CHARACTERIZATION OF WAVE TRANSPORT IN NON-CONSERVATIVE MEDIA WITH RANDOM AND CORRELATED DISORDER

by

BENJAMIN HENRY PAYNE

A DISSERTATION

Presented to the Faculty of the Graduate School of the

MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

In Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

in

PHYSICS

2012

Approved by

Dr. Alexey Yamilov, Advisor
Dr. Hui Cao
Dr. Paul Parris
Dr. John Story
Dr. Thomas Vojta

PUBLICATION DISSERTATION OPTION

This dissertation has been prepared in the form of six papers.

Paper 1. Pages 21–57 have been published as *Relation between transmission and energy stored in random media with gain*, Physical Review B **82** 104204 (2010) with Jonathan Andreasen, Hui Cao, and Alexey Yamilov.

Paper 2. Pages 58–69 have been published as *Classification of regimes of wave transport in non-conservative random media*, Journal of Modern Optics (2010) with Alexey Yamilov.

An earlier letter, *Criterion for light localization in random amplifying media*, has been published in Physica B **405**, 3012–3015 (2010) with Hui Cao and Alexey Yamilov. This is reported in Paper 1.

ABSTRACT

Passive quasi-one-dimensional random media exhibit one of the three regimes of transport – ballistic, diffusive, or Anderson localization – depending on system size. The ballistic and diffusion approximations assumes particle transport, whereas Anderson Localization occurs when wave self-interference effects are dominant. When the system contains absorption or gain, then how the regimes can be characterized becomes unclear. By investigating theoretically and numerically the ratio of transmission to energy in a random medium in one dimension, we show this parameter can be used to characterize localization in random media with gain.

Non-conservative media implies a second dimension for the transport parameter space, namely gain/absorption. By studying the relations between the transport mean free path, the localization length, and the gain or absorption lengths, we enumerate fifteen regimes of wave propagation through quasi-one-dimensional non-conservative random media. Next a criterion characterizing the transition from diffusion to Anderson localization is developed for random media with gain or absorption. The position-dependent diffusion coefficient, which is closely related to the ratio of transmission to energy stored in the system, is investigated using numerical models.

ACKNOWLEDGMENTS

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Alexey Yamilov. Brilliant and motivated. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

Dr. Paul Parris, Dr. Greg Story, and Dr. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. , Cavendish McKay, and Thad Walker Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Dr. Scott Eckel, Beth Torrison,

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

TABLE OF CONTENTS

Page

APPENDICES

LIST OF ILLUSTRATIONS

LIST OF TABLES

Table Page

1. INTRODUCTION

1.1. MESOSCOPIC LIGHT TRANSPORT

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[1, 2]. However, when effects due to self-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, Anderson localization (AL)[3].Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [4, 5, 6]. For finite systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [7, 8, 9].Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo;Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L$ from $\langle T \rangle \propto \ell_{tmfp}/L$ for diffusion to $\langle T \rangle \propto \exp(-L/\xi)$ for AL (e.g., [10]). Here, ℓ_{tmfp} is the transport mean free path, and ξ is the localization length (c.f. Appendix ALorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo).

Historically,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. This scale refers to a system length LLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-based predictions.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoL_ϕ is greater than L, the effect of de Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [1, 11, 12, 13]. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-electron and electron-phonon interactions.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo; more importantly, however,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, including electromagnetic waves[14, 15].

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [13], whereas for light, there is no such constraint.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Since actual experiments [1, 16, 17, 18]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

1.2. CRITERIA FOR DIFFUSION-LOCALIZATION TRANSITION

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(LC) in nonconservative random media. To investigate the transition process,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo λ is much less than ℓ_{tmfp}, whereas AL is expected when $k\ell_{tmfp} \approx 1$ in three dimensional (3D) random media [19]. Here, k is equal to $2\pi/\lambda$. Thus, AL cannot use the same particle-based models as diffusion. Although much work has been done with 3D systems, finite quasi-one-dimensional (quasi-1D)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, as described below,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1.5. Interest in quasi-1D systems is driven by experiments [20]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Before establishing an LC,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(see section 1.6).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In experiments with random media [21, 22]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [23].Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, the results apply to any self-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [24, 25, 26].

To determine whether AL or diffusion (or neither)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, three regimes are defined.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(ℓ_{scat}).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(see definition of ℓ_{tmfp} in Appendix A),Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Finally,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoξ. In this case,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. A single-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The term "parameter"Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. For transport of electrons, dimensionless conductance* g [7] is the parameter,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T$ is equivalent to g.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1; conductance of less than 1 indicates AL. For passive random media, single-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-to-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo g. Transmission T in photonic systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[27, 28, 29], is

$$T = \sum_{a,b} |t_{ab}|^2 = g \tag{1.1}$$

where t_{ab}Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$a$ at output b of the waveguide. For electronic systems, gLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-channel resolved transmission T_a and speckle T_{ab}:

$$\begin{aligned} T_a &= \sum_{b} |t_{ab}|^2 \\ T_{ab} &= |t_{ab}|^2 \end{aligned} \tag{1.2}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-to-one correspondence [30].Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Transmission greater than 1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas

*Conductance $G = \frac{e^2}{h} Tr(tt^{+}) = \frac{e^2}{h} g$ [27]

ultrices egestas commodo [31, 32], and transmission of less than 1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [16, 33]. Thus, a two-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, i.e.,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. A criterion,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-defined,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

1.3. PASSIVE CRITERIA CURRENTLY AVAILABLE

Currently,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. For example, Thouless [34]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\delta\omega$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\Delta\omega$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo:

$$\frac{\delta\omega}{\Delta\omega} = g. \tag{1.3}$$

Just as for g, when $\delta\omega/\Delta\omega$ is less than 1, then AL occurs. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The self-consistent theory of AL [35]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Without self-interference of waves, the diffusion coefficient D_0 is constant throughout the medium. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-interfere, the diffusion coefficient decreases.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$D(z)$. Thus, the change from constant D_0 to position-dependent $D(z)$ signifies the transition to AL. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Besides conductance, $D(z)$, and the Thouless criteria,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [36] of observables, the inverse participation ratio, and transmission fluctuations.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoparameter scaling. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. For example Ref. [16] presents a ratio $\mathrm{var}(T/\langle T\rangle)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle T\rangle$ is not well defined. When gain is present in media,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, a few will lase, and the average or higher moments of T are ill-defined. To avoid this issue,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. A second problem with the $\mathrm{var}(T/\langle T\rangle)$ ratio as a criterion is that TLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Section 1.4Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

1.4. $T/\mathcal{E}$ AS DIFFUSION-LOCALIZATION CRITERION

In media with gain, transmission TLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(RLT). (Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.). To eliminate the divergence TLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\mathcal{E}$.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [37]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ approaches a constant. Starting from conservation of energy ($\mathcal{E} = \int_0^L \mathcal{W}(z)dz$) with respect to flux J,

$$\frac{\partial \mathcal{W}}{\partial t} + \vec{\nabla} \cdot \vec{J} = \frac{\mathcal{W}c}{\ell_g} + J_0\delta(z - z_p) \tag{1.4}$$

where z_p is penetration depth, J_0 is incident flux, ℓ_g is gain length, and c is the speed of light.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,

$$\frac{dJ_z}{dz} = \frac{\mathcal{W}c}{\ell_g} + J_0\delta(z - z_p). \tag{1.5}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$z$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo:

$$T + R - 1 = \mathcal{E}\frac{c}{\ell_g J_0}. \tag{1.6}$$

In the limit that gain length ℓ_g approaches RLT (critical gain length $\ell_{g_{cr}}$), both T and R go to infinity. Assuming $T \approx R$,

$$\frac{T}{\mathcal{E}} = \frac{c}{2\ell_{g_{cr}} J_0}. \tag{1.7}$$

This constant is disorder-specific due to $\ell_{g_{cr}}$, so $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

For gain below RLT, by comparing the $\langle T/\mathcal{E}\rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(and thus constitute a signature of AL). For passive media,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle T/\mathcal{E}\rangle$ is related [37] to the well-established [38] $D(z)$ based on the self-consistent theory of AL (see Appendix C):

$$\left\langle \frac{T}{\mathcal{E}} \right\rangle \approx \frac{1}{J_0}\frac{2D_0}{L^2}\left(\frac{1}{L}\int_0^L \frac{D_0}{D(z)}dz\right)^{-1}. \tag{1.8}$$

Since $\langle T/\mathcal{E}\rangle$ is related to $D(z)$, then experimentally $\langle T/\mathcal{E}\rangle$ should behave as $D(z)$ does with respect to D_0 for passive media; that is, it should decrease as self-interference of waves increases. Therefore, $\langle T/\mathcal{E}\rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, does not diverge in media with gain,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$D(z)$.

1.5. METHOD OF STUDY OF CRITERIA FOR DIFFUSION- LOCALIZATION TRANSITION

When P. W. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-interference of waves,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, the Anderson tight-binding Hamiltonian [3],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. For quasi-1D geometry, random matrix theory (RMT) [39, 40, 41] is widely used. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(and thus the total energy $\mathcal{E}$) inside a random medium.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The first is a one-dimensional (1D)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(T) and energy inside the medium ($\mathcal{E}$) as a possible criterion $T/\mathcal{E}$ for nonconservative media [37, 42]. The ratio $T/\mathcal{E}$ has been verified as nondivergent,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(as expected). The 1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo; thus, the effects cannot be due to diffusion. Since diffusion is not possible in 1D systems, a planar quasi-1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo($D(z)$, inverse participation ratio, $T/\mathcal{E}$). This model is necessary since, even for passive systems, the literature offers no plot of $D(z)$ in the diffusive regime (c.f. Fig. 1.1).

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [8, 43, 44] for the entire waveguide. Essentially,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Not only is the quasi-1D geometry experimentally viable [45],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [46, 47]. Here, waveguides described as "quasi-1D" have the following characteristics: (1) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, expressed as $E(y = 0, W; \forall z) = 0$ as for metallic edges, (2) waveguide width W less than ℓ_{tmfp}Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, and (3) aspect ratio ($L : W$) is not fixed (i.e., W is constant when

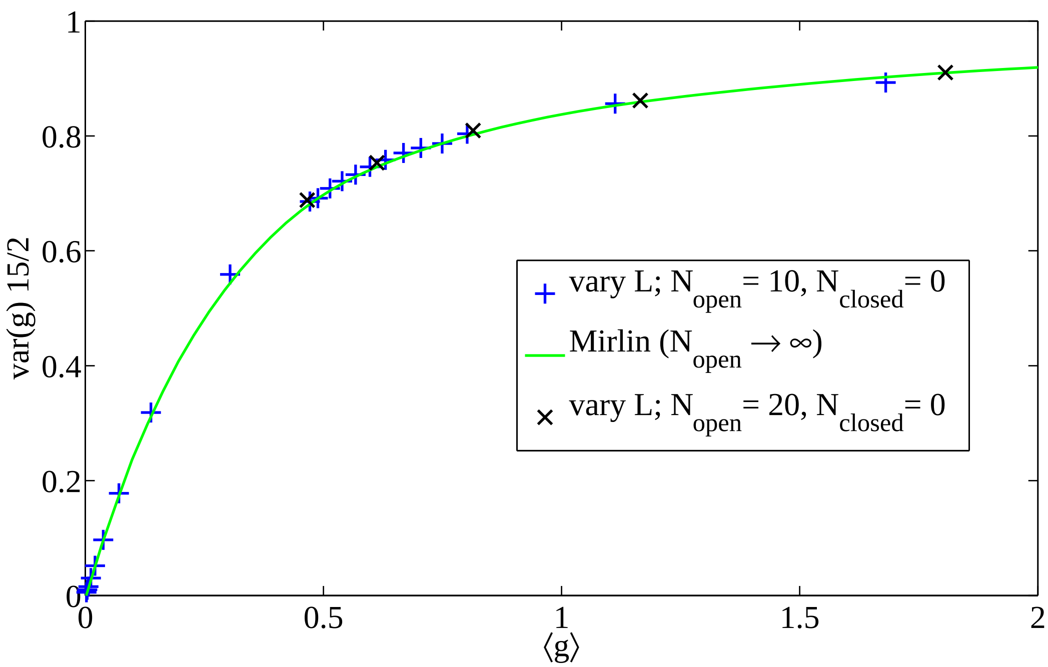

Figure 1.1: Position-dependent diffusion coefficient $D(z)$ as predicted by self-consistent theory of localization (smooth red curves) and numeric results (rough blue lines) for quasi-1D waveguides with randomly-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo L, constant scatterer density, and width W. Very good agreement for ballistic ($L = 100\lambda$), diffusive, and localized ($L = 800\lambda$) regimes. The term ℓ is transport mean free path, and z_0 is penetration depth.

L is increased, with a fixed disorder density). Further,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

As shown in Appendix B, the differential wave equation

$$\nabla^2 E(\vec{r}) = -\frac{\omega^2}{c^2} E(\vec{r}) \tag{1.9}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(resolving wave vector $\vec{k}$ into $k_\perp$ and $k_\|$).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, scattering potentials are introduced, initially as δ functions.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo*ab initio* based on Maxwell's equations [48],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

For light waves,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo form of a vector.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(c.f. Appendix B). In 1D,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoE_0 and its derivative E_0'Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoΔx:

$$\begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \begin{pmatrix} E_{\Delta x} \\ E_{\Delta x}' \end{pmatrix}. \tag{1.10}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\hat{T}_{total} = \prod_i \hat{T}_i$.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing

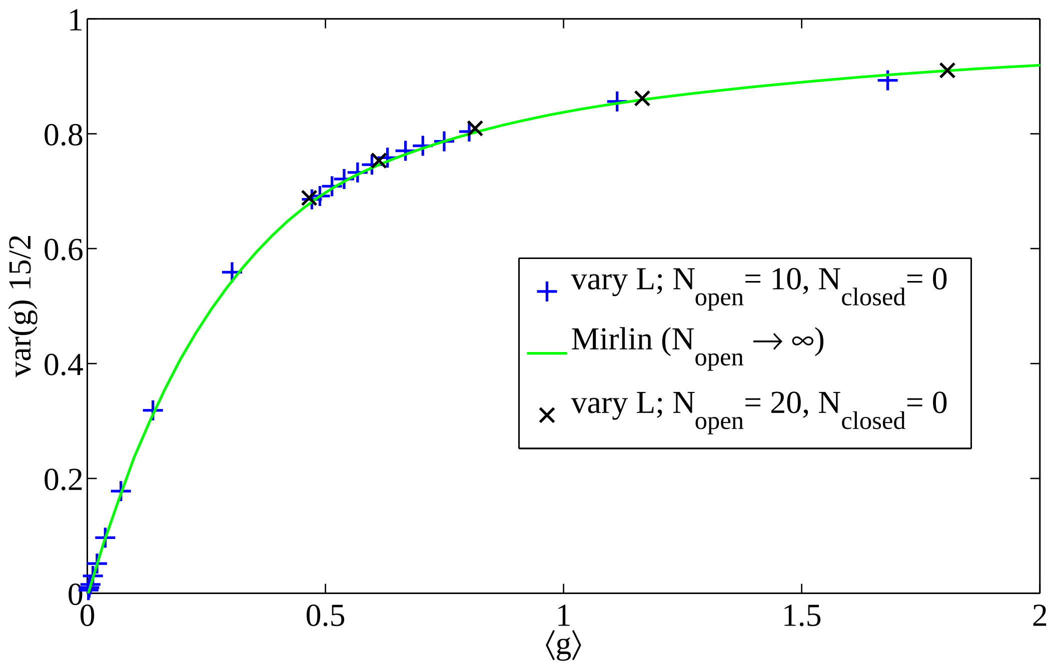

Figure 1.2: Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Midsection of waveguide is shown (from $z/L = 80/200$ to $z = 120/200$) for a resonant frequency (higher than average transmission). Spatially varying field intensity (with continuous wave incident flux)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoδ function.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, the resulting electric field magnitude, plotted in Fig. 1.2, is a secondary benefit.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [49],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [50]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo $\det(\hat{\mathrm{A}})\det(\hat{\mathrm{B}}) = \det(\hat{\mathrm{A}}\hat{\mathrm{B}})$. A self-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [51, 52]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle g\rangle$ versus variance $\mathrm{var}(g)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-based approach [53]. With no fitting parameters, there is very good agreement (c.f. Fig. 1.3). Similarly,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$D(z)$ (c.f. Fig. 1.1).

1.6. OUTLINE OF TRANSPORT REGIMES

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, a two-parameter diagram (c.f. Fig. 1.4) enumerates types of transport behavior. The first parameter is system length L,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The two-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ in nonconservative random media.

A single-valued parameter such as $T/\mathcal{E}$ is useful even in this two-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. However, not all single-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo $T/\mathcal{E}$,Lorem ipsum dolor sit amet, consectetur

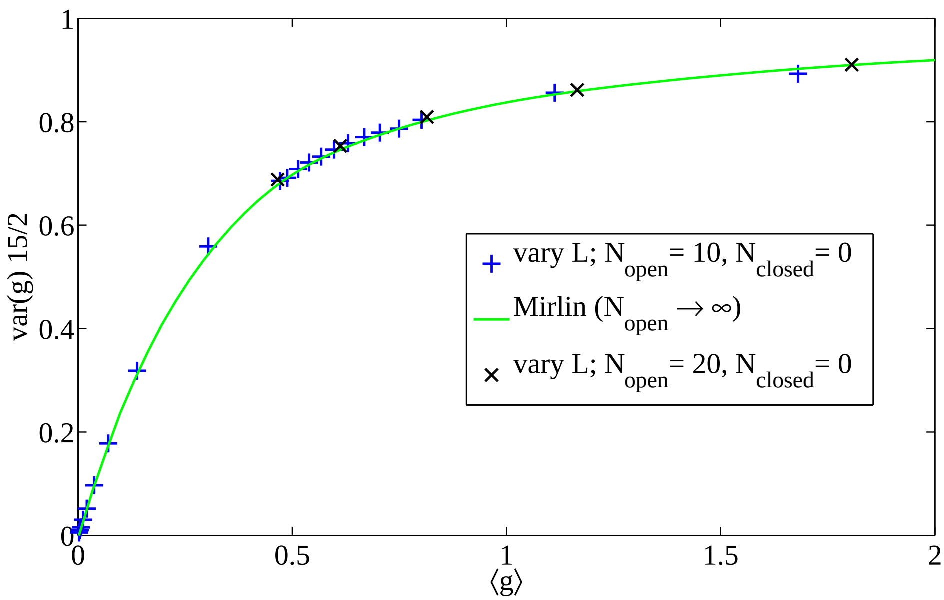

Figure 1.3: Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$g$ versus variance of g for quasi-1D waveguide [53]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo 1.5.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The 15/2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle g \rangle$ and var(g)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(with the number of open channels N_{open} determined by W) and varying system length L. The supersymmetry-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, but N_{open} equal to 10 and 20 is sufficient.

adipiscing elit; Maecenas ultrices egestas commodo. Currently,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Figure 1.4 describes types of transport in quasi-1D waveguides with random media; it has three passive regimes: ballistic (**B**), diffusive (**D**), localized (**L**) on the horizontal axis and gain (**G**) or absorption (**A**) strength on the vertical axis. The two-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The passive regime transitions (B/D/L)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo ℓ_{tmfp} and localization length ξ, as described in section 1.2.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo λ.

The absorption (gain) rate $\gamma_{a,g}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(doubled) along a specific path. The absorption (gain) rate is the inverse of the absorption (gain) lifetime, $\gamma_{a,g} = \frac{1}{\tau_{a,g}}$, where $\tau_{a,g}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(doubled).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Given a characteristic time $\tau_{a,g}$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo $\ell_{a,g} = \tau_{a,g}c$, where cLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(doubling) with respect to the path length. The $\ell_{a,g}$ is determined from the time-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,

$$D\frac{\partial^2 I}{\partial z^2} = \frac{\partial I}{\partial t}, \tag{1.11}$$

to be

$$\ell_{a,g} = \left(\frac{d}{\pi^2}\right)\frac{L^2}{\ell_{tmfp}}. \tag{1.12}$$

However, $\ell_{a,g}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(doubled when gain is present). The system length L (Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo) should be replaced by a new diffusive-regime length, $\xi_{a,g}$. Eq. 1.12 can then be solved for $\xi_{a,g}$:

$$\xi_{a,g} = \sqrt{\frac{\ell_{a,g}\ell_{tmfp}}{d}}. \tag{1.13}$$

Physically, $\xi_{a,g}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-scattering system. To distinguish the two absorption (gain) lengths, $\xi_{a,g}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L$ (rather than path length L_D), whereas $\ell_{a,g}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoL_D. If L is equal to L_D, then no diffusion is occurring and $\ell_{a,g}$ is equal to $\xi_{a,g}$. Usually,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoL_DLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L$.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo: first, experimentally, L_D is harder to measure than L; second,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

For localized systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(i.e., ray optics do not apply). In this regime $\xi_{a,g}$ is used, but it is not defined in terms of $\ell_{a,g}$ as in Eq. 1.13.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\xi_a = \xi$ (the horizontal line between AD_3 and AL_3 in Fig. 1.4).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 1.13 to $\xi_{a,g} = \xi = N_{open}\ell_{tmfp}$ and solving

$$N_{open}^2 \ell_{tmfp}^2 = \frac{\ell_{tmfp}\ell_{a,g}}{d} \tag{1.14}$$

to get $\ell_{a,g} = dN_{open}^2 \ell_{tmfp}$. For the diffusive regime,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(gain)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The remaining curves in Fig. 1.4Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, rather than the characteristic lengths.

For passive media,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo($\delta\omega$ of the Thouless criterion in Eq. 1.3)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo). To account for absorption or gain, an additional term is needed [36] in the form of a rate: $\delta\omega + \gamma_{a,g}$. Although the width of DOS $\Delta\omega$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Kronig relation [48],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[54]) or absorption rate $\gamma_{a,g} = \mp\, c/\ell_{a,g}$:

$$\delta' = \frac{\delta\omega + \gamma_{a,g}}{\Delta\omega}; \tag{1.15}$$

it is plotted as the red curve $\delta' = 1$. Physically,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-mode of the system, as plotted by the black curve $\pm\gamma = \delta\omega$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 1.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, two-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

In the ballistic regime GB_1,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(and similarly for AB_1 when $\ell_a < L$).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoAD_1 and GD_1,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The use of conditional statistics [36]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. With sufficient absorption, signatures of diffusion are reduced (AD_2) and suppressed (AD_3). In contrast, gain enhances fluctuations (GL_1) and leads to lasing (GL_2) on average for many realizations [54]. Transport in region GD_2 is the equivalent of “negative absorption” in region AD_2.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(AL_1)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(AL_2)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(AL_3).

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 1.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$D(z)$, correlation functions, and the inverse participation ratio,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo [55], wave front shaping [56]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, eigenmodes of transmission [57],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

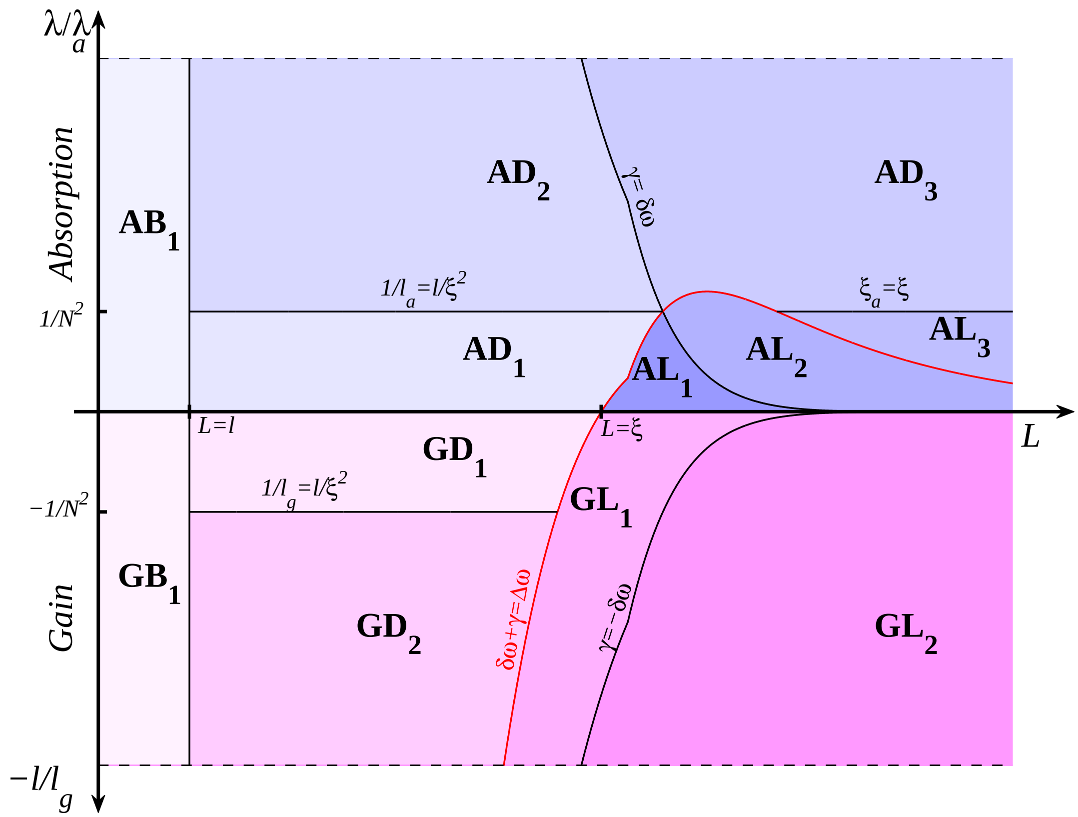

Figure 1.4: Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-letter abbreviations (see text for explanation).Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Passive (conservative)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L$. Plotted vertically, amounts of absorption or gain (nonconservative media)Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

2. LOREM IPSUM DOLOR SIT AMET, CONSECTETUR ADIPISCING ELIT; MAECENAS ULTRICES EGESTAS COMMODO

Ben Payne[1], Jonathan Andreasen[2], Hui Cao[2], and Alexey Yamilov[1]

[1] *Department of Physics, Missouri University of Science & Technology, Rolla, MO 65409*

[2] *Department of Applied Physics, Yale University, New Haven, CT 06520*

ABSTRACT*

In this work, we investigate a possibility of using the ratio between optical transmission, T, and energy stored inside the system, $\mathcal{E}$, as a quantitative measure of the enhanced mesoscopic corrections to diffusive transport of light through a random medium with gain. We obtain an expression for $T/\mathcal{E}$ as a function of amplification strength in the diffusive approximation and show that it does not a have tendency to diverge when the threshold for random lasing is approached, as both T and $\mathcal{E}$ do. Furthermore, we find that a change in $T/\mathcal{E}$ signifies a change in the electric field distribution inside the random medium. In the localization regime, we also investigate the correlations between transmission and energy stored in the medium with and without amplification. Our results suggest that $T/\mathcal{E}$ is a promising parameter which can help characterize the nature of wave transport in random medium with gain.

*Published in Physical Review B **82** 104204 (2010).

2.1. INTRODUCTION

Anderson localization [3] (AL) is a wave phenomenon [14, 15, 19] that leads to a breakdown of diffusion[13, 35]. First conceived in electronic systems, it originates from a repeated self-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Conservation of number of carriers,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Understanding the effect of absorption [14], ubiquitous in optical systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo*light* localization [16, 17, 58, 59, 60, 61]. It also prompted the search[16]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Coherent amplification,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[22],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

In the case of absorption, an alternative criterion, based on the magnitude of *fluctuations*Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, was put forward [16]. In random media with gain, this quantity (Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo) becomes ill-defined [36]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[22]. Without saturation effects,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo- and material-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. To avoid such dependence,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, in Ref. [36]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[31, 32, 36]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo–Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

In this work,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T$ and the energy inside a random medium $\mathcal{E}$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, e.g. perforated dielectric films,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-field scanning optical microscopy.

In Section 2.2,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(no saturation mechanism is assumed). However, when taken in a form of a ratio,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Secondly,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-dependent diffusion constant $D(z)$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[38, 62] in the self-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The connection between $T/\mathcal{E}$ and $D(z)$ demonstrates that the former can, indeed,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(interference) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

In Section 2.2.4 we obtain an expression for $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(using the slab geometry) with gain. This establishes a baseline – a decrease of the parameter $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ as a localization criterion, in Sec. 2.3Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. This introduces an additional (geometrical) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Interestingly, this effect leads to profound gain-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.4.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. **??**.

2.2. ANALYSIS OF $T/\mathcal{E}$: DIFFUSIVE REGIME

2.2.1. Model Description. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(electrons, photons, etc) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-less diffusion equation. Under this condition, the ensemble-averaged diffusive flux $\langle \mathbf{J}(\mathbf{r},t)\rangle$ and the energy density $\langle \mathcal{W}(\mathbf{r},t)\rangle$ are related via[63]

$$\langle \mathbf{J}(\mathbf{r},t)\rangle = -D(\mathbf{r})\boldsymbol{\nabla}\langle \mathcal{W}(\mathbf{r},t)\rangle. \tag{2.1}$$

The diffusion approximation amounts to $D(\mathbf{r}) \equiv D_0 = v_E \ell/3$, where ℓ is transport mean free path and v_E is the energy transport velocity [64]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodov_ELorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Vollhardt and Wölfle [35]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-like formalism. With recent refinements [38, 62],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$D(\mathbf{r})$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. At the same time, deviation (reduction) of $D(\mathbf{r})$ from the constant value of D_0Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Eq. (2.1) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo

$$\frac{\partial \langle \mathcal{W}(\mathbf{r},t)\rangle}{\partial t} + \operatorname{div}\langle \mathbf{J}(\mathbf{r},t)\rangle = \frac{c}{l_g}\langle \mathcal{W}(\mathbf{r},t)\rangle + J_0 \delta(z - z_p), \tag{2.2}$$

where l_gLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoJ_0 at $z = z_p \sim \ell$, the penetration depth [65], near the left boundary. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(e.g. acoustic) waves with $\mathcal{W}(\mathbf{r},t) = \varepsilon(\mathbf{r}) \left| d\psi(\mathbf{r},t)/dt \right|^2/(2c^2) + \left| \boldsymbol{\nabla}\psi(\mathbf{r},t) \right|^2/2$ and the electromagnetic waves with $\mathcal{W}(\mathbf{r},t) = \varepsilon(\mathbf{r}) \left| \mathbf{E}(\mathbf{r},t) \right|^2/2 + \mu \left| \mathbf{H}(\mathbf{r},t) \right|^2/2$ [63]. In the former case $c/\varepsilon^{1/2}(\mathbf{r})$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\psi(\mathbf{r})$ and in the latter, $\varepsilon(\mathbf{r})$ is the dielectric function and $\mathbf{E}(\mathbf{r},t)$, $\mathbf{H}(\mathbf{r},t)$ are the electric and magnetic fields.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[22, 66, 67, 68, 69] become crucial.

We consider a 3Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$z$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\boldsymbol{\rho}$ as $\mathbf{r} = (\boldsymbol{\rho}, z)$. Under a plane-wave illumination at normal incidence, the dependence on $\boldsymbol{\rho}$ can be neglected. In this case $J_z(z)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-sectional area of the slab. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(CW) regime where the energy density $\mathcal{W}(z)$ is stationary: $\partial\langle\mathcal{W}(z)\rangle/\partial t = 0$. Under the above assumptions, the boundary conditions

$$J_z(z=0) = -J_0 R, \;\; J_z(z=L) = J_0 T \tag{2.3}$$

together with Eqs. (2.1,2.2) completely specify the problem.

2.2.2. Asymptotic Behavior of $T/\mathcal{E}$: Passive System Limit. In Appx. 2.6.1 we demonstrate that Eqs. (2.1,2.2,2.3) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T$ and $\mathcal{E}$ in the passive limit $1/l_g \to 0$:

$$\frac{T}{\mathcal{E}} \simeq \frac{1}{J_0} \times \frac{2D_0}{L^2} \times \left[\frac{1}{L}\int_0^L \frac{D_0}{D(z)} dz\right]^{-1}, \tag{2.4}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\mathcal{E}$ is formally defined as

$$\mathcal{E} = \int_0^L \langle\mathcal{W}(z)\rangle dz. \tag{2.5}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.4), terms on the order of $\sim \ell/L \ll 1$ were dropped.

Eq. (2.4) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-consistent diffusion coefficient. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[35, 38],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ parameter below its diffusion-predicted value,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.4) by assuming $D(z) \equiv D_0$ gives the expected result

$$\frac{T}{\mathcal{E}} \simeq \frac{1}{J_0} \times \frac{2D_0}{L^2}. \tag{2.6}$$

The quantity D_0/L^2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodot_D –Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The reduction of $D(z) < D_0$ leads to an increase of $T/\mathcal{E}$, as expected in passive systems.

2.2.3. Asymptotic Behavior of $T/\mathcal{E}$: Strong Gain Limit. Under the assumptions specified in Sec. 2.2.1, the integration of Eq. (2.2) over the interval $z \in [0, L]$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.3) gives

$$(T + R - 1) \times J_0 = \mathcal{E} \times (c/l_g). \tag{2.7}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Hence,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. We note that in absence of gain $1/l_g \to 0$, Eq. (2.7) reduces to familiar $T + R = 1$ (see Eq. (2.3) for the definitions of T, R).

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.2.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$l_{g,cr}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Importantly,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, T and R become comparable $T \simeq R \gg 1$. Under such conditions, Eq. (2.7) yields

$$\frac{T}{\mathcal{E}} \simeq \frac{1}{J_0} \times \frac{c}{2l_g} \to \frac{1}{J_0} \times \frac{c}{2l_{g,cr}}. \tag{2.8}$$

This shows that the studied ratio $T/\mathcal{E}$, indeed, remains finite. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$1/l_{g,cr}$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(such as $T \simeq R$ above) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo– Eq. (2.8) and Eq. (2.6) respectively.

2.2.4. Intermediate Gains. In this section,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ for an optically thick ($\ell \ll L$) slab of 3Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.2.1. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[19], i.e. $k\ell \gg 1$, so that the condition $D(z) = D_0$ can be reasonably well satisfied. A combination of Eqs. (2.1,2.2) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo

$$0 = D_0 \nabla_z^2 \langle \mathcal{W}(z) \rangle + \frac{c}{l_g} \langle \mathcal{W}(z) \rangle + J_0 \delta(z - z_p). \tag{2.9}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. [2, 65]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$l_a = -l_g$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo“negative absorption” model. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[54, 70, 71].

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. [2, 65]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[63]

$$\langle J_\pm(z) \rangle = \frac{c}{4} \langle \mathcal{W}(z) \rangle \mp \frac{D_0}{2} \frac{d\langle \mathcal{W}(z) \rangle}{dz} \tag{2.10}$$

where $\langle J_- \rangle$ and $\langle J_+ \rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$z$-directions respectively. Evaluating $\langle J_-(0) \rangle$ and $\langle J_+(L) \rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo

$$\langle J_-(0) \rangle = J_0 \frac{\sin(\alpha(L - z_p)) + \alpha z_0 \cos(\alpha(L - z_p))}{(1 - \alpha^2 z_0^2) \sin(\alpha L) + 2\alpha z_0 \cos(\alpha L)} \tag{2.11}$$

$$\langle J_+(L) \rangle = J_0 \frac{\sin(\alpha z_p) + \alpha z_0 \cos(\alpha z_p)}{(1 - \alpha^2 z_0^2) \sin(\alpha L) + 2\alpha z_0 \cos(\alpha L)}, \tag{2.12}$$

where $z_0 = 2\ell/3 \sim \ell$ is the extrapolation length [72], and $\alpha^{-1} = \sqrt{\ell l_g / 3}$. Ensemble-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, c.f. Eq. (2.5)

$$\mathcal{E} = \frac{J_0}{D_0\alpha^2}\left[\frac{\sin(\alpha z_p) + \sin(\alpha(L - z_p)) + \alpha z_0(\cos(\alpha z_p) + \cos(\alpha(L - z_p))}{(1 - \alpha^2 z_0^2)\sin(\alpha L) + 2\alpha z_0 \cos(\alpha L)} - 1\right]. \tag{2.13}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.7) is satisfied.

In Fig. 2.1a,b we plot Eqs. (2.11,2.12,2.13) for a slab of thickness $L/\ell = 100$. As expected, we observe the divergence of the transmission and reflection fluxes, c.f. Fig. 2.1a, when diffusive random lasing threshold (RLT) is approached ($\alpha \to \alpha_{cr} = \pi/(L + 2z_0) \simeq \pi/L$) with an increase of gain parameter α or, equivalently, a decrease of gain length l_g toward $l_{g,cr} \simeq 3L^2/(\pi^2\ell)$. The asymptotic dependence is then

$$\langle J_-(0)\rangle \simeq \langle J_+(L)\rangle \simeq J_0\frac{z_0 + z_p}{\pi}\frac{\alpha_{cr}^2}{\alpha_{cr} - \alpha}. \tag{2.14}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L$ is increased towards critical length, $L_{cr}(\alpha)$,while keeping the gain parameter α fixed.

In Fig. 2.1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle J_-(0)\rangle/\mathcal{E} \equiv J_0R/\mathcal{E}$ and transmission to energy $\langle J_+(L)\rangle/\mathcal{E} \equiv J_0T/\mathcal{E}$ obtained from Eqs. (2.11,2.12,2.13). One can observe that both $R/\mathcal{E}$ and $T/\mathcal{E}$ indeed remain finite at α_{cr}Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.8). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$(\alpha_{cr} - \alpha)$, Eq. (2.14),Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, the second order terms (not shown) are not. This explains different slopes of $R/\mathcal{E}$ and $T/\mathcal{E}$ in approach to lasing threshold in Fig. **??**b.

By making assumption that $\ell/L \ll 1$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ with *an arbitrary value of gain*

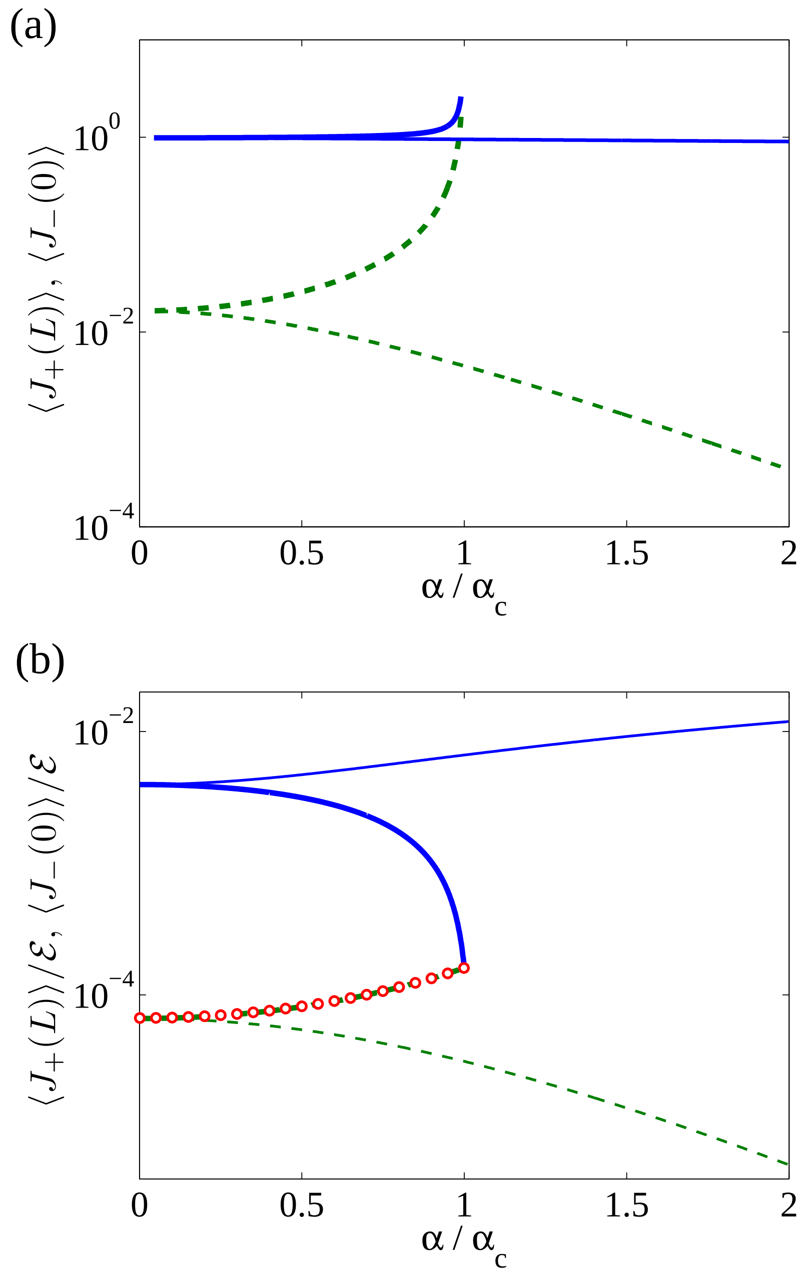

Figure 2.1: (a) Transmission $\langle J_+(L)\rangle$ (dashed line) and reflection $\langle J_-(0)\rangle$ (solid line) given by Eqs. (2.11,2.12) are plotted for increasing gain (thick lines) and absorption (thin lines) coefficients for a slab of random medium of thickness $L/\ell = 100$. In panel (b) we plot the same quantities as in (a) but normalized by the value of total energy stored inside random medium $\mathcal{E}$, c.f. Eq. (2.13). The divergence in the vicinity of RLT is prevented as both curves approach the same limiting value given by Eq. (2.8). $T/\mathcal{E}$ obtained by evaluating the approximate expression Eq. (2.15) is shown with open circles. For the chosen $L/\ell = 100 \gg 1$ the deviation from the exact result is indiscernible.

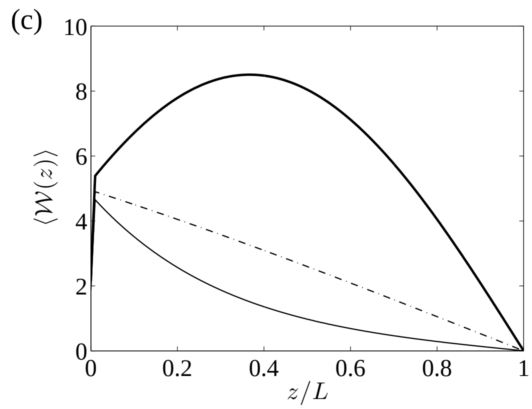

Figure 2.2: Diffuse energy density distribution $\langle \mathcal{W}(z) \rangle$ inside the slab of random medium with thickness $L/\ell = 100$ from Eq. (2.13). Thick solid line corresponds to the sample with gain ($\alpha/\alpha_{cr} = 0.8$), dashed line – to passive sample, whereas thin solid line – to the sample with absorption($|\alpha/\alpha_{cr}| = 1$). Absorption curve is shown for comparison.

parameter α:

$$\frac{T}{\mathcal{E}} \simeq \frac{D_0 \alpha^2}{2 J_0 \sin^2 (\alpha L/2)}. \tag{2.15}$$

Fig. 2.1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(open circles) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.12,2.13). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ Eqs. (2.6,2.8) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Based on our observations above,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3:

(a) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, c.f. Eq. (2.14). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, without relying on the incident flux;

(b) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas

commodo. (2.5), both $R/\mathcal{E}$ and $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Instead, they converge to the finite value of $c/(2l_{g,cr}) \equiv 2D_0/L^2 \times (\pi^2/4)$, c.f. Eq. (2.8). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\pi^2/4 \simeq 2.5$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, c.f. Eq. (2.6);
(c) The change of the quantities $R/\mathcal{E}$ and $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, c.f. Fig. ??c. When energy density $\langle \mathcal{W}(z) \rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle \mathcal{W}(z) \rangle \propto \sin(\pi z/L)$ the ratios $R/\mathcal{E}$ and $T/\mathcal{E}$ saturate;
(d) In Sec. 2.2.2 we showed, c.f. Eq. (2.4),Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T/\mathcal{E}$ smaller then its diffusion prediction, Eq. (2.6). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Thus, Eq. (2.15) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo:
(i) The diffusion approximation (Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo;
(ii) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[36] and random lasing threshold [73]. Thus $T/\mathcal{E}$ in the form of a ratio between the *average*Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}$ and energy-stored $\tilde{\mathcal{E}}$ in any given realization. Instead, it

may need to be replaced with $\langle \tilde{T}/\tilde{\mathcal{E}} \rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo;

(iii) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, the divergence of fluctuations of $\tilde{T}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\left\langle \left(\tilde{T}/\tilde{\mathcal{E}}\right)^n \right\rangle$ or, perhaps, its entire distribution;

(iv) At the onset of random lasing, nonlinear and dynamical processes[22, 66, 67, 68, 69]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, thus, a CW quantity such as $T/\mathcal{E}$ may no longer be suitable.

2.3. ANALYSIS OF $T/\mathcal{E}$: LOCALIZED REGIME

2.3.1. Model Description. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(i-iii) from Sec. 2.2.4 influence $\tilde{T}/\tilde{\mathcal{E}}$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T$ and $\mathcal{E}$ are reserved for the ensemble-averaged quantities. Compared to Sec. 2.2, we consider the other extreme case – the regime of localized transport – Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. For this purpose a one-dimensional (1D) model is already sufficient. Indeed, long enough 1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, therefore,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Despite the reduced dimensionality,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[74, 75, 76, 77] for which the considered one-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo($\epsilon = 1$ and 1.2), and width a. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1000 pairs. One last $\epsilon = 1$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo2001 layers. Then the total sample has length L. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\epsilon = 1.2$ layer. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo($a \ll \xi$); the localization length $\xi \sim L/5$ to $L/10$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[78]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo:

$$\begin{cases} E_L(x < 0, \omega) = \exp[i\omega x/c] + r_L(\omega)\exp[-i\omega x/c] \\ E_L(x > L, \omega) = t_L(\omega)\exp[i\omega x/c] \end{cases} \tag{2.16}$$

$$\begin{cases} E_R(x < 0, \omega) = t_R(\omega)\exp[-i\omega x/c] \\ E_R(x > L, \omega) = \exp[-i\omega x/c] + r_R(\omega)\exp[i\omega x/c] \end{cases} \tag{2.17}$$

Here, $r_{L,R}(\omega), t_{L,R}(\omega)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo; $\tilde{T} = |t|^2$ and $\tilde{R} = |r|^2$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo2×2 transfer matrices[79, 80, 81]

$$\widehat{t_i} = \begin{bmatrix} \cos(kn_ia_i) & n_i^{-1}\sin(kn_ia_i) \\ -n_i\sin(kn_ia_i) & \cos(kn_ia_i) \end{bmatrix} \tag{2.18}$$

which act on the two-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Here $n_i = \epsilon_i^{1/2}$ is the refractive index and a_i is the width of the i'th slab.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Fig. 2.3Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\mathcal{E}$ obtained in a single realization. Subsequently,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In sections 2.3.2-2.3.3 the system is assumed to be passive. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo 2.3.4.

2.3.2. Correlations Between $\tilde{T}$ and $\tilde{\mathcal{E}}$. Motivated by our analysis in Section 2.2.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-model. Fig. (2.3b) shows $\tilde{T}$ and $\tilde{\mathcal{E}}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Indeed,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}/\tilde{\mathcal{E}}$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. At the off-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. They are illustrated in Fig. 2.4.

In the first scenario, c.f. bold line in Fig. 2.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_0 and falls off after

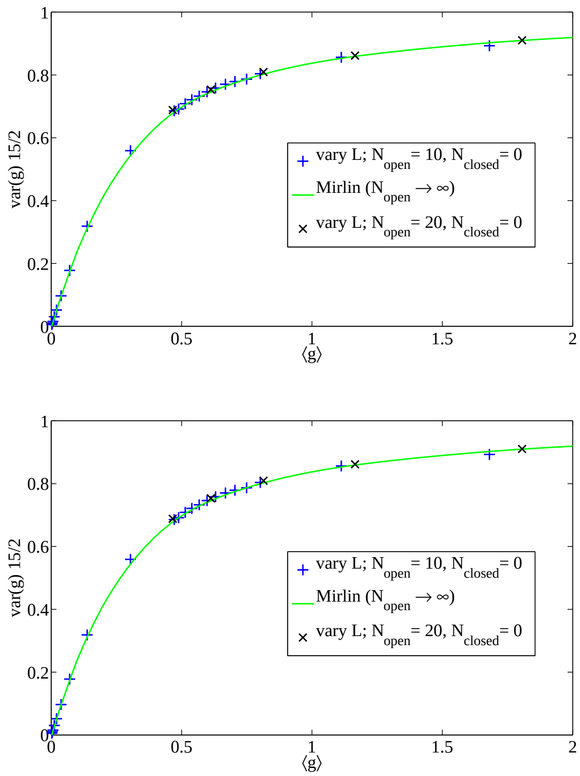

Figure 2.3: (a) The spatially-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (b) Transmission (solid line, right y-axis) as a function of frequency, $\tilde{T}(\omega)$, is compared to the total energy (dashed line, left y-axis) in the sample $\tilde{\mathcal{E}}(\omega)$ for one random realization of disorder. No one-to-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

it. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Such behavior is attributed[82]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_0.

In the other case, c.f. thin line in Fig. 2.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, see figure caption). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{\mathcal{E}}$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4 were studied in Ref. [82],Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

We note that multi-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-called necklace states[74, 83, 84]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(almost) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Such realizations, however,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L \gg \xi$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

We find that, on average,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{\mathcal{E}}$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Indeed,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas

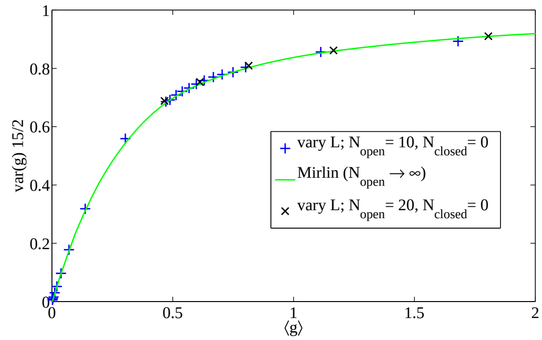

Figure 2.4: Two types of the on-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_0 in the first half (bold lines) and the second half (thin lines) of the sample ($x_0/L \approx 0.25$). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.16) to Eq. (2.17). Due to reciprocity,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{\mathcal{E}}(\omega)$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoexp($\pm x/\xi$) spatial dependences. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.1.

ultrices egestas commodo. 2.4. In contrast,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(or non-distinguishable) peak in energy,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(closer to the exit boundary). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. A sample with a localized state $0 < x_0 < L/2$ automatically yields the $L/2 < x_0 < L$ state in the mirror-image sample or, equivalently,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, thus, energy stored, is dramatically different. In Appx. 2.6.2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Indeed,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.6.2 (c.f. Figs. 2.4,2.9) can be also obtained in other models. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.1, but with no disorder. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, appear equivalent to that in Fig. 2.9.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo: (i) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(x_0) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (ii) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (iii) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. And otherwise,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

2.3.3. Behavior of $\tilde{T}/\tilde{\mathcal{E}}$ in Passive Random Medium: Spectral Vicinity of a Resonance. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.2 and Appx. 2.6.2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}/\tilde{\mathcal{E}}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

In the localization regime,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, via resonant tunneling. Thus,Lorem

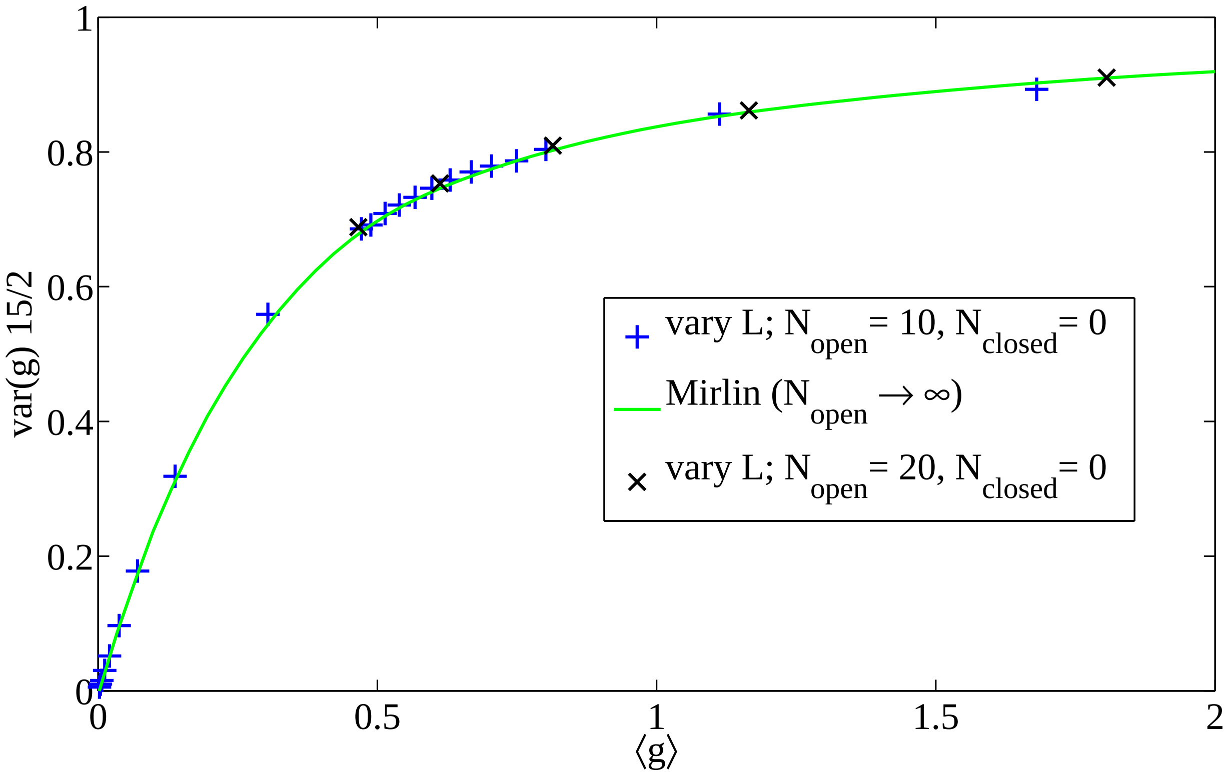

Figure 2.5: The dependencies of $\tilde{T}(\omega)$ (a,b); envelope of the electric field $E(x)$ (c,d); and energy in the system $\tilde{\mathcal{E}}(\omega)$ (e,f). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The plots (a,c,e) and (b,d,f) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$x_0 = L/4$ and $x_0 = 3L/4$ respectively. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.17) – wave incident from the right – for easy comparison with Figs. 2.4,2.9. In the first case,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Three sets of $E(x)$ in (c,d) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(a,b,e,f). The envelopes illustrate the on- and off-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.20,2.21,2.22,2.23). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In this case, unlike $\tilde{T}(\omega)$, $\tilde{\mathcal{E}}(\omega)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoω_0, compare (e) and (f). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$0 < x_0 < L/2$ and $L/2 < x_0 < L$ in the $\tilde{T}/\tilde{\mathcal{E}}$ as discussed in Sec. 2.3.2 and Appx. 2.6.2.

ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo:

$$\tilde{T}(\omega) = \frac{t_0^2}{[2(k-k_0)\Delta]^2 + t_0^2 \cosh^2 \frac{|L-2x_0|}{\xi}} \tag{2.19}$$

where $k = \omega/c$ and $t_0 = \exp(-L/\xi)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodok_0. ΔLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo"cavity"[85]. ΔLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, see e.g. [79, 86]. In the case of random media, ΔLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In this system ΔLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[87].

Of course,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.6.2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.19). However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$k-k_0, x_0, \xi$ and ,L: Eq. (2.19) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$|k-k_0| \leq \delta k$ and $L \gg \xi$ (which also leads to $\delta k \ll k_0$ condition). Here δk is the spectral width of the resonance.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.6.2 was demonstrated in Ref. [87]. Therefore, we expect Eq. (2.19) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.1.

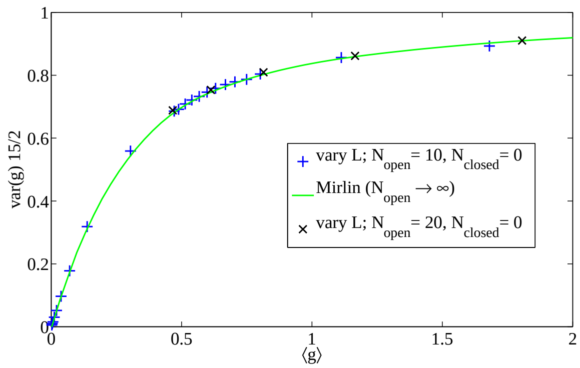

Figure 2.6: The solid line plots $\tilde{\mathcal{E}}(\omega_0, x_0)$ from Eq. (2.23). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(exponentially) when the center of localization x_0 increases beyond $L/2$ and reaches the off-resonant value for $x_0 > 2L/3$. The dashed line with the right y-axis represents the ratio between $\tilde{T}(\omega_0, x_0)$ in Eq. (2.19) and $\tilde{\mathcal{E}}(\omega_0, x_0)$ in Eq. (2.23). The ratio peaks at the same value of $x_0/L = 2/3$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoξ or the system length L.

We note two important properties of Eq. (2.19). First, the maximum (resonant) value of the transmission at $\omega = \omega_0$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}(\omega_0) = \cosh^{-2}\left(|L - 2x_0|/\xi\right)$. It turns to unity when $x_0 = L/2$. Secondly,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$|\omega - \omega_0| \gg \delta\omega$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo–Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.2 and Appx. 2.6.2, c.f. Figs. 2.4,2.9, we approximate the envelope of the *on-resonance* electric field distribution as

$$E(x, \omega_0) = \begin{cases} B(\omega_0, x_0) \exp[(x - x_0)/\xi] & 0 < x < x_0 \\ C(\omega_0, x_0) \exp[-(x - x_0)/\xi] & x_0 < x < L \end{cases} \tag{2.20}$$

for $x_0 < L/2$; and

$$E(x,\omega_0) = \begin{cases} A(\omega_0,x_0)\exp[-x/\xi] & 0 < x < x_T(\omega_0) \\ B(\omega_0,x_0)\exp[(x-x_0)/\xi] & x_T(\omega_0) < x < x_0 \\ C(\omega_0,x_0)\exp[-(x-x_0)/\xi] & x_0 < x < L \end{cases} \tag{2.21}$$

for $x_0 > L/2$. Here A, B, CLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. At the boundaries we set $E(x=0) = 1$ and $E(x=L) = \tilde{T}^{1/2}(\omega_0)$. Noticing that $E(x=L,\omega_0) \approx \exp[-|2x_0 - L|/\xi]$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, $x_T(\omega_0) = 2x_0 - L$, in the case of Eq. (2.21). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, such as in Sec. 2.3.1, the Eqs. (2.20,2.21) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$(\omega - \omega_0), x_0, \xi$, and L.

In both cases above, away from the resonant frequency k_0, we see three distinct regions:

$$E(x) = \begin{cases} A(\omega,x_0)\exp[-x/\xi] & 0 < x < x_T(\omega) \\ B(\omega,x_0)\exp[(x-x_0)/\xi] & x_T(\omega) < x < x_0 \\ C(\omega,x_0)\exp[-(x-x_0)/\xi] & x_0 < x < L \end{cases} \tag{2.22}$$

where $x_T(\omega)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$E(x=0) = 1$ and $E(x=L) = T^{1/2}(\omega)$. Fig. 2.5Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$x_0 < L/2$ and $x_0 > L/2$ cases. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_0. Indeed, integrating Eqs. (2.20,2.21) gives us the sought expression for $\tilde{\mathcal{E}}(\omega)$. At $\omega = \omega_0$ it simplifies to

$$\tilde{\mathcal{E}}(x_0) \propto \begin{cases} 2\tilde{T}(\omega)\exp[2(L-x_0)/\xi] - 1 - \tilde{T}(\omega) & 0 < x_0 < L/2 \\ 2\tilde{T}(\omega)\exp[2(L-x_0)/\xi] + 1 - 3\tilde{T}(\omega) & L/2 < x_0 < L \end{cases} \tag{2.23}$$

This expression is plotted in Fig. 2.6. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$x_0 < L/2$ and $x_0 > L/2$. Eq. (2.23) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-resonant case for $x_0 > 2L/3$. The latter value of x_0Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoξ, L, etc.

When expressions in Eqs. (2.19,2.23) are combined to form the ratio $\tilde{T}/\tilde{\mathcal{E}}$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_0, c.f. Fig. 2.6. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$0 < x_0 < 2L/3$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$2L/3 < x_0 < L$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}/\tilde{\mathcal{E}}$ on x_0. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(linear) optical amplification on this quantity.

2.3.4. Behavior of $\tilde{T}/\tilde{\mathcal{E}}$ in Active Random Medium. As we observed in Sec. 2.2.4, a change in $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}/\tilde{\mathcal{E}}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Interestingly,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, we employ the deterministic (non-random) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(see Sec. 2.3.3). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\epsilon(x) \to \epsilon(x) + i\alpha$. As previously discussed in Sec. 2.1,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

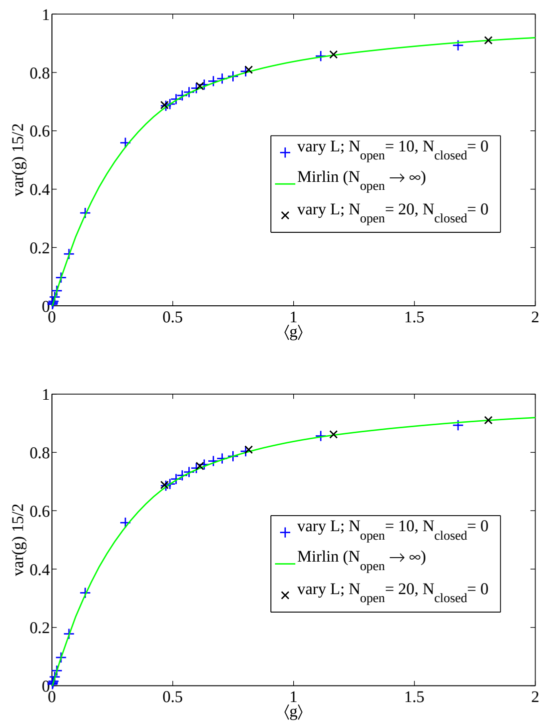

Figure 2.7: Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1D random medium. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$x_0 = L/4$ is considered. As discussed in Sec. 2.3.2,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(such as those in Fig. 2.4) in random media. Panels (a) and (b) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(at resonant frequency) from the left and right respectively. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. The solid curves (from bottom up) are obtained for $l_{g,cr}/l_g$ equal to 0.5, 0.9, 0.99 in (a) and 0.85, 0.95, 0.98, 0.99 in (b). The field distribution in (b) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.7. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_T,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, towards the sample boundary. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$E(x)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.2.

At first glance,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. [88, 89, 90] where (in localized regime) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In our work,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, whereas in the previous works [88, 89, 90]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Under such excitation conditions, the situation shown in Fig. 2.7a is always realized [42]. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.7Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.7Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas

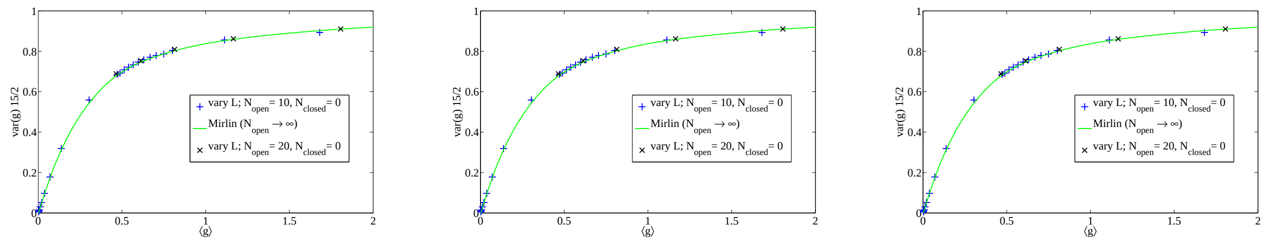

Figure 2.8: Panel (a) plots $E_{L,R}(x,\omega_c) \equiv E^{(L,R)}(x,\omega_c,\alpha = 0)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.16,2.17), see text for notations. For non-zero gain, $\alpha > 0$, the electric field distributions $E^{(L,R)}(x,\omega_c,\alpha)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.7. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(a), and the resulting coefficients $C^{(L)}_{L,R}(\alpha)$ and $C^{(R)}_{L,R}(\alpha)$, defined in Eq. (2.25), are plotted in (b) and (c) respectively. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$E_L(x,\omega_c)$ and $E^{(c)}(x,\omega_c)$ (the solution in the closed system) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In all three panels, thick / thin curve and large / small symbols refer to $E_L(x,\omega_c)$ / $E_R(x,\omega_c)$.

ultrices egestas commodo(reflected and transmitted) signals. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(CW) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(linearly independent) solution for the same frequency. In general, any two (because Maxwell'Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoω.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.16,2.17). They correspond to the left- and right-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.2 and Appx. 2.6.2 with $r_{L,R}(\omega), t_{L,R}(\omega)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(it is independent of x in our model of disorder $\epsilon(x)$) is non-zero for one particular value of x. At $x = 0$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.16,2.17) as $-2it_R(\omega)\omega/c \neq 0$.

At some special frequencies ω_c, a linear combination $E^{(c)}(x, \omega_c) = C_L^{(c)} E_L(x, \omega_c) + C_R^{(c)} E_R(x, \omega_c)$ can be formed such that conditions $E^{(c)}(x = 0, \omega_c) = 0$ and $E^{(c)}(x = L, \omega_c) = 0$ are satisfied simultaneously. Such ω_c's correspond to the true eigen-modes of the closed system – the system defined by $\epsilon(0 \leq x \leq L)$ with zero (reflecting) boundary conditions at $x = 0, L$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo1Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(similar to $E_L(x, \omega_c)$ depicted with thick lines in Figs. (2.4,2.9)) makes the dominant contribution to $E^{(c)}(x, \omega_c)$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(similar to $E_R(x, \omega_c)$ depicted with thin lines in Figs. (2.4,2.9)) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. [88, 89, 90].

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, $\alpha > 0$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, $E^{(L)}(x, \omega_c, \alpha)$, and from the right $E^{(R)}(x, \omega_c, \alpha)$, c.f. Fig. 2.7. In this case, the distributions can no longer, strictly speaking, be expressed in terms of $E_{L,R}(x, \omega_c) \equiv E^{(L,R)}(x, \omega_c, \alpha = 0)$. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, $\alpha < \alpha_{cr} \ll \omega_c$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$E_{L,R}(x, \omega_c)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$E(x, \omega_c, \alpha)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoα-dependent $C_L(\alpha)$,$C_R(\alpha)$:

$$E(x, \omega_c, \alpha) \simeq C_L(\alpha) E_L(x, \omega_c) + C_R(\alpha) E_R(x, \omega_c). \tag{2.24}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.24) is verified numerically by computing

$$C_{L,R}^{(L,R)}(\alpha) = \int_0^L E^{(L,R)}(x, \omega_c, \alpha) E_{L,R}^*(x, \omega_c) dx. \tag{2.25}$$

Figs. 2.8b,c show $C_{L,R}^{(L)}(\alpha)$ and $C_{L,R}^{(R)}(\alpha)$ respectively. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$x_0 \sim L/4$ where passive profiles, depicted in Fig. 2.8a,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4,2.9,2.7. When gain is added to the system, we observe that $E^{(L)}(x, \omega_c, \alpha) \propto E_L(x, \omega_c)$ for all values of gain, c.f. Fig. 2.8b. In contrast, $E^{(R)}(x, \omega_c, \alpha)$ exhibited a crossover behavior from $E^{(R)}(x, \omega_c, \alpha) \propto E_R(x, \omega_c)$ for small α, to $E^{(R)}(x, \omega_c, \alpha) \propto E_L(x, \omega_c)$ in the vicinity of lasing threshold. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.4, c.f. Fig. 2.7,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(at the same frequency),Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-boundaries. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Finally, we note that the solutions $E_{L,R}(x, \omega)$ should not be confused with the quasi-mode $E(x, \omega_c + i\varepsilon)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\omega_c + i\varepsilon$ where e.g. transmission becomes singular. Such quasi-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. However, for a uniform gain in the form $\epsilon(x)(1 + i\alpha)$ the dominant mode profile $E^{(L)}(x, \omega_c, \alpha_{cr})$ does indeed coincide with the quasi-mode due to equivalence between $\epsilon(x)(1 + i\alpha)\omega_c/c$ and $\epsilon(x)(\omega_c + i\varepsilon)/c$.

2.4. DISCUSSION AND OUTLOOK

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, thus,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[31, 32, 36].

In Sec. 2.2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-averaged quantities $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, Eq. (2.4), in a passive random medium. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(decrease) of the $T/\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-renormalized diffusion coefficient $D(z) \equiv D_0 = c\ell/3$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

In Sec. 2.2.4 we obtained the expressions for T and $\mathcal{E}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, we conjecture that the decrease in $T/\mathcal{E}$ below the level established by Eq. (2.15) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-to-sample fluctuations,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, the ratio between $\tilde{T}$ and $\tilde{\mathcal{E}}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, c.f. Sec. 2.3. Although the ratio $\tilde{T}/\tilde{\mathcal{E}}$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(or, equivalently, the direction of the incident wave) even in the passive system, c.f. Sec. 2.3.2. In Sec. 2.3.3,2.6.2Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. This is unlike the transmission $\tilde{T}$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(because of reciprocity) –Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo: $\tilde{T}_G(\alpha) = \left(\tilde{T}(\alpha)/\tilde{\mathcal{E}}(\alpha)\right) \times \tilde{\mathcal{E}}_0$. Here $\tilde{\mathcal{E}}_0 \equiv \tilde{\mathcal{E}}(\alpha = 0)$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}_G$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. By construction, $\tilde{T}_G(\alpha)$ reduces to the transmission in a 1D system without gain ($\alpha \to 0$) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(average) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Hence, one can refer to T_G as generalized transmission (conductance).

In future,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\tilde{T}(\alpha)/\tilde{\mathcal{E}}(\alpha)$ and $T_G(\alpha)$ in random medium with gain. Experimentally,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-field scanning measurements in two-dimensional random media – Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

2.5. ACKNOWLEDGMENTS

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. DMR-0704981 and DMR-0808937. The numerical results obtained at the Tera-Grid, Grants No. DMR-090132 and No. DMR-100030. AY is grateful to S. Skipetrov, A. Lagendijk and B. van Tiggelen for valuable comments.

2.6. APPENDIX

2.6.1. Derivation of Equation (2.4). We consider a slab geometry,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$z$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoρ as $\mathbf{r} = (\rho, z)$. Assuming no dependence on ρ allows us to rewrite Eq. (2.1) in the form

$$\langle J_z(z)\rangle = -D(z)\frac{d\langle \mathcal{W}(z)\rangle}{dz}. \tag{2.26}$$

Independence of ρ is insured for the plane-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\mathcal{W}(z)$ is stationary, $\partial\langle \mathcal{W}(z)\rangle/\partial t = 0$, it follows from Eq. (2.2) that the z-component of flux is constant for $z > z_p \sim \ell$. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$z = L$ as

$$\langle J_z(z)\rangle = \begin{cases} \langle J_z(L)\rangle \equiv J_0 T, & z_p < z < L \\ \langle J_z(0)\rangle \equiv -J_0 R, & 0 < z < z_p \end{cases} \tag{2.27}$$

where T is the transmission coefficient. Furthermore, by integrating Eq. (2.2) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\langle J_z(L)\rangle - \langle J_z(0)\rangle = J_0T - (-J_0R) = J_0(T+R) = J_0$.

After establishing Eq. (2.27),Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. (2.26). An integration over z gives

$$\int_z^L \frac{\langle J_z(z')\rangle dz'}{D(z')} = -\langle \mathcal{W}(L)\rangle + \langle \mathcal{W}(z)\rangle. \tag{2.28}$$

The energy density $\langle \mathcal{W}(L)\rangle$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo- and left-propagating fluxes $\langle J_+(L)\rangle = J_0T$, $\langle J_-(L)\rangle = 0$ using Eqs. (2.10). We obtain $\langle \mathcal{W}(L)\rangle = 2J_0T/c$. To take advantage of the fact that $\langle J_z(z)\rangle$ is piecewise constant, c.f. Eq. (2.27), we have to neglect by $0 < z < z_p$ contribution. This introduces an error $\propto z_p/L \sim \ell/L \ll 1$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$T$ and $D(z)$ contributions as

$$J_0T\left[\int_z^L \frac{dz'}{D(z')} + 2/c\right] = \langle \mathcal{W}(z)\rangle. \tag{2.29}$$

Before proceeding further,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$\sim \ell/L$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Hence, $2/c$ contribution has to be dropped as well.

A subsequent integration of Eq. (2.29) gives

$$J_0T\int_0^L\int_z^L \frac{1}{D(z')}dz'dz = \int_0^L \langle \mathcal{W}(z)\rangle dz \equiv \mathcal{E} \tag{2.30}$$

Taking advantage of the system symmetry, $D(z) = D(L-z)$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo

$$\begin{aligned} \int_0^L \int_z^L \frac{1}{D(z')} dz' dz &= \frac{1}{2} \int_0^L \int_0^L \frac{1}{D(z')} dz' dz \\ &= \frac{L}{2} \int_0^L \frac{1}{D(z)} dz. \end{aligned} \tag{2.31}$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, $D(z) = D_0 \equiv c\ell/3$, we obtain Eq. (2.4).

2.6.2. Dependence of $\tilde{T}/\tilde{\mathcal{E}}$ on Position of the Defect State: Non-random Model. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-dependent energy content as in Fig. 2.4Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

A clarification is in order. There is no one-to-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoödinger and Helmholtz equations[91]. Indeed, John [92]Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo, strictly speaking, applied to the electromagnetic waves. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo[93] (e.g. disordered photonic crystals), the negative energy and, thus, tunneling regime can be recovered. To achieve this formally,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-medium transformation, as in e.g. Refs [87, 94, 95].

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. In particular,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(at $x_0 = L/4$) and the second (at $x_0 = 3L/4$) half of of the system. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.9. As in Sec. 2.3.2, $x_0 = 3L/4$Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Table 2.1: Nearest and next-nearest pairings kazoo

$c_1^{(nn)}$	$c_2^{(nn)}$	$c_3^{(nn)}$	$c_1^{(nnn)}$	$c_2^{(nnn)}$	$c_3^{(nnn)}$
0.004105	0.000915	0.000238	0.002034	0.000736	0.000261

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodoöLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Fig. 2.9Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo$L/4$ and $3L/4$,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-on-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo2.3.1,2.3.2. In the case of $x_0 = 3L/4$ defect,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_T, where x_T is the turning point. The position of x_TLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.9,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

The non-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(in the electromagnetic problem,Lorem

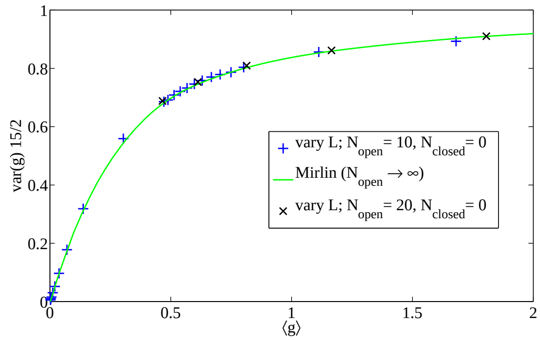

Figure 2.9: Solution of the SchröLorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo-Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo(thick line) and the right (thin line) Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.4.

ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo). Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. It appears that in the $x_0 = 3L/4$ case,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodox_T. In contrast,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit; Maecenas ultrices egestas commodo. 2.3.

3. CLASSIFICATION OF REGIMES OF WAVE TRANSPORT IN QUASI ONE-DIMENSIONAL NON-CONSERVATIVE RANDOM MEDIA

Alexey Yamilov[1] and Ben Payne[1]

[1]*Department of Physics, Missouri University of Science & Technology, Rolla, MO 65409*

ABSTRACT*

Passive quasi-one-dimensional random media are known to exhibit one of the three regimes of transport – ballistic, diffusive or localized – depending on the system size. In contrast, in non-conservative systems the physical parameter space also includes the gain/absorption length scale. Here, by studying the relationships between the transport mean free path, the localization length, and the gain/absorption length, we enumerate fifteen regimes of wave propagation through quasi-one-dimensional random media with gain or absorption. The results are presented graphically in a form of a phase diagram. Of particular experimental importance, in absorbing random medium we identify three different regimes which bear signatures of the localized regime of the passive counterpart. We also review the literature and, when possible, assign experimental systems to a particular regime on the diagram.

*Published in Journal of Modern Optics (2010)

3.1. INTRODUCTION

Discovery of Anderson localization (AL) [3]Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [1]. AL is a wave phenomenon [96] that results in cessation of diffusion [97]. First conceived in electronic systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Conservation of number of carriers,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, lies in the foundation of the concept of AL [13].

Understanding the effect of absorption [14], ubiquitous in optical systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.*light* [16, 17, 58, 59, 60, 61] and other classical waves such as ultrasound [26, 98]. It also prompted [16]Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Furthermore, the effect opposite to the absorption, coherent amplification,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [22, 23]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [42].

In this work,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.(quasi-1D) random media, such as disordered waveguides,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. In quasi-1D geometry the transition to AL lacks sharp features (mobility edges) observed in even more complex three-dimensional systems. Thus, Section 3.2Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.1D random media. In Section 3.3Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.1D non-conservative systems. In Section 3.4,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Furthermore, we review the available publications on the subject and, when published data is sufficient, assign them to a particular region on our phase-diagram. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.**??**.

3.2. LOCALIZATION IN QUASI-1D NON-CONSERVATIVE RANDOM MEDIA

3.2.1. Localization in Finite Passive Random Media. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [99]. In experimentally relevant situations,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Thus,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, but microscopically different disorder realizations, g,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [34]. According to scaling theory of localization [7], gLorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, formally described by the scaling function [100]. Thus,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

are listed in Table 3.1. Out of six possible couplings only three appear to be important including one next-nearest coupling. The latter corresponds to coupling between the diagonal cavities in the diamond arrangement.

3.2.2. Localization in Finite Random Media With Gain or Absorption. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices

Table 3.1: Coupling coefficients for nearest and next-nearest pairings

$c_1^{(nn)}$	$c_2^{(nn)}$	$c_3^{(nn)}$	$c_1^{(nnn)}$	$c_2^{(nnn)}$	$c_3^{(nnn)}$
0.004105	0.000915	0.000238	0.002034	0.000736	0.000261

egestas commodo. [27]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.(LC) based on g in optical systems. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [10]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [30]. Indeed, in absorbing systems $g \ll 1$ may not be indicative of the presence of localization [16, 33], and $g \gg 1$Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [31, 32]. Therefore,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [16, 32, 101, 102, 103, 104, 105], rounding of the coherent back scattering cone [106, 107, 108], anomalous diffusion [17, 26, 45, 58, 61, 109] and others.

In the case of absorption, a quantitative criterion, based on the magnitude of *fluctuation* of transmission normalized by its average, was put forward [16]. Although it described the experiment well,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [30],Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

In random media with gain,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Without saturation effects,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. To regularize the statistical ensemble,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [36]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. [31, 32, 36]. It was found that the correlation linewidth $\delta\omega$ [110]Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.$\delta = \delta\omega/\Delta\omega$ in random media with gain. Here $\Delta\omega$Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Reduction of δ correlates well [36]Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, the relationship $g = \delta$ is no longer valid in non-conservative media. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo(or absorption),Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo(see e.g. Refs. [111, 112]) are not accounted for.

3.2.3. Disordered Waveguide (Wire) Geometry. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, the transport properties in quasi-1D (waveguide) Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [40]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [81] which yielded some exact analytical results [53, 113].

In passive quasi-1Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo$L$ only (and not the strength of disorder as in three-dimensions, 3D), even if the system is weakly scattering $k\ell \gg 1$. The diffusion regime is only a transitive regime which, unlike in 3D systems, does not persists in the limit $L \to \infty$. Therefore, quasi-1D systems [16, 32, 33, 49, 103, 105, 114, 115, 116, 117, 118, 119, 120, 121]Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. However,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo$L \to \infty$ may not be easily defined, quasi-1Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo*finite size*. As shown below, it is expected to exhibit very complex parameter space, c.f. Fig. 3.3. Furthermore,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo1D non-conservative random media.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo(see e.g. Refs. [31, 32, 36, 62, 67, 68, 69, 73, 77, 101, 102, 103, 104, 106, 122, 123, 124, 125, 126]),Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Below,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo1D non-conservative random media.

3.3. DEFINITIONS OF PARAMETERS IN QUASI-1D NON- CONSERVATIVE RANDOM MEDIA

In passive volume-disordered waveguides,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodoℓ and $\xi = N \times \ell$ respectively. Here ℓ is the transport mean free path, N is the number of waveguide channels, and ξ is the localization length [40]. In waveguides filled with a non-conservative random medium, the parameter space becomes two-dimensional: beside the system size L, it also includes the gain or absorption length scale $\ell_{g,a}$. Fig. 3.3 shows this two-parameter phase space.

The boundaries between different regions in Fig. 3.3Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. The length

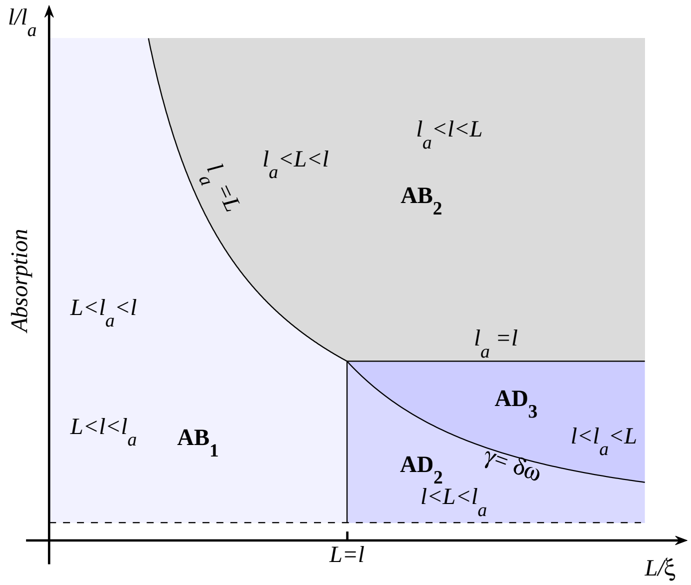

Figure 3.1: Classification of regimes of wave transport in quasi-1D non-conservative random media; upper panel.

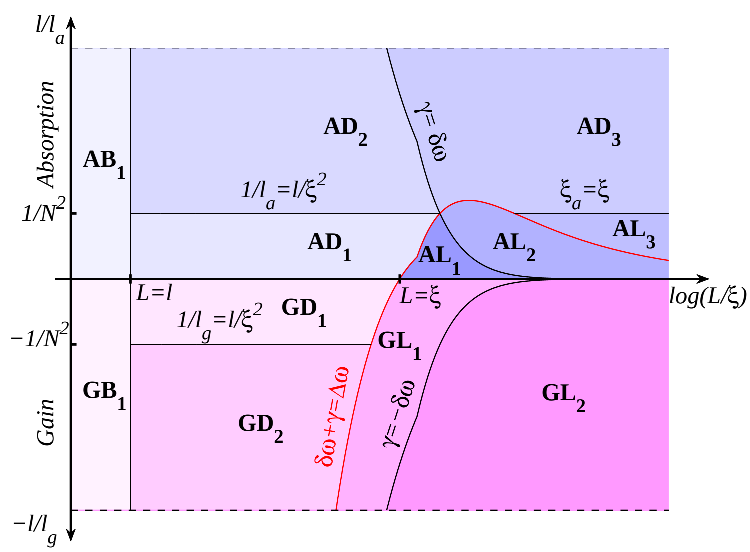

Figure 3.2: Classification of regimes of wave transport in quasi-1D non-conservative random media; middle panel.

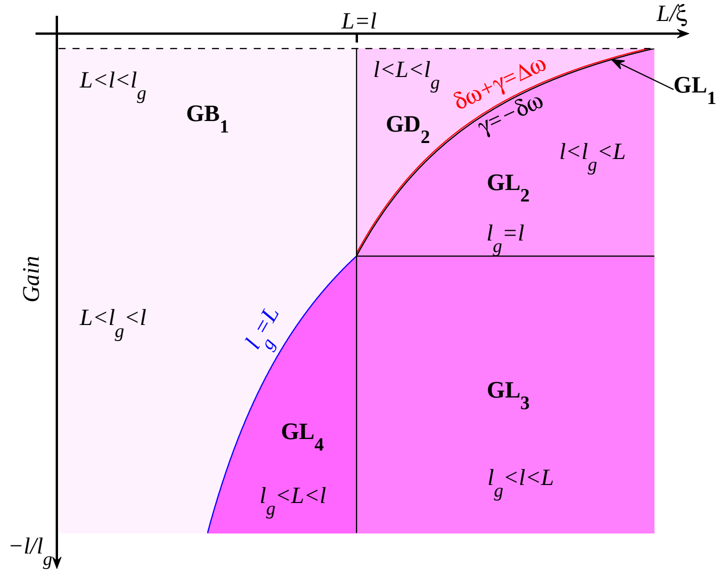

Figure 3.3: Classification of regimes of wave transport in quasi-1D non-conservative random media; lower panel. X and Y axes correspond to the system length L and absorption/gain length $\ell_{a,g}$, see text for labeling convention. Due to large disparity in the characteristic length scales, the plot is separated into three panels which correspond to strong absorption $1/\ell_a \sim 1/\ell$ (upper panel), weak absorption and gain $1/\ell_{a,g} \sim \ell/\xi^2$ (middle panel), and strong gain $1/\ell_g \sim 1/\ell$ (lower panel) regimes.

parameters include L, ℓ, ξ, and (ballistic) absorption/gain lengths ℓ_a/ℓ_g. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo: the average mode spacing $\Delta\omega \propto (NL)^{-1}$; the passive average mode linewidth $\delta\omega$ ($\propto DL^{-2}$ in diffusive regime $\ell < L < \xi$); and gain or absorption rate $\gamma_{g,a} = \mp c/\ell_{g,a} \equiv \mp\tau_{g,a}^{-1}$ (negative in the case of gain). Here, c is speed of light and D is the diffusion constant. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo:

- $L \sim \ell$Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo/gain $\ell < \ell_{g,a}$ shown in the middle panel in Fig. 3.3;
- Generalized Thouless parameter $\delta\omega(\gamma)/\Delta\omega \simeq (\delta\omega{+}\gamma)/\Delta\omega$ describes [36]Lorem

ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Here $\delta\omega \equiv \delta\omega(\gamma = 0)$. In the case of passive system $\gamma = 0$, the ratio reduces to $\delta = g$;

• $|\gamma| = \delta\omega$Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo;

•Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. In quasi-1D, the probability of such paths becomes equal to unity at $L = \xi$, their length is given by $L^2/\ell = \xi^2/\ell$. Therefore,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo$\ell_{g,a}$ becomes comparable to this length scale;

• Condition $\ell = \ell_{g,a}$ marks the onset of the regimes of very strong absorption/gain shown in the upper/lower panel in Fig. 3.3. Here, the ballistic regimes become limited by the condition $\ell_{g,a} = L$.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. When gain is present, the statistical ensemble is assumed to be conditional [36], which excludes the non-physical solutions [127]. Furthermore, the considered (open) system is of a finite size and, therefore, the transitions between different “phases” are expected to be smooth. Hence,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo.

3.4. “PHASES” OF WAVE TRANSPORT THROUGH NON-CONSERVATIVE RANDOM MEDIA

The regions in Fig. 3.3 are labeled with two letters and a subscript. The first letter, A/G, stand for absorption/gain and is common for all regions above/below the horizontal axis. The second letter in the labels, B, D or L, is attributed to the regimes where some signatures of the ballistic, diffusive, and localized transport are expected to occur. Based on the list of separatrices listed above, one can identify the following regions:

- GB_1, AB_1:Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Note that in the regime of very strong gain or absorption, $\ell_{g,a}^{-1} > \ell^{-1}$, the ballistic region becomes bounded by $L < \ell_{g,a}$;
- GD_1, AD_1: With exception of anomalously localized states [13, 53, 122, 128, 129, 130],Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo;
- GD_2: Such systems were successfully treated with the "negative absorption"Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [21, 54, 70, 71, 131, 132, 133, 134, 135]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [31, 32, 36];
- GL_1: Random media with such strong gain, $\delta\omega(\gamma)/\Delta\omega < 1$,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [73, 136]. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [127, 137] for the systems with the parameters in this region;
- GL_2: The condition $\gamma_g = -\delta\omega(\gamma = 0)$ signifies lasing of an average mode and, in diffusive systems,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [54];
- AL_1, AL_2, AL_3:Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Of these, AL_1Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, $\delta\omega(\gamma)/\Delta\omega \simeq (\delta\omega(\gamma = 0) + \gamma)/\Delta\omega < 1$,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo(possibly,

experimental systems of [16] belong to this parameter "phase"). The latter is no longer true for AL_2 regime. AL_3Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, but still exhibiting the weak localization corrections;

• AD_2, AD_3:Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [33]. For strong absorption,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo;

• AB_2:Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodoℓ_a is the shortest of all length-scales. Because it also implies $\ell_a^{-1} > \ell^{-1}$, diffusion-like propagation does not sets in;

• GL_3, GL_4: In these regimes, similar to GL_2, it is more meaningful to ascribe L notation to lasing. In contrast to the very strong absorption counterpart AB_2, we separated $\ell_g^{-1} > \ell^{-1}$ region into $\ell_g < \ell < L$ (GL_3) and $\ell_g < L < \ell$ (GL_4). In the latter regime, one can justify neglecting scattering. Thus, GL_4 encompasses lasing phenomena in Fabry-Perot geometry. In contrast, in GL_3Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [138, 139].

3.5. DISCUSSION AND OUTLOOK

As it was discussed in the previous section,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Importantly, the coherent amplification/Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo, thus, can promote/suppress localization phenomena.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [31, 32, 36, 37, 42, 73, 136]. Furthermore, in the experimental studies of localization of light,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo [16, 17, 18, 59, 109].

In finite *passive* random media,Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo: averaged dimensional conductance, its mesoscopic fluctuations relative to the mean value, Thouless parameter, renormalization of the diffusion coefficient, inverse participation ratio, spatial correlations and others. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. We believe that our analysis of the parameter space in Sec. 3.4Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo*non-conservative* random media [37, 42].

3.6. ACKNOWLEDGMENTS

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Maecenas ultrices egestas commodo. DMR-0704981.

4. CONCLUSIONS

4.1. CHARACTERIZATION OF TRANSPORT REGIME TRANSITIONS IN NON-CONSERVATIVE RANDOM MEDIA

The purpose of the first portion of this dissertation was to characterize the effect of absorption and gain in ballistic, diffusive, and localized transport regimes. Particle and wave-based transport models were studied in one dimension (1D) and quasi-1D geometries.

An investigation of the ratio of transmission to energy in the system, $T/\mathcal{E}$, was performed in 1D using theoretical and numerical methods [37]. The numerical model uses the transfer matrix method [8] with self-embedding [51] to simulate layers of dielectric material with random widths. Since diffusion cannot occur in 1D, the response of the parameter $T/\mathcal{E}$ in the regime of Anderson localization when gain is present was found. A decrease of $T/\mathcal{E}$ from the value given by the classical un-renormalized diffusion coefficient may be attributed to wave-interference localization effects. Although $T/\mathcal{E}$ does not diverge as the random lasing threshold is approached, there is a dependence on the position of the center of localization. This position dependence is closely related to the existance of necklace states [83].

To investigate the transition from diffusion to Anderson localization, the previous numerical model was extended to quasi-1D geometry. To guide our efforts we developed a phase space diagram with 15 different transport regimes [140]. A way of characterizing which regime a given system is in was clearly needed. In the process of using the numerical model in this geometry, we showed that evanescant channels do not need to be included in simulations of passive media [55]. The effect of evanescant channels is renormalize the transport mean free path while conforming to single parameter scaling.

A parameter related to $T/\mathcal{E}$, the position dependent diffusion coefficient $D(z)$, was investigated for use in characterizing the multiple transport regimes in quasi-1D non-conservative random media [62]. Our results indicate that $D(z)$ may serve as a useful criterion for the enumerated transport regimes.

4.2. ANALYSIS OF DETERMINISTIC APERIODIC STRUCTURES

Although random media produces novel features, these systems lack easy reproducibility. For applications such as photonic integrated circuits we are interested in novel features unavailable to periodic media while maintaining reproducibility. Thus media correlated disorder is a natural avenue of pursuit. The numerical model used in this dissertation can simulate any arrangement of scatters, making it amenable to aperiodic media.

In the studies of deterministic aperiodic structures (DAS) we focused on the Thue-Morse pattern. This generation algorithm yeilds a singular continuous Fourier transform spectrum with self-similar features. We demonstrated the possibility of mapping the array of micro-cavities in the two dimensional (2D) Thue Morse DAS onto a periodic square lattice [141]. Such mapping allowed us to uniquely identify and enumerate the configurations of nearest and next-nearest neighbors. Thus the original aperiodic structure is reduced to the periodic structure with aperiodic arrangement of the limited set of pairings.

Once this step was completed, we demonstrated the applicability of the tight binding approach in a deterministic aperiodic array of photonic micro-cavities. Under realistic conditions, we observed hybridization of the modes of individual micro-cavities into the eigenstates of the entire array. Our work adds the tight binding approach to the arsenal of theoretical tools for studying of 2D Thue-Morse structures as well as for design and analysis of experiments.

The tight binding model allows us to investigate the size scaling of the density of the optical states in large arrays of optical micro-cavities; monitor the evolution of the spectra; and to study spatial properties of the eigenstates via e.g. the inverse participation ratio. The inverse participation ratio shows coexistence of localized and extended states in the same spectral regions. Some of the extended states have nearly constant intensity across the entire sample. This property makes the considered system extremely promising for practical applications in optical control of light propagation via e.g. wave-front shaping.

APPENDIX A

CHARACTERISTIC LENGTH SCALES

Table A.1: Length scales used in this dissertation

Symbol	Name	Description
λ	wavelength	Wavelength of incident light
L	system length	Length of waveguide along direction of propagation (z-axis)
W	system width	Dimension of waveguide perpendicular to direction of propagation (y-axis)
L_ϕ	phase coherence length	Length over which phase remains coherent. Equivalent to $L_{inelastic}$[27, 57]. Applicable only to electron transport.
L_D	path length	How far a particle (i.e. ray optics) travels in the media in ballistic and diffusive regimes
ℓ_{scat}	scattering length	Average distance between scattering events. Often referred to as ℓ_{mfp} (mean free path) or the inelastic length[14], or extinction length[10].
ℓ_{tmfp}	transport mean free path	Average distance over which phase and direction are randomized. $\ell_{tmfp} = \frac{\ell_{scat}}{1-\langle cos\ \theta\rangle}$. Sometimes referred to as elastic mfp[92]. Measured with respect to L.
ξ	localization length	Probability of diffusive path forming loop is 1. $\xi = N\ \ell_{tmfp}$. Measured with respect to L.
$\ell_{a,g}$	ballistic absorption/gain length	Average distance over which intensity decreases by two/increases by two.
$\xi_{a,g}$	absorption/gain length	How far, on average, a particle travels in the diffusive regime before being absorbed (or doubled), measured with respect to path length L_D in the diffusive regime
z_p	penetration depth	Applies to diffusive regime only. $z_p \approx \ell_{tmfp}$

All length scales (except λ) are normalized by wavelength.

APPENDIX B

TRANSFER MATRICES FOR ELECTRIC FIELD PROPAGATION

In the following derivation, the transfer matrix method[8][43][44] is developed from Maxwell's equations[48]. Before starting, the assumptions necessary for the derivation are enumerated.

- No leakage of electric field at edge ($y = 0$, $y = W$) of waveguide (i.e. metallic boundaries). Gives boundary conditions of zero field at edges. Incident and output edges are open (no restrictions).
- δ-function scattering potentials, later reduced to a finite sum of Fourier components. Use of this scattering potential has generalized results
- No inelastic scattering: no energy loss due to scattering when passive, and phase remains coherent [scatterers only affect amplitude].
- No noise (spontaneous emission). We are interested primarily in the AL/diffusion phenomenon. Also, experimentally noise can be suppressed.
- The gain mechanism is purely mathematical: no atomic level modeling is included. This is part of being mesoscopic regime: independent of atomic-based scattering mechanisms.
- No input beam properties are assumed (can be plane wave, but that is not necessary).

The wave equation is derived from Maxwell's equations (not show). In the following, only s-polarized waves (for transverse-magnetic (TM) waves) are assumed incident: electric field oscillates perpendicular to the plane of the 2D waveguide. [142]

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \tag{B.1}$$

where $\mu_0 \epsilon_0 = \frac{1}{c^2}$.

TIME INDEPENDENT WAVE EQUATION

Assuming electric field variables are separable,

$$E(\vec{r},t) = E(\vec{r})e^{i\omega t} \tag{B.2}$$

the field is simplified by also assuming monochromatic and continuous wave (CW). Substituting Eq. B.2 into the right side of Eq. B.1, time dependence can be canceled.

$$\nabla^2 E(\vec{r}) = -\frac{\omega^2}{c^2}E(\vec{r}) \tag{B.3}$$

where $\frac{\omega}{c} = k$. Although the following results will appear to be "time independent," the time dependence can be reintroduced by multiplying both sides by $e^{i\omega t}$. Effectively the same as assuming $t = 0$.

SEPARATION OF VARIABLES

Convert from general $\vec{r}$ to two-dimensional Cartesian coordinates (since the transfer matrices for a planar quasi-1D waveguide are desired): $\vec{r} = z\hat{i} + y\hat{j}$. Let $W \equiv$ width and $L \equiv$ length of waveguide.

The z and y components of the field are independent, the separation of variables applies spatially.

$$E(\vec{r}) = E(z,y) = \sum_{n=1}^{\infty} E_n(z)\chi_n(y) \tag{B.4}$$

where the sum is over all channels. For δ-function scatterers, there can be an infinite number of closed channels.

Now the wave equation (Eq. B.3) is

$$\nabla^2 E(z,y) = -\frac{\omega^2}{c^2} E(z,y) \tag{B.5}$$

Apply Laplacian and separation (Eq. B.4)

$$\sum_{n=1}^{\infty} \left[\frac{\partial^2 E_n(z)}{\partial z^2} \chi_n(y) + E_n(z) \frac{\partial^2 \chi_n(y)}{\partial y^2} \right] = -\frac{\omega^2}{c^2} \sum_{n=1}^{\infty} E_n(z) \chi_n(y) \tag{B.6}$$

PERPENDICULAR COMPONENT SOLUTION

The solution to the differential equation perpendicular to the direction of propagation is found from the auxiliary equation for each channel

$$\left(\frac{\partial^2}{\partial y^2} + k_{\perp n}^2 \right) \chi_n(y) = 0 \tag{B.7}$$

Boundary conditions for metallic waveguide: Electric field E is zero at the boundaries, $\chi_n(0) = \chi_n(W) = 0$. The normalized solution is the familiar

$$\chi_n(y) = \sqrt{\frac{2}{W}} \sin(k_{\perp n} y) \tag{B.8}$$

where $k_{\perp n} \equiv \frac{n\pi}{W}$. As a check of normalization, for $m = n$

$$\int_0^W \chi_n^2(y) dy = \frac{2}{W} \int_0^W \sin^2(k_{\perp n} y) = \frac{2}{W} \frac{1}{2} W = 1 \tag{B.9}$$

and if $m \neq n$, solutions are orthogonal

$$\int_0^W \chi_n(y) \chi_m(y) dy = 0 \tag{B.10}$$

Thus, for general n and m,

$$\int_0^W \chi_n(y)\chi_m(y)dy = \delta_{n,m} \tag{B.11}$$

PARALLEL COMPONENT SOLUTION

For the solution parallel to the direction of propagation of Eq. B.5, the z-component starts with

$$\frac{\partial^2 E_n(z)}{\partial z^2} - k_{\perp n}^2 E_n(z) = -\frac{\omega^2}{c^2} E_n(z) \tag{B.12}$$

Re-arrange and introduce a new variable

$$\frac{\partial^2 E_n(z)}{\partial z^2} + k_{\parallel n}^2 E_n(z) = 0 \tag{B.13}$$

where

$$k_{\parallel n}^2 \equiv \frac{\omega^2}{c^2} - k_{\perp n}^2 \tag{B.14}$$

Note: $k_{\parallel n}^2$ can be positive (corresponding to open channels) or negative (closed channels). If negative, then $k_{\parallel n}$ is imaginary, denoted $k_{\parallel n} = i\kappa_{\parallel n}$ for $n > N_{open}$. Open channels propagate forward, with velocity decreasing as channel index increases. Closed channels decrease in amplitude exponentially.

Separate electric field components into left(-) and right (+) traveling plane waves (two solutions to the second order differential equation)

$$\begin{aligned} &\text{Open:} \quad E_n(z) = E_n^+ \exp(ik_{\parallel n} z) + E_n^- \exp(-ik_{\parallel n} z) \\ &\text{Closed:} \quad E_n(z) = E_n^+ \exp(-\kappa_{\parallel n} z) + E_n^- \exp(\kappa_{\parallel n} z) \end{aligned} \tag{B.15}$$

where $i\kappa \equiv k$

WAVEGUIDE WITH SCATTERERS

Up to this point, an empty waveguide has been considered. For scattering, replace $\frac{\omega^2}{c^2}$ of the wave equation B.5 with a spacial Sellmeier equation

$$\frac{\omega^2}{c^2}(1 + \alpha\delta(z - z_0, y - y_0)) \tag{B.16}$$

where $\delta(z - z_0, y - y_0) \equiv \delta(z - z_0)\delta(y - y_0)$ is the scattering potential and α is the scattering strength. α can be complex; then the real part is the strength and the imaginary component is gain or absorption.

To determine transport of light past a scattering potential, apply continuity of electric field E and its derivative. The following carries out matching component-wise derivative.

Assuming the scattering potential is located at cross-section z (inside the waveguide $0 < z < L$), and the electric field just before or after the scatterer (at $z \pm \Delta$) is a sum of independent channel components.

$$E(z \pm \Delta, y) = \sum_{n=1}^{\infty} E_n(z \pm \Delta)\chi_n(y) \tag{B.17}$$

Applying Eq. B.16 to Eq. B.13, the wave equation becomes

$$\sum_{n=1}^{\infty} \left(E_n''\chi_n + k_{\parallel n}^2 E_n\chi_n + \alpha\frac{\omega^2}{c^2}\delta(z - z_0, y - y_0)E_n\chi_n \right) = 0 \tag{B.18}$$

Multiply Eq. B.18 by χ_m and $\int_0^W dy$. By applying Eq. B.11 and letting $A_{m,n}(y_0) = \chi_m(y_0)\chi_n(y_0)$,

$$\sum_{n=1}^{\infty} \left(E_n''\delta_{nm} + k_{\parallel n}^2 E_n\delta_{nm} + \alpha\frac{\omega^2}{c^2}E_n\delta(z_0)A_{nm}(y_0) \right) = 0 \tag{B.19}$$

Apply the summation over n, which eliminates the Kronecker deltas.

$$E''_m + k^2_{\|m} E_m + \alpha \frac{\omega^2}{c^2} E_n \delta(z - z_0) \sum_{n=1}^{\infty} A_{nm}(y_0) = 0 \tag{B.20}$$

Integrate over z from $(z - \Delta)$ to $(z + \Delta)$ and let $\Delta \to 0$.

$$\begin{gathered} \int_{z_0-\Delta}^{z_0+\Delta} E''_m(z)dz + k^2_{\|m} \int_{z_0-\Delta}^{z_0+\Delta} E_m(z)dz + \\ \alpha \frac{\omega^2}{c^2} \sum_{n=1}^{\infty} A_{n,m}(y_0) \int_{z_0-\Delta}^{z_0+\Delta} \delta(z_0) E_n dz = 0 \end{gathered} \tag{B.21}$$

To do the second term integration, assume that for small Δ, $E(z) \approx E(z_0)$.

$$E'_m(z_0 + \Delta) - E'_m(z_0 - \Delta) + k^2_{\|m} E_m(z_0) 2\Delta + \alpha \frac{\omega^2}{c^2} \sum_{n=1}^{\infty} A_{n,m}(y_0) E_n(z_0) = 0 \tag{B.22}$$

Since $\Delta \to 0$, then 2Δ is really small, so that term is dropped.

To conclude, for a given channel m, electric field and the field derivative on both sides of the scatterer must match

$$\begin{gathered} E_m(z_0 + \Delta) = E_m(z_0 - \Delta) \\ E'_m(z_0 + \Delta) = E'_m(z_0 - \Delta) - \alpha \frac{\omega^2}{c^2} \sum_{n=1}^{\infty} A_{n,m}(y_0) E_n(z_0) \end{gathered} \tag{B.23}$$

Note that the δ function scatterer has been eliminated, and $A_{n,m}$ can form an array (the "scattering matrix").

$$\begin{gathered} \begin{pmatrix} \hat{I} & 0 \\ -\alpha \frac{\omega^2}{c^2} A_{mn}(y_0) & \hat{I} \end{pmatrix} \begin{pmatrix} E_{1..N_{max}}(z_0 - \Delta) \\ \frac{1}{\kappa_{\|1..N_{max}}} E'_{1..N_{max}}(z_0 - \Delta) \end{pmatrix} = \\ \begin{pmatrix} E_{1..N_{max}}(z_0 + \Delta) \\ \frac{1}{\kappa_{\|1..N_{max}}} E'_{1..N_{max}}(z_0 + \Delta) \end{pmatrix} \end{gathered} \tag{B.24}$$

Due to the form of the matrix, the determinant is always unity (only the diagonal contributes non-zero terms) regardless of the elements in the lower left quadrant. Elements of the lower left quadrant are

$$-\alpha\frac{\omega^2}{c^2}\frac{2}{W}\sin(k_{\perp m}y_0)\sin(k_{\perp n}y_0) \tag{B.25}$$

Note that the scattering matrix is real unless α or ω are complex.

FREE SPACE PROPAGATION OF OPEN CHANNELS

For open channels ($n \leq N_o$), field E_n and derivative of field $\frac{1}{k_{\parallel n}}E'_n$ are more convenient basis than "left traveling" $E_n^-(z)$ and "right traveling" $E_n^+(z)$. First, the connection between the two basis is found. Starting from Eq. B.15, electric field $E(z)$ is the solution to a second order differential equation, so it has two solutions.

$$\begin{aligned} E_n(z) &= E_n^+\exp(ik_{\parallel n}z) + E_n^-\exp(-ik_{\parallel n}z) \\ E'_n(z) &= ik_{\parallel n}E_n^+\exp(ik_{\parallel n}z) - ik_{\parallel n}E_n^-\exp(-ik_{\parallel n}z) \end{aligned} \tag{B.26}$$

Solving for left- and right-traveling wave components,

$$\begin{aligned} E_n^+(z) &= \frac{1}{2}\left(E_n(z) + \frac{1}{i}\frac{1}{k_{\parallel n}}E'_n(z)\right)\exp(-ik_{\parallel n}z) \\ E_n^-(z) &= \frac{1}{2}\left(E_n(z) - \frac{1}{i}\frac{1}{k_{\parallel n}}E'_n(z)\right)\exp(ik_{\parallel n}z) \end{aligned} \tag{B.27}$$

To preemptively clear up notation confusion, in previous steps Δ was used to denote a small distance away from the scatterer. Here Δz will be used to signify a not infinitesimal displacement in position along the z axis. The field and derivative of field is translated over distance Δz from the original position z. First, substitute the shift into Eq. B.26

$$E_n(z+\Delta z) = E_n^+ \exp(ik_{\|n}(z+\Delta z)) + E_n^- \exp(-ik_{\|n}(z+\Delta z)) \tag{B.28}$$

Then substitute Eq. B.27

$$\begin{gathered} E_n(z+\Delta z) = \frac{1}{2}\left(E_n(z) + \frac{1}{i}\frac{1}{k_{\|n}}E_n'(z)\right)\exp(ik_{\|n}z) + \\ \frac{1}{2}\left(E_n(z) - \frac{1}{i}\frac{1}{k_{\|n}}E_n'(z)\right)\exp(-ik_{\|n}z) \end{gathered} \tag{B.29}$$

Reducing leaves how to shift an electric field over distance Δz.

$$E_n(z+\Delta z) = E_n(z)\cos(k_{\|n}\Delta z) + \frac{1}{k_{\|n}}E_n' \sin(k_{\|n}\Delta z) \tag{B.30}$$

Similarly,

$$\frac{1}{k_{\|n}}E_n'(z+\Delta z) = iE_n^+ \exp(ik_{\|n}(z+\Delta z)) - iE_n^- \exp(-ik_{\|n}(z+\Delta z)) \tag{B.31}$$

Then substitute Eq. B.27

$$\begin{gathered} \frac{1}{k_{\|n}}E_n'(z+\Delta z) = \frac{i}{2}\left(E_n(z) + \frac{1}{i}\frac{1}{k_{\|n}}E_n'(z)\right)\exp(ik_{\|n}z) - \\ \frac{i}{2}\left(E_n(z) - \frac{1}{i}\frac{1}{k_{\|n}}E_n'(z)\right)\exp(-ik_{\|n}z) \end{gathered} \tag{B.32}$$

$$\frac{1}{k_{\|n}}E'(z+\Delta z) = -E_n(z)\sin(k_{\|n}\Delta z) + \frac{1}{k_{\|n}}E_n' \cos(k_{\|n}\Delta z) \tag{B.33}$$

FREE-SPACE PROPAGATION OF CLOSED CHANNELS

For closed channels ($n > N_o$), change of i results in hyperbolic trig functions.

$$\begin{aligned} E_n(z) &= E_n^+ \exp(-\kappa_{\|n} z) + E_n^- \exp(\kappa_{\|n} z) \\ E_n'(z) &= -\kappa_{\|n} E_n^+ \exp(-\kappa_{\|n} z) + \kappa_{\|n} E_n^- \exp(\kappa_{\|n} z) \end{aligned} \tag{B.34}$$

Recalling that $k_{\|n} = i\kappa_{\|n}$, then

$$\begin{aligned} E_n^+(z) &= \frac{1}{2}\left(E_n(z) - \frac{1}{\kappa_{\|n}} E_n'(z)\right) \exp(\kappa_{\|n} z) \\ E_n^-(z) &= \frac{1}{2}\left(E_n(z) + \frac{1}{\kappa_{\|n}} E_n'(z)\right) \exp(-\kappa_{\|n} z) \end{aligned} \tag{B.35}$$

Shifting the field by Δz

$$E_n(z + \Delta z) = E_n(z) \cosh(\kappa_{\|n} \Delta z) + \frac{1}{\kappa_{\|n}} E_n'(z) \sinh(\kappa_{\|n} \Delta z) \tag{B.36}$$

and

$$\frac{1}{\kappa_{\|n}} E_n'(z + \Delta z) = E_n(z) \sinh(\kappa_{\|n} \Delta z) + \frac{1}{\kappa_{\|n}} E_n'(z) \cosh(\kappa_{\|n} \Delta z) \tag{B.37}$$

To summarize,

$$\begin{aligned} E_n(z + \Delta z) &= E_n(z) \cosh(\kappa_{\|n} \Delta z) + \frac{1}{\kappa_{\|n}} E_n'(z) \sinh(\kappa_{\|n} \Delta z) \\ \frac{1}{\kappa_{\|n}} E_n'(z + \Delta z) &= E_n(z) \sinh(\kappa_{\|n} \Delta z) + \frac{1}{\kappa_{\|n}} E_n'(z) \cosh(\kappa_{\|n} \Delta z) \end{aligned} \tag{B.38}$$

From Eq. B.30, B.33, and B.38 the "free space propagation matrix" can be constructed. The array would be of rank $2n_{max}$ ($n_{max} = N_o + N_c$). The determinant of this matrix is alway unity (regardless of argument) because terms can be factored into $\sin^2 x + \cos^2 = 1$ for each channel. Thus, for both free and scattering matrices, the determinant is unity regardless of free space separation Δz or real (passive) and complex (active media) dielectric values.

APPENDIX C

RELATION OF $T/\mathcal{E}$ TO $D(Z)$

This is an expansion of Appendix section 2.6.1. As in that section we assume a slab geometry. The z coordinate normal to the slab is separated from the perpendicular component ρ as $\mathbf{r} = (\rho, z)$. Again assuming no dependence on ρ allows us to give the ensemble-averaged diffusive flux $\langle \vec{J}(\vec{r},t)\rangle$ and the energy density $\langle \mathcal{W}(\vec{r},t)\rangle$ are related via [63]

$$\langle \vec{J}(\vec{r},t)\rangle = -D(\vec{r})\vec{\nabla}\langle \mathcal{W}(\vec{r},t)\rangle \tag{C.1}$$

The diffusion approximation amounts to $D(\vec{r}) \equiv D_0 = c\ell_{tmfp}/3$, where c is the speed of light and ℓ_{tmfp} is the transport mean free path.

We consider a 3D random medium in a shape of a slab of thickness L, where we explicitly separate the coordinate z normal to the slab from the perpendicular component ρ as $\mathbf{r} = (\rho, z)$. Under a CW plane-wave illumination at normal incidence, the dependence on ρ and t can be neglected.

$$\langle \vec{J}_z(z)\rangle = -D(z)\frac{d}{dz}\langle \mathcal{W}(z)\rangle \tag{C.2}$$

Integration over z gives

$$\int_z^L \frac{\langle J_z(z')\rangle dz'}{D(z')} = -\langle \mathcal{W}(L)\rangle + \langle \mathcal{W}(z)\rangle \tag{C.3}$$

where the energy stored inside the random medium $\mathcal{E}$ is formally defined as

$$\langle \mathcal{E}\rangle = \int_0^L \langle \mathcal{W}(z)\rangle dz. \tag{C.4}$$

thus

$$\langle \mathcal{E}\rangle = \int_0^L \left(\langle \mathcal{W}(L)\rangle + \int_z^L \frac{\langle J_z(z')\rangle}{D(z')}dz'\right) dz \tag{C.5}$$

The remaining work is to factor out transmission T in order to find the relation between $T/\mathcal{E}$ and $D(z)$. The energy density $\langle \mathcal{W}(L)\rangle$ at the right boundary can be expressed in terms of right- and left-propagating fluxes. From the definition of diffusive flux [63]

$$\langle J_{\pm}(z)\rangle = \frac{c}{4}\langle \mathcal{W}(z)\rangle \mp \frac{D_0}{2}\frac{d\langle \mathcal{W}(z)\rangle}{dz} \tag{C.6}$$

where $\langle J_{-}\rangle$ and $\langle J_{+}\rangle$ are the fluxes propagating along negative and positive z-directions respectively. Since $\langle J_{+}(L)\rangle = J_0 T$ and $\langle J_{-}(L)\rangle = 0$, using Eqs. C.6 to eliminate D_0 yields

$$\langle J_{+}(L)\rangle + \langle J_{-}(L)\rangle = 2\frac{c}{4}\langle \mathcal{W}(L)\rangle \tag{C.7}$$

Therefore $\langle \mathcal{W}(L)\rangle = 2J_0T/c$ and the energy can be re-written as

$$\langle \mathcal{E}\rangle = \int_0^L \left(2J_0T/c + \int_z^L \frac{\langle J_z(z')\rangle}{D(z')}dz'\right) dz \tag{C.8}$$

Next, we reduce $\langle J_z(z')\rangle$ to find an approximately equivalent transmission.

In the CW regime when the energy density $\mathcal{W}(z)$ is stationary, $\partial\langle \mathcal{W}(z)\rangle/\partial t = 0$, it follows from energy conservation condition for flux $\vec{J}$ and energy $\mathcal{W}$

$$\frac{\partial\langle \mathcal{W}(\vec{r},t)\rangle}{\partial t} + \vec{\nabla}\cdot\langle \vec{J}(\vec{r},t)\rangle = \frac{c}{\ell_g}\langle \mathcal{W}(\vec{r},t)\rangle + J_0\delta(z-z_p) \tag{C.9}$$

that the z component of flux is constant for $z > z_p \sim \ell$. The value of the constant can be obtained from the boundary condition at $z = L$ as

$$\langle J_z(z)\rangle = \begin{cases} \langle J_z(L)\rangle \equiv J_0\langle T\rangle, & z_p < z < L \\ \langle J_z(0)\rangle \equiv -J_0\langle R\rangle, & 0 < z < z_p \end{cases} \tag{C.10}$$

where T (R) is the transmission (reflection) coefficient. As a check, by integrating Eq. (C.9) over the entire system we obtain the standard (passive) flux conservation $\langle J_z(L)\rangle - \langle J_z(0)\rangle = J_0\langle T\rangle - (-J_0\langle R\rangle) = J_0(\langle T\rangle + \langle R\rangle) = J_0$. To take advantage of the fact that $\langle J_z(z)\rangle$ is piecewise constant, c.f. Eq. (C.10), we have to neglect by

$0 < z < z_p$ contribution. Then a constant can be substituted for $J_z(z')$,

$$\langle \mathcal{E} \rangle = \int_0^L \left(2J_0 \langle T \rangle / c + \int_z^L \frac{J_0 \langle T \rangle}{D(z')} dz' \right) dz \tag{C.11}$$

This introduces an error $\propto z_p/L \sim \ell/L \ll 1$. Factoring T from the integrands,

$$\langle \mathcal{E} \rangle = J_0 \langle T \rangle \int_0^L \left(\int_z^L \frac{dz'}{D(z')} + 2/c \right) dz \tag{C.12}$$

Note that the second term is of the same order $\sim \ell/L$ as the term omitted in arriving to the above expression. Hence, $2/c$ contribution has to be dropped as well.

$$\langle \mathcal{E} \rangle = J_0 T \int_0^L \int_z^L \frac{1}{D(z')} dz' dz \tag{C.13}$$

Taking advantage of the system symmetry, $D(z) = D(L - z)$, the double integration can be further simplified as

$$\begin{aligned} \int_0^L \int_z^L \frac{1}{D(z')} dz' dz &= \frac{1}{2} \int_0^L \int_0^L \frac{1}{D(z')} dz' dz \\ &= \frac{L}{2} \int_0^L \frac{1}{D(z)} dz. \end{aligned} \tag{C.14}$$

After normalizing the integral so that it yields unity in the case when the wave interference effects are neglected, $D(z) = D_0 \equiv c\ell/3$, for passive media

$$\frac{\langle T \rangle}{\langle \mathcal{E} \rangle} \simeq \frac{1}{J_0} \frac{2D_0}{L^2} \left(\frac{1}{L} \int_0^L \frac{D_0}{D(z)} dz \right)^{-1}, \tag{C.15}$$

We note that in process of deriving Eq. (C.15), we dropped the terms on the order of $\sim \ell/L \ll 1$.

Dropping the localization corrections leaves

$$\frac{\langle T\rangle}{\langle \mathcal{E}\rangle} \simeq \frac{1}{J_0}\frac{2D_0}{L^2} \tag{C.16}$$

Any deviation from Eq. C.16 in passive diffusive media can be attributed to localization corrections.

BIBLIOGRAPHY

[1] A. Lagendijk, B. van Tiggelen, and D. S. Wiersma. Fifty years of anderson localization. *Phys. Today*, 62:24–29, 2009.

[2] M. C. van Rossum and Th. M. Nieuwenhuizen. Multiple scattering of classical waves: microscopy, mesoscopy, and diffusion. *Rev. Mod. Phys.*, 71:313–371, 1999.

[3] P. W. Anderson. Absence of diffusion in certain random lattices. *Phys. Rev.*, 109:1492–1505, 1958.

[4] J. Frohlich and T. Spencer. Absence of diffusion in the anderson tight binding model in large disorder or low energy. *Comm. in Math. Phys.*, pages 151–184, 1983.

[5] I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur. *Introduction to the Theory of Disordered Systems*. John Wiley & Sons, 1988.

[6] H. von Dreifus and A. Klein. A new proof of localization in the anderson tight binding model. *Comm. in Math. Phys.*, 124:285–299, 1989.

[7] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan. Scaling theory of localization: Absence of quantum diffusion in two dimensions. *Phys. Rev. Lett.*, 42:673–676, 1979.

[8] A. MacKinnon and B. Kramer. One-parameter scaling of localization length and conductance in disordered systems. *Phys. Rev. Lett.*, 47:1546–1549, 1981.

[9] P. Markos. Numerical analysis of the anderson localization. *Acta Physica Slovaca*, 56:561–685, 2006.

[10] B. van Tiggelen. Localization of waves. In J. P. Fouque, editor, *Diffuse waves in complex media*, pages 1–60. Kluwer Academic, Dordrecht, The Netherlands, 1999.

[11] P. Lee and T. Ramakrishnan. Disordered electronic systems. *Rev. Mod. Phys.*, 57:287–337, 1985.

[12] R. A. Webb and S. Washburn. Quantum interference fluctuations in disordered metals. *Phys. Today*, pages 46–53, 1988.

[13] B. L. Altshuler, P. A. Lee, and R. A. Webb, editors. *Mesoscopic Phenomena in Solids*. North Holland, Amsterdam, 1991.

[14] S. John. Electromagnetic absorption in a disordered medium near a photon mobility edge. *Phys. Rev. Lett.*, 53:2169–2172, 1984.

[15] P. W. Anderson. The question of classical localization: a theory of white paint? *Philos. Mag. B*, 52:505–509, 1985.

[16] A. A. Chabanov, M. Stoytchev, and A. Z. Genack. Statistical signatures of photon localization. *Nature*, 404:850–853, 2000.

[17] D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini. Localization of light in a disordered medium. *Nature*, 390:671–673, 1997.

[18] A. Z. Genack and N. Garcia. Observation of photon localization in a three-dimensional disordered system. *Phys. Rev. Lett.*, 66:2064–2067, 1991.

[19] A. F. Ioffe and A. R. Regel. Non-crystalline, amorphous, and liquid electronic semiconductors. *Prog. Semicond.*, 4:237–291, 1960.

[20] P. Sheng. *Introduction to wave scattering, localization and mesoscopic phenomena.* Springer, 2006.

[21] H. Cao, Y. Zhao, S. T. Ho, E. W. Seelig, Q. H. Wang, and R. P. H. Chang. Random laser action in semiconductor powder. *Phys. Rev. Lett.*, 82:2278–2281, 1999.

[22] H. Cao. Review on latest developments in random lasers with coherent feedback. *J. Phys. A: Math. Gen.*, 38:10497–10535, 2005.

[23] D. S. Wiersma. The physics and applications of random lasers. *Nature Physics*, 4:359–367, 2008.

[24] T. R. Kirkpatrick. Localization of acoustic waves. *Phys. Rev. B*, 31:5746–5755, 1985.

[25] A. G. Yamilov, R. L. Weaver, and O. Lobkis. Ultrasound amplification by stimulated emission of radiation. *Photonics Spectra*, pages 90–94, 2006.

[26] H. Hu, A. Strybulevych, J. H. Page, S. E. Skipetrov, and B. A. van Tiggelen. Localization of ultrasound in a three-dimensional elastic network. *Nat. Phys.*, 4:945–948, 2008.

[27] A. D. Stone and A. Szafer. What is measured when you measure a resistance? the landauer formula revisited. *IBM J. Res. Devel.*, 32:384–413, 1988.

[28] D. S. Fisher and P. A. Lee. Relation between conductivity and transmission matrix. *Phys. Rev. B*, 23:6851–6854, 1981.

[29] E. N. Economou and C. M. Soukoulis. Static conductance and scaling theory of localization in one dimension. *Phys. Rev. Lett.*, 46:618–621, 1981.

[30] V. D. Freilikher, M. Pustilnik, and I. Yurkevich. Effect of absorption on the wave transport in the strong localization regime. *Phys. Rev. Lett.*, 73:810–813, 1994.

[31] A. Yamilov and H. Cao. Effects of localization and amplification on intensity distribution of light transmitted through random media. *Phys. Rev. E*, 70, 2004.

[32] A. Yamilov and H. Cao. Effect of amplification on conductance distribution of a disordered waveguide. *Phys. Rev. E*, 74, 2006.

[33] P. W. Brouwer. Transmission through a many-channel random waveguide with absorption. *Phys. Rev. B*, 57:10526–10536, 1998.

[34] D. J. Thouless. Maximum metallic resistance in thin wires. *Phys. Rev. Lett.*, 39:1167–1169, 1977.

[35] D. Vollhardt and P. Wölfle. Diagrammatic, self-consistent treatment of the anderson localization problem in $d \leq 2$ dimensions. *Phys. Rev. B*, 22:4666–4679, 1980.

[36] A. Yamilov, S. H. Chang, A. Burin, A. Taflove, and H. Cao. Field and intensity correlations in amplifying random media. *Phys. Rev. B*, 71, 2005.

[37] B. Payne, J. Andreasen, H. Cao, and A. Yamilov. Relation between transmission and energy stored in random media with gain. *Phys. Rev. B*, 82, 2010.

[38] N. Cherroret and S. E. Skipetrov. Microscopic derivation of self-consistent equations of anderson localization in a disordered medium of finite size. *Phys. Rev. E*, 77, 2008.

[39] E. P. Wigner. On the statistical distribution of the widths and spacings of nuclear resonance levels. *Proc. Cambridge Philo. Soc.*, 47:790–798, 1951.

[40] C. W. Beenakker. Random-matrix theory of quantum transport. *Rev. Mod. Phys.*, 69:731–808, 1997.

[41] C. W. J. Beenakker. *Applications of random matrix theory to condensed matter and optical physics*, chapter 35,36. Oxford University Press, 2009.

[42] B. Payne, H. Cao, and A. Yamilov. Criterion for light localization in random amplifying media. *Physica B*, 405:3012–3015, 2010.

[43] J. B. Pendry and A. MacKinnon. Calculation of photon dispersion relations. *Phys. Rev. Lett.*, 69:2772–2775, 1992.

[44] T. Brandes and S. Kettemann. *Anderson Localization and Its Ramifications.* Springer, 2003.

[45] Z. Q. Zhang, A. A. Chabanov, S. K. Cheung, C. H. Wong, and A. Z. Genack. Dynamics of localized waves: Pulsed microwave transmissions in quasi-one-dimensional media. *Phys. Rev. B*, 79, 2009.

[46] O. N. Dorokhov. Transmission coefficient and the localization length of an electron in n bound disordered chains. *Pisma Zh. Eksp. Teor. Fiz., JETP Lett.*, 36:259–262, 1982.

[47] P. A. Mello, P. Pereyra, and N. Kumar. Macroscopic approach to multichannel disordered conductors. *Ann. of Phys.*, 181:290–317, 1988.

[48] J. D. Jackson. *Classical Electrodynamics*. Wiley, 1999.

[49] L. S. Froufe-Pérez, M. Yépez, P. A. Mello, and J. J. Sáenz. Statistical scattering of waves in disordered waveguides: From microscopic potentials to limiting macroscopic statistics. *Phys. Rev. E*, 75, 2007.

[50] V. I. Osedelec. A multiplicative ergodic theorem: Lyapunov characteristic exponents for dynamical systems. *Trans. Mosc. Math. Soc.*, 19:197–231, 1968.

[51] L. I. Deych, A. Yamilov, and A. A. Lisyansky. Effects of resonant tunneling in electromagnetic wave propagation through a polariton gap. *Phys. Rev. B*, 59:11339–11348, 1999.

[52] R. Bellman and G. Wing. *An Introduction to Invariant Embedding*. Wiley, 1976.

[53] A. Mirlin. Statistics of energy levels and eigen-functions in disordered systems. *Phys. Rep.*, 326:259–382, 2000.

[54] V. S. Letokhov. Generation of light by a scattering medium with negative resonance absorption. *Sov. Phys. JETP*, 26:835–840, 1968.

[55] B. Payne, T. Mahler, and A. Yamilov. Effect of evanescent modes on conductance distribution in disordered waveguides. 2010. unpublished.

[56] I. M. Vellekoop and A. P. Mosk. Universal optimal transmission of light through disordered materials. *Phys. Rev. Let.*, 101, 2008.

[57] Y. Imry. Active transmission channels and universal conductance fluctuations. *Europhys. Lett.*, pages 249–256, 1986.

[58] J. M. Drake and A. Z. Genack. Observation of nonclassical optical diffusion. *Phys. Rev. Lett.*, 63:259–262, 1989.

[59] F. Scheffold, R. Lenke, R. Tweer, and G. Maret. Localization or classical diffusion of light? *Nature*, 398:206–207, 1999.

[60] C. Aegerter, M. Storzer, S. Fiebig, W. Buhrer, and G. Maret. Observation of anderson localization of light in three dimensions. *J. Opt. Soc. Am. A*, 24:A23–A27, 2007.

[61] T. Schwartz, G. Bartal, S. Fishman, and M. Segev. Transport and anderson localization in disordered two-dimensional photonic lattices. *Nature*, 446:52–55, 2007.

[62] B. Payne, A. Yamilov, and S. E. Skipetrov. Anderson localization as position-dependent diffusion in disordered waveguides. *Phys. Rev. B*, 82, 2010.

[63] P. M. Morse and H. Feshbach. *Methods of Theoretical Physics*. McGraw-Hill, New York, 1953.

[64] M. P. van Albada, B. A. van Tiggelen, A. Lagendijk, and A. Tip. Speed of propagation of classical waves in strongly scattering media. *Phys. Rev. Lett.*, 66:3132–3135, 1991.

[65] A. A. Lisyansky and D. Livdan. Influence of internal reflection on correlation of intensity fluctuation in random media. *Phys. Rev. B*, 47:14157–14164, 1993.

[66] O. Zaitsev, L. I. Deych, and V. Shuvayev. Statistical properties of one-dimensional random lasers. *Phys. Rev. Lett.*, 102, 2009.

[67] H. E. Tureci, L. Ge, S. Rotter, and A. D. Stone. Strong interactions in multimode random lasers. *Science*, 320:643–646, 2008.

[68] C. Conti and A. Fratalocchi. Dynamic light diffusion, three-dimensional anderson localization and lasing in inverted opals. *Nat. Phys.*, 4:794–798, 2008.

[69] R. Frank, A. Lubatsch, and J. Kroha. Light transport and localization in diffusive random lasers. *J. Opt. A*, 11, 2009.

[70] N. M. Lawandy, R. M. Balachandran, A. S. L. Gomes, and E. Sauvain. Laser action in strongly scattering media. *Nature*, 368:436–438, 1994.

[71] G. van Soest. *Experiments on random lasers*. PhD thesis, Universiteit van Amsterdam, 2001.

[72] J. X. Zhu, D. J. Pine, and D. A. Weitz. Internal reflection of diffusive light in random media. *Phys. Rev. A*, 44:3948–3959, 1991.

[73] A. Y. Zyuzin. Transmission fluctuations and spectral rigidity of lasing states in a random amplifying medium. *Phys. Rev. E*, 51:5274–5278, 1995.

[74] P. Sebbah, B. Hu, J. M. Klosner, and A. Z. Genack. Extended quasimodes within nominally localized random waveguides. *Phys. Rev. Lett.*, 96, 2006.

[75] J. A. Scales and M. Batzle. Millimeterwave analysis of the dielectric properties of oil shales. *Appl. Phys. Lett.*, 89, 2006.

[76] G. A. Luna-Acosta, H. Schanze, U. Kuhl, and H. J. Stickmann. Impurity effects on the band structure of one-dimensional photonic crystals: Experiment and theory. *New J. Phys.*, 10, 2008.

[77] V. Milner and A. Z. Genack. Photon localization laser: Low-threshold lasing in a random amplifying layered medium via wave localization. *Phys. Rev. Lett.*, 94, 2005.

[78] L. I. Deych, A. A. Lisyansky, and B. L. Altshuler. Single parameter scaling in one-dimensional localization revisited. *Phys. Rev. Lett.*, 84:2678–2681, 2000.

[79] J. B. Pendry. Symmetry and transport of waves in one-dimensional disordered systems. *Adv. Phys.*, 43:461–542, 1994.

[80] P. Yeh. *Optical Waves in Layered Media*. Wiley-Interscience, 2nd edition, 2005.

[81] P. A. Mello and N. Kumar. *Quantum Transport in Mesoscopic Systems*. Oxford University Press, Oxford, UK, 2004.

[82] M. Azbel. Resonance tunneling and localization spectroscopy. *Solid State Commun.*, 45:527–530, 1983.

[83] J. B. Pendry. Quasi-extended electron states in strongly disordered systems. *J. Phys. C*, 20:733–742, 1987.

[84] J. Bertolotti, S. Gottardo, D. S. Wiersma, M. Ghulinyan, and L. Pavesi. Optical necklace states in anderson localized 1d systems. *Phys. Rev. Lett.*, 94, 2005.

[85] K. Yu. Bliokh, Yu. P. Bliokh, V. Freilikher, S. Savel'ev, and F. Nori. Unusual resonators: Plasmonics, metamaterials, and random media. *Rev. Mod. Phys.*, 80:1201–1213, 2008.

[86] L. I. Deych, A. Yamilov, and A. A. Lisyansky. Local polariton modes and resonant tunneling of electromagnetic waves through periodic bragg multiple quantum well structures. *Phys. Rev. B*, 64, 2001.

[87] K. Y. Bliokh, Y. P. Bliokh, and V. D. Freilikher. Resonances in one-dimensional disordered systems: localization of energy and resonant transmission. *J. Opt. Soc. Am. B*, 21:113–120, 2004.

[88] X. Jiang and C. M. Soukoulis. Localized random lasing modes and a path for observing localization. *Phys. Rev. E*, 65, 2002.

[89] P. Sebbah and C. Vanneste. Random laser in the localized regime. *Phys. Rev. B*, 66, 2002.

[90] C. Vanneste and P. Sebbah. Localized modes in random arrays of cylinders. *Phys. Rev. E*, 71, 2005.

[91] D. Dragoman and M. Dragoman. *Quantum Classical Analogies*. Springer, 2004.

[92] S. John. Localization of light. *Phys. Today*, 44:32–40, 1991.

[93] S. John. Strong localization of photons in certain disordered dielectric superlattices. *Phys. Rev. Lett.*, 58:2486–2489, 1987.

[94] J. E. Sipe, L. Poladian, and C. M. de Sterke. Propagation through nonuniform grating structures. *J. Opt. Soc. Am. A*, 11:1307–1320, 1994.

[95] A. Yamilov, M. Herrera, and M. F. Bertino. Slow-light effect in dual-periodic photonic lattice. *J. Opt. Soc. Am. B*, 25:599–608, 2008.

[96] E. Akkermans and G. Montambaux. *Mesoscopic Physics of Electrons and Photons*. Cambridge University Press, Cambridge, UK, 2007.

[97] P. Wölfle and D. Vollhardt. Self-consistent teory of anderson localization: general formalism and applications. *Int. J. Mod. Phys. B*, 24:1526–1554, 2010.

[98] O. Lobkis and R. Weaver. Anderson localization of ultrasound in plate: Further experimental results. *J. Acoust. Soc. Am*, 124:3528–3533, 2008.

[99] T. Spencer. Mathematical aspects of anderson localization. *Int. J. Mod. Phys. B*, 24:1621–1639, 2010.

[100] B. Kramer, A. MacKinnon, T. Ohtsuki, and K. Slevin. Mathematical aspects of anderson localization. *Int. J. Mod. Phys. B*, 24:1841–1854, 2010.

[101] P. Pradhan and N. Kumar. Localization of light in coherently amplifying random media. *Phys. Rev. B*, 50:9644–9647, 1994.

[102] Z. Q. Zhang. Light amplification and localization in randomly layered media with gain. *Phys. Rev. B*, 52:7960–7964, 1995.

[103] J. C. J. Paasschens, T. Sh. Misirpashaev, and C. W. J. Beenakker. Localization of light: Dual symmetry between absorption and amplification. *Phys. Rev. B*, 54:11887–11890, 1996.

[104] V. D. Freilikher, M. Pustilnik, and I. Yurkevich. Statistical properties of the reflectance and transmittance of an amplifying random medium. *Phys. Rev. B*, 56:5974–5977, 1997.

[105] F. Scheffold and G. Maret. Universal conductance fluctuations of light. *Phys. Rev. Lett.*, 81:5800–5803, 1998.

[106] W. Deng, D. S. Wiersma, and Z. Q. Zhang. Coherent backscattering of light from random media with inhomogeneous gain coefficient. *Phys. Reb. B*, 56:178–181, 1997.

[107] G. Labeyrie, F. De Tomasi, J. Bernard, C. A. Muller, C. Miniatura, and R. Kaiser. Coherent backscattering of light by cold atoms. *Phys. Rev. Lett.*, 83:5266–5269, 1999.

[108] C. M. Aegerter and G. Maret. Coherent backscattering and anderson localization of light. *Prog. in Optics*, 52:1–62, 2009.

[109] C. M. Aegerter, M. Störzer, and G. Maret. Experimental determination of critical exponents in anderson localisation of light. *Europhys. Lett.*, 75:562–568, 2006.

[110] P. Sebbah, R. Pnini, and A. Z. Genack. Field and intensity correlation in random media. *Phys. Rev. E*, 62:7348–7352, 2000.

[111] M. Patra and C. W. J. Beenakker. Excess noise for coherent radiation propagating through amplifying random media. *Phys. Rev. A*, 60:4059–4066, 1999.

[112] V. Yu. Fedorov and S. E. Skipetrov. Photon noise in a random laser amplifier with fluctuating properties. *Phys. Rev. A*, 79, 2009.

[113] C. W. J. Beenakker and B. Rajaei. Exact solution for the distribution of transmission eigenvalues in a disordered wire and comparison with random-matrix theory. *Phys. Rev. B*, 49:7499–7510, 1994.

[114] E. Kogan and M. Kaveh. Random-matrix-theory approach to the intensity distributions of waves propagating in a random medium. *Phys. Rev. B*, 52:R3813–R3815, 1995.

[115] A. Bulatov, D. Livdan, and J. L. Birman. Statistical properties of the cumulative phase of electromagnetic radiation transmitted through a waveguide with random scatterers. *Phys. Rev. E*, 57:3574–3578, 1998.

[116] K. A. Muttalib and J. R. Klauder. Generalized fokker-planck equation for multichannel disordered quantum conductors. *Phys. Rev. Lett.*, 82:4272–4275, 1999.

[117] L. S. Froufe-Pérez, P. García-Mochales, P. A. Serena, P. A. Mello, and J. J. Sáenz. Conductance distributions in quasi-one-dimensional disordered wires. *Phys. Rev. Lett.*, 89, 2002.

[118] P. Markos and C. M. Soukoulis. Intensity distribution of scalar waves propagating in random media. *Phys. Rev. B*, 71, 2005.

[119] A. Z. Genack and A. A. Chabanov. Signatures of photon localization. *J. Phys. A*, 38:10465–10488, 2005.

[120] L. C. Botten, A. A. Asatryan, N. A. Nicorovici, R. C. McPhedran, and C. M. de Sterke. Generalisation of the transfer matrix formulation of the theory of electron and photon conductance. *Physica B*, 394:320–324, 2007.

[121] C. J. S. de Matos, L. de Menezes, A. M. Brito-Silva, M. A. M. Gamez, A. S. L. Gomes, and C. B. de Araujo. Random fiber laser. *Phys. Rev. Lett*, 99, 2007.

[122] H. Cao, J. Y. Xu, D. Z. Zhang, S. H. Chang, S. T. Ho, E. W. Seelig, X. Liu, and R. P. H. Chang. Spatial confinement of laser light in active random media. *Phys. Rev. Lett.*, 84:5584–5587, 2000.

[123] Q. Li, K. Ho, and C. Soukoulis. Mode distribution in coherently amplifying random media. *Physica B: Cond. Matt.*, 296:78–84, 2001.

[124] C. Vanneste and P. Sebbah. Selective excitation of localized modes in active random media. *Phys. Rev. Lett.*, 87, 2001.

[125] V. M. Apalkov, M. E. Raikh, and B. Shapiro. Light intensity correlations in optically active media. *J. Phys. A*, 38:10545–10548, 2005.

[126] J. Heinrichs. Transmission, reflection and localization in a random medium with absorption or gain. *J. Phys. Cond. Matter*, 18:4781–4792, 2006.

[127] C. K. Nam and Z. Q. Zhang. Light amplification and localization in random amplifying layered media: Statistics from physical solutions. *Phys. Rev. B*, 66, 2002.

[128] B. A. Muzykantskii and D. E. Khmelnitskii. Nearly localized states in weakly disordered conductors. *Phys. Rev. B*, 51:5480–5483, 1995.

[129] V. M. Apalkov, M. E. Raikh, and B. Shapiro. Random resonators and prelocalized modes in disordered dielectric films. *Phys. Rev. Lett.*, 89, 2002.

[130] A. L. Burin, H. Cao, G. C. Schatz, and M. A. Ratner. High-quality optical modes in low-dimensional arrays of nanoparticles: Application to random lasers. *J. Opt. Soc. Am. B*, 21:121–131, 2004.

[131] S. John and G. Pang. Theory of lasing in a multiple-scattering medium. *Phys. Rev. A*, 54:3642–3652, 1996.

[132] D. S. Wiersma and A. Lagendijk. Light diffusion with gain and random lasers. *Phys. Rev. E*, 54:4256–4265, 1996.

[133] M. Siddique, R. R. Alfano, G. A. Berger, M. Kempe, and A. Z. Genack. Time-resolved studies of stimulated emission from colloidal dye solutions. *Opt. Lett.*, 21:450–452, 1996.

[134] S. V. Frolov, Z. V. Vardeny, and K. Yoshino. Stimulated emission in high-gain organic media. *Phys. Rev. B*, 59:R5284–R5287, 1999.

[135] L. Florescu and S. John. Photon statistics and coherence in light emission from a random laser. *Phys. Rev. Lett.*, 93, 2004.

[136] A. A. Burkov and A. Y. Zyuzin. Correlations in transmission of light through a disordered amplifying medium. *Phys. Rev. B*, 55:5736–5741, 1997.

[137] X. Jiang, Q. Li, and C. M. Soukoulis. Symmetry between absorption and amplification in disordered media. *Phys. Rev. B*, 59:R9007–R9010, 1999.

[138] X. Wu, W. Fang, A. Yamilov, A. Chabanov, A. A. Asatryan, L. C. Botten, and H. Cao. Lasing with coherent feedback in weakly scattering media. *Phys. Rev. A*, 74, 2006.

[139] X. Wu, W. Fang, A. Yamilov, A. Chabanov, and H. Cao. Laser resonators formed by two nanoparticles. *Proc. SPIE*, 2006.

[140] A. Yamilov and B. Payne. Classification of regimes of wave transport in quasi-one-dimensional non-conservative random media. *J. Mod. Opt.*, 57:1916–1921, 2010.

[141] B. Payne, H. Noh, H. Cao, and A. Yamilov. Mapping algorithm between two-dimensional aperiodic thue-morse structure and a square lattice. *Comp. Phys. Comm.*, 2012. submitted.

[142] M. M. Sigalas, C. M. Soukoulis, C. T. Chan, and D. Turner. Localization of electromagnetic waves in two-dimensional disordered systems. *Phys. Rev. B*, 53:8340–8348, 1996.

VITA

Ben Payne was born on the planet Earth. As a Boy Scout he achieved the rank of Eagle Scout. From 2001 to 2008 he served as a crew chief on F-16s in the Air National Guard, earning an Associates degree in Aircraft Maintenance Technology from the Community College of the Air Force. At the University of Wisconsin-Madison he served as President and then Secretary of the UW-Madison Physics club from 2005 to 2007. He earned a Bachelors of Science from UW-Madison, majoring in both "Applied Math, Engineering, and Physics" and also in Physics. During this time he earned his private pilots license and skydiving license. While an undergraduate in Madison, Ben worked as a Network Administrator at the UW-Madison Office of the Registrar from 2002 to 2007.

After he moved to Missouri, Ben was the founding employee at Micom LLC, again working as a Network Administrator. At Missouri University of Science and Technology he earned his Masters of Science in 2009 and Doctorate of Philosophy in May 2012. While there he greatly enjoyed his work as a research assistant to Dr. Alexey Yamilov for five years. He served as a teaching assistant in a unique role, helping to rewrite the undergraduate course content. In addition to publishing the papers presented in this dissertation, Ben presented his research at scientific meetings and participated in conferences discussing challenges in high performance computing. This was possible as a result of his securing competitive funding from the National Science Foundation. Ben earned multiple competition prizes for presentations given at the Missouri S&T Physics department. He won awards from the Council of Graduate Students for "Best Representative" for two consecutive years. While in Rolla he served as the primary officer of the Missouri S&T Skydiving club from 2007 to 2012.