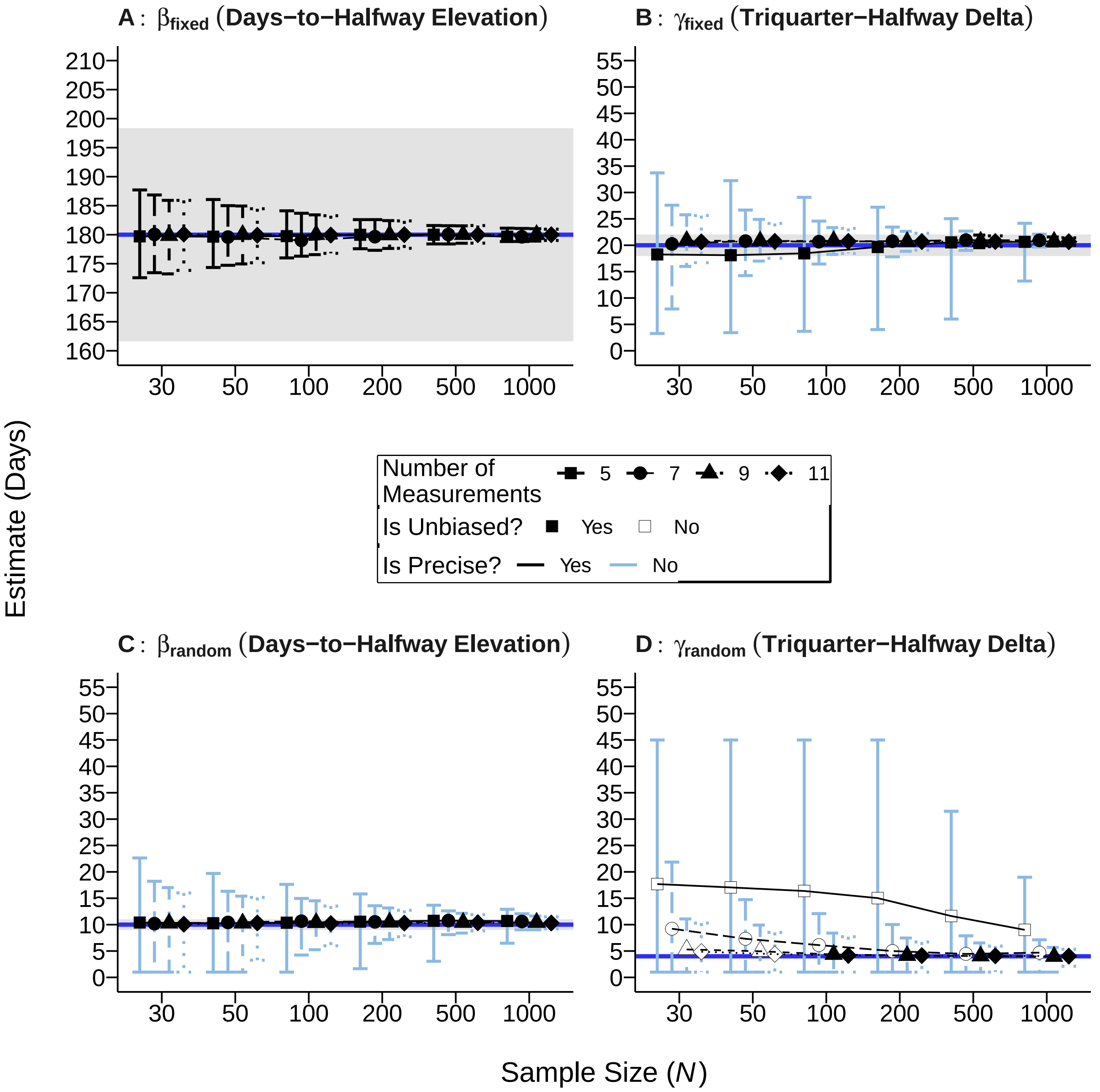
- fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$; Figure 4.4A): no cells.
- fixed-effect halfway-triquarter delta parameter ($\gamma_{fixed}$; Figure 4.4B): no cells.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$; Figure 4.4C): no cells.

- random-effect triquarter-halfway elevation parameter ($\gamma_{random}$; Figure 4.4D): five and seven measurements across all sample sizes and nine and 11 measurements with $N \leq 100$.

In summary, with time-structured data, estimation of all the day-unit parameters across all manipulated nature-of-change values is unbiased using at least nine measurements with $N \geq 200$, which is indicated by the emboldened text in the 'Unbiased' column of Table 4.1.

**Figure 4.4**

*Parameter Estimation Plots for Day-Unit Parameters With Time-Structured Data in Experiment 3*




*Note.* Panel A: Parameter estimation plot for the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$). Panel B: Parameter estimation plot for the fixed-effect triquarter-halfway elevation parameter ($\gamma_{fixed}$). Panel C: Parameter estimation plot for the random-effect days-to-halfway elevation parameter ($\beta_{random}$). Panel D: Parameter estimation plot for the random-effect triquarter-halfway elevation parameter ($\gamma_{random}$). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed}$ = 180.00, $\beta_{random}$ = 10.00, $\gamma_{fixed}$ = 20.00, $\gamma_{random}$ = 4.00. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff

(i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table ?? for specific values estimated for each parameter and Table 4.2 for $\omega^2$ effect size values.

**Table 4.2**

*Partial $\omega^2$ Values for Manipulated Variables With Time-Structured Data in Experiment 3*

| | Effect | | |
|---|---|---|---|
| Parameter | NM | S | NM x S |
| $\beta_{fixed}$ (Figure 4.4A) | 0.00 | 0.02 | 0.00 |
| $\beta_{random}$ (Figure 4.4B) | 0.14 | 0.27 | 0.03 |
| $\gamma_{fixed}$ (Figure 4.4C) | 0.25 | 0.12 | 0.07 |
| $\gamma_{random}$ (Figure 4.4D) | 0.18 | 0.03 | 0.01 |

NM = number of measurements (5, 7, 9, 11), S = sample size (30, 50, 100, 200, 500, 100), NM x S = interaction between number of measurements and sample size.

#### 4.2.3.0.2 Precision

With respect to precision for time-structured data, estimates are imprecise (i.e., error bar length with at least one whisker length exceeding 10% of a parameter's population value) in the following cells for each day-unit parameter:

- fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$; Figure 4.4A): no cells.
- fixed-effect halfway-triquarter delta parameter ($\gamma_{fixed}$; Figure 4.4B): five and seven measurements across all sample sizes and nine and 11 measurements with $N \leq 200$.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$; Figure 4.4C): all cells.
- random-effect halfway-triquarter delta parameter [$\gamma_{random}$] in Figure 4.4D): all cells.

In summary, with time-structured data, precise estimation can be obtained for the fixed-effect day-unit parameters using at least nine measurements with $N \geq 500$, but no manipulated measurement number-sample size pairing results in precise estimation of the random-effect day-unit parameters (see the 'Precise' column of Table 4.1).

#### 4.2.3.0.3 Qualitative Description

For time-structured data in Figure 4.4, although no manipulated measurement number results in precise estimation of all the day-unit parameters, the largest improvements in precision (and bias) result from using moderate measurement number-sample size pairings. With respect to bias under time-structured data, the largest improvements in bias result with the following measurement number-sample size pairings for the random-effect triquarter-halfway delta parameter ($\gamma_{fixed}$):

- random-effect triquarter-halfway delta parameter ($\gamma_{random}$): seven measurements with $N \geq 100$ or nine measurements with $N \leq 50$.

With respect to precision under time-structured data, the largest improvements in precision for the estimation of all the day-unit parameters (except the fixed-effect days-to-halfway elevation parameter [$\beta_{fixed}$]) result from using the following measurement number-sample size pairings:

- fixed-effect triquarter-halfway delta parameter ($\gamma_{fixed}$): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$, which results in a maximum error bar length of 9.79 days.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$): seven measurements across all manipulated sample sizes, which which results in a error bar length of 17.22 days.
- random-effect triquarter-halfway delta parameter ($\gamma_{random}$): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$, which results in a maximum error bar length of 10.08 days.

For an applied researcher, one plausible question might be what measurement number-sample size pairing(s) results in the greatest improvements in bias and precision in the estimation of all day-unit parameters with time-structured data. In looking across the measurement number-sample size pairings in the above lists, it becomes apparent that greatest improvements in bias and precision in the estimation of all day-unit parameters result with the following measurement number-sample size pairing(s): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$ (see the emboldened text in the 'Qualitative Description' column of Table 4.1).

#### 4.2.3.1 Summary of Results

In summarizing the results for time-structured data, estimation of all day-unit parameters is unbiased using least nine measurements with $N \geq 200$ (see bias). Precise estimation is never obtained in the estimation of all day-unit parameters with any manipulated measurement number-sample size pairing (see precision). Although it may be discouraging that no manipulated measurement number-sample size pairing under equal spacing results in precise estimation of all day-unit parameters, the largest improvements in precision (and bias) across all day-unit parameters are obtained with moderate measurement number-sample size pairings. With time-structured data, the largest improvements in bias and precision in the estimation of all day-unit parameters are obtained using seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$ (see qualitiative description).

### 4.2.4 Time-Unstructured Data Characterized by a Fast Response Rate

For time-unstructured data characterized by a fast response rate, Table 4.3 provides a concise summary of the results for the day-unit parameters (see Figure 4.5 for the corresponding parameter estimation plots). The sections that follow will present the results for each column of Table 4.3 and provide elaboration when necessary (for a description of Table 4.3, see concise summary).

**Table 4.3**

*Concise Summary of Results for Time-Unstructured Data (Fast Response Rate) in Experiment 3*

| | | | Description | |
|---|---|---|---|---|
| Parameter | Unbiased | Precise | Qualitative Description | Error Bar Length |
| $\beta_{fixed}$ (Figure 4.5A) | All cells | All cells | Unbiased and precise estimation in all cells | 15.35 |
| $\gamma_{fixed}$ (Figure 4.5B) | All cells | NM $\geq$ 9 with $N \geq$ 500 | Largest improvements in precision using **NM = 7 with *N* $\geq$ 200** or **NM = 9 with *N* $\leq$ 100** | 10.25 |
| $\beta_{random}$ (Figure 4.5C) | All cells | No cells | Largest improvements in precision with NM = 7 | 17.47 |
| $\gamma_{random}$ (Figure 4.5D) | **NM $\geq$ 7 with *N* = 1000** or **NM $\geq$ 9 with *N* $\geq$ 200** or **NM = 11 with *N* = 100** | No cells | Largest improvements in precision using **NM = 7 with *N* $\geq$ 200** or **NM = 9 with *N* $\leq$ 100** | 10.51 |

*Note.* Text in the 'Unbiased' and 'Precise' columns indicates the measurement number-sample size pairings that, respectively, result in unbiased and precise estimation. Emboldened text in the 'Unbiased' and 'Qualitative Description' columns indicates the number of measurements needed to, respectively, obtain unbiased estimates and the greatest improvements in bias and precision across all day-unit parameters (acceptable precision not achieved in the estimation of all day-unit parameters with equal spacing). 'Error Bar Length' column indicates the maximum error bar length that results from using the measurement number-sample size recommendation listed in the 'Qualitative Description' column. Parameter names and population values are as follows: $\beta_{fixed}$ = fixed-effect days-to-halfway elevation parameter = 180; $\gamma_{fixed}$ = fixed-effect halfway-triquarter delta parameter = 20; $\beta_{random}$ = random-effect days-to-halfway elevation parameter = 10; $\gamma_{random}$ = random-effect halfway-triquarter delta parameter = 4. NM = number of measurements.

#### 4.2.4.0.1 Bias

With respect to bias for time-unstructured data characterized by a fast response rate, estimates are biased (i.e., above the acceptable 10% cutoff) for each day-unit parameter in the following cells:

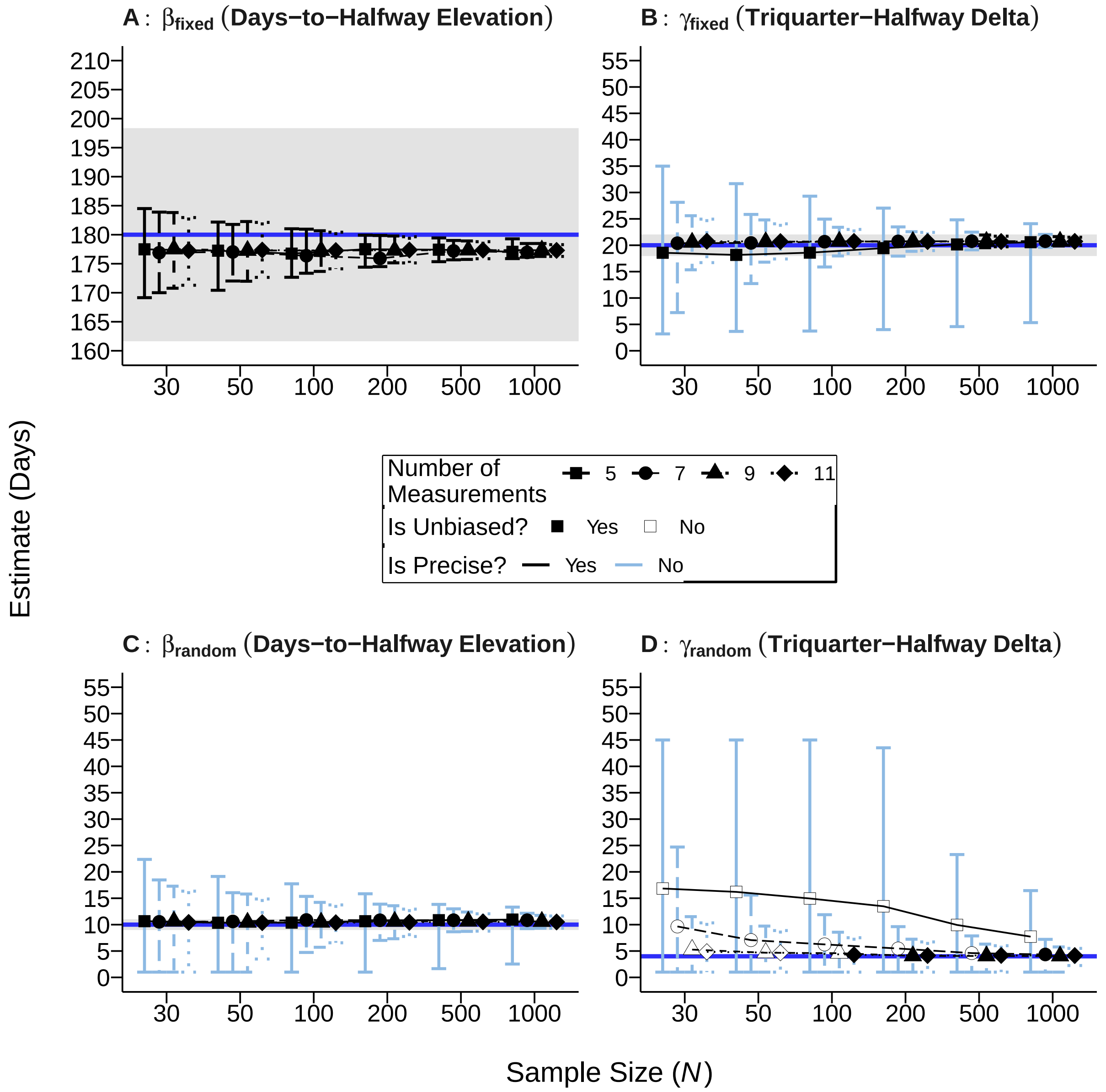
- fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$; Figure 4.5A): no cells.
- fixed-effect halfway-triquarter delta parameter ($\gamma_{fixed}$; Figure 4.5B): no cells.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$; Figure 4.5C): no cells.
- random-effect triquarter-halfway elevation parameter ($\gamma_{random}$; Figure 4.5D): five measurements across all sample sizes, seven measurements with $N \leq 500$, nine measurements with $N \geq 100$, and 11 measurements with $N \leq 50$.

Note that, for the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$), although bias is still within the acceptable margin of error, bias appears to be constant across all manipulated measurement number-sample size pairings.

In summary, with time-unstructured data characterized by a fast response rate, estimation of all the day-unit parameters across all manipulated nature-of-change values is unbiased using at least seven measurements with $N = 1000$, nine measurements with $N \geq 200$, or 11 measurements with $N \geq 100$, which is indicated by the emboldened text in the 'Unbiased' column of Table 4.3.

**Figure 4.5**

*Parameter Estimation Plots for Day-Unit Parameters With Time-Unstructured Data Characterized by a Fast Response Rate in Experiment 3*




*Note.* Panel A: Parameter estimation plot for the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$). Panel B: Parameter estimation plot for the fixed-effect triquarter-halfway elevation parameter ($\gamma_{fixed}$). Panel C: Parameter estimation plot for the random-effect days-to-halfway elevation parameter ($\beta_{random}$). Panel D: Parameter estimation plot for the random-effect triquarter-halfway elevation parameter ($\gamma_{random}$). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed}$ = 180.00, $\beta_{random}$ = 10.00, $\gamma_{fixed}$ = 20.00, $\gamma_{random}$ = 4.00. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff

(i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table ?? for specific values estimated for each parameter and Table 4.4 for $\omega^2$ effect size values.

**Table 4.4**

*Partial $\omega^2$ Values for Manipulated Variables With Time-Structured Data in Experiment 3*

| | Effect | | |
|---|---|---|---|
| Parameter | NM | S | NM x S |
| $\beta_{fixed}$ (Figure 4.5A) | 0.00 | 0.02 | 0.00 |
| $\beta_{random}$ (Figure 4.5B) | 0.15 | 0.27 | 0.03 |
| $\gamma_{fixed}$ (Figure 4.5C) | 0.29 | 0.14 | 0.08 |
| $\gamma_{random}$ (Figure 4.5D) | 0.17 | 0.04 | 0.01 |

NM = number of measurements (5, 7, 9, 11), S = sample size (30, 50, 100, 200, 500, 100), NM x S = interaction between number of measurements and sample size.

#### 4.2.4.0.2 Precision

With respect to precision for time-unstructured data characterized by a fast response rate, estimates are imprecise (i.e., error bar length with at least one whisker length exceeding 10% of a parameter's population value) in the following cells for each day-unit parameter:

- fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$; Figure 4.5A): no cells.
- fixed-effect halfway-triquarter delta parameter ($\gamma_{fixed}$; Figure 4.5B): five and seven measurements across all sample sizes and nine and 11 measurements with $N \leq 200$.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$; Figure 4.5C): all cells.
- random-effect halfway-triquarter delta parameter [$\gamma_{random}$] in Figure 4.5D): all cells.

In summary, with time-unstructured data characterized by a fast response rate, precise estimation can be obtained for the fixed-effect day-unit parameters using at least nine measurements with $N \geq 500$, but no manipulated measurement number-sample size pairing results in precise estimation of the random-effect day-unit parameters (see the 'Precise' column of Table 4.3).

##### 4.2.4.0.3 Qualitative Description

For time-unstructured data characterized by a fast response rate (see Figure 4.5), although no manipulated measurement number results in precise estimation of all the day-unit parameters, the largest improvements in precision (and bias) result from using moderate measurement number-sample size pairings. With respect to bias under time-unstructured data characterized by a fast response rate, the largest improvements in bias result with the following measurement number-sample size pairings for the random-effect triquarter-halfway delta parameter ($\gamma_{fixed}$):

- random-effect triquarter-halfway delta parameter ($\gamma_{random}$): seven measurements with $N \geq 100$ or nine measurements with $N \leq 50$.

With respect to precision under time-unstructured data characterized by a fast response rate, the largest improvements in precision for the estimation of all the day-unit parameters (except the fixed-effect days-to-halfway elevation parameter [$\beta_{fixed}$]) result from using the following measurement number-sample size pairings:

- fixed-effect triquarter-halfway delta parameter ($\gamma_{fixed}$): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$, which results in a maximum error bar length of 10.25 days.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$): seven measurements across all manipulated sample sizes, which which results in a error bar length of 17.47 days.

- random-effect triquarter-halfway delta parameter ($\gamma_{random}$): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$, which results in a maximum error bar length of 10.51 days.

For an applied researcher, one plausible question might be what measurement number-sample size pairing(s) results in the greatest improvements in bias and precision in the estimation of all day-unit parameters with time-unstructured data characterized by a fast response rate. In looking across the measurement number-sample size pairings in the above lists, it becomes apparent that greatest improvements in bias and precision in the estimation of all day-unit parameters result with the following measurement number-sample size pairing(s): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$ (see the emboldened text in the 'Qualitative Description' column of Table 4.3).

#### 4.2.4.1 Summary of Results

In summarizing the results for time-unstructured data characterized by a fast response rate, estimation of all day-unit parameters is unbiased using least seven measurements with $N = 1000$, nine measurements with $N \geq 200$, or 11 measurements with $N \geq 100$ (see bias). Importantly, bias for some day-unit parameters is constant across manipulated measurement number-sample size pairings. Precise estimation is never obtained in the estimation of all day-unit parameters with any manipulated measurement number-sample size pairing (see precision). Although it may be discouraging that no manipulated measurement number-sample size pairing under time-unstructured data characterized by a fast response rate results in precise estimation of all day-unit parameters, the largest improvements in precision (and bias) across all day-unit parameters are obtained with moderate measurement number-sample size pairings. With time-unstructured data characterized by a fast response rate, the largest improvements in bias and precision in the estimation of all day-unit parameters are obtained using seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$ (see qualitiative description).

### 4.2.5 Time-Unstructured Data Characterized by a Slow Response Rate

For time-unstructured data characterized by a slow response rate, Table 4.5 provides a concise summary of the results for the day-unit parameters (see Figure 4.6 for the corresponding parameter estimation plots). The sections that follow will present the results for each column of Table 4.5 and provide elaboration when necessary (for a description of Table 4.5, see concise summary).

**Table 4.5**
*Concise Summary of Results for Time-Unstructured Data (Slow Response Rate) in Experiment 3*

| Parameter | Unbiased | Precise | Summary | |
|---|---|---|---|---|
| | | | Qualitative Summary | Error Bar Length |
| $\beta_{fixed}$ (Figure 4.6A) | All cells | All cells | Low bias and high precision in all cells | 16.68 |
| $\gamma_{fixed}$ (Figure 4.6B) | All cells except NM = 5 with $N$ = 50 | NM = 7 with $N$ = 200 or NM = 9 with $N \leq$ 500 | Largest improvements in precision using **NM = 7 with $N \geq$ 200** or **NM = 9 with $N \leq$ 100** | 10.53 |
| $\beta_{random}$ (Figure 4.6C) | No cells except NM = 5 with $N$ = 30 and NM = 11 with $N \leq$ 50 | No cells | Largest improvements in precision with NM = 7 | 18.44 |
| $\gamma_{random}$ (Figure 4.6D) | No cells | No cells | Largest improvements in bias and precision using NM = 7 with $N \geq$ 200 or M = 9 with $N \leq$ 100 | 10.9 |

*Note.*

Bolded text in the ‘Low Bias’ and ‘Qualitative Summary’ columns indicates the measurement number-sample size pairing needed to, respectively, achieve low bias and the greatest improvements in bias and precision across all day-unit parameters (high precision not achieved in the estimation of all day-unit parameters with time-unstructured data characterized by a slow response rate). ‘Error Bar Length’ indicates the longest error bar length that results from using the measurement number-sample size pairings in the ‘Qualitative Summary‘ column. Parameter names and population values are as follows: $\beta_{fixed}$ = fixed-effect days-to-halfway elevation parameter = 180; $\gamma_{fixed}$ = fixed-effect halfway-triquarter delta parameter = 20; $\beta_{random}$ = random-effect days-to-halfway elevation parameter = 10; $\gamma_{random}$ = random-effect halfway-triquarter delta parameter = 4.

#### 4.2.5.0.1 Bias

With respect to bias for time-unstructured data characterized by a slow response rate, estimates are biased (i.e., above the acceptable 10% cutoff) for each day-unit parameter in the following cells:

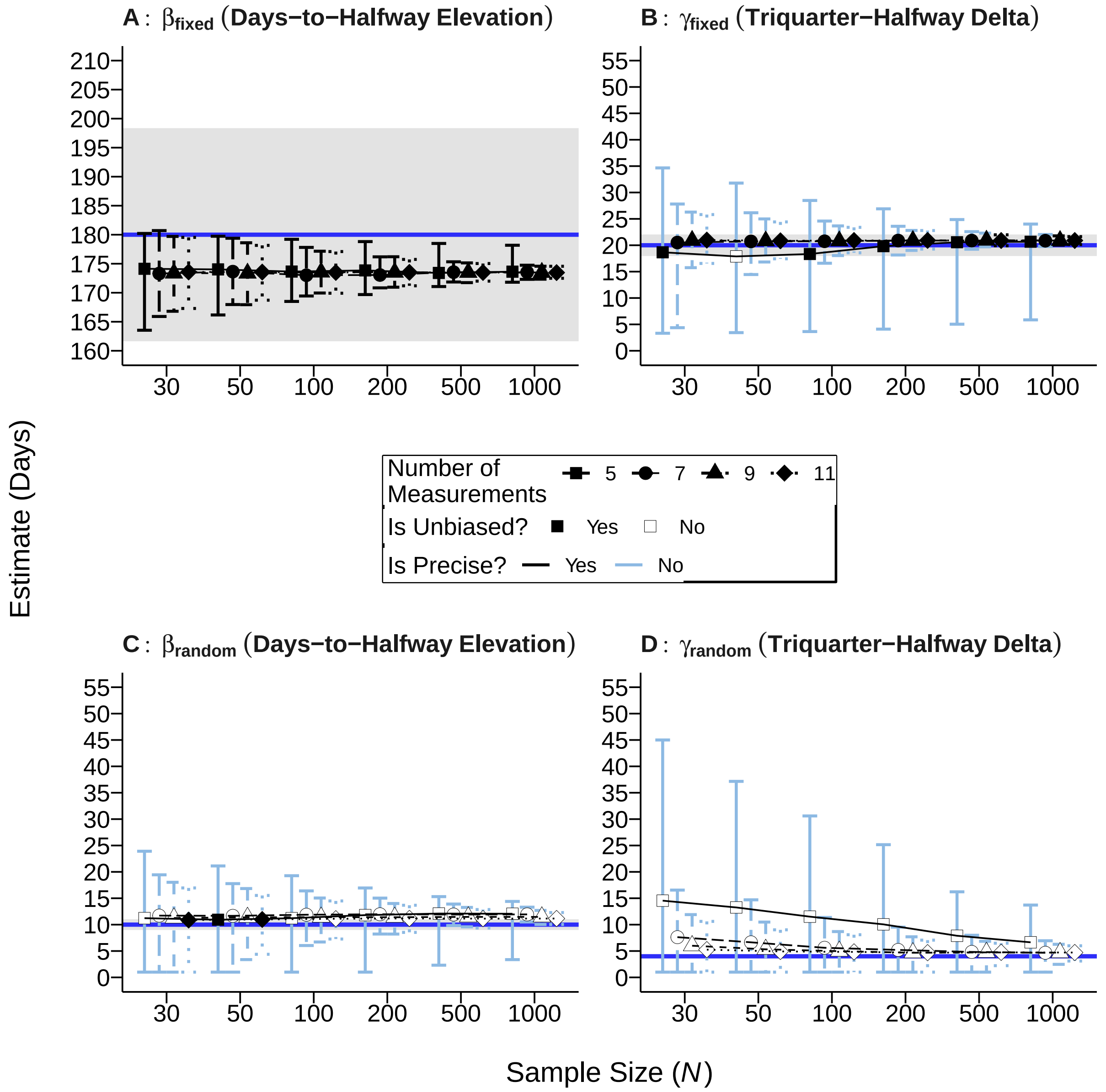
- fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$; Figure 4.6A): no cells.
- fixed-effect halfway-triquarter delta parameter ($\gamma_{fixed}$; Figure 4.6B): no cells.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$; Figure 4.6C): no cells.
- random-effect triquarter-halfway elevation parameter ($\gamma_{random}$; Figure 4.6D): five measurements across all sample sizes, seven measurements with $N \leq 500$, nine measurements with $N \geq 100$, and 11 measurements with $N \leq 50$.

Note that, for all parameters except the halfway-triquarter delta parameter ($\gamma_{fixed}$), bias appears to be constant across all manipulated measurement number-sample size pairings Liu et al. (2021)

In summary, with time-unstructured data characterized by a slow response rate, estimation of all the day-unit parameters across all manipulated nature-of-change values is unbiased using at least seven measurements with $N = 1000$, nine measurements with $N \geq 200$, or 11 measurements with $N \geq 100$, which is indicated by the emboldened text in the 'Unbiased' column of Table 4.5.

**Figure 4.6**

*Parameter Estimation Plots for Day-Unit Parameters With Time-Unstructured Data Characterized by a Slow Response Rate in Experiment 3*




*Note.* Panel A: Parameter estimation plot for the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$). Panel B: Parameter estimation plot for the fixed-effect triquarter-halfway elevation parameter ($\gamma_{fixed}$). Panel C: Parameter estimation plot for the random-effect days-to-halfway elevation parameter ($\beta_{random}$). Panel D: Parameter estimation plot for the random-effect triquarter-halfway elevation parameter ($\gamma_{random}$). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed}$ = 180.00, $\beta_{random}$ = 10.00, $\gamma_{fixed}$ = 20.00, $\gamma_{random}$ = 4.00. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff

(i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table ?? for specific values estimated for each parameter and Table 4.6 for $\omega^2$ effect size values.

**Table 4.6**
*Partial $\omega^2$ Values for Manipulated Variables With Time-Unstructured Data Characterized by a Slow Response Rate in Experiment 3*

| | Effect | | |
|---|---|---|---|
| Parameter | NM | S | NM x S |
| $\beta_{fixed}$ (Figure 4.6A) | 0.00 | 0.02 | 0.00 |
| $\beta_{random}$ (Figure 4.6B) | 0.15 | 0.27 | 0.03 |
| $\gamma_{fixed}$ (Figure 4.6C) | 0.29 | 0.14 | 0.08 |
| $\gamma_{random}$ (Figure 4.6D) | 0.17 | 0.04 | 0.01 |

NM = number of measurements (5, 7, 9, 11), S = sample size (30, 50, 100, 200, 500, 100), NM x S = interaction between number of measurements and sample size.

#### 4.2.5.0.2 Precision

With respect to precision for time-unstructured data characterized by a slow response rate, estimates are imprecise (i.e., error bar length with at least one whisker length exceeding 10% of a parameter's population value) in the following cells for each day-unit parameter:

- fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$; Figure 4.6A): no cells.
- fixed-effect halfway-triquarter delta parameter ($\gamma_{fixed}$; Figure 4.6B): five and seven measurements across all sample sizes and nine and 11 measurements with $N \leq 200$.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$; Figure 4.6C): all cells.
- random-effect halfway-triquarter delta parameter [$\gamma_{random}$] in Figure 4.6D): all cells.

In summary, with time-unstructured data characterized by a slow response rate, precise estimation can be obtained for the fixed-effect day-unit parameters using at least nine measurements with $N \geq 500$, but no manipulated measurement number-sample size pairing results in precise estimation of the random-effect day-unit parameters (see the 'Precise' column of Table 4.5).

#### 4.2.5.0.3 Qualitative Description

For time-unstructured data characterized by a slow response rate (see Figure 4.6), although no manipulated measurement number results in precise estimation of all the day-unit parameters, the largest improvements in precision (and bias) result from using moderate measurement number-sample size pairings. With respect to bias under time-unstructured data characterized by a slow response rate, the largest improvements in bias result with the following measurement number-sample size pairings for the random-effect triquarter-halfway delta parameter ($\gamma_{fixed}$):

- random-effect triquarter-halfway delta parameter ($\gamma_{random}$): seven measurements with $N \geq 100$ or nine measurements with $N \leq 50$.

With respect to precision under time-unstructured data characterized by a slow response rate, the largest improvements in precision for the estimation of all the day-unit parameters (except the fixed-effect days-to-halfway elevation parameter [$\beta_{fixed}$]) result from using the following measurement number-sample size pairings:

- fixed-effect triquarter-halfway delta parameter ($\gamma_{fixed}$): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$, which results in a maximum error bar length of 10.53 days.
- random-effect days-to-halfway elevation parameter ($\beta_{random}$): seven measurements across all manipulated sample sizes, which which results in a error bar length of 18.44 days.

- random-effect triquarter-halfway delta parameter ($\gamma_{random}$): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$, which results in a maximum error bar length of 10.9 days.

For an applied researcher, one plausible question might be what measurement number-sample size pairing(s) results in the greatest improvements in bias and precision in the estimation of all day-unit parameters with time-unstructured data characterized by a fast response rate. In looking across the measurement number-sample size pairings in the above lists, it becomes apparent that greatest improvements in bias and precision in the estimation of all day-unit parameters result with the following measurement number-sample size pairing(s): seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$ (see the emboldened text in the 'Qualitative Description' column of Table 4.5).

#### 4.2.5.1 Summary of Results

In summarizing the results for time-unstructured data characterized by a slow response rate, estimation of all day-unit parameters is least seven measurements with $N = 1000$, nine measurements with $N \geq 200$, or 11 measurements with $N \geq 100$ (see bias). Importantly, bias for most day-unit parameters is constant across manipulated measurement number-sample size pairings. Precise estimation is never obtained in the estimation of all day-unit parameters with any manipulated measurement number-sample size pairing (see precision). Although it may be discouraging that no manipulated measurement number-sample size pairing under time-unstructured data characterized by a slow response rate results in precise estimation of all day-unit parameters, the largest improvements in precision (and bias) across all day-unit parameters are obtained with moderate measurement number-sample size pairings. With time-unstructured data characterized by a slow response rate, the largest improvements in bias and precision in the estimation of all day-unit parameters are obtained using seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$ (see qualitiative description).

#### 4.2.6 How Does Time Structuredness Affect Modelling Accuracy?

In Experiment 3, I was interested in how decreasing time structuredness affected modelling accuracy. Table 4.7 summarizes the results for each spacing schedule in Experiment 3. Text within the 'Unbiased' and 'Precise' columns indicates the measurement number-sample size pairing needed to, respectively, obtain unbiased an precise estimation for all the day-unit parameters. The 'Error Bar Length' column indicates longest error bar lengths that result in the estimation of each day-unit parameter from using the measurement number-sample size pairings listed in the 'Qualitative Description' column. In looking at the 'Qualitative Description' column, the greatest improvements in bias and precision for all time structuredness levels result from using either seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$.

Although the same measurement number-sample size pairing can be used to obtain the greatest improvements in modelling accuracy under any time structuredness level, two results suggest that modelling accuracy decreases as the time structuredness decreases. First, the error bar lengths in Table **??** increase as time structuredness decreases. As an example, the error bar length of the fixed-effect days-to-halfway elevation parameter is 15.13 days with time-structured data and increases to 16.68 days with time-unstructured data characterized by a slow response rate. Second, and more alarming, the bias incurred as time structuredness decreases is constant across all measurement number-sample size pairings (see Figure **??**). That is, the increase in bias that results from time-unstructured data cannot be reduced by increasing the number of measurements or sample size. An an example, the fixed-effect days-to-halfway elevation parameter is underestimated by roughly 6 days across all measurement number-sample size pairings ($\beta_{fixed}$; see Figure ??A).

**Table 4.7**

*Concise Summary of Results Across All Time Structuredness Levels in Experiment 3*

| | | | | Error Bar Summary | | | |
|---|---|---|---|---|---|---|---|
| Time Structuredness | Unbiased | Precise | Qualitative Description | $\beta_{fixed}$ | $\gamma_{fixed}$ | $\beta_{random}$ | $\gamma_{random}$ |
| Time structured (see Figure 4.4 and Table 4.1) | NM $\geq$ 9 with $N \geq$ 200 | No cells | Largest improvements in precision using **NM = 7 with *N* $\geq$ 200** or **NM = 9 with *N* $\leq$ 100** | 15.13 | 9.79 | 17.22 | 10.08 |
| Time unstructured (fast response rate; see Figure 4.5 and Table 4.3) | NM $\geq$ 7 with $N$ = 1000 or NM $\geq$ 9 with $N \geq$ 200 or NM = 11 with $N$ = 100 | No cells | Largest improvements in precision using **NM = 7 with *N* $\geq$ 200** or **NM = 9 with *N* $\leq$ 100** | 15.35 | 10.25 | 17.47 | 10.51 |
| Time unstructured (slow response rate; see Figure 4.6 and Table 4.5) | No cells | No cells | Largest improvements in precision using **NM = 7 with *N* $\geq$ 200** or **NM = 9 with *N* $\leq$ 100** | 16.68 | 10.53 | 18.44 | 10.90 |

*Note.* 'Qualitative Description' column indicates the number of measurements that obtains the greatest improvements in bias and precision across all day-unit parameters. 'Error Bar Summary' columns list the error bar lengths that result for each day-unit parameter using the measurement number listed in the 'Qualitative Description' column. Parameter names and population values are as follows: $\beta_{fixed}$ = fixed-effect days-to-halfway elevation parameter $\in \{80, 180, 280\}$; $\gamma_{fixed}$ = fixed-effect halfway-triquarter delta parameter = 20; $\beta_{random}$ = random-effect days-to-halfway elevation parameter = 10; $\gamma_{random}$ = random-effect halfway-triquarter delta parameter = 4. NM = number of measurements.

To understand why bias is constant as time structure decreases, it is important to first understand latent growth curve models more deeply. By default, latent growth curve models assume time-structured data. As a reminder, data are time structured when participants provide data at the exact same moment at each time point (e.g., if a study collects data on the first day of each month for a year, then time-structured data would only be obtained if participants all provide their data at the exact same moment each time data are collected). Consider a random-intercept-random-slope model shown in Figure ?? that is used to model stress ratings collected on the first day of each month over the course of five months from $j$ people. Stress ratings at each $i$ time point for each $j$ person are predicted by person-specific intercepts ($b_{0j}$) and slopes ($b_{1j}$; in addition to a residual term [$\epsilon_{ij}$]) as shown below in Equation 4.10 (which is often called Level-1 equation):

$$Stress_{ij} = b_{0j} + b_{1j}(Stress_{ij}) + \epsilon_{ij}. \tag{4.10}$$

The person-specific intercepts and slopes are the sum of a fixed-effect parameter whose value is constant across all people ($\gamma_{00}$ and $\gamma_{10}$) and a random-effect parameter that represents the variance of the person-specific variables (i.e., $\sigma_{00}$ and $\sigma_{10}$). The fixed-effect intercept and slope, respectively, represent the mean starting stress value (i.e., average stress value at Time $= 0$) and the average slope value. Importantly, by estimating a random-effect parameter (in addition to the fixed-effect parameters), deviations from the mean intercept an slope values can be obtained for each $j$ person ($\sigma_{0j}$ and $\sigma_{1j}$) and these values then allow the person-specific intercepts and slopes to be computed as shown in

Equations 4.11–4.12 (which are often called Level-2 equations):

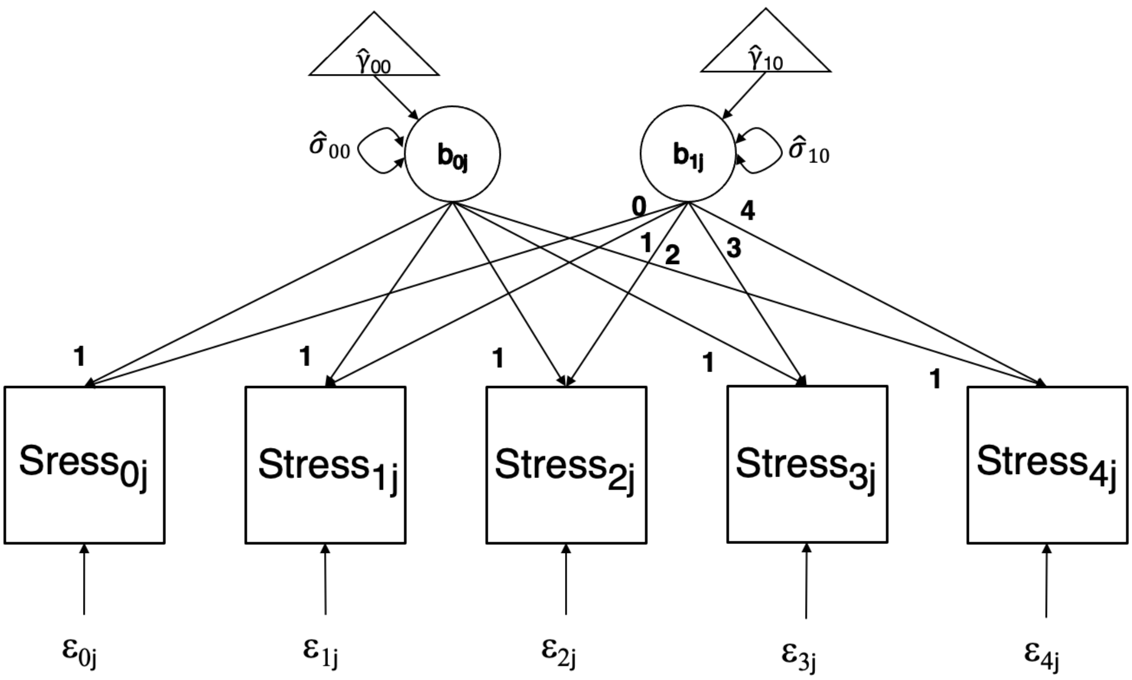
$$b_{0j} = \hat{\gamma_{00}} + \sigma_{0j} \tag{4.11}$$

$$b_{1j} = \hat{\gamma_{10}} + \sigma_{1j} \tag{4.12}$$

Note that the fixed- and random-effect parameters in Figure 4.7 are superscribed with a caret ( $\hat{}$ ) to indicate that the values of these parameters are estimated by the latent growth curve model. Also note that, in Figure 4.7, circles indicate latent variables, triangles indicate constants, and squares indicate observed (or manifest variables).

**Figure 4.7**

*Path Diagram for a Random-Intercept-Random-Slope Latent Growth Curve Model*



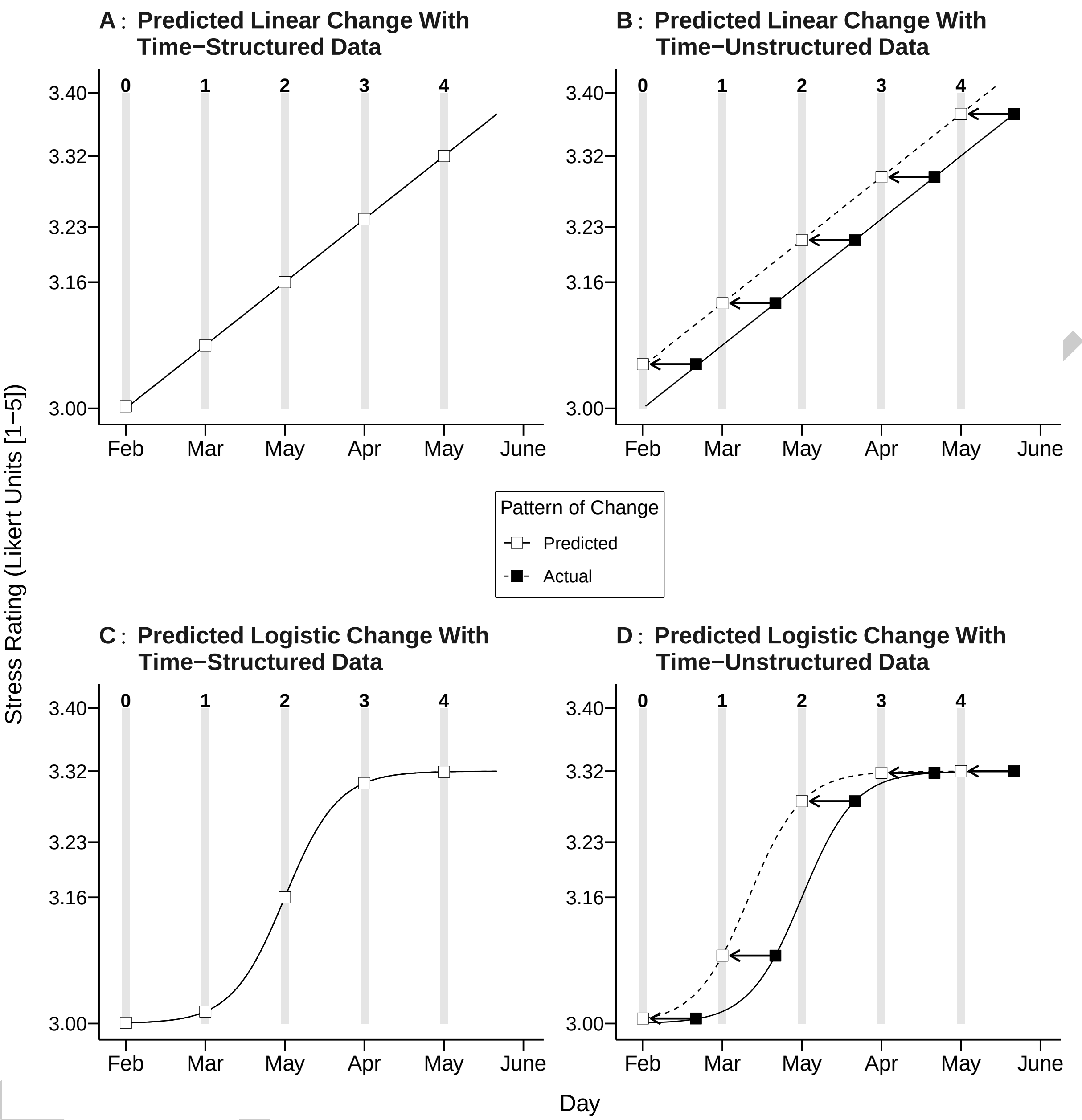

*Note.* Stress at each $i$ time point for each $j$ person is predicted by a person-specific slope ($b_{0j}$), person-specific intercept ($b_{1j}$), and residual ($\epsilon_{ij}$; see Equation 4.10 [Level-1 equation]). The person-specific effects are also called *random effects* and each is the sum of a fixed-effect parameter whose value is constant across all people ($\gamma_{00}$ and $\gamma_{10}$) and a random-effect parameter that represents the variance of the person-specific variables (i.e., $\sigma_{00}$ and $\sigma_{10}$; see Equations 4.11–4.12 [Level-2 equations]). Note that the fixed- and random-effect parameters are superscribed with a caret ( $\hat{}$ ) to indicate that the values of these parameters are estimated by the latent growth curve model. Also note that circles indicate latent variables, triangles indicate constants, and squares indicate observed (or manifest variables).

To understand why bias in parameter estimation increases as time structuredness decreases, it is important to discuss one component of the latent growth curve model not yet discussed: loadings. In latent variable models, *loadings* comprise numbers that indicate how a latent variable should be modelled. The numbers in loadings satisfy two needs of latent variables. First, loadings give latent variables a unit; latent variables are inherently unitless, and so require a unit so that they can be meaningfully interpreted. By fixing at least one pathway between a latent and observed variable with a loading, the latent variable takes on the units of the observed variable. In the current example, the intercept and slope latent variables take on the units of the stress ratings (e.g., Likert units). Second, in latent growth curve models, latent variables need their effect to be specified, and loadings satisfy this need. In the current example, the intercept has a constant effect at each time point, and this is represented by setting its loadings at each time point to 1. The slope represents linearly increasing change over time, and so its loadings are set to increase by an integer value of 1 after each time point.

Although loadings allow latent variables to model change over time, their values are constant across participants and it is this characteristic that causes modelling accuracy to decrease as time structuredness decreases. In focusing on the slope variable in Figure 4.7, the loadings of 0, 1, 2, 3, and 4 assume that only one response pattern describes how each participant provides their data over the five-month period. Specifically, the loadings assume that each participant provides data on the first day of each month, which is indicated by the gray rectangles (along with the loading number above each gray rectangle) in each panel of Figure **??**. With time-structured data, constant loadings do not decrease modelling accuracy because each participant provides their data on the first day of each month. As examples of modelling accuracy with time-structured data, panels A and C of Figure **??** show the predicted and actual patterns for individual participants with linear and logistic patterns of change, respectively. Because each individual participant displays a response pattern identical to the one specified by the loadings, the predicted and actual patterns of change are identical. With time-unstructured data, the predicted and actual patterns of change no longer overlap because response patterns in participants differ from the one assumed by the loadings. As examples of modelling accuracy with time-unstructured data, panels B and D of Figure **??** show the predicted and actual patterns for individual participants with linear and logistic patterns of change, respectively. Although each participant provides data many days after the first day of each month, the constant loadings set in the model lead the it to assume that data were collected on the first day of each month. Because the model misattributes the time at which data are recorded, the predicted patterns of change are shifted leftward, leading to a decrease in modelling accuracy. In Figure **??**B, the intercept ($b_{0j}$) increases due to time-unstructured data. In Figure **??**D, the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$) decreases due to time-unstructured data. Therefore, the loading structured specified by default in latent growth curve model causes modelling accuracy to decrease when data are time unstructured.

**Figure 4.8**

*Modelling Accuracy Decreases as Time Structuredness Decreases*

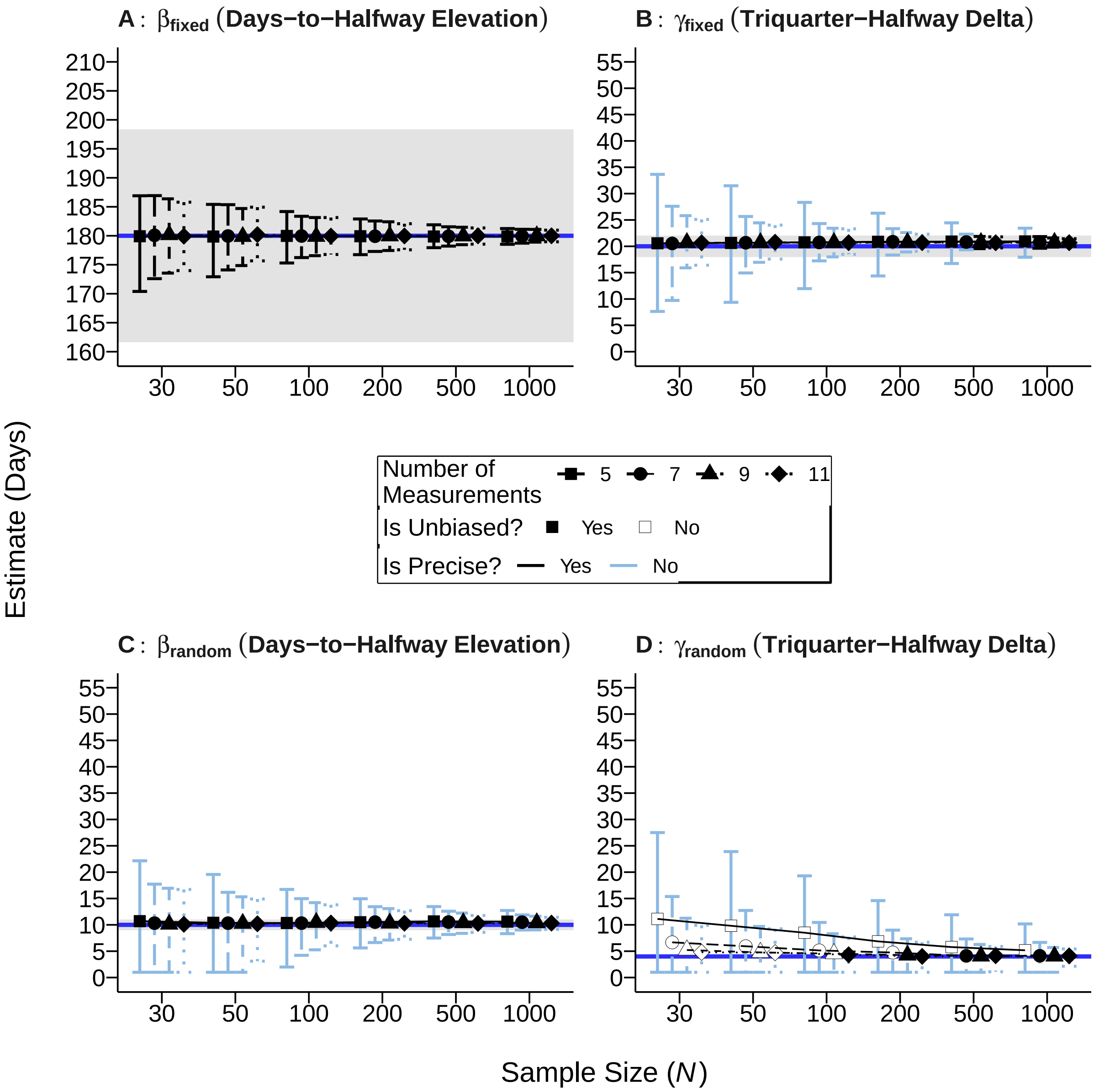



*Note.* Panel A: Predicted and actual linear patterns of change are identical because of time-structured data. Panel B: Predicted and actual linear patterns of change are different because of time-untructured data decreases modelling accuracy. Panel C: Predicted and actual logistic patterns of change are identical because of time-structured data. Panel D: Predicted and actual logistic patterns of change differ because of time-unstructured data decreases modelling accuracy. Shaded vertical rectangles indicate the response pattern expected across all participants by the loadings set in the latent growth curve model depicted in Figure 4.7.

### 4.2.7 Eliminating the Bias Caused by Time Unstructuredness: Using Definition Variables

**Figure 4.9**

*Parameter Estimation Plots for Day-Unit Parameters When Using Definition Variables To Model Time-Unstructured Data Characterized by a Slow Response Rate*




*Note.* Panel A: Parameter estimation plot for the fixed-effect days-to-halfway elevation parameter ($\beta_{fixed}$). Panel B: Parameter estimation plot for the fixed-effect triquarter-halfway elevation parameter ($\gamma_{fixed}$). Panel C: Parameter estimation plot for the random-effect days-to-halfway elevation parameter ($\beta_{random}$). Panel D: Parameter estimation plot for the random-effect triquarter-halfway elevation parameter ($\gamma_{random}$). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed}$ = 180.00, $\beta_{random}$ = 10.00, $\gamma_{fixed}$ = 20.00, $\gamma_{random}$ = 4.00. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table ?? for specific values estimated for each parameter and Table ?? for $\omega^2$ effect size values.

## 4.3 Summary

# 5 References


Coulombe, P., Selig, J. P., & Delaney, H. D. (2016). Ignoring individual differences in times of assessment in growth curve modeling. *International Journal of Behavioral Development*, *40*(1), 76–86. https://doi.org/10.1177/0165025415577684

Dillman, D. A., Smyth, J. D., & Christian, L. M. (2014). *Internet, phone, mail, and mixed-mode surveys: The tailored design method*. John Wiley & Sons.

Liu, J., Perera, R. A., Kang, L., Sabo, R. T., & Kirkpatrick, R. M. (2021). Obtaining interpretable parameters from reparameterized longitudinal models: Transformation Mmtrices between growth factors in Tto parameter spaces. *Journal of Educational and Behavioral Statistics*, *47*(2), 167–201. https://doi.org/10.3102/10769986211052009

Pan, B. (2010). Online travel surveys and response patterns. *Journal of Travel Research*, *49*(1), 121–135. https://doi.org/10.1177/0047287509336467