**Hyperplane manual**

**1. Introduction to hyperplane**

Given *n*-dimensional space such as Euclidean space or more generally, affine space or vector space, it is conventional that elements in space are called points. Although concepts “*point*” and “*vector*” are different in space when point is coordinate of a given vector in , each affine space like real number space is associated to another *n*-dimensional real vector space and so we can considered a point as a vector and otherwise. Without loss of generality, is considered as real space *.*

Given two points ***x*** and ***y*** belonging to , they are denoted as column vectors:

Note that *xi* and *yi* (s) which are scalar number are called elements or components of ***x*** and ***y***, respectively. Followings are definitions of the distance *d*(***x***, ***y***), the operations such as addition ***x*** + ***y***, subtraction ***x*** – ***y***, scalar product ***x****T****y*** and cross product ***x*** ***y*** of two given points or vectors ***x*** and ***y***.

Note that the letter *T* denotes transposition operator which changes column vector to row vector and otherwise as following:

Given *n*-dimensional Euclidean space , a flat is defined as subset of lower dimension and is congruent to . For example, flats in 2-dimensional space are points and lines; flats in 3-dimensional space are points, lines and planes. In general, flats in *n*-dimensional space are subsets of dimension from 0 to *n* – 1. *The hyperplane of n-dimensional space is defined as a flat of dimension n –* 1. The *general equation* of hyperplane of dimension *n* – 1 is:

Where *ai* (s) are not all zero; in other words, there exists index *i* such that .

Note that *ai* (s) and *α* are real coefficients and is real vector variable. The vector denoted *p* which is perpendicular to hyperplane is called *normal vector* of hyperplane.

The normal vector is also gradient vector of function *f*(***x***) = *a*1*x*1 + *a*2*x*2 + … + *a*n*x*n + *β* when equation (1) is essentially constructed by setting *f*(***x***) to be zero. Note that *f*(***x***) is called *underlying function*.

The linear equation (1) is re-written in normal form which is scalar product of variable ***x*** and the normal vector:

Equation (2) is called *normal equation* of hyperplane. It is easy to recognize that normal equation and general equation are the same and they are basic form of equation of hyperplane. Hyperplane with normal vector *p* through point is specified by so-called *normal-point* equation:

The parameter equation of hyperplane which is equivalent to general equation is defined as a set of *n* linear equations:

Without loss of generality, suppose , the *parameter equation* of hyperplane becomes:

The hyperplane of dimension *n* – 1 requires *n* – 1 parameters *ti* (s) where .

The hyperplane degrades to straight line and plane in 2-dimensional and 3-dimensional space, respectively. The general equation of 2-dimensional straight line, derived from equation (1) is:

Where *x* and *y* are scalar variables.

The underlying function of equation (5) is *f*(*x, y*) = *ax* + *by + c*. The normal vector (also gradient vector of underlying function) perpendicular to the straight line is

The normal equation of 2-dimensional straight line is expanded as following:

The normal-point equation through point (*x*0, *y*0) is:

The vector perpendicular to normal vector is called *directional vector* of 2-dimensional straight line. Let *q* denote directional vector, we have:

Suppose , dividing both sides of equation (5) by *b*, we have:

This implies that straight line can be specified in so-called *slope-intercept equation* as following:

Where slope is not infinity and *r* = –*c*.

The parameters *s* and *r* are called slope and intercept of straight line, respectively and *s* is also derivative of *y* with regard to *x*. The normal vector *p* and directional vector *q* of slope-intercept equation are:

Given slope *s*, the straight line goes through point (*x*0, *y*0) is expressed in so-called *slope-point equation* which is a variant of slope-intercept equation as following

The intercept of equation (9) is *r* = *y*0 – *sx*0.

Suppose the straight line goes through two points (*x*0, *y*0) and (*x*1, *y*1), this line can be specified by so-called *point-point equation* which is a variant of slope-point equation as following.

Where,

The slope and intercept of point-pointequation (10) are:

Back slope-intercept equation (8), let *t* be equal to *x*, we have:

The equation (11) is parameter equation of 2-dimensonal straight line, which is equivalent to slope-intercept equation with the same formulas for normal vector and directional vector.

The general equation of 3-dimensional plane, derived from equation (1) is:

Where *x*, *y* and *z* are scalar variables.

The normal vector perpendicular to the plane is:

The normal equation of plane is:

The normal-point equation of plane through point (*x*0, *y*0, *z*0) is:

**2. Hyperplane properties**

**2. Separating hyperplane and support vector machine**