The Leader Election Criterion for Decentralized Economic Dispatch Using Incremental Cost Consensus Algorithm

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***Abstract* -- In a smart grid, effective distributed control algorithms could be embedded in distributed controllers to properly allocate electrical power among connected buses autonomously. By selecting the incremental cost of each generation unit as the consensus variable, the Incremental Cost Consensus (ICC) algorithm is able to solve the conventional centralized economic dispatch problem in a distributed manner. The mathematical formulation of the algorithm is presented in this paper. The results of several case studies are also presented to show that the difference between network topologies will influence the convergence rate of the ICC algorithm.**

***Index Terms*—Distributed control, leader-follower consensus, economic dispatch, multi-agent system, smart grid**

I. INTRODUCTION

The current US power system has been serving us for more than 60 years. It has encountered many challenges which it was not initially designed to handle. There is always a trade-off between efficiency and reliability in most systems, and a power system is no exception. Since the first priority of a power system is to provide quality electrical power, the energy management should not affect the stability of the power system under any circumstance. Most of the conventional control algorithms and energy management techniques were designed to be operated with very limited communication requirements, so the overall efficiency of a conventional power system is limited.

One of the major upgrades from current grid to smart grid is the improvement of the communication network. State-of-the-art Information and Communication Technologies (ICT) can provide a much more reliable communication network with improved throughput. The improvements of ICT can be used to develop more powerful energy management techniques.

In the last few years, new operating techniques have been developed for smart grid operation. The conventional Optimal Power Flow (OPF) has been extended to distribution level [1] and also has been revised under a smart grid environment [2]. The Unit Commitment (UC) has also been modified for the smart grid in order to push the performance further to the system limit [3]. Some of the new optimization techniques have been applied to smart grid operation such as Risk-limiting Dispatch [4] and Jump and Shift Method [5]. Auction based algorithms also have been developed such as intelligent auction scheme for smart grid market [6] and flow-gate bidding [7].

A power system is inherently distributed, so one of the possible control approaches is to apply distributed control algorithms to power system problems. The smart grid can be viewed as a Large-Scale Networked Control system (LSNCS). In a LSNCS, the components of the system, such as sensors, controllers and actuators are connected by the communication network [8].

The conventional centralized control scheme may encounter severe challenges when applying a LSNCS. The central controller is required to have a high level of connectivity, which may impose a substantial computational burden. The centralized control scheme is also more sensitive to failures than distributed control schemes [9]. Another challenge is that the topology of the smart grid is unknown, not only because of the variety of configurations of the power grid and communication network topologies, but also because "Plug-and-Play" technologies will make the topology time-

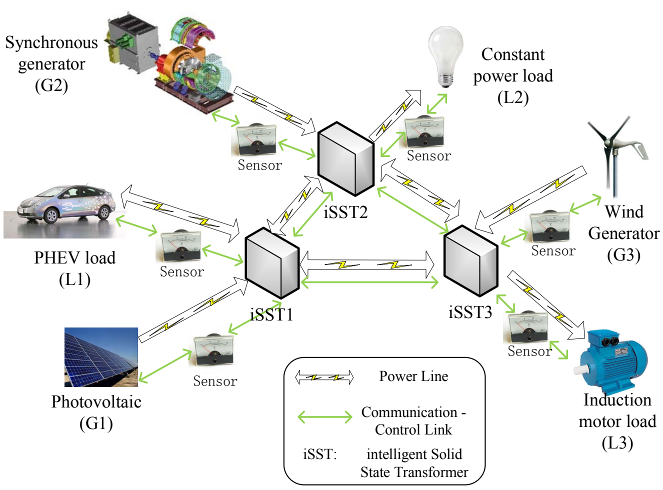

Fig. 1. A Prototype of future Microgrid

varying [10]. Thus, in order to control the smart grid, a robust algorithm should be able to operate correctly in the presence of limited and unreliable communication capabilities, and often in the absence of a central control mechanism. Fig.1 is a prototype of a future power system. In this Multi-Agent System (MAS), effective distributed control algorithms could be embedded in distributed controllers to properly allocate electrical power among connected buses autonomously. The agent based distributed control for microgrids are also discussed in [11] and [12].

We have proposed an Incremental Cost Consensus (ICC) algorithm as an example to illustrate the use of distributed control on a smart grid in [13]. Several preliminary simulations are also shown in [13] to demonstrate the feasibility of the ICC algorithm. The ICC algorithm has been extended to a more distributed fashion in [14], which the system mismatch can be acquired by the average consensus algorithm. Since most of the distributed algorithms have sacrificed efficiency in order to gain robustness, how to improve the efficiency of distributed controlled systems is always one of the most important problems of such systems. There are many parameters that can affect the convergence rate of an ICC algorithm, and in this paper, we explored the relationship between the convergence rate and the location of the leader. We have evaluated the performance of the ICC algorithm with different nodes acting as leader. The results are shown in several representative case studies. We have also compared our simulation results with several centrality indices. And we suggest selecting the leader based on the Eigenvector Centrality indices for our ICC algorithm.

The paper is organized as follows. In Section II, we introduce basic graph theory concepts and ICC algorithm preliminaries. The definition of four centrality indices is in Section III. Simulation results based on different leader location are given in Section IV. Finally, some conclusions are presented in Section V.

II. INCREMENTAL COST CONSENSUS ALGORITHM

In this section, the mathematic presentation of Incremental Cost Consensus (ICC) algorithm will be introduced. The detailed discussion of the ICC algorithm can be found in [13] and [14].

A. Graph Theory Preliminaries

A graph G will be used to model the network topology of the system. The graph G is a pair of sets (V, E), where V is a finite non-empty set of elements called vertices or nodes, and E is a set of unordered pairs of distinct vertices called edges. A simple graph is an unweighted, undirected graph containing no graph loops or multiple edges [15]. Unless stated otherwise, the unqualified term "graph" usually refers to a simple graph. A graph is connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. The directed spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph contains a directed spanning tree if there is a directed spanning tree that is a subset of the graph.

The adjacency matrix A of a finite graph G on n vertices is the $n \times n$ matrix where the off-diagonal entry a_{ij} is the number of edges from vertex i to vertex j. In the special case of a finite simple graph, the adjacency matrix is a (0,1)-matrix with zeros on its diagonal. If the graph is undirected, the adjacency matrix is symmetric. Let matrix $L=[l_{ij}]$ be defined as:

$$l_{ii} = \sum_{i \neq j} a_{ij}, \text{ for on-diagonal elements;}$$

$$l_{ij} = -a_{ij}, \text{ for off-diagonal elements.}$$

For an undirected graph, L is called the Laplacian matrix, which has the property that it is symmetric positive semi-definite.

B. Incremental Cost Consensus Algorithm

The objective of economic dispatch is to minimize the fuel cost of operation. Assume the generation units have the quadratic cost function:

$$C_i\left(P_{Gi}\right) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2. \tag{1}$$

The objective of the EDP is to minimize the total cost of operation for an n generator system:

$$C_{total} = \sum_{i=1}^{n} C_i\left(P_{Gi}\right). \tag{2}$$

under the power balance constraint:

$$P_D - \sum_{i=1}^{n} P_{Gi} = 0, \tag{3}$$

where P_{Gi} denotes the output power of unit i and P_D denotes the total power demand.

Using the consensus algorithm as the basic framework, the Economic Dispatch Problem can be solved in a distributed manner. The definition of *Incremental Cost* for each generator is the same as the conventional economic dispatch:

$$IC_i = \frac{\partial C_i\left(P_{Gi}\right)}{\partial P_{Gi}} = \lambda_i \quad i = 1, 2, \ldots, n. \tag{4}$$

Select IC as the consensus variable, using the first-order discrete consensus algorithm described in [16]:

$$\lambda_i[k+1] = \sum_{j=1}^{n} d_{ij} \lambda_j[k], i = 1, ..., n, \tag{5}$$

where d_{ij} is the (i ,j) entry of the row-stochastic matrix D_n. (5) is the updating rule for followers.

In order to satisfy the power balance constraint, define ΔP to indicate the mismatch between the total demand and overall power generated:

$$\Delta P = P_D - \sum_{i=1}^{n} P_{Gi}. \tag{6}$$

The update rule for the leader generator becomes:

$$\lambda_i[k+1] = \sum_{j=1}^{n} d_{ij}\lambda_j[k] + \varepsilon\Delta P\,, \tag{7}$$

where ε is a positive scalar. We call ε the convergence coefficient and it controls the convergence speed of the leader generator. (7) is the update rule of the leaders.

Comparing (5) and (7), it is obvious that the leader has the external information. The information ΔP can be acquired though a distributed average consensus network [12]:

$$\Delta P_i[k+1] = \sum_{j=1}^{n} w_{ij}\Delta P_j[k], i = 1,...,n, \tag{8}$$

where ΔP_i is the power mismatch at each node, and ΔP is the total power mismatch which equals to that defined in (6), w_{ij} is the $(i\,,j)$ entry of the updating matrix W. $W=I-\mu L$, $0 < \mu < 1/d_{max}$. μ is the updating weight, and d_{max} is the maximum degree of the graph G. The optimal setting of μ for every specific topology is given in [17]:

$$\mu^* = \frac{2}{\lambda_2(L) + \lambda_n(L)} \tag{9}$$

where $\lambda_2(L)$ is the algebraic connectivity and $\lambda_n(L)$ the largest eigenvalue of L. By using an average consensus algorithm to solve the value of ΔP, every node can acquire the value of total power mismatch, thus there could be multiple leaders in the network to improve the convergence rate [12].

The power generation constraint also needs to be considered:

$$\begin{cases} \lambda_i = \lambda_{i\_lower}, \; when\; P_{Gi} < P_{Gi,\min} \\ \lambda_i[k+1] = \sum_{j=1}^{n} d_{ij}\lambda_j[k], \; when\; P_{Gi,\min} \le P_{Gi} \le P_{Gi,\max} \\ \lambda_i = \lambda_{i\_upper}, \; when\; P_{Gi} > P_{Gi,\max} \end{cases} \tag{10}$$

Equations (5) - (10) are the mathematical representations of the ICC algorithm. Fig. 2 is a flow chart which represents the procedure of the ICC algorithm.

C. Convergence Rate of ICC

The convergence rate of the ICC algorithm is important, as a slow convergence rate not only leads to poor performance by the system but also may affect the stability of the system. There are several parameters that can affect the convergence rate such as the algebraic connectivity of the communication network, the convergence coefficient ε [11], the weight of the updating matrix and the location of leader. In this paper, we focus on the relationship between the location of leader and the convergence rate of *IC*.

III. CENTRALITY INDICES

Algebraic connectivity can quantify the convergence speed for the leaderless consensus network. However, in the leader-follower consensus architecture, the location of the leader can affect the convergence rate. Thus, in this paper, we are interested in the importance of the individual node in a given graph.

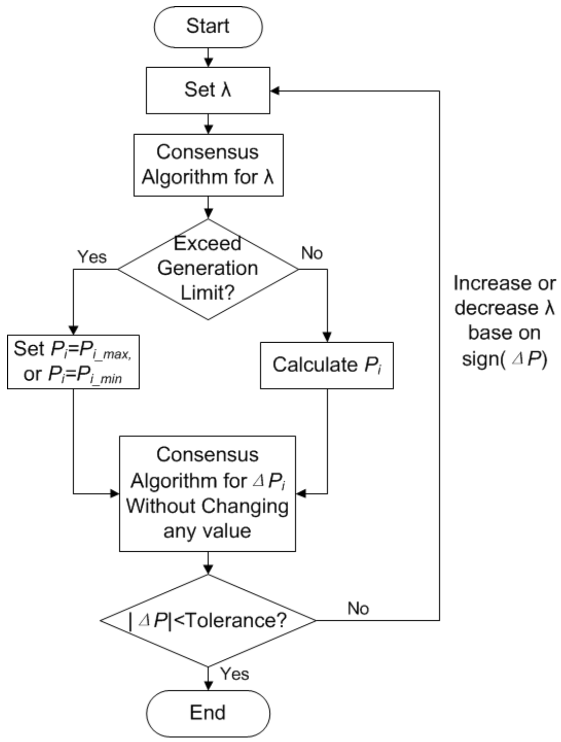

Fig. 2. Flowchart of ICC algorithm

In graph theory, “centrality” indicates the importance of one vertex or edge among the rest of the vertices or edges in the graph. In this paper, the centrality we have discussed only refers to node centrality. Researchers have been proposing different metrics for centrality for years, but there is no single accepted definition. Almost all metrics are empirical. There are four commonly used indices, Degree Centrality, Betweenness Centrality, Closeness Centrality and Eigenvector Centrality.

A. Degree Centrality

This is the simplest measure, for undirected graph $G = (V,E)$ with n nodes (vertices), the degree centrality $C_D(v)$ for vertex v is:

$$C_D(v) = \frac{\deg(v)}{n-1}, \tag{11}$$

where n-1 is the possible maximum degree for a vertex. Since we are considering the simple graph, there is no self loop. Degree Centrality is very easy to calculate. However, the disadvantage is that it doesn’t include density information for the rest of the network.

B. Betweenness Centrality

Betweenness Centrality is a shortest path enumeration-based metric introduced by Freeman [18]:

$$C_B(v) = \sum_{s \ne v \ne t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}, \tag{12}$$

where σ_{st} is the number of shortest paths from any vertex s to any vertex t, and $\sigma_{st}(v)$ is the number of shortest paths from s to t that pass through the vertex v.

Unlike the Degree Centrality the Betweenness Centrality contains the density information of the entire network. However, Betweenness Centrality needs to calculate the shortest path for every pair of nodes, so it requires a lot of computation that may not be suitable for real-time application.

C. Closeness Centrality

The Closeness Centrality for vertex v is the reciprocal of the sum of shortest distance to all other vertices of V [19]:

$$C_C(v) = \frac{1}{\sum_{t \in V}^{n} dis(v,t)}, \quad (13)$$

where $dis(v, t)$ denotes the shortest distance from any vertex v to any vertex t.

Closeness Centrality is relatively easy to calculate compared to Betweenness Centrality. However, it can only calculate connect graph, because one unconnected node will make every node $C_C=1/\infty$. Moreover, the Closeness Centrality suffers the same issue as Degree Centrality: it doesn't include density information for the rest of the network.

D. Eigenvector Centrality

This is a more sophisticated index introduced in [20]; where Degree Centrality gives a simple count of the number of connections a vertex has, Eigenvector Centrality acknowledges that not all connections are equal. If we denote the centrality of vertex i by x_i, then we can allow for this effect by making x_i proportional to the average of the centralities of i's network neighbors:

$$x_i = \frac{1}{\lambda}\sum_{j=1}^{n} A_{ij}x_j, \quad (14)$$

where λ is the constant. Equation (14) can be rewritten in matrix form as

$$Ax = \lambda x \quad (15)$$

Thus we can see that x is an eigenvector of the adjacency matrix with eigenvalue λ. Assuming that we wish the centralities to be non-negative, it can be shown that λ must be the largest eigenvalue of the adjacency matrix and $\boldsymbol{x}$ the corresponding eigenvector [21].

The Eigenvector Centrality depends on the number and the quality of its connections: having a large number of connections still counts, but a vertex with a smaller number of high-quality contacts will be better than the one with a larger number of low-quality contacts.

With these four centrality indices, we can test which one is more suitable for our leader election application.

IV. Leader Election Criterion

In this section, we will discuss the simulation results of the ICC algorithm with different location of leader. Fig. 3 shows the communication topology of the five-unit system. There is a generation unit and load in each area. By just looking at the topology in Fig. 3, it is hard to tell whether G2 (and G5) or G3 is more important to the network.

The ranking of each node based on the four centrality indices is shown in Table I. The Degree Centrality and Closeness Centrality suggest that the ranking of 5 nodes from the most important to the least important are *G2 = G3 = G5 >G1 > G4.* And the Betweenness Centrality suggests that: *G3 > G2 = G5 >G1 > G4.* Finally the Eigenvector Centrality suggests that *G2 = G5 > G3>G1 > G4.*

TABLE I Centrality Ranking of Five Nodes

Node	C_D	C_B	C_C	C_E
1	0.5	0	0.14	0.41
2	0.75	0.2	0.2	0.54
3	0.75	0.6	0.2	0.47
4	0.25	0	0.13	0.18
5	0.75	0.2	0.2	0.54

In this case, the C_D and C_C are not able to separate the difference between G2 and G3, so they have already failed in this particular case. C_B and C_E give the opposite results.

The system contains five generation units serving an electrical load P_D. The parameters and initial conditions for the five units are shown in Table II:

TABLE II Parameters of five unit system

Unit	α_i	β_i	γ_i
1	561	7.92	0.001562
2	310	7.85	0.00194
3	78	7.8	0.00482
4	561	7.92	0.001562
5	78	7.8	0.00482

A. Case 1

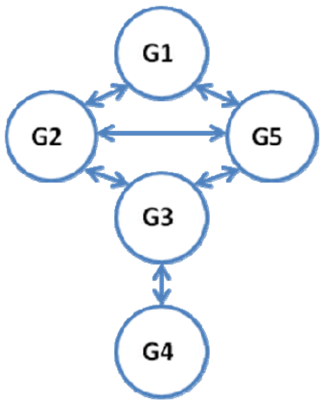

Fig. 3. Case study communication topology

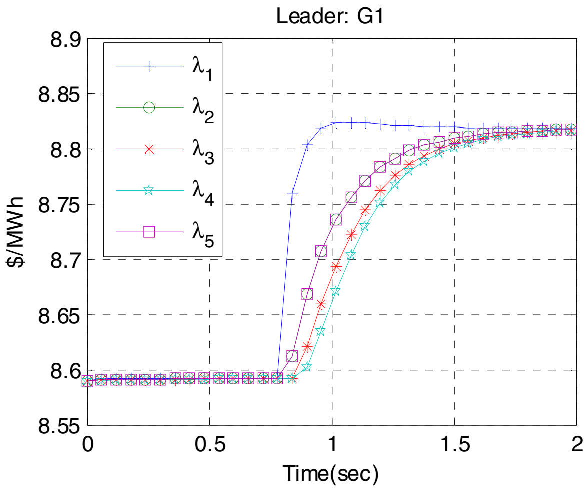

Fig. 4. The convergence result when G1has been selected as leader

First we selected G1 as the leader; the result is shown in Fig .4. Five units' incremental costs (λs) have been plotted. As Fig. 4 shows, the system started with an equilibrium point where all generators have the same λ. We simulated the load change as a step input at Time = 0.8 sec. Since G1 had been selected as the leader, it sensed the load change and started raising its λ first. G2 and G5 are the neighboring nodes of G1, so they followed the G1's change, which was then followed by G3 and G4. The system reached its new equilibrium point around 2 seconds.

B. *Case 2*

Then we selected G2 as the leader and kept the rest of the parameters unchanged. Fig. 5 shows the simulation result. The system converged faster than *Case 1*. Similar to what *Case 1* shows, the leader (G2 in this case) first responded to the load change and rest of the group followed. The system reached consensus before 1.5 seconds.

C. *Case 3*

We selected G3 as the leader; the simulation result is shown in Fig. 6. Since G4 is an end node and it only has one connection to the rest of the group, it will follow its only neighbor very close. In this case, G4's only neighbor is the leader, so it even followed the overshoot of G3. If we further increase the value of convergence coefficient ε, G3 and G4 will start to oscillate.

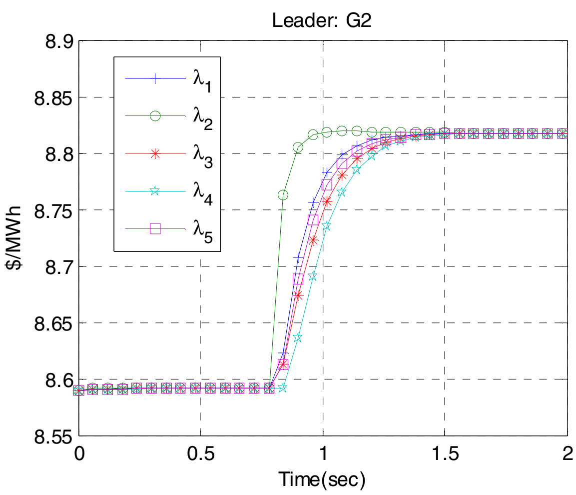

Fig. 5. The convergence result when G2 has been selected as leader

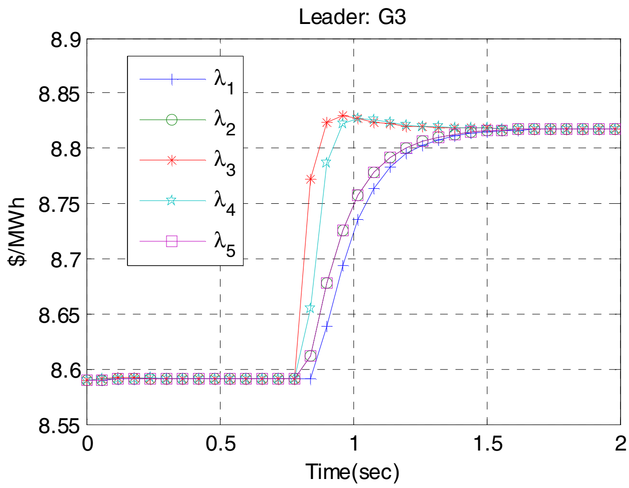

Fig. 6. The convergence result when G3 has been selected as leader

More importantly, the convergence rate in *Case 3* is slightly slower than the convergence rate shown in *Case 2,* which indicates that the system performance when we select G2 as the leader is better than the system performance when we select G3 as the leader. Thus the ranking given by the Eigenvector Centrality is the same as the simulation result.

If we select G5 as the leader, the results are similar with this case since the graph is symmetric.

D. *Case 4*

Lastly we selected G4 as the leader. As Fig 7 shows, the convergence rate is the slowest one among the five nodes, which verified that G4 has the lowest ranking in all centrality indices.

Based on the comparison of simulation results and ranking given by the four centrality indices, we suggest using Eigenvector

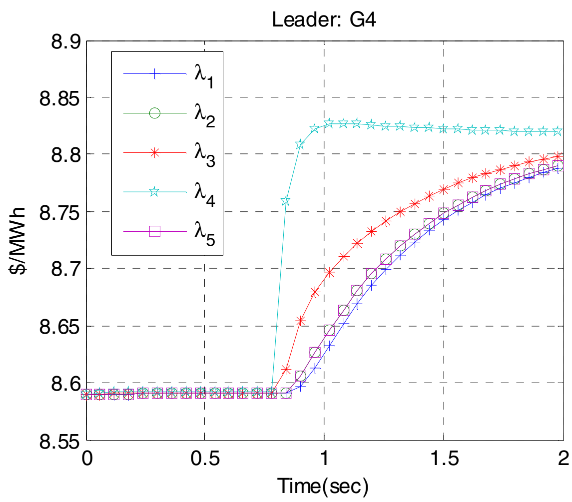

Fig. 7. The convergence result when G4 has been selected as leader

Centrality index of the node as the leader election criterion for the ICC algorithm.

V. CONCLUSIONS

In this paper, a practical EDP has been solved in a distributed manner to illustrate the use of distributed control on a smart grid. The simulation results demonstrate the effectiveness and robustness of the ICC algorithms even in the absence of a centralized control center. The ICC algorithm guarantees that all of the generation units can converge to the optimal IC asymptotically, as long as there exists a common optimal IC corresponding to the minimum fuel cost point subject to the power balance constraint. The convergence is also guaranteed under different communication topologies as long as a minimal spanning tree exists in the communication topology.

We have evaluated the performance of the ICC algorithm with different nodes acting as the leader. The results are shown in several representative case studies. We have also compared our simulation results with several centrality indices. We suggest selecting the leader based on the Eigenvector Centrality indices for our ICC algorithm.

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