The Discriminant

$$b^2 - 4ac$$

This equation is part of the quadratic formula $(x = \frac{-b \pm \sqrt{b^2-4ac}}{2a})$ which is used to find values of x for a quadratic formula (a formula in which the highest power of x is 2). The value of the discriminant gives us an indication as to how many solutions our equation has. If we know the value of the discriminant, we know how many solutions our equation will give.
If:
1) $b^2 - 4ac > 0$ then there are two real solutions
2) $b^2 - 4ac = 0$ then there is one repeated real solution
3) $b^2 - 4ac < 0$ then there are no real solutions

Empirical Proof

If we need to solve the equation $2x^2 + 8x - 24 = 0$ (using the format $ax^2 + bx + c$) we can use the quadratic formula

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 2 \times -24}}{2 \times 2}$$

Therefore

$$x = \frac{-8 \pm \sqrt{256}}{4}$$

and so $x = 2\,, x = -6$

Note that the value of the discriminant was 256, so $b^2 - 4ac = 256$.

Here we can see 1) is true, as two outcomes are produced when adding or subtracting the same number (in this case 16, or $\sqrt{256}$). The second is obvious when the following equation is considered.

$$x = \frac{-b \pm \sqrt{0}}{2a}$$

The square root of 0 is 0, and adding or subtracting 0 has no effect on the value and only one value is produced. However, this value is produced twice and so the solution is repeated.

If the discriminant is negative ($b^2 - 4ac < 0$) then there are clearly no real solutions as you cannot square root a negative number to create a real number.

See also

- Quadratic formula