$$\int_{\mathbb{R}} f \, d\mu$$

Propiedades de la Integral

Para definidas como indefinidas

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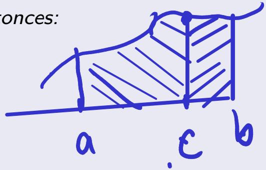
Teorema (Propiedades de la integral definida)

Supongamos que f y g son integrables sobre $I = [a, b]$, entonces:



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1. $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$
2. $\displaystyle\int_a^a f(x)dx = 0$
3. *Si* $c \in [a, b] \implies \displaystyle\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$n \geq 1$

$$\Delta x_1 = \frac{b-a}{n} = \frac{2}{n}$$

$$\Delta x_2 = \frac{a-b}{2} = -\frac{2}{n}$$

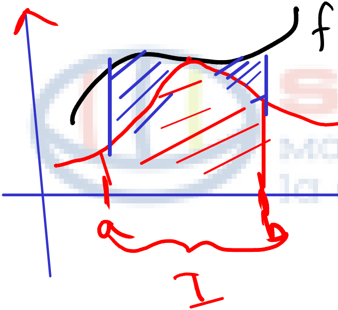
$a = 1, b = 3$

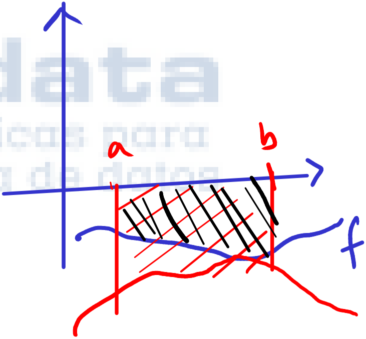
$b - a + c - c$

$b - c + c - a$

Propiedades de la integral definida

4. $\displaystyle\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
5. Si $g(x) \leq f(x)$ para toda $x \in I$, $\Longrightarrow \displaystyle\int_a^b g(x)dx \leq \int_a^b f(x)dx$

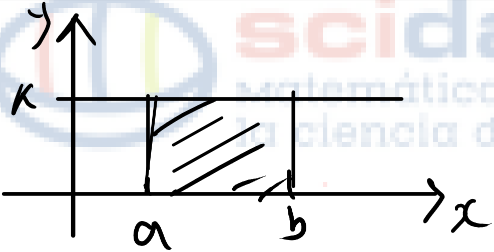


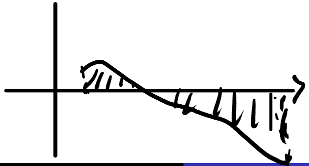


$-10 \leq -8$

Propiedades de la integral definida

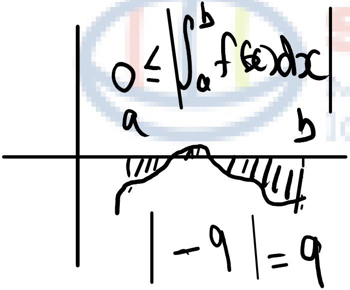
6. Si $f(x) = k$ es constante, $\implies \displaystyle\int_a^b k\,dx = k(b-a)$
7. $\displaystyle\int_a^b k f(x)\,dx = k \int_a^b f(x)\,dx$ k constante.
8. $\displaystyle\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$

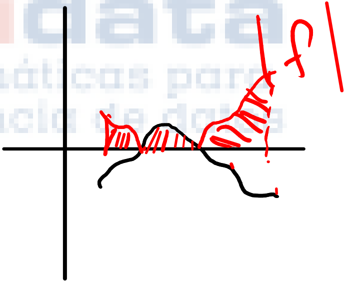




Propiedades de la integral definida

6. Si $f(x) = k$ es constante, $\implies \int_a^b k\,dx = k(b-a)$
7. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$
8. $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$

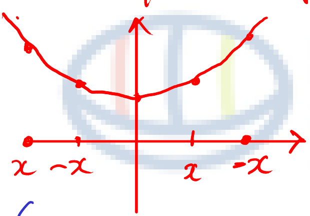


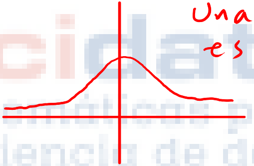


Propiedades de la integral definida

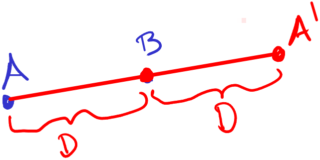
9. Si f es par, entonces $\displaystyle\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
10. Si f es impar, entonces $\displaystyle\int_{-a}^{a} f(x)dx = 0$

Decimos que f es par si $f(-x) = f(x)$





Una función f es par si su gráfica es simétrica con "y"



A es simétrico a A' con respecto a B.

Propiedades de la integral definida

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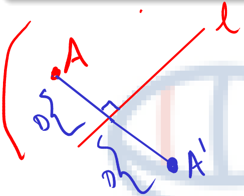
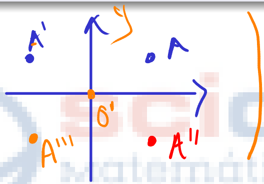
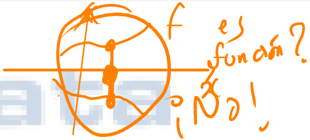
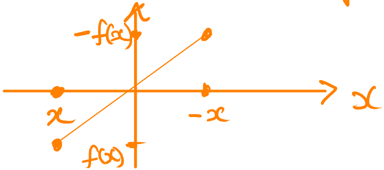
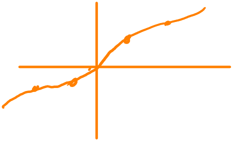






Def. Decimos que f es impar si $f(-x) = -f(x)$
simétrica f con respecto a $(0,0)$.





Propiedades de la integral definida

9. Si f es par, entonces $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
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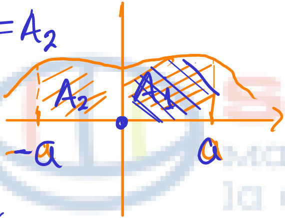
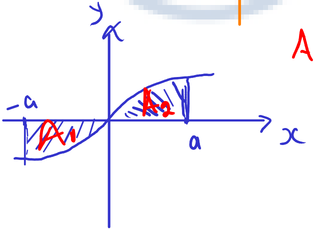
$A_1 = A_2$



$[-a, a]$ intervalo centrado en y.



$A_1 = -A_2 \therefore A_1 + A_2 = -A_2 + A_2 = 0.$

Ej: funciones pares.

$f(x) = x^n$ con n par

Funciones impares

$f(x) = \text{sen}(x)$ es impar

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$\mathrm{Sen}\left(-\frac{\pi}{4}\right) = -\mathrm{Sen}\left(\frac{\pi}{4}\right)$. impar

$\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$ par.