STA305/1004-Class 21

March 29, 2017

Today's Class

- Fractional factorial design

Exam review session

Exam Coverage material after mid term.

Wednesday, April 5th

Date: ~~Thursday~~, ~~April 6th~~

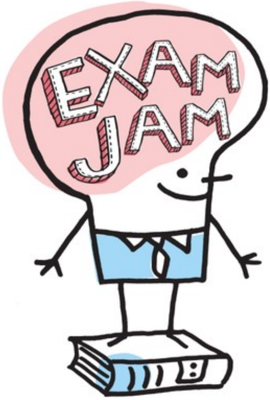
during Class time.

Time: ~~10 am - 11 am~~

In class

Location: ~~SS 2118~~

Stop
by the SS lobby to take a few photos
in the Photobooth, enjoy some free
coffee and snacks and engage in other
fun activities (lobby activities run 11-3).



Fractional factorial designs

- A 2^k full factorial requires 2^k runs.
- Full factorials are seldom used in practice for large k (k>=7).
- For economic reasons fractional factorial designs, which consist of a fraction of full factorial designs are used.

Example - Effect of five factors on ~~six~~ five. properties of film in eight runs

For a full factorial require 2^5 runs.

Five factors were studied in 8 runs (Box, Hunter, and Hunter (2005)). The factors were:

effect −5.75

Average difference in Stability between high Conc. and low Conc. is −5.75.

1. Catalyst concentration (A)
2. Amount of additive (B)
3. Amounts of three emulsifiers (C, D, E)

AB = .25

Polymer solutions were prepared and spread as a film on a microscope slide. Six different responses were recorded.

run	A	B	C	D	E	y1	y2	y3	y4	y5	y6
1	-1	-1	-1	1	-1	no	no	yes	no	slightly	yes
2	1	-1	-1	1	1	no	yes	yes	yes	slightly	yes
3	-1	1	-1	-1	1	no	no	no	yes	no	no
4	1	1	-1	-1	-1	no	yes	no	no	no	no
5	-1	-1	1	-1	1	yes	no	no	yes	no	slightly
6	1	-1	1	-1	-1	yes	yes	no	no	no	no
7	-1	1	1	1	-1	yes	no	yes	no	slightly	yes
8	1	1	1	1	1	yes	yes	yes	yes	slightly	yes

AB interaction:

The Average difference in the effect of B for A at it's high level and the effect of B for A at it's low level is 0.25.

Run	A	B	AB	
1	−	−	+	y_1
2	+	−	−	y_2
3	−	+	−	y_3
4	+	+	+	y_4

$$\left(\frac{y_1+y_4}{2}\right)-\left(\frac{y_2+y_3}{2}\right)$$

Example - Effect of five factors on six properties of film in eight runs

- The eight run design was constructed beginning with a standard table of signs for a 2^3 design in the factors A, B, C.
- The column of signs associated with the BC interaction was used to accommodate factor D, the ABC interaction column was used for factor E.
- A full factorial for the five factors A, B, C, D, E would have needed $2^5 = 32$ runs.
- Only 1/4 were run. This design is called a quarter fraction of the full 2^5 or a 2^{5-2} design (a two to the five minus two design).
- In general a 2^{k-p} design is a $\frac{1}{2^p}$ fraction of a 2^k design using 2^{k-p} runs. This design can study k factors in $\frac{1}{2^p}$ fraction of the runs.

$p = 2$, $k = 5$

Standard table of Signs for 2^3 design

Run	A	B	C	AB	AC	BC = D	ABC = E
1	−	−	−			+	−
2	+	−	−			+	+
3	−	+	−			−	+
4	+	+	−			−	−
5	−	−	+			−	+
6	+	−	+			−	−
7	−	+	+			+	−
8	+	+	+			+	+

This is a fractional factorial design.
We can study five factors in 8 runs.

Effect Aliasing and Design Resolution

- A chemist in an industrial development lab was trying to formulate a household liquid product using a new process.
- The liquid had good properties but was unstable.
- The chemist wanted to synthesize the product in hope of hitting conditions that would give stability, but without success.
- The chemist identified four important influences: A (acid concentration), B (catalyst concentration), C (temperature), D (monomer concentration).

Effect Aliasing and Design Resolution

- His 8 run fractional factorial design is shown below.

= ABC

test	A	B	C	D	y
1	-1	-1	-1	-1	20
2	1	-1	-1	1	14
3	-1	1	-1	1	17
4	1	1	-1	-1	10
5	-1	-1	1	1	19
6	1	-1	1	-1	13
7	-1	1	1	-1	14
8	1	1	1	1	10

- The signs of the ABC interaction is used to accommodate factor D. The tests were run in random order. He wanted to achieve a stability value of at least 25.

Effect Aliasing and Design Resolution

D = ABC
I = ABCD
⇒ A = BCD
B = ACD
C = ABD

AB = CD
AC = BD
AD = BC

Aliasing

```
fact.prod <- lm(y~A*B*C*D,data=tab0602)
fact.prod1 <- aov(y~A*B*C*D,data=tab0602)
round(2*fact.prod$coefficients,2)
```

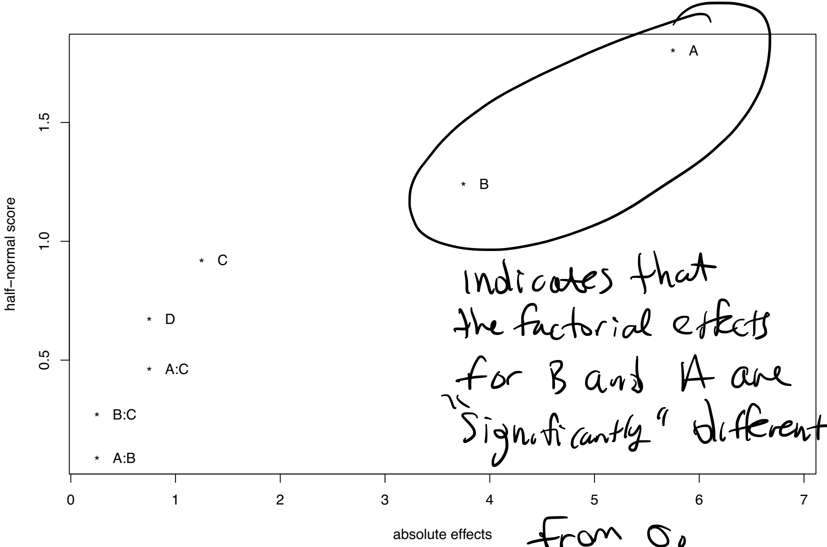
```
(Intercept)           A           B           C           D         A:B
      29.25       -5.75       -3.75       -1.25        0.75        0.25
        A:C         B:C         A:D         B:D         C:D       A:B:C
       0.75       -0.25          NA          NA          NA          NA
      A:B:D       A:C:D       B:C:D     A:B:C:D
         NA          NA          NA          NA
```

Even though the stability never reached the desired level of 25, two important factors, A and B, were identified.

The standard errors would all be NA
∵ the expt. is unreplicated.

Effect Aliasing and Design Resolution

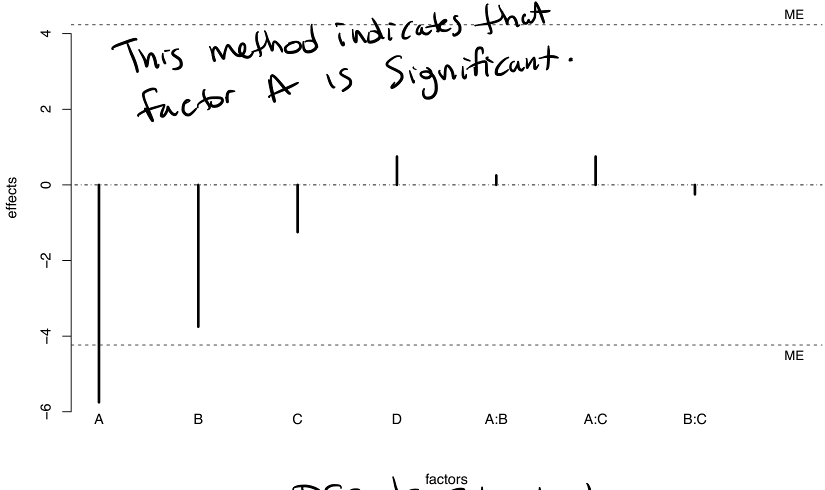
```
BsMD::DanielPlot(fact.prod,half = T)
```



Indicates that the factorial effects for B and A are "significantly" different from 0.

Effect Aliasing and Design Resolution

```
BsMD::LenthPlot(fact.prod1)
```



This method indicates that factor A is significant.

Pseudo Standard error.

```
##     alpha       PSE        ME       SME
##  0.050000  1.125000  4.234638 10.134346
```

ME - pseudo margin of error not Corrected for multiple testing.

SME - Simutaneous pseudo margin of error Corrected for multiple testing. (using Bonferroni Correction).

e.g.) $H_0: \mu_1 = \mu_2 = \mu_3$

$H_a: \mu_i \neq \mu_j$ Some $i \neq j$.

Poll Question

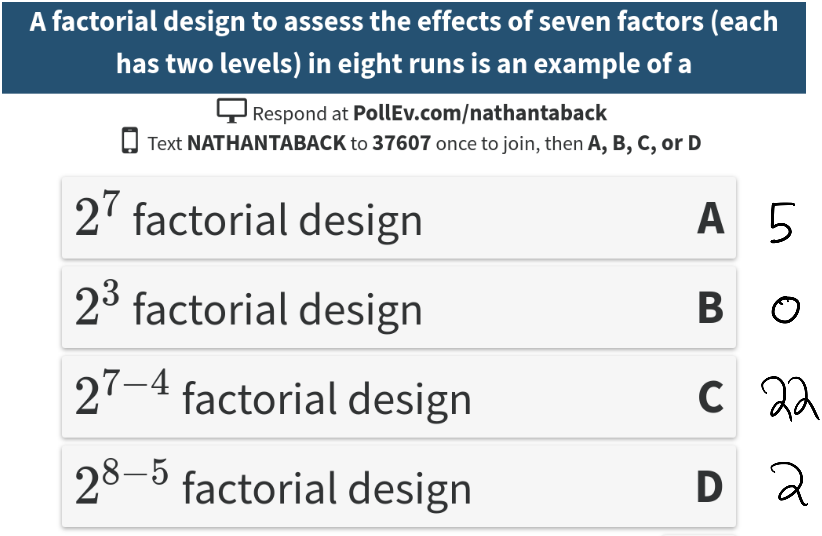


Figure 1:

A full factorial design is 2^7, but we only have 8 runs to study 7 factors $\therefore$ 2^{7-4}.

Effect Aliasing and Design Resolution

What information could have been obtained if a full 2^5 design had been used?

Factors	Number of effects	
Main	5	$\binom{5}{1}$
2-factor	10	$\binom{5}{2}$
3-factor	10	$\binom{5}{3}$
4-factor	5	$\binom{5}{4}$
5-factor	1	$\binom{5}{5}$

16 df

lower order interactions are more important than higher order.

- 31 degrees of freedom in a 32 run design.
- 16 used for estimating three factor interactions or higher.
- Is it practical to commit half the degrees of freedom to estimate such effects?
- According to effect hierarchy principle three-factor and higher not usually important.
- Thus, using full factorial wasteful. It's more economical to use a fraction of full factorial design that allows lower order effects to be estimated.

Effect Aliasing and Design Resolution

Consider a design that studies five factors in 16 run. A half fraction of a 2^5 or 2^{5-1}.

$$\overset{K}{2^4} \text{ runs} \; \overset{\shortparallel}{16}$$

Run	B	C	D	E	Q
1	-1	1	1	-1	-1
2	1	1	1	1	-1
3	-1	-1	1	1	-1
4	1	-1	1	-1	-1
5	-1	1	-1	1	-1
6	1	1	-1	-1	-1
7	-1	-1	-1	-1	-1
8	1	-1	-1	1	-1
9	-1	1	1	-1	1
10	1	1	1	1	1
11	-1	-1	1	1	1
12	1	-1	1	-1	1
13	-1	1	-1	1	1
14	1	1	-1	-1	1
15	-1	-1	-1	-1	1
16	1	-1	-1	1	1

- The factor E is assigned to the column BCD.
- The column for E is used to estimate the main effect of E and also for BCD.
- The main factor E is said to be **aliased** with the BCD interaction.

Effect Aliasing and Design Resolution

- This aliasing relation is denoted by

$$E = BCD \text{ or } I = BCDE,$$

  where I denotes the column of all $+$'s.
- This uses same mathematical definition as the confounding of a block effect with a factorial effect.
- Aliasing of the effects is a price one must pay for choosing a smaller design.
- The 2^{5-1} design has only 15 degrees of freedom for estimating factorial effects, it cannot estimate all 31 factorial effects among the factors B, C, D, E, Q.

Effect Aliasing and Design Resolution

$B = CDE$

$C = BDE$, $D = BCE$, $E = BCD$, $BC = DE$, $BD = CE$,

$BE = CD$, $Q = BCDEQ$, $BQ = CDEQ$, $CQ = BDEQ$,

$DQ = BCEQ$, $EQ = BCDQ$, $BCQ = DEQ$, $BDQ = CEQ$

$BEQ = CDQ$

- The equation $I = BCDE$ is called the **defining relation** of the 2^{5-1} design.
- The design is said to have resolution IV because the defining relation consists of the "word" BCDE, which has "length" 4.
- Multiplying both sides of $I = BCDE$ by column B

$$B = B \times I = B \times BCDE = CDE,$$

  the relation $B = CDE$ is obtained.
- B is aliased with the CDE interaction. Following the same method all 15 aliasing relations can be obtained.

all the aliasing relations.

Effect Aliasing and Design Resolution

- To get the most desirable alias patterns, fractional factorial designs of highest resolution would usually be employed.
- There are important exceptions to this rule that we will not cover in the course.

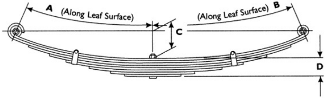
Class Question

Consider a 2^{5-1} fractional factorial design.

1. How many factors does this design have? 5
2. How many runs are involved in this design? 16
3. How many levels for each factor? 2
4. The factor E is assigned to the four-way interaction ($ABCD$). What is the defining relation? What is the design resolution? What are the aliasing relations?

Example - Leaf spring experiment

An experiment to improve a heat treatment process on truck leaf springs (Wu and Hamada (2009)). The height of the unloaded spring is an important quality characteristic.



Example - Leaf spring experiment

Five factors that might affect height were studied in this 2^{5-1} design.

Factor	Level
B. Temperature	1840 (-), 1880 (+)
C. Heating time	23 (-), 25 (+)
D. Transfer time	10 (-), 12 (+)
E. Hold down time	2 (-), 3 (+)
Q. Quench oil temperature	130-150 (-), 150-170 (+)

Example - Leaf spring experiment

= BCD

B	C	D	E	Q	y
-1	1	1	-1	-1	7.7900
1	1	1	1	-1	8.0700
-1	-1	1	1	-1	7.5200
1	-1	1	-1	-1	7.6333
-1	1	-1	1	-1	7.9400
1	1	-1	-1	-1	7.9467
-1	-1	-1	-1	-1	7.5400
1	-1	-1	1	-1	7.6867
-1	1	1	-1	1	7.2900
1	1	1	1	1	7.7333
-1	-1	1	1	1	7.5200
1	-1	1	-1	1	7.6467
-1	1	-1	1	1	7.4000
1	1	-1	-1	1	7.6233
-1	-1	-1	-1	1	7.2033
1	-1	-1	1	1	7.6333

Example - Leaf spring experiment

The factorial effects are estimated as before.

$Q = BCDEQ$

```
fact.leaf <- lm(y~B*C*D*E*Q,data=leafspring)
fact.leaf2 <- aov(y~B*C*D*E*Q,data=leafspring)
round(2*fact.leaf$coefficients,2)
```

```
(Intercept)           B           C           D           E           Q
      15.27        0.22        0.18        0.03        0.10       -0.26
        B:C         B:D         C:D         B:E         C:E         D:E
       0.02        0.02       -0.04          NA          NA          NA
        B:Q         C:Q         D:Q         E:Q       B:C:D       B:C:E
       0.08       -0.17        0.05        0.03          NA          NA
      B:D:E       C:D:E       B:C:Q       B:D:Q       C:D:Q       B:E:Q
         NA          NA        0.01       -0.04       -0.05          NA
      C:E:Q       D:E:Q     B:C:D:E     B:C:D:Q     B:C:E:Q     B:D:E:Q
         NA          NA          NA          NA          NA          NA
    C:D:E:Q   B:C:D:E:Q
         NA          NA
```

Interpretation of Interaction effects.

e.g., 2^3

Run	A	B	C	AB	
1	$-$	$-$	$-$	$+$	y_1
2	$+$	$-$	$-$	$-$	y_2
3	$-$	$+$	$-$	$-$	y_3
4	$+$	$+$	$-$	$+$	y_4
5	$-$	$-$	$+$	$+$	y_5
6	$+$	$-$	$+$	$-$	y_6
7	$-$	$+$	$+$	$-$	y_7
8	$+$	$+$	$+$	$+$	y_8

fix $A = -$ Calc. the effect of B.

$$\underbrace{\left(\frac{y_3+y_7}{2}\right)}_{B=+} - \underbrace{\left(\frac{y_1+y_5}{2}\right)}_{B=-}$$

$A=+$ Calc. effect of B.

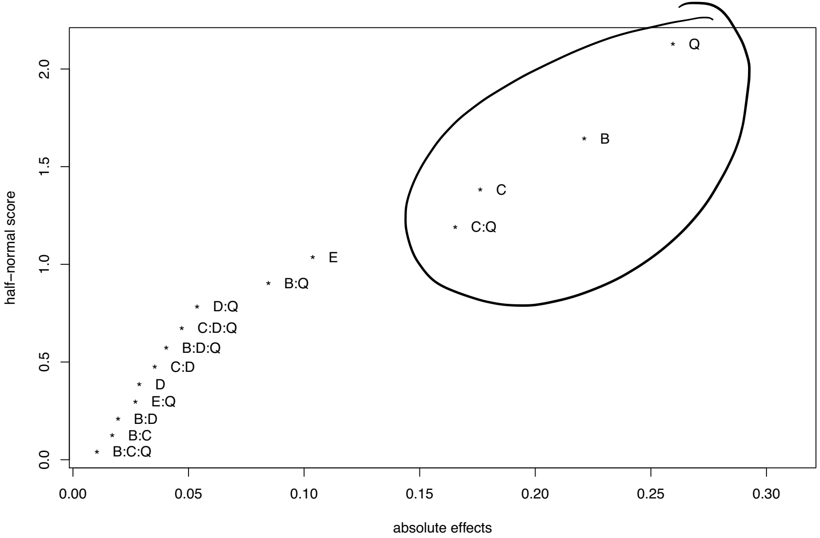
$$\underbrace{\left(\frac{y_4+y_8}{2}\right)}_{B=+} - \underbrace{\left(\frac{y_2+y_6}{2}\right)}_{B=-}$$

$$\frac{\left[\left(\frac{y_3+y_7}{2}\right)-\left(\frac{y_1+y_5}{2}\right)\right]-\left[\left(\frac{y_4+y_8}{2}\right)-\left(\frac{y_2+y_6}{2}\right)\right]}{2}$$

interaction effect:

Example - Leaf spring experiment

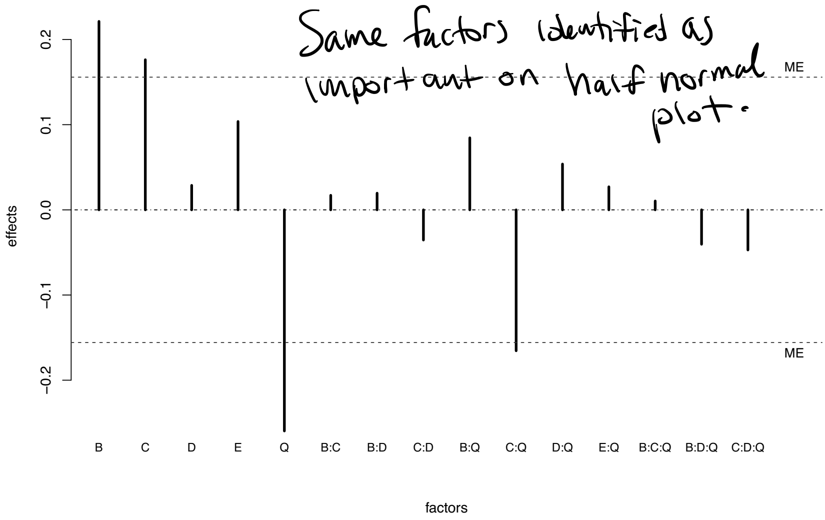
```
BsMD::DanielPlot(fact.leaf,half = T)
```



Example - Leaf spring experiment

```
BsMD::LenthPlot(fact.leaf2,cex.fac = 0.8)
```

Same factors identified as important on half normal plot.




```
    alpha       PSE        ME       SME
0.0500000 0.0606000 0.1557773 0.3162503
```