Integration by Parts

$$\int u\frac{dv}{dx}dx = \text{uv} - \int v\frac{du}{dx}dx$$

This equation can be useful to find the integral of an expression - especially when two terms are multiplied together, which we call u and $\frac{dv}{dx}$.

For example, if we need to integrate the expression $y = x\cos x$,
we can let

$$u = 3x, \frac{dv}{dx} = \cos x$$

Then we know

$$\frac{du}{dx} = 3, v = -\sin x$$

And so we can input this

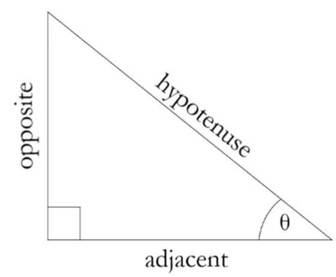
$$\int x \times cos\, x\, dx = 3x \times -sin\, x - \int -\sin x \times 3\, dx$$

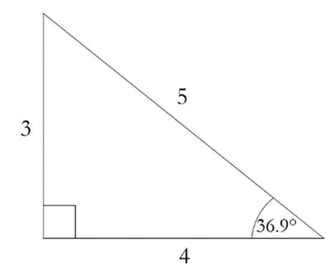
This can be simplified

$$\int x\cos x\,\text{dx} = -3\text{x}\sin x - \int -3\sin x\, dx$$

We still need to integrate, but this new integral is much simpler, so we get

$$\int x\cos x\,\text{dx} = -3\text{x}\sin x - 3\cos x$$





Proof

We should know the product rule, which is

$$\frac{d}{dx}uv = u\frac{dv}{dx} + \text{v}\frac{du}{dx}$$

If we integrate both sides

$$\int \frac{d}{dx}uv\, dx = \int \left(u\frac{dv}{dx} + \text{v}\frac{du}{dx}\right)dx$$

The left hand side cancels, as we have an integral of a derivative

$$uv = \int \left(u\frac{dv}{dx} + \text{v}\frac{du}{dx}\right)\, dx$$

We can split the right side into two smaller expressions

$$uv = \int u\frac{dv}{dx}dx + \int \text{v}\frac{du}{dx}dx$$

We can re-arrange this to get

$$\int u\frac{dv}{dx}dx = \text{uv} - \int v\frac{du}{dx}dx$$

See also

- Product Rule
- Integration
- Differentiation

References

Attwood, G. et al. (2017). *Edexcel A level Mathematics - Pure - Year 2*. London: Pearson Education. p.307.