



# Geometría Analítica

## Rectas y planos

Dr. Juan Luis Palacios Soto

palacios.s.j.l@gmail.com



scidata  
matemáticas para  
la ciencia de datos

## Definición

La ecuación general de una recta en el plano se define como:

$$ax + by + c = 0,$$

donde  $x, y$  son las variables y  $a, b, c \in \mathbb{R}$  son constantes no todas cero.

Wolframalpha

Dos puntos:

line `[//math:(2,1)//] [//math:(3,4)//]`

Doble intersección:

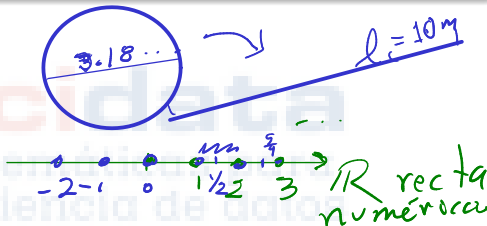
line, x-intercept=2, y-intercept=1

Pendiente, intersección

line, slope = 1, y-intercept=5

Punto, pendiente

?



$\leftarrow$  ordenada

$\rightarrow$  wolfram alpha?

?

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \dots\}$$

$$\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0 \right\}$$

$$\mathbb{Q}^c = \mathbb{Q}' = \mathbb{I} = \{\sqrt{2}, e, \sqrt{3}, \pi, \dots\}$$

$$\pi = \frac{l_c}{d} = \frac{10}{d}$$

$$\sqrt{2} \neq \frac{p}{q}$$

$$-2 = \left( \frac{-2}{1} \right)$$

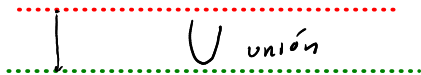
1 2 3 4 ...  $\mathbb{N}$

... -2 -1 0 1 2 ...

$\mathbb{Z}$  zahlen

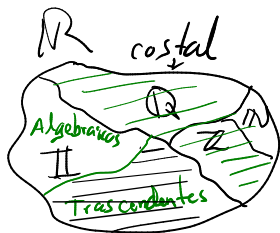
$\mathbb{Q}$  Racionales

$\mathbb{I}$  Irracionales



$$\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$$

$$\mathbb{Q} \cap \mathbb{I} = \{ \} = \emptyset$$



$$\text{Alg} \rightarrow \sqrt{2}, \sqrt{3}, \dots$$

$$\text{Trasc} \rightarrow e, \pi, \dots, e^{\pi}, \pi e, e^{\pi} + \pi, \pi + e^{\pi}$$

2. 71434343...

rational.

8.

$$\pi \approx 3.141592\dots$$

irrational.

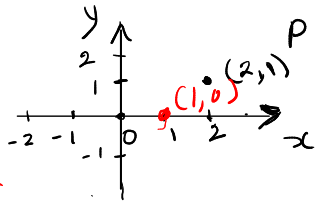
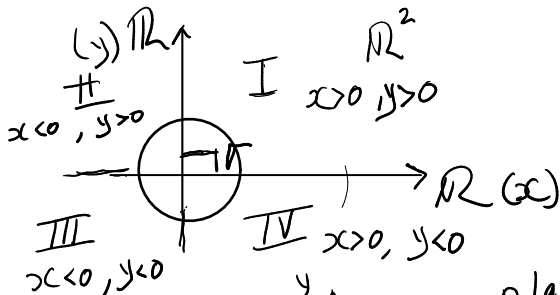
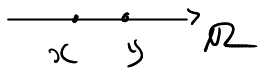
O . O O O O O O O O !

rational

0 . 06 6 6666 06 06 06 6666

[illegible]

$\mathbb{R}^2 = \{ (x, y) : x, y \in \mathbb{R} \}$   
 tal que  $\uparrow$  pertencem



plano cartesiano

$(2, 1)$   
 $(x, 1)$

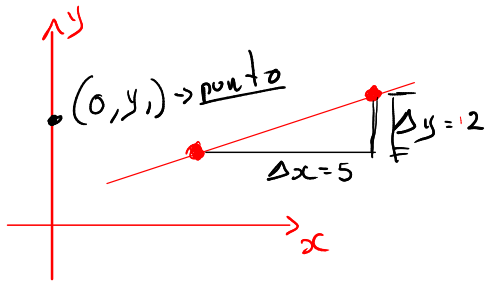
$(-3, 2) \rightarrow \text{II}$

$(1, 0)$

$(-1, -4) \rightarrow \text{II}$

$$ax + by + c = 0$$

① Dos puntos.



$$m = \frac{\Delta y}{\Delta x} \rightarrow \text{pendiente de la recta.}$$

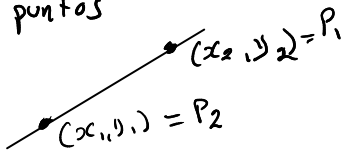
②  $m$  y un punto.

③ Ordenada al origen  $y = y_1$   
y  $m$ .

④  $x = a$ ,  $y = b$   
 $(a, 0)$ ,  $(0, b)$

intersección con  
ejes

① 2 puntos

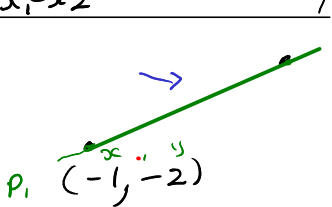


A line segment is drawn through two points labeled  $(x_1, y_1) = P_2$  and  $(x_2, y_2) = P_1$ . To the right of the line, the slope formula is given as  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$y = m(x - x_1) + y_1$  Fórmula de la recta dado 2 puntos

$$y = \frac{y_1 - y_2}{x_1 - x_2} (x - x_1) + y_1$$



A line is drawn through two points labeled  $P_1 (-1, -2)$  and  $P_2 (5, 3)$ . A blue arrow points upwards along the line, indicating a positive slope. The coordinates are written as  $(x, y)$  for  $P_2$ .

$(5, 3) P_2 \rightarrow \text{gral.}$

$$ax + by + c = 0$$

①  ~~$5x - 6y + 3 = 0$~~  X

②  $-5x + 6y + 7 = 0$  ✓

$$m = \frac{-2 - 3}{-1 - 5} = \frac{-5}{-6} = \frac{5}{6} > 0$$



$$m = \frac{3 - (-2)}{5 - (-1)} = \frac{3 + 2}{5 + 1} = \frac{5}{6} > 0$$

$$y = \frac{5}{6}(x - (-1)) + (-2) = \frac{5}{6}(x + 1) - 2$$

$$\rightarrow (6) \quad y = \frac{5}{6}(x + 1) - 2$$

$$\Rightarrow 6y = \frac{5 \cdot 6}{6}(x + 1) - 2(6)$$

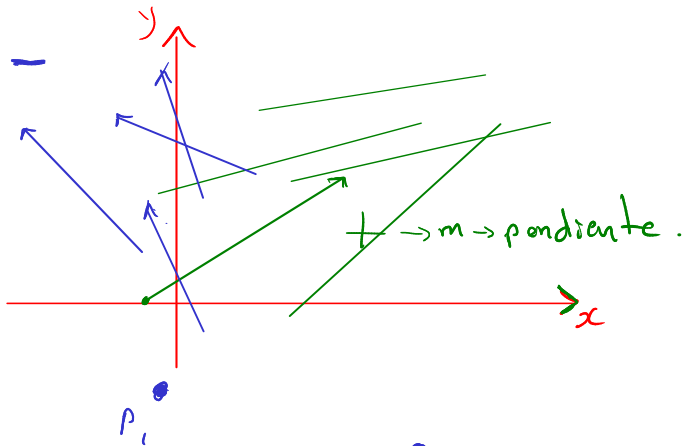
$$\Rightarrow 6y = 5(x + 1) - 12$$

$$6y = 5x + \underline{5 - 12}$$

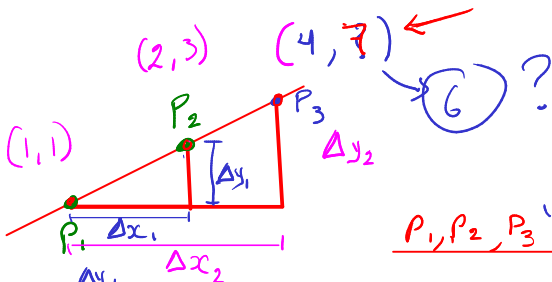
$$6y - 5x = -7$$

$$\underline{-5x + 6y + 7 = 0} \quad \checkmark$$

pend



$\rightarrow \underbrace{(1, -3)}_{IV}, \underbrace{(-2, 1)}_{II} \rightarrow m < 0$



$P_1, P_2, P_3$  "colineales"

$$P_1 \rightarrow P_2 \quad m_1 = \frac{\Delta y_1}{\Delta x_1}$$

$$P_1 \rightarrow P_3 \quad m_2 = \frac{\Delta y_2}{\Delta x_2}$$

$$¿m_1 = m_2?$$

Retos

$$m_1 = \frac{3-1}{2-1} = \frac{2}{1} = 2.$$

$$m_2 = \frac{6-1}{4-1} = \frac{5}{3} ?$$

## Definición

*La ecuación general de un plano en el espacio se define como:*

$$ax + by + cz + d = 0,$$

*donde  $x, y, z$  son las variables y  $a, b, c, d \in \mathbb{R}$  son constantes no todas cero.*



scidata  
matemáticas para  
la ciencia de datos

## Definición

*La ecuación general de un hiperplano en un espacio de  $n$  dimensiones se define como:*

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_{n-1}x_{n-1} + a_nx_n + b = 0,$$

*donde  $x_1, x_2, \dots, x_n$  son las variables y  $a_1, a_2, \dots, a_n, b \in \mathbb{R}$  son constantes no todas cero.*



scidata  
matemáticas para  
la ciencia de datos