International Astronomy and Astrophysics Competition 2025

Qualification Round Solutions

Solutions by Md Irfan

Solution of problem A:

The components of the reflector telescope labeled A to I are:

- A: Eyepiece
- B: Focuser
- C: Optical Tube
- D: Telescope tube
- E: Mirror-cell primary
- F: Mount base
- G: Outer tripod leg
- H: Accessory tray
- I: Tripod leg extension

Solution of problem B:

We will use following values to find our answer

- Diameter of Sun = 1,400,000 km $\Rightarrow$ 22 cm
- Diameter of Earth = 12,750 km
- Earth-Sun distance = 1 AU = 1.496×10^8 km
- Distance to Alpha Centauri = 4.25 light-years
- 1 light-year $\approx 9.461 \times 10^{12}$ km

Scale factor:

$$\text{Scale} = \frac{22 \text{ cm}}{1.4 \times 10^6 \text{ km}} = 1.5714 \times 10^{-5} \text{ cm/km}$$

(i) Size of Earth:

$$12750 \times 1.5714 \times 10^{-5} = 0.2 \text{ cm}$$

(ii) Distance Earth-Sun:

$$1.496 \times 10^8 \times 1.5714 \times 10^{-5} \approx 2.35 \times 10^3 \text{ cm}$$

(iii) Distance to Alpha Centauri:

$$4.25 \times 9.461 \times 10^{12} \times 1.5714 \times 10^{-5} \approx 6.32 \times 10^8 \text{ cm}$$

Solution of problem C:

(a) Derivation

Using the gravitational acceleration:

$$g = \frac{GM}{R^2} \Rightarrow M = \frac{gR^2}{G}$$

where G=γ
$Volume of sphere: V = \frac{4}{3}\pi R^3$ Density:

$$\rho = \frac{M}{V} = \frac{gR^2}{G \cdot \frac{4}{3}\pi R^3} = \frac{3g}{4\pi GR}$$

Thus:

$$\rho(g, R) = \frac{3g}{4\pi\gamma R}$$

(b) Calculation of density

Given:

$$g = 9.81 \text{ m/s}^2, \quad R = 12750/2 = 6.375 \times 10^6 \text{ m}, \quad \gamma = 6.674 \times 10^{-11} \text{ m}^3/\text{kg/s}^2$$

$$\rho = \frac{3 \times 9.81}{4\pi \cdot 6.674 \times 10^{-11} \cdot 6.375 \times 10^6} \approx 5515 \text{ kg/m}^3$$

Solution of problem D:

From the question ,we have:

- Scale factor: $a(t) = \lambda t^\beta$, with λ, β *being real numbers*
- Hubble parameter: $H(t) = \frac{\dot{a}(t)}{a(t)}$

- Deceleration parameter:

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right)$$

- Age of the universe: $t_0 = 13.7$ billion years $\approx 4.32 \times 10^{17}$ seconds
- Current Hubble parameter: $H_0 \approx 72.6\,\text{km/s/Mpc} \approx 2.35 \times 10^{-18}\,\text{s}^{-1}$

Compute $H(t)$

Given $a(t) = \lambda t^{\beta}$, we differentiate:

$$\dot{a}(t) = \lambda \beta t^{\beta-1}$$

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{\lambda \beta t^{\beta-1}}{\lambda t^{\beta}} = \frac{\beta}{t}$$

Compute $\dot{H}(t)$

$$\dot{H}(t) = \frac{d}{dt}\left(\frac{\beta}{t}\right) = -\frac{\beta}{t^2}$$

Compute $\frac{\dot{H}}{H^2}$

$$H^2 = \left(\frac{\beta}{t}\right)^2 = \frac{\beta^2}{t^2}, \quad \Rightarrow \quad \frac{\dot{H}}{H^2} = \frac{-\beta/t^2}{\beta^2/t^2} = -\frac{1}{\beta}$$

Compute the deceleration parameter q

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) = -\left(1 - \frac{1}{\beta}\right) = \frac{1}{\beta} - 1$$

Determine the sign of q

- Accelerating universe: $q < 0 \Rightarrow \frac{1}{\beta} - 1 < 0 \Rightarrow \beta > 1$
- Decelerating universe: $q > 0 \Rightarrow \beta < 1$

Estimate β

$$\beta = H_0 t_0 = (2.35 \times 10^{-18}) \times (4.32 \times 10^{17}) \approx 1.015$$

Compute q numerically

$$q = \frac{1}{\beta} - 1 = \frac{1}{1.015} - 1 \approx 0.9852 - 1 = -0.0148$$

Conclusion

- $\beta \approx 1.015$
- $q \approx -0.015$
- Since $q < 0$, the expansion of the universe is **accelerating**.

Solution of problem E:

Composition of a comet: Comets consist of the following materials:

- Water ice
- Carbon dioxide, methane, ammonia
- Dust and rocky particles

Explanation of formation of the bright tail:

- As a comet approaches the Sun, solar radiation causes the ice to sublimate into gas.
- Gases and dust are ejected, forming two types of tails:
  - **Ion tail**: pushed directly away from the Sun by solar wind.
  - **Dust tail**: curves due to solar radiation pressure.
- These tails reflect sunlight, making them visible from Earth, sometimes even in daylight.