IMN-359

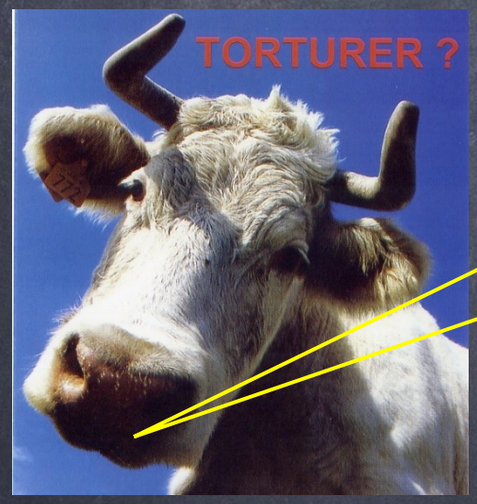
Labo
Transformée de Fourier Discrète et
FFT (Fast Fourier Transform)

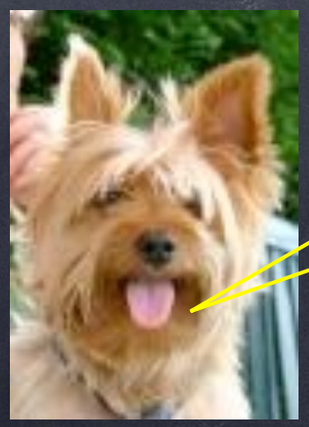
T.F discrète

- La transformée de Fourier

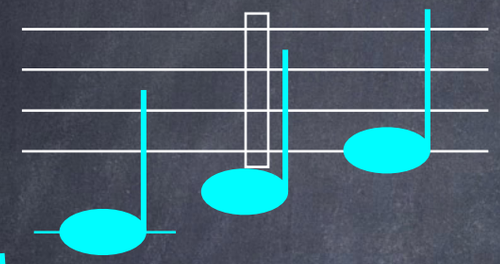
Notion de transformée de Fourier (TF)

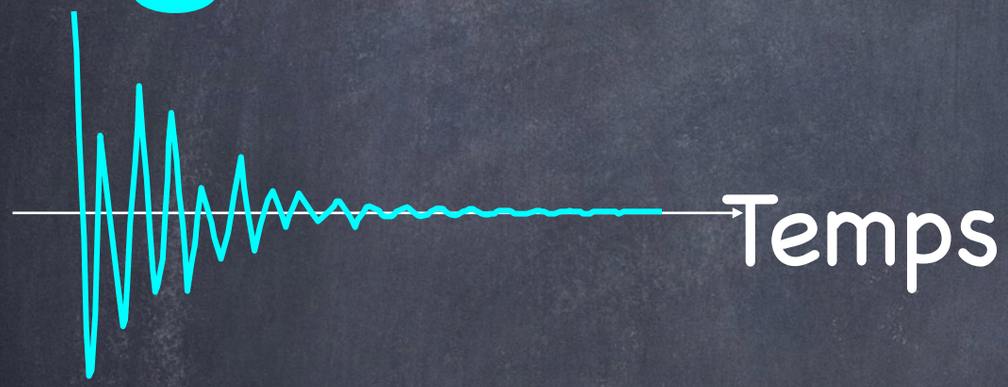
Analogie : son = vibration qui se propage dans le temps

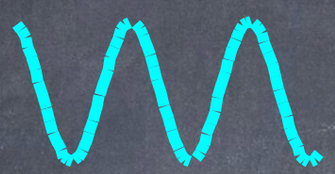


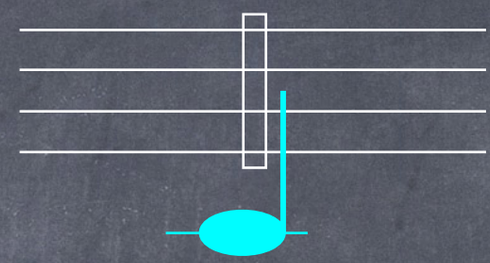


On entend non pas une vibration (fonction du temps) mais une note donc une fréquence

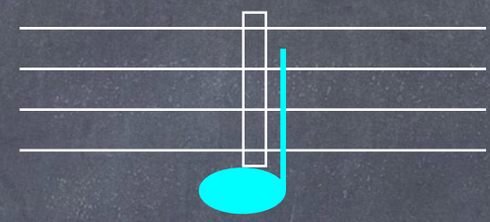




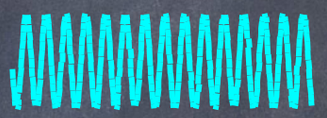


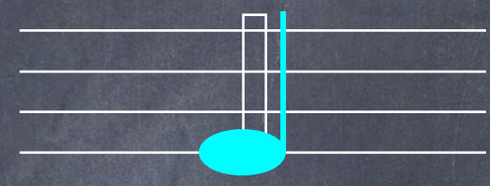


DO



MI

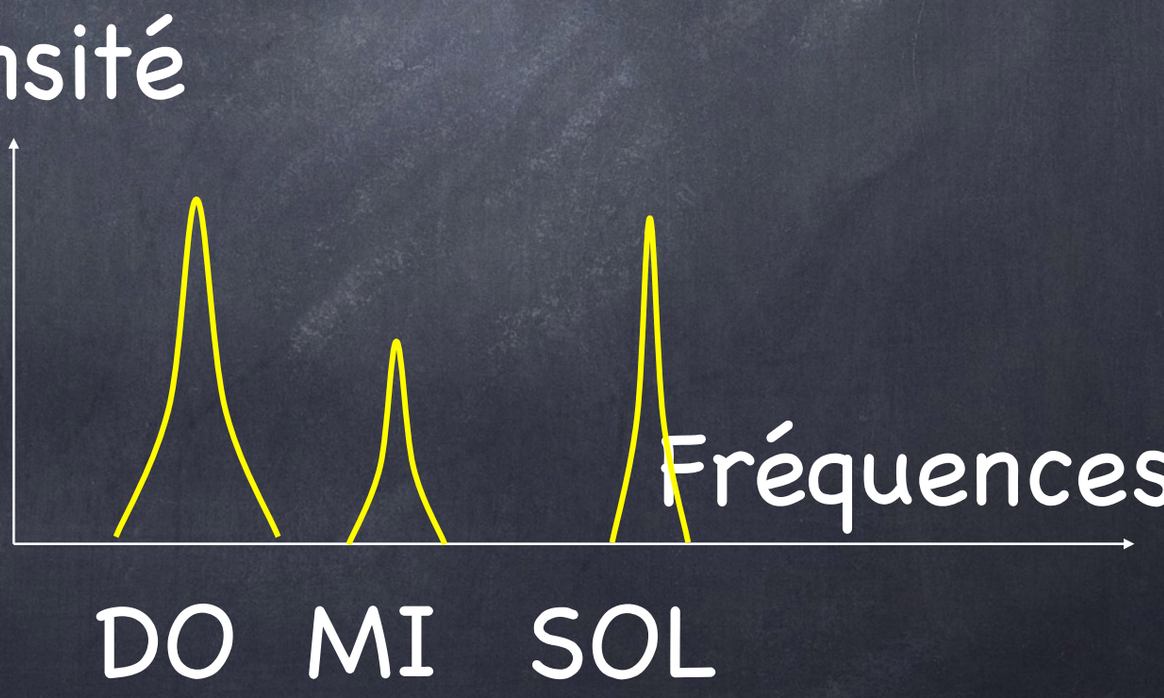




SOL

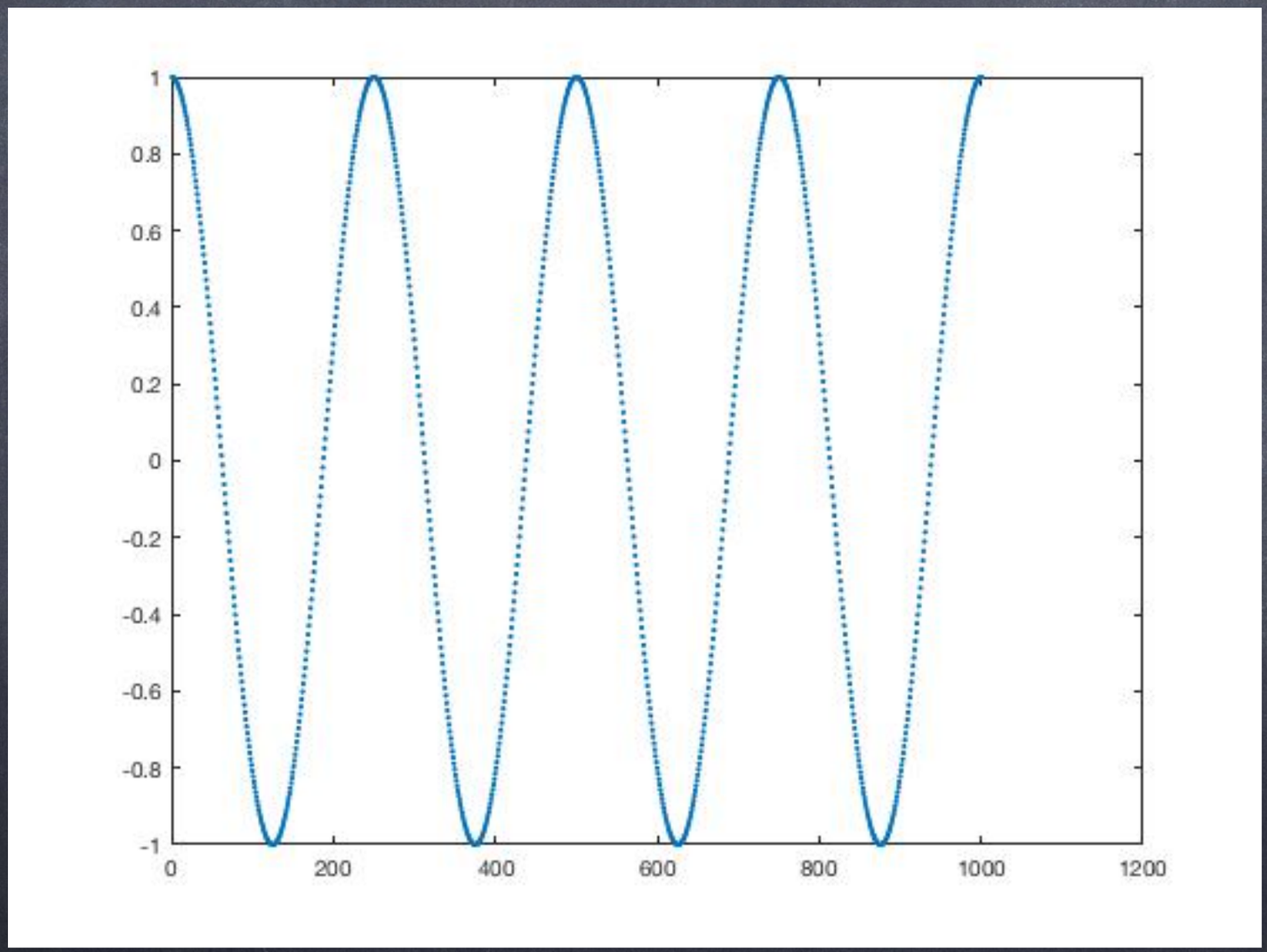
Transformée de Fourier : donner le « poids » relatif d'une fréquence dans un signal

TF



Exercice 1D

1. Faites-moi le plot d'un cos qui oscille k0 fois par période de N pts (ici N = 4)
   y = cos(2*pi/N*k0*n)



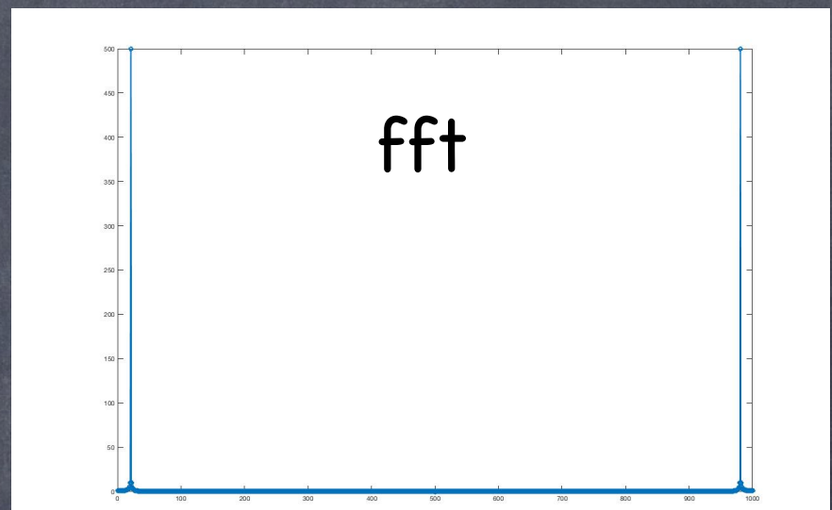
Exercice 1D

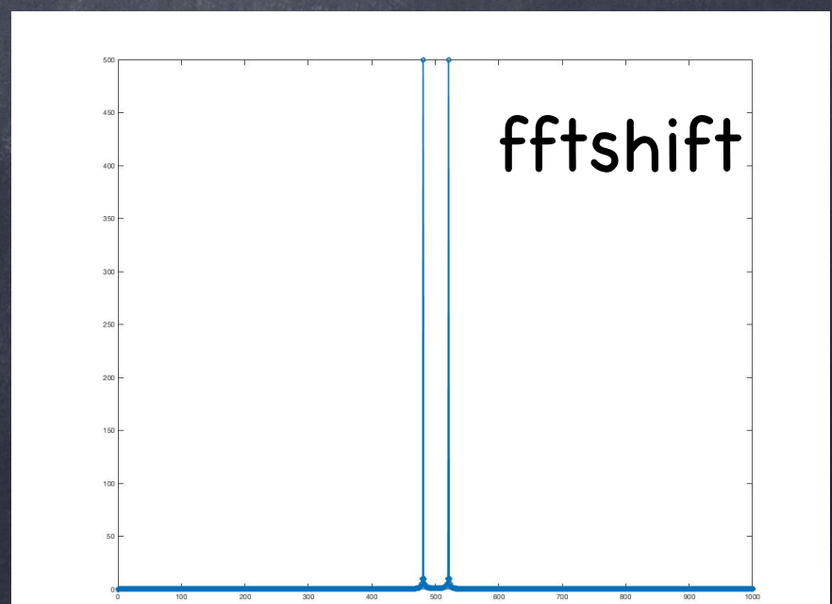
1. En theorie, la TF analytique de:
   TF[cos(k0*t)] = dirac(w-k0) + dirac(w+k0)
   (ici k0 = 20)

2. Faites la FFT de votre cos numérique

- Visualiser-le (real, abs, imag)

- Visualiser la version fftshift





Observations

- Il y a toujours des petits complexes qui trainent. Le real ou abs seront nécessaires pour faire les plot
- La TFD du cosinus qui oscille à la fréquence k0 sur N points allume une fréquence à k0 et à -k0 (N - k0 dans un tableau python)

Q1: En utilisant la TF, comment faire un plot de

$$f(x) = \cos\left(\frac{2\pi}{N} k_0 x\right) \quad ?$$

k_0 → fréquence

$$\hat{f}(k_1) = \sum_{n=0}^{N-1} \cos\left(\frac{2\pi}{N} k_0 n\right) e^{-\frac{2\pi i n k_1}{N}}$$

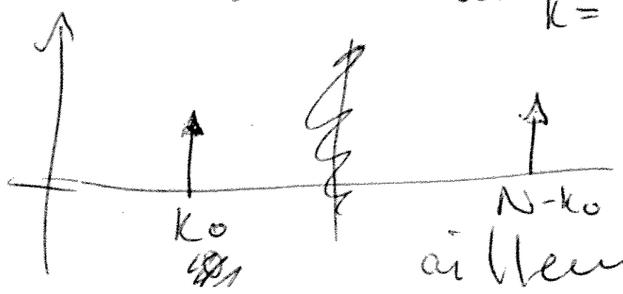
$$= \sum_{n=0}^{N-1} \left(\frac{e^{i\frac{2\pi k_0 n}{N}} + e^{-i\frac{2\pi k_0 n}{N}}}{2} \right) e^{-\frac{2\pi i n k_1}{N}}$$

$$= \frac{1}{2} \sum_{n=0}^{N-1} e^{\frac{i 2\pi n}{N}(k_1 - k_0)} + e^{-\frac{2\pi i n}{N}(k_1 + k_0)}$$

$$= \frac{1}{2} \sum_{n=0}^{N-1} \left(1 + e^{-\frac{2\pi i n 2k_1}{N}} \right)$$

→ Orthogonalité de la base de Fourier

$$= \frac{N}{2} \quad \text{si } k = k_0 \text{ ou } k = N - k_0$$



Exercice 1D #2

- Générez-moi un cos qui oscille N fois par période sans utiliser la fonction cos !

  - Remplir un vecteur de Fourier **vide**
  - Allumer les bonnes fréquences
  - Visualiser la ifft

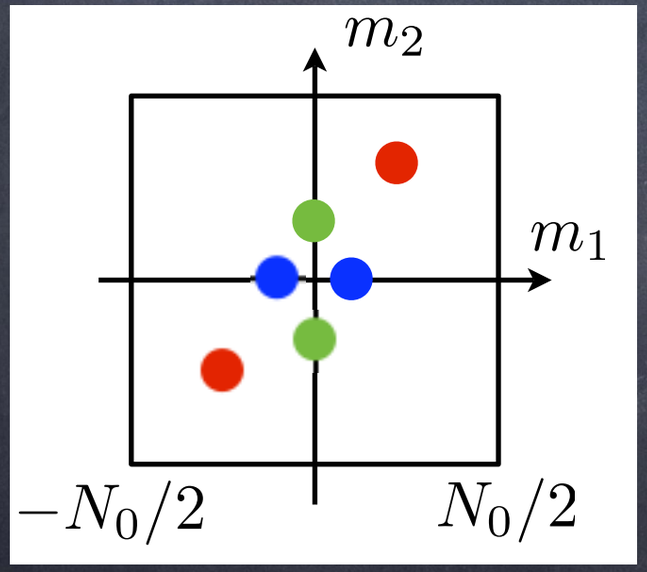
  (attention au shift et imaginaires)

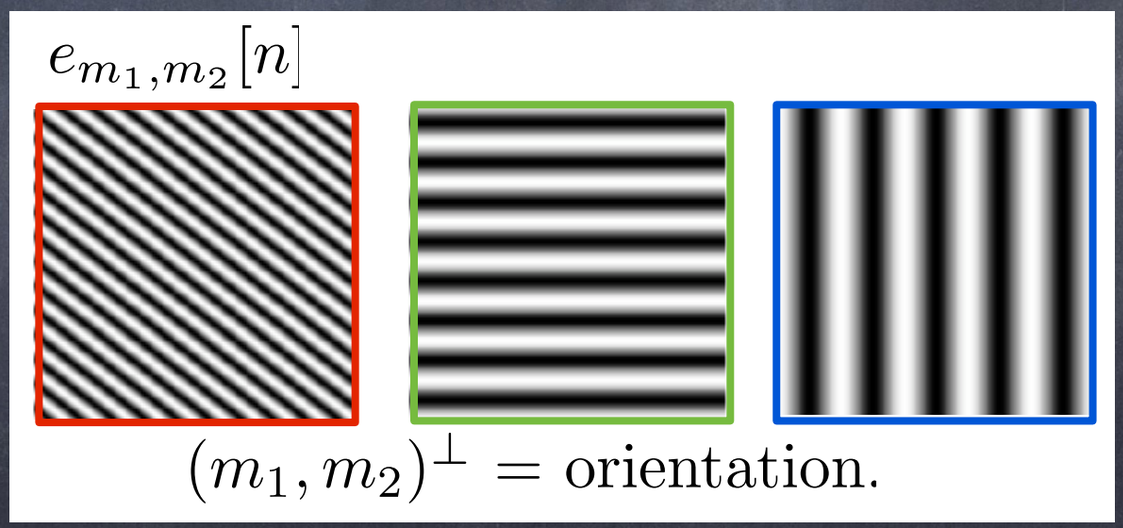
2D discrete Fourier Basis

2D discrete Fourier basis: $N = N_0 \times N_0$ pixels

$$e_m[n] = \frac{1}{\sqrt{N}} e^{\frac{2i\pi}{N_0} m_1 n_1 + \frac{2i\pi}{N_0} m_2 n_2} = e_{m_1}[n_1] e_{m_2}[n_2]$$

Frequency $m = (m_1, m_2) \in \{0, \ldots, N_0 - 1\} \times \{0, \ldots, N_0 - 1\}$



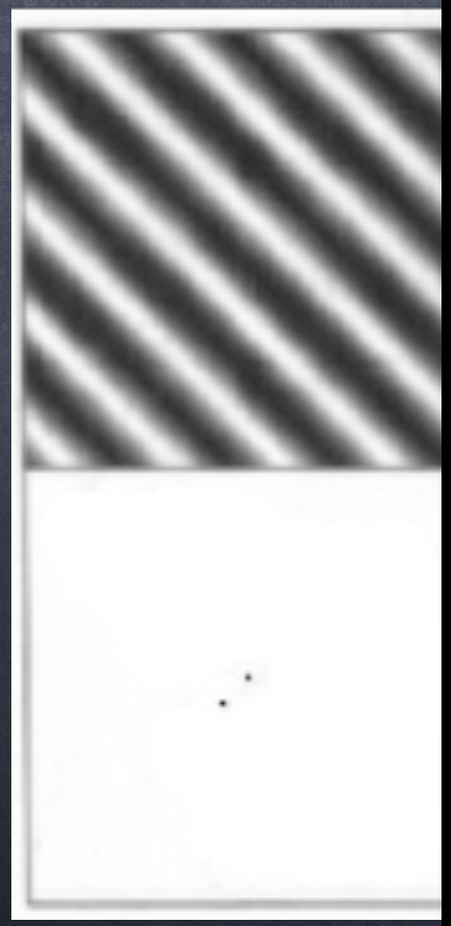


[Peyré, Numerical Tour of Signal Processing]

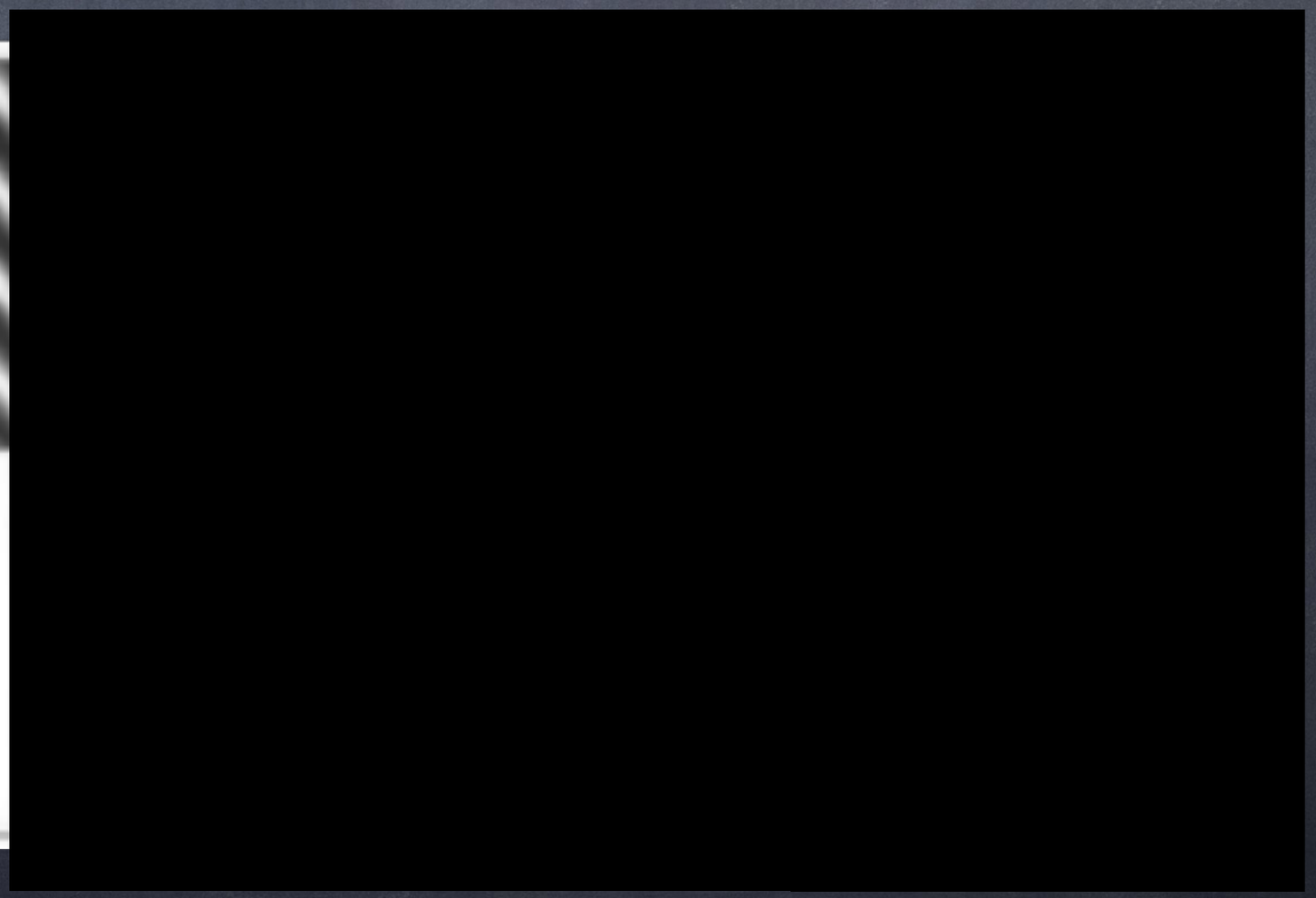
Et l'image, elle?

- Une image peut elle aussi être vue comme une somme d'ondes qui oscillent à différentes fréquences

Image

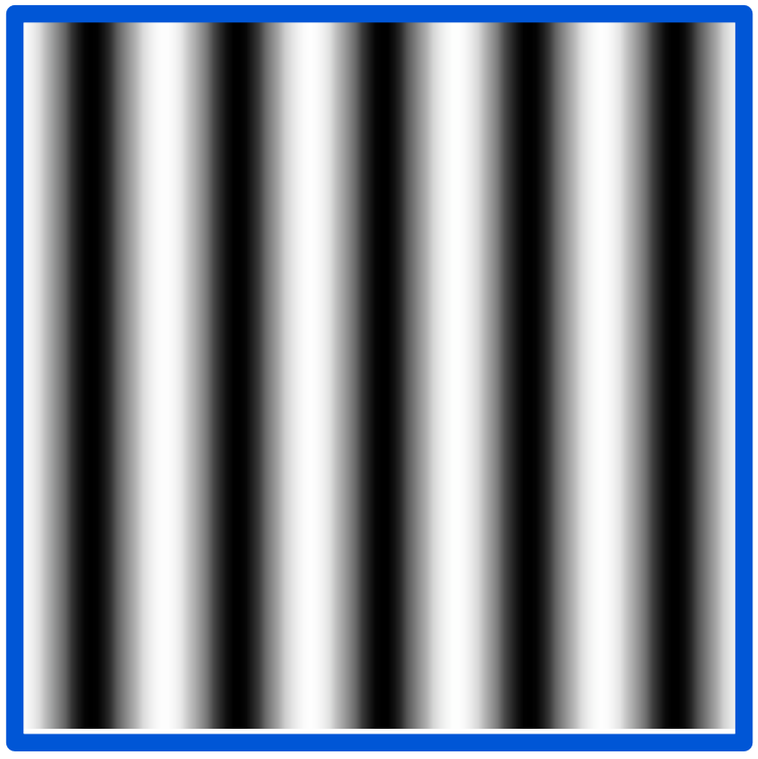


T.F.



TP pratique

Générer une
texture horizontale
en utilisant la
ifft

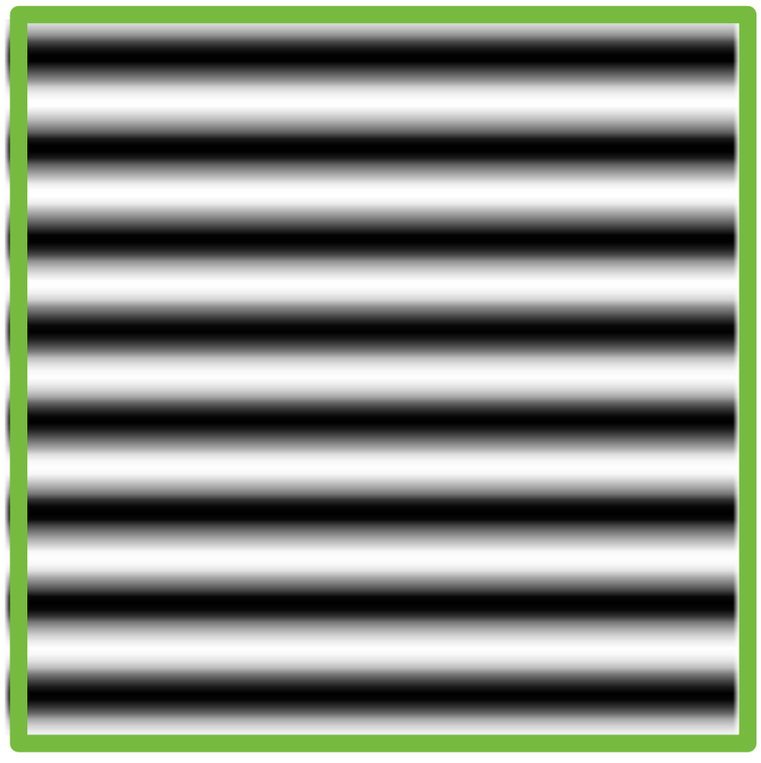


Pour les curieux: une vague en 2D

http://www.math.ubc.ca/~cass/courses/m309-03a/a1/clayton/part4.html

TP pratique

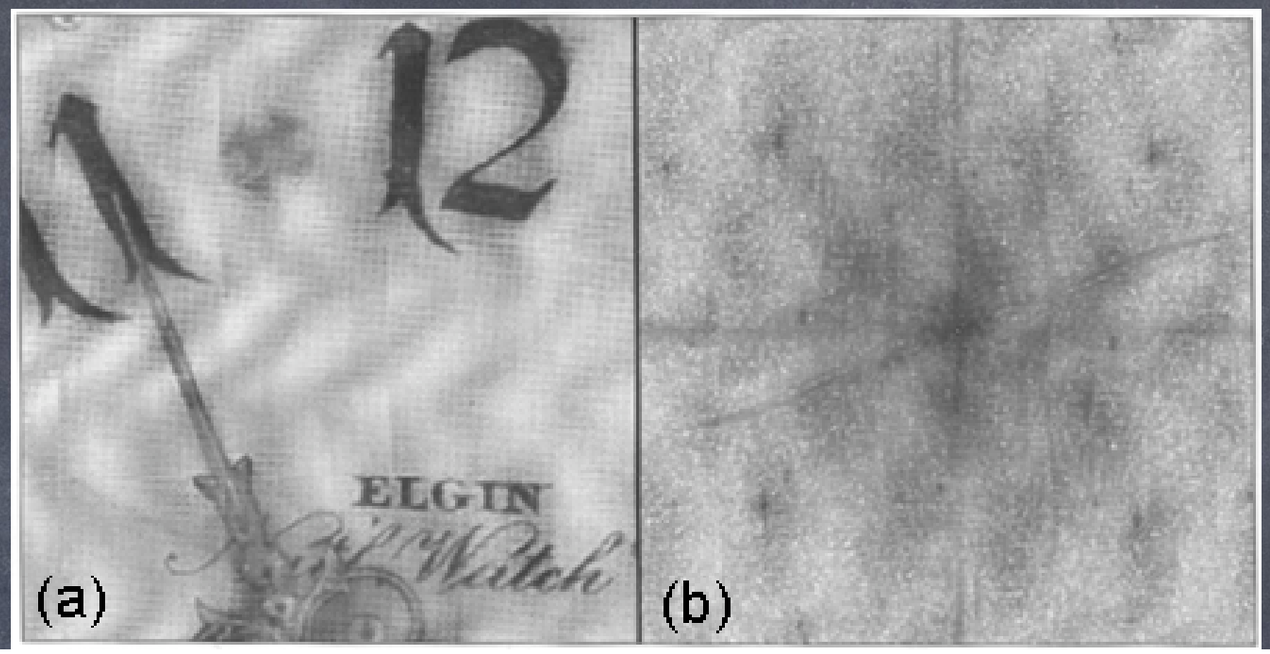
Générer une texture verticale en utilisant la ifft



TP pratique

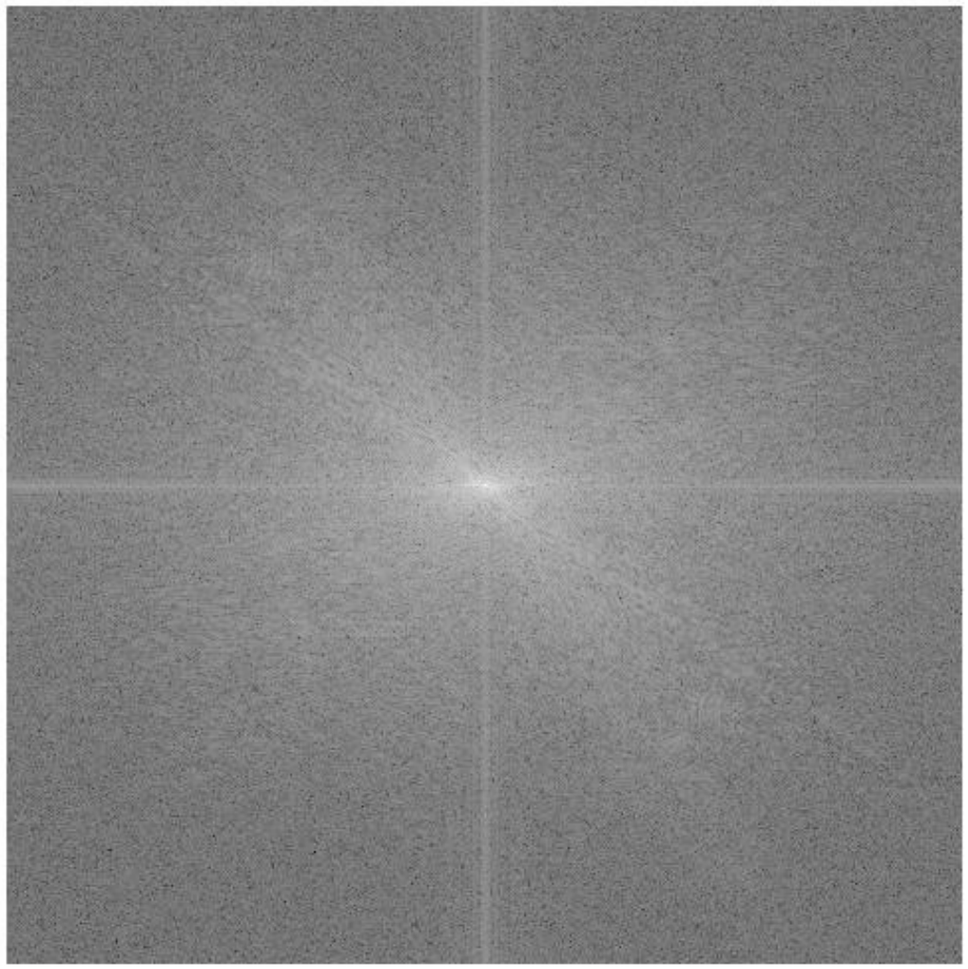
Générer une texture diagonale en utilisant la ifft

Photocopie originale

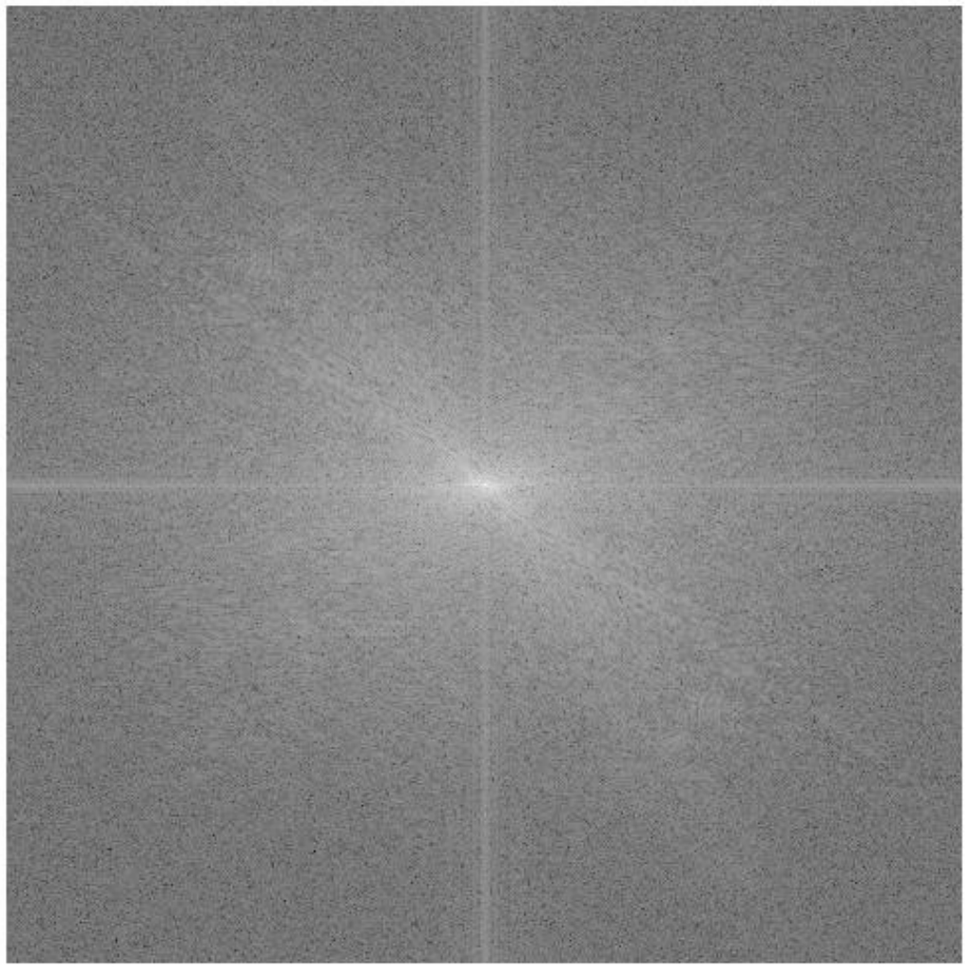


T.F.
On voit les étoiles!

TP pratique



TP pratique
La FFT2 de Lena



TF Lena (demo06)

- load
- imshow

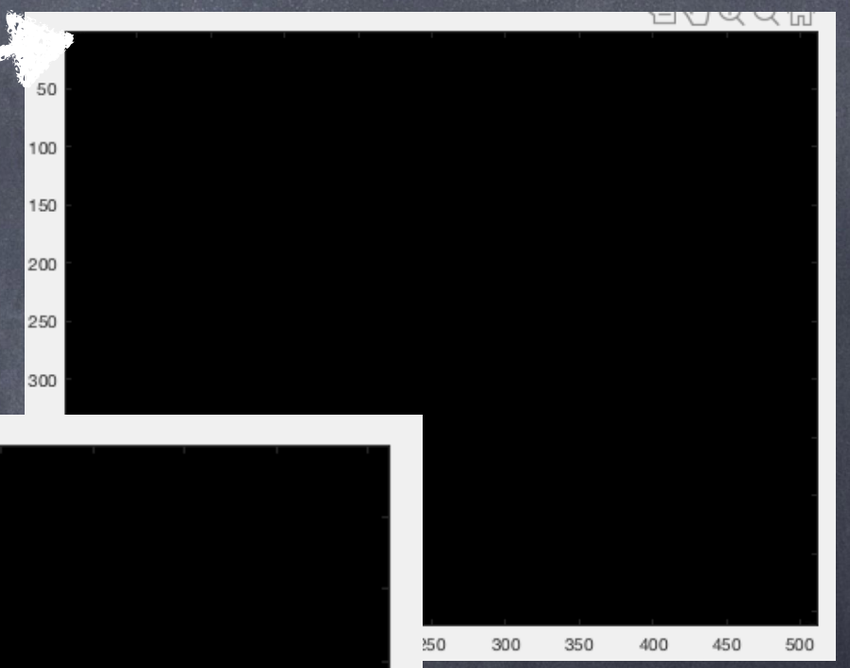
TF Lena

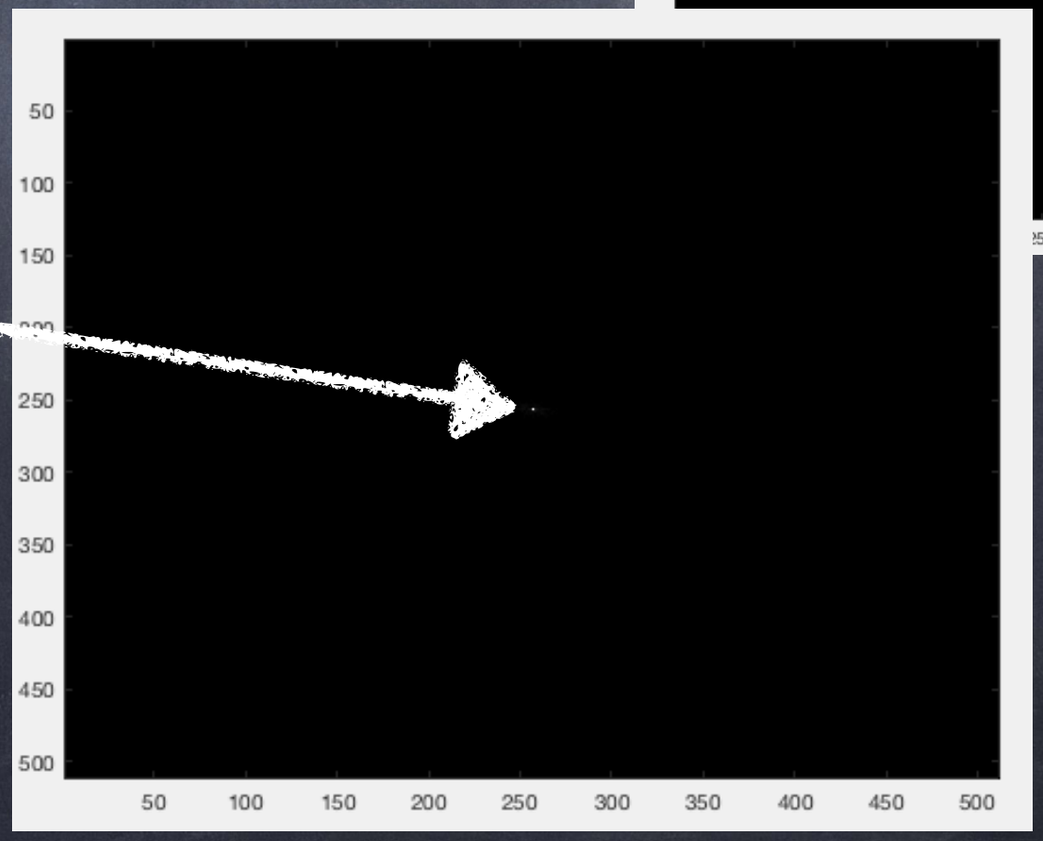
- fft2(M) (pleins de complexes)
- abs(fft2(M)) donne le module de la TF
- C'est quoi le min, max de la abs(fft2(M))?

- L = abs(fft2(M));
- min() max()

- (0.0081, 118010)

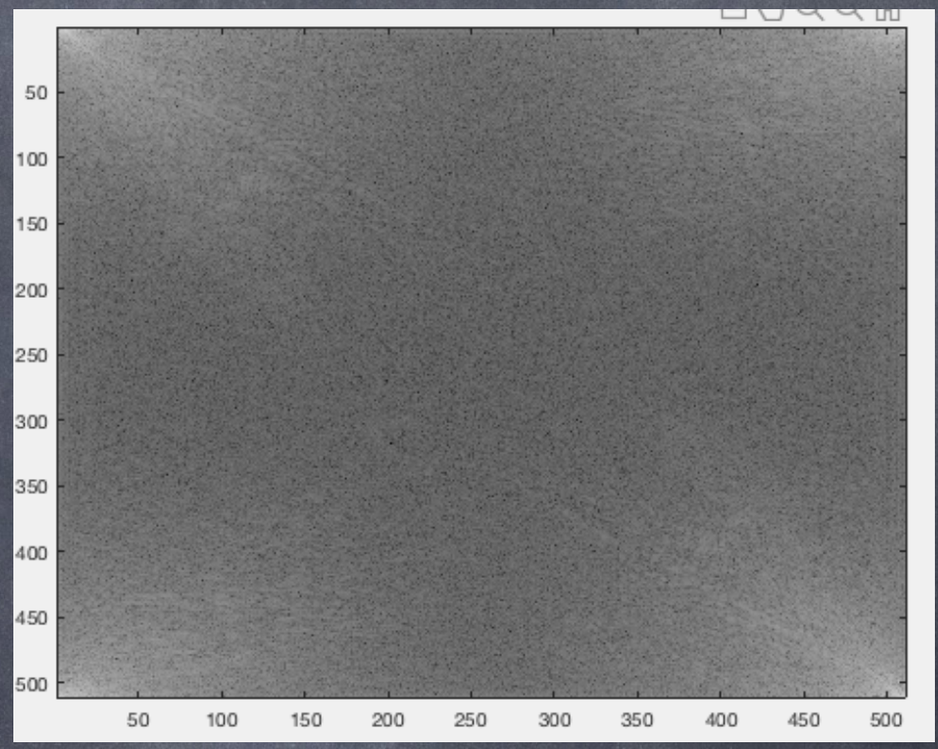
- imshow (on voit rien)
  - On ne voit rien! (min, max) trop loins
  - Le max est ou?
  - (0,0) le coefficient_0,0
- imagesc(fftshift(L))
- L_shift = fftshift(L)
- L_shift(256,256)





Petit trick de visu de TF

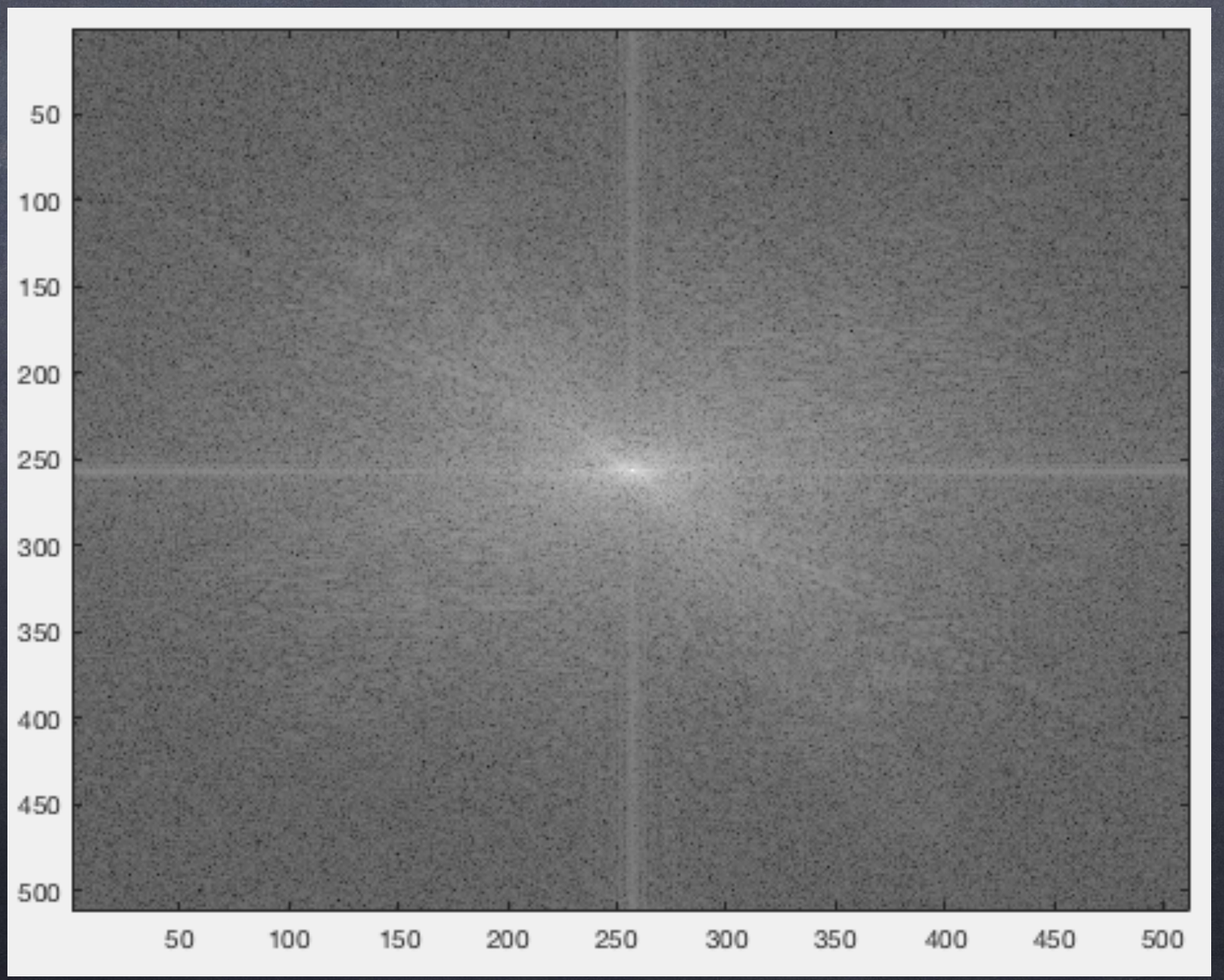
- On prend le log
  - log(118010) = 11.67
  - log(0.0081) = -4.8144



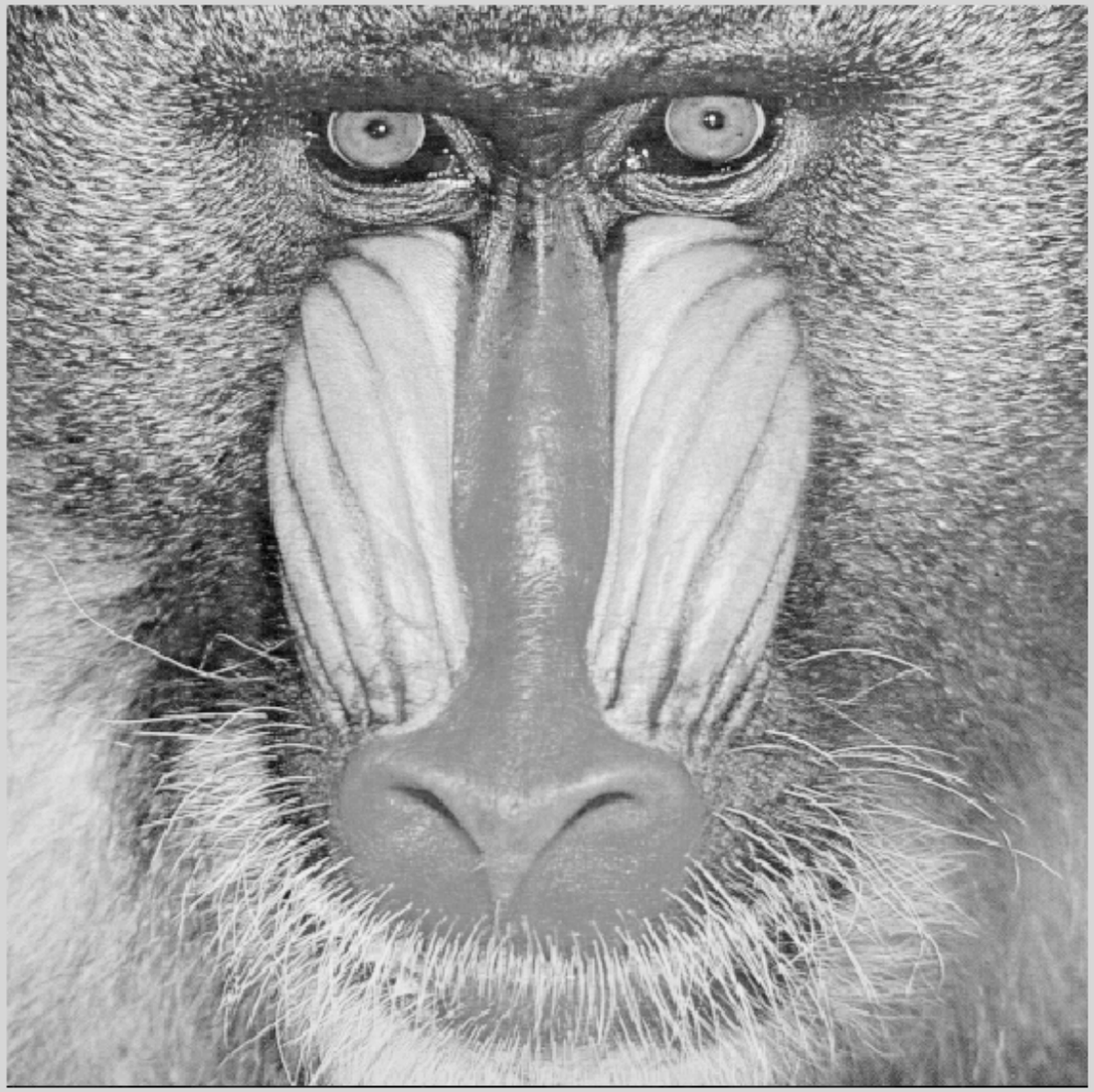
- Du coup, la dynamique est maintenant entre -5 et 12 environ
- imshow(log(abs(fft2(lena))))

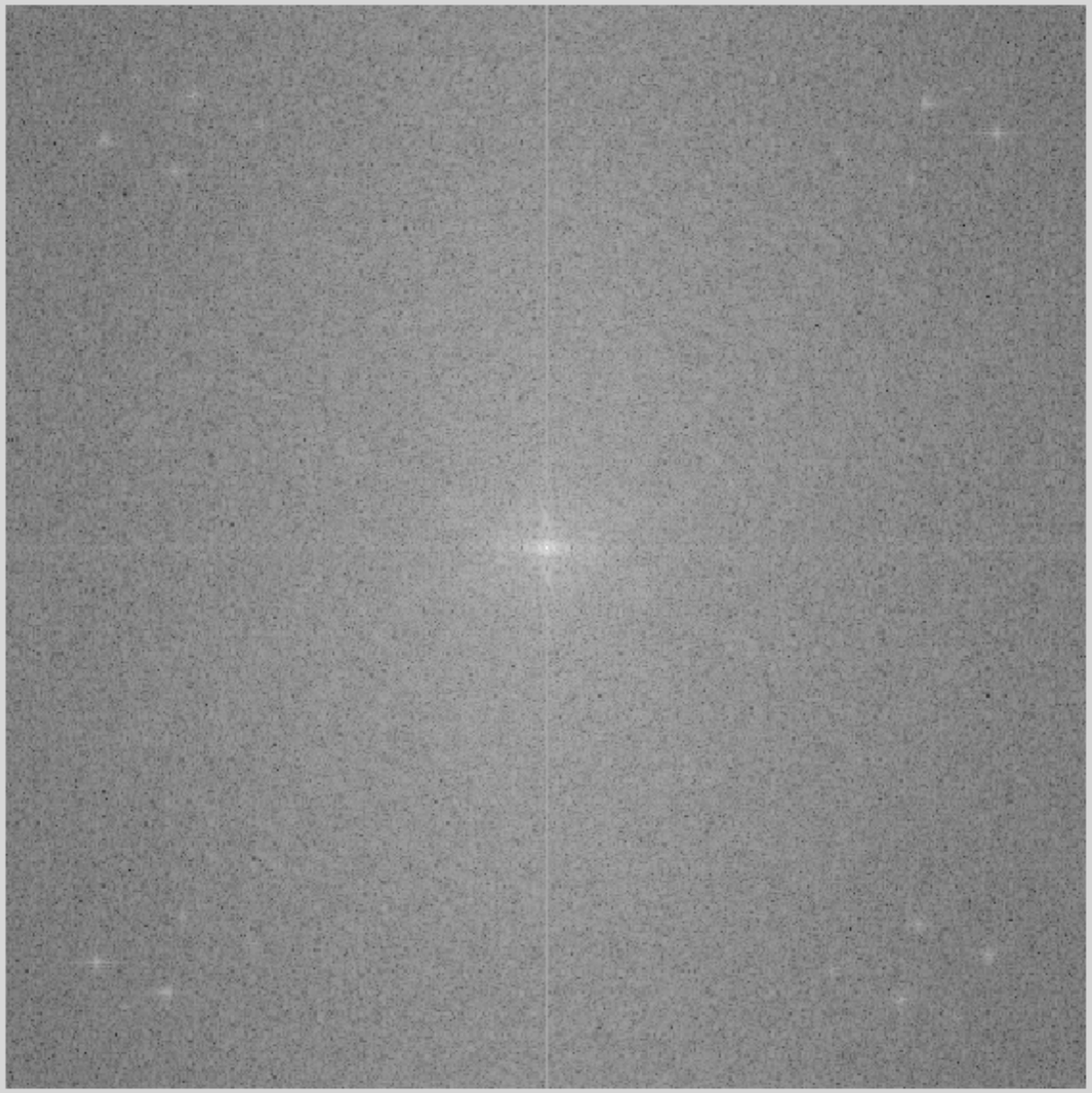
Petit trick de visu de TF

- imshow(log(abs(fftshift(fft2(lena)))))



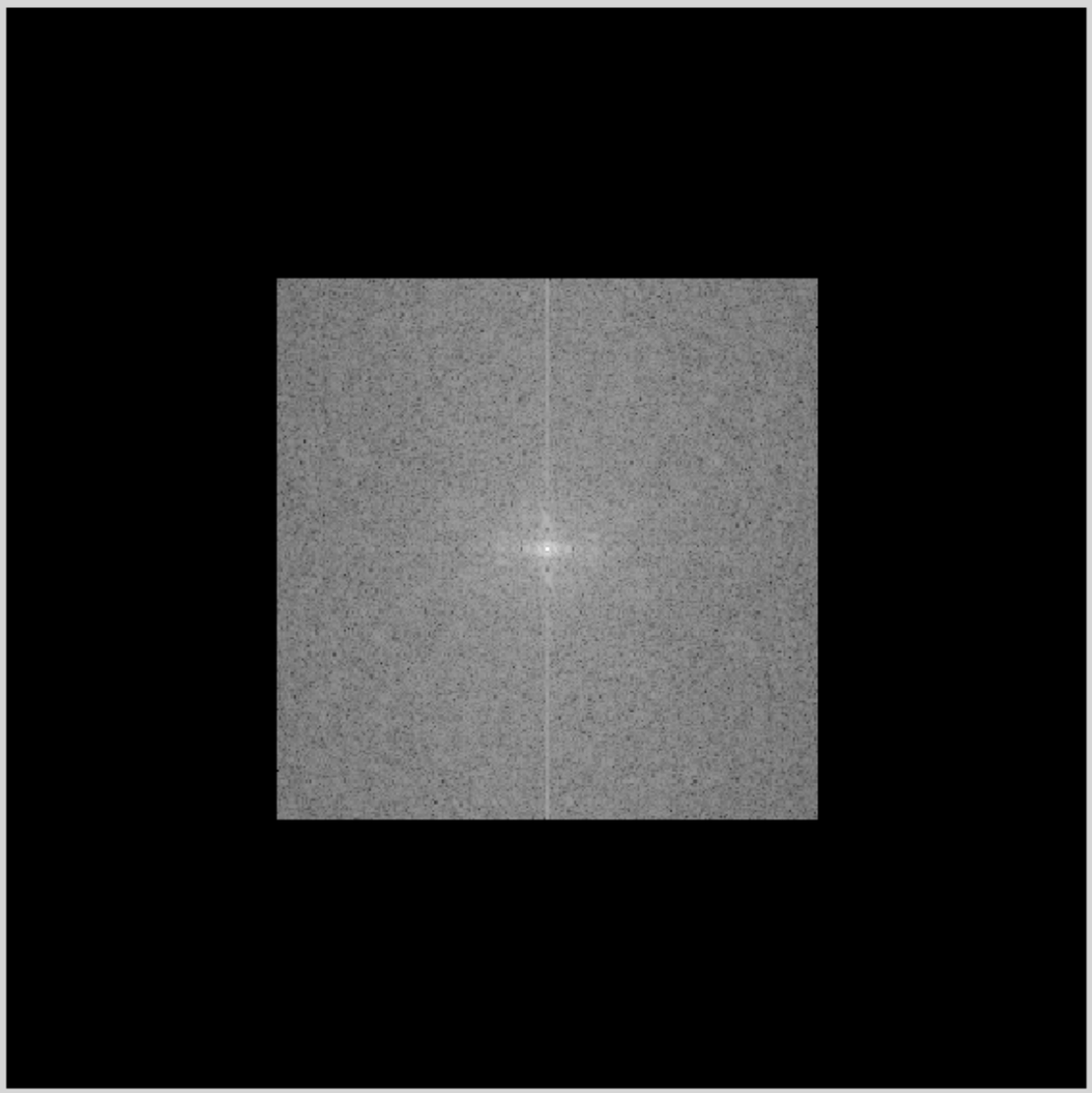
Mandrill



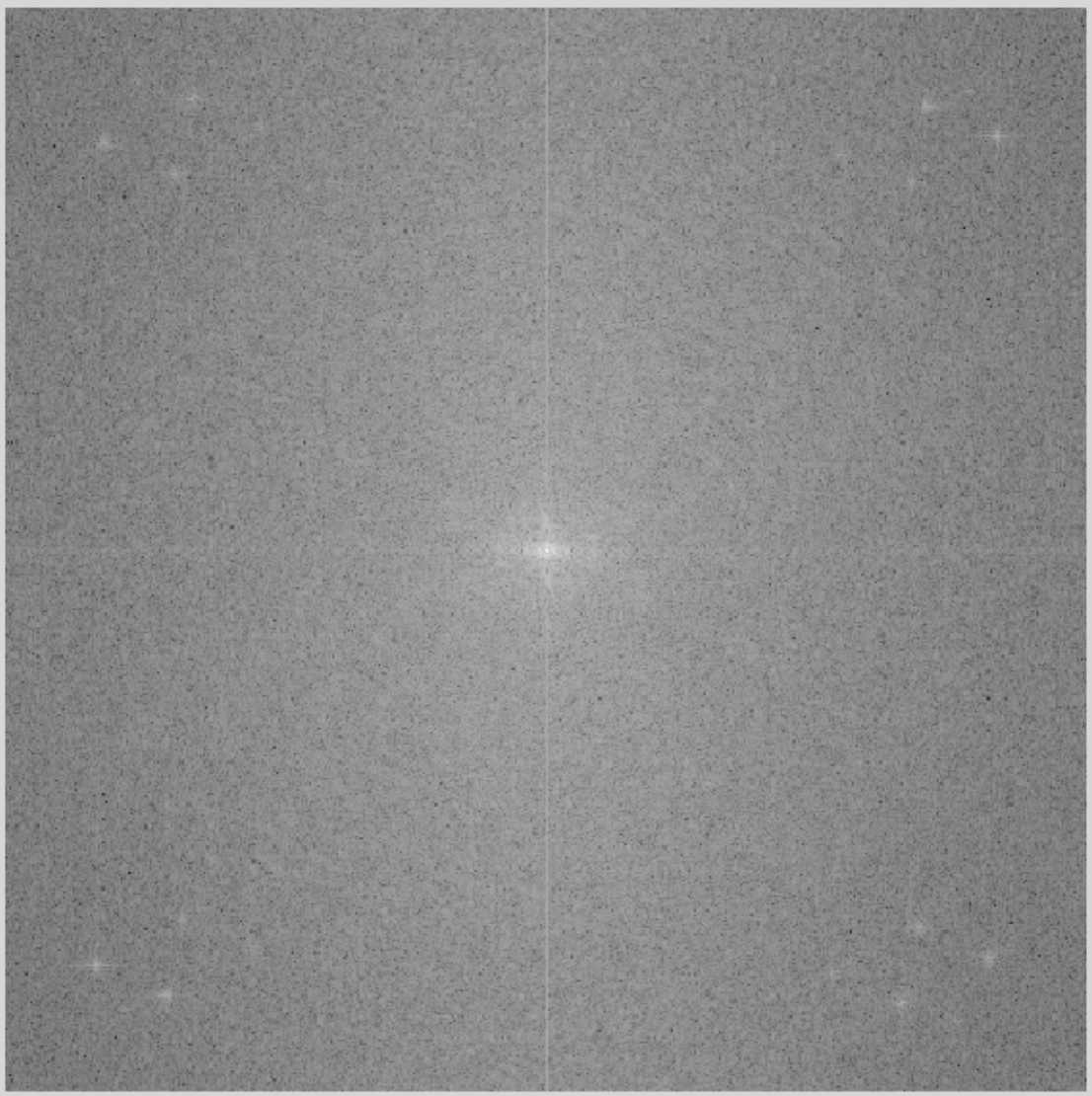


Mandrill

Cropped spectrum : 25% of coefficients

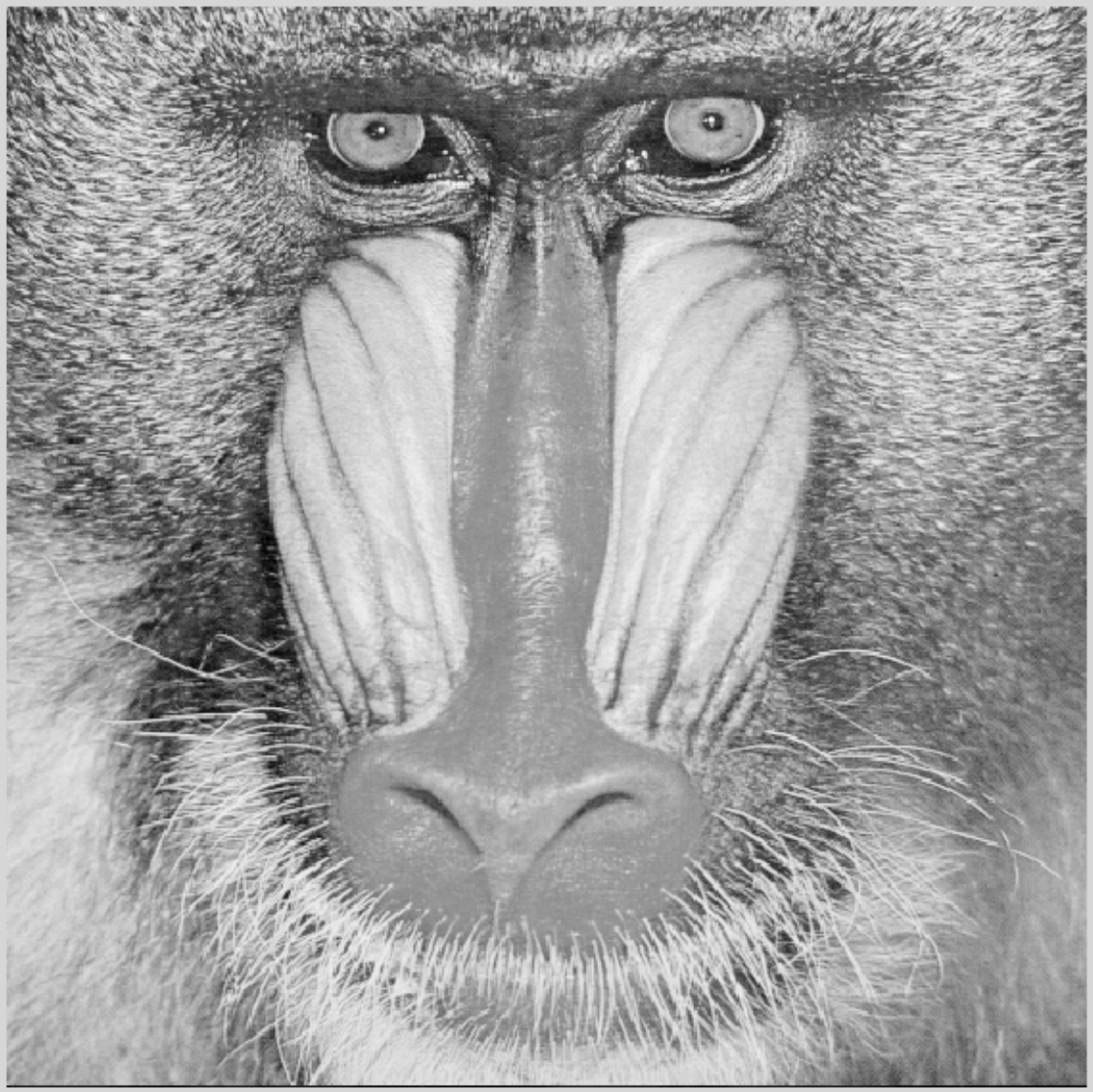


Original spectrum

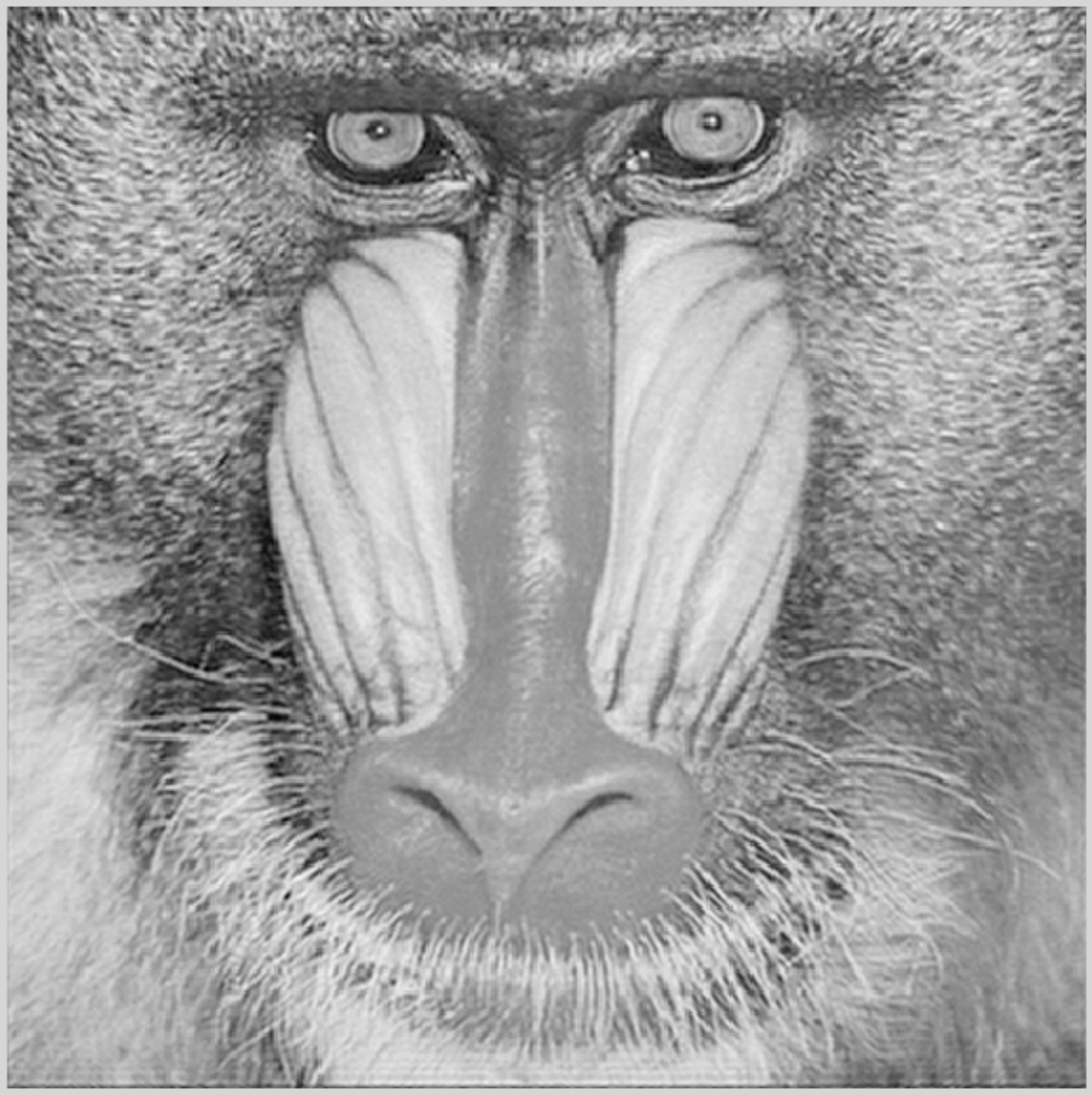


Mandrill

Original image

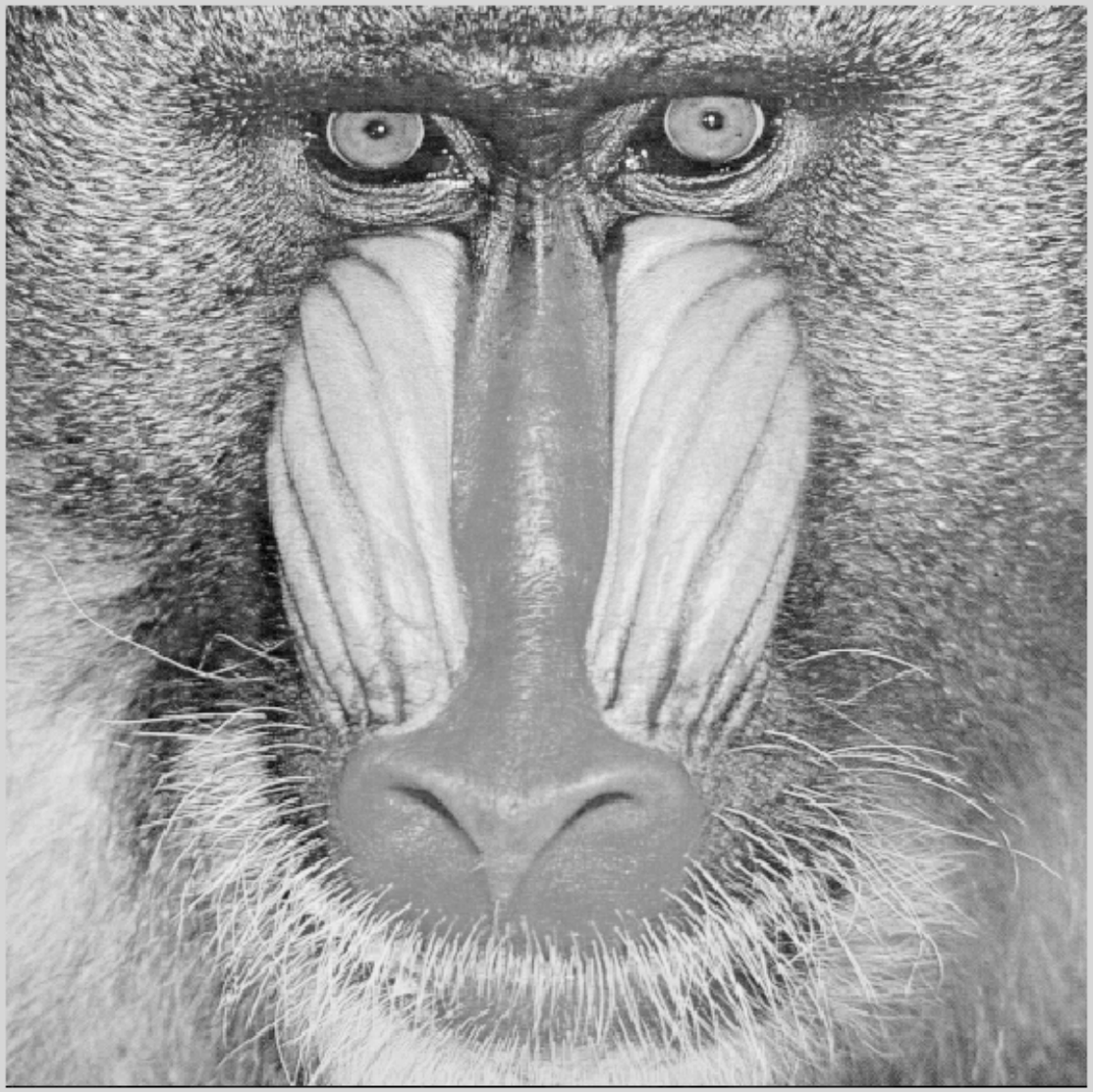


Approx, SNR=18.92dB



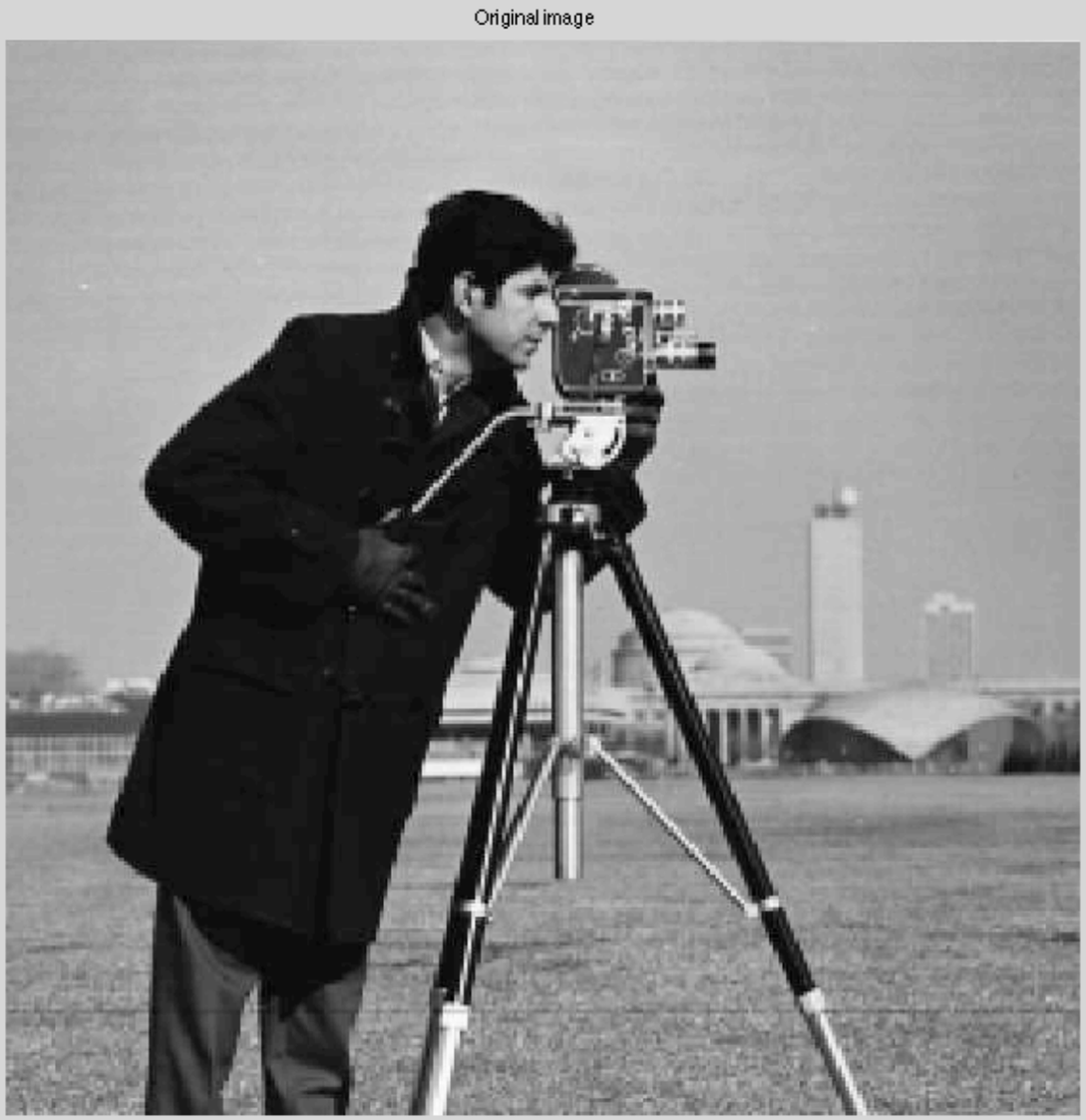
Mandrill

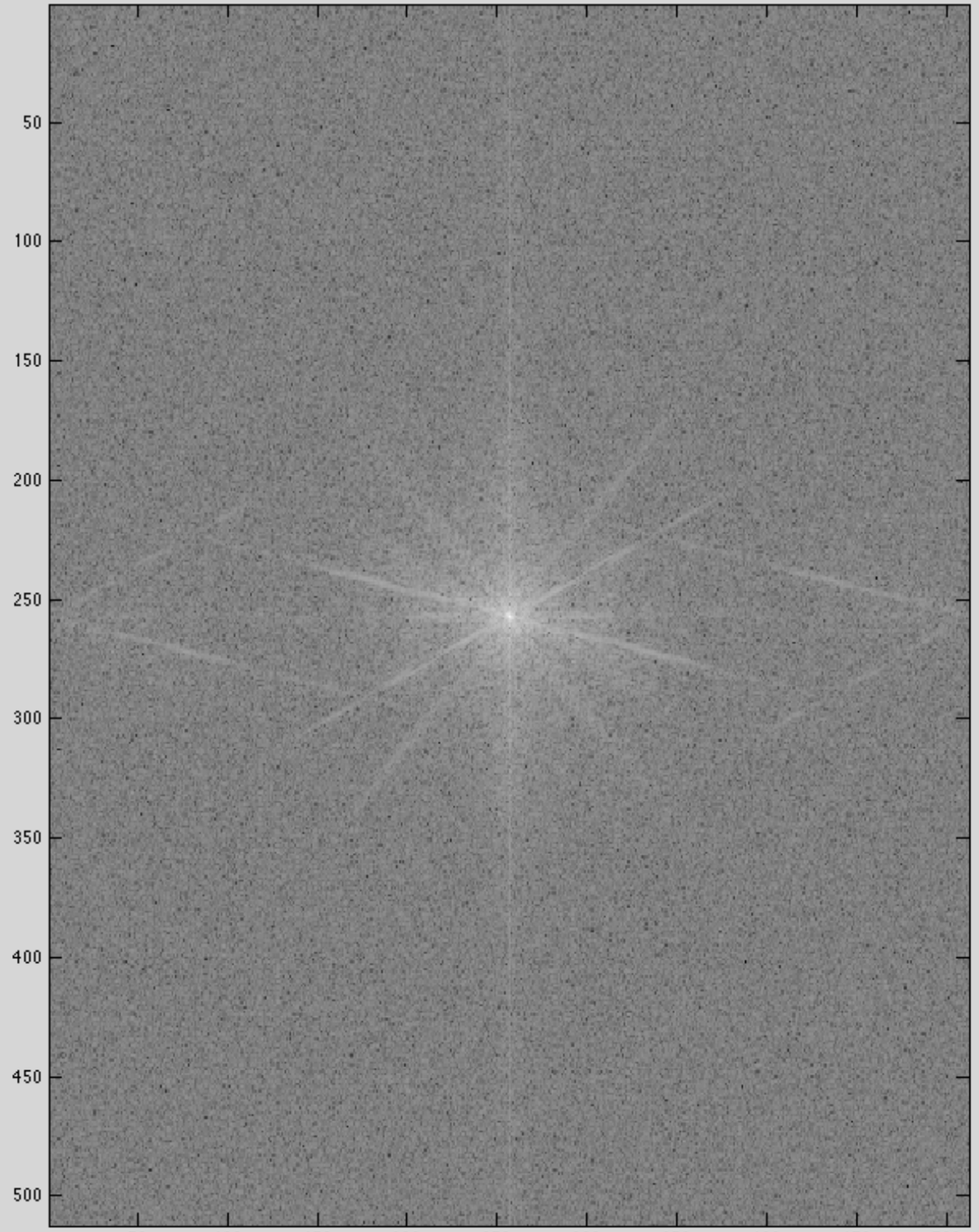
Original image



Approx, SNR=13.8dB

Camera Man


Original image


50
100
150
200
250
300
350
400
450
500
50
100
150
200
250
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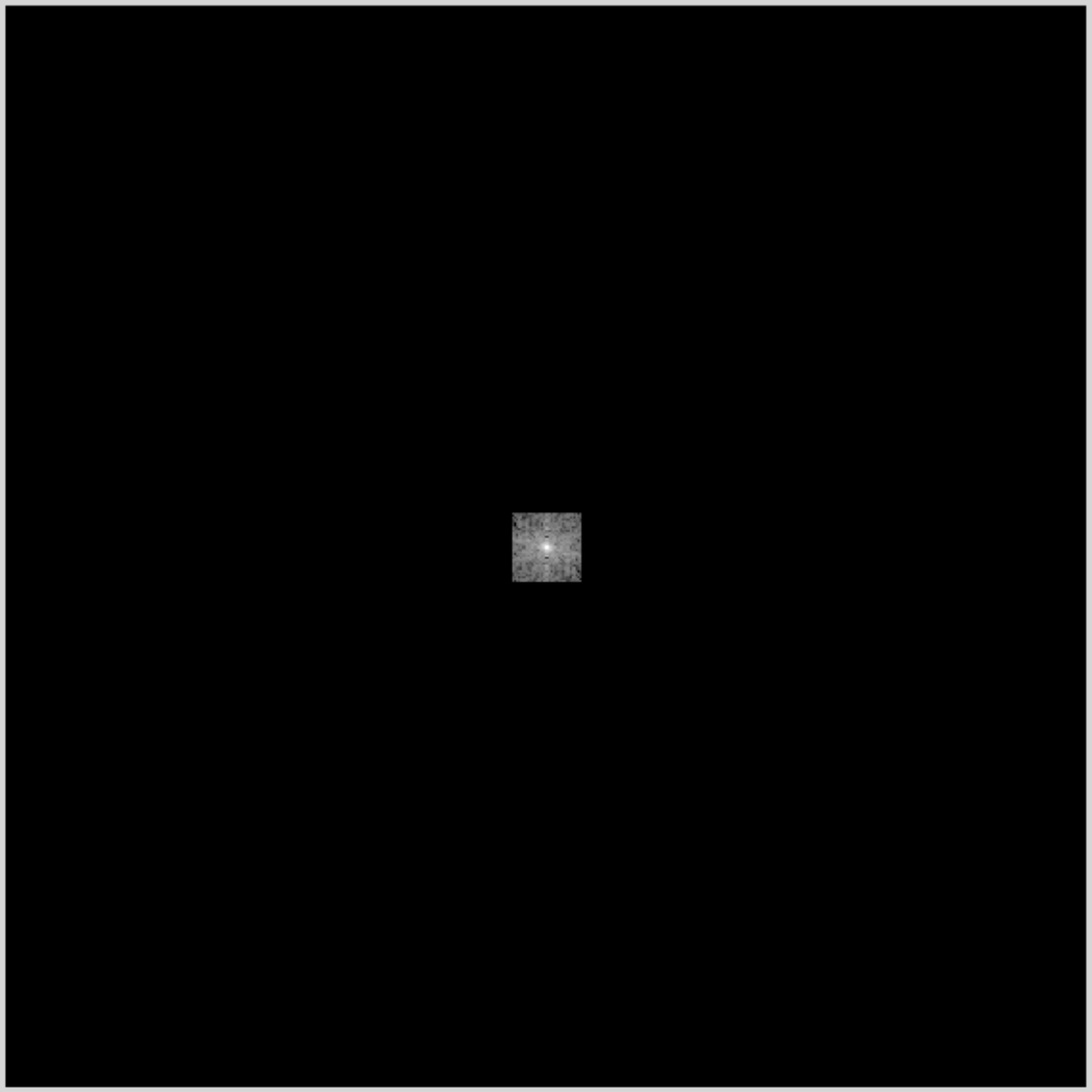
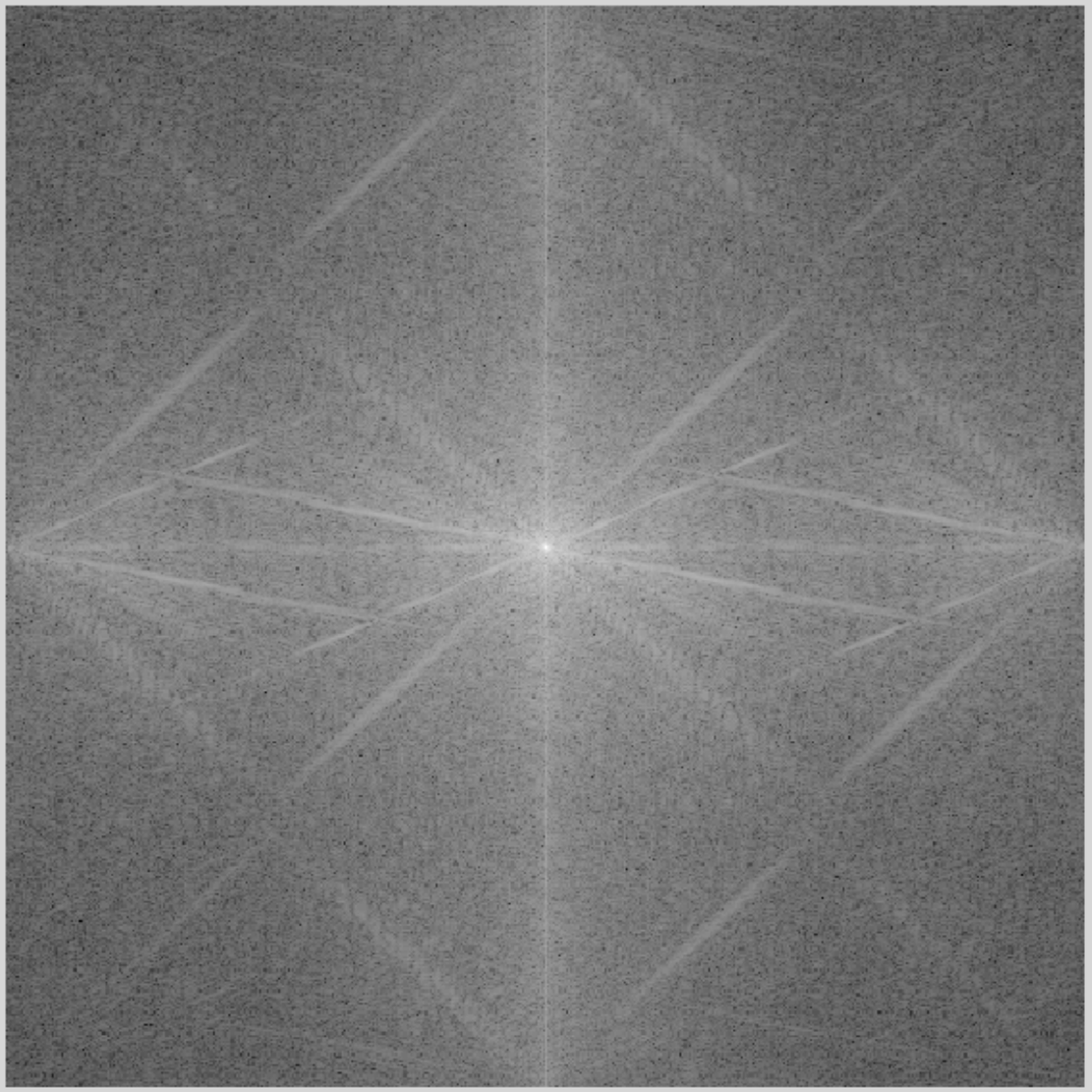
350
400
450
500

Camera Man

Cropped spectrum : 0.39062% of coefficients



Original spectrum

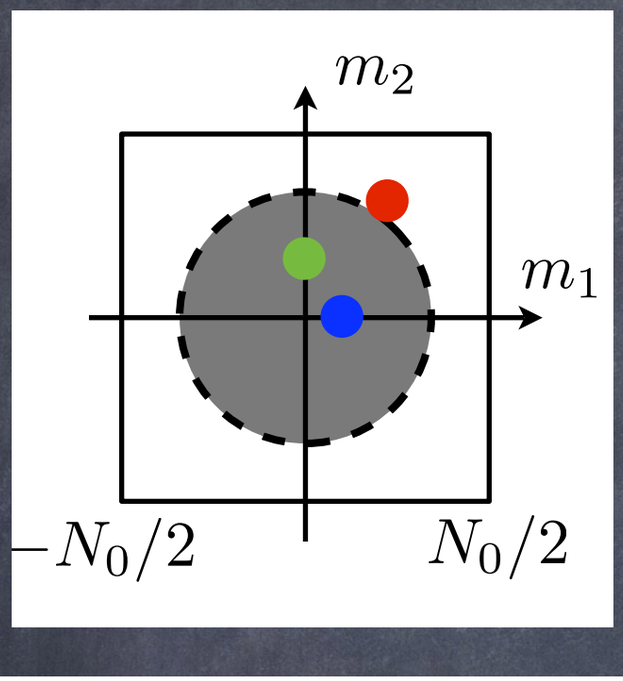


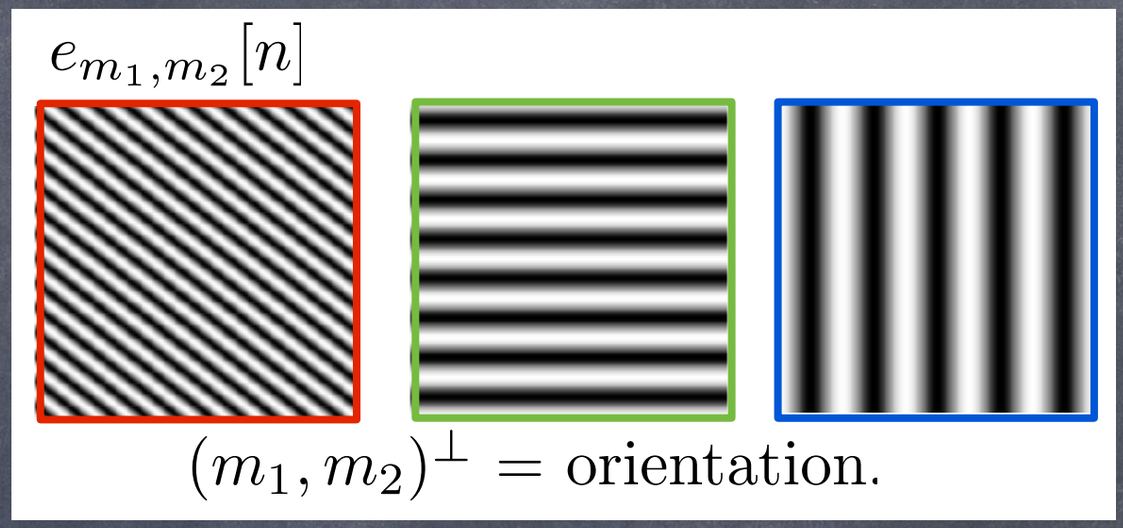
Cropped Fourier : 25% of coefficients

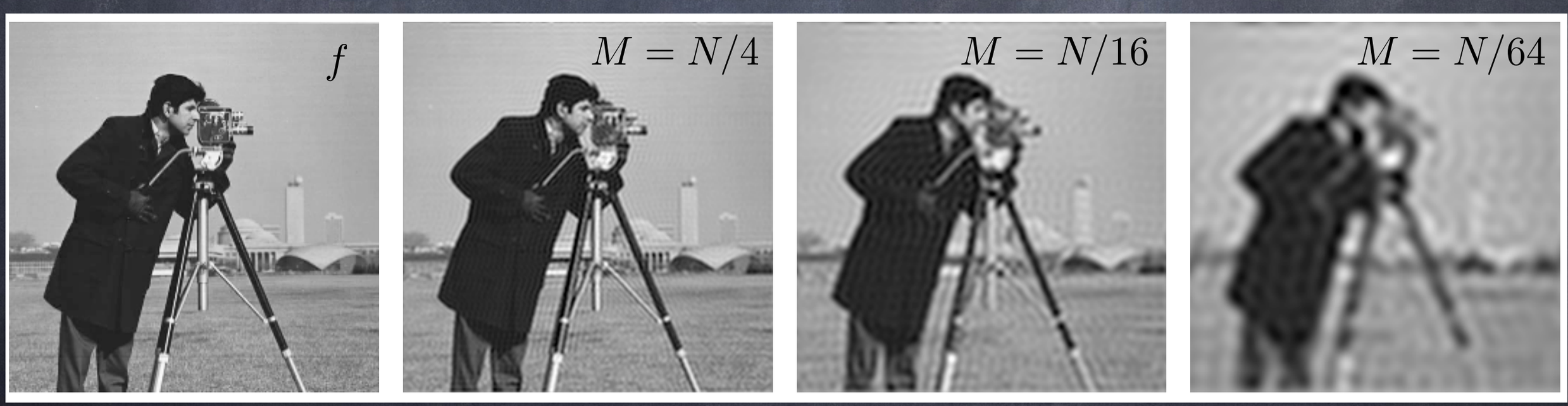
Full Fourier noisy

Original

Fourier & discontinuities





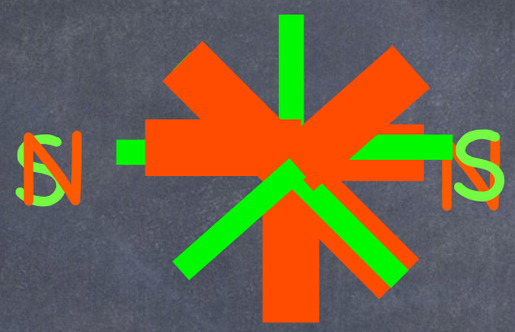


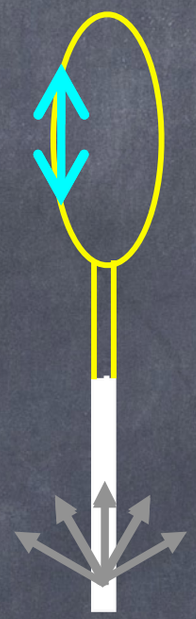
[Peyré, Numerical Tour of Signal Processing]

IRM - principe

- Dispositif d'acquisition : aimant qui permet de créer des champs magnétiques
- Imag
- On r
- Nécessite [illegible] uction
  - Fo

Futu [illegible] nnée (compressed sensing - IMN764)





Courant

0

Temps

Collecte de l'information fréquentielle

175	175	176	175	175	171	166	164	163	161	158	157	155	154	150	147	143	141	139	136	134	131	128	125	122	120	118
178	178	178	178	177	174	173	168	167	164	162	160	157	156	153	151	147	145	143	140	138	135	131	127	125	125	122
180	180	180	180	180	178	176	173	171	167	167	164	162	160	156	155	151	149	146	143	140	139	136	132	130	128	127
185	184	184	184	182	180	179	177	175	171	170	167	165	163	160	160	155	153	151	148	145	142	139	137	136	133	129
194	194	191	187	185	185	183	182	180	176	175	174	170	170	153	163	161	158	155	152	150	147	144	141	139	136	136
199	199	197	194	192	188	185	184	183	180	178	176	173	171	145	165	163	160	157	155	152	149	147	144	141	139	137
202	201	203	199	199	195	192	188	187	184	183	180	177	176	116	171	167	166	161	159	158	155	153	149	146	143	141
206	205	205	204	201	200	198	197	183	144	169	184	177	178	81	174	172	168	160	164	162	159	156	153	151	149	145
209	210	211	209	207	205	204	205	151	72	97	123	89	156	72	173	169	157	140	172	169	167	163	160	157	154	151
215	215	213	213	213	209	209	208	147	65	86	98	66	76	79	169	165	122	85	154	174	172	169	166	163	160	158
225	225	223	220	219	217	215	214	127	50	60	88	50	27	27	85	108	83	57	127	178	178	175	171	168	165	163
230	230	229	227	226	225	224	223	141	62	73	63	28	13	7	38	33	41	52	119	123	150	174	172	166	170	168
233	232	166	107	200	231	187	182	115	91	65	47	17	5	5	37	13	7	52	89	63	56	87	52	49	63	167
231	224	73	21	39	124	42	38	77	35	17	20															
		51	36	47	60	42	39	36	12	5	13															
		92	67	38	48	51	81	47	38	44	50															
		110	114	61	66	56	113	70	18	102	113															
		105	107	84	96	100	127	107	42	125	124															
		112	112	98							113															
		114	112	110							87															

T.F inverse