Baroclinic Instabilities

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1 Introduction

Baroclinic instability is a fluid dynamical instability that occurs in stably stratified, rotating fluids. They are used to explain the generation and growth of extratropical cyclones. The main source of a baroclinic instability is caused by vertical shear of the background wind profile. On Earth, this shearing is generated through the differential heating of the Earth from the Sun causing a temperature gradient between the pole and equator. The existence of a meridional temperature gradient indicates a source of available potential energy (APE) released into the flow. The process of converting the APE produced from the meridional temperature gradient into kinetic energy (KE) is the essence of baroclinic instability.

1.1 Basic Mechanism

To make this idea clearer, a qualitative description of this process is given by Pedlosky, 1979. In figure 1 below we have the following situation, a constant potential temperature (θ_*) surface tilts upward in the meridional plane by an angle α. This creates a meridional the meridional temperature gradient. Now consider a fluid parcel starting at position A, be displaced to position B. By considering the change in density when moving from A to B, the restoring force becomes

$$E_* = \frac{g}{\theta_*}\frac{\partial \theta_*}{\partial z_*}\sin\phi\left[d_{z_*} - d_{y_*}\left(\frac{\partial z_*}{\partial y_*}\right)_{\theta_*}\right] \tag{1}$$

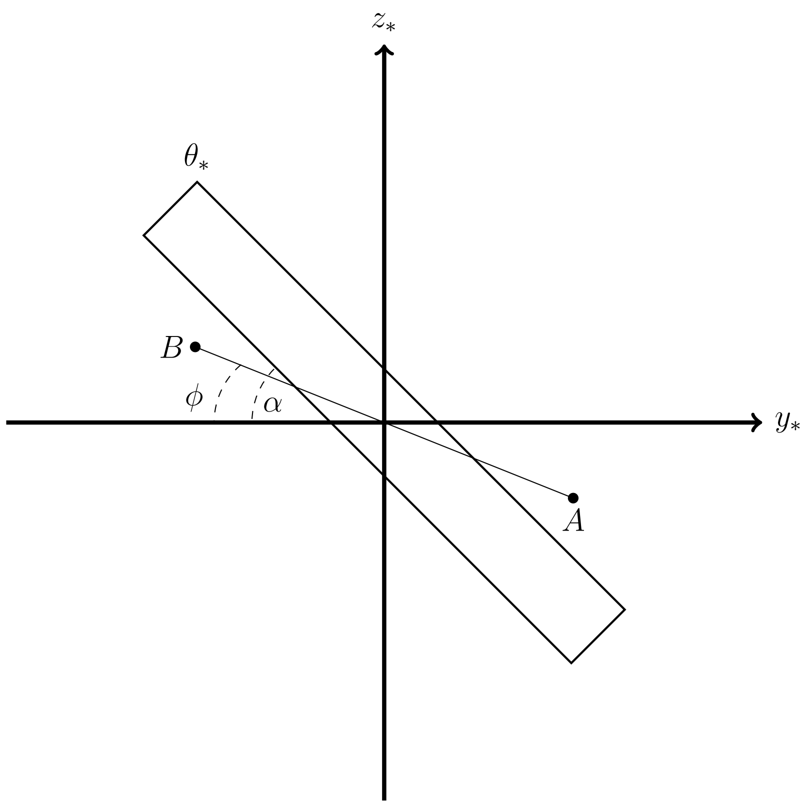

Figure 1: The tilting of the potential temperature creates a region of instability. Fluid parcels on a trajectory within this region will convert the APE into KE, causing them to accelerate away from there initial position.

where d_{y_*} and d_{z_*} are displacements in the y and z planes respectively and $\phi = \tan^{-1}(d_{z_*}/d_{y_*})$ is the angle of displacement. From this we can deduce that any vertical displacement ($d_{y_*} = 0$ and $\sin\phi = 1$) reduces E_* to the Brunt-Väsälä frequency. For positive restoring forces we see that the system will return to an equilibrium state, but for a negative restoring force occurring when the fluid element satisfies

$$0 < \tan\phi < \left(\frac{\partial z_*}{\partial y_*}\right), \tag{2}$$

will cause the buoyancy force to accelerate the fluid parcel further and further away from its initial position. This is the main idea behind a baroclinic instability. For the fluid within this section defined by the angle ϕ, the lower density fluid will rise, and the higher density fluid will sink, in turn releasing potential energy. The idea of varying densities can be directly linked variations in temperature of the fluid, meaning that a baroclinic instability is a form of thermal convection.

2 The Eady Model

The process that causes baroclinic instabilities to grow is the process of converting the APE into KE. One of the earliest and simplest mathematical models that indicate as to which particular cases will lead to a growing instability is the model proposed by Eady, 1949. Before describing the model we have to mention the criteria for an instability, that being the Charney-Stern criteria. The criteria states

$$\frac{\partial \overline{q}}{\partial y} \text{ must change sign somewhere in the } (y, z) \text{ plane,}$$

where $\overline{q}$ is the zonal potential vorticity profile.

2.1 Setup

Following the description given by Hoskins and James, 2013, the model makes the following assumptions

- There is no meridional variation in the Coriolis force i.e. The flow is on the f plane,
- The reference density ρ_R and the stratification N^2 are constant,
- The flow is bounded above and below by rigid surfaces at $z = \pm H/2$.

The flow of the basic state is independent has a uniform vertical shear $\overline{u}(z) = \Lambda z$ and there is a constant meridional buoyancy gradient

$$-\frac{\partial \overline{b}}{\partial y} = f_0 \frac{\partial \overline{u}}{\partial z} = \Lambda f_0, \tag{3}$$

from the thermal wind and f_0 is the Coriolis force. These assumptions for the basic state satisfy the criteria for an instability with the boundary conditions at $z = H/2 \quad \partial\overline{u}/\partial z > 0$ and at $z = -H/2 \quad \partial\overline{u}/\partial z < 0$. The gradient of $\overline{q}$ between $z = \pm H/2$ is 0. A schematic of the model set up is shown in figure 2. Flow in this model is governed by the quasi-geostrophic potential vorticty equation

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right) q' = 0 \tag{4}$$

where q' is a perturbation of the potential vorticity given by,

$$q' = \nabla_H^2 \psi' + \frac{f_0^2}{N^2}\frac{\partial^2 \psi'}{\partial z^2} \tag{5}$$

where ψ' is a perturbation of the stream function and ∇_H is the horizontal Laplacian. At the top and bottom boundaries the thermodynamic energy equation is

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right) T' = 0 \tag{6}$$

where T' is a temperature perturbation.

2.2 Wave solutions

Since the equations governing the flow are linear, we can understand how the growth of the perturbations with waves. So take the following wave-like solutions for our equations to be

$$\psi' = \Re\left[\Psi(z)e^{i(k(x-ct)+ly)}\right], \tag{7}$$

where k, l are the zonal and meridional wave numbers and Ψ, c are the complex amplitude and phase speed. Using the fact that the equation governing the flow of the potential vorticty perturbation has to be zero throughout the domain, substituting the wave solution into our equations above, we get the following equation for the amplitude,

$$\frac{d^2\Psi}{dz^2} - K\Psi = 0, \tag{8}$$

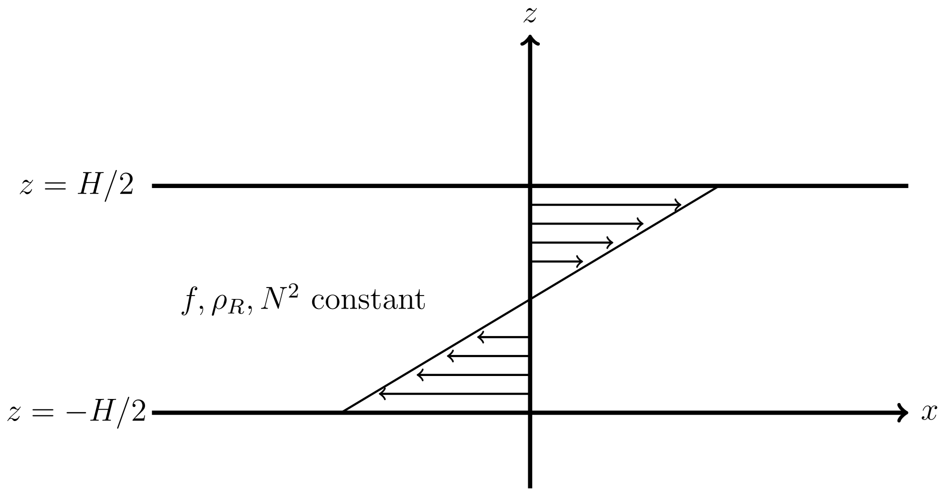

Figure 2: Sketch of the model described by Eady.

where $K = \sqrt{k^2 + l^2}$. Solutions to this equation take the form

$$\Psi(z) = a\cosh(Kz) + b\sinh(Kz) \tag{9}$$

where a and b are arbitrary constants that are determined through the boundary conditions. Through some algebraic manipulation eliminating these constants, we can obtain the following form of the phase speed,

$$c^2 = \frac{1}{K^2}\left(\frac{K}{2} - \coth\left(\frac{K}{2}\right)\right)\left(\frac{K}{2} - \tanh\left(\frac{K}{2}\right)\right). \tag{10}$$

From this we can determine the criteria for the stable and unstable waves based on when the right hand side of the equation for the phase speed is positive or negative. These two cases are:

1. For $K < 2.399$ we have $c^2 < 0$ which gives an imaginary phase speed. This corresponds to pairs of growing and decaying modes.

2. For $K > 2.399$ we have $c^2 > 0$ giving a real phase speed. This gives a pair of stable modes that propagate at phase speeds $\pm c$.

In figure 3 we have examples of unstable and stable perturbations of ψ, buoyancy b and v. Where b and v are obtained from differentiating ψ with respect to z and x respectively. In the stable case we can see from the plot of b that we have no temperature being transported vertically, hence we have no growth. Now in the unstable case we can see transport occuring between the top and bottom boundaries. We also have a relationship between b and v in that they tilt in opposing directions, b is tilting west to east and v east to west. What we see based on these contours that warm air moves poleward and rises and similarly cold air moves equatorward and sinks.

Extensions of the Eady model, such as the Charney model, consider a more realistic atmosphere by taking into account other parameters such as the β-effect for a varying latitude. But interestingly, they yield similar results to what Eady was able to show.

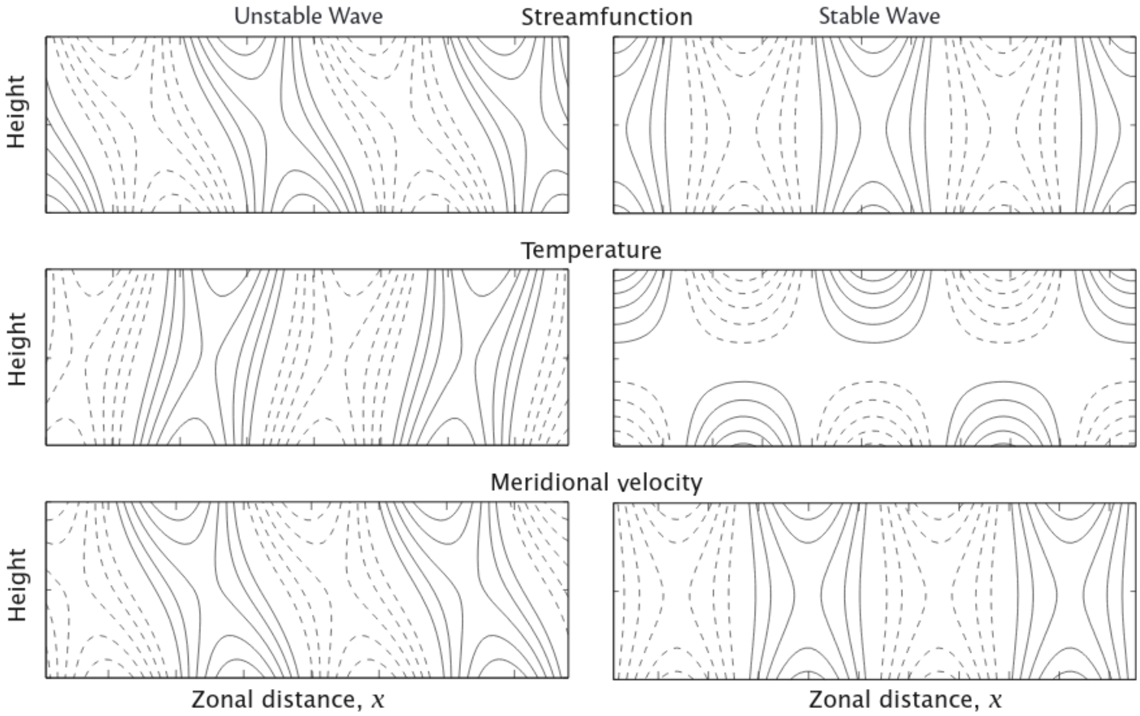

Figure 3: Vertical structure of the unstable and stable Eady waves taken from Vallis, 2019. Dashed lines represent negative values.

3 Cyclogenesis

Baroclinic instabilities can be seen physically in the process of cyclogenesis. Cyclogenesis is the strengthening and growth of a cyclonic circulation leading to the formation of an extratropical cyclone. This process is initiated through a baroclinic instability occurring along a stationary or slow-moving front of cold and warm air. This is linked to what we saw in figure 3, with the warm air moving poleward and cold air moving equatorward, this process intensifies due to the instability.

From the Eady model, considering only the imaginary part of 10 we can obtain the Eady growth rate,

$$\sigma = 0.31\frac{f}{N}\frac{\partial u}{\partial z} \tag{11}$$

which can be used to diagnose regions on Earth of the most unstable waves indicating favourable regions of cyclogenesis. For example, figure 4 shows σ over the North Atlantic and we can see a distinct maximum occurring just off of the east coast of North America. This area is often called the beginning of the storm track in the North Atlantic. An example of an extratropical cyclone that has occurred in this region is shown in figure 5. This cyclone leads to an extreme blizzard that disrupted travel and power outages to cities up and down the east coast.

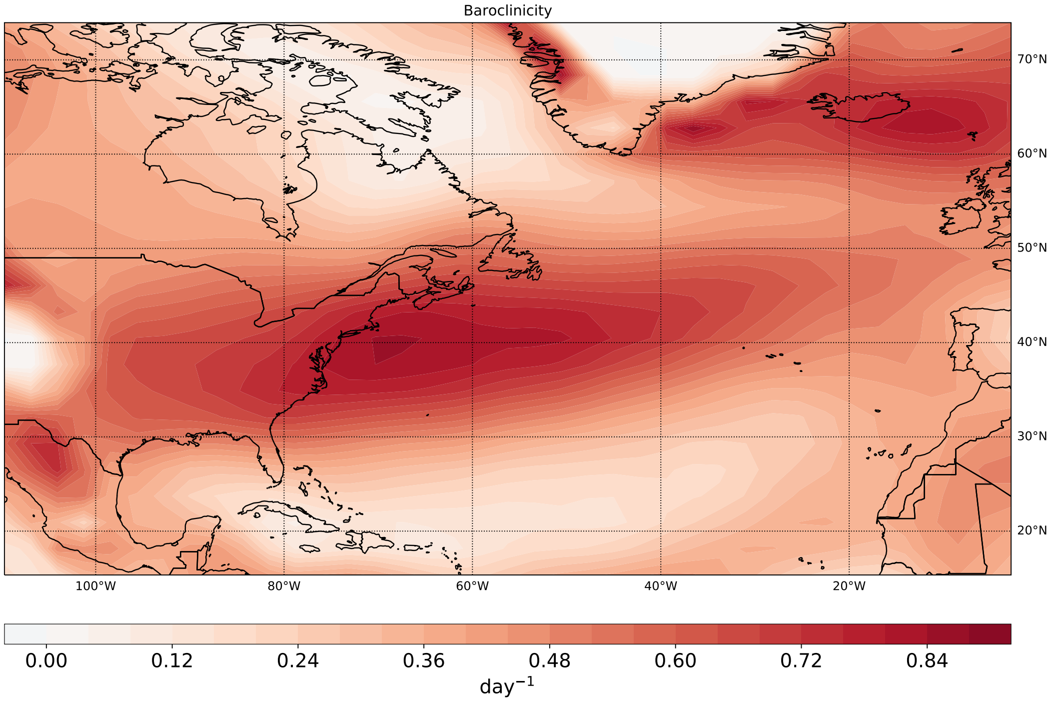

Figure 4: Contour plot of the Eady growth rate over the North Atlantic. Plot produced from data from the IGCM4 climate model by the author as part of his PhD work.

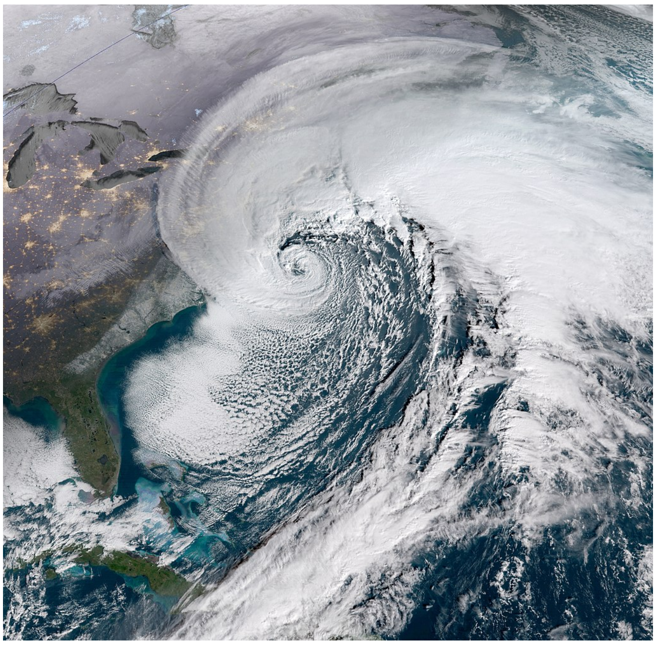

Figure 5: Formation of a cyclone off the coast of North Ameria in January of 2018.

4 Summary

Here we have broken down a baroclinic instability into three parts. The first part is understanding that the tilt of a constant potential temperature surface creates APE due to changing height which (if enough is available) creates an instability where APE is converted into KE and accelerates a fluid parcel away from its initial position. To understand the waves further, we looked at the Eady model which is one of the simplest models in which a baroclinic instability occurs due to opposing signs in vertical shear at the top and bottom boundaries. We found that wave-like solutions to the Eady model display tilting as we saw in the example from Pedlosky. This tilting that occurs in the case of an unstable wave, represents a poleward heat flux. Finally, we looked at the process of cyclone intensification cyclogenesis. The presence of a baroclinic instability triggers the intensification of a cyclone. We then looked at the Eady growth rate over the North Atlantic, which indicates regions of unstable waves and hence favourable regions for cyclogenesis to occur. We saw that this region occurred on the east coast of North America in the North Atlantic.

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