SESSION 6

CACHE BLOCKING & TILING

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Computing the matrix transpose

Given N-by-N matrices A and B, we can compute

$$B_{ij} \leftarrow A_{ji}$$

with

```
double *A, *B;
...
for (int i = 0; i < N; i++)
  for (int j = 0; j < N; j++)
    B[i*N + j] = A[j*N + i];
```

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What is the performance of this code?

Matrix transpose: simple performance model

- N^2 stores
- N^2 loads
- no computation

What do you expect?

Matrix transpose: simple performance model

- N^2 stores
- N^2 loads
- no computation

What do you expect?

Matrix size	bandwidth [GB/s]
128×128	22
256×256	13
512×512	13
1024×1024	5
2048×2048	1.6
4096×4096	0.9

What went wrong?

```
for (int i = 0; i < N; i++)
    for (int j = 0; j < N; j++)
        B[i*N + j] = A[j*N + i];
```

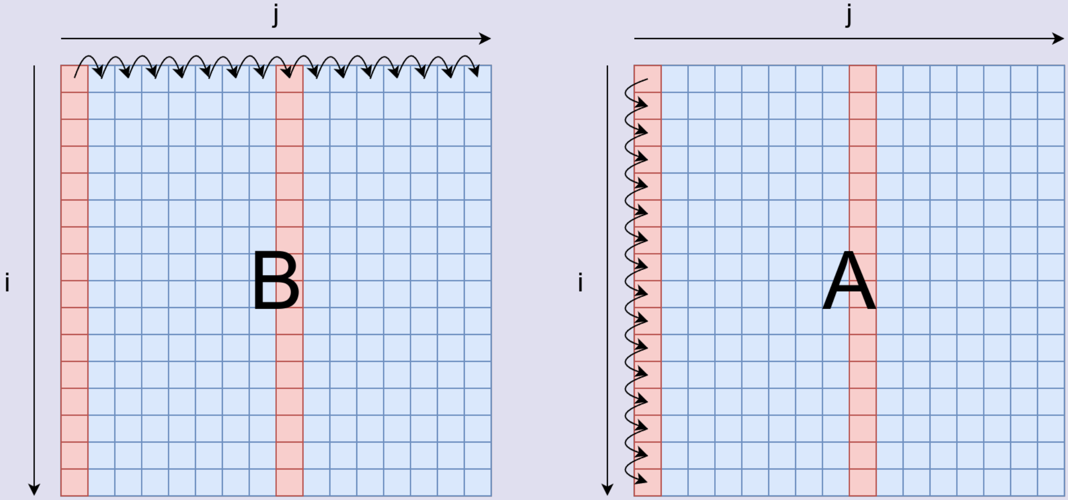
- Contiguous access to `B`, stride-N access to `A`
- If both matrices fit in cache, a reasonable model could be

$$T_{\mathrm{cache}} = N^2(t_{\mathrm{read}} + t_{\mathrm{write}})$$

- Reads of `A` load a full cache line, but use only 8 bytes:

$$T_{\mathrm{mem}} = N^2(8t_{\mathrm{read}} + t_{\mathrm{write}})$$

A picture



Cache locality

- Matrices are stored by rows
- Cache line size is L
- `A` has strided access
- We need LN/8 cache to get reuse
- We can *reorder* the iterations to preserve spatial locality

Idea

- Break loop iteration space into blocks
    1. *strip mining*
    2. *loop reordering*

Strip mining

Before

```
for ( int i = 0; i < N; i++ )
  A[i] = f(i);
```

After

```
for ( int ii = 0; ii < N; ii += stride)
  for ( int i = ii; i < min(N, ii + stride); i++)
    A[i] = f(i);
```

Mostly useful for nested loops

Strip mining nested loops

Before

```
for (int i = 0; i < N; i++)
  for (int j = 0; j < N; j++)
    B[i*N + j] = A[j*N + i];
```

After

```
for (int ii = 0; ii < N; ii += stridei)
  for (int i = ii; i < min(N, ii+stridei); i++)
    for (int jj = 0; jj < N; jj += stridej)
      for (int j = jj; j < min(N, jj+stridej); j++)
        B[i*N + j] = A[j*N + i];
```

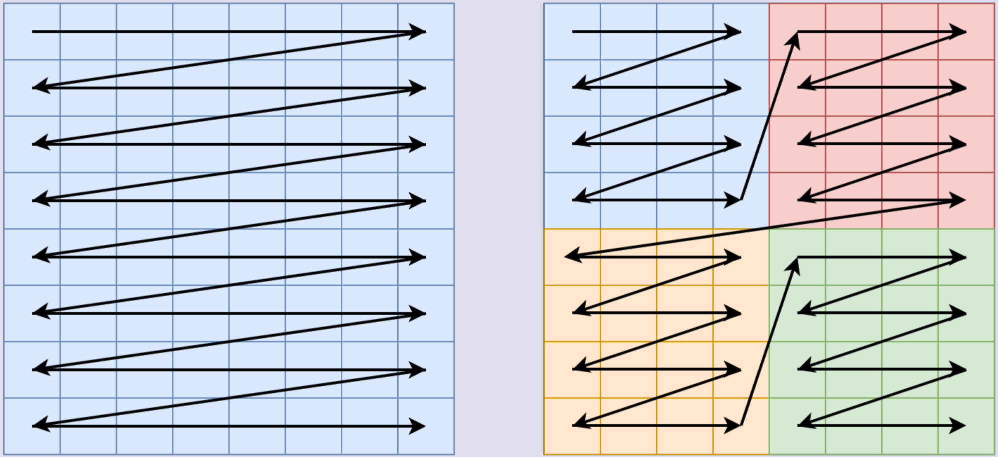
Reordering loops

After permuting `i` and `jj` loops

```
for (int ii = 0; ii < N; ii += stridei)
  for (int jj = 0; jj < N; jj += stridej)
    for (int i = ii; i < min(N, ii+stridei); i++)
      for (int j = jj; j < min(N, jj+stridej); j++)
        b[i*N + j] = a[j*N + i];
```

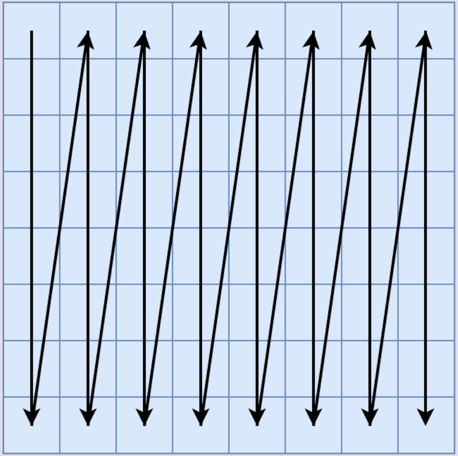
- Two free parameters `stridei` and `stridej`
- Need to choose according cache hierarchy
- Ideally block for L1, L2, L3
- The extra logic adds some overhead

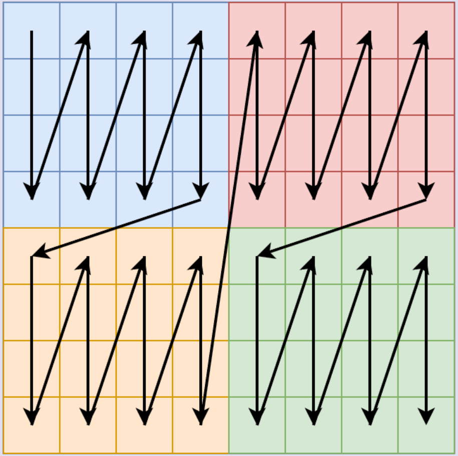
Iteration over B



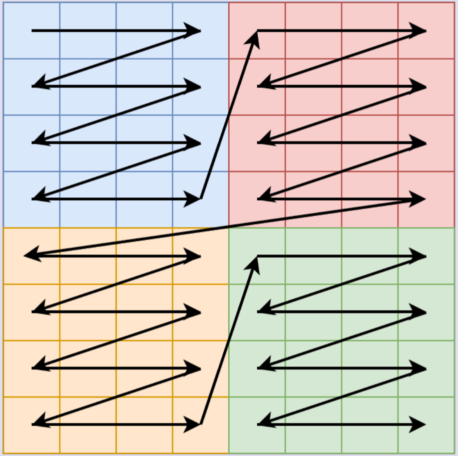
Before After

Iteration over A

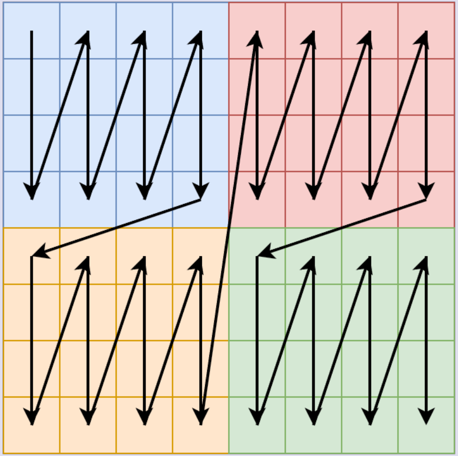

Before


After

Comparison



Tiled B



Tiled A

Exercise 7: Tiled matrix transpose

1. Split into small groups
2. Download the two versions of the code
3. Measure bandwidth as matrix size changes
4. Try different tile sizes
5. Ask questions!