Note: Answers at the back aka solutions

CS 5117
SIGNALS AND SYSTEMS
Long Exam 2

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Instructions: Read the items carefully. Enclose your final answer in a box, where applicable. Show your solutions in your answer sheet and send a photo / scanned copy of this to your instructor.

1. [Total: 9 pts.] Define $x(t)$ as:

$$x(t) = 5\cos(\omega t) + 5\cos(\omega t + 120°) + 5\cos(\omega t - 120°)$$

   a. [5 pts.] Simplify $x(t)$ into standard sinusoidal form: $x(t) = A\cos(\omega_0 t + \phi)$, that is, evaluate the sum of these three signals. Use phasors to do the algebra. $x(t) = 0 = 0\cos(\omega t + \phi)$

   b. [4 pts.] Make a plot of the three signals represented by vectors (phasors) in the complex plane, and also draw the vector sum. Don't forget to label the respective phases for each vector. Plot at the back

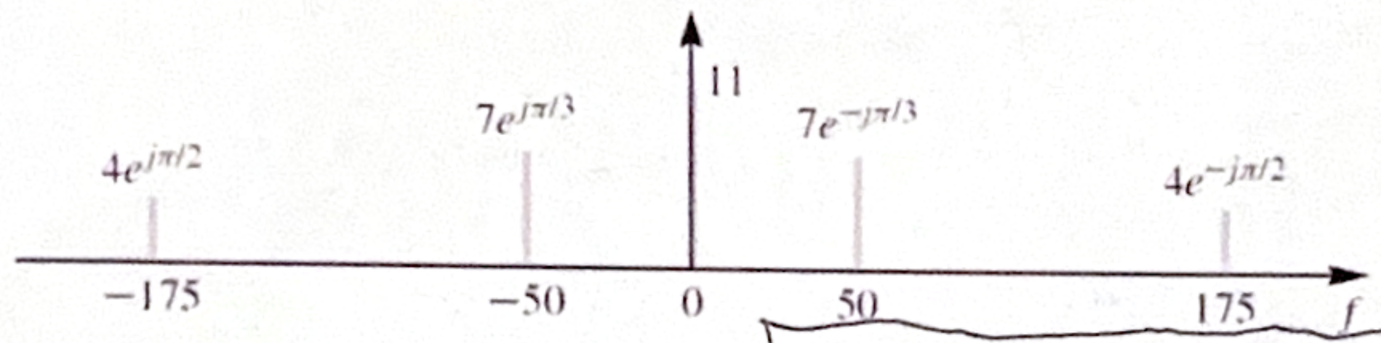
2. [Total: 12 pts.] A signal composed of sinusoids is given by the equation

$$x(t) = 16\cos(500\pi t + \pi/4) + 9\cos(1000\pi t - \pi/3) - 5\cos(750\pi t).$$

   a. [6 pts.] List the set of rotating phasors represented by the (f_k, a_k), or $(f_k, \frac{1}{2}X_k)$, pairs. $\{(250\text{ Hz}, 8e^{j\pi/4}), (500\text{ Hz}, 8e^{-j\pi/4}), (500\text{ Hz}, 4e^{-j\pi/3}), (500\text{ Hz}, 4.5e^{j2\pi/3}), (375\text{ Hz}, 2.5e^{j\pi}), (375\text{ Hz}, 2\pi e^{j\pi})\}$

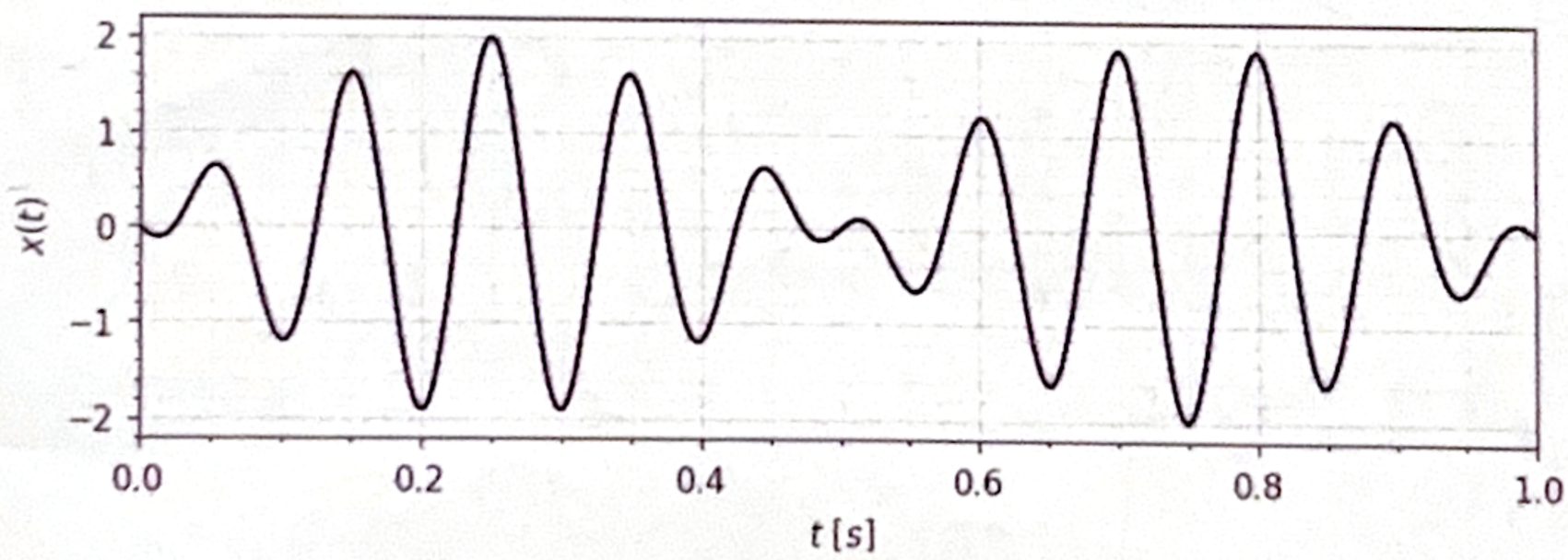
   b. [6 pts.] Plot a two-sided spectrum representation of the signal indicating the relative magnitudes of each of the complex amplitudes and their respective frequency positions. plot at the back

3. [5 pts.] A signal $x(t)$ has the two-sided spectrum representation shown below:



Write an expression for $x(t)$ as a sum of cosines. $x(t) = 11 + 14\cos(100\pi t - \pi/3) + 8\cos(350\pi t - \pi/2)$

4. [Total: 4 pts.] The "beating" of two cosine signals of frequency f_1 and f_2 is shown in the graph below:



   a. [2 pts.] By carefully analyzing the waveform, determine the central (carrier) frequency f_c and the deviation (beat) frequency f_Δ. $f_\Delta = \frac{1}{T_\Delta} = \frac{1}{0.5} = 2\text{ Hz}$ $f_\Delta > 2\text{ Hz}$ $f_c = 22\text{ Hz}$

   b. [2 pts.] From your results in item 4a, find f_1 and f_2. $f_s = f_c \pm \frac{f_\Delta}{2}$ $f_1 = 22 + 1 = 23\text{ Hz}$ $f_2 = 22 - 1 = 21\text{ Hz}$

■ END

1. a. $x_1 = 5\cos(\omega t) = 5\angle 0^\circ$

$x_2 = 5\cos(\omega t + 120^\circ) = 5\angle 120^\circ$

$x_3 = 5\cos(\omega t - 120^\circ) = 5\angle -120^\circ$

$120^\circ = \frac{120\pi}{180} = \frac{2\pi}{3}$

* $z = r\cos\theta + j\, r\sin\theta = re^{j\theta}$

$x_1 = 5\cos(0) + j5\sin(0) = 5 + j0$

$x_2 = 5\cos\left(\frac{2\pi}{3}\right) + j5\sin\left(\frac{2\pi}{3}\right) = -\frac{5}{2} + j\,5\,\frac{\sqrt{3}}{2} = -2.5 + j4.33$

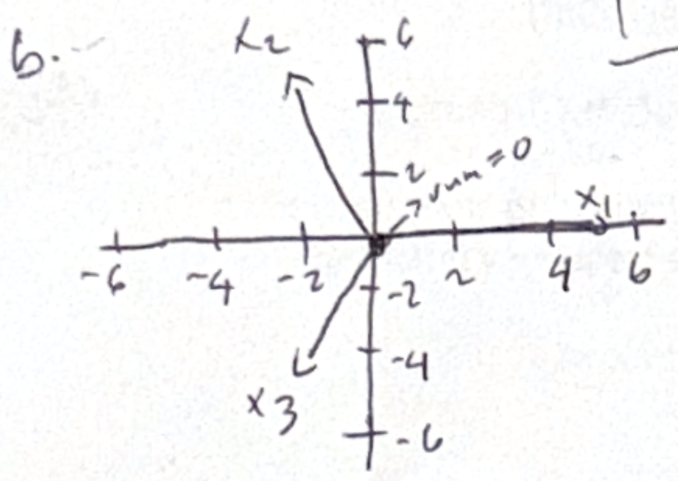
$x_3 = 5\cos\left(-\frac{2\pi}{3}\right) + j5\sin\left(-\frac{2\pi}{3}\right) = -\frac{5}{2} - j\,5\,\frac{\sqrt{3}}{2} = -2.5 - j4.33$

$x(t) = 5 + 2.5 + (-2.5) + j(0) + j(4.33) + j(-4.33)$

$x(t) = 0 + j0$

$\boxed{x(t) = 0\cos(\omega t + \phi) = 0}$

b.



2. a. $x_1 = 16\cos(500\pi t + \pi/4)$

$f_1 = \frac{500\pi}{2\pi} = 250\ Hz$, $A_1 = 16$

$x_1 = 16e^{j\pi/4}$, $\frac{1}{2}x_1 = 8e^{j\pi/4}$

$x_3 = 5\cos(750\pi) = 5\cos(750\pi + \pi)$

$f_3 = \frac{750\pi}{2\pi} = 375\ Hz$, $A_3 = -5$ or 5

$x_3 = 5e^{j\pi}$, $\frac{1}{2}x_3 = 2.5e^{j\pi}$

$x_2 = 9\cos(1000\pi t - \pi/3)$

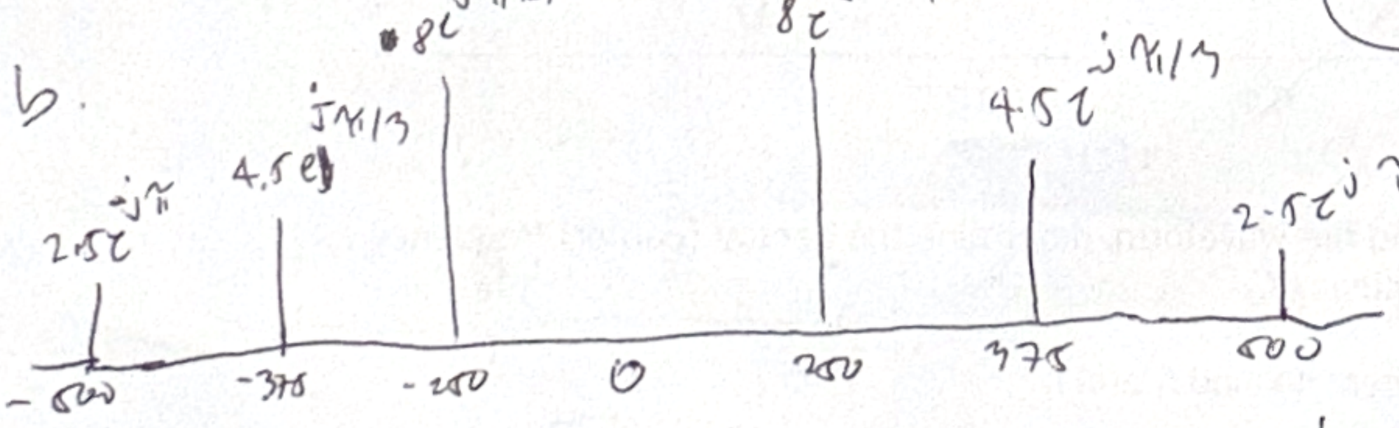
$f_2 = \frac{1000\pi}{2\pi} = 500\ Hz$, $A_2 = 9$

$x_2 = 9e^{-j\pi/3}$, $\frac{1}{2}x_2 = 4.5e^{-j\pi/3}$

lot of rotating phasors

$\{(250\ Hz, 8e^{j\pi/4}), (-250\ Hz, 8e^{-j\pi/4}), (500\ Hz, 4.5e^{-j\pi/4}), (-500\ Hz, 4.5e^{j\pi/4}), (375\ Hz, 2.5e^{j\pi}), (-375\ Hz, 2.5e^{j\pi})\}$

b.



3. $\{(50, 7e^{-j\pi/3}), (-50, 7e^{j\pi/3}), (175, 4e^{-j\pi/2}), (-175, 4e^{j\pi/2}), (0, 11)\}$

$(f_1, \frac{1}{2}x_1) = 2 \times 7\cos(2\pi(50)t + (-\pi/3))$

$= 14\cos(100\pi t - \pi/3)$

$(f_1, \frac{1}{2}x_2) = 2 \times 4\cos(2\pi(175)t + (-\pi/2))$

$= 8\cos(350\pi t - \pi/2)$

$x(t) = 11 + 14\cos(100\pi t - \pi/3) + 8\cos(350\pi t - \pi/2)$