

### 8.1.1 Minimal Coherence Functional and Selection Rule

#### Formal layer — Choose one coherence functional

Define the projected state at stroboscopic step  $n$  as  $\rho_n^{\text{loc}} = \Lambda \rho_n \Lambda^\dagger$ . Use a single coherence functional in a fixed reference basis  $\{|i\rangle\}$ :

$$\mathcal{C}(\rho_n^{\text{loc}}) \equiv \sum_{i \neq j} \left| \rho_{ij}^{\text{loc}}(n) \right|.$$

This is the  $\ell_1$  coherence measure (minimal, basis-explicit, and computable).

#### Formal layer — Selection functional (free-energy style)

Define a stability-selection functional over one recurrence window:

$$\mathcal{F}_n = E_n - \kappa \mathcal{C}(\rho_n^{\text{loc}}), \quad \kappa > 0,$$

where  $E_n = \text{Tr}[H_{\text{eff}} \rho_n^{\text{loc}}]$  is effective projected energy. A sector is preferred if it minimizes time-averaged selection cost:

$$\bar{\mathcal{F}}_N = \frac{1}{N} \sum_{n=1}^N \mathcal{F}_n.$$

#### Interpretation — Interpretation

The model now has one explicit knob: coherence-weight  $\kappa$ . Larger  $\kappa$  rewards sectors that maintain off-diagonal structure under recurrence; smaller  $\kappa$  reduces to energy-dominant selection.

#### Test hook — Operational use

Scan candidate sectors and estimate  $(E_n, \mathcal{C}_n)$  from projected trajectories. Stable classical-looking sectors should occupy low-variance bands of  $\bar{\mathcal{F}}_N$  over long windows.