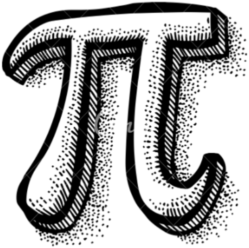

Geometría Analítica

Rectas y planos

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Definición

La ecuación general de una recta en el plano se define como:

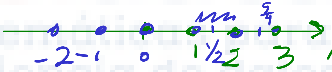
$$ax + by + c = 0,$$

donde x, y son las variables y $a, b, c \in \mathbb{R}$ son constantes no todas cero.

Wolframalpha

Dos puntos:
line [//math:(2,1)//] [//math:(3,4)//]

Doble intersección:
line, x-intercept=2, y-intercept=1



$\mathbb{R}$ recta numérica

Pendiente, intersección
line, slope = 1, y-intercept=5
ordenada

Punto, pendiente → wolframalpha?
?

$\mathbb{N} = \{1, 2, 3, \dots\}$

$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \dots\}$

$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z},\ q \neq 0\}$

$\pi = \frac{l_c}{D} = \frac{10}{}$

$\mathbb{Q}^c = \mathbb{Q}' = \mathbb{I} = \{\sqrt{2}, e, \sqrt{3}, \pi, \dots\}$

$\sqrt{2} \neq \frac{p}{q}$

$-2 = \frac{-2}{1}$

$1 \quad 2 \quad 3 \quad 4 \quad \ldots \quad \mathbb{N}$.

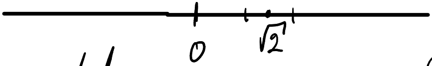
$\ldots \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad \ldots$

$\mathbb{Z}$ zahlen

$\mathbb{Q}$ Racionales

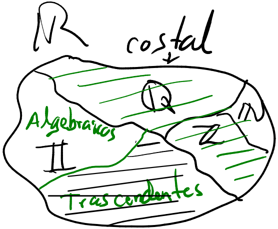
$\mathbb{I}$ Irracionales

$\cup$ unión



$\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$

$\mathbb{Q} \cap \mathbb{I} = \{\ \} = \emptyset$



Alg $\to \sqrt{2}, \sqrt{3}, \ldots$

Trasc $\to e, \pi, \to$ ¿e^{π}, π^{e}?

$e^{\pi}, \pi + e$ ¿$\pi + e$?

$2.714343\underline{43}\ldots$ racional.

$8.$

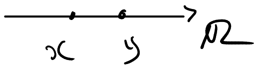
$\pi \cong 3.141592\ldots$ irracional.

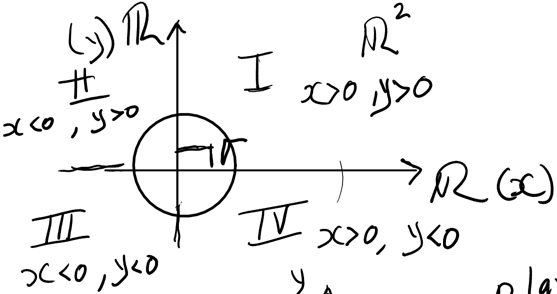
0.0000000001 racional

0.00000000000000000001

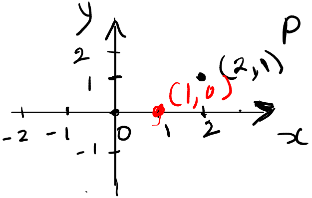
$1 \times 10^{-30} = 0.00000000000000000000\ldots01$

$$\mathbb{R}^2 = \{ \underbrace{(x,y)} \underset{\text{tal que}}{:} x, y \underset{\text{pertenecen}}{\in} \mathbb{R} \}$$





plano cartesiano



$(2,1)$

$(x,1)$

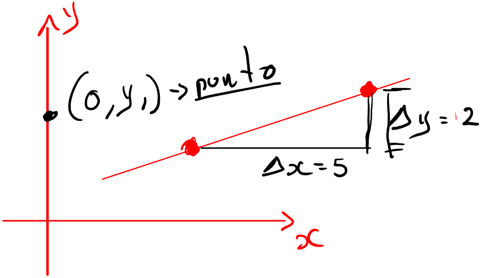
$(-3,2) \to \mathrm{II}$

$(1,0)$

$(-1,-4) \to \mathrm{III}$

$ax + by + c = 0$

① Dos puntos.



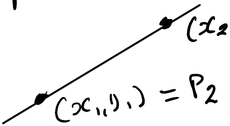
$m = \frac{\Delta y}{\Delta x}$ -> pendiente de la recta.

② m y un punto.

③ Ordenada al origen $y = y_1$
y m.

④ $x = a$, $y = b$ intersección con ejes
$(a, 0)$, $(0, b)$

① 2 puntos



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

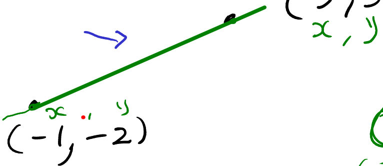
$$y = m(x - x_1) + y_1$$

Fórmula de la recta dado 2 puntos

$$\boxed{y = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1) + y_1}$$

gral.

$$\boxed{ax + by + c = 0}$$



① $\cancel{5x - 6y + 3 = 0}$ ✗

② $-5x + 6y + 7 = 0$ ✓

$$m = \frac{-2 - 3}{-1 - 5} = \frac{-5}{-6} = \frac{5}{6} > 0$$

$$m = \frac{3-(-2)}{5-(-1)} = \frac{3+2}{5+1} = \frac{5}{6} > 0$$

$$y = \frac{5}{6}(x-(-1)) + (-2) = \frac{5}{6}(x+1) - 2$$

$$\to (6) \quad y = \frac{5}{6}(x+1) - 2$$

$$\Rightarrow \quad 6y = \frac{5 \cdot \cancel{6}}{\cancel{6}}(x+1) - 2(6)$$

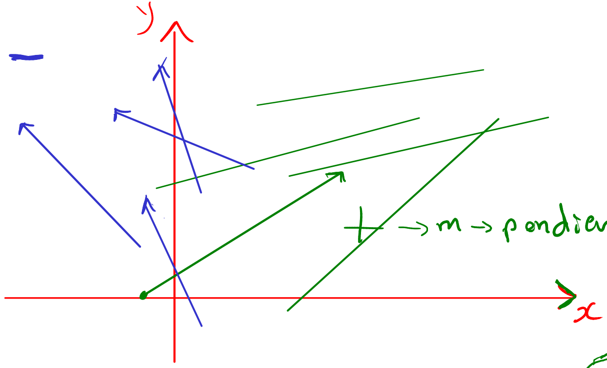
$$\Rightarrow \quad 6y = 5(x+1) - 12$$

$$6y = 5x + \underline{5-12}$$

$$6y - 5x = -7$$

$$\underline{-5x + 6y + 7 = 0}$$

pend $-$

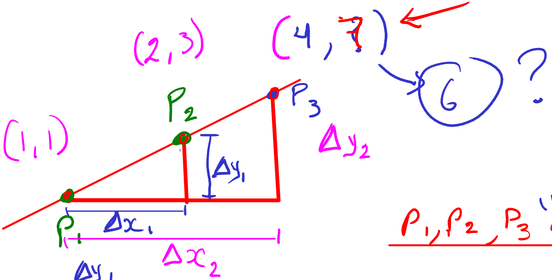


$+ \to m \to$ pendiente.

P_1

P_2

$\longrightarrow \underbrace{(1,-3)}_{IV}, \underbrace{(-2,1)}_{II} \longrightarrow m<0$



P_1, P_2, P_3 "colineales"

$P_1 \to P_2 \quad m_1 = \frac{\Delta y_1}{\Delta x_1}$

$P_1 \to P_3 \quad m_2 = \frac{\Delta y_2}{\Delta x_2}$

¿$m_1 = m_2$?

Retos

$$m_1 = \frac{3-1}{2-1} = \frac{2}{1} = 2.$$

$$m_2 = \frac{6-3}{4-2} = \frac{3}{2} \ ?$$

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Ej. ① Determinar E.g. $ax+by+c=0$ de la recta dado $(-3,-1)$, $(5,-1)$.

$y=-1$

② $m=-2$, pasa por $\overset{x_0,\ y_0}{(-2,-1)}$

$y+2x+5=0 \Rightarrow y=m(x-x_0)+y_0$

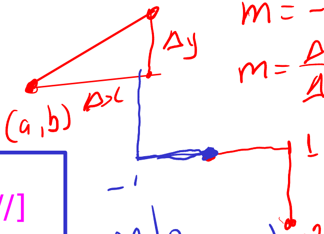
③ Intersección con $\underbrace{x=3 \text{ y } y=2}_{(3,0) \qquad (0,2)}$.

$2x+3y=6$.

$m=-2=\frac{-2}{1}=\frac{2}{-1}$

$m=\frac{\Delta y}{\Delta x}$

5 min

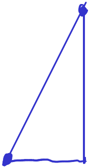


slope -2 [//math:(-2,-1)//]

punto, pendiente.

line [//math:(1,2)//] [//math:(2,5)//]

$(1,2)$, $m = 3 = \frac{3}{1} = \frac{6}{2} = \frac{\Delta y}{\Delta x}$



$(2,5)$

$(1,2) \to (1+1, 2+3)$

Definición

La ecuación general de un plano en el espacio se define como:

$(ax+by+c=0)$

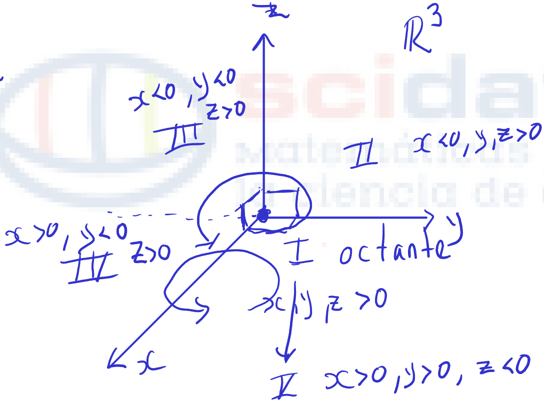
$$ax + by + cz + d = 0,$$

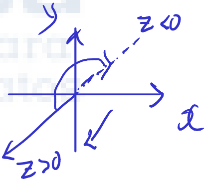
$x+0=x$

$1\cdot x=x$

donde x, y, z *son las variables y* $a, b, c, d \in \mathbb{R}$ *son constantes no todas cero.*

$x'=x$





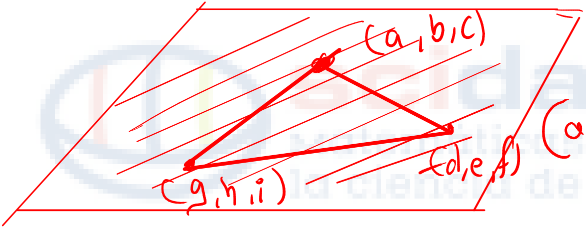
?

Definición

La ecuación general de un plano en el espacio se define como:

$$ax + by + cz + d = 0,$$

donde x, y, z *son las variables y* $a, b, c, d \in \mathbb{R}$ *son constantes no todas cero.*



$(a,b,c,d) \in \mathbb{R}^4$

$(3,7,-1,0,4,8,9) \in \mathbb{R}^7$

Definición

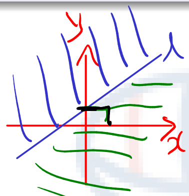
La ecuación general de un plano en el espacio se define como:

$$ax + by + cz + d = 0,$$

$ax+by+z+d > 0$

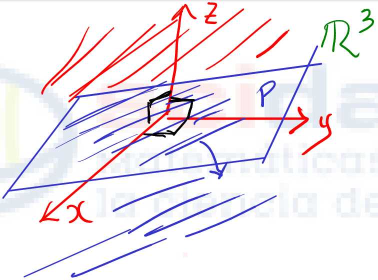
< 0

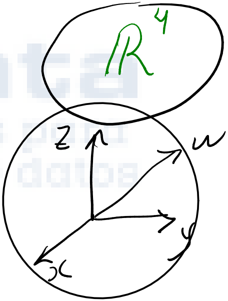
donde x, y, z *son las variables y* $a, b, c, d \in \mathbb{R}$ *son constantes no todas cero.*



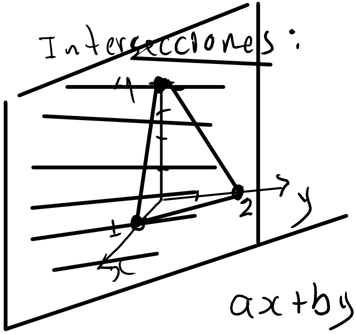
$ax+by+d>0$

$ax+by+d<0$





Intersecciones: $\underbrace{x = 1 \;,\; y = 2,\; z = 4}$



Ecuación del plano

$\underbrace{(1,0,0),\; (0,2,0)\; (0,0,4)}$

$ax + by + cz + d = 0$

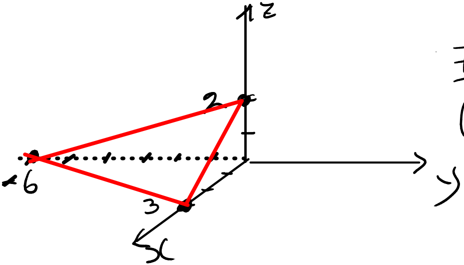
3 min

Ej. $2x - y + 3z = 6 \; . \rightarrow$

① $x = y = 0 \Rightarrow 3z = 6 \Rightarrow z = \frac{6}{3} = 2 \Rightarrow (0,0,2)$

② $x = z = 0 \Rightarrow -y = 6 \Rightarrow y = -6 \Rightarrow (0,-6,0)$

③ $y = z = 0 \Rightarrow 2x = 6 \Rightarrow x = 3 \Rightarrow (3,0,0)$



Ecuación del plano

$(1,0,0),\ (0,2,0)\ \ (0,0,4)$

$$\underbrace{ax + by + cz}\ = d$$

$\cancel{x + 2y + 4z =}$

① $x = y = 0$ $\quad z = 4 \rightarrow$ ④, 8, 12, ...

② $x = z = 0 \Rightarrow y = 2 \rightarrow$ 2, ④, 6

③ $y = z = 0 \Rightarrow x = 1$

$$x + \frac{1}{2}y + \frac{1}{4}z = 1$$

$$\boxed{4x + 2y + z = 4}$$

$$ax + by + cz = ?$$

③ $ax = 1$

② $by = 2$

① $cz = 4$

Definición

La ecuación general de un híperplano en un espacio de n dimensiones se define como:

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_{n-1}x_{n-1} + a_nx_n + b = 0,$$

donde $x_1, x_2, ..., x_n$ son las variables y $a_1, a_2, ..., a_n, b \in \mathbb{R}$ son constantes no todas cero.

$\mathbb{R}^5$

video X

$(N_1, N_2, N_3, N_4, N_5)$

visual, likes, susc. compartido