Partial Derivatives

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1 Definition

Partial Derivative represents how the multivariate function changes with respect to one variable while keeping all other variables constant. It is represented by $\frac{\partial}{\partial x}$

Now, let's consider a multivariate function $f(x_1, x_2, ..., x_n)$ which depends on n variables. We can express the partial derivation of the function with respect to the variable x_i as a limit,

$$\frac{\partial}{\partial x_i} f(x_1, x_2, ..., x_i, ..., x_n) = \lim_{h \to 0} \frac{f(x_1, x_2, ..., x_i + h, ..., x_n) - f(x_1, x_2, ..., x_i, ..., x_n)}{h} \tag{1}$$

This partial derivative can also be represented as $\frac{\partial f}{\partial x_i}$, $\partial_{x_i} f$ or f'_{x_i}.

2 Total Derivative

Total derivatives represents how the multivariate function changes with respect to one variable considering the changes in all variables that the function depends on. It is represented by $\frac{d}{dx}$

We can express the total derivative of f with respect to a parameter t using the partial derivatives with respect to each of its variables,

$$\frac{d}{dx}f(x_1, x_2, ..., x_n) = \frac{\partial f}{\partial x_1}\frac{dx_1}{dx} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dx} + ... + \frac{\partial f}{\partial x_n}\frac{dx_n}{dx} \tag{2}$$

2.1 Univariate Functions

The concept of Partial and Total Derivatives is only for multivariate functions because for univariate functions, both are the same and are just referred to as "derivatives".

Proof:

Applying the total derivative formula for an univariate function $p(x)$,

$$\frac{dp}{dx} = \frac{\partial p}{\partial x}\frac{dx}{dx}$$
$$\Rightarrow \frac{dp}{dx} = \frac{\partial p}{\partial x}$$

3 Equality of Mixed Partials

Exchanging the order of partial derivatives of a multivariate function does not change the result if certain conditions are satisfied. See Wikipedia.

$$\frac{\partial}{\partial x_i}(\frac{\partial f}{\partial x_j}) = \frac{\partial}{\partial x_j}(\frac{\partial f}{\partial x_i}) \tag{3}$$

The mixed partial $\frac{\partial}{\partial x_i}(\frac{\partial f}{\partial x_j})$ can also be represented as,

$$\frac{\partial^2 f}{\partial x_i \partial x_j},\ \partial_{x_i}\partial x_j f \text{ or } f'_{x_i x_j}$$