Knapsack is NP-Complete

- **(Decision) KNAPSACK**: Given a finite set X, nonnegative weights w_i, nonnegative values v_i, a weight limit W, and a target value V, is there a subset $S \subseteq X$ such that:

$$\sum_{i \in S} w_i \leq W$$
$$\sum_{i \in S} v_i \geq V$$

- **SUBSET-SUM**: Given a finite set Y, nonnegative values u_i, and an integer U, is there a subset $S' \subseteq Y$ whose elements sum to exactly U?

- Claim. SUBSET-SUM $\leq_P$ KNAPSACK.
- Reduction. Given instance (u_1, …, u_n, U) of SUBSET-SUM, create KNAPSACK instance:

$$v_i = w_i = u_i \qquad \sum_{i \in S} u_i \leq U$$
$$V = W = U \qquad \sum_{i \in S} u_i \geq U$$