

### 8.3.1 Emergent Geometry from Distinguishability

#### Formal layer — Metric choice on projected states

For neighboring projected states, define infinitesimal distance with quantum Fisher metric:

$$ds^2 = \frac{1}{4} \mathcal{I}_Q(\rho_\theta^{\text{loc}}) d\theta^2,$$

where  $\theta$  indexes effective coordinates in the projected sector. For discrete samples  $(\rho_a^{\text{loc}}, \rho_b^{\text{loc}})$  use Bures proxy:

$$d_{ab}^2 = 2 \left( 1 - \sqrt{F(\rho_a^{\text{loc}}, \rho_b^{\text{loc}})} \right),$$

with fidelity  $F(\rho_a, \rho_b) = (\text{Tr} \sqrt{\sqrt{\rho_a} \rho_b \sqrt{\rho_a}})^2$ .

#### Formal layer — Graph-to-geometry protocol

Build a weighted graph over sampled projected states with edge weights

$$w_{ab} = \exp(-d_{ab}^2/\sigma^2).$$

Estimate effective local metric tensor  $g_{\mu\nu}$  via neighborhood regression (diffusion-map/Laplacian embedding), then extract curvature scalars from  $g_{\mu\nu}$  on the learned manifold.

#### Interpretation — Interpretation

Geometry is reconstructed from state distinguishability, not assumed a priori. Distances, neighborhoods, and curvature emerge from how hard it is to transform one projected state into another under coherence constraints.

#### Test hook — Falsifiable signature

If the framework is correct, regions with persistent coherence bottlenecks should map to positive effective curvature zones and correlate with enhanced recurrence stability under Floquet updates.