Change-point detection in a Poisson process

S. Robin

joint work with E. Lebarbier, C. Dion-Blanc [DBLR23]

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Example

Bat cries (night of the 17 jul. 2019)

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Point process on $t \in [0, 1]$.

Event times:

$$0 < T_1 < \dots T_i < \dots T_n < 1$$

Counting process:

$$N(t) = \sum_{i=1}^{n} \mathbb{I}\{T_i \leqslant t\}$$

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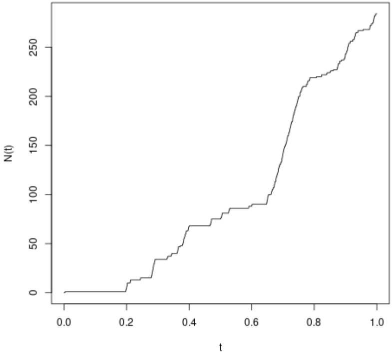

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Poisson Process.

$$\{N(t)\}_{0 \leqslant t \leqslant 1} \sim PP(\lambda(t))$$

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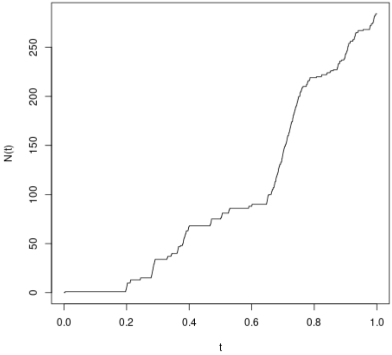

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Intensity function $\lambda(t)$:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\mathbb{P}\{N(t + \Delta t) - N(t) = 1\}}{\Delta t}, \qquad \mathbb{E}N(s) - \mathbb{E}N(t) = \int_t^s \lambda(u)\, \mathrm{d}u$$

Change-point detection

Piecewise constant intensity function.

Change-points

$$(\tau_0 =)\ 0 < \tau_1 \cdots < \tau_{K-1} < 1\ (= \tau_K)$$

For $t \in I_k =]\tau_{k-1}, \tau_k]$:

$$\lambda(t) = \lambda_k$$

→ Continuous piecewise linear cumulated intensity function

$$\Lambda(0,t) = \int_0^t \lambda(s)\ ds.$$

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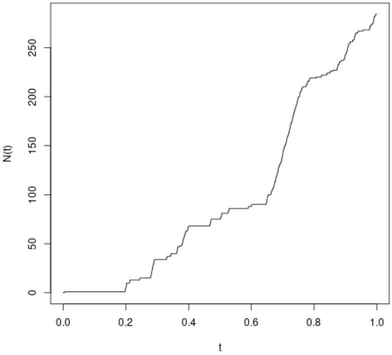

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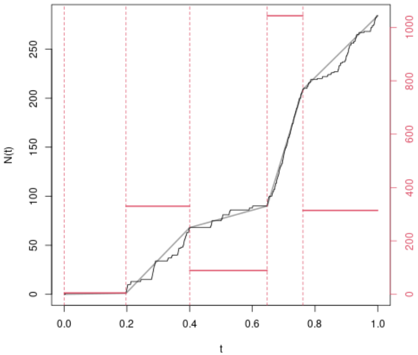

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- Segmentation: estimate (τ, λ) in a reasonnably fast manner
- Model selection: choose K

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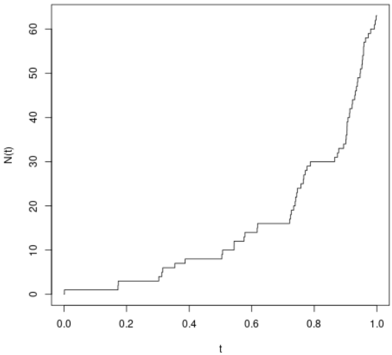

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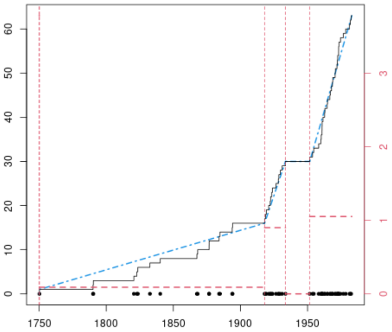

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Outline

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First useful property of Poisson processes: Independence of disjoint intervals.

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- $\Delta\tau_k$ the length of the k-th interval $(= \tau_k - \tau_{k-1})$,
- ΔN_k the number of events within the k-th interval $(= N(\tau_k) - N(\tau_{k-1}))$:

$$-\log p_{\tau,\lambda}(N) = \sum_{k=1}^{K} \lambda_k \Delta\tau_k - \Delta N_k \log \lambda_k,$$

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Additive contrast. General form = sum over the segments

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Optimization problem.

$$(\widehat{\tau}, \widehat{\lambda}) = \operatorname*{arg\,min}_{\tau \in \mathcal{T}^K, \lambda \in (\mathbb{R}^+)^K} \gamma(\tau, \lambda).$$

Minimizing the contrast function

Optimal λ. Because the contrast is additive, we may define

$$\widehat{\lambda}_k = \widehat{\lambda}_k(\tau) = \arg\min_{\lambda_k \in \mathbb{R}^+} C(\Delta N_k, \Delta \tau_k, \lambda_k)$$

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Optimal τ. We are left with the minimization problem

$$\widehat{\tau} = \arg\min_{\tau \in \mathcal{T}^K} \quad \widehat{\gamma}(\tau), \qquad \text{where} \quad \widehat{\gamma}(\tau) = \gamma(\tau, \widehat{\lambda}(\tau))$$

where $\mathcal{T}^K$ is the continuous segmentation space:

$$\mathcal{T} = \left\{\tau \in [0,1]^{K+1} : 0 = \tau_0 < \tau_1 \cdots < \tau_{K-1} < \tau_K = 1\right\}.$$

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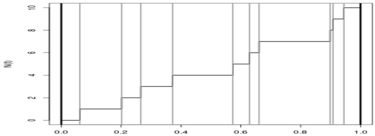
Main issue: The contrast $\widehat{\gamma}(\tau)$ is neither convex nor continuous wrt τ.

Shape of the contrast fonction

Observed $N(t)$: $n = 10$,



[1] gray borders come by pair

Shape of the contrast fonction

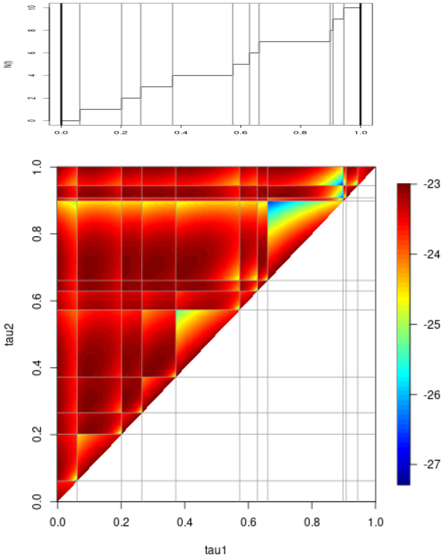
Observed $N(t)$: $n = 10$,

Contrast $\widehat{\gamma}(\tau)$ for $K = 3$ segments:

$$\tau = (\tau_1, \tau_2).$$

One 'block' =
one specific value for the vector [1]

$$\Delta N = (\Delta N_1, \Delta N_2, \Delta N_3)$$



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Partitioning the segmentation space

Partitioning the number of events. Define $\mathcal{N}^K = \left\{\nu \in \mathbb{N}^K : \sum_{k=1}^K \nu_k = n\right\}$.
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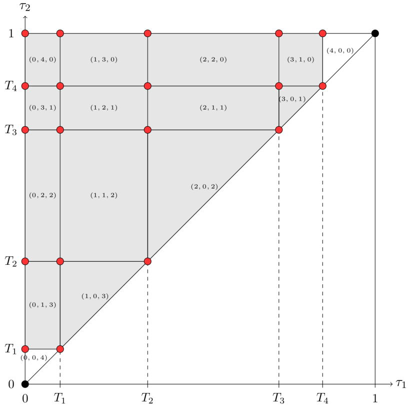
$$\mathcal{T}_\nu^K = \left\{\tau \in \mathcal{T}^K : \Delta N = \nu\right\}.$$

$\rightarrow$ $\mathcal{T}_\nu^K$ = set of segmentation satisfying the prescribed $\nu = (\nu_1, \dots \nu_K)$.

We have

$$\min_{\tau \in \mathcal{T}^K} \widehat{\gamma}(\tau) = \min_{\nu \in \mathcal{N}^K} \min_{\tau \in \mathcal{T}_\nu^K} \widehat{\gamma}(\tau,).$$



Optimal segmentation

Proposition 1. If $K \leqslant n$ and if $\widehat{\gamma}(\tau)$ is strictly concave wrt $\tau \in \mathcal{T}_\nu^K$ for each $\nu \in \mathcal{N}^K$, then

$$\widehat{\tau} = \arg\min_{\tau \in \mathcal{T}^K} \widehat{\gamma}(\tau) \subset \{T_1^-, T_1, T_2^-, T_2^-, \dots T_n^-, T_n\}.$$

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Proposition 2. If each $\widehat{C}(\nu_k, \Delta\tau_k) := C(\nu_k, \Delta\tau_k, \widehat{\lambda}_k)$ is strictly concave wrt $\Delta\tau_k$, $\widehat{\gamma}(\tau)$, then is strictly concave wrt $\tau \in \mathcal{T}_\nu^K$.

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Consequence. $\widehat{\tau}$ can be obtained by dynamic programming over the $2n+2$ possible change-points

$$\mathcal{S} = \{0, T_1^-, T_1, T_2^-, T_2, \dots T_n^-, T_n, 1\}$$

with complexity at most $O(n^2)$.

Admissible contrasts

Poisson contrast. $\widehat{C}_P(\nu_k, \Delta\tau_k) = \nu_k(1 - \log \nu_k + \log \Delta\tau_k)$ is concave wrt $\Delta\tau$.

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Poisson-Gamma model. For each segment $1 \leqslant k \leqslant K$:

$$\Lambda_k \text{ iid } \sim \mathcal{G}\text{am}(a, b), \qquad \{N(t)\}_{t \in I_k} \mid \Lambda_k \sim PP(\Lambda_k).$$

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Contrast for one segment:

$$C_{PG}(\Delta N_k, \Delta\tau_k) = \text{cst} - \log\Gamma(a + \Delta N_k) + (a + \Delta N_k)\log(b + \Delta\tau_k)$$

$\rightarrow$ Strictly concave wrt $\Delta\tau_k$.

Desirable contrast

Remark. The Poisson contrast $\widehat{C}_P(\nu_k, \Delta\tau_k) = \nu_k(1 - \log \nu_k + \log \Delta\tau_k)$ satisfies

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- The optimal solution will involve segments with null length and containing only one event.
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Poisson-Gamma contrast. $C_{PG}(\nu_k, \Delta\tau_k) = -\log \Gamma(a + \nu_k) + (a + \nu_k) \log(b + \Delta\tau_k)$.

- Satisfies the concavity property ($\rightarrow$ admissible),
- but avoids segments with null length ($\rightarrow$ desirable).

Outline

Model selection

Second useful property of Poisson processes: Thining.

- $\{N(t)\} \sim PP(\lambda(t))$
- Sample event times (with prob. v)
- Store the remaining events

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- $\lambda^L(t)$ piecewise constant with change-points (τ_k) and intensities $(v\lambda_k)$,
- $\lambda^T(t)$ piecewise constant with change-points (τ_k) and intensities $((1-v)\lambda_k)$,
- $\{N^L(t)\} \perp \{N^T(t)\}$.

Cross validation

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Cross-validation procedure. For $1 \leqslant K \leqslant K_{\max}$,

- Repeat for $1 \leqslant m \leqslant M$:

  1 – Sample the event times to form $\{N^{L,m}(t)\}$ (learn) and $\{N^{T,m}(t)\}$ (test),

  2 – Estimate $\widehat{\tau}^{L,m}$ and $\widehat{\lambda}^{L,m}$ from $\{N^{L,m}(t)\}$,

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- Select

$$\widehat{K} = \arg\min_K \overline{\gamma}_K$$

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Practical implementation.

Contrasts. During the CV process, we use

- a sampling rate of $v = 4/5$,
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to fit the observed total number of events.

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R package `CptPointProcess` available on `github.com/Elebarbier/CptPointProcess`.

Some simulations

Simulation setting. $K = 6$ segments with varying length. Tuning parameters:

- $\overline{\lambda}$ average intensity ($\rightarrow$ total number of events),
- λ_R = height of the steps ($\rightarrow$ contrast between segments).

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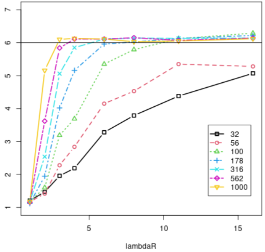
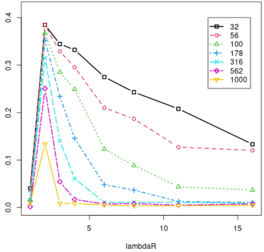
Results. Choose K via CV, then refit the parameters to the whole dataset.

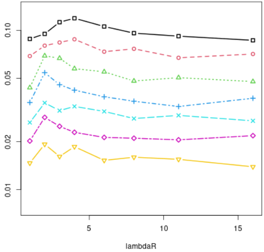
Model selection
$\widehat{K}$



Change-point location
Haussdorf$(\widehat{\tau}, \tau^*)$



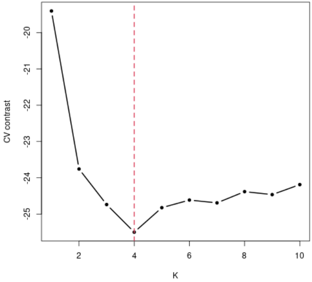
Cumulated intensity
$\ell_2(\widehat{\Lambda}, \Lambda^*)$



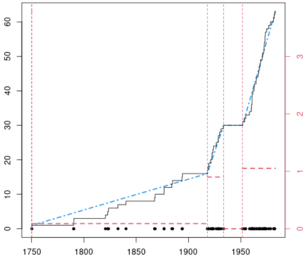
Kilauea eruptions

$n = 63$ eruptions reported between the mid 18th and the late 20th century.

Model selection via CV



Resulting segmentation



Outline

Estimation

Model selection

Illustrations

Extensions

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Marked Poisson Process.

- $\{Y(t)\}_{0 \leqslant t \leqslant 1} \sim MPP(\lambda(t), \mu(t))$:

$$\{N(t)\}_{0 \leqslant t \leqslant 1} \sim PP(\lambda(t)), \qquad \text{at each } T_i: \quad X_i \sim \mathcal{F}(\mu(T_i))$$

- Works the same way, provided that concavity holds.
- Bat cries: Mark = bat species or cry duration.
- Poisson-Gamma events + Exponential-Gamma durations is both admissible and desirable.

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Segmentation-clustering.

- Each segment belongs to a class $1 \leqslant q \leqslant Q$ (with probability π_q and intensity $\lambda_k = \ell_q$),
- Combination of EM and DP algorithms [PRLD07],
- Bat cries: Class = animal behaviour (hunt, transit, ...)

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Marked Poisson Process.

- $\{Y(t)\}_{0 \leqslant t \leqslant 1} \sim MPP(\lambda(t), \mu(t))$:

$$\{N(t)\}_{0 \leqslant t \leqslant 1} \sim PP(\lambda(t)), \qquad \text{at each } T_i: \quad X_i \sim \mathcal{F}(\mu(T_i))$$

- Works the same way, provided that concavity holds.
- Bat cries: Mark = bat species or cry duration.
- Poisson-Gamma events + Exponential-Gamma durations is both admissible and desirable.

Segmentation-clustering.

- Each segment belongs to a class $1 \leqslant q \leqslant Q$ (with probability π_q and intensity $\lambda_k = \ell_q$),
- Combination of EM and DP algorithms [PRLD07],
- Bat cries: Class = animal behaviour (hunt, transit, ...)

And also.

- Theoretically grounded model selection criterion (BIC),
- Consistency of the estimated change-points,
- Other desirable contrasts, ...

References I

C Dion-Blanc, E Lebarbier, and S Robin. Multiple change-point detection for poisson processes. Technical Report 2302.09103, arXiv, 2023.

C-H Ho and M Bhaduri. A quantitative insight into the dependence dynamics of the Kilauea and Mauna Loa volcanoes, Hawaii. *Mathematical Geosciences*, 49(7):893–911, 2017.

F. Picard, S. Robin, E Lebarbier, and J-J Daudin. A segmentation/clustering model for the analysis of array CGH data. *Biometrics*, 63(3):758–766, 2007.

Appendix

Number of elements in the partition of the segmentation space.

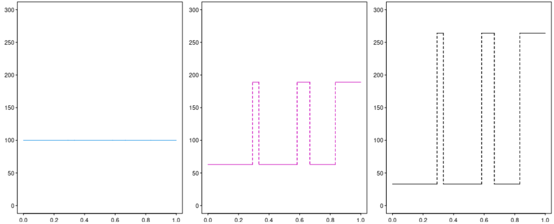
$$|\mathcal{N}_K| = \sum_{h=\lfloor (K-1)/2 \rfloor}^{K} \binom{n-1}{h-1} \binom{h+1}{K-h}$$

Appendix

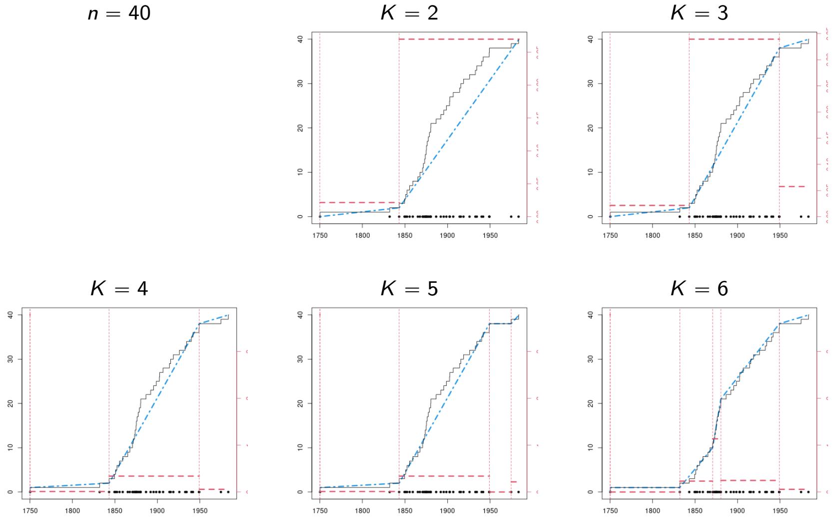
Number of elements in the partition of the segmentation space.

$$|\mathcal{N}_K| = \sum_{h=\lfloor (K-1)/2 \rfloor}^{K} \binom{n-1}{h-1} \binom{h+1}{K-h}$$

Simulations: Shape of the intensity function $\lambda(t)$. $K = 6$, $\overline{\lambda} = 100$, $\lambda_R = 1, 3, 8$.



Poisson process: Mauna Loa eruptions



Marked Poisson process: Etna eruptions

Count and marks: Events = eruptions, marks = duration of each eruption.

Model.

- Piecewise-constant intensity Poisson process for the events
- Exponential distribution (with segment specific parm.) for the durations

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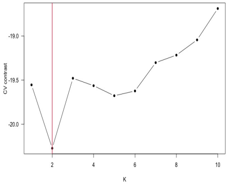
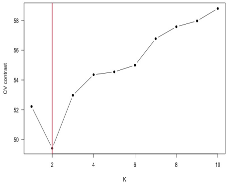
CV for the selection of K.

Poisson



Marked Poisson



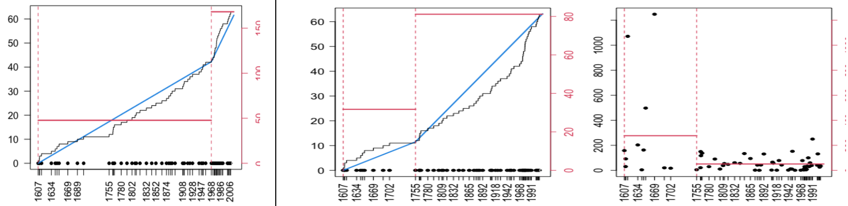
Marked Poisson process: Etna eruptions

Poisson ($\widehat{K} = 2$)

Events

Marked Poisson ($\widehat{K} = 2$)

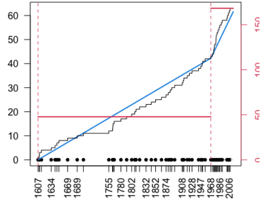
Events — Marks



Marked Poisson process: Etna eruptions

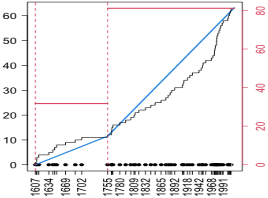
Poisson ($\widehat{K} = 2$)

Events

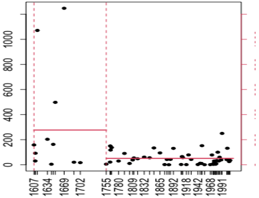


Marked Poisson ($\widehat{K} = 2$)

Events

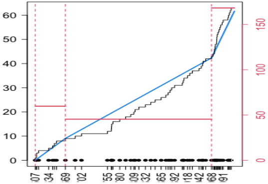


Marks



Marked Poisson ($K = 3$)

Events



Marks