Difference of two squares

What is the difference of two squares?

As you will already know, 'difference' is a term used in mathematics, which is the same process as subtraction.

i.e., What is the difference between 5 and 2?

$5 - 2 = 3$

The difference is 3. (An alternate answer could be a difference of -3 if you do 2 – 5).

"Square" in this context means a number that has been squared, i.e., a number that has been raised to the power of two. We can express this algebraically. Let there be two numbers which we shall call *x* and *y*. From above, the difference of two squares is:

$$x^2 - y^2$$

Factorising the difference of two squares

Factorising is writing expressions as a product of their factors. For example, we can factorise 10 as 2×5. The difference of two squares can be factorised thus:

$$(x - y)(x + y)$$

As expanding is the opposite of factorising, we should get the original difference of two squares if we expand this expression. To do so, we shall multiply the terms in the first bracket (x and -y) with the terms of the second bracket (x and y). We must multiply each term by the other two terms in the other bracket.

$x \times x = x^2$
$(-y) \times x = -xy$
$x \times y = xy$
$(-y) \times y = -y^2$

We then add up all of the new terms to get:

$$x^2 - xy + xy - y^2$$

xy – xy = 0, so then we have

$$x^2 - y^2 + 0$$

which is

$$x^2 - y^2$$

This is what we wanted. This is a proof that $(x - y)(x + y) \equiv x^2 - y^2$

Points to consider

Is $x^4 - y^4$ a difference of two squares?

Yes, it is. It can also be expressed in the form $(x^2)^2 - (y^2)^2$ which is x to the power of 2 to the power of 2 and y to the power of 2 to the power of 2. Therefore, this can be factorised:

$$(x^2 - y^2)(x^2 + y^2)$$

The $x^2 + y^2$ cannot be factorised, but as we already know $x^2 - y^2$ can. A completely factorised format would therefore be:

$$(x^2 + y^2)(x + y)(x - y)$$

In fact, any equation of the form $x^n - y^n$, where n is an even number, is a difference of two squares.

Why can we not factorise $x^2 + y^2$?

There is no way to multiply two terms together to get this result. Your immediate thoughts may be that if $(x + y)(x - y) = x^2 - y^2$, perhaps $(x + y)(x + y)$ (which is the same thing as $(x + y)^2$) might provide us with this format. But if we try to expand this:

$x \times x = x^2$
$x \times y = xy$
$y \times x = xy$
$y \times y = y^2$

We therefore have the expression:
$x^2 + 2xy + y^2$

This also rules out $(x - y)^2$ as a possibility.
There is no way to get rid of this 2xy by means of multiplication, so we cannot factorise the above expression.

Example

A simple example you might see in an exam would be $x^2 - 9$, x^2 is clearly a square number. $9 = 3^2$, therefore, we have a difference of two squares.

See also

- Difference of three squares
- Laws of indices