Integration

$$\lim_{\delta x \to 0} \sum_{x=a}^{b} f(x)\,\delta x = \int_a^b f(x)\,dx$$

Integration is the process of finding the area underneath a graph. In the above equation, $f(x)$ is the function that is being integrated and we want to find the area of the graph between a and b. The left hand side of this equation tells us what integration is. It is the adding (otherwise known as summation – which is what the $\sum$ denotes) of lots of very thin strips under the curve together to find the area. They are infinitesimally thin (you could say that each strip is of thickness $0.000 \ldots 1$, which is what $\lim_{\delta x \to 0}$ tells us – δx 'tends to' 0). The $\int$ symbol is really just a tidier way of saying that something is being integrated, so usually this symbol is what you will use if you want to integrate something.

Integration is the reverse of differentiation, so if you have an expression for the gradient of a curve you can integrate that to find the expression for the original curve. As integration is the inverse of differentiation, integration has its own rules. For each term, increase the power by one and divide the co-efficient by the new power. This is the standard method of integration.

For example, let the function $f(x) = 2x$, and say we want to find the area between $x = 2$ and $x = 4$ This means that

$$\lim_{\delta x \to 0} \sum_{x=2}^{4} 2x\,\delta x = \int_2^4 2x\,dx$$

Using the standard method of integration

$$\int_2^4 2x\,dx = \left[\frac{2x^{1+1}}{1+1}\right]_2^4$$

This can be simplified to the form

$$\int_2^4 2x\,dx = \left[\frac{2x^2}{2}\right]_2^4$$

So

$$\int_2^4 2x\,dx = [x^2]_2^4$$

Then minus the smaller value from the larger,

$$\int_2^4 2x\,dx = (4^2) - (2^2)$$

So the area is 12.

Note

What we have found above is a definite integral. But what of an indefinite integral? This is the general expression we get when we do not know what x-values we want to find the area between.

The process is quite similar.

$$\int 2x\,dx = \frac{2x^{1+1}}{1+1} + c$$

And this simplifies to

$$\int 2x\,dx = x^2 + c$$

Why do we have the $+c$? Consider the expression $y = x^2 + 3$. If we differentiate this we find that $\frac{dy}{dx} = 2x$. This is because when differentiating constants always go to 0. So this means when we integrate 2x, it could be x^2 or $x^2 + 1$ or $x^2 - \frac{2}{3}$ or $x^2 + 3$ and so on.

Often, we might be told that when $x = 3$, $\int f(x)\,dx = 11$, so in this instance…

$$\int 2x\,dx = x^2 + c$$
$$(3)^2 + c = 12$$
$$c = 12 - 9 = 3$$
$$\int 2x\,dx = x^2 + 3$$

<u>See also</u>
- Differentiation
- Series

<u>References</u>

Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. pp.288-289.