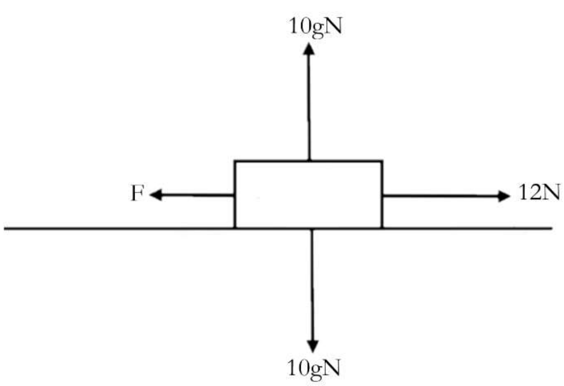
Friction Equation

$$F_r \leq \mu R$$

This equation is used very frequently in mechanics, whenever a surface is modelled as being rough[1] there will be friction involved. This equation means (friction) ≤ (co-efficient of friction)×(normal reaction). Friction is written F_r to avoid it being confused with F which is force. $F_r = \mu R$ when an object is in limiting equilibrium (on the point of motion) or is moving. Otherwise, $F_r < \mu R$. The co-efficient of friction is constant if the same two materials are being used against each other. For example, a steel box on an oak floor or an oak box on a steel floor will always have the same value for μ. $0 \leq \mu < 1$, μ will only equal 0 if the surfaces are both completely smooth (this is not possible in reality, but is used for modelling purposes. μ has no units, because both friction and the normal reaction are forces so have the same unit – the newton.

For example, a horizontal force of 12N (12 Newtons) pulls a box along a rough surface. The co-efficient of friction is 0.05. The mass of the box is 10kg. Find its acceleration.

Firstly, we should draw a diagram. This will give us a better understanding of what is going on.



There are two horizontal forces that are acting on the box, the pulling force and the frictional force. The box is being moved in the direction of the pulling force, and friction acts in the opposite direction to motion. There are also two vertical forces acting on the box, weight and the normal reaction[2]. Because the box is neither accelerating upwards nor accelerating downwards, these forces must be equal to each other. (Weight) = (mass)×(gravitational field strength), gravitational field strength gives us the acceleration due to gravity. This makes sense because $F = ma$. Weight is a force, mass is mass and g.f.s.[3] is acceleration. $G.f.s. = 9.8$[4], but is often written as g. So $10gN = 98N$.

We therefore know that the normal reaction is $98N$. We also know that since the box is in motion, $F_r = \mu R$, so $F_r = 0.05 \times 98 = 4.9N$. Therefore, the resultant force is $12 - 4.9 = 7.1N$ so by $\Sigma F = ma$, $7.1 = 10a$ therefore $a = 0.71ms^{-2}$

Proof

The only way to "prove" such an equation is by experimental observation. This equation is often used in mechanics, rather than pure mathematics and so cannot be derived.

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[1] Meaning objects in contact with the surface experience a frictional force if they are moving or are acted on by a force

[2] The normal reaction is the force the ground exerts on the box, so that it does not fall through the floor

[3] Gravitational field strength

[4] The unit for g.f.s. is N/kg or Nkg^{-1} or m/s^2 or ms^{-2}, remember that a negative power of a value means the reciprocal of the positive power of a value, so $s^{-2} = \frac{1}{s^2}$, so $ms^{-2} = m/s^2$, the same is true for N/kg and Nkg^{-1}

Note

You may have been wondering why $F_r = \mu R$ in some circumstances, but $F_r < \mu R$ in others.
Imagine a large brick on a road. If you were to tap its side, it would not move. This is because of friction. But the friction is only large enough to stop the brick from moving, otherwise the brick would fire towards you whenever you tap it! If you tap it harder, the brick may still not move, but friction has increased to counteract the larger force. At this point, we know that friction is not its maximum possible value, so $F_r < \mu R$. Eventually, you will reach a point where the force you apply to the brick will mean it is on the cusp of moving. This is limiting equilibrium. But the friction cannot increase any more than this, you have reached the maximum friction – this is when $F_r = \mu R$.

See also

- Newton's Second Law

References

Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Statistics and Mechanics - Year 1*. London: Pearson Education. p.100.