

To solve the compatibility condition for the **COSMOS** model, we define the precise relationship between global stroboscopic evolution and its local projection.

Here is the LaTeX subsection for **Section 8.2.1**:

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## 8.2.1 Precise Compatibility & Stability Thresholds

### Formal layer — The Commutativity Constraint

Let  $\mathcal{B}(S)$  be the algebra of bounded operators on the substrate. We define the **Stability Error**  $\epsilon$  as the operator norm of the mismatch between the global update and the induced local dynamics:

$$\epsilon := \|\Lambda\mathcal{F} - \mathcal{F}_{\text{loc}}\Lambda\|_{\text{op}} = \sup_{|\psi\rangle \in S, \langle\psi|\psi\rangle=1} |(\Lambda\mathcal{F} - \mathcal{F}_{\text{loc}}\Lambda)|\psi\rangle|$$

The emergent sector  $S_{\text{loc}}$  is **stably coherent** if there exists a local propagator  $\mathcal{F}_{\text{loc}}$  such that:

$$\epsilon < \delta_{\text{coh}}$$

where  $\delta_{\text{coh}}$  is the *coherence margin* defined by the integration operator  $\Delta$  (typically related to the spectral gap of the global generator  $H_{\text{tot}}$ ).

### Interpretation — Dynamical Drift and Decoherence

The condition  $\epsilon \approx 0$  implies that the local "map" is a faithful representation of the global "land." If  $\epsilon$  exceeds the threshold  $\delta_{\text{coh}}$ , the local sector undergoes **representation drift**. In this state, the effective operator  $\Omega$  can no longer consistently organize histories; the projected sector "decouples" from the global rhythm. This is the origin of decoherence: not necessarily an interaction with an external environment (as there is no "outside" the substrate), but a failure of the local projection to track the global periodic update.

### Child intuition — The Movie Projector

Imagine a movie projector ( $\mathcal{F}$ ) spinning a film. The projection operator ( $\Lambda$ ) is the lens that puts the light on the screen. If the lens and the film-strip are perfectly synchronized, the movie looks sharp ( $\epsilon < \delta_{\text{coh}}$ ). But if the lens starts wobbling or the film slips, the picture gets blurry and eventually disappears into static. That "blurriness" is  $\epsilon$ .

### Test hook — Phase Transitions in Time Crystals

What would count as evidence? A sharp transition in the "classicality" of a system. If we tune a system's parameters such that  $\epsilon$  crosses  $\delta_{\text{coh}}$ , we should observe a sudden loss of periodic recurrence—a "melting" of the emergent time crystal into a state of maximum entropy (thermalization) where  $\Omega$  no longer yields stable geometry.

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