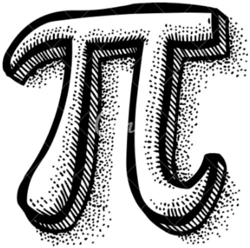

Geometría Analítica

Rectas y planos

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Definición

La ecuación general de una recta en el plano se define como:

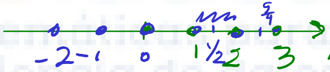
$$ax + by + c = 0,$$

donde x, y son las variables y $a, b, c \in \mathbb{R}$ son constantes no todas cero.

Wolframalpha

Dos puntos:
line [//math:(2,1)//] [//math:(3,4)//]

Doble intersección:
line, x-intercept=2, y-intercept=1



$\mathbb{R}$ recta numérica

Pendiente, intersección
line, slope = 1, y-intercept=5
ordenada

$\mathbb{N} = \{1, 2, 3, \ldots\}$

Punto, pendiente →wolframalpha?
?

$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \ldots\}$

$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z},\ q \neq 0\}$

$\pi = \frac{l_c}{D} = \frac{10}{\ }$

$\mathbb{Q}^c = \mathbb{Q}' = \mathbb{I} = \{\sqrt{2}, e, \sqrt{3}, \pi, \ldots\}$

$\sqrt{2} \neq \frac{p}{q}$

$-2 = \frac{-2}{1}$

1 2 3 4 . . . . $\mathbb{N}$.

. . . -2 -1 0 1 2 . . . .

$\mathbb{Z}$ zahlen

$\mathbb{Q}$ Racionales

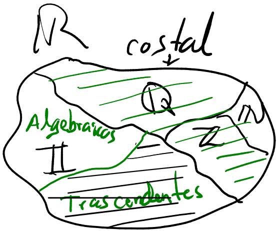
$\mathbb{I}$ Irracionales

$\cup$ unión

0 $\sqrt{2}$

$\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$

$\mathbb{Q} \cap \mathbb{I} = \{\ \} = \emptyset$



Alg $\to \sqrt{2}, \sqrt{3}, \ldots$

Trasc $\to e, \pi, \to$ ¿e^{π}, π^{e}?

$e^{\pi}, \pi + e$ ¿$\pi + e$?

$2.714343\underline{43}\ldots$ racional.

$8.$

$\pi \cong 3.141592\ldots$ irracional.

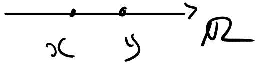
0.0000000001 racional

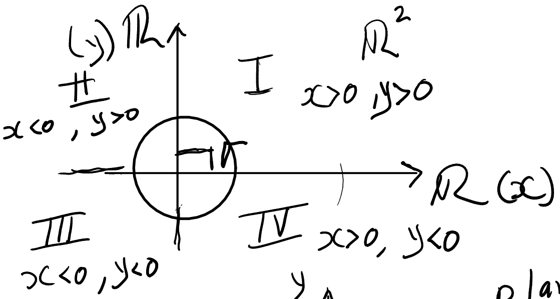
0.00000000000000000001

$1 \times 10^{-30} = 0.00000000000000000000000\ldots01$

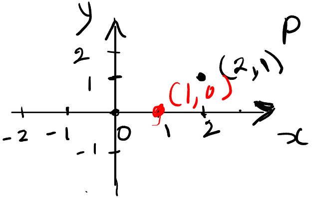
$\mathbb{R}^2 = \{ \underbrace{(x,y)}_{} \; : \; x, y \in \mathbb{R} \}$

tal que   pertenecen





plano cartesiano



$(2,1)$

$(x,1)$

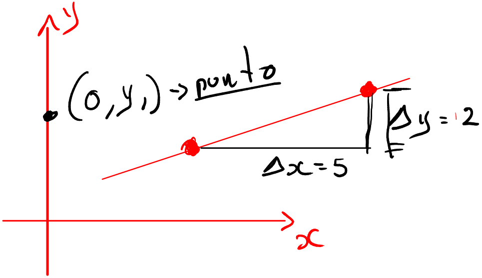
$(-3,2) \to \mathrm{II}$

$(1,0)$

$(-1,-4) \to \mathrm{II}$

$ax + by + c = 0$

① Dos puntos.



$m = \frac{\Delta y}{\Delta x}$ -> pendiente de la recta.

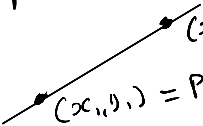
② m y un punto.

③ Ordenada al origen $y = y_1$ y m.

④ $x = a$, $y = b$ intersección con ejes

$(a, 0)$, $(0, b)$

(1) 2 puntos



$(x_2, y_2) = P_1$

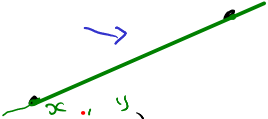
$(x_1, y_1) = P_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$y = m(x - x_1) + y_1$$

Fórmula de la recta dado 2 puntos

$$\boxed{y = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1) + y_1}$$



$(5, 3)$ P_2

x, y

$\rightarrow$ gral.

$$\boxed{ax + by + c = 0}$$

P_1 $(-1, -2)$

x, y

(1) ~~$5x - 6y + 3 = 0$~~ ✗

(2) $-5x + 6y + 7 = 0$ ✓

$$m = \frac{-2 - 3}{-1 - 5} = \frac{-5}{-6} = \frac{5}{6} > 0$$

$$m = \frac{3-(-2)}{5-(-1)} = \frac{3+2}{5+1} = \frac{5}{6} > 0$$

$$y = \frac{5}{6}(x-(-1)) + (-2) = \frac{5}{6}(x+1) - 2$$

$$\to (6) \quad y = \frac{5}{6}(x+1) - 2$$

$$\Rightarrow \quad 6y = \frac{5 \cdot \cancel{6}}{\cancel{6}}(x+1) - 2(6)$$

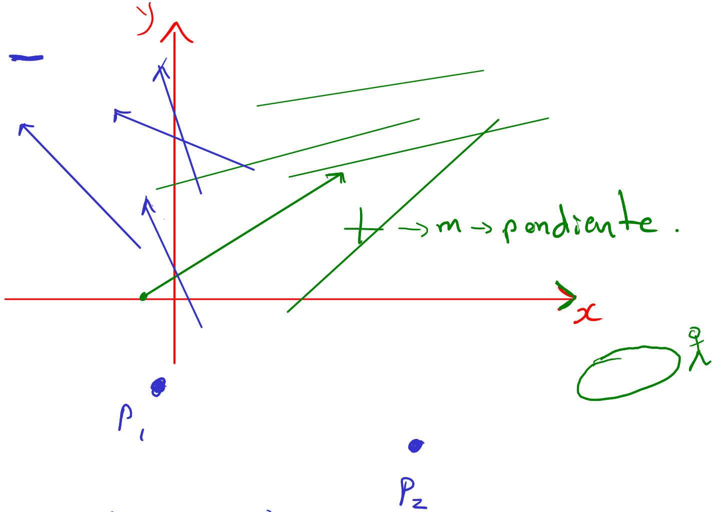
$$\Rightarrow \quad 6y = 5(x+1) - 12$$

$$6y = 5x + \underbrace{5 - 12}$$

$$6y - 5x = -7$$

$$\underline{-5x + 6y + 7 = 0}$$

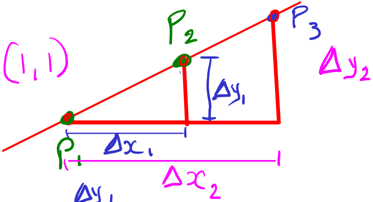
pend —



$\rightarrow \underbrace{(1,-3)}_{IV}, \underbrace{(-2,1)}_{II} \rightarrow m<0$

(2, 3) (4, 7)

6 ?



P_1, P_2, P_3 "colineales"

$P_1 \to P_2 \quad m_1 = \dfrac{\Delta y_1}{\Delta x_1}$

$P_1 \to P_3 \quad m_2 = \dfrac{\Delta y_2}{\Delta x_2}$

¿$m_1 = m_2$?

Retos

$$m_1 = \frac{3-1}{2-1} = \frac{2}{1} = 2.$$

$$m_2 = \frac{6-3}{4-2} = \frac{3}{2} \quad ?$$

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Ej. ① Determinar E. g. $ax+by+c=0$
de la recta dado $(-3,-1)$, $(5,-1)$

$y=-1$

② $m=-2$, pasa por $(\overset{x_0}{-2}, \overset{y_0}{-1})$

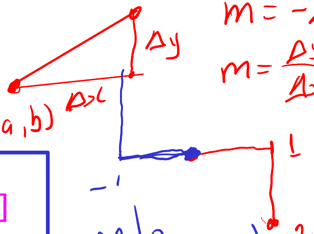
$y+2x+5=0 \Rightarrow y=m(x-x_0)+y_0$

③ Intersección con $\underbrace{x=3 \text{ y } y=2}$.

$2x+3y=6$. $(3,0)$ $(0,2)$

$m=-2=\frac{-2}{1}=\frac{2}{-1}$

$m=\frac{\Delta y}{\Delta x}$



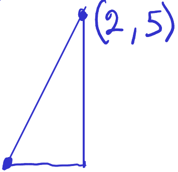
5 min

slope -2 [//math:(-2,-1)//]

punto, pendiente.

line [//math:(1,2)//] [//math:(2,5)//]

$(1,2)$, $m = 3 = \frac{3}{1} = \frac{6}{2} = \frac{\Delta y}{\Delta x}$



$(1,2) \rightarrow (1+1, 2+3)$

Definición

La ecuación general de un plano en el espacio se define como: $(ax+by+c=0)$

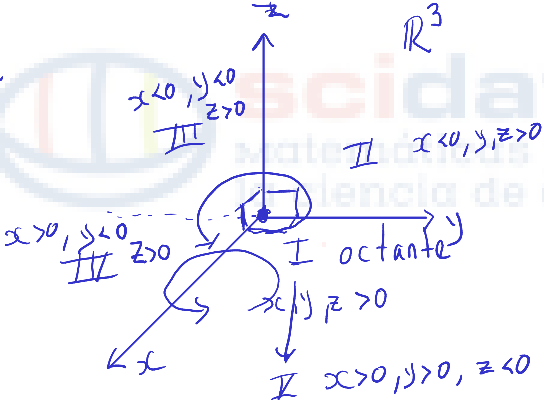
$$ax + by + cz + d = 0,$$

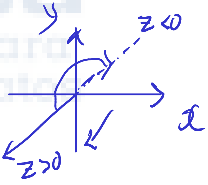
donde x, y, z *son las variables y* $a, b, c, d \in \mathbb{R}$ *son constantes no todas cero.*

$x+0=x$

$1 \cdot x = x$

$x' = x$





?

Definición

La ecuación general de un plano en el espacio se define como:

$$ax + by + cz + d = 0,$$

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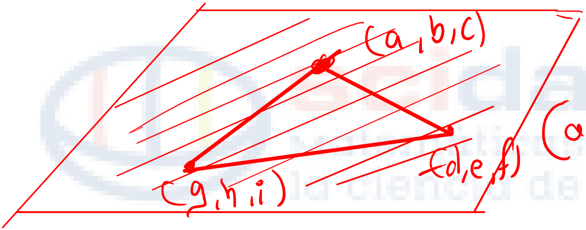


$(a,b,c,d) \in \mathbb{R}^4$

$(3,7,-1,0,4,8,9) \in \mathbb{R}^7$

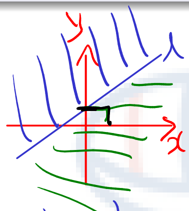
Definición

La ecuación general de un plano en el espacio se define como:

$$ax + by + cz + d = 0,$$

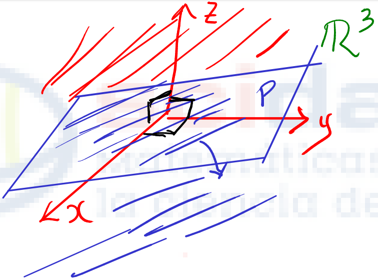
$ax+by+cz+d > 0$
< 0

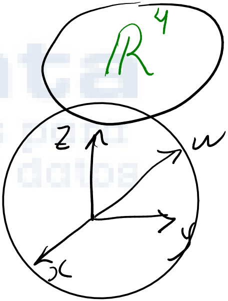
donde x, y, z *son las variables y* $a, b, c, d \in \mathbb{R}$ *son constantes no todas cero.*



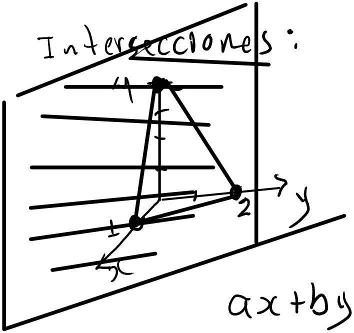
$ax+by+d>0$

$ax+by+d<0$





Intersecciones: $x=1$, $y=2$, $z=4$



Ecuación del plano

$(1,0,0)$, $(0,2,0)$ $(0,0,4)$

$ax+by+cz+d=0$

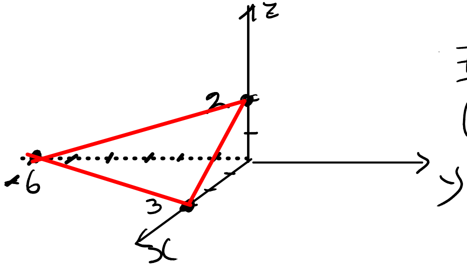
3 min

Ej. $2x-y+3z=6$ $\rightarrow$

(1) $x=y=0 \Rightarrow 3z=6 \Rightarrow z=\frac{6}{3}=2 \Rightarrow (0,0,2)$

(2) $x=z=0 \Rightarrow -y=6 \Rightarrow y=-6 \Rightarrow (0,-6,0)$

(3) $y=z=0 \Rightarrow 2x=6 \Rightarrow x=3 \Rightarrow (3,0,0)$



Ecuación del plano

$(1,0,0),\ (0,2,0)\ \ (0,0,4)$

$\underbrace{ax+by+cz}=d$

$\cancel{x+2y+4z=}$

$Z=4$ → ④, 8, 12 . . .

① $x=y=0$

② $x=z=0 \Rightarrow y=2$ → 2, ④, 6

③ $y=z=0 \Rightarrow x=1$

$x+\frac{1}{2}y+\frac{1}{4}z=1$

$\boxed{4x+2y+z=4}$

$ax+by+cz=\,?$

③ $ax=1$

② $by=2$

① $cz=4$

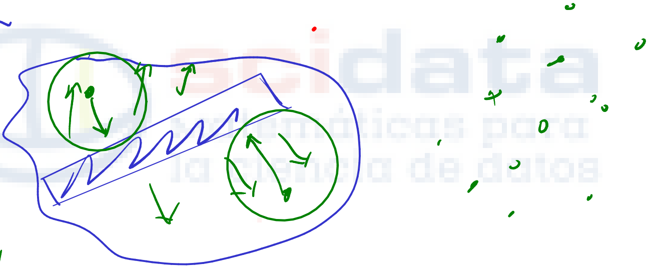
Definición

La ecuación general de un híperplano en un espacio de n dimensiones se define como:

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_{n-1}x_{n-1} + a_nx_n + b = 0,$$

donde $x_1, x_2, \ldots, x_n$ son las variables y $a_1, a_2, \ldots, a_n, b \in \mathbb{R}$ son constantes no todas cero.

$\mathbb{R}^5$



Video X $\to$

$(N_1, N_2, N_3, N_4, N_5)$

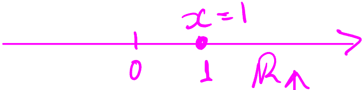
Visual, likes, susc. compartido

plane Cartesian equation 3x-2y+z=3

plane through three points

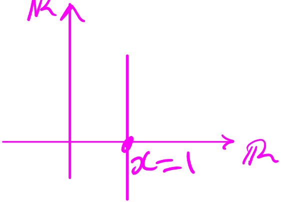
Wolfram alpha.

¿x=1? En $\mathbb{R}^n$ $n \geq 1$ $n=1$ $\mathbb{R}^1 = \mathbb{R}$

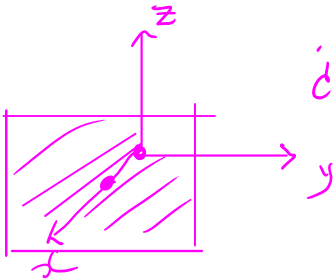


$n=2$ $\mathbb{R}^2$

$x=1$



$n = 3 \quad \mathbb{R}^3 \quad x = 1\ ?$



¿ $x = 1$ en $\mathbb{R}^4$?