The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation is used to find values of x for a quadratic formula (a formula in which the highest power of x is 2). The value of the discriminant gives us an indication as to how many solutions our equation has.

For example, if we need to solve the equation $2x^2 + 8x - 24 = 0$ (using the format $ax^2 + bx + c$) we can use the quadratic formula

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 2 \times -24}}{2 \times 2}$$

Therefore

$$x = \frac{-8 \pm \sqrt{256}}{4}$$

and so $x = 2$, $x = -6$

Proof

Let there be an equation of the form $ax^2 + bx + c = 0$. If we complete the square:

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$

Multiply both sides by a

$$a^2\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4} - ac$$

Multiply both sides by 4

$$4a^2\left(x + \frac{b}{2a}\right)^2 = b^2 - 4ac$$

Divide both sides by 4a²

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Square root both sides

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

Subtract $\frac{b}{2a}$ from both sides

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

See also
- The Discriminant

References
Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. p.21