8.3.1 Emergent Geometry from Distinguishability

Formal layer — Metric choice on projected states

For neighboring projected states, define infinitesimal distance with quantum Fisher metric:

$$\mathrm{d}s^2 = \frac{1}{4}\,\mathcal{I}_Q\big(\rho_\theta^{\mathrm{loc}}\big)\,\mathrm{d}\theta^2,$$

where θ indexes effective coordinates in the projected sector. For discrete samples $(\rho_a^{\mathrm{loc}}, \rho_b^{\mathrm{loc}})$ use Bures proxy:

$$d_{ab}^2 = 2\Big(1 - \sqrt{F(\rho_a^{\mathrm{loc}}, \rho_b^{\mathrm{loc}})}\Big),$$

with fidelity $F(\rho_a, \rho_b) = \big(\mathrm{Tr}\,\sqrt{\sqrt{\rho_a}\rho_b\sqrt{\rho_a}}\big)^2$.

Formal layer — Graph-to-geometry protocol

Build a weighted graph over sampled projected states with edge weights

$$w_{ab} = \exp\big(-d_{ab}^2/\sigma^2\big).$$

Estimate effective local metric tensor $g_{\mu\nu}$ via neighborhood regression (diffusion-map/Laplacian embedding), then extract curvature scalars from $g_{\mu\nu}$ on the learned manifold.

Interpretation — Interpretation

Geometry is reconstructed from state distinguishability, not assumed a priori. Distances, neighborhoods, and curvature emerge from how hard it is to transform one projected state into another under coherence constraints.

Test hook — Falsifiable signature

If the framework is correct, regions with persistent coherence bottlenecks should map to positive effective curvature zones and correlate with enhanced recurrence stability under Floquet updates.