Vertex cover reduces to set cover

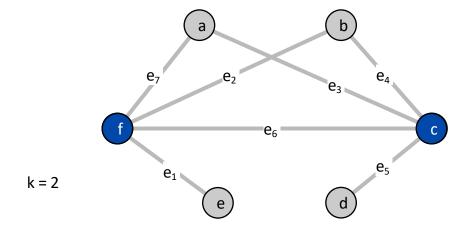


Lemma. G = (V, E) contains a vertex cover of size k iff (U, S, k) contains a set cover of size k.

That is, $VC(i) = yes \Leftrightarrow SC(f(i)) = yes$

Pf. \Rightarrow Let $X \subseteq V$ be a vertex cover of size k in G.

Then $Y = \{ S_v : v \in X \}$ is a set cover of size k.



vertex cover instance (k = 2)

$$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$S_a = \{ 3, 7 \} \qquad S_b = \{ 2, 4 \}$$

$$S_c = \{ 3, 4, 5, 6 \} \qquad S_d = \{ 5 \}$$

$$S_e = \{ 1 \} \qquad S_f = \{ 1, 2, 6, 7 \}$$

set cover instance (k = 2)