Convergence Analysis of the Incremental Cost Consensus Algorithm Under Different Communication Network Topologies in a Smart Grid

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***Abstract*—In a smart grid, effective distributed control algorithms could be embedded in distributed controllers to properly allocate electrical power among connected buses autonomously. By selecting the incremental cost of each generation unit as the consensus variable, the incremental cost consensus (ICC) algorithm is able to solve the conventional centralized economic dispatch problem in a distributed manner. The mathematical formulation of the algorithm has been presented in this paper. The results of several case studies have also been presented to show that the difference between network topologies will influence the convergence rate of the ICC algorithm.**

***Index Terms*—Distributed control, economic dispatch, leader-follower consensus, multi-agent system, smart grid.**

I. Introduction

ECONOMIC dispatch is one of the fundamental problems in power systems. It is essentially an optimization problem with the objective of reducing the total generation cost, subject to several constraints. Previous efforts to solve the economic dispatch problem (EDP) have been made by implementing various numerical methods and optimization techniques. The conventional methods include the lambda-iteration method [1] and the gradient search method [2]. These methods provide a feasible solution when the fuel cost function is convex. More sophisticated techniques have been employed to solve the nonconvex cost function, such as genetic algorithms (GA) [3], particle swarm optimization (PSO) [4], and more recently the improved particle swarm optimization (IPSO) [5]. The performance and applicability of economic dispatch has been improved by these optimization techniques. However, it is essential to have a single control center that can access the state of the entire system. This centrally controlled framework may cause some performance limitations in the future power grid. In this paper, we explore the solution of the EDP from another angle. We propose using a distributed algorithm to solve the EDP in a distributed fashion, rather than the conventional central controller.

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The conventional centralized control scheme may encounter severe challenges in applying a Network Control System (NCS). First, in order to achieve the optimal operating conditions, the central controller will be required to have a high level of connectivity, which will impose a substantial computational burden and is typically more sensitive to failures and modeling errors than distributed control schemes [7]. Another challenge is that the topology of the smart grid is unknown, not only because of the variety of configurations of the power grid and communication network topologies, but also because "plug-and-play" technologies will make the topology time-varying [8]. Thus, in order to control this kind of NCS, or, specifically speaking, the smart grid, a robust algorithm should be able to operate correctly in the presence of limited and unreliable communication capabilities, and often in the absence of a central control mechanism [9].

Thus, distributed control algorithms are more suitable for solving this problem. A fundamental problem in distributed control systems is the need for all the nodes to reach a consensus. The consensus problem has been studied in several areas, such as social science, animal science [10], and computer science [11]. Consensus algorithms and their application have been studied extensively in the system and control area [12]–[18]. One of the potential algorithms that can be used to improve the power system is the consensus algorithm of a multi-vehicle system introduced in [12]. The multi-vehicle system performs cooperative tasks, and the vehicles are able to communicate with some set of other vehicles through the communication network. Similarly, the future power grid can be considered as an aggregation of controllable power electronics devices which controlled by distributed agents through the communication network [8]. This future power system is composed of two networks: a communication network and a power network.

We proposed an incremental cost consensus (ICC) algorithm as an example to illustrate the use of distributed control on a smart grid in [19]. Several preliminary simulations without generator dynamics also have been shown in [19] to demonstrate the feasibility of the ICC algorithm. In this paper, we focus on convergence analyses, such as the relationship between communication topology and convergence rate. We extended the ICC algorithm by adding the generator capacity into consideration. We also suggest using the node centrality measurement to help the "leader election" in the leader-follower consensus algorithm. In order to study the transient response of the ICC algorithm, we also extend the simulation model by including generator dynamics. The performance of ICC algorithm under large scale systems is discussed in this paper. Since the characteristics of consensus tracking behavior vary under different network topologies, we have evaluated the performance under different network topologies. The results are shown in several representative case studies.

The paper is organized as follows. In Section II, we introduce basic graph theory concepts and consensus algorithm preliminaries. The problem formulation of the ICC algorithm is in Section III. Simulation results based on different topologies are given in Section IV. Section V contains the convergence analysis. Finally, some conclusions are presented in Section VI.

II. Graph Theory and Consensus Algorithms

A. Graph Theory Notations

A graph G will be used to model the network topology of the system. The graph G is a pair of sets (V, E), where V is a finite non-empty set of elements called vertices or nodes and E is a set of unordered pairs of distinct vertices called edges. A simple graph is an unweighted, undirected graph containing no graph loops or multiple edges [20]. Unless stated otherwise, the unqualified term "graph" usually refers to a simple graph. A graph is connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node has exactly one parent except for one node, called the root, which has no parent but has a directed path to every other node . The directed spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph contains a directed spanning tree if there is a directed spanning tree that is a subset of the graph.

The adjacency matrix A of a finite graph G on n vertices is the $n \times n$ matrix where the off-diagonal entry a_{ij} is the number of edges from vertex i to vertex j. In the special case of a finite simple graph, the adjacency matrix is a (0, 1)-matrix with zeros on its diagonal. If the graph is undirected, the adjacency matrix is symmetric. Let matrix $L = [l_{ij}]$ be defined as

$$l_{ii} = \sum_{i \neq j} a_{ij}, \quad \text{for on-diagonal elements}$$
$$l_{ij} = -a_{ij}, \quad \text{for off-diagonal elements.}$$

For an undirected graph, L is called the Laplacian matrix and has the property of being symmetric positive semi-definite. It is well known that the eigenvalue of the L matrix contains a lot of information on network performance . For instance, the second smallest eigenvalue of Laplacian λ_2 is called the algebraic connectivity of the graph. The algebraic connectivity of the network topology is a measure of the convergence speed of consensus algorithms [12].

B. First-Order Consensus Algorithm

Let $x_i \in \mathbb{R}$ denote the state valuable of node i. The state value of a node might represent physical quantities such as voltage, output power, incremental cost, etc. We say that the nodes of a network have reached a consensus if and only if $x_i = x_j$ for all i, j [11]. Assuming each agent has a first-order dynamic

$$\dot{x}_i = x_i, \quad i = 1, \ldots, n. \tag{1}$$

A continuous-time consensus algorithm is given in [12], [17] as

$$\dot{x}_i = -\sum_{j=1}^{n} a_{ij}(x_i - x_j), \quad i = 1, \ldots, n \tag{2}$$

where a_{ij} is the (i, j) entry of the adjacency matrix A. The consensus algorithm also can be written in matrix form as

$$\dot{x} = -L_n x \tag{3}$$

where L_n is the $n \times n$ graph of the Laplacian matrix.

When the measurement information is available instantaneously, a continuous-time model can be used to describe the dynamics of the consensus network. When information takes a fixed time T to travel between nodes, we need to model the consensus network dynamics as a discrete-time dynamic system [17] to facilitate analysis. A discrete-time consensus algorithm is described by

$$x_i[k+1] = \sum_{j=1}^{n} d_{ij} x_j[k], \quad i = 1, \ldots, n \tag{4}$$

where k is the discrete-time index and d_{ij} is the (i, j) entry of the row-stochastic matrix D, which can be defined by the following:

$$d_{ij} = |l_{ij}| \Bigg/ \sum_{j=1}^{n} |l_{ij}|, \quad i = 1, \ldots, n. \tag{5}$$

Since each data packet in a communication network always arrives discretely, the discrete consensus algorithm, which is also known as the quantized consensus [15], has been selected for further development.

III. Incremental Cost Consensus Algorithm

Using the consensus algorithm as the basic framework, the conventionally centralized control problem can be solved in a distributed manner. In this section, we use the economic dispatch problem as an example to illustrate the use of the distributed consensus algorithm.

A. Problem Statement

The objective of solving the EDP is to minimize the total cost of operations. Traditionally, all of the generator parameters have to be sent to the control center. The control center calculates the optimal system operation point based on the information acquired from the entire system.

a) Conventional Approach: By using the Lagrange multiplier method to solve the EDP, assuming no generator has reached its generation limit, each generator will have the same incremental cost (IC) at the optimal operating point. An appropriate consensus algorithm can guarantee that all of the consensus variables converge to a common value asymptotically [12]. Thus, the IC (also known as λ) has been selected as the consensus variable.

For example, consider a three-bus system where each bus has its own generator and load. Fig. 1(a) shows the system communication topology when using the conventional central control.

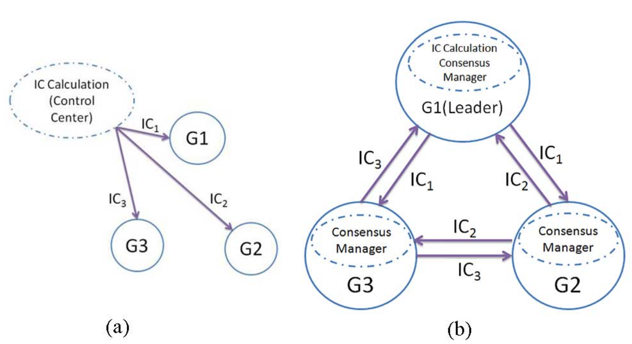

Fig. 1. (a) Conventional centralized control communication topology for a three-unit system. (b) Distributed control for an incremental cost consensus network.

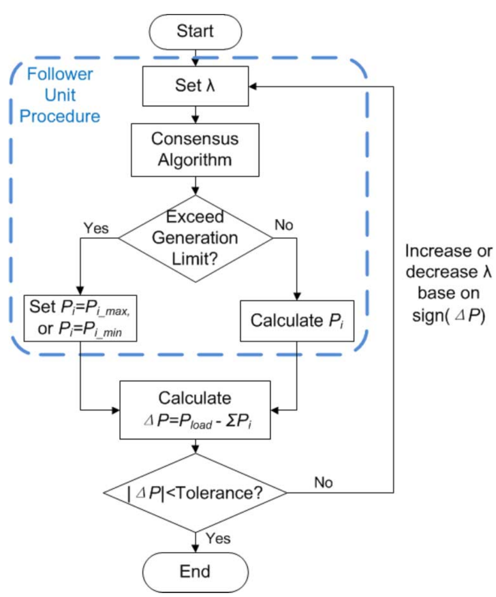

Fig. 2. Flowchart of ICC algorithm.

The control center acquires all the information (loads, generator output powers, etc.) and calculates the IC for each generator ($G1, G2$, and $G3$).

b) Decentralized Approach: By using the consensus algorithm and selecting the IC as the consensus variable, the EDP can be solved in a distributed manner. Fig. 1(b) shows a consensus network with distributed control: the local controller (embedded in each generation unit) will update its own IC based on its neighbors' ICs. In addition, a "leader unit" has to be selected, which will control whether to increase or decrease the group IC. That is, if the sum of total power generation is larger than the actual load, then decrease the group IC; and vice versa. In the example shown in Fig. 1(b), G1 has been selected as the leader generator.

Fig. 2 shows the flowchart of the ICC algorithm. The leader unit will run through the entire procedure in every iteration; the follower units only need to run the basic consensus algorithm and generation limitation checking process.

B. ICC Algorithm Formulation

Assume the generation units have a quadratic cost function

$$C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2. \tag{6}$$

The objective of solving the EDP is to minimize the total cost of operation for an n generator system

$$C_{\text{total}} = \sum_{i=1}^{n} C_i(P_{Gi}). \tag{7}$$

Under the power balance constraint and power generation constraint

$$P_D - \sum_{i=1}^{n} P_{Gi} = 0, \tag{8}$$

$$P_{Gi,\min} \le P_{Gi} \le P_{Gi,\max} \tag{9}$$

where P_{Gi} denotes the output power of unit i and P_D denotes the total power demand. Assume that all the generation units are operating within their generation constraints. In the ICC algorithm, the definition of IC for each generator is the same as the conventional economic dispatch

$$\text{IC}_i = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = \lambda_i \quad i = 1, 2, \ldots, n. \tag{10}$$

Select IC as the information state, using the first-order discrete consensus algorithm described in [17]:

$$\lambda_i[k+1] = \sum_{j=1}^{n} d_{ij} \lambda_j[k], i = 1, \ldots, n \tag{11}$$

where d_{ij} is the (i, j) entry of the row-stochastic matrix D_n. In Fig. 1(b), (10) is the update rule for $G2$ and $G3$.

By following the update rule described in (10), the system will converge to a common IC asymptotically. The convergence rate of the average consensus is based on the topology of the system's communication network. Precisely speaking, the second smallest eigenvalue of the graph Laplacian, which is also known as the algebraic connectivity, is a measure of the convergence speed of the consensus algorithms [15].

Additionally, in order to satisfy the power balance constraint (8), define ΔP to indicate the mismatch between the total demand and the overall power generated:

$$\Delta P = P_D - \sum_{i=1}^{n} P_{Gi}. \tag{12}$$

The update rule for the lead generator becomes

$$\lambda_i[k+1] = \sum_{j=1}^{n} d_{ij} \lambda_j[k] + \varepsilon \Delta P \tag{13}$$

where ε is a positive scalar. We call ε the convergence coefficient and it controls the convergence speed of the lead generator. The increases/decreases of λ will follow the sign of ΔP. If $\Delta P > 0 (P_D > P_G)$, then we need generate more power, so the current λ should be increased. This is very similar to the concept of proportional control where the ε can be considered as the proportional gain. In Fig. 1(b), (12) is the update rule of

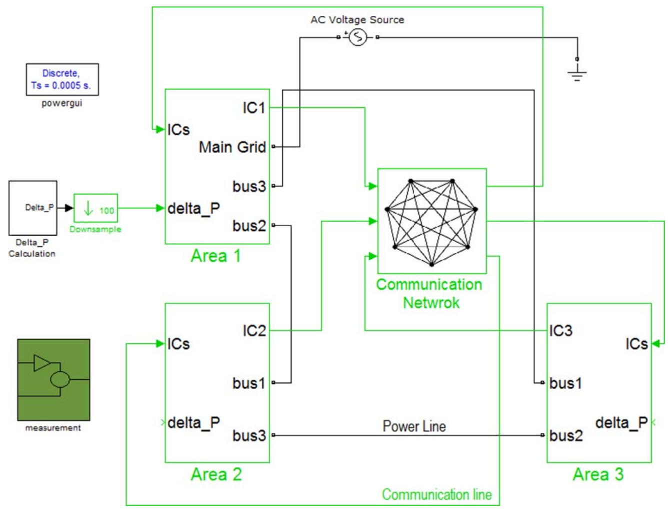

Fig. 3. Three-bus micro grid simulation.

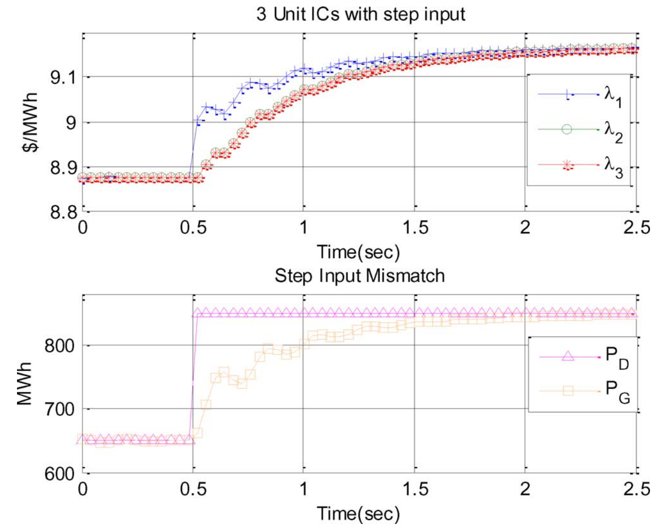

Fig. 4. Case study 1 IC and mismatch.

TABLE I
PARAMETERS OF THE THREE-UNIT SYSTEM

Unit	α_i	β_i	γ_i	$P_{Gi}(0)$
1	561	7.92	0.001562	300
2	310	7.85	0.00194	250
3	78	7.8	0.00482	100

the leader; in this case, G1 is the leader. Meanwhile, the power generation constraint also needs to be considered:

$$\begin{cases} \lambda_i = \lambda_{i\_\text{lower}}, \text{ when } P_{Gi} < P_{Gi,\min} \\ \lambda_i[k+1] = \sum_{j=1}^{n} d_{ij}\lambda_j[k], \text{ when } P_{Gi,\min} \le P_{Gi} \le P_{Gi,\max} \\ \lambda_i = \lambda_{i\_\text{upper}}, \text{ when } P_{Gi} > P_{Gi,\max} \end{cases} \tag{14}$$

Equations (11)–(14) are the mathematical representations of the ICC algorithm. Fig. 2 is a flow chart which represents the procedure of the ICC algorithm.

IV. Simulation Results

In this section, four representative case studies are presented to analyze the performance of the ICC algorithm under different load conditions and communication topologies. Fig. 3 shows the three-bus micro grid system we have developed in Simulink to test the performance of the ICC algorithm. There are a generation unit and load in each area. Each area is connected with rest of the areas through power lines and communication links. The synchronous generator has been modeled by using the basic swing equation:

$$J\frac{d\omega}{dt} = T_m - T_e \tag{15}$$

where J is the moment of inertia of the rotor, T_m is the mechanical torque supplied by the prime mover, T_e is the electrical torque output of the alternator, and ω is the angular speed of the rotor.

Case Study 1: Three-Unit System With Step Input Load: The system contains three generation units serving an electrical load P_D. The parameters and initial conditions for these three units are from [2] and are shown in Table I.

By using the Lagrange multiplier method to solve the system's IC, when $P_D = 850$ MW, the optimal incremental cost $\lambda^* = 9.148$ \$/MWh. The corresponding output powers for three generators are 393.2 MW, 334.6 MW, and 122.2 MW. A detailed derivation is available in [2].

The ICC algorithm described by (10) and (12) was used to solve *Case study 1*. The communication network topology is the same as in Fig. 1(b). The simulation was configured with a discrete 0.02-s fixed step size and a convergence coefficient of $\varepsilon = 0.0005$. Initially, $P_D = 650$ MW. The convergences of the three ICs are shown in Fig. 4. λ_1, λ_2, and λ_3 represent the incremental cost for Unit 1, Unit 2, and Unit 3, respectively. As the plot shows, initially, the three units all have the same value for λ and the system is in steady state.

After 0.5 s, the power demand increased from 650 MW to 850 MW. As Fig. 4 shows, the leader, Unit 1 (λ_1), first responded to the mismatch after the load increased. Then, the two follower units (Unit 2 and Unit 3) followed the IC value of Unit 1. After 2.5 s, all the λ's converge to the new optimal IC and P_D follows P_G asymptotically. This illustrates one of the advantages of the ICC algorithm: the ICC will drive the λ values of all the units to the optimal $\lambda*$ automatically if an optimal $\lambda*$ exists. Fig. 5 shows the corresponding output power of each generation unit in *Case study 1*.

Case Study 2: Five-Unit System With Star Connection: As mentioned in the previous section, one of the challenges is that the topology of the smart grid is unknown. Thus, in this case study, we extend the problem to a five-unit system serving an electrical load P_D to simulate the performance of the ICC algorithm. Fig. 6 shows the communication topology of the five-unit system.

This connection is known as a star connection. It is also a centralized control-like topology if we consider Unit 1 as the central controller. Since a star connection is a simplified version of a tree connection, this topology is similar to the topology that was discussed in [1]. The parameters and initial conditions for the five units are shown in Table II.

The simulation results are shown in Fig. 7. Similar to the three-unit system, Unit 1 was selected as the leader with a fixed

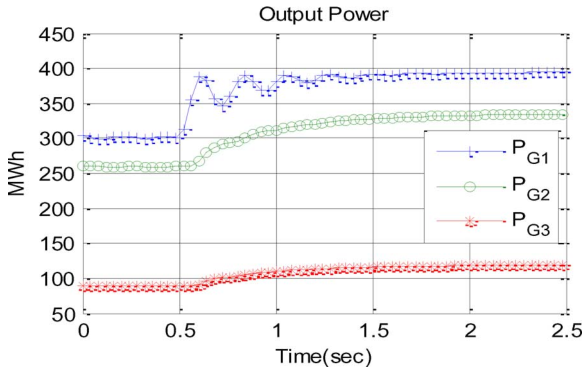

Fig. 5. Case study 1 output power.

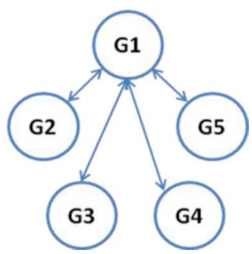

Fig. 6. Communication topology of the five-unit system: star connection.

TABLE II
PARAMETERS OF FIVE-UNIT SYSTEM

Unit	α_i	β_i	γ_i	$P_{Gi}(0)$
1	561	7.92	0.001562	200
2	310	7.85	0.00194	250
3	78	7.8	0.00482	100
4	561	7.92	0.001562	200
5	78	7.8	0.00482	100

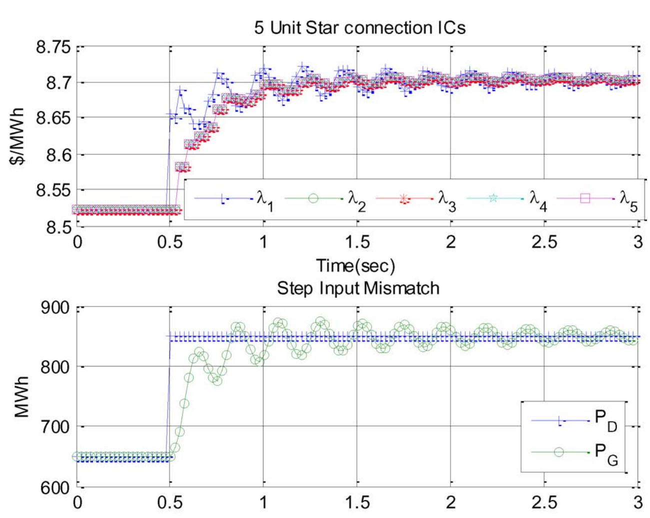

Fig. 7. Case study 2 ICs and mismatch.

step size of 0.02 s and a convergence coefficient of $\varepsilon = 0.0005$. As Fig. 7 shows, all the generation units were able to converge to the optimal IC asymptotically under the star topology.

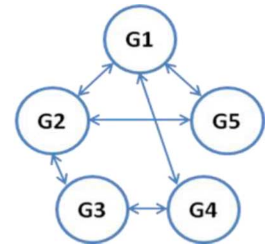

Fig. 8. Communication topology of the five-unit system: random connection.

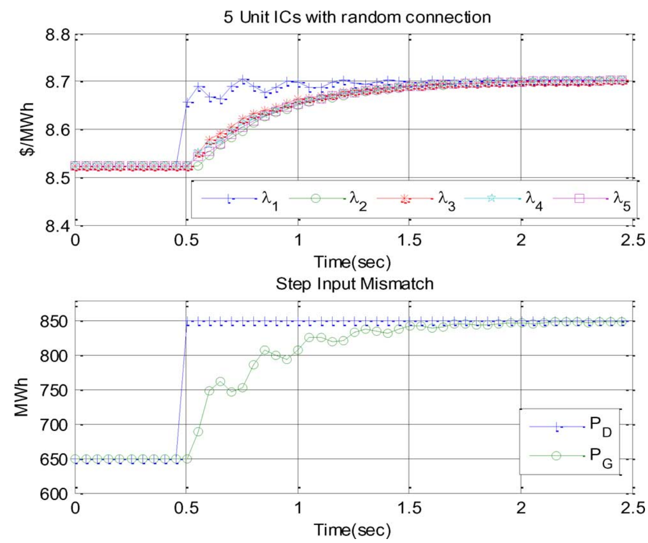

Fig. 9. Case study 3 ICs and mismatch.

Case Study 3: Five-Unit System With Random Connection: In order to demonstrate the capability of the ICC algorithm, a random network topology, as shown in Fig. 8, was selected to test the algorithm. Unlike *Case study 2*, no single unit is connected to the entire network in this topology. The generator parameter and initial conditions are same as in the previous case, and Unit 1 was selected as the leader. As the simulation result shows in Fig. 9, even with the absence of central control all of the generation units were still able to converge to the optimal IC asymptotically under this random topology.

Case Study 4: Five-Unit System With Generation Limitation: The idea of adding generation limitation into consideration in the ICC is similar to using the Lagrange multiplier method to solve the EDP. Conventionally, once one of the units reaches its limit, the maximum output power of that unit is subtracted from the total demand, and the EDP is solved for the remaining power demand using the rest of the generation units.

To demonstrate one of the generation units exceeding its limit, we use the same network configuration and parameters from *Case study 3*, with $P_{G2,\max}$ set to 300 MW. As Fig. 10 shows, Unit 2 reached its upper limit at 0.7 s. The corresponding λ_2 was set to its maximum value after Unit 2 reached the limit. These real ICs have been used to calculate ΔP. Thus, the leader will continue to raise the group λ until the demand has been reached. As Fig. 10 shows, the final group IC is 8.714$/MWh, which is larger than the final group IC in *Case study 3* of 8.7$/MWh.

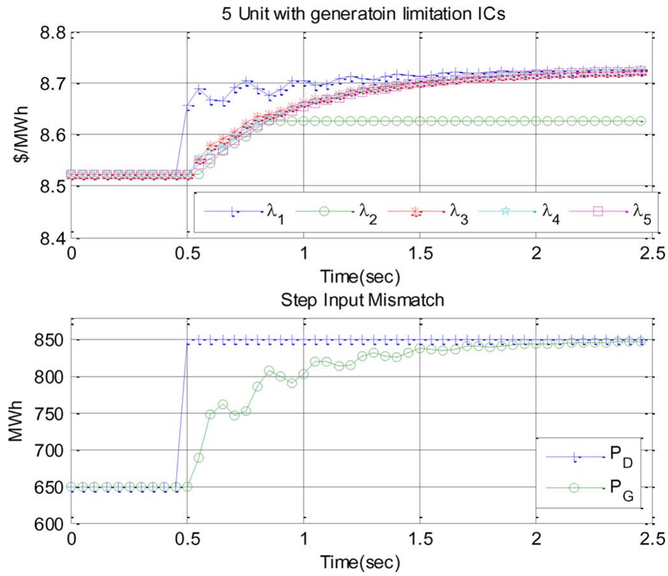

Fig. 10. Case study 4 ICs and mismatch.

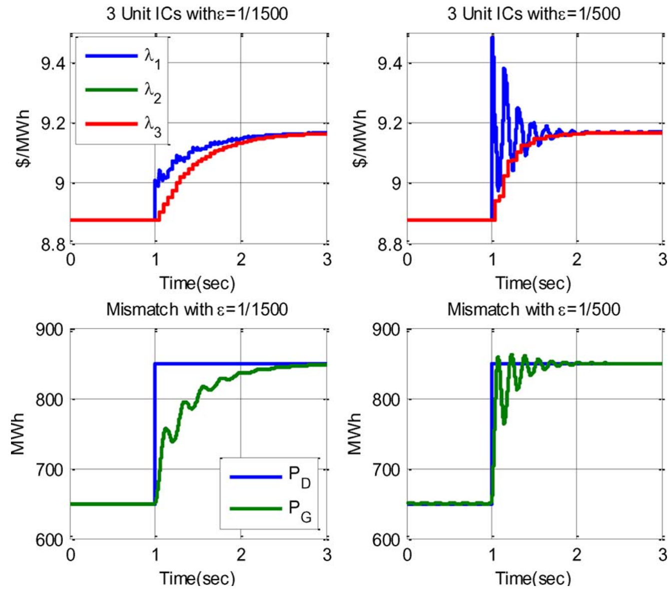

Fig. 11. Convergence analysis with different ε.

These four case studies show that, if a common optimal IC exists, the system will be able to converge to the optimal IC by using the ICC algorithm. Furthermore, the convergence is guaranteed as long as the topology satisfies the sufficient conditions of the consensus algorithm. It is obvious that the convergence rate is one of the critical parameters of the consensus network system. Failure to converge to the target IC in time will reduce system performance and may also affect the stability of the entire system. The topology of the communication network is one of the most important factors influencing the convergence rate of the ICC algorithm.

V. Convergence Analysis

There are many types of system configurations that can affect the convergence rate of EDP algorithms, such as the inertia of the synchronous generators, the power grid topology, the system sampling rate, and the signal transmission delay. These general configurations can affect the convergence rate of most of the economic dispatch algorithm including the lambda-iteration method and the gradient search method. Because the ICC algorithm can handle a wider range of communication networks, the topology of the communication network and the location of the leader will also affect the convergence rate.

The network topology may be determined by the geographical location of the system and is not easily modified. However, there are several other factors that can affect the convergence rate of the ICC algorithm, and these factors can be changed without any reconfiguration of the physical system.

Two convergences are involved when using the ICC algorithm. One is the total power generated, P_G, which converges to the total power demand P_D; the other convergence is that of the follower units' ICs to the leader unit's IC. Since the ICC algorithm is based on the discrete consensus algorithm, the common parameter that will affect these two convergence speeds is the discrete sampling time. Theoretically, decreasing the system's sampling time will increase both of the convergence speeds [17]. However, in practice, increasing the sampling rate also increases the burden on the communication network. Therefore, decreasing the system's sampling time is not a suitable solution.

A. P_G Converges to P_D

As we mentioned in Section III, the leader will increase or decrease the group IC based on the mismatch, ΔP. This convergence rate can be controlled by adjusting the convergence coefficient ε.

Fig. 11 shows two different convergence rates using different values for ε and the parameters from *Case study 1*. Using the same step size, the system with $\varepsilon = 1/500$ converges faster than the system with $\varepsilon = 1/1500$. Thus, the larger ε is related to a faster convergence speed. However, as Fig. 11 shows, λ_1 starts oscillating when $\varepsilon = 1/500$. Thus, if ε is increased above a certain limit, it will cause the system to become unstable. Since the entire distributed control system can be viewed as a loosely coupled linear system. Varying the value of ε is essentially varying the state transition matrix of this discrete-time linear system. Thus, the approximated optimal ε can be selected by examining the corresponding dominant eigenvalue of the state transition matrix. For the model we have used in this section, the variation of 2% settling time under different ε and its corresponding dominant eigenvalue are shown in Fig. 12. Since different system configurations may lead to the different state transition matrices such as different generator dynamics, different power grid topology and communication topology, etc., consequently, different system will have different optimal value of ε.

B. Following Units Converge to Group IC

The group λ is provided by the leader unit. In essence, this convergence speed is similar to most of the consensus algorithms under the leader-follower scenario. It has been studied extensively in the system and control area [16], [17]. As the previous section mentioned, this convergence rate is based on the topology of the system's communication network. Intuitively,

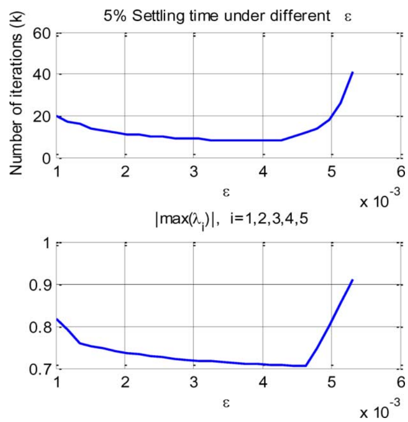

Fig. 12. Settling time with different ε.

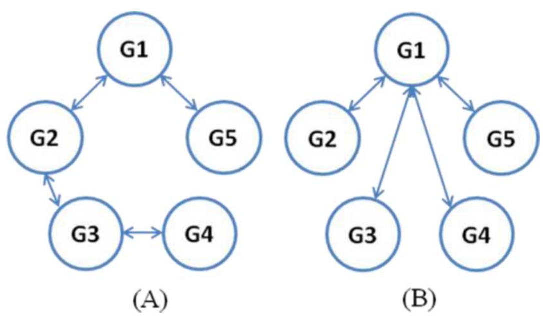

Fig. 13. Two different communication topologies.

one of the parameters that can quantify the convergence rate for a certain topology is algebraic connectivity.

Consider the five-unit system with a chain connection, which is shown in Fig. 13(A), and a star connection, which shown in Fig. 13(B).

The Laplacian graphs of topology A and topology B are

$$L_A = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_B = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The corresponding algebraic connectivities are 0.3820 and 1, respectively. Thus, topology B should reach consensus faster than topology A. Fig. 14 shows the simulation results under different communication topologies when selecting G2 as leader. Although algebraic connectivity can represent the convergence rate of the average consensus algorithm under different topologies, topology A has a faster convergence rate than topology B, as shown in Fig. 14. This result suggests that the algebraic connectivity may not be a suitable measurement for the convergence rate of the ICC algorithm.

Since the ICC algorithm essentially is a leader-follower consensus algorithm, it has some special properties that the leaderless consensus algorithm (also known as the average consensus

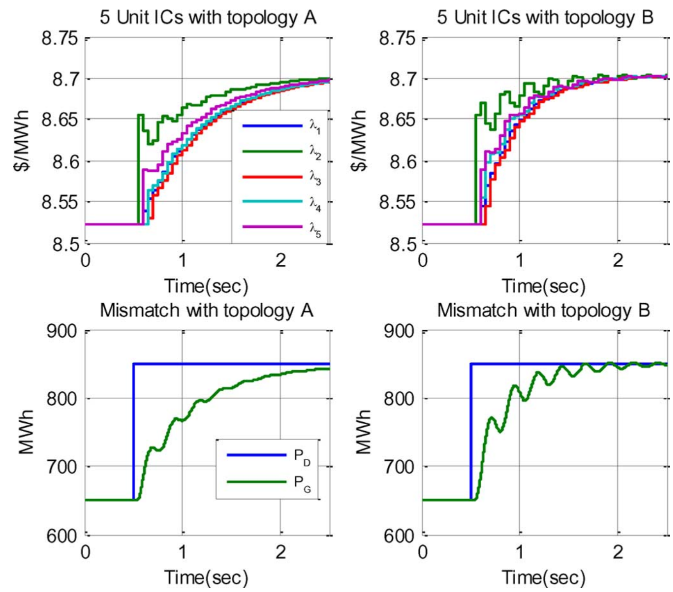

Fig. 14. Convergence analysis with different topologies.

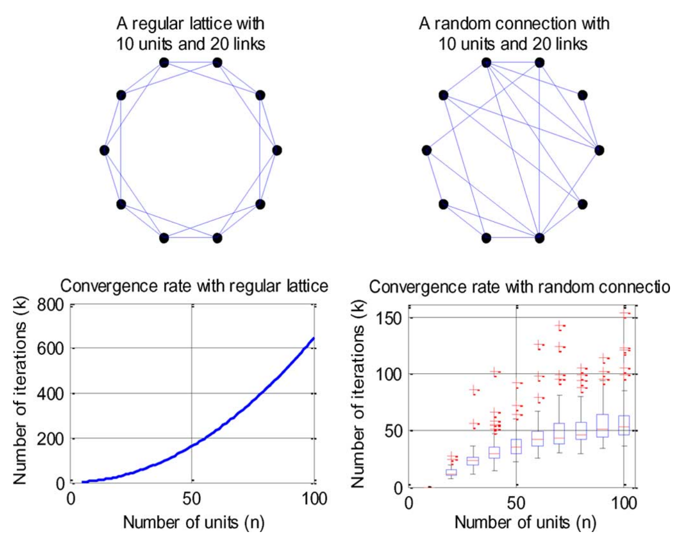

Fig. 15. Convergence rate with a different number of units.

algorithm) does not have. For instance, determining how to select the appropriate node as the leader to achieve a fast convergence speed is a typical problem for this type of algorithm. A heuristic approach for leader selection is to use the existing node centrality indices. Currently, there are multiple centrality indices in use by researchers. To name a few, they are degree centrality, betweenness centrality, closeness centrality, eigenvector centrality, etc. These centrality indices can be used as indicators for selecting the leader node.

C. Convergence Rate Under a Different Number of Units

As mentioned in the previous section, scalability is one of the advantages of distributed control systems over a centralized control system. Generally speaking, when the number of units increases, the ICC algorithm needs more iterations to reach a consensus. Thus, the convergence rate under a different number of units is a very critical performance measure. Two types of topologies have been selected for the convergence rate test and the results are shown in Fig. 14. In a regular lattice network, each unit can communicate with its one-hop neighbor and its two-hop neighbor. Thus, for an n-units network, the total number of links is $2n$. For the convergence rate test, 96 lattices with a number of nodes from 5 to 100 have been generated. As a comparison, a group of random networks also has been generated. In order to make a fair comparison, the number of links in each of the random networks is also $2n$.

As Fig. 15 shows, when the communication topology is a regular lattice, the number of iterations needed before it reaches consensus increases exponentially. However, with the random communication topology, the median of the convergence rate only increases linearly. Thus, the ICC algorithm is able to handle a relatively large number of units in a reasonable period of time. The current simulation is conducted by a PC with an Intel(R) Core™ i5 CPU at 2.3 GHz with 2 GB of RAM, and the simulation time for the 100-unit system is less than 0.04 s. It is worth mentioning that while the current simulation is conducted on a single machine, the computational load can be assigned to distributed controllers in the real-world application.

VI. Conclusion

This paper has explored the use of consensus algorithms embedded in generation units as an effective means of distributed control to minimize the total cost of operations in the power system. A practical EDP has been solved in a distributed manner to illustrate the use of distributed control on a smart grid. The simulation results demonstrate the effectiveness and robustness of the ICC algorithms even in the absence of a centralized control center. The ICC algorithm guarantees that all of the generation units can converge to the optimal IC asymptotically, as long as there is a common optimal IC corresponding to the minimum fuel cost point that is subject to the power balance constraint. The convergence is also guaranteed under different communication topologies as long as a minimal spanning tree exists in the communication topology.

The relationship between the convergence rate and network topology has been discussed through several case studies. The influence of the convergence coefficient ε and leader unit selection on the convergence rate also has been discussed through the simulation results.

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