scidata

.

$$\int_{\mathbb{R}} f\,d\mu$$

Integración por Partes

Dr. Juan Luis Palacios Soto
palacios.s.j.l@gmail.com

$$\frac{d}{dx}\left(f(x)g(x)\right) = \frac{d}{dx}(f(x))\cdot g(x) + f(x)\cdot\frac{d}{dx}(g(x))$$

$$\int (f\cdot g)' = \int f'g + \int fg'$$

$$\frac{f}{g} = f\cdot g^{-1}$$

$$f\cdot g = \int f'g + \int fg'$$

Definición (Integración por partes para integral indefinida)

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx,$$

o bien si $u = f(x)$ *y* $dv = g'(x)$ dx *la fórmula queda*

$$\int udv = uv - \int vdu.$$

Definición (Integración por partes para integral definida)

$$\int_a^b f(x)g'(x)dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)dx,$$

o bien si $u = f(x)$ *y* $dv = g'(x)$ *la fórmula queda*

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu.$$

Ejemplo

$$\int x\sin(x)dx = \frac{x^2}{2}\operatorname{sen} x - \int \frac{x^2}{2}\cos(x)\,dx$$

más complicada

$\cancel{f = \operatorname{sen}(x) \qquad f' = \cos(x)\,dx}$

$\cancel{g' = x\,dx \quad \Rightarrow \quad g = \frac{x^2}{2}}$

No conviene

$f = x \quad \Rightarrow \quad f' = dx$

$g' = \operatorname{sen}(x)\,dx \qquad g = -\cos(x)$

$$\Rightarrow = -x\cos(x) - \int -\cos(x)\,dx = x\cos(x) + \int \cos(x)\,dx$$

$$\int x\operatorname{sen}(x)\,dx = -x\cos(x) + \operatorname{sen}(x) + C$$

$$= \operatorname{sen}(x) - x\cos(x) + C$$

Ejemplo

$$\int \ln(x)dx \quad = \int e^u u\,du$$

$f = \ln(x)$ $\qquad f' = \frac{dx}{x}$

$g' = dx$ $\qquad g = x$

$$\Rightarrow \int \ln(x)\,dx = x\ln(x) - \int \cancel{x}\,\frac{dx}{\cancel{x}} = x\ln(x) - \int dx$$

$$= x\ln(x) - x + C.$$

$$= x\left(\ln(x) - 1\right) + C$$

Ejemplo

$$\int xe^x\,dx = xe^x - 1\int e^x dx$$

$f = x \Rightarrow f' = dx$

$g' = e^x dx \Rightarrow g = e^x$

$= xe^x - e^x + C.$

$= (x-1)e^x + C.$

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - 2\int x e^x dx$$

$f = x^2 \Rightarrow f' = 2x\,dx$

$g' = e^x dx \Rightarrow g = e^x$

Ejemplo

Ejercicio - Tarea Foro

$$\int x^n e^x \, dx$$

Determinar fórmula gral. para $n \in \mathbb{N}$

$$\int x^n e^x \, dx = \underbrace{x^n e^x - n\int x^{n-1} e^x \, dx}_{\text{recursividad}} = \dots \; ?$$

$f = x^n \qquad f' = n x^{n-1} \, dx$

$g' = e^x \, dx \Rightarrow g = e^x$

$$\int e^x \operatorname{sen}(x)\,dx = e^x \operatorname{sen} x - \int e^x \cos(x)\,dx$$

$f = \operatorname{sen}(x)$ $\quad f' = \cos(x)\,dx$

$g' = e^x\,dx$ $\Rightarrow$ $g = e^x$

$f = e^x$ $\Rightarrow$ $f' = e^x\,dx$

$g' = \cos(x)\,dx$ $\quad g = \operatorname{sen}(x)$

$$\int e^x \operatorname{sen}(x)\,dx = e^x \operatorname{sen}(x) - \left[e^x \operatorname{sen}(x) - \int \operatorname{sen}(x)\, e^x\,dx \right]$$

$$= \cancel{e^x \operatorname{sen}(x)} - \cancel{e^x \operatorname{sen}(x)} + \int \operatorname{sen}(x)\, e^x\,dx$$

$$\int e^x \operatorname{sen}(x)\,dx = e^x \operatorname{sen}(x) - \int e^x \cos(x)\,dx$$

$f = \operatorname{sen}(x)$ $\quad f' = \cos(x)\,dx$ $\qquad$ $f = \cos(x)$ $\quad f' = -\operatorname{sen}(x)\,dx$

$g' = e^x\,dx \Rightarrow g = e^x$ $\qquad$ $g' = e^x\,dx \Rightarrow g = e^x$

$$\int e^x \operatorname{sen}(x)\,dx = e^x \operatorname{sen}(x) - \left[e^x \cos(x) - \int e^x \operatorname{sen}(x)\,dx \right]$$

$$\underbrace{\int e^x \operatorname{sen}(x)\,dx}_{a} = \underbrace{e^x \operatorname{sen}(x)}_{b} - \underbrace{e^x \cos(x)}_{c} - \underbrace{\int e^x \operatorname{sen}(x)\,dx}_{a}$$

$$a = b + c - a \Rightarrow 2a = b + c$$

$$a = \frac{b+c}{2}$$

$$2\int e^x \cos(x)\,dx = e^x\left(\operatorname{sen}(x) - \cos(x)\right)$$

$$\int e^x \cos(x)\,dx = \frac{e^x}{2}\left(\operatorname{sen}(x) - \cos(x)\right) + C.$$

$$\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$$

$$f = x^n \qquad f' = nx^{n-1}\,dx$$

$$g' = e^x\,dx \quad \Rightarrow \quad g = e^x$$

$$\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx = x^n e^x - n\left[x^{n-1}e^x - (n-1)\int x^{n-2}e^x\,dx\right]$$

$$= x^n e^x - nx^{n-1}e^x + n(n-1)\int x^{n-2}e^x\,dx$$

$$= x^n e^x - nx^{n-1}e^x + n(n-1)\left[x^{n-2}e^x - (n-2)\int x^{n-3}e^x\,dx\right]$$

$$= x^n e^x - nx^{n-1}e^x + n(n-1)x^{n-2}e^x - n(n-1)(n-2)\int x^{n-3}e^x\,dx$$

$$= x^n e^x - n x^{n-1} e^x + n(n-1) x^{n-2} e^x - \underline{n(n-1)(n-2)}\left[x^{n-3} e^x - (n-3)\int x^{n-4} e^x dx \right]$$

$$= \underbrace{x^n e^x}_{P_1} \underbrace{- n x^{n-1} e^x}_{P_2} + \underbrace{n(n-1) x^{n-2} e^x}_{P_3} - \underline{n(n-1)(n-2) x^{n-3} e^x} + \underline{n(n-1)(n-2)(n-3)\int x^{n-4} e^x dx}$$

$x^0 = 1$

$+$

$-$

$P_{10} \Rightarrow$ sig $= ?$

$P_{10} \Rightarrow$ sig negativo

$n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$

se lee n factorial

Si $n = 4$ 5 términos

Si n es potencia de x

$\Rightarrow$ $n+1$ términos en la sol.

$$= x^n e^x - n x^{n-1} e^x + n(n-1) x^{n-2} e^x + \cdots + \underline{(-1)^n n! e^x}$$

$$(-1)^2 = (-1)(-1) = 1$$

$$(-1)^3 = (-1)(-1)(-1) = -1$$

$n = 5$

$$\int x^5 e^x dx = x^5 e^x - 5x^4 e^x + 5\cdot 4 x^3 e^x - 5\cdot 4\cdot 3\, x^2 e^x + 5\cdot 4\cdot 3\cdot 2\, x e^x$$
$$- 5\cdot 4\cdot 3\cdot 2\cdot 1\, e^x$$

$$\underbrace{\int \operatorname{sen}(\ln(x))\,dx}_{\text{intentar}} \leftarrow \text{Por partes}$$

$f =$

$g' =$