Technical University of Denmark
DTU Engineering

Final Assignment

Introduction to Financial Engineering

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1 Portfolio optimization

Part A: Diversification

In order to develop the first exercise, we have chosen the following financial stock indices (described in `www.sectorspdr.com/sectorspdr`):

1. **Financials in the S&P 500** (`XLF`): A wide array of diversified financial service firms are featured in this sector with business lines ranging from investment management to commercial and investment banking. Among the companies included in the Index are JPMorgan Chase, Wells Fargo, and BankAmerica Corp. Estimate weight of the sector components in the S&P 500 (in the following abbreviated as weight) = 16.61%.
2. **Energy in the S&P 500** (`XLE`): Energy companies in this Index primarily develop and produce crude oil and natural gas, and provide drilling and other energy-related services. Leaders in the group include ExxonMobil Corp., Chevron Corp, and ConocoPhillips. Weight=7.03%.
3. **Consumer Staples in the S&P 500** (`XLP`): The companies in this sector are primarily involved in the development and production of consumer products that cover food and drug retailing, beverages, food products, tobacco, household products, and personal products. Component stocks include Wal-Mart, Proctor & Gamble, Philip Morris International, and Coca-Cola. Weight=9.54%.
4. **Technology in the S&P 500** (`XLK`): Stocks primarily covering products developed by internet software and service companies, IT consulting services, semiconductor equipment and products, computers and peripherals, diversified telecommunication services and wireless telecommunication services are included in this Index. Components include Microsoft Corp., AT&T, International Business Machines Corp., and Cisco. Weight=23.25%.
5. **Industrials in the S&P 500** (`XLI`): General Electric Co., Minnesota Mining & Manufacturing Co., and United Parcel are among the largest components by market capitalization in this sector. Industries in the Index include aerospace and defense, building products, construction and engineering, electrical equipment, conglomerates, machinery, commercial services and supplies, air freight and logistics, airlines, marine, road and rail, etc. Weight=10.07%.
6. **Health Care in the S&P 500** (`XLV`): Companies in this sector primarily include health care equipment and supplies, health care providers and services, biotechnology, and pharmaceuticals industries. Pfizer Inc., Johnson & Johnson, and Abbott Labs are included in this sector's mix. Weight=14.71%.
7. **Materials in the S&P 500** (`XLB`): This Index is primarily composed of companies involved in industries such as chemicals, construction materials, containers and packaging, metals and mining, and paper and forest products. Among its largest components are Monsanto, E.I. DuPont de Nemours & Co., and Dow Chemical Co. Weight=2.85%.
8. **Utilities in the S&P 500** (`XLU`): The Utilities Index primarily provides companies that produce, generate, transmit or distribute electricity or natural gas.

The component companies include Exelon Corp., Southern Co., and Dominion Resources Inc. Weight=2.85%.

For the market, the following index will be used:

- **S&P 500** (`^GSPC`): An American stock market index based on the market capitalizations of 500 large companies.

The data that will be analysed has been monthly collected for an interval of 16 years (from 1999 to 2015) from Yahoo! Finance's database. All the values are in the same domestic currency (USD). Therefore, it was not necessary to convert the currencies.

Part B: Estimate

The expected yearly returns and the covariance matrix have been calculated using 11 rolling windows. Since the recording for the chosen indices started in 1999, the rolling windows have been formed in the following way: the 7 first windows cover a period of 10 years (2005-2015 to 1999-2009) and from window 8 to 11 (1999-2008, 1999-2007, 1999-2006 and 1999-2005) they cover 9, 8, 7 and 6 years respectively. Both the mean return and the covariance matrix for each window have been computed using monthly returns. Figure 1 shows the explained compositions of each rolling window.

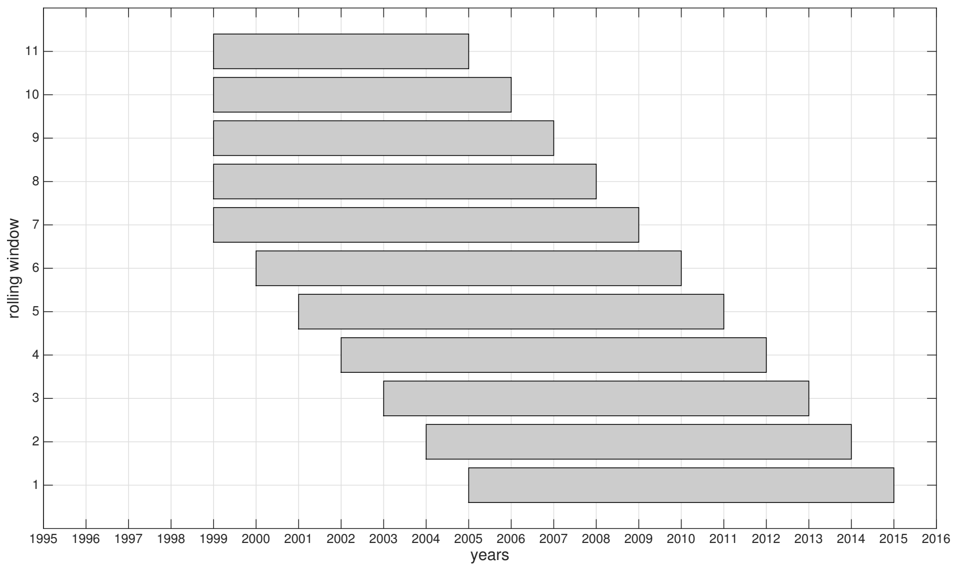

Figure 1: Rolling windows composition

First, for each index the monthly return has been computed with the following formula:

$$R_m(1)_{dec\,2014} = \log\left(\frac{P(1)_{jan\,2015}}{P(2)_{dec\,2014}}\right)$$

$$R_m\,(2)_{nov\,2014} = \log\Big(\frac{P(2)_{dec\,2014}}{P(3)_{nov\,2014}}\Big)$$

$$R_m\,(n)_{month} = \log\Big(\frac{P(n)_{month}}{P(n+1)_{month-1}}\Big) \tag{1}$$

- R_m: Monthly return
- P: Adjusted close price of the index

Then, for each of the windows, the expected yearly return (2), the covariance matrix (3) and the correlation matrix (4) have been calculated with the data from the corresponding time period.

$$\bar{R}_y = 12 * \bar{R}_m \tag{2}$$

$$Cov_y = 12 * Cov_m \tag{3}$$

$$Corr_y = Corr_m \tag{4}$$

- R_y: Yearly return
- Cov_m, Cov_y: Covariance matrix (resp. monthly and yearly)
- $Corr_m$, $Corr_y$: Correlation matrix (resp. monthly and yearly)

The resulting expected yearly return for each asset and window are shown in Table 1.

window	XLF	XLE	XLP	XLK	XLI	XLV	XLB	XLU
1	0.01%	8.70%	9.97%	8.62%	8.10%	10.51%	7.57%	8.79%
2	-1.02%	12.40%	8.70%	6.29%	8.13%	7.63%	7.83%	8.65%
3	0.01%	14.24%	8.85%	8.34%	9.15%	6.50%	9.63%	10.38%
4	-3.90%	11.68%	4.62%	2.39%	5.18%	4.45%	7.63%	6.18%
5	-3.61%	9.83%	2.94%	-2.67%	3.16%	2.17%	8.46%	3.70%
6	-2.59%	8.49%	3.13%	-8.01%	1.72%	1.56%	4.94%	4.19%
7	-7.31%	9.21%	-0.17%	-8.83%	-0.17%	0.64%	2.29%	3.18%
8	4.19%	14.45%	1.81%	-4.83%	6.04%	3.28%	9.37%	6.50%
9	7.40%	13.86%	1.51%	-5.27%	6.18%	3.78%	9.08%	6.11%
10	5.97%	15.55%	-0.47%	-7.48%	5.00%	3.10%	8.07%	4.66%
11	5.46%	10.79%	-0.84%	-10.19%	4.86%	1.84%	7.45%	2.86%

Table 1: Yearly expected returns

The resulting covariance and correlation matrices of the first rolling window (2005-2015) are shown in (5) and (6) respectively.

$cov(2005 \div 2015) = cov\_matrix\{1\} =$

$$
= \begin{pmatrix}
0.061 & 0.026 & 0.018 & 0.029 & 0.040 & 0.022 & 0.039 & 0.012 \\
0.026 & 0.053 & 0.012 & 0.024 & 0.028 & 0.013 & 0.036 & 0.016 \\
0.018 & 0.012 & 0.012 & 0.013 & 0.016 & 0.011 & 0.015 & 0.009 \\
0.029 & 0.024 & 0.013 & 0.029 & 0.026 & 0.014 & 0.029 & 0.011 \\
0.040 & 0.028 & 0.016 & 0.026 & 0.036 & 0.017 & 0.035 & 0.012 \\
0.022 & 0.013 & 0.011 & 0.014 & 0.017 & 0.017 & 0.017 & 0.010 \\
0.039 & 0.036 & 0.015 & 0.029 & 0.035 & 0.017 & 0.044 & 0.013 \\
0.012 & 0.016 & 0.009 & 0.011 & 0.012 & 0.010 & 0.013 & 0.019
\end{pmatrix} \tag{5}
$$

$corr(2005 \div 2015) = corr\_matrix\{1\} =$

$$
= \begin{pmatrix}
1.000 & 0.456 & 0.669 & 0.692 & 0.842 & 0.667 & 0.739 & 0.352 \\
0.456 & 1.000 & 0.469 & 0.617 & 0.629 & 0.447 & 0.740 & 0.497 \\
0.669 & 0.469 & 1.000 & 0.676 & 0.748 & 0.739 & 0.652 & 0.597 \\
0.692 & 0.617 & 0.676 & 1.000 & 0.817 & 0.648 & 0.821 & 0.460 \\
0.842 & 0.629 & 0.748 & 0.817 & 1.000 & 0.687 & 0.870 & 0.450 \\
0.667 & 0.447 & 0.739 & 0.648 & 0.687 & 1.000 & 0.628 & 0.565 \\
0.739 & 0.740 & 0.652 & 0.821 & 0.870 & 0.628 & 1.000 & 0.441 \\
0.352 & 0.497 & 0.597 & 0.460 & 0.450 & 0.565 & 0.441 & 1.000
\end{pmatrix} \tag{6}
$$

In order to develop this assignment, several combination of indices where analysed. However, all the correlation matrices resulted in highly positive correlations between them. Therefore, the results of the next parts where unusual. Finally, indices for different sectors of the S&P were selected, as they represent the lowest correlation of the ones that where analysed. Nevertheless, this correlation is still positive and of considerable value. Thus, the diversification benefit will not be so favorable.

Part C: Efficient frontier

The function called `efficient_frontier` has been developed in order to calculate the efficient frontier for each rolling window. With three inputs, respectively the expected yearly returns of each asset and the covariance and correlation matrices, the function computes the corresponding efficient frontier as an output.

Two optimal portfolios with the maximum slope $\frac{return - R_f}{standard\, deviation}$ for two different risk free rates R_f are calculated (`portfolio.composition` and `portfolio2.composition`). In order to find these optimal portfolios, the technique explained in the course's slides and the function `highest_slope_portfolio`, provided in the solutions to the exercises, have been used. Afterwards, another optimal portfolio `port_comp` is computed as a combination of the two previous ones, as in the following equation (7):

$$X_3 = \alpha \cdot X_1 + (1 - \alpha) \cdot X_2 \tag{7}$$

In order to draw the efficient frontier, α has been varied in the range of -30 and 30 with steps of 10^{-4}.

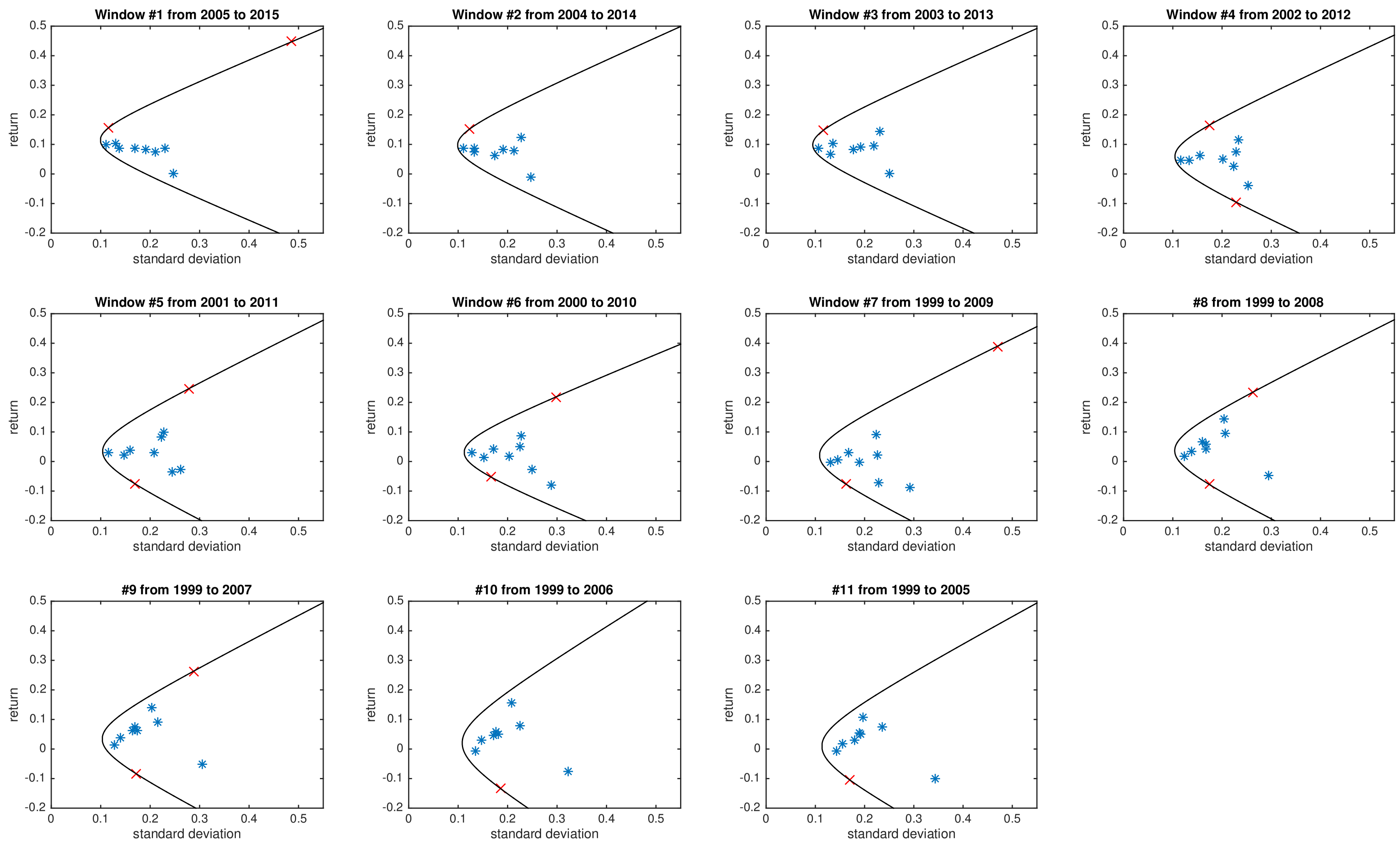

Figure 2: Efficient frontiers for the eleven rolling windows. Single indices displayed as $\star$, portfolios used to draw the efficient frontier as $\times$. Efficient frontier depicted as full line.

Diversification benefit

The term diversification in finances describes the splitting of an investment in a broad range of assets in order to reduce the unsystematic risk. Following this strategy results in a more stable portfolio, due to the fact that even when some assets in your portfolio are underperforming in relation to the market as a whole, it is still possible to have positive returns. This is the case because the assets in the portfolio are not totally correlated.

The benefits of portfolio diversification become obvious when looking at Figure 2. Comparing the efficient frontier to the position of the single indices in the return-volatility plots, it can be seen that it is possible to obtain a higher return assuming the same risk, when holding a portfolio of different indices. Moreover, the higher the correlation between the indices is, the lower the diversification benefit becomes.

Part D: Tobin separation

Once unlimited riskless lending and borrowing is accounted for in the investment models, the efficient frontier changes significantly. In particular, the efficient frontier becomes the upper one of two straight lines that are the result of the Tobin separation. Both of these lines pass through the point with null standard deviation and risk free rate as return and are tangents to the efficient frontier curve found in the previous section.

If the risk free rate is given as an additional input, the function `efficient_frontier` is used to calculate the Tobin separation. The values of the return `mu_front`, standard deviation `sigma_front` and the portfolio composition `port_comp` for each point of the efficient frontier (1...n) have been saved in matrix `A`. This matrix has been composed in the following way.

$$port\_comp(i) = \begin{pmatrix} Xstock1_i \\ Xstock2_i \\ \vdots \\ Xstock8_i \end{pmatrix} \tag{8}$$

$$A = \begin{pmatrix} R_1 & R_2 & \dots & R_n \\ \sigma_1 & \sigma_2 & \dots & \sigma_n \\ Xstock1_1 & Xstock1_2 & \dots & Xstock1_n \\ Xstock2_1 & Xstock2_2 & \dots & Xstock2_n \\ \vdots & \vdots & \ddots & \vdots \\ Xstock8_1 & Xstock8_2 & \dots & Xstock8_n \end{pmatrix} \tag{9}$$

- R_i: Return of the i-th portfolio in the selected window
- σ_i: Standard deviation of the i-th portfolio
- $Xstock\,j_i$: percentage invested in the j-th asset in the i-th portfolio

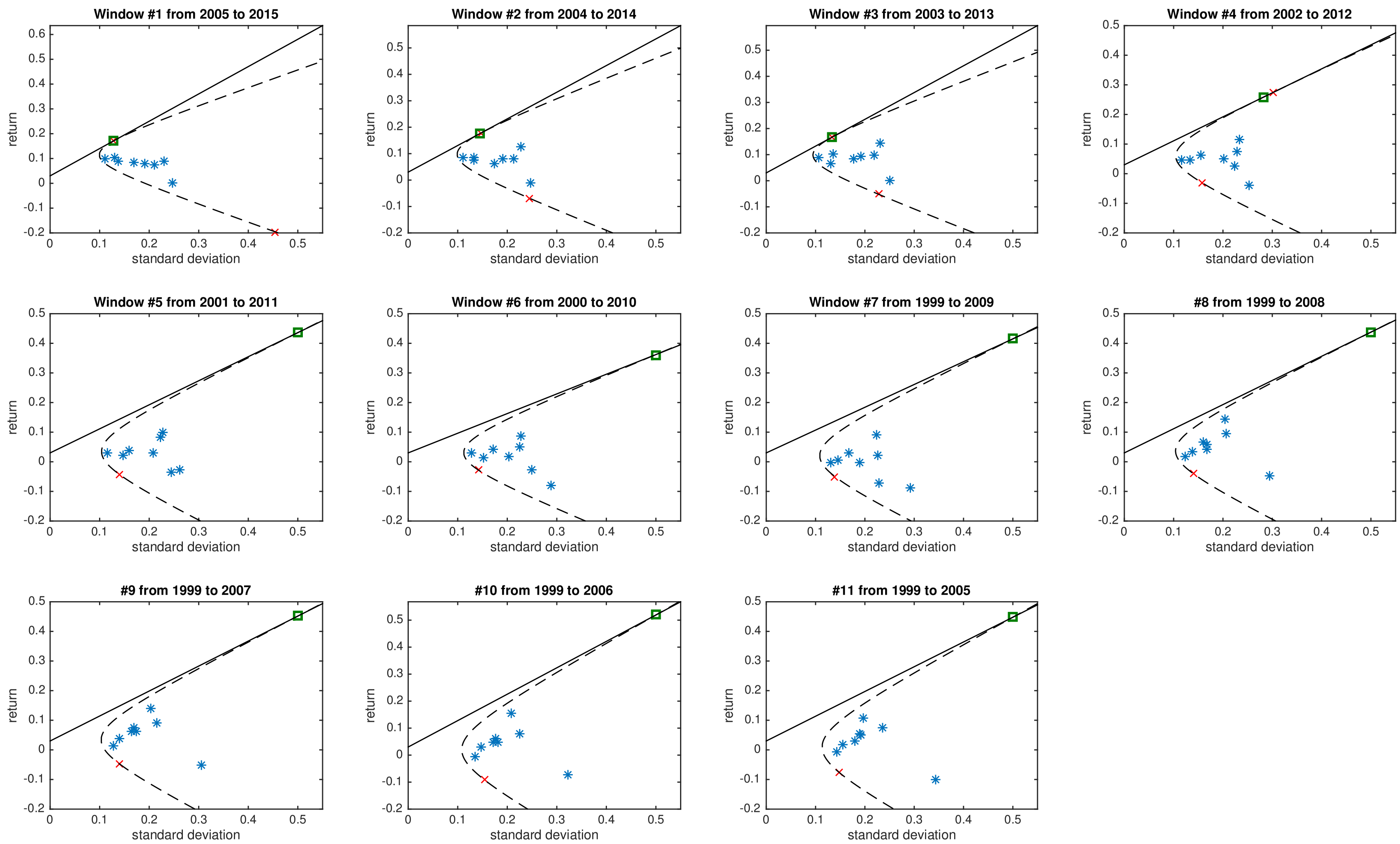

Figure 3: Efficient frontiers for the eleven rolling windows, calculated via Tobin separation (cf Table 12). Single indices displayed as $\star$, portfolios used to draw the previous efficient frontier as $\times$, optimal portfolio as $\square$. Previous efficient frontier depicted as dashed line, market line as full line.

A numerical technique has been developed in order to find the point of the efficient frontier where the Sharpe ratio is maximal. This step has been conducted because the analytical model that can be used to find the maximum slope gave negative returns for the optimal portfolio in some cases. Therefore, matrix `B` has been computed by sorting matrix `A`'s columns in ascendant order with respect to the return value.

$$B = \begin{pmatrix} R_{min} & \dots & R_{max} \\ \sigma(R_{min}) & \dots & \sigma(R_{max}) \\ Xstock1(R_{min}) & \dots & Xstock1(R_{max}) \\ Xstock2(R_{min}) & \dots & Xstock2(R_{max}) \\ \vdots & \ddots & \vdots \\ Xstock8(R_{min}) & \dots & Xstock8(R_{max}) \end{pmatrix} \tag{10}$$

Once the different points of the efficient frontier have been sorted in matrix `B`, a loop is run from the point of the curve with minimum standard deviation, to the point where the difference between the previous slope and the current one is not higher than 10^{-6} or standard deviation is equal to 0.5. This point of the efficient frontier will be the one where the market line is tangent to the efficient frontier.

The results of the Tobin Separation for each rolling window with a risk free rate of 3% are shown in Figure 3. Moreover, the composition of each optimal portfolio can be found in the Appendix in Table 12. Two other representations have been chosen to visualize the results. On the one hand, Figure 4 depicts the volatility and returns of the optimal portfolios, as they are shown in Figure 3. On the other hand, Figure 5 shows a comparison of the efficient frontiers for different rolling windows.

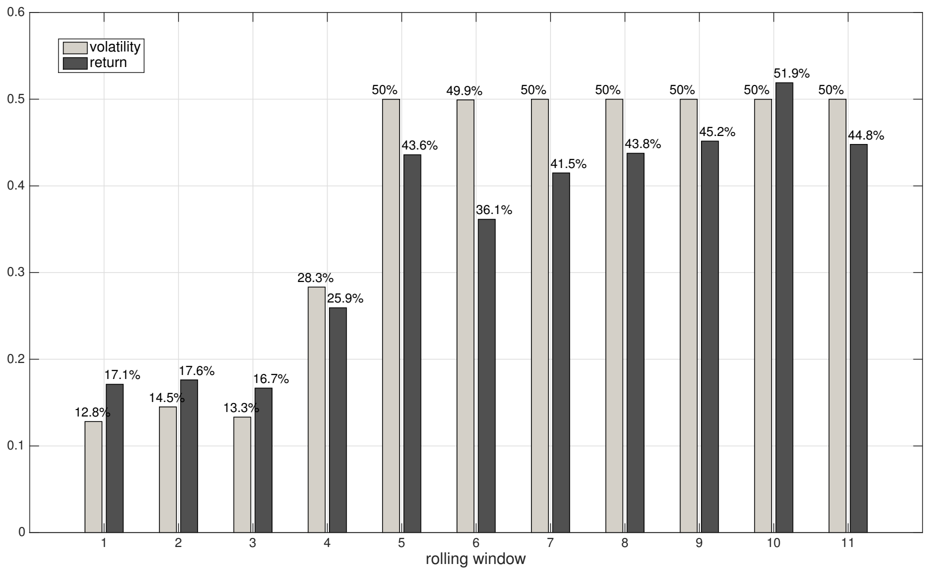

Figure 4: Volatility and returns of the optimal portfolios for the eleven rolling windows.

As one can see in Figure 3, the portfolio diversification benefits the investors in the form of a significant increment in returns for an equal amount of risk. By including the possibility to invest money in riskfree assets with a return of 3%, investors can adjust the amount of risk they want to take by splitting their investement between the optimal portfolio and the riskfree asset. Additionaly it is feasible to increase risk and return over the amount of the optimal portfolio by borrowing money and investing it in the optimal portfolio. In this case it has been assumed that money can be borrowed and lended at the same rate of 3% and one can borrow an infinite amount of money.

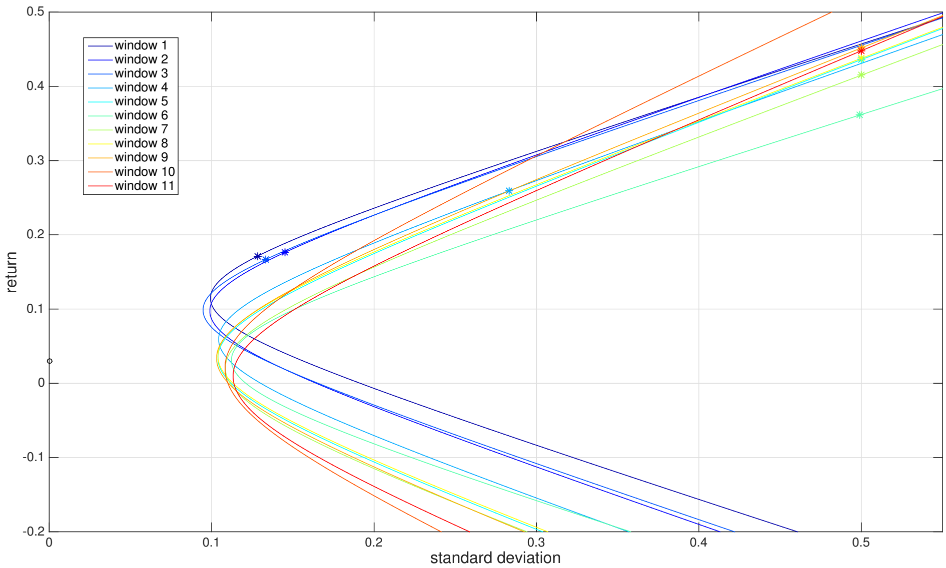

Figure 5: Efficient frontier comparison for the eleven rolling windows.

Comparing the different rolling windows (Figure 5), it has to be pointed out that the optimal portfolios in the last seven windows (windows 5-11) have significantly higher volatilities and expected returns than the ones in the first four portfolios. The explanation for that can be divided into an analytical and an economical part. Analitically, the average of the indices' returns in the latter windows is lower than for the other windows. Thus it is impossible to draw a tangent whose intersection with the previous efficient frontier has a low standard deviation. Therefore, the termination criterion constrains the optimal portfolio to be at a standard deviation of 0.5. Economically, this means that you cannot build a portfolio with return that overperforms the risk free rate without taking a high risk. For the chosen timeframe, this situation can perhaps be attributed to the Dot-com bubble and its collapse in 2000.

Part E: Asset allocation

To calculate the optimal asset allocation between the risk free asset and the computed optimal portfolios when requesting constant returns $\bar{R}$, the following equation (11) has been used. Additionally, Figure 6 visualizes the calculations made below.

$$\frac{100\% - X}{100\% - 0\%} = \frac{R_{optimal} - \bar{R}}{R_{optimal} - R_F} \tag{11}$$

- X: Percentage invested in the portfolio
- $\bar{R}$: requested constant return

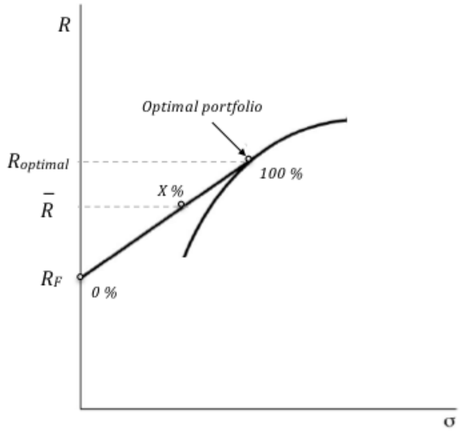

Figure 6: Asset allocation as a composition of risk free asset and optimal portfolio.

The turnover expresses change in portfolio composition from one year to another and has been calculated according to formula (12), when only the changes between the percentage invested in each asset are taken into consideration.

$$Turn(t) = \sum_{i=1}^{8} \left| w(i)_t - w(i)_{t-1} \right| \tag{12}$$

- $w(i)_t$: percentage of asset i at time t in the optimal portfolio
- $Turn(t)$: turnover at time t

In a more general case, where the difference between the investment in the risk free asset is also considered, the following formula (13) has been used:

$$Turn'(t) = \sum_{i=1}^{8} \left| X_t w(i)_t - X_{t-1} w(i)_{t-1} \right| + \big((1 - X_t) - (1 - X_{t-1})\big) \tag{13}$$

Table 2 shows the percentages invested in the optimal portfolio and the portfolio turnover for each window and two different choices for the requested return, respectively 5% and 10%. It can be pointed out that most of the fund is invested in the risk free rate, which means that the optimal choice for the investor in order to gain the requested return will be to lend money at the risk free rate.

window	year	opt portfolio (no R_f) σ	R	Turn	$\bar{R}$=5% % port	Turn'	$\bar{R}$=10% % portf	Turn'
1	-	12.8%	17.1%	-	14.2%	-	49.6%	-
2	2015	14.5%	17.6%	170.0%	13.7%	23.2%	47.9%	81.1%
3	2014	13.3%	16.7%	127.1%	14.6%	18.3%	51.2%	63.9%
4	2013	28.3%	25.9%	427.3%	8.7%	43.0%	30.5%	150.6%
5	2012	50.0%	43.6%	515.8%	4.9%	32.0%	17.2%	112.1%
6	2011	49.9%	36.1%	475.9%	6.0%	26.2%	21.1%	91.7%
7	2010	50.0%	41.5%	368.1%	5.2%	19.0%	18.2%	66.3%
8	2009	50.0%	43.8%	587.9%	4.9%	30.5%	17.2%	106.6%
9	2008	50.0%	45.2%	331.6%	4.7%	16.1%	16.6%	56.5%
10	2007	50.0%	51.9%	201.0%	4.1%	10.1%	14.3%	35.5%
11	2006	50.0%	44.8%	161.2%	4.8%	7.0%	16.8%	24.5%

Table 2: Percentage invested in the portfolio and turnover for the optimal portfolios of each rolling window.

Part F: Backtest

To conduct a backtest, the expected return in the following year has to be calculated for each rolling window. To do so, it is assumed that the portfolio composition of each rolling window is invested in the next year market and subsequently the corresponding revenues are computed. Whenever all money is invested in the optimal portfolio and therefore not split between this one and the risk free asset, the return out of sample will be computed in the following way:

$$Ros_{2015} = w_{(2004 \div 2014)} * R_{2015}$$

$$Ros_{2014} = w_{(2003 \div 2013)} * R_{2014}$$

$$\vdots$$

$$Ros_{2007} = w_{(1999 \div 2006)} * R_{2007}$$

$$Ros_{2006} = w_{(1999 \div 2005)} * R_{2006}$$

- Ros_i: Return out of sample `ret_out_of_sample` in the year i.
- w: Optimal portfolio composition for one rolling window.

However, if the percentage invested in the R_f is taken into account (considering the results of the optimal asset allocation for the case of the Tobin separation efficient frontier calculated in the previous sections and assuming a requested return $\bar{R} = 10\%$), the return out of sample will be calculated in the following way:

$$Ros_{rf,2015} = (100 - X_{(2004 \div 2014)}) * R_f + X_{(2004 \div 2014)} * w_{(2004 \div 2014)} * R_{2015}$$

$$Ros_{rf,2014} = (100 - X_{(2004 \div 2014)}) * R_f + X_{(2004 \div 2014)} * w_{(2003 \div 2013)} * R_{2014}$$
$$\vdots$$
$$Ros_{rf,2007} = (100 - X_{(2004 \div 2014)}) * R_f + X_{(2004 \div 2014)} * w_{(1999 \div 2006)} * R_{2007}$$
$$Ros_{rf,2006} = (100 - X_{(2004 \div 2014)}) * R_f + X_{(2004 \div 2014)} * w_{(1999 \div 2005)} * R_{2006}$$

- $Ros_{rf,i}$: Return out of sample considering the optimal asset allocation for the case of the Tobin separation efficient frontier `ret_out_of_sample_rf` in the year i.
- X: Percentage invested in the portfolio.
- $(100 - X)$: Percentage invested in the R_f.

The mean and standard deviation of the results of each case are shown in Figure 7. It is possible to see that the first choice, only investing in the optimal portfoilio as in Figure 3, corresponds to a situation where there is a high volatility in the returns out of sample ($\sigma = 0.414$). On the other hand a shared investment in the portfolio and the risk free asset is associated with a relatively low volatility ($\sigma = 0.075$). However, both options show a positive average return out of sample, respectively 0.031 (portfolio) and 0.037 (portfolio and risk free asset). Therefore it is advantageous to split your investment, since the average return is almost similar, but the prevision on future earnings is more reliable in the latter case.

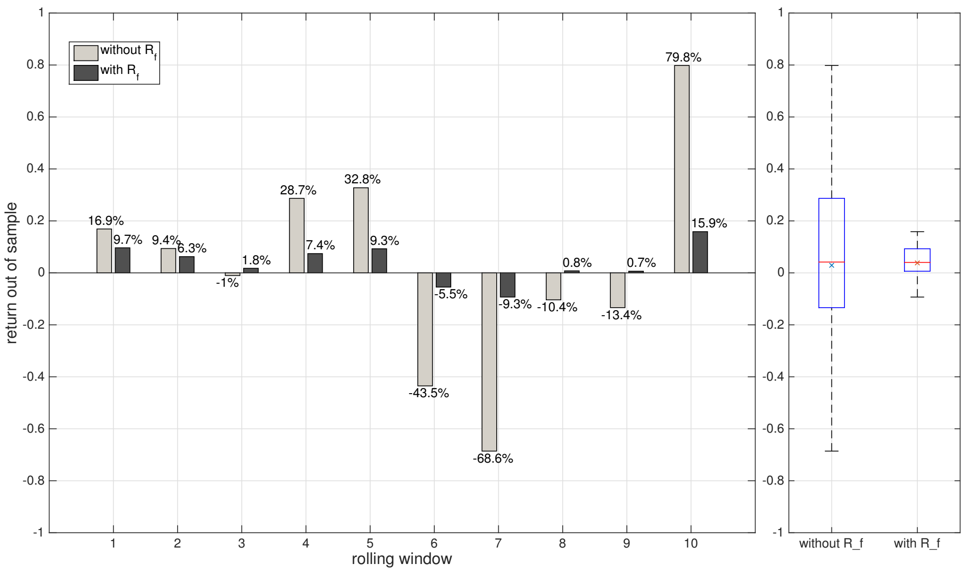

Figure 7: Out of sample returns for different investments for rolling windows 1-10 and respective statistics (mean $\times$, median (red line), 25th and 75th percentiles, whiskers).

Part G: Beta

To test whether the portfolio is in line with the CAPM prediction (14), the broad stock index that has been used is the S&P 500.

$$\bar{R}_P = R_f + (\bar{R}_M - R_f) \cdot \beta \tag{14}$$

- $\bar{R}_P$: Return out of sample.
- $\bar{R}_M$: Market return.

First of all, the yearly return of the market for the last 10 years (for which the return out of sample has been calculated) has been computed. Afterwards, a lineal regression has been developed to get the $\hat{\alpha}$ and $\hat{\beta}$ parameter estimates of equation (15).

$$(\bar{R}_P - R_f) = \alpha + (\bar{R}_M - R_f) \cdot \beta \tag{15}$$

Parameter	Estimate	pValue	Confidence Interval
α	-0.021	0.86697	$-0.307 \div 0.264$
β	0.890	0.17469	$-0.488 \div 2.269$

Table 3: CAPM test parameters (using the return out of sample calculated without taking into account the optimal asset allocation).

Parameter	Estimate	pValue	Confidence Interval
α	0.003	0.90303	$-0.049 \div 0.054$
β	0.168	0.15699	$-0.080 \div 0.416$

Table 4: CAPM test parameters (using the return out of sample taking into account the optimal asset allocation).

With respect to the question, our model has created an α, but it is not significant, since its p-value is higher than 0.05. Therefore it can be assumed to be null. With this result it is possible to say that our portfolio is in line with the CAPM prediction. If the optimal asset allocation is taken into account (Table 4), α becomes even less significant and the confidence interval becomes even closer to zero.

Part H: Black Litterman

In order to avoid some "classical problems" of the modern portfolio theory, e.g. extreme portfolio weights and a tendency to overweight assets (compare Part D), the Black-Litterman model has been developed.

As a first step, this model creates a neutral asset allocation based on the relative market capitalization of the different stocks in the portfolio. Subsequently, personal views on the evolution of the market are included. The following formulas have been used to calculate the new expected returns of the assets.

$$\Pi = \gamma \Omega w \tag{16}$$

$$E(R) = \Pi + \tau \Omega P^T (P \tau \Omega P^T)^{-1} (V - P\Pi) \tag{17}$$

- Π: Vector of reference excess returns
- γ: Risk aversion, in this simulation $\gamma = 4$
- Ω: Covariance matrix of expected returns
- w: Relative market capitalization
- $E(R)$: Expected returns
- τ: Precision, in this simulation $\tau = 0.3$
- P: Prediction matrix, in this simulation

$$P = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

- V: Vector of amount of predicted variation, in this simulation

$$V = \begin{pmatrix} 0.03 \\ 0.01 \end{pmatrix}$$

The weight of risky assets is negatively proportional to γ. This means, the higher the γ, the less risky assets one will hold. Since γ should be in the intervall of 2 to 4, a relatively conservative approach has been chosen. This is reflected in the efficient frontier in Figure 8 for the last seven windows. The composition of each optimal portfolio can be found in the Appendix in Table 13. With the Black-Litterman model the optimal portfolio is not as extreme as in part D. The views on future market development, reflected in the prediction matrix P and the vector V, are based on an underperformance of the technology sector in relation to the consumer staples sector in 3% and an overperformance of the heatlth care sector over the financial sector in 1%.

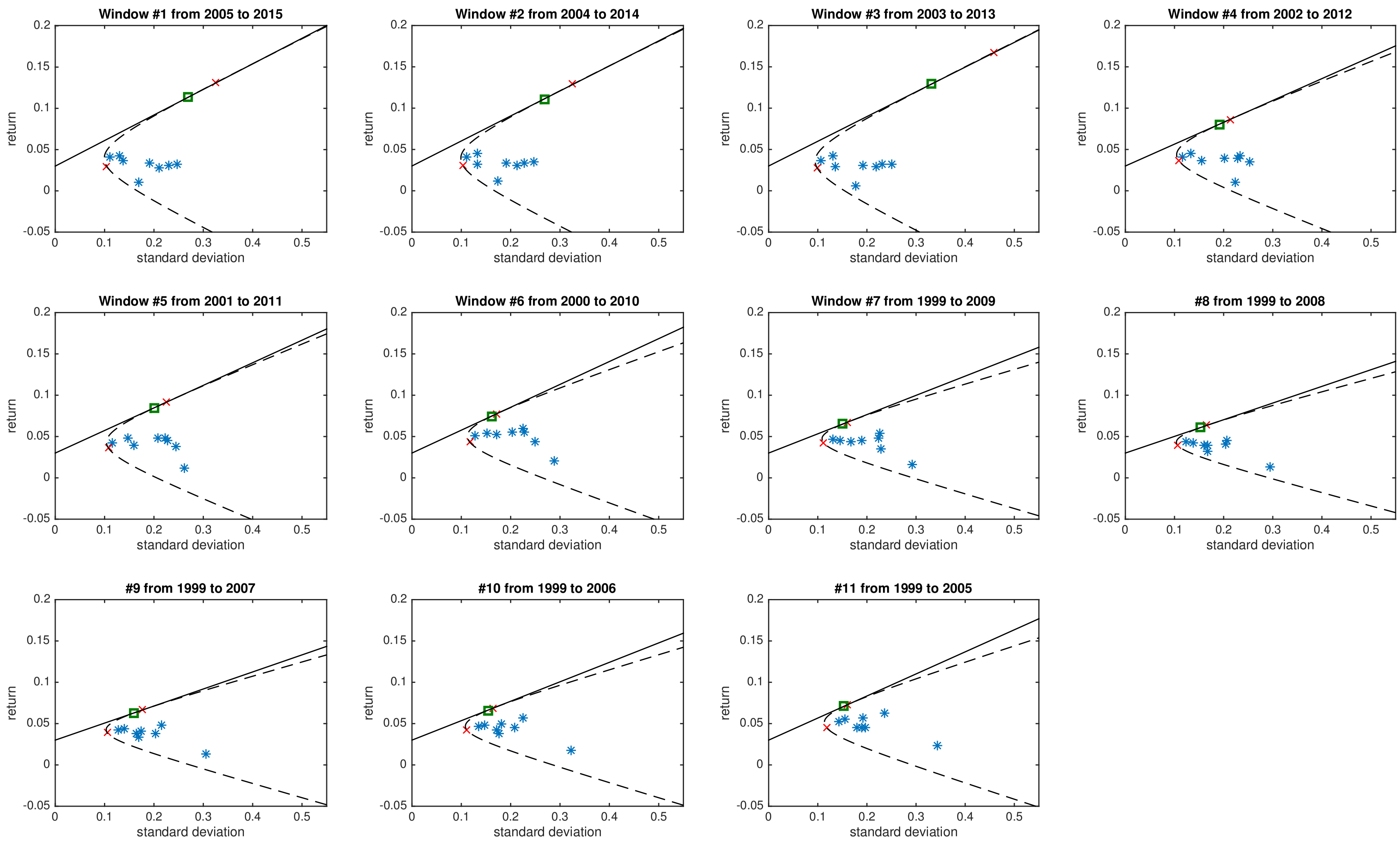

Figure 8: Efficient frontiers for the eleven rolling windows, calculated via Tobin separation and Black-Litterman. Single indices displayed as $\star$, portfolios used to draw the previous efficient frontier as $\times$, optimal portfolio as $\square$. Previous efficient frontier depicted as dashed line, market line as full line.

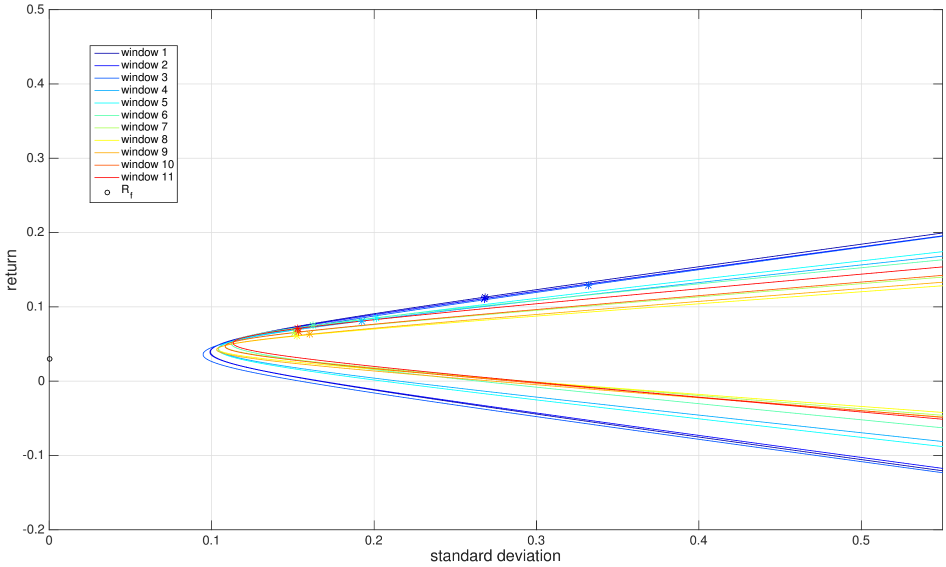

Figure 9: Efficient frontier comparison for the eleven rolling windows, expected returns computed with Black-Litterman model.

Figure 9 shows all new efficient frontiers that are computed with the Black-Litterman model. In comparison with the efficient frontiers depicted in Figure 5, these efficient frontiers seem to be much more stable over the timeframe of the eleven rolling windows. This result could be explained due to the fact that the same views have been chosen for all the rolling windows. Additionally, the optimal portfoilios show less extreme results.

		opt portfolio (no R_f)			$\bar{R}$=5%		$\bar{R}$=10%	
window	year	σ	R	Turn	% port	Turn'	% port	Turn'
1	-	26.9%	11.3%	-	24.0%	-	84.1%	-
2	2015	26.8%	11.1%	99.7%	24.7%	23.1%	86.3%	80.9%
3	2014	33.2%	12.9%	136.5%	20.1%	23.7%	70.5%	83.0%
4	2013	19.2%	8.1%	417.7%	39.4%	56.9%	137.8%	199.2%
5	2012	20.1%	8.5%	84.0%	36.4%	37.1%	127.2%	130.0%
6	2011	16.2%	7.5%	118.2%	44.5%	39.6%	155.9%	138.6%
7	2010	15.1%	6.5%	72.7%	56.9%	66.4%	199.2%	232.3%
8	2009	15.2%	6.1%	35.0%	65.1%	45.6%	227.8%	159.7%
9	2008	16.0%	6.3%	31.6%	60.5%	21.5%	211.7%	75.4%
10	2007	15.3%	6.6%	50.5%	55.4%	45.9%	193.9%	160.8%
11	2006	15.3%	7.1%	34.3%	48.9%	38.7%	171.1%	135.3%

Table 5: Percentage invested in the portfolio and turnover for the optimal portfolios of each rolling window, where the expected returns have been computed with the Black-Litterman model.

In comparison with Table 2, Table 5 shows a higher percentage invested in the portfolio. This also reflects in a change from lending to borrowing money in most of the cases because our optimal portfolios are characterized by less risk and are thereby more realistic. The turnovers for an investment only in the portfolio are lower, because the optimal portfolios are more similar to each other. In the portfolios for the fixed returns the turnovers are generally higher, but no easy explanation for this pattern was found.

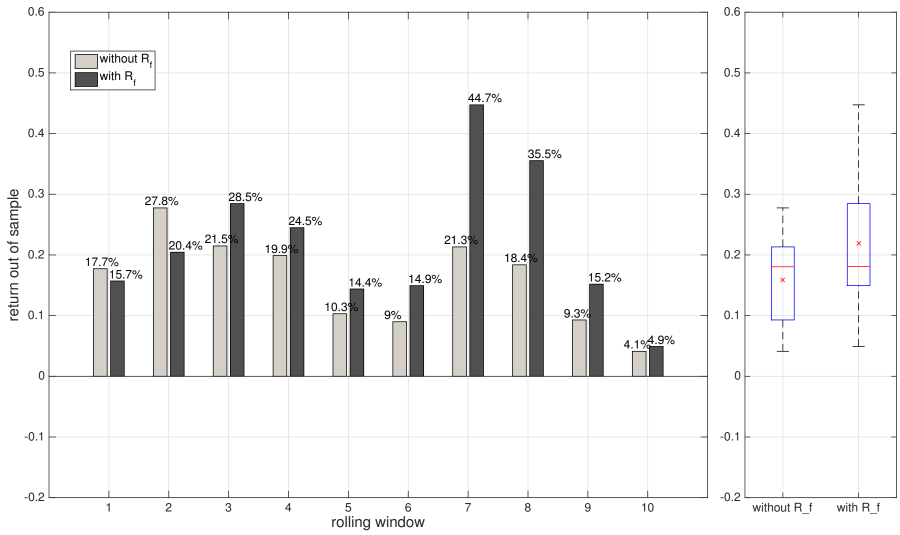

Figure 10: Out of sample returns for different investments for rolling windows 1-10 and respective statistics (mean, median, 25th and 75th percentiles, whiskers), when expected returns have been computed with the Black-Litterman model.

In Figure 10 it is possible to see that all out of sample returns are positive (respectively, the first case without the risk free asset with a mean of 0.1593 and a standard deviation of 0.0736 and the second case considering the risk free asset with a mean of 0.2188 and a standard deviation of 0.1172). This situation could be explained due to the fact that a risk aversion of $\gamma = 4$ has been chosen, which is relatively high, and our views could reflect the real development in the market during the years.

By using the Black-Litterman model, the Jensen's alpha becomes significantly different from zero (see tables 6 and 7). The differences in comparison to the CAPM model can be accounted to the higher complexity of the Black-Litterman model.

Parameter	Estimate	pValue	Confidence Interval
α	0.132	5.3877e-04	$0.077 \div 0.187$
β	-0.102	0.39993	$-0.366 \div 0.162$

Table 6: CAPM test parameters (Using the return out of sample without taking into account the optimal asset allocation), when expected returns have been computed with the Black-Litterman model.

Parameter	Estimate	pValue	Confidence Interval
α	0.200	4.3419e-05	$0.142 \div 0.257$
β	-0.421	0.0080866	$-0.698 \div -0.143$

Table 7: CAPM test parameters (Using the return out of sample when taking into account the optimal asset allocation), when expected returns have been computed with the Black-Litterman model.

Part I: Timing

The Treynor-Mazuy measure has been calculated by computing a regression over equation (18).

$$(\bar{R}_P - R_f) = a + b * (\bar{R}_M - R_f) + c * (\bar{R}_M - R_f)^2 + e \tag{18}$$

- $\bar{R}_P$: Return out of sample. The regression has been realized in two different ways. In the first case, the return out of sample has been considered without asset allocation. In the second case, the return out of sample considering the optimal asset allocation of the Tobin separation efficient frontier has been used.
- $\bar{R}_M$: Market return.
- a, b, c and e: Regression coefficients.

By analysing the results of the regressions shown in Figures 11 and 12 and considering the position of the different points with respect to the regression line, it can be concluded that it is not possible to beat the market in all cases with this portfolio.

Parameter	Estimate	pValue	Confidence Interval
a	0.221	0.20927	$-0.157 \div 0.599$
b	-0.760	0.45636	$-3.038 \div 1.519$
c	-4.695	0.083677	$-10.204 \div 0.813$

Table 8: Treynor-Mazuy test parameters (Using the return out of sample without taking into account the optimal asset allocation)

Parameter	Estimate	pValue	Confidence Interval
a	0.043895	0.1821	$-0.0262 \div 0.1140$
b	-0.11195	0.55083	$-0.5344 \div 0.3105$
c	-0.79606	0.10787	$-1.8174 \div 0.2253$

Table 9: Treynor-Mazuy test parameters (Using the return out of sample taking into account the optimal asset allocation, with a requested return of $\bar{R} = 10\%$).

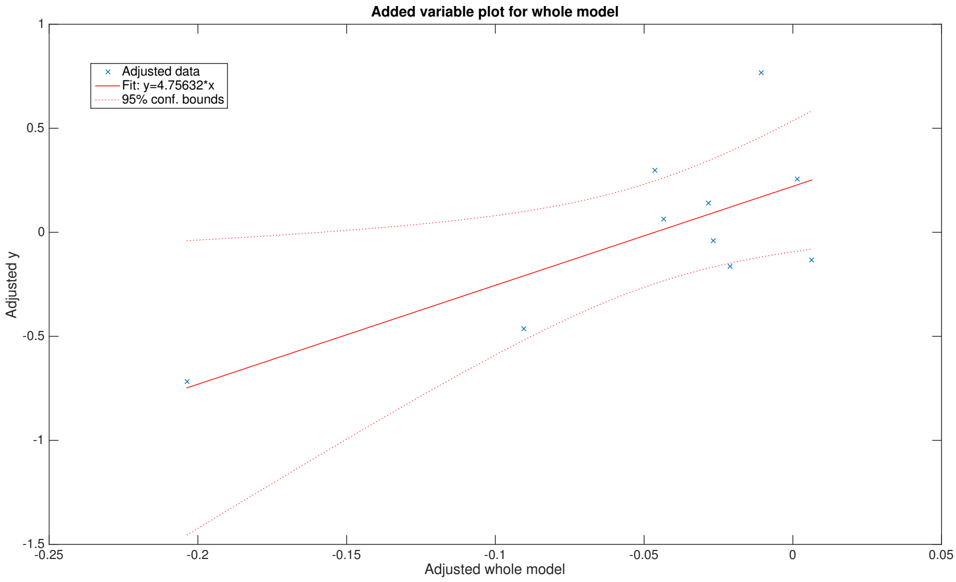

Figure 11: Treynor-Mazuy regression (see Table 8).

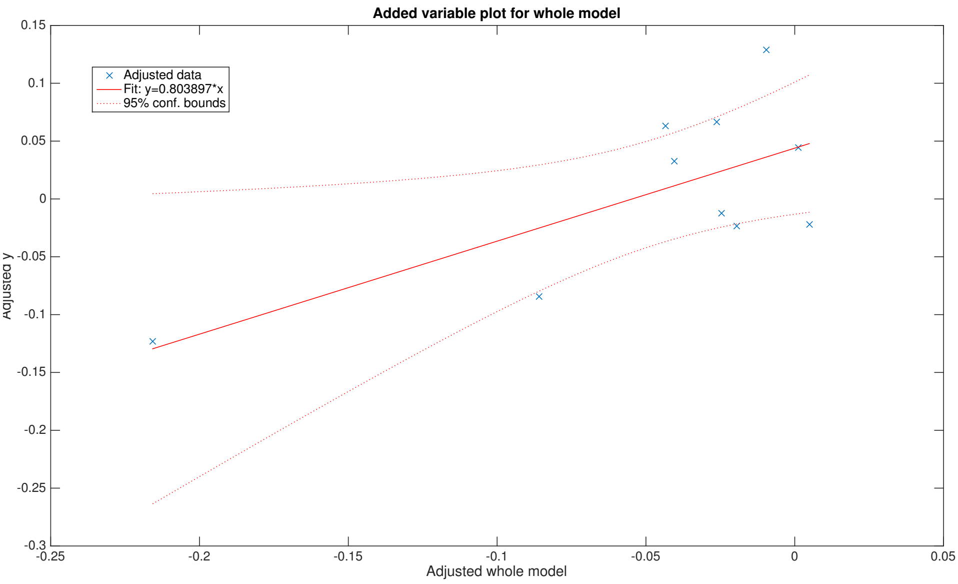

Figure 12: Treynor-Mazuy regression (see Table 9).

2 Bonds

Part A

The following five bonds have been chosen randomly to conduct the second exercise:

1. **GENERAL ELECTRIC CAPITAL CORP**, with a clean price of 107.50 USD and a coupon of 4.5%.
2. **NATIONSBANK CORP**, with a clean price of 127.38 USD and a coupon of 7.25%.
3. **CITIGROUP INC**, with a clean price of 113.13 USD and a coupon of 5.5%.
4. **FEDERAL NATL MTG ASSN**, with a clean price of 100.95 USD and a coupon of 3%.
5. **MERCK & CO INC**, with a clean price of 131.75 USD and a coupon of 6.3%.

Each of these bonds got a semiannual payout. The full data available in Yahoo! Finance for each bond can be found in Table 14 in the Appendix. The datasets have been collected at the 11th November 2015.

Thereupon, the yield to maturity has been computed using formula (19), considering the different number of payouts in the evaluated 10 years (see Table 10). In particular, the right side of the equation is composed as a sum of the payout for the previous owner of the bond as a compensation for the next coupon, the future semiannual payouts and the final payout of the bond.

$$CP = -\frac{\frac{C(t)}{2} \cdot (1 - \frac{n}{180})}{(1 + \frac{y}{2})^{\frac{n}{180}}} + \sum_{t=1}^{T} \frac{\frac{C(t)}{2}}{(1 + \frac{y}{2})^{t-1+\frac{n}{180}}} + \frac{FP}{(1 + \frac{y}{2})^{T-1+\frac{n}{180}}} \tag{19}$$

- CP: Clean price
- C: Annual cash flow (exluding the final payment)
- n: Days to payout
- y: Annual yield to maturity
- FP: Final payment

	bond 1	bond 2	bond 3	bond 4	bond 5
clean price in USD (CP)	107.50	127.38	113.13	100.95	90.00
days to payout (n)	4	156	123	10	13
remaining payouts (T)	21	20	20	21	21

Table 10: Situation for the chosen bonds at the 11th of November

Based on the computed yield to maturity, it is possible to calculate durations and convexities for each bond. These values are a measure of the sensitivity of the bond price to a change in interest rates, where the first one is computed as the first derivative of the price with respect to the yield and the latter one as the second derivative of the same arguments.

The following equations (20) and (21) were used for calculating these values, as suggested in Chapter 22 of the course-related textbook, which does not consider the effect of the dirty price. The results can be found in Table 11.

$$D = \frac{\sum_{t=1}^{T} \frac{t \cdot \frac{C}{2}}{(1+\frac{y}{2})^t} + \frac{T \cdot FP}{(1+\frac{y}{2})^T}}{CP} \tag{20}$$

$$Co = \frac{\sum_{t=1}^{T} \frac{t \cdot (t+1) \cdot \frac{C}{2}}{(1+\frac{y}{2})^t} + \frac{T \cdot (T+1) \cdot FP}{(1+\frac{y}{2})^T}}{2 \cdot CP} \tag{21}$$

- D: duration
- Co: convexity

	bond 1	bond 2	bond 3	bond 4	bond 5
yield to maturity	3.64%	4.01%	3.94%	2.89%	2.82%
duration	17.13	15.13	15.89	18.18	16.40
convexity	175.06	143.29	153.51	190.17	164.50

Table 11: Computed yield to maturity, durantion and convexity.

Part B

Now it is supposed to create a portfolio of these bonds, investing 100.000 USD in each of them. The portfolio duration and convexity can be easily calculated as the weighted mean of the bond durations and convexities. Since in the suggested porfolio the bonds are all equally weighted, the used equations and values are:

$$D_{portfolio} = \frac{1}{5} \sum_{bonds} D_{bond_i} = 16.55 \tag{22}$$

$$Co_{portfolio} = \frac{1}{5} \sum_{bonds} Co_{bond_i} = 165.31 \tag{23}$$

Part C

Given the previous results, it is possible to compute the variation of the market value of the given portfolio whenever the yield increases by 150 basis points.

$$\Delta CP = -D \cdot \Delta y + C \cdot (\Delta y)^2 = -22.96\% \tag{24}$$

As it can be seen from the negative result of equation 24, the increase in yield results in a decrease of the portfolio market value, as expected. In particular, starting from a portfolio of 500.000 USD, the new price will be 385.200 USD.

Appendix

window	XLF	XLE	XLP	XLK	XLI	XLV	XLB	XLU
1	-71.6%	1.6%	91.4%	6.7%	32.6%	64.1%	-8.3%	-16.6 %
2	-84.1%	18.4%	100.4%	-28.4%	72.1%	27.1%	-8.6%	3.2 %
3	-70.9%	12.5%	106.0%	-9.8%	55.4%	-13.9%	-2.8%	23.5 %
4	-170.6%	39.9%	68.5%	-65.1%	106.5%	52.6%	65.7%	2.3 %
5	-203.8%	66.2%	99.5%	-152.8%	29.6%	37.2%	266.4%	-42.4 %
6	-157.5%	114.4%	36.2%	-171.8%	85.2%	112.1%	110.8%	-29.4 %
7	-187.8%	152.3%	-10.7%	-146.8%	103.4%	198.3%	3.9%	-12.7 %
8	3.2%	204.7%	-131.5%	-143.0%	53.3%	78.2%	50.6%	-15.5 %
9	169.0%	187.5%	-164.5%	-146.9%	45.7%	77.9%	-5.2%	-63.5 %
10	136.4%	222.6%	-149.6%	-130.8%	7.8%	112.2%	-17.8%	-80.8 %
11	127.5%	192.8%	-128.1%	-143.5%	54.6%	97.8%	-5.5%	-95.6 %

Table 12: Portfolio composition, as in Figure 3

window	XLF	XLE	XLP	XLK	XLI	XLV	XLB	XLU
1	-33.6%	18.1%	158.0%	-254.4%	73.6%	126.0%	28.4%	-16.1%
2	-32.7%	22.1%	147.3%	-229.6%	54.4%	139.9%	34.6%	-36.1%
3	-25.2%	30.6%	141.6%	-290.9%	77.8%	165.6%	37.8%	-37.3%
4	-32.8%	14.6%	72.8%	-112.7%	52.3%	95.7%	16.9%	-6.6%
5	-39.8%	10.6%	44.5%	-97.7%	72.2%	98.4%	21.2%	-9.4%
6	-44.3%	7.5%	37.5%	-48.0%	48.2%	84.2%	14.9%	0.0%
7	-43.8%	9.2%	59.8%	-42.8%	25.0%	80.5%	21.7%	-9.4%
8	-54.6%	7.8%	61.6%	-42.0%	20.5%	93.1%	20.9%	-7.2%
9	-57.8%	4.3%	59.9%	-44.2%	30.2%	98.7%	21.3%	-12.4%
10	-49.4%	6.7%	56.8%	-36.7%	37.1%	84.4%	13.8%	-12.8%
11	-42.6%	-1.0%	56.8%	-31.2%	41.1%	79.1%	9.6%	-12.0%

Table 13: Portfolio composition with Black-Litterman model, as in Figure 8

	bond 1	bond 2	bond 3	bond 4	bond 5
Price	107.50	127.38	113.13	100.95	131.75
Coupon	4.50%	7.25%	5.50%	3.00%	6.300%
Maturity date	15 Nov 2025	15 Oct 2025	13 Sep 2025	21 Nov 2025	1 Jan 2026
Yield to maturity	3.66%	4.10%	3.99%	2.90%	2.923%
Current yield	4.19%	5.69%	4.86%	2.97%	4.782%
Fitch ratings	AA	BBB	BBB	AAA	AA
Coupon frequency	semi-annual	semi-annual	semi-annual	semi-annual	semi-annual
First coupon	15 May 2011	15 Apri 1996	13 Mar 2014	21 Nov 2012	1 Jul 1996
Type	corporate	corporate	corporate	corporate	corporate
Callable	no	no	no	yes	no

Table 14: Bonds data