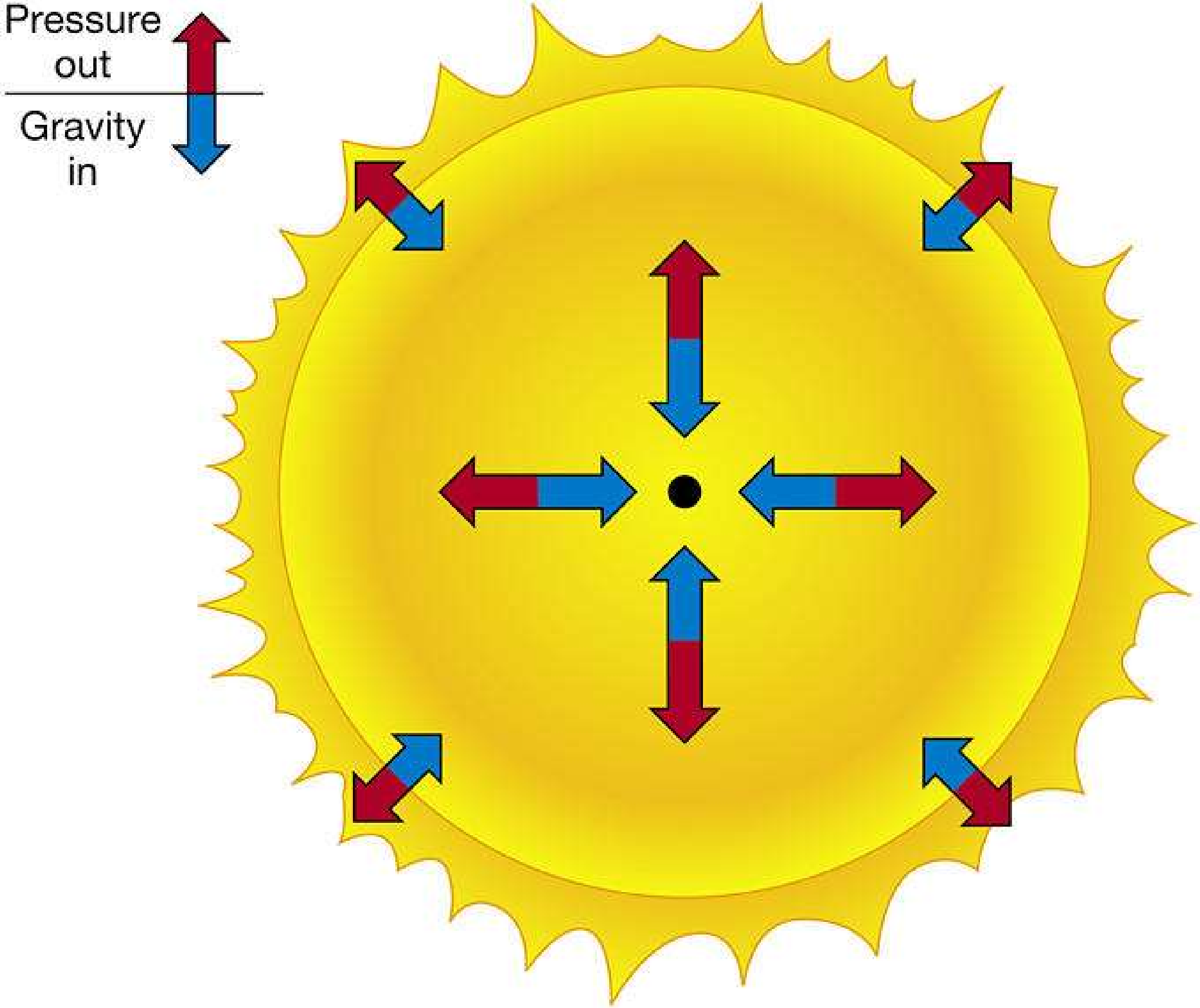
2. The Evolution of Stars (별의 진화)

2-1 Energy Generation (에너지 생성)



Hydrostatic equilibrium (정 유체역학적 평형)

https://pages.uoregon.edu/jimbrau/BraulmNew/Chap16/FG16_06.jpg

김상철
(Sang Chul KIM)

Main Sequence (MS, 주계열성)

- Nuclear fusion (proton-proton chain, 양성자-양성자 연쇄반응) → energy generation and radiation
- Mass decreases slowly → almost constant luminosity (equilibrium between pressure and gravity)
- core : N(H) decreases, N(He) increases

- $M > 1.5\ M_\odot$, $T_{center} > 1.5 \times 10^7$ K → H burning by CNO cycle (탄소 순환반응)

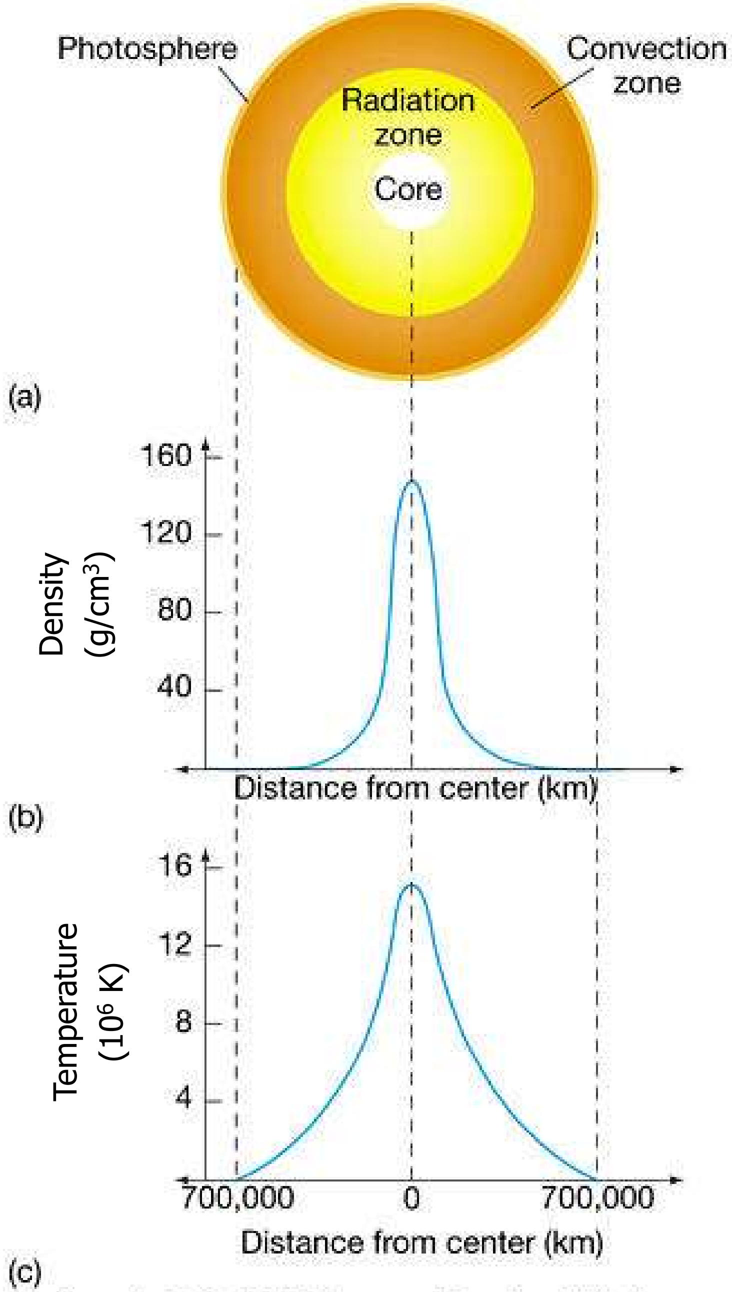
Hydrostatic equilibrium (정 유체역학적 평형)

- If somehow core H-burning accelerates --> Core T ↑, P ↑ → star expands

- Expansion makes T ↓ → H-burning decelerates

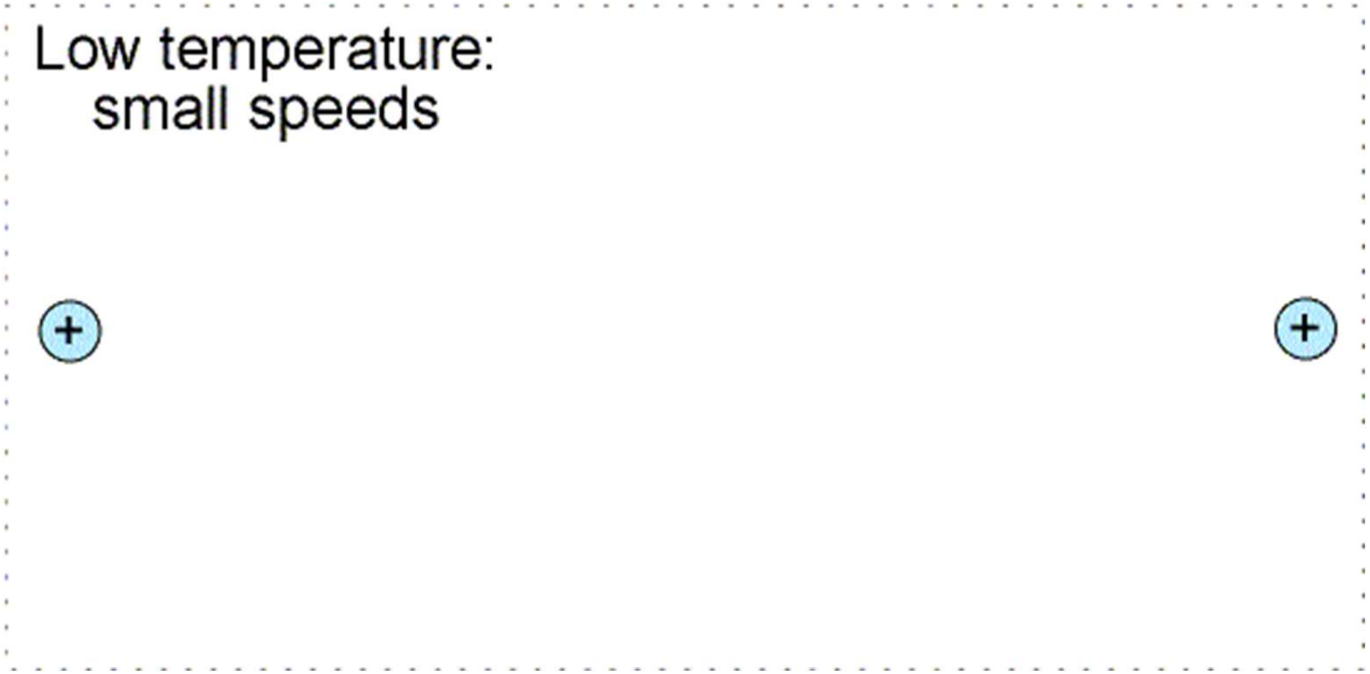
- Hydrostatic equilibrium makes H-burning speed keep ~constant

Interior structure of the Sun (태양의 내부 구조)



http://pages.uoregon.edu/jimbrau/BrauImNew/Chap16/FG16_08.jpg

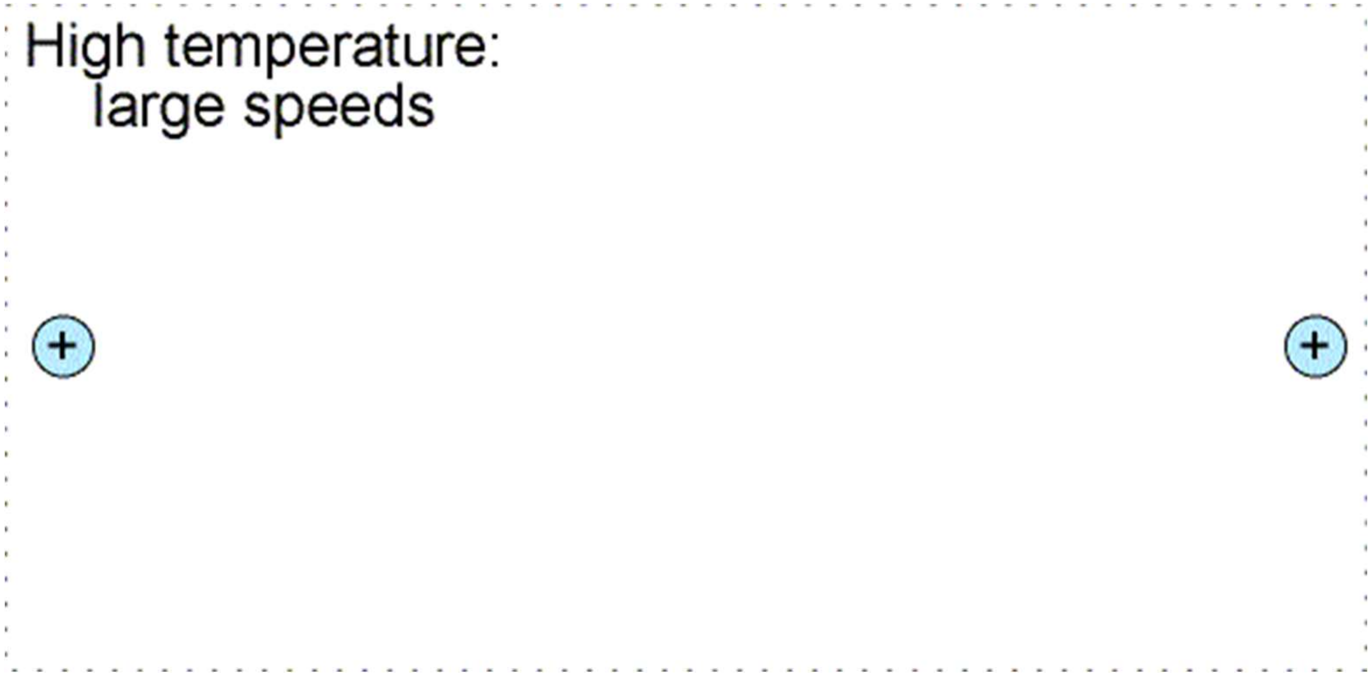
Why H-burning occurs at the core - 1



Low Temp, small speeds

→ Electrical repulsion > proton-proton fusion

Why H-burning occurs at the core - 2



High Temp, large speeds

→ Strong nuclear force (강한 핵력)

→ Nuclei fuse, energy released

http://www.astronomynotes.com/starsun/s3.htm

symbol

$$^{A}_{Z}X$$

X : the chemical symbol of the element (e.g. : H, He, ...)
Z : the number of protons (the total positive charge, in units of e)
A : the mass number (the total number of nucleons, protons plus neutrons)

- A = Z+N
  (mass number = Z protons + N neutrons)

(e.g.)

$$^{1}_{1}H \qquad ^{4}_{2}He$$

Energy from proton-proton chain

H atomic weight : 1.0078

4 H → 4.0312

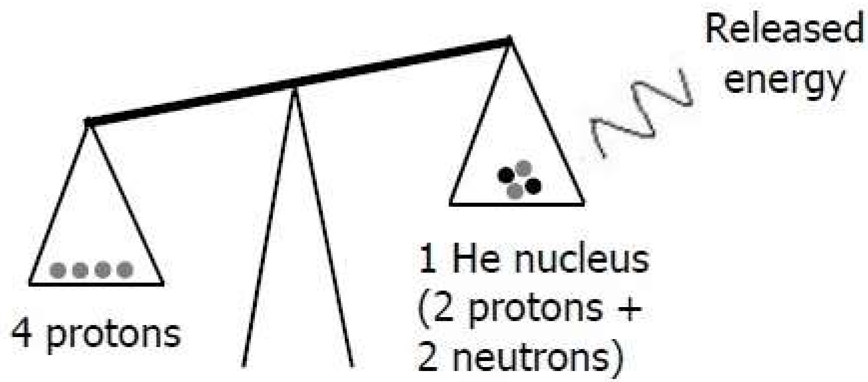
1 He → 4.0026

$\triangle m$ = 0.0286 (mass deficit/defect, 질량 결손)

E = mc² (Einstein's equation for the equivalence of mass and energy)

$$\begin{aligned}\triangle m &= 4\times m(p) - m(He)\\ &= (0.0286\times 1.67\times 10^{-24}\text{g})\times(3\times 10^{10}\ \text{cm/s})^2\\ &= 4.3\times 10^{-5}\ \text{erg}\\ &= 4.3\times 10^{-12}\ \text{J}\end{aligned}$$

$$\begin{aligned}1\ \text{Joule} &= 1\ \text{kg}\cdot\text{m}^2/\text{s}^2\\ &= 1\ \text{N}\cdot\text{m}\\ &= \mathbf{10^7\ erg}\\ &= 6.24\times 10^{18}\ \text{eV}\\ &= 0.2390\ \text{cal}\\ &= 2.78\times 10^{-7}\ \text{kW}\cdot\text{h}\end{aligned}$$



Energy from proton-proton chain

- Energy from coal 1 kg ~ 5000 kcal = 2.1×10^7 J

- Use H 1 kg to produce He

→ 0.993 kg He produces, 0.007 kg changes into E

→ $E = mc^2 = 7\text{ g} \times (3 \times 10^{10}\text{ cm/s})^2 = 7 \times 9 \times 10^{20}\text{ g cm}^2/\text{s}^2$

$= 6.3 \times 10^{21}\text{ erg} = 6.3 \times 10^{14}\text{ J}$

→ same as E from 3×10^7 kg = 30,000 tons of coal

Sun : 4×10^{38} protons/sec = 6.68×10^{14} g/sec are consumed.

2×10^{38} neutrinos/sec are generated.

Sun : $1\ L_\odot = 3.9 \times 10^{33}$ erg/s = 3.9×10^{26} J/s

Energy from proton-proton chain

$1\ L_\odot = 3.9 \times 10^{33}$ erg/s $= 3.9 \times 10^{26}$ J/s

= amount of electricity for all human beings can use for 10^7 years

Assuming :

Mankind populations = $5 \times 10^9 \rightarrow 10^9$ families (assume 1 family = 5 members)

Monthly family electricity uses, home use : 200~250 kW·h

Let's assume ~1000 kW·h/month including commercial use, etc

$= 10^6\ \text{W} \times 3600$ s/month $= 10^6\ \text{J/s} \times 3600$ s/month $= 3.6 \times 10^9$ J/month

1 W = 1 J/s

Monthly mankind electricity uses $\sim 3.6 \times 10^9\ \text{J} \times 10^9$ families $= 3.6 \times 10^{18}$ J

$$\rightarrow \quad \frac{3.9 \times 10^{26}\,\text{J}}{3.6 \times 10^{18}\,\text{J}} = 1.1 \times 10^8 \text{ months} = 9.2 \times 10^6 \text{ years} \sim 10^7 \text{ years}$$

Energy from proton-proton chain

Atomic weight (원자량, 原子量) or relative atomic mass (상대 원자 질량) :

H $\rightarrow$ 1

$^{12}C \rightarrow 12$ ($1.998467052 \times 10^{-23}$ g) (since 1962),

then H $\rightarrow$ 1.0078

He $\rightarrow$ 4.0026

α-ray, α-particle : He nucleus

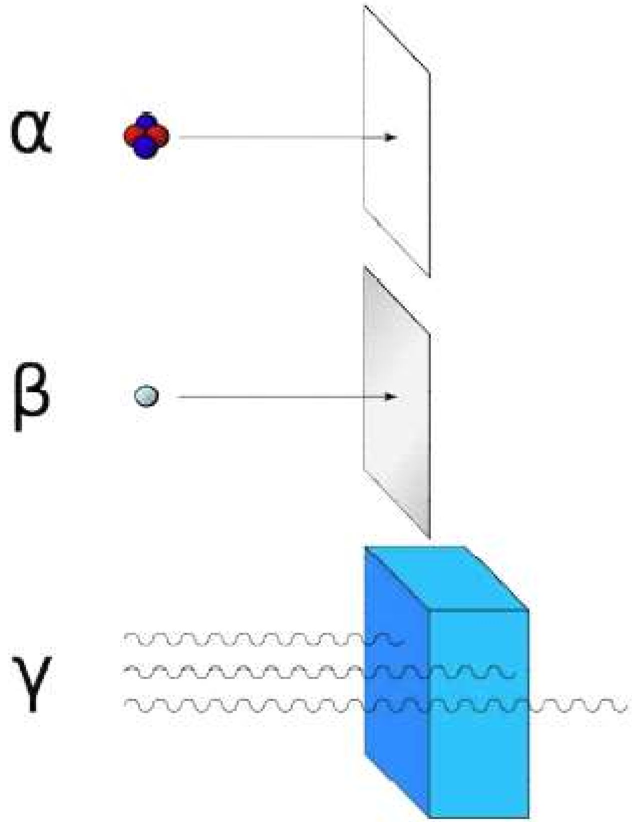
→ cannot penetrate a paper

β-ray : electron

→ cannot penetrate aluminum

γ-ray : high-energy photon

→ absorbed by dense materials (water, lead)



https://ko.wikipedia.org/wiki/알파_입자

β(beta)-decay

β^- decay : $n \rightarrow p + \beta^- + \overline{\nu_e}$

β^+ decay : $p \rightarrow n + \beta^+ + \nu_e$

※ $\beta^- = e^-$ (electron)

※ $\beta^+ = e^+$ (positron)

inverse-β decay : $p^+ + e^- \rightarrow n + \nu$

(IBD)

Periodic Table of the Elements

Atomic Number
Symbol
Name
Atomic Mass

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H Hydrogen 1.01																	2 He Helium 4.00
3 Li Lithium 6.94	4 Be Beryllium 9.01											5 B Boron 10.81	6 C Carbon 12.01	7 N Nitrogen 14.01	8 O Oxygen 16.00	9 F Fluorine 19.00	10 Ne Neon 20.18
11 Na Sodium 22.99	12 Mg Magnesium 24.31											13 Al Aluminum 26.98	14 Si Silicon 28.09	15 P Phosphorus 30.97	16 S Sulfur 32.06	17 Cl Chlorine 35.45	18 Ar Argon 39.95
19 K Potassium 39.10	20 Ca Calcium 40.08	21 Sc Scandium 44.96	22 Ti Titanium 47.88	23 V Vanadium 50.94	24 Cr Chromium 51.99	25 Mn Manganese 54.94	26 Fe Iron 55.85	27 Co Cobalt 58.93	28 Ni Nickel 58.69	29 Cu Copper 63.55	30 Zn Zinc 65.38	31 Ga Gallium 69.72	32 Ge Germanium 72.63	33 As Arsenic 74.92	34 Se Selenium 78.97	35 Br Bromine 79.90	36 Kr Krypton 84.80
37 Rb Rubidium 85.47	38 Sr Strontium 87.62	39 Y Yttrium 88.91	40 Zr Zirconium 91.22	41 Nb Niobium 92.91	42 Mo Molybdenum 95.95	43 Tc Technetium 98.91	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.91	46 Pd Palladium 106.42	47 Ag Silver 107.87	48 Cd Cadmium 112.41	49 In Indium 114.82	50 Sn Tin 118.71	51 Sb Antimony 121.76	52 Te Tellurium 127.6	53 I Iodine 126.90	54 Xe Xenon 131.29
55 Cs Cesium 132.91	56 Ba Barium 137.33	57-71 Lanthanides	72 Hf Hafnium 178.49	73 Ta Tantalum 180.95	74 W Tungsten 183.85	75 Re Rhenium 186.21	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.97	80 Hg Mercury 200.59	81 Tl Thallium 204.38	82 Pb Lead 207.20	83 Bi Bismuth 208.98	84 Po Polonium [208.98]	85 At Astatine 209.98	86 Rn Radon 222.02
87 Fr Francium 223.02	88 Ra Radium 226.03	89-103 Actinides	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [278]	110 Ds Darmstadtium [281]	111 Rg Roentgenium [280]	112 Cn Copernicium [285]	113 Nh Nihonium [286]	114 Fl Flerovium [289]	115 Mc Moscovium [289]	116 Lv Livermorium [293]	117 Ts Tennessine [294]	118 Og Oganesson [294]

57 La Lanthanum 138.91	58 Ce Cerium 140.12	59 Pr Praseodymium 140.91	60 Nd Neodymium 144.24	61 Pm Promethium 144.91	62 Sm Samarium 150.36	63 Eu Europium 151.96	64 Gd Gadolinium 157.25	65 Tb Terbium 158.93	66 Dy Dysprosium 162.50	67 Ho Holmium 164.93	68 Er Erbium 167.26	69 Tm Thulium 168.93	70 Yb Ytterbium 173.06	71 Lu Lutetium 174.97
89 Ac Actinium 227.03	90 Th Thorium 232.04	91 Pa Protactinium 231.04	92 U Uranium 238.03	93 Np Neptunium 237.05	94 Pu Plutonium 244.06	95 Am Americium 243.06	96 Cm Curium 247.07	97 Bk Berkelium 247.07	98 Cf Californium 251.08	99 Es Einsteinium [254]	100 Fm Fermium 257.10	101 Md Mendelevium 258.10	102 No Nobelium 259.10	103 Lr Lawrencium [262]

Alkali Metal | Alkaline Earth | Transition Metal | Basic Metal | Metalloid | Nonmetal | Halogen | Noble Gas | Lanthanide | Actinide

http://sciencenotes.org/printable-periodic-table/

Energy generation rate

- H atomic weight : 1.0078

4 H → 4.0312

1 He → 4.0026

△m = 0.0286

- 0.0286/4.0312 = 0.0071 → 0.71%

- Only H in the core → T, P high enough to permit nuclear reactions → ~10 % of the mass of the Sun is available for energy conversion
- Thermonuclear energy available in the Sun :

$$E_{Total} = mc^2 = (\frac{4\,{}^1H - {}^4He}{4\,{}^1H} \times 0.1 M_\odot) \times (c^2)$$

$$= 0.0071 \times (1.99 \times 10^{32}\ \mathrm{g}) \times (3 \times 10^{10}\ \mathrm{cm/s})^2$$
$$\approx 10^{51}\ \mathrm{g \cdot cm^2/s^2}$$
$$\approx 10^{51}\ \mathrm{erg}$$
$$\approx 10^{44}\ \mathrm{J}$$

1 Joule = 1 kg·m^2/s^2
= 1 N·m
= 10^7 erg
= 6.24×10^{18} eV
= 0.2390 cal
= 2.78×10^{-7} kW·h

$L_\odot = 3.90 \times 10^{33}$ erg/s = 3.90×10^{26} J/s

→ $E/L_\odot$ = Solar lifetime = 8.11×10^9 yr ~10 Gyr

Energy generation rate

※ AGN → efficiency of conversion from mass to energy ~10% (η=0.1)
(see, e.g., Fabian & Rees 1995, MNRAS, 277, L55,
http://phys.huji.ac.il/~joaw/winterschool/heckman_1.pdf)

※ Gravitational wave (e.g, by merging of two identical black holes) → conversion efficiency ~30% (η~0.3)
(『Black Hole Physics』 1991, Edited by Venzo de Sabbata, Zhenjiu Zhang, NATO ASI Series, Vol. 364, p. 139)

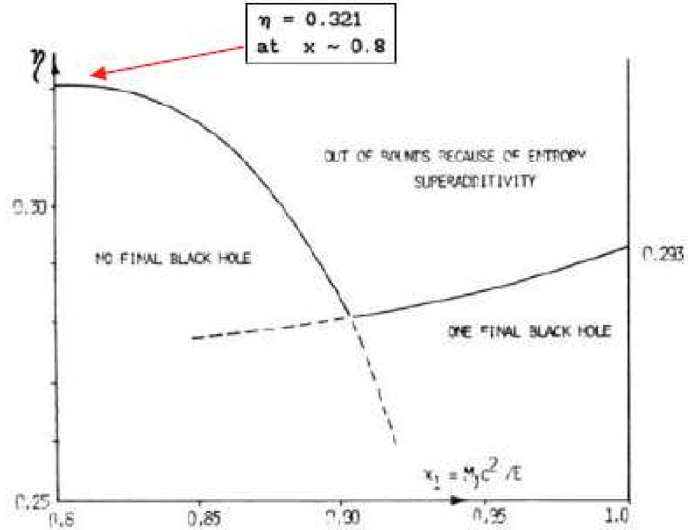
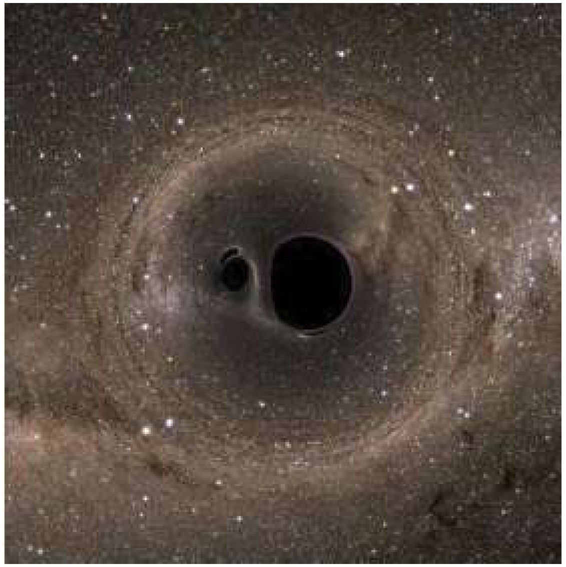
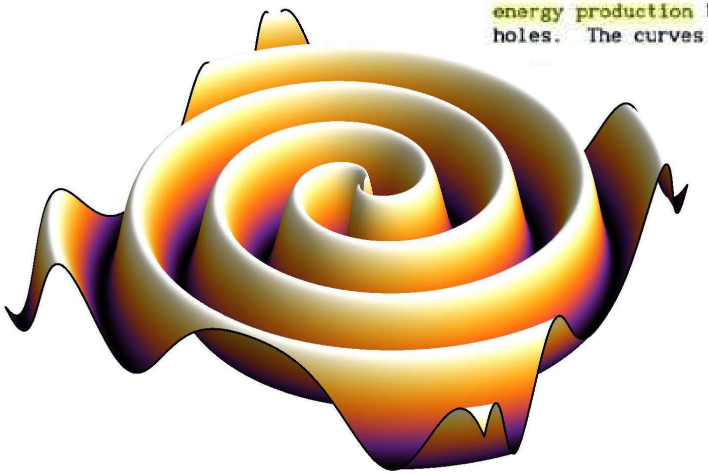


Figure 8. The maximum possible efficiency of gravitational wave energy production by the merging of two identical Schwarzschild black holes. The curves intersect at $x_1 \sim 0.903$, $\eta \sim 0.281$.



http://news.nationalgeographic.com/2016/02/160211-gravitational-waves-found-spacetime-science/



http://www.universetoday.com/127329/gravitational-wave-sources/

Energy generation rate

MS Lifetime : $\tau = 10^{10}(M/L)$ yr

(M=stellar mass, L=stellar luminosity)

※ Mass-Luminosity Relation :

$L/L_{\odot} = (M/M_{\odot})^4$ for $M > 0.4M_{\odot}$

$L/L_{\odot} = (M/M_{\odot})^{2.3}$ for $M < 0.4M_{\odot}$

→ $\tau = 10^{10}(M/M^4)$ yr $= 10^{10} / [M(M_{\odot})]^3$ yr

$1\ M_{\odot}$ → 10^{10} yr

$10\ M_{\odot}$ → 10^7 yr

O stars (the most massive stars) → a million years

"we are seeing some O stars!" = SF is still occurring in the MW

Main-sequence Lifetimes

http://www.ifa.Hawaii.edu/~szapudi/astro110/2007/ch21.pdf

Mass ($M_{\odot}$)	Effective Temperature (K)	Spectral Class	Luminosity ($L_{\odot}$)	MS Lifetime (10^6 yr)
25	35,000	O	80,000	3
15	30,000	B	10,000	15
3	11,000	A	60	500
1.5	7000	F	5	3000
1.0	6000	G	1	10,000
0.75	5000	K	0.5	15,000
0.50	4000	M	0.03	200,000

http://ircamera.as.arizona.edu/astr_250/Lectures/Lecture_20.htm

Proton-proton (PP) fusion chain process

1st step : $^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e$ (→1.44 MeV)

$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e$ ← weak nuclear force

(It takes 5×10^9 years for a given proton to undergo this reaction.)

2nd step : $^2H + {}^1H \rightarrow {}^3He + \gamma$ (→5.49 MeV)

$^2H + {}^1H \rightarrow {}^3He + \gamma$ (occurs in ~1 s)

3rd step : $^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$ (→12.9 MeV)

← strong nuclear force

(It takes $\sim 3 \times 10^5$ years.)

^1H = p : proton
^2H : deuterium (중수소) = p + n
e^+ : positron (e^-의 반물질 antimatter, same mass, positive charge)
ν: neutrino (energy, spin, very small mass)
ν_e : electron neutrino
^3He : helium-3 nucleus = 2p + 1n
γ : photon
^4He : helium-4 nucleus = 2p + 2n

→ 91% of the time in the Sun

Proton-proton (PP) fusion chain process

$$^{1}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} \longrightarrow ^{2}_{1}\mathrm{H} + e^{+} + \nu_e$$

$$^{2}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} \longrightarrow ^{3}_{2}\mathrm{He} + \gamma$$

69%

$$^{3}_{2}\mathrm{He} + ^{3}_{2}\mathrm{He} \longrightarrow ^{4}_{2}\mathrm{He} + 2^{1}_{1}\mathrm{H}$$

(PP I)

31%

$$^{3}_{2}\mathrm{He} + ^{4}_{2}\mathrm{He} \longrightarrow ^{7}_{4}\mathrm{Be} + \gamma$$

99.7%

$$^{7}_{4}\mathrm{Be} + e^{-} \longrightarrow ^{7}_{3}\mathrm{Li} + \nu_e$$

$$^{7}_{3}\mathrm{Li} + ^{1}_{1}\mathrm{H} \longrightarrow 2^{4}_{2}\mathrm{He}$$

(PP II)

0.3%

$$^{7}_{4}\mathrm{Be} + ^{1}_{1}\mathrm{H} \longrightarrow ^{8}_{5}\mathrm{B} + \gamma$$

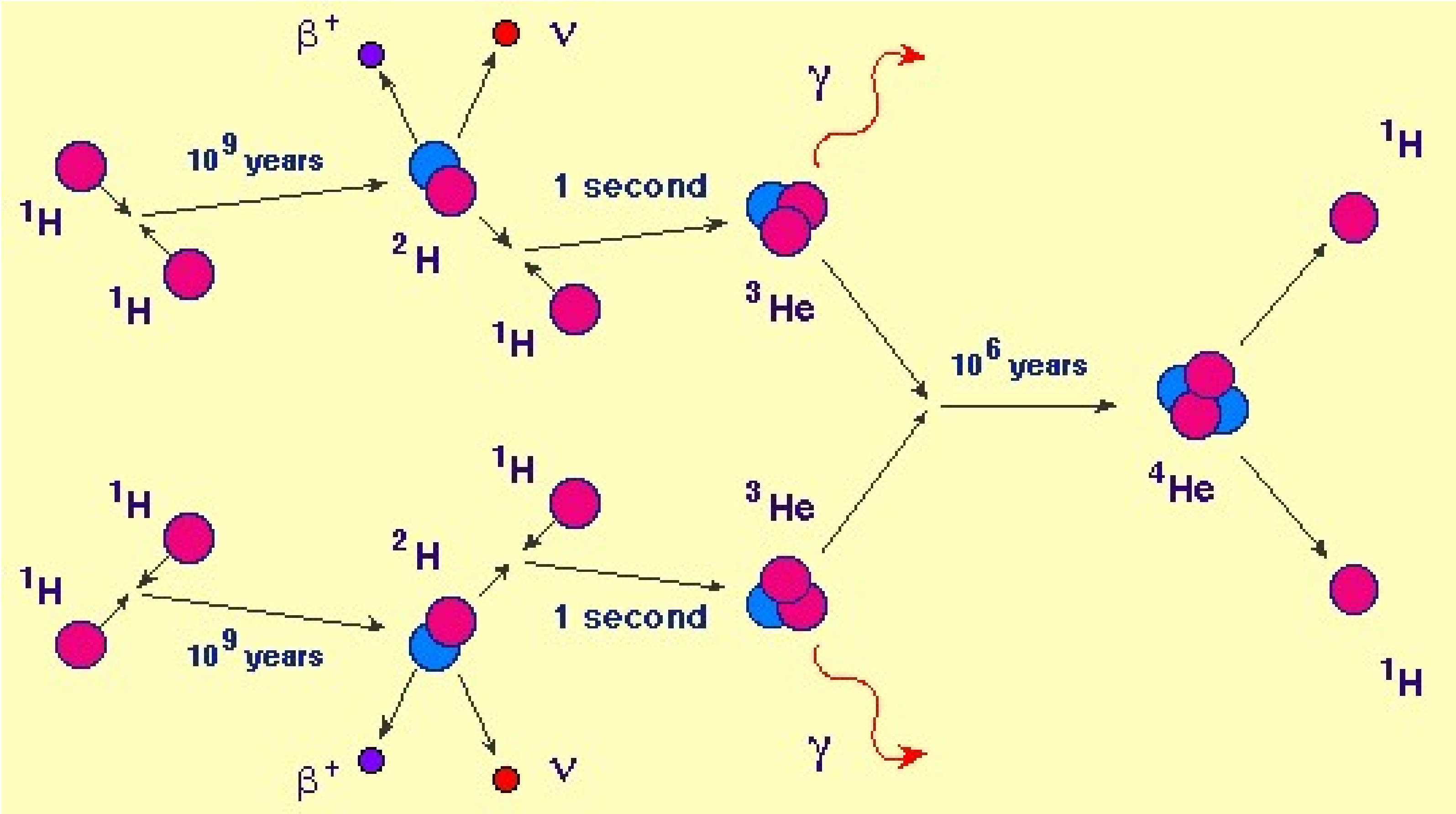
$$^{8}_{5}\mathrm{B} \longrightarrow ^{8}_{4}\mathrm{Be} + e^{+} + \nu_e$$

$$^{8}_{4}\mathrm{Be} \longrightarrow 2^{4}_{2}\mathrm{He}$$

(PP III)

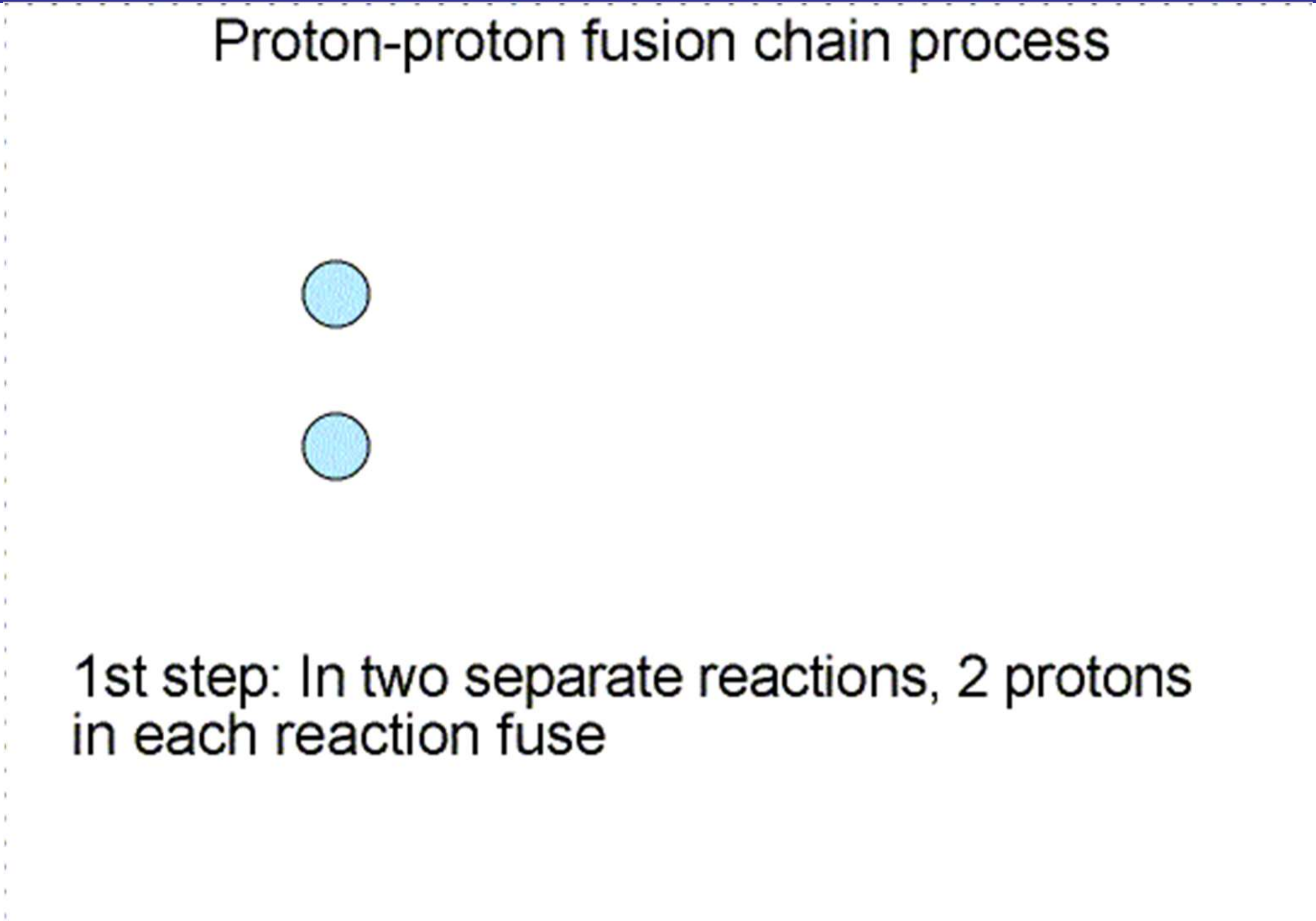
Why this complicated process?

- Direct collision of 4 H atoms to produce He → probability is very low
- Relatively, 2-body collision is more often !



http://atropos.as.arizona.edu/aiz/teaching/a250/pp.html

proton-proton (PP) fusion chain process



- 1^{st} step : p + p --> β^{+} + ν + ^{2}H
- 2^{nd} step : ^{2}H + p --> ^{3}He + γ
- 3^{rd} step : ^{3}He + ^{3}He --> ^{4}He + p + p

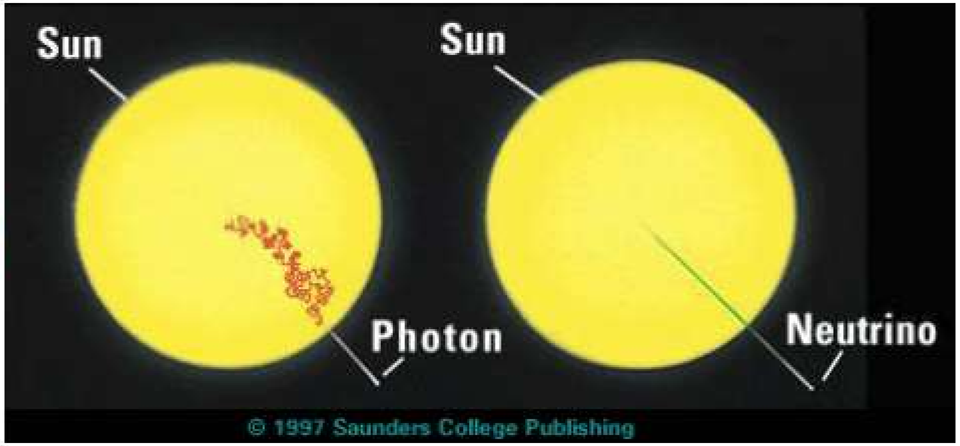
http://www.astronomynotes.com/starsun/s3.htm

Photon, neutrino – escape from the Sun

Photon and neutrino – travel times out of the center of the Sun

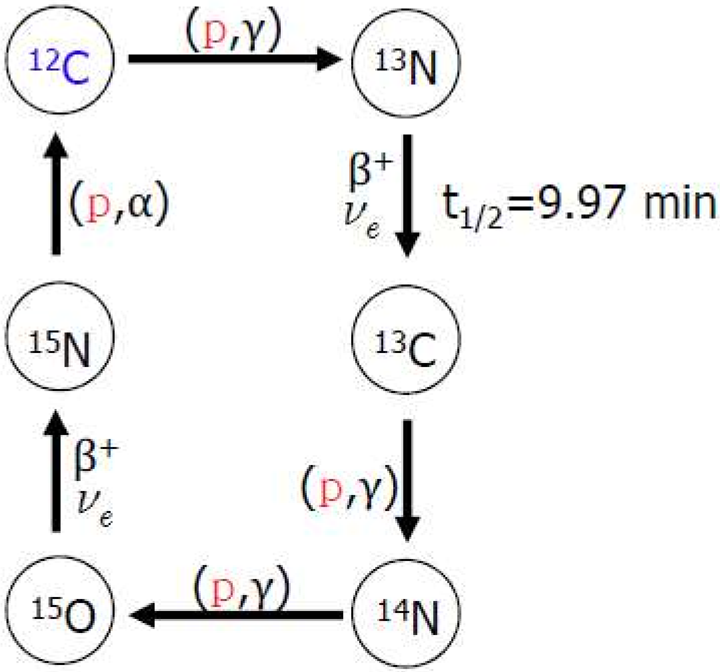
10^4 -10^5 yr   2 sec

"Random walk"



CNO Cycle

- Carbon = catalyst : a substance that causes a chemical reaction to happen more quickly, while the substance not consumed and remain unchanged
- 촉매(觸媒) : 반응과정에서 소모되거나 변화되지 않으면서 반응속도를 빠르거나 느리게 변화시키는 물질



β+ decay : $p \rightarrow n + \beta^+ + \nu_e$
※ $\beta^+ = e^+$ (positron)

- Start : C + p
- At high-temp stars : $T > 1.6 \times 10^7$ K, $M > 1.1\ M_\odot$

CNO cycle process

$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$

$^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$ ^{13}N : unstable isotope

$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$

$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$

$^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e$ ^{15}O : unstable isotope

$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$

CNO cycle process

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma \qquad (1.95\ \text{MeV})$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e \qquad (1.20\ \text{MeV},\ {}^{13}N\ \text{half-life}=9.965^{m})$$

$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma \qquad (7.54\ \text{MeV})$$

$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma \qquad (7.35\ \text{MeV})$$

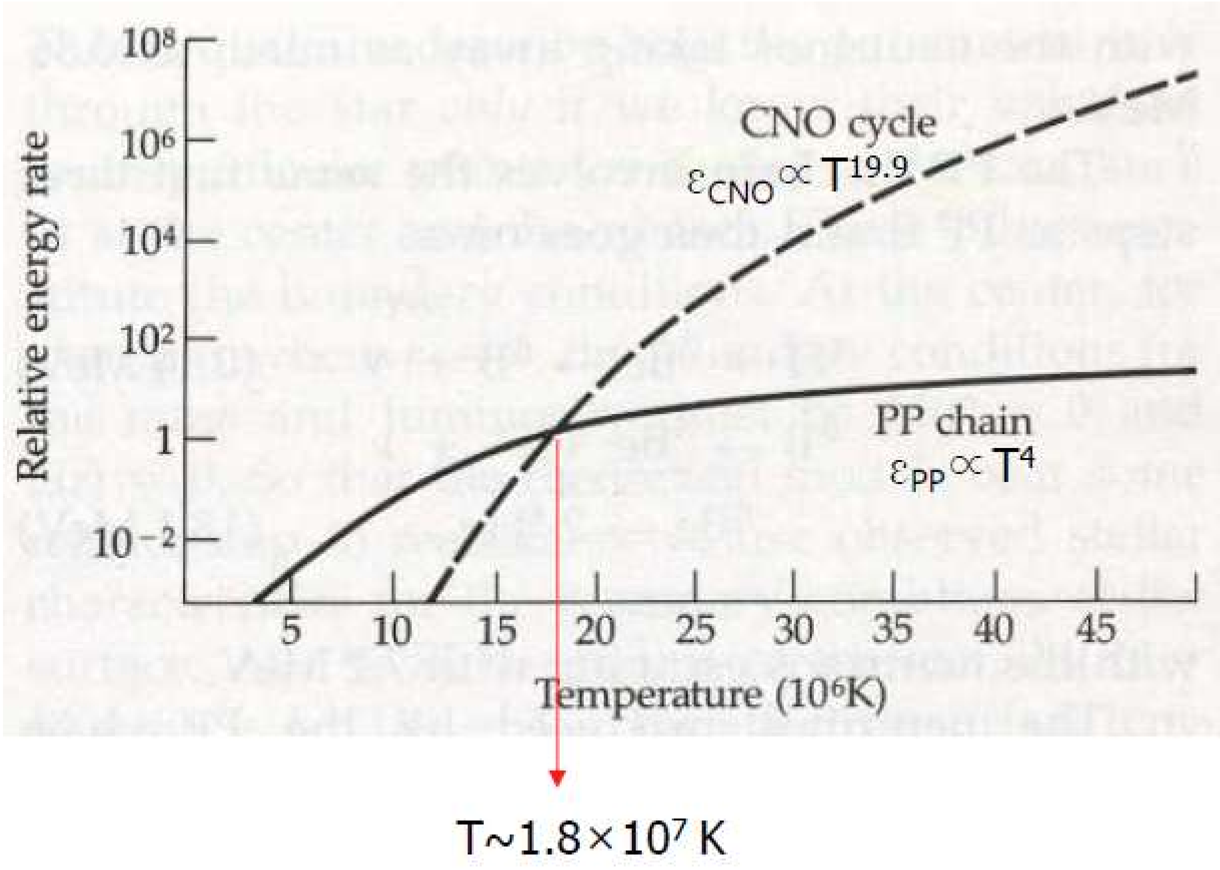
$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e \qquad (1.73\ \text{MeV},\ {}^{15}O\ \text{half-life}=122.24^{s})$$

$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He \qquad (4.96\ \text{MeV})$$

※ 2 positrons annihilate with 2 ambient electrons → produce 2.04 MeV

Total 24.73 + 2.04 = 26.73 MeV

Energy-generation rates



Introductory Astronomy and Astrophysics (4th edition)
Michael Zeilik & Stephen A. Gregory (1998), Fig 16-2

Triple-alpha (3α) process

Nucleosynthesis using He :

$T \geq 10^8$ K, $\rho \geq 10^5$ g cm^{-3} : He → heavier elements

$$^4He + {}^4He \Leftrightarrow {}^8Be + \gamma$$

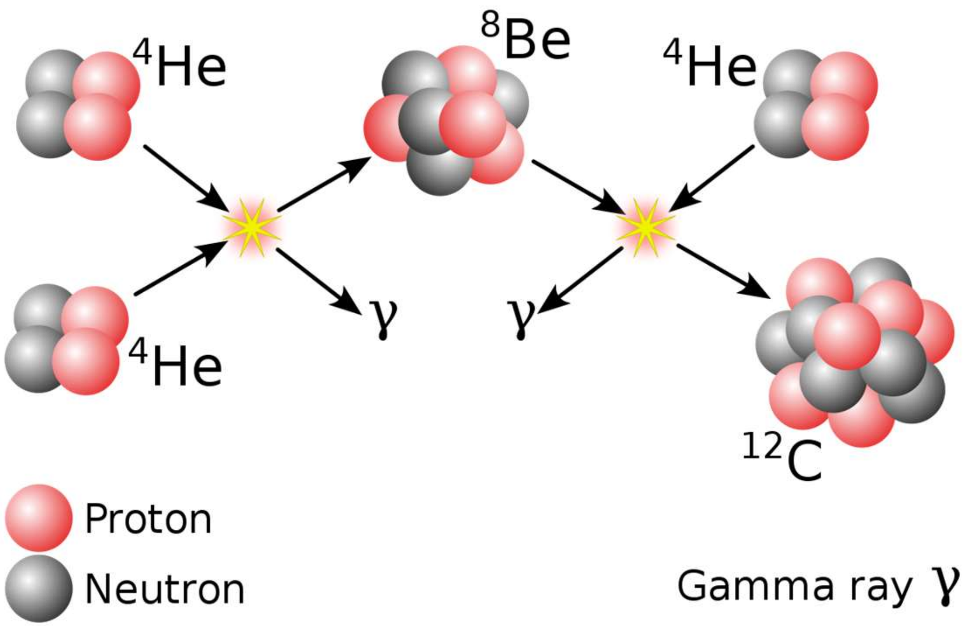
※ ^{8}Be lifetime ~ 2.6×10^{-16} s

> mean collision (scattering) time of α-particle ($T_{scattering}$)

Huge amount of He, n(^{8}Be) : n(^{4}He) = 1 : 10^9

$$^8Be + {}^4He \rightarrow {}^{12}C + \gamma$$

"Triple- α process"



https://en.wikipedia.org/wiki/Triple-alpha_process

C, O-burning

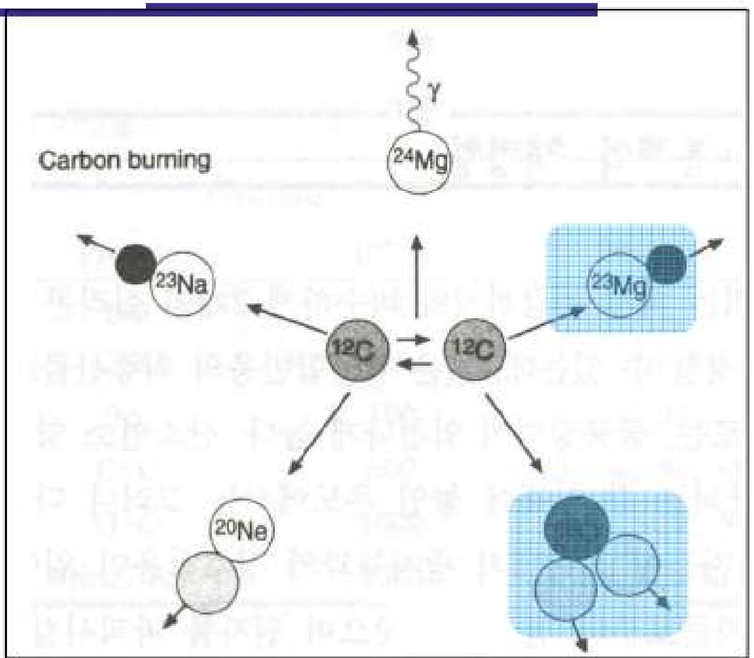
$T \geq 5\times10^8$ K → C-burning

$$^{12}C + ^{12}C \rightarrow ^{24}Mg + \gamma \quad (13.933 \text{ MeV})$$
$$\rightarrow ^{23}Mg + n \quad (-2.599 \text{ MeV}) \text{ (endothermic)}$$
$$\rightarrow ^{23}Na + p \quad (2.241 \text{ MeV})$$
$$\rightarrow ^{20}Ne + \alpha \quad (4.617 \text{ MeV})$$
$$\rightarrow ^{16}O + 2\alpha \quad (-0.113 \text{ MeV}) \text{ (endothermic)}$$

⟨E⟩

평균 방출에너지 ≈ 13 MeV

~5.2 x 10^{13} J/kg = 5.2 x 10^{17} erg/g

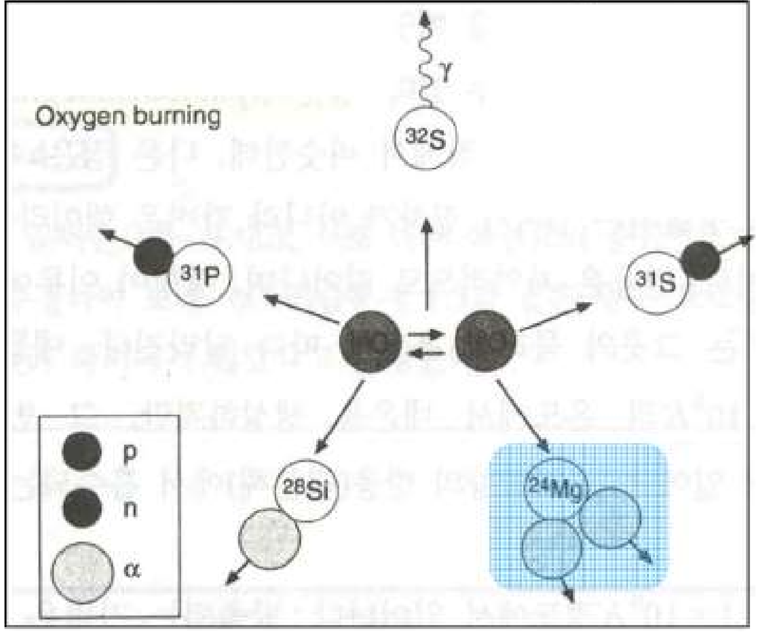


$T \geq 1\times10^9$ K → O-burning

$$^{16}O + ^{16}O \rightarrow ^{28}Si + \alpha \quad (9.954 \text{ MeV}) \ (\sim60\%)$$
$$\rightarrow ^{31}P + p \quad (7.678 \text{ MeV}) \ (\sim40\%)$$
$$\rightarrow ^{31}S + n \quad (1.500 \text{ MeV})$$
$$\rightarrow ^{32}S + \gamma$$
$$\rightarrow ^{24}Mg + 2\alpha \quad \text{(endothermic)}$$

평균 방출에너지 ≈ 16 MeV

~4.8 x 10^{13} J/kg = 4.8 x 10^{17} erg/g



항성내부구조 및 진화, Dina Prialnik 지음, 김용기 옮김, 2007 (An Introduction to the Theory of Stellar Structure and Evolution, 2000), p 85

Si-burning

- After O-burning, core = Si + S
- T ~ (2.7-3.5)$\times 10^9$ K (depends on mass)
- Duration ~ 2 weeks
- No further fusion is possible.

$$^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{32}_{16}\mathrm{S}$$

$$^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{36}_{18}\mathrm{Ar}$$

$$^{36}_{18}\mathrm{Ar} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{40}_{20}\mathrm{Ca}$$

$$^{40}_{20}\mathrm{Ca} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{44}_{22}\mathrm{Ti}$$

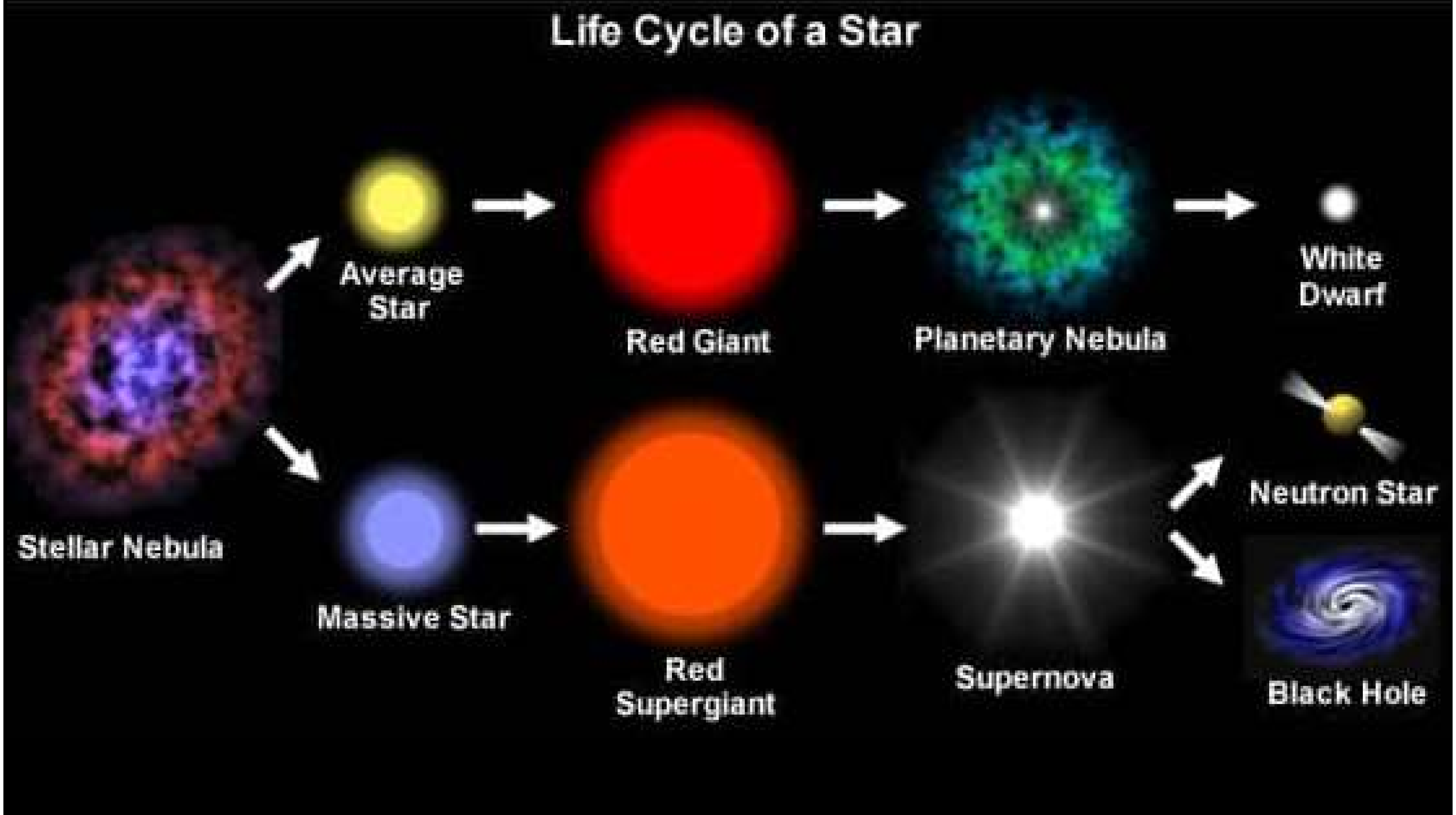
$$^{44}_{22}\mathrm{Ti} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{48}_{24}\mathrm{Cr}$$

$$^{48}_{24}\mathrm{Cr} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{52}_{26}\mathrm{Fe}$$

$$^{52}_{26}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{56}_{28}\mathrm{Ni}$$

https://en.wikipedia.org/wiki/Silicon-burning_process

2. The Evolution of Stars (별의 진화)
2-2 Stellar Evolution (항성 진화)



http://schoolworkhelper.net/types-of-stars/

Sun + Star

Solar surface composition :
X=0.73, Y=0.25, Z=0.02

Star = atmosphere + interior

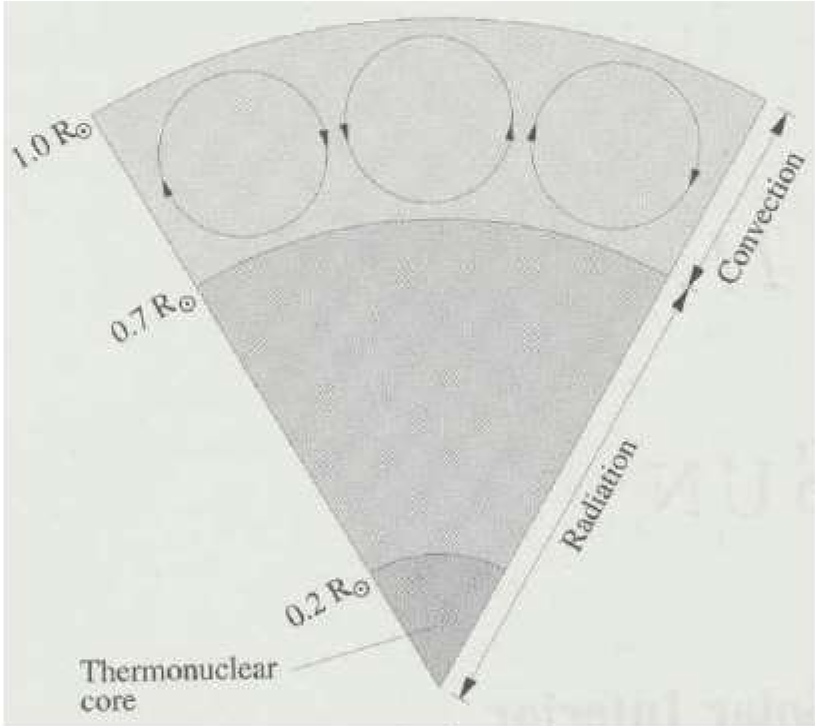
Optical depth < 1	Optical depth > 1 nuclear reaction

※ Optical depth, τ_λ : $d\tau_\lambda = -\kappa_\lambda \rho \, ds$

$\tau_\lambda \gg 1$: optically thick

$\tau_\lambda \ll 1$: optically thin

※ κ_λ : absorption coefficient (opacity)



Sun's interior

An Introduction to Modern Astrophysics (2nd edition), Bradley W. Carroll & Dale A. Ostlie (1996), p. 382

※ Mass fractions

Mass ratios (rather than numbers of particles)

Mass fractions of hydrogen : $X \equiv \frac{total\ mass\ of\ hydrogen}{total\ mass\ of\ gas}$

Mass fractions of helium : $Y \equiv \frac{total\ mass\ of\ helium}{total\ mass\ of\ gas}$

Mass fractions of metals : $Z \equiv \frac{total\ mass\ of\ metals}{total\ mass\ of\ gas}$

X + Y + Z = 1

For stars, usually, X ~ 0.7, 0 < Z < 0.03

$$[\mathrm{Fe/H}] = \log \frac{(\mathrm{Fe/H})_*}{(\mathrm{Fe/H})_\odot} = \log (\mathrm{Fe/H})_* - \log (\mathrm{Fe/H})_\odot$$

$$\log Z = 0.977[\mathrm{Fe/H}] - 1.699 \qquad [\mathrm{Fe/H}] = 1.024 \log Z + 1.739$$

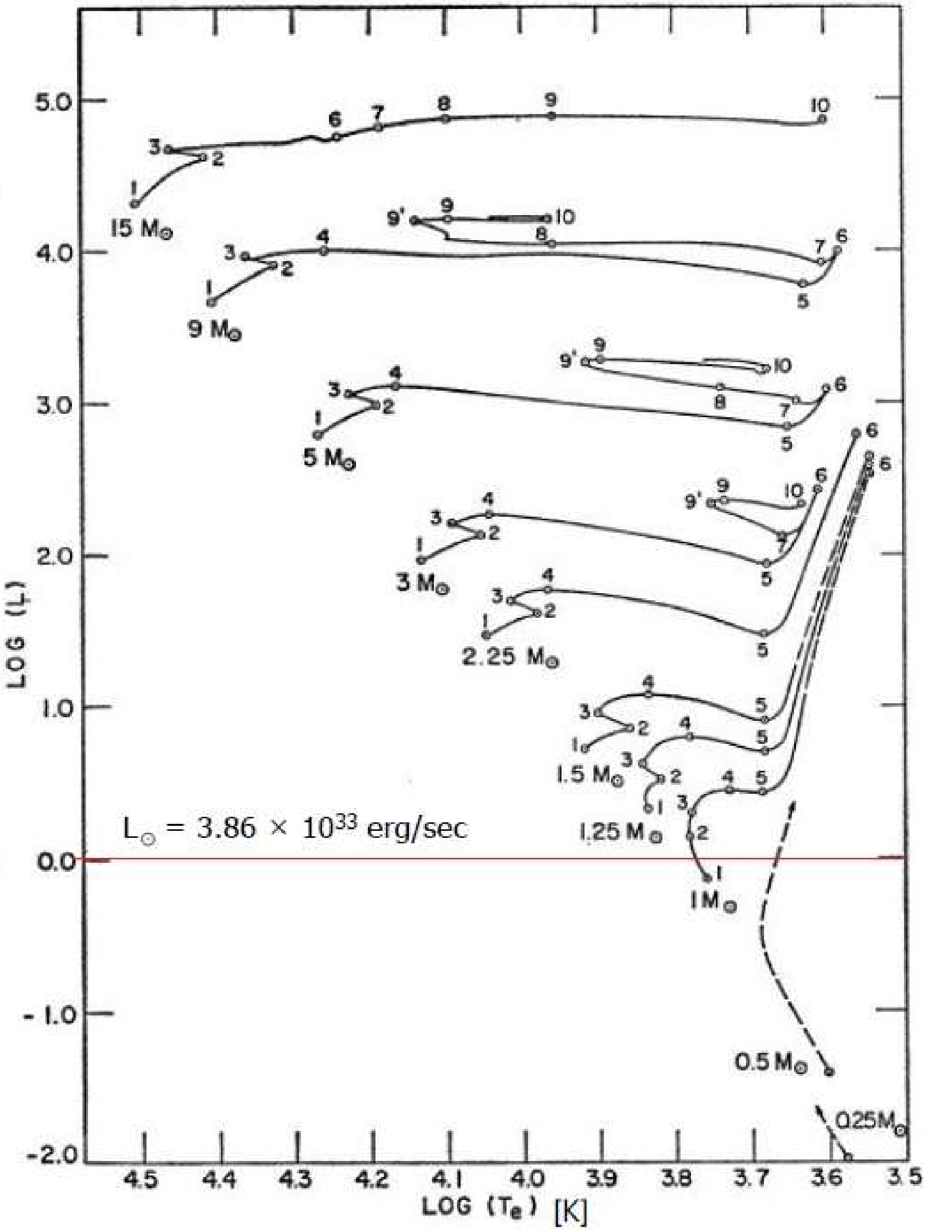
(e.g.) $-2.2 < [\mathrm{Fe/H}] < -0.7$ dex

An Introduction to Modern Astrophysics (2nd edition) Bradley W. Carroll & Dale A. Ostlie, (1996) p. 325
https://en.wikipedia.org/wiki/Metallicity
http://burro.case.edu/Academics/Astr222/Galaxy/Structure/metals.html

Bertelli et al. 1994, A&AS, 106, 275 - Theoretical isochrones from models with new radiative opacities

Post-MS Stellar Evolution

Hertzsprung-Russell Diagram of Metal-rich stars
(X=0.708, Y=0.272, Z=0.020)
M = 15, 9, 5, 3, 2.25, 1.5, 1.25, 1, 0.5, 0.25 $M_\odot$
Dashed lines : estimates



Iben (1967, ARA&A, 5, 571) Fig 3

- Sun's L, R, T : increased steadily since it reached the zero-age main sequence (ZAMS) ~4.5 Gyr ago

Solar Evolution

Sun's center, during its lifetime :

X : 0.71 → 0.34
Y : 0.27 → 0.64

changing the composition and structure

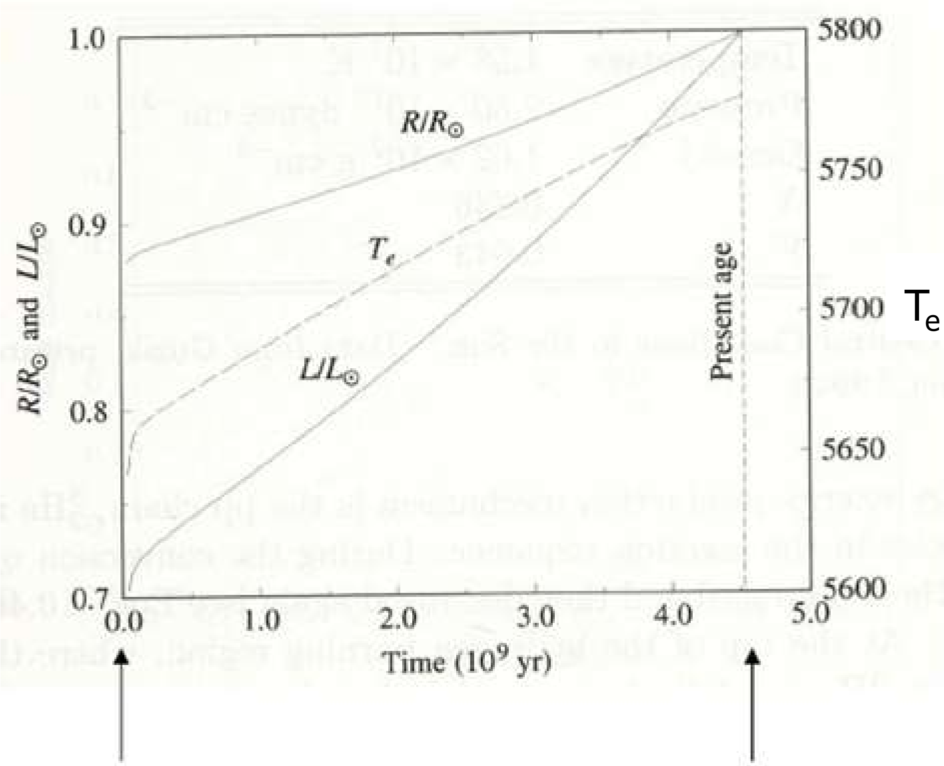
The change in the Sun's central composition
→direct influence on observable L, T_{eff}, and R

L increased by 40%
R increased by > 10%

※ Physical parameters of the Solar center
(standard solar model of Joyce Guzik)

Central conditions in the Sun

T_C	(1.56~) 1.58 $\times 10^7$ K
P_C	2.50×10^{17} dyne/cm^2
ρ_C	162 (150-160) g/cm^3
X_C	0.336
Y_C	0.643



Birth (beginning of the nuclear burning)

Present

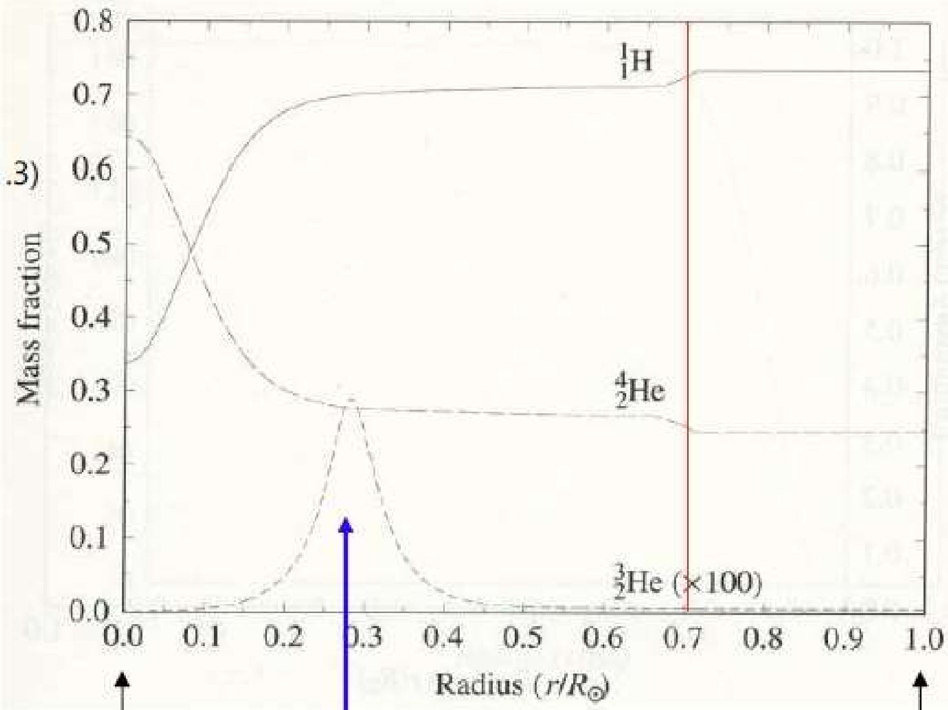
L : 1.43 times increase

An Introduction to Modern Astrophysics (2nd edition),
Bradley W. Carroll & Dale A. Ostlie (1996), p. 383

Solar composition structure

^3_2He : intermediate species in the pp chain reaction sequence

(Fig 11.3)



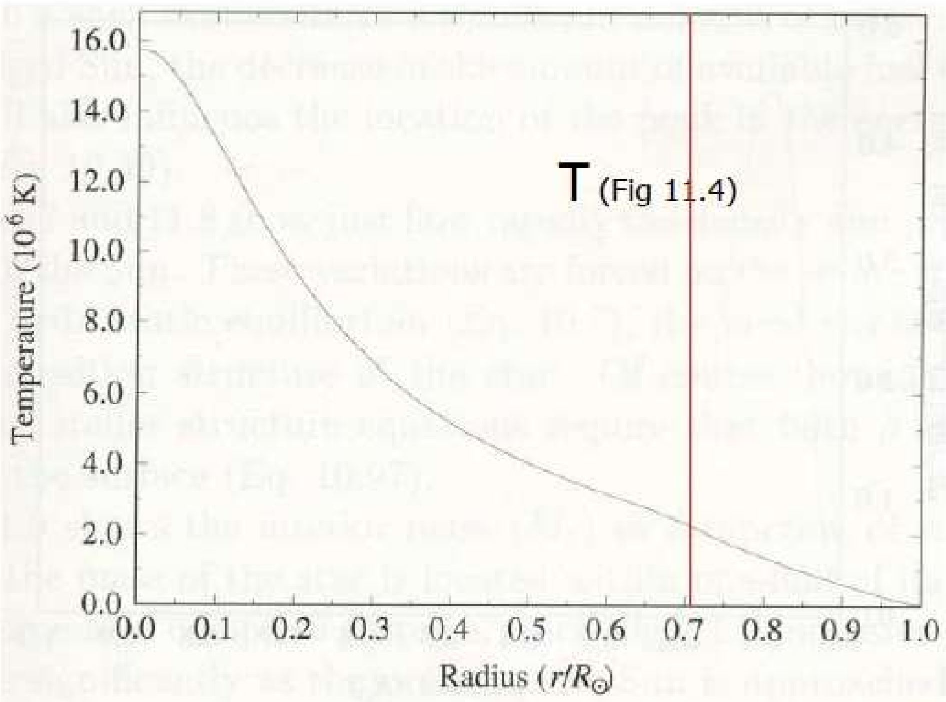
center

surface

Produced more easily than destroyed

Solar Temp structure

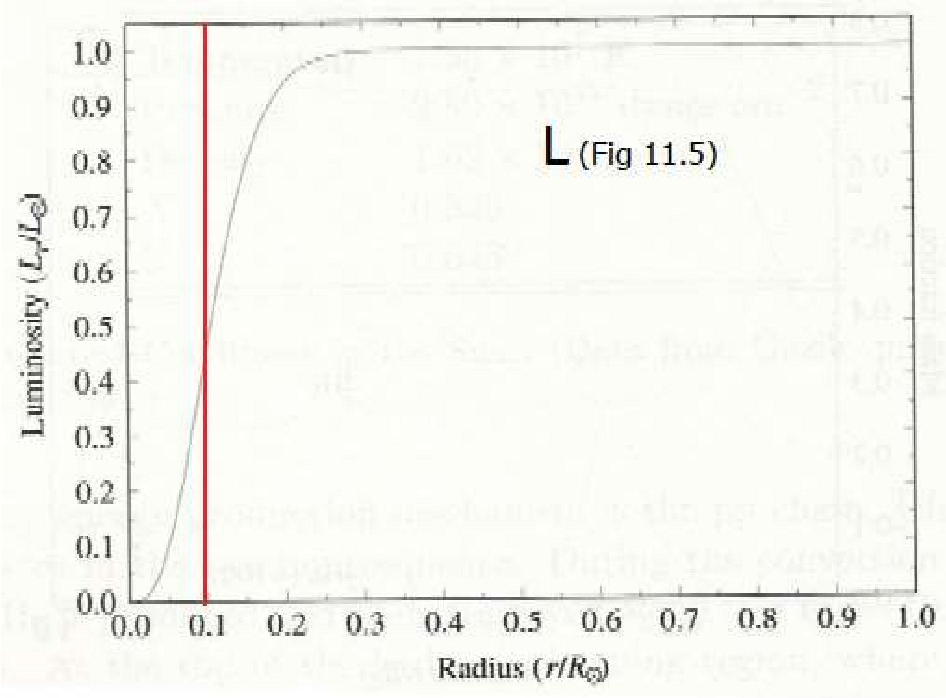
T (Fig 11.4)

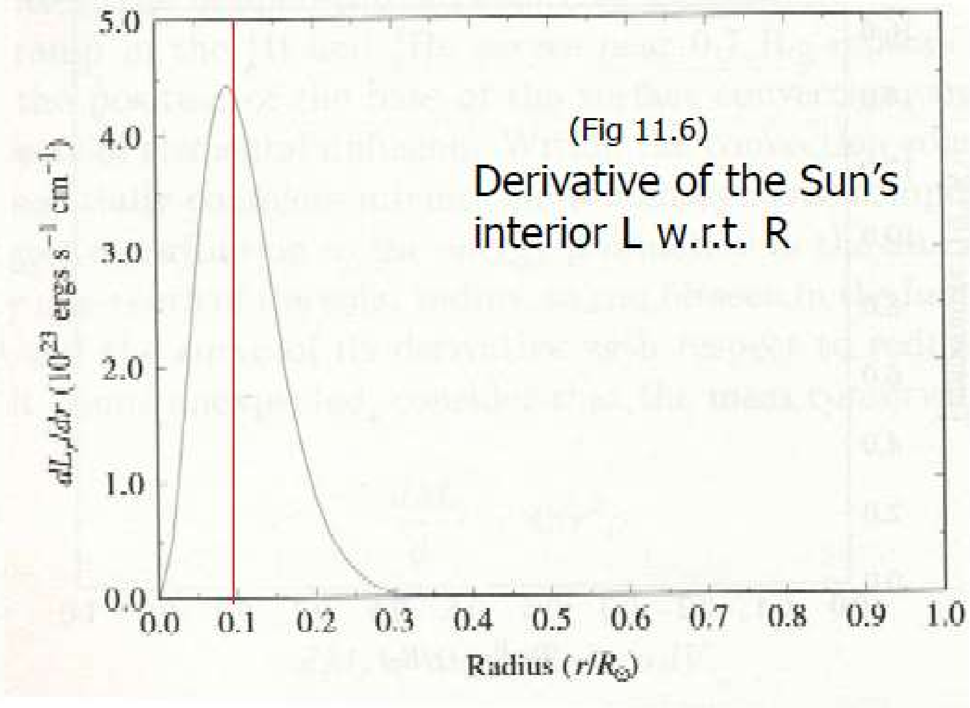


An Introduction to Modern Astrophysics (2nd edition), Bradley W. Carroll & Dale A. Ostlie (1996), p. 385

Interior Lum of the Sun

Location of the greatest contribution to the E output





Though amount of E liberated per gram of material (ε) decreases steadily from R=0 outward
→ Largest contribution to the total L – in a shell that contains a significant amount of mass

An Introduction to Modern Astrophysics (2nd edition),
Bradley W. Carroll & Dale A. Ostlie (1996), p. 386

Post-MS Stellar Evolution

TABLE III Stellar Lifetimes (yr)[a]

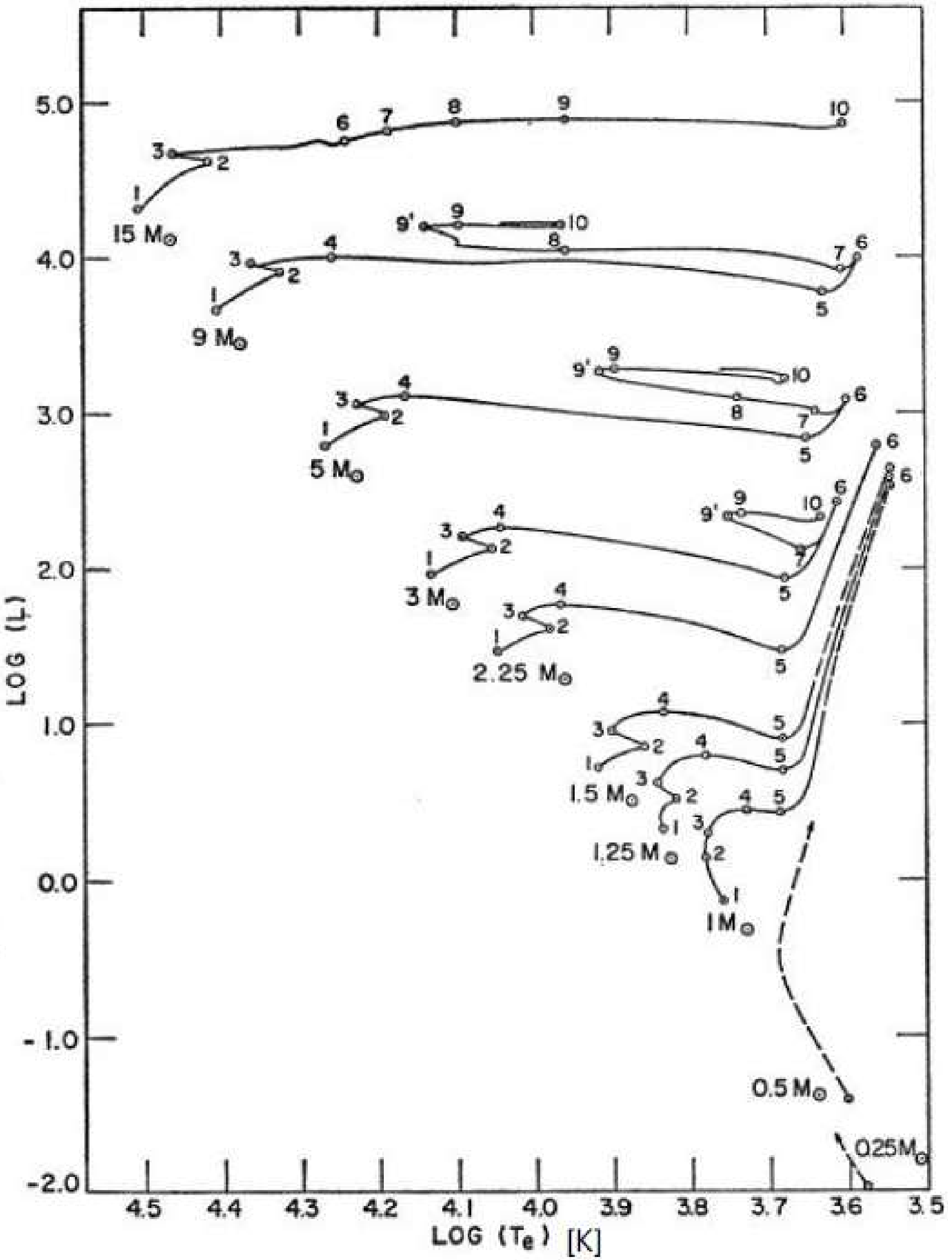
Mass ($M_\odot$) \ Interval ($i-j$)	(1–2)	(2–3)	(3–4)	(4–5)	(5–6)
15	1.010 (7)	2.270 (5)		7.55 (4)	
9	2.144 (7)	6.053 (5)	9.113 (4)	1.477 (5)	6.552 (4)
5	6.547 (7)	2.173 (6)	1.372 (6)	7.532 (5)	4.857 (5)
3	2.212 (8)	1.042 (7)	1.033 (7)	4.505 (6)	4.238 (6)
2.25	4.802 (8)	1.647 (7)	3.696 (7)	1.310 (7)	3.829 (7)
1.5	1.553 (9)	8.10 (7)	3.490 (8)	1.049 (8)	≥2 (8)
1.25	2.803 (9)	1.824 (8)	1.045 (9)	1.463 (8)	≥4 (8)
1.0	7 (9)	2 (9)	1.20 (9)	1.57 (8)	≥1 (9)

[a] Numbers in parentheses beside each entry give the power of ten to which that entry is to be raised.

TABLE IV Stellar Lifetimes (yr)[a]

Mass ($M_\odot$) \ Interval ($i-j$)	(6–7)	(7–8)	(8–9)	(9–10)
15	7.17 (5)	6.20 (5)	1.9 (5)	3.5 (4)
9	4.90 (5)	9.50 (4)	3.28 (6)	1.55 (5)
5	6.05 (6)	1.02 (6)	9.00 (6)	9.30 (5)
3	2.51 (7)	4.08 (7)		6.00 (6)

[a] Numbers in parentheses beside each entry give the power of ten to which that entry is to be raised.



Iben (1967, ARA&A, 5, 571) Fig 3

Post-MS Stellar Evolution

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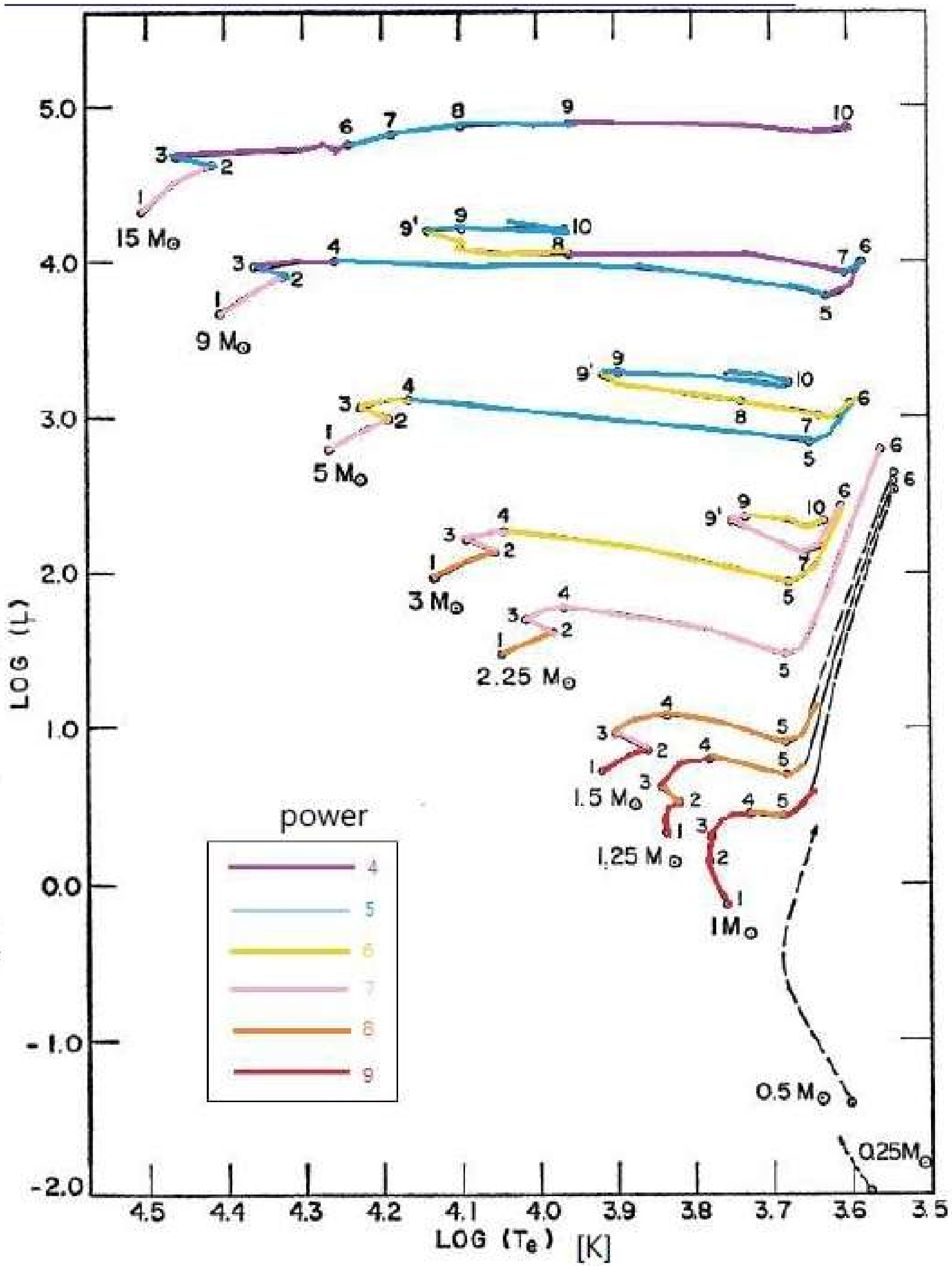


Iben (1967, ARA&A, 5, 571) Fig 3

Stellar Evolution – zoom in

- Evolutionary paths for pop I stars

M = 1.5, 1.25, 1 $M_\odot$

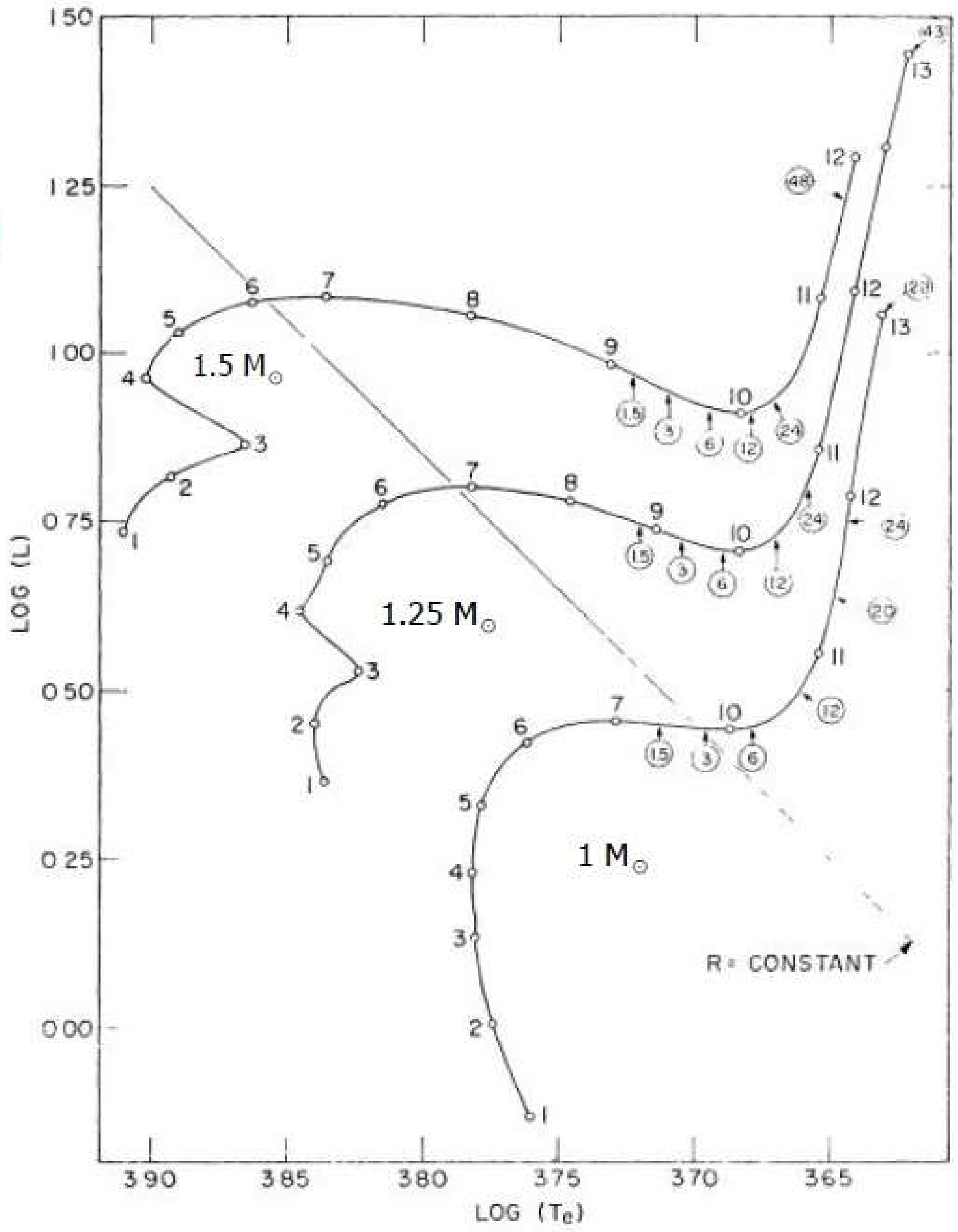
- Circled numbers : factors by which surface ^{7}Li abundance has been depleted (relative to its MS value)

6708Å, 6104Å

(Times to reach circled points)

TABLE 1 EVOLUTIONARY LIFETIMES (10^9 yr)

Point	1 $M_\odot$	1.25 $M_\odot$	1.50 $M_\odot$
1	0 05060	0 02954	0 01821
2	3 8209	1 4220	1 0277
3	6 7100	2 8320	1 5710
4	8 1719	3 0144	1 6520
5	9 2012	3 5524	1 8261
6	9 9030	3 9213	1 9666
7	10 195	4 0597	2 0010
8		4 1204	2 0397
9	... ...	4 1593	2 0676
10	10 352	4 2060	2 1059
11	10 565	4 3427	2 1991
12	10 750	4 4505	2 2628
13	10 875	4 5349	



Iben (1967, ApJ, 147, 624)

Fig 1

Stellar Evolution – zoom in

- Evolutionary paths for pop I stars

M = 1.5, 1.25, 1 $M_\odot$

- Circled numbers : factors by which surface ^{7}Li abundance has been depleted (relative to its MS value)

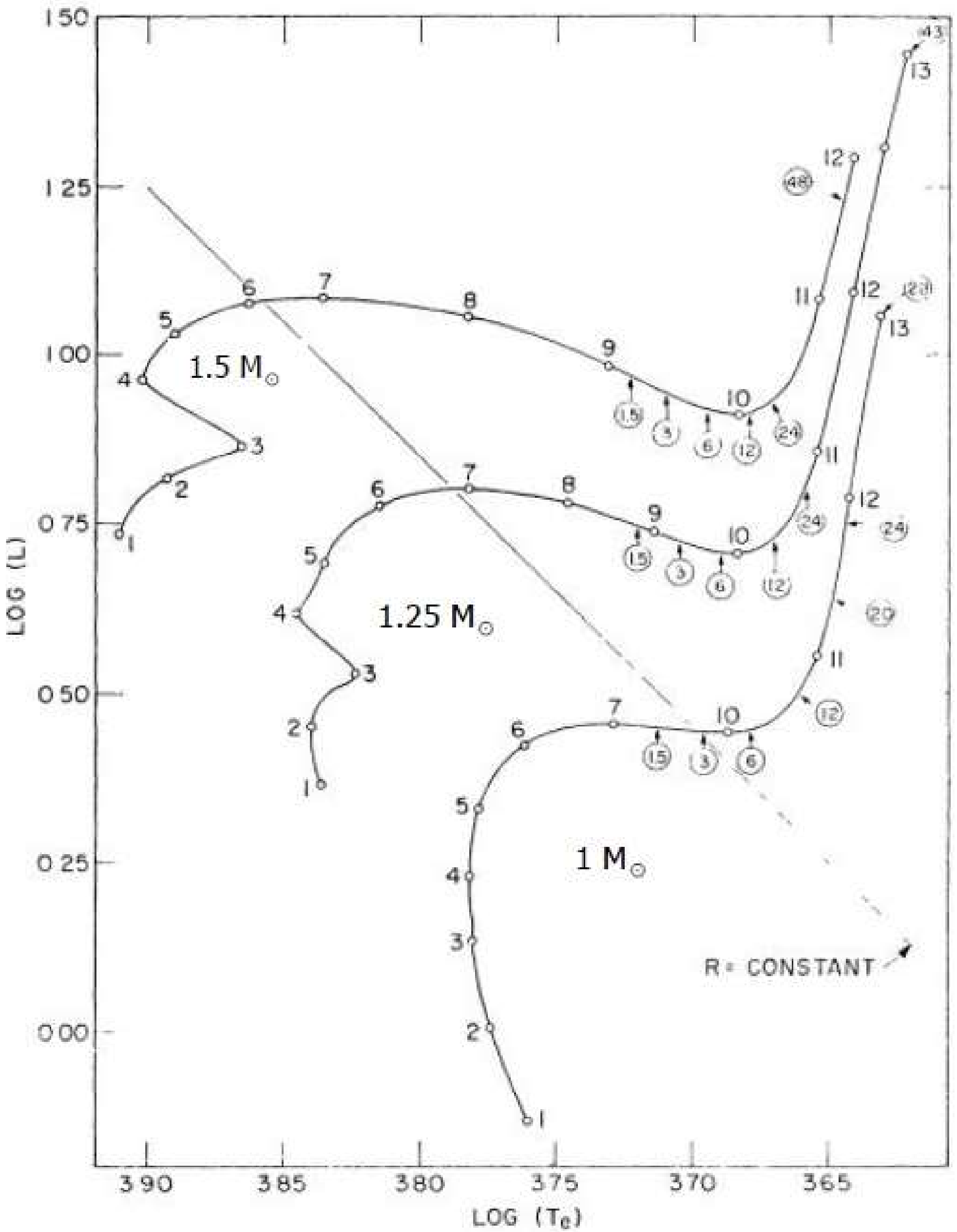
6708Å, 6104Å

(1) Solar surface lithium abundance : 140× less than the protosolar value

(2) Temperature at the base of the surface convective zone : not hot enough to burn lithium

→ Li depletion process is needed

(Israelian+09 Nature 462 189 - Enhanced lithium depletion in Sun-like stars with orbiting planets)



Iben (1967, ApJ, 147, 624)

Fig 1

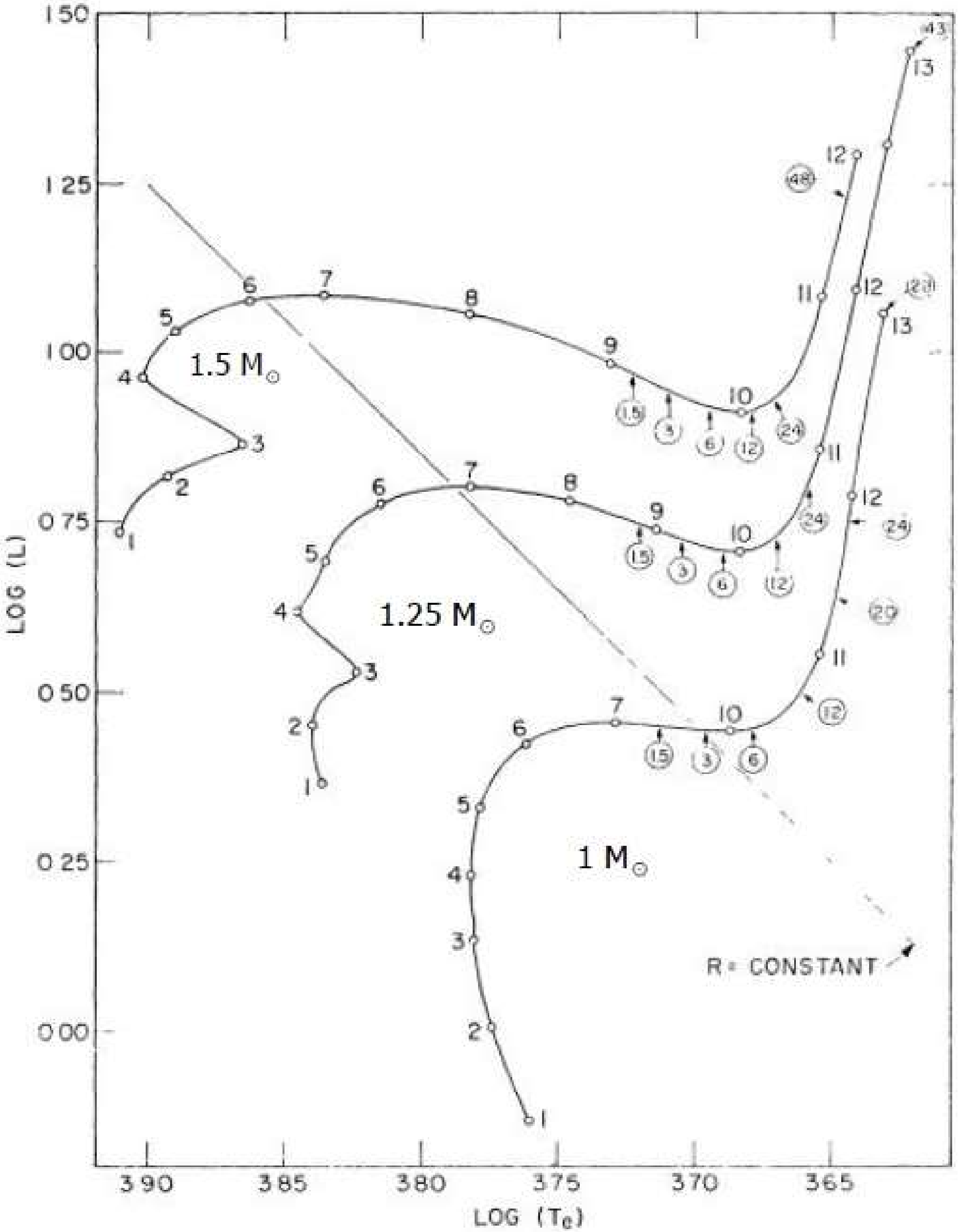
Stellar Evolution – zoom in

- As stellar mass ↓, the direction of evolution off the MS shifts from movement to the red to movement to the blue
- The phase of over-all contraction and increasing surface temperature (points 3 → 4) : becomes less pronounced with M ↓, disappearing between 1.25 $M_\odot$ and 1 $M_\odot$ (core convection during the H-burning phase vanishes in this mass range)
- During the shell-narrowing phase (points 7 → 10), the drop in L decreases with M ↓

$$-\Delta(\log L) = 0.15,\quad 0.10,\quad 0.015$$
$$1.5\,M_\odot,\ 1.25\,M_\odot,\ 1\,M_\odot$$

- As mass ↓, the △L (increase) between points 1 and 7 : increases

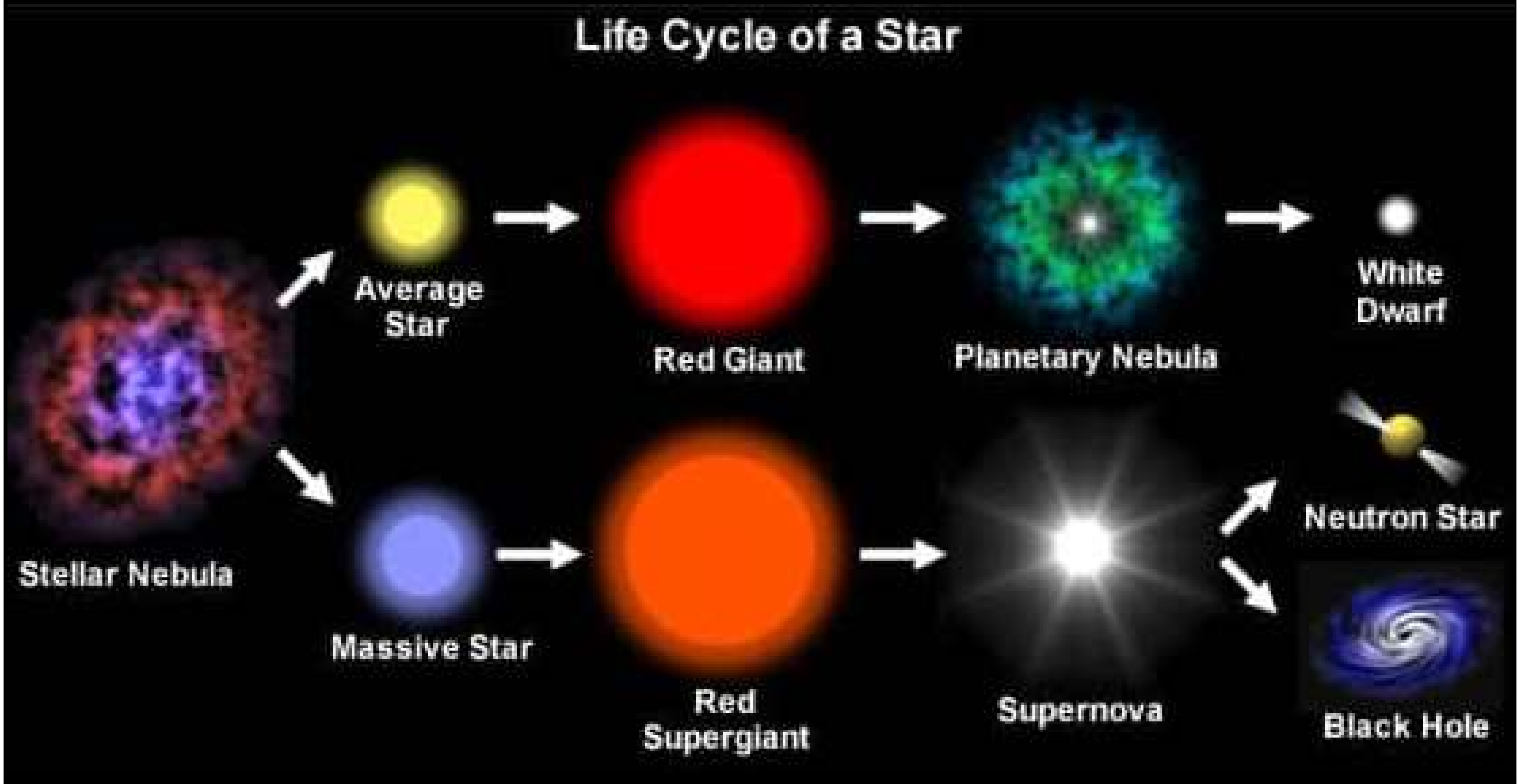
$$\Delta(\log L) = 0.35,\quad 0.44,\quad 0.69$$
$$1.5\,M_\odot,\ 1.25\,M_\odot,\ 1\,M_\odot$$



Iben (1967, ApJ, 147, 624)

Fig 1

2. The Evolution of Stars (별의 진화)
2-3 Post-Main-Sequence Evolution (주계열 이후 진화)



http://schoolworkhelper.net/types-of-stars/

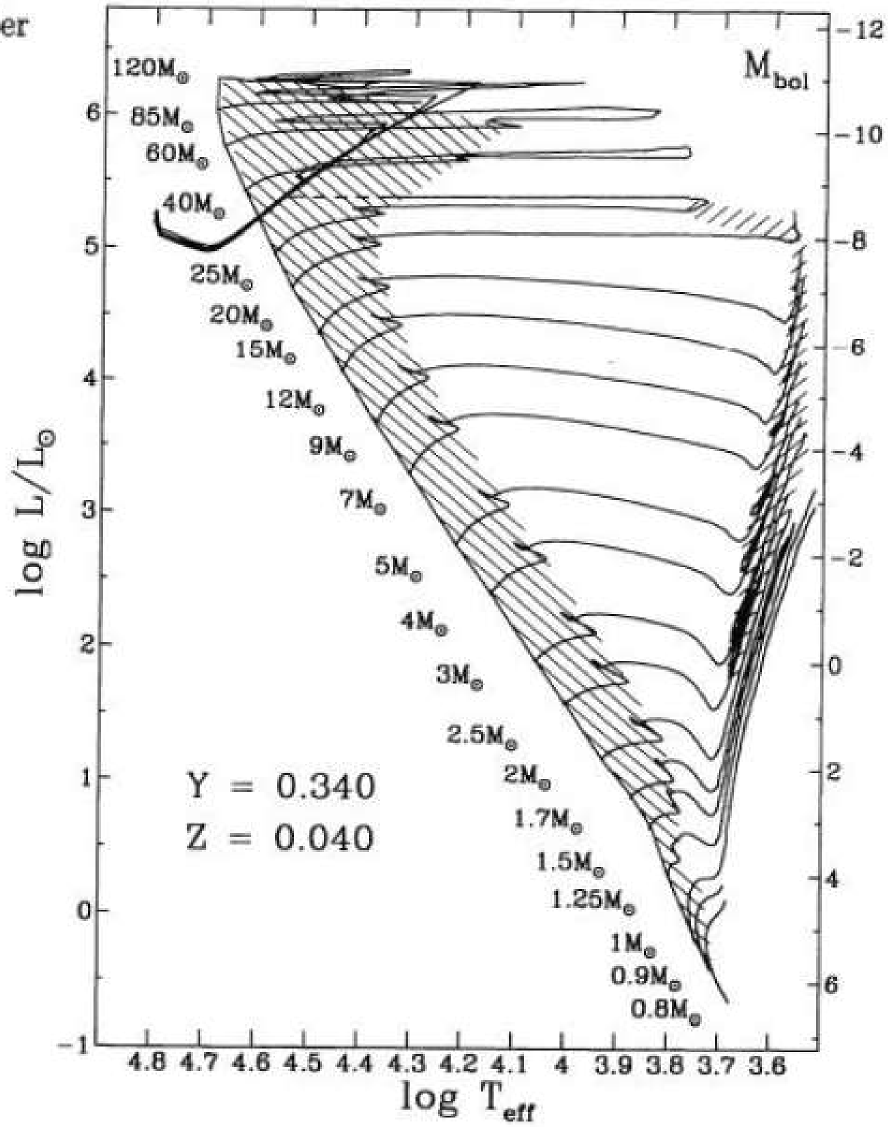
Post-MS Stellar Evolution

D. Schaerer, C. Charbonnel, G. Meynet, A. Maeder and G. Schaller
1993 *Astron. Astrophys. Suppl. Ser.* **102,** 339-342

Grids of stellar models.
IV. From 0.8 to 120 $M_\odot$ at $Z = 0.040$

Stellar evolution isochrones
- Padova
- Geneva
- VandenBerg
- Yonsei-Yale (Y^2)
- Dartmouth

Hatched areas : slow phases of nuclear burning



Schaerer et al (1993, A&AS, 102, 339) – Fig 1

Post-MS Stellar Evolution

D. Schaerer, C. Charbonnel, G. Meynet, A. Maeder and G. Schaller

1993 *Astron. Astrophys. Suppl. Ser.* **102**, 339-342

Grids of stellar models.
IV. From 0.8 to 120 $M_\odot$ at $Z = 0.040$

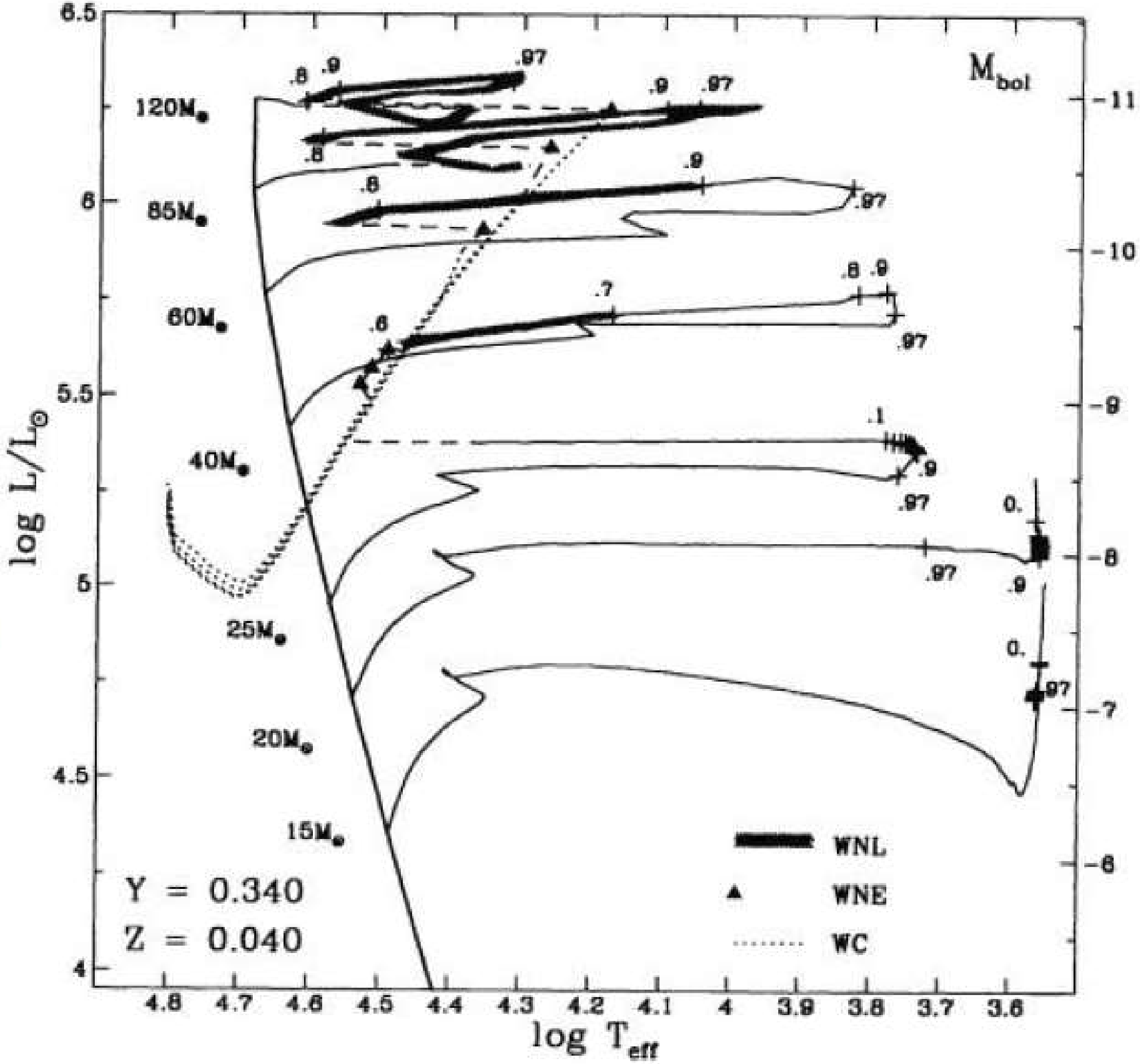
HRD of massive stars (15 – 120 $M_\odot$) with Z=0.040

Yc (central He content) : 0.97, 0.90, 0.80, 0.70, 0.60, ---, 0.10, 0

Wolf-Rayet stars

- broad emission lines of highly ionized He and N or C
- very high surface enhancement of heavy elements, depletion of hydrogen
- strong stellar winds
- T_{eff} = 30,000 K - 200,000 K

WN Early (WN2 – WN6)
WN Late (WN6 – WN9)
WC

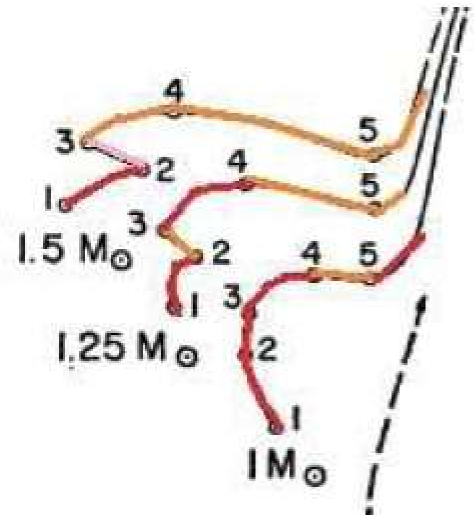


Schaerer et al (1993, A&AS, 102, 339) – Fig 3

Evolution beyond point 3

Maximum fraction of a star's mass
that can exist in an isothermal core + still support the overlying layers
= $\boxed{\text{Schönberg-Chandrasekhar limit}}$ (1942)

$$\left(\frac{M_{ic}}{M}\right)_{\mathrm{SC}} \simeq 0.37 \left(\frac{\mu_e}{\mu_{ic}}\right)^2$$



where, μ = 〈m〉/m_H : mean molecular weight

(the average mass 〈m〉 in units of the mass of a hydrogen atom m_H)

An Introduction to Modern Astrophysics (2nd edition),
Bradley W. Carroll & Dale A. Ostlie (1996), p. 485, 488

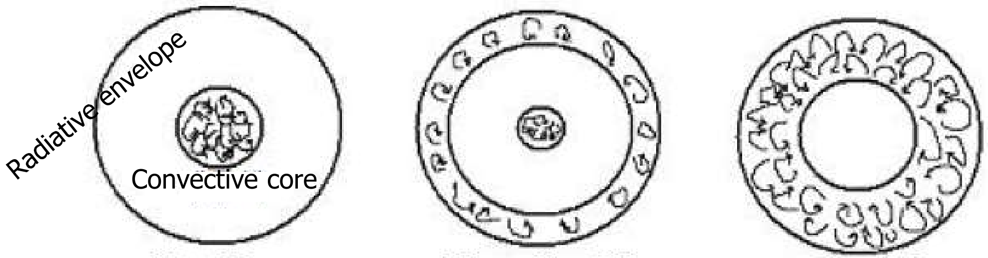
Characteristics of MS Stars

$M > 1.5\,M_\odot$	$M < 1.5\,M_\odot$
$T_c > 2 \times 10^7$ K	$T_c < 2 \times 10^7$ K
CNO cycle	PP chain
Energy production rate : $\varepsilon_{CNO} \propto T^{19.9}$ --> Production of most E near the center	$\varepsilon_{CNO} \propto T^4$ --> E source is not concentrated to the center
core : convective	core : radiative
envelope : radiation	envelope : H-convection

The more massive, the larger convective core.

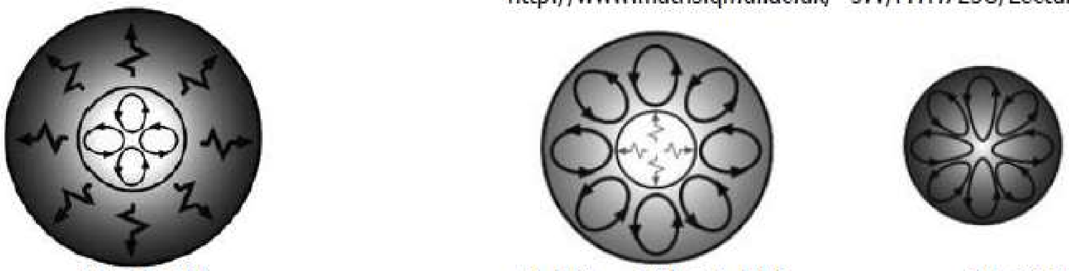
mass

As mass decreases, H-convection zone deepens.



$M > 2M_\odot$ $\quad$ $1M_\odot < M < 2M_\odot$ $\quad$ $M < 1M_\odot$

http://www.maths.qmul.ac.uk/~svv/MTH725U/Lecture8.htm

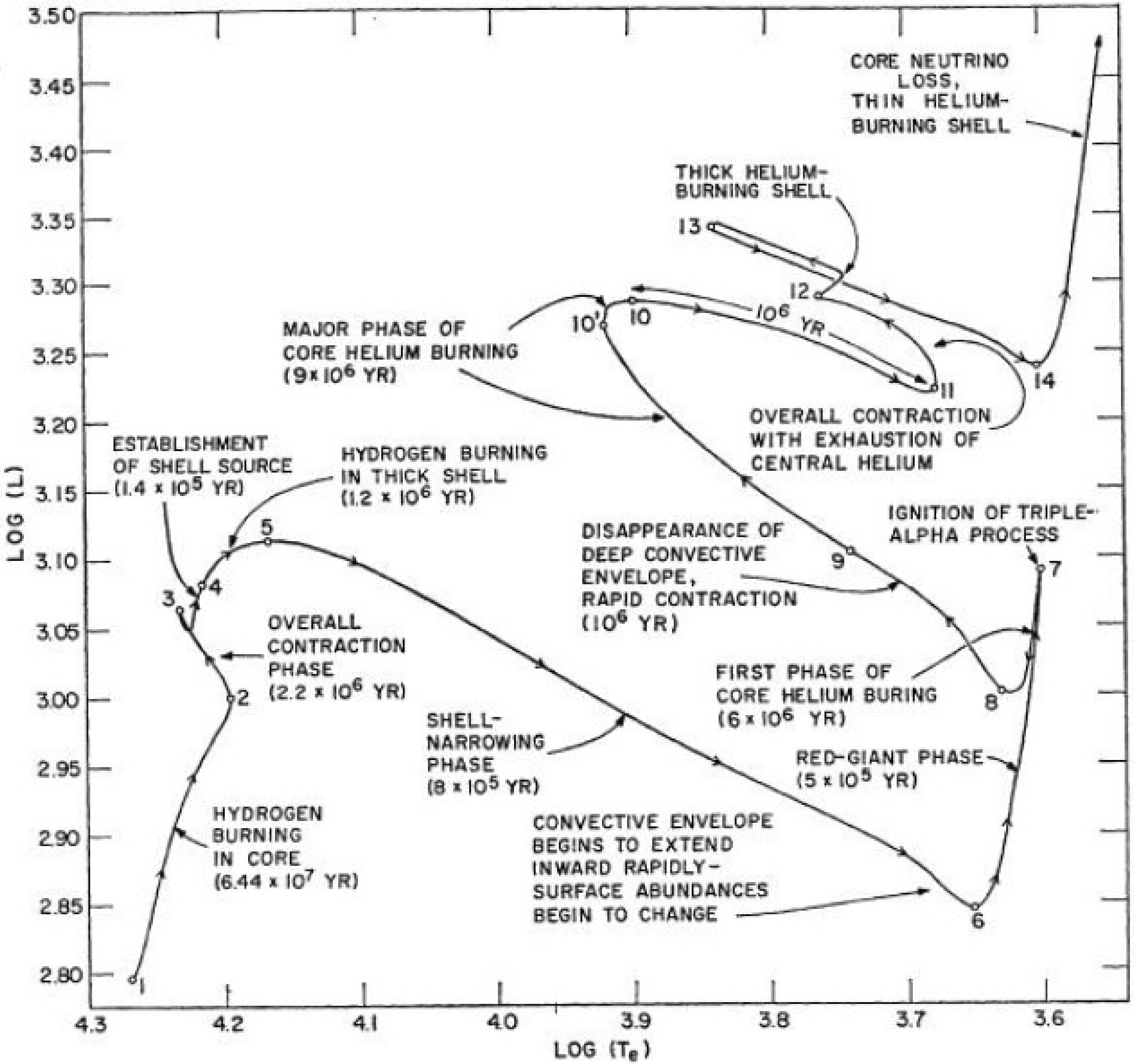


$M > 1.5M_\odot$ $\quad$ $0.5M_\odot < M < 1.5M_\odot$ $\quad$ $M < 0.5M_\odot$

https://ase.tufts.edu/cosmos/view_picture.asp?id=1409

Post-MS Stellar Evolution

- Evolutionary path of a 5 $M_\odot$ metal-rich star in the HRD
- From ZAMS to the asymptotic giant branch (AGB)

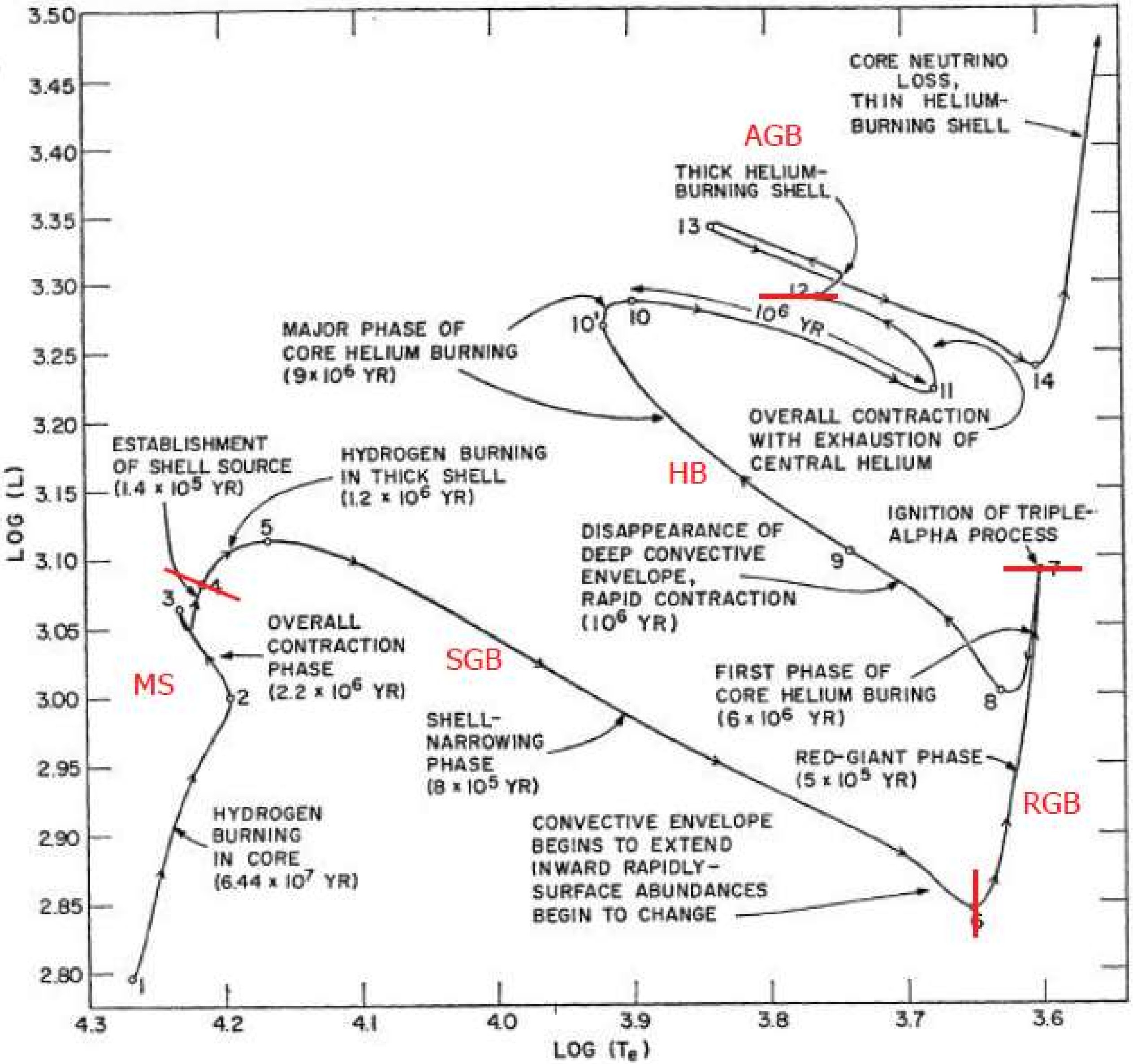


Iben (1967, ARA&A, 5, 571) Fig. 1

Post-MS Stellar Evolution

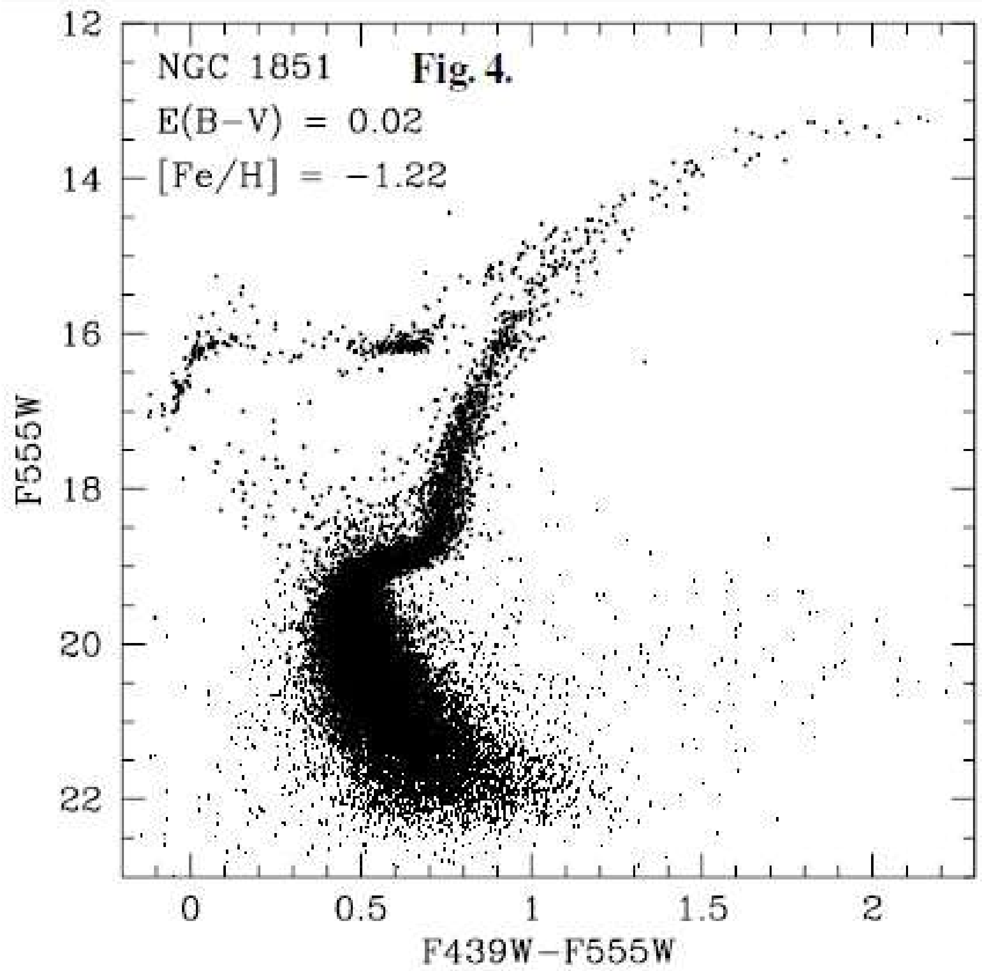
- Evolutionary path of a 5 $M_\odot$ metal-rich star in the HRD

- From ZAMS
to the asymptotic giant branch (AGB)

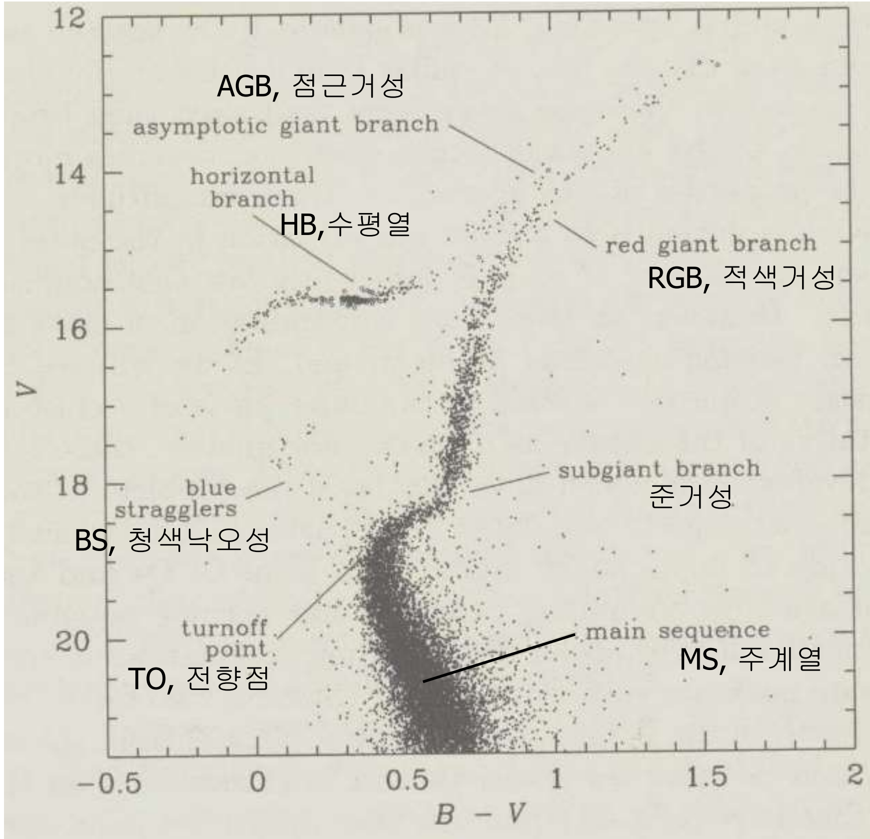


Iben (1967, ARA&A, 5, 571) Fig. 1

Color-Magnitude Diagrams for globular clusters (GCs)



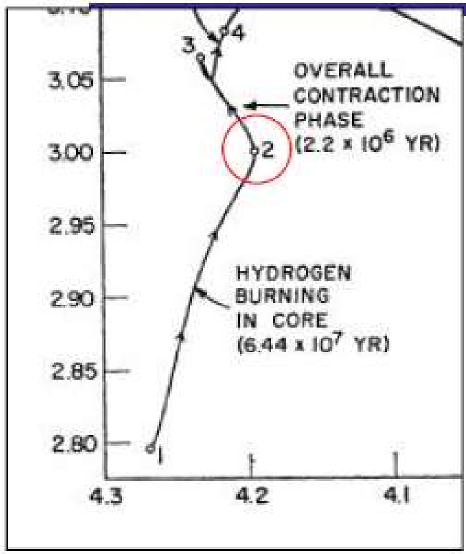
Piotto et al. (2002, A&A, 391, 945)



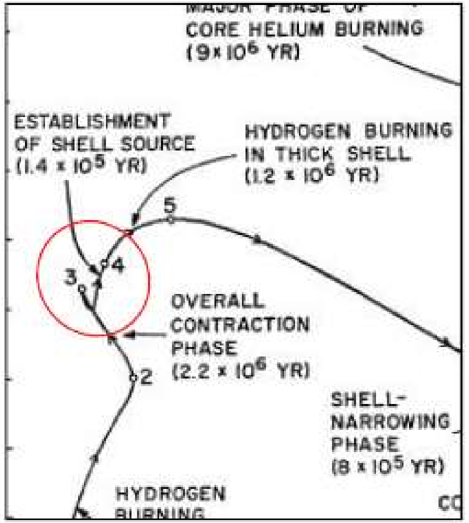
GC M3 (NGC 5272)
(variables=open circles)

Galactic Astronomy (J. Binney & M. Merrifield, 1998) p. 334

5 $M_\odot$ star - structure



- At point 2 : near depletion of H fuel in the core
- During 2 → 3 : overall contraction (Core H mass fraction X : reduced to 0.01)

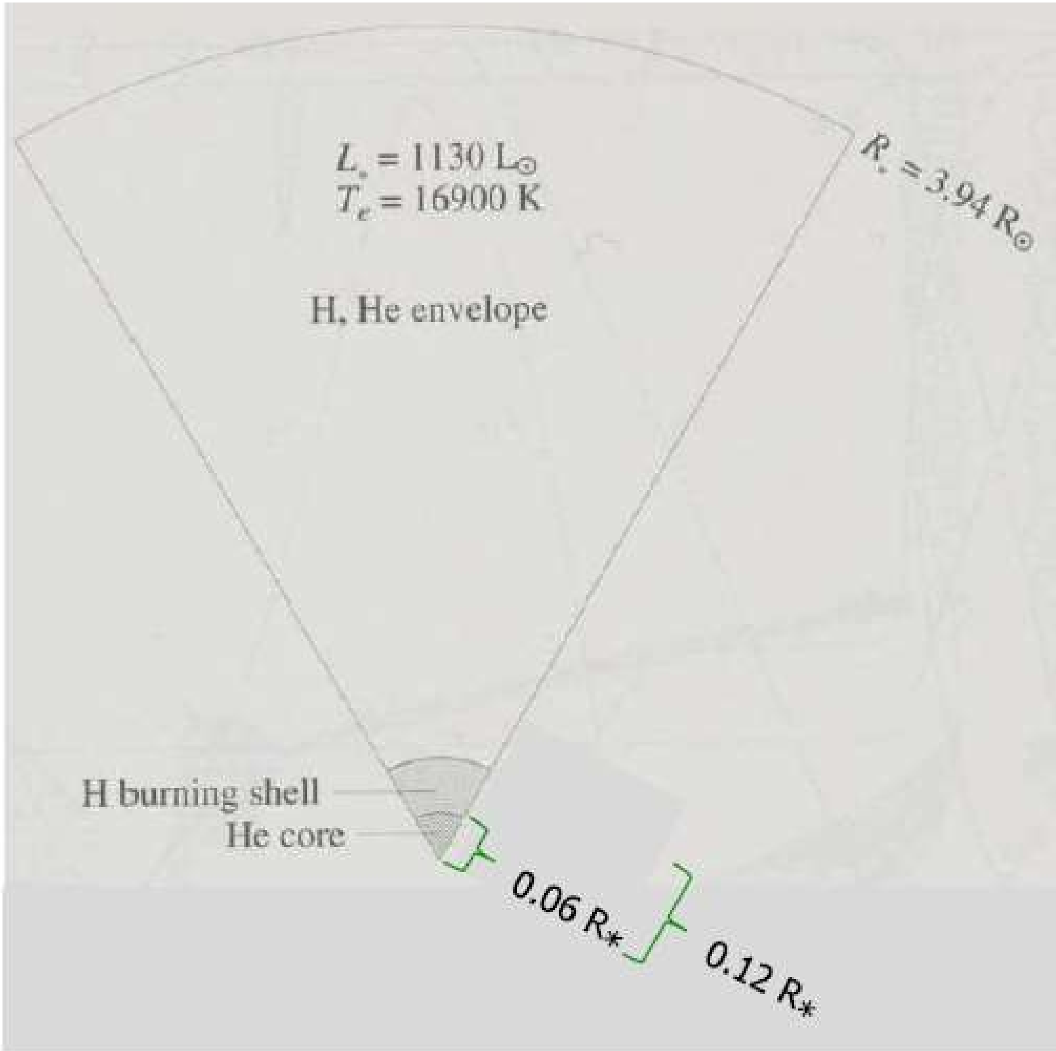


- During 3 → 4 : establishment of shell source - H burning begins in a thick shell
  - Rapid shell ignition → overlying **envelope** absorbs some or much of the E release by the shell → envelope slightly **expands**

5 $M_{\odot}$ star approaching the RGB

Shortly after point 3 :

H shell burning



An Introduction to Modern Astrophysics (2nd edition), Bradley W. Carroll & Dale A. Ostlie (1996), p. 498

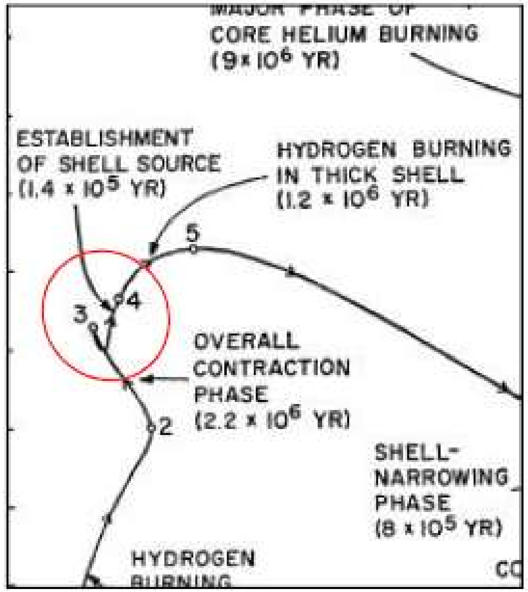
Evolution beyond point 3

- At this point,
- L (generated in the thick shell) > L (produced by the core during the core-H-b)

→ Evolutionary track continues to rise beyond point 3 $\boxed{L \uparrow}$

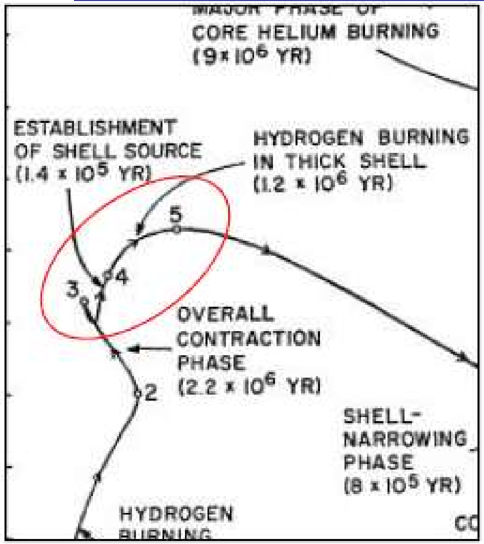
- Some of the energy generated → goes into a slow expansion of the envelope
- T_{eff} decrease slightly, evolutionary track bends to the right

$$\frac{\text{(shell H-b time)}}{\text{(core H-b time)}} \propto \frac{1}{\text{mass}}$$



H-b shell continues to consume its nuclear fuel
→ Isothermal He-core grows in mass
Star moves farther to the red in the HRD
→ Redward locus = subgiant branch

5 $M_\odot$ star approaching the RGB

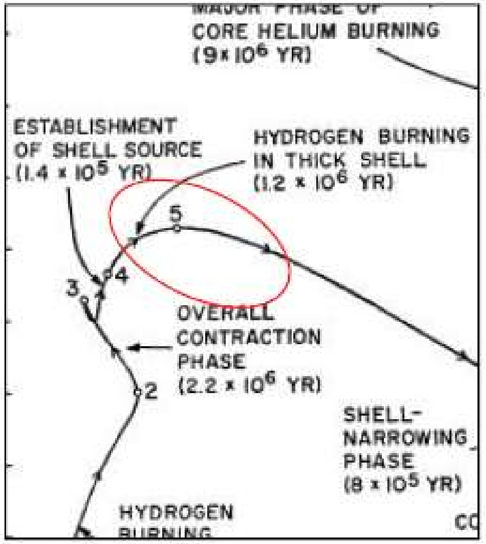


- During 3 → 5 : Shell continues to consume H

→ He-core : mass and size steadily increase

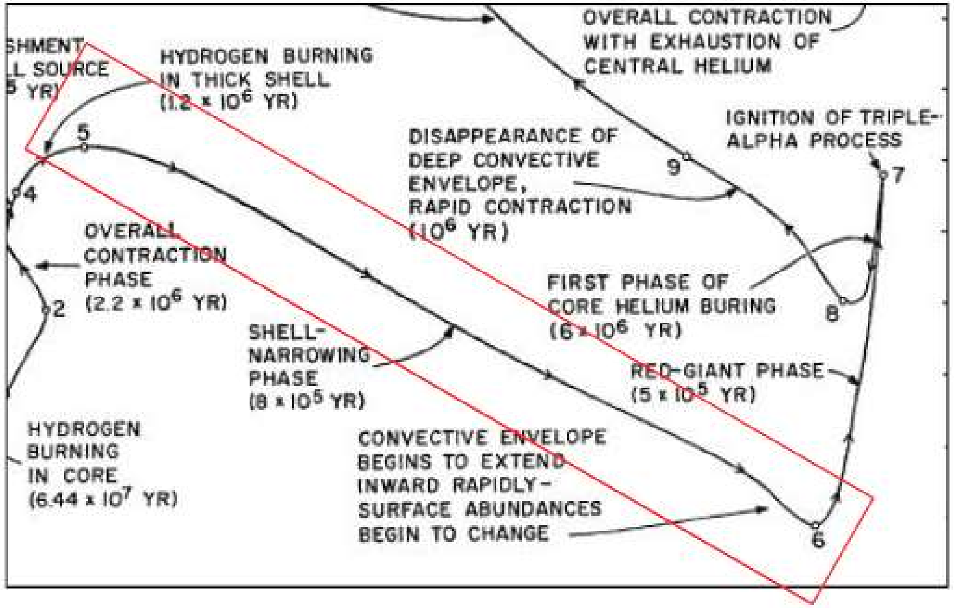
→ becomes nearly isothermal core

- At Point 5 → Schönberg-Chandrasekhar limit is reached
  - → Core begins to collapse
  - → Much faster evolution



- At points from 5 :
  - Core begins to collapse → gravitational potential E releases
    → T gradient re-appears
  - H-b shell : T, ρ increase
  - Though shell begins to narrow significantly / shell E-generation rate increases rapidly

5 $M_\odot$ star approaching the RGB



- During point 5 → 6 :
  - **Envelope expands** (absorbing much of the shell's E, before it reaches the surface)
  - → L decreases, effective T decreases, photospheric optacity increases (H^- ion)
  - → convection zone begins to develop near the surface

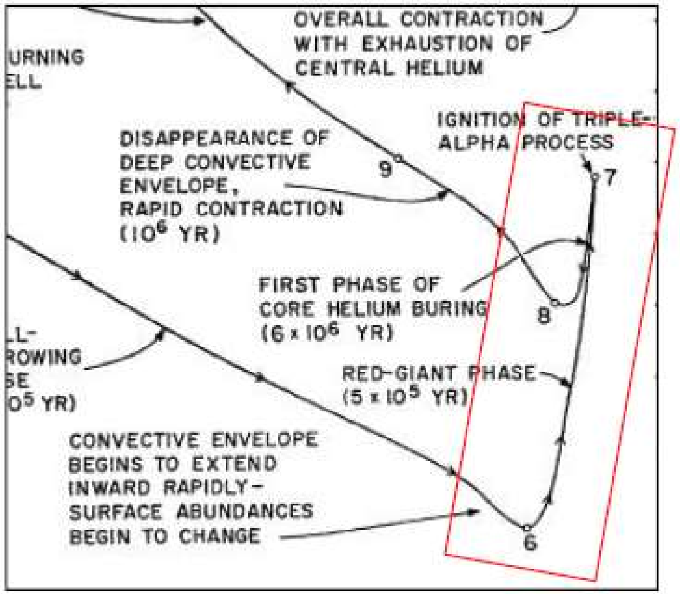
- Base of the convection zone → extends down into regions where the chemical composition has been modified by nuclear processes
- Surface convection zone encounters these chemically modified regions

→Processed material become mixed

→Photosphere composition change (Li ↓, ${}^3_2$He ↑)

"First dredge-up" begins at point 6

5 $M_\odot$ star at the RGB



- Surface of the envelope continues to cool → recombination → N(free e^-) ↓ → contribution of the H^- ion to the opacity ↓ → more E release

- During 6 → 7

 : core continues to contract – degenerate He

→ (narrowing) H-b shell E-production rate ↑

→ L↑, R ↑

→ vertical evolution, "**red giant branch (RGB)**"

- At point 7 :

- $T_c = 1.3 \times 10^8$ K

- $\rho_c = 7700$ g/cm³

- High central T and density → quantum-mechanical tunneling through the Coulomb barrier (acting between $^4_2$He nuclei) becomes effective

 → triple-α process begins

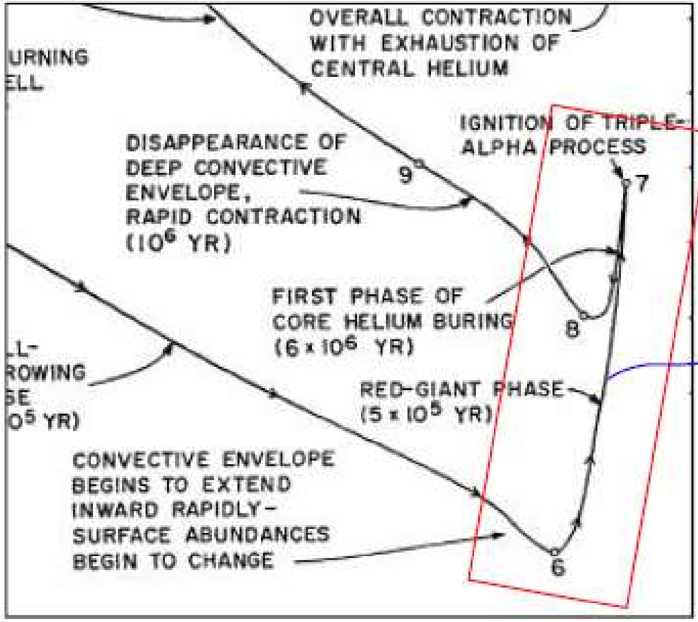
- Slow ignition of He-burning phase!
- New source of E → core expands

→ Shell E-generation rate decreases

→ L ↓

and envelope contracts, T_{eff} ↑

※ For low mass stars of $M < 2\ M_\odot$



- As He-core continues to collapse

→ Core becomes strongly electron-degenerate

- T, ρ become high enough

to initiate the triple-α process (10^8 K, 10^4 g/cm^3)

→ Explosive E release!

(core) helium flash

- Luminosity $\sim 10^{11}\ L_\odot$

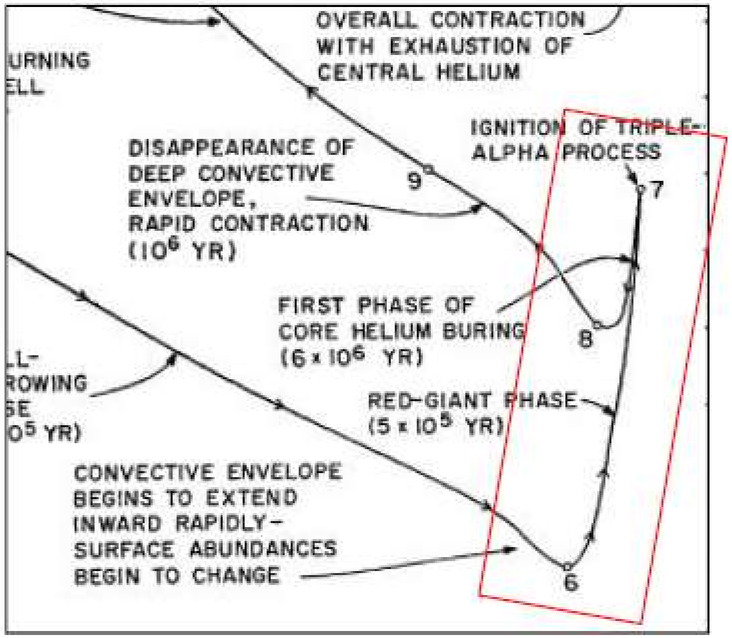
→ For only a few seconds!

(most of the E never reaches the surface)

→ E "lifts" the degeneracy and

absorbed by the overlying layers of the envelope (possibly cause some mass loss)

※ For low mass stars of M < 2 $M_\odot$



- **(core) helium flash**

: Nearly constant mass – function of metallicity
~1.6 $M_\odot$ for population I stars
~1 $M_\odot$ for population II stars

- For pop II stars (ages 2-15 Gyr),

if metallicity is same
→ bolometric luminosity is ~ constant
(varies only by ~ 0.1 mag)

→ TRGB (tip of the red giant branch)
: distance indicator for old, resolved galaxies

TRGB (tip of the red giant branch)

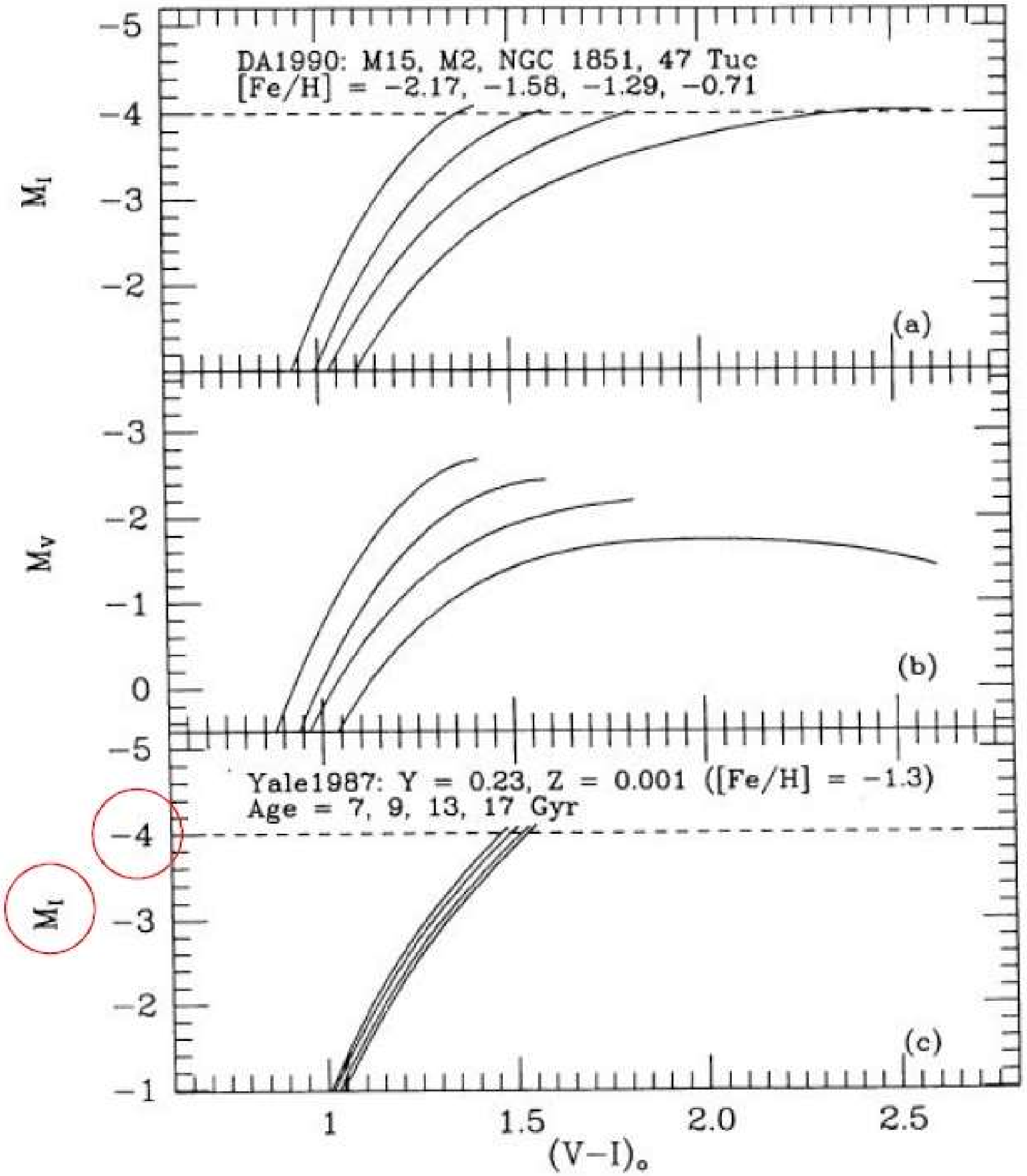


Fig. 1

Lee et al. (1993, ApJ, 417, 553)

TRGB (tip of the red giant branch)

TABLE 1

Distance Estimates for Resolved Galaxies Based on Primary Distance Indicators

Galaxy (1)	Type[a] (2)	$E(B-V)$ (3)	$(m-M)_0$ Cepheid (4)	RR Lyrae (5)	I_{TRGB} (6)	I_{TRGB} (7)	Reference[b] (8)	[Fe/H][c] (9)	M_B (10)	M_V (11)
LMC	SBmII	0.10	18.50	18.28	18.42	14.6	1, 2, 3	−1.2	−17.93	−18.36
NGC 6822	ImIV–V	0.28	23.62	...	23.46	20.05	4, 4	−1.8:	−15.13	−16.42
NGC 185	dE3pec	0.19	...	24.01	23.94	20.30	5, 6	−1.2	−14.63	−15.52
NGC 147	dE5	0.17	...	24.06	24.13	20.4	7, 8	−0.9	−14.39	−15.17
IC 1613	IMV	0.02	24.42	24.27	24.27	20.25	1, 9, 10	−1.3	−14.51	−15.16
M31	SbI–II	0.08	24.44	24.36	24.44	20.55	1, 11, 12	−0.8	−20.98	−21.74
M33	Sc(s)II–III	0.10	24.63	24.71	24.70	20.95	1, 13, 12	−2.0	−18.94	−19.40
WLM	ImIV–V	0.02	24.92	...	24.81	20.85	14, 14	−1.6:	−14.28	−14.62
NGC 205	S0/dE5pec	0.035	...	24.76	24.42	20.45	15, 16	−0.8	−15.80	−16.62
NGC 3109	SmIV	0.04	25.5	...	25.45	21.55	17, 18	−1.6	−15.95	−16.25

[a] From Sandage & Tammann 1987.

[b] References: (1) Madore & Freedman 1991, (2) Walker 1988, (3) Reid & Mould 1987, (4) Lee, Freedman, & Madore 1993, (5) Saha & Hoessel 1990, (6) Lee, Freedman, & Madore 1992, 1993c; (7) Saha, Hoessel, & Mossman 1990; (8) Mould, Kristian, & Da Costa 1983; (9) Saha et al. 1992; (10) Freedman 1988; (11) Pritchet & van den Bergh 1987, 1988; (12) Mould & Kristian 1986; (13) Pritchet 1988; (14) Lee, Freedman, & Madore 1993a; (15) Mould, Kristian, & Da Costa 1984; (16) Saha, Hoessel, & Krist 1991; (17) Capaccioli, Piotto, & Bresolin 1992; (18) Lee 1993.

[c] The metallicity [Fe/H] has been determined using the color $(V-I)_{-3.5}$.

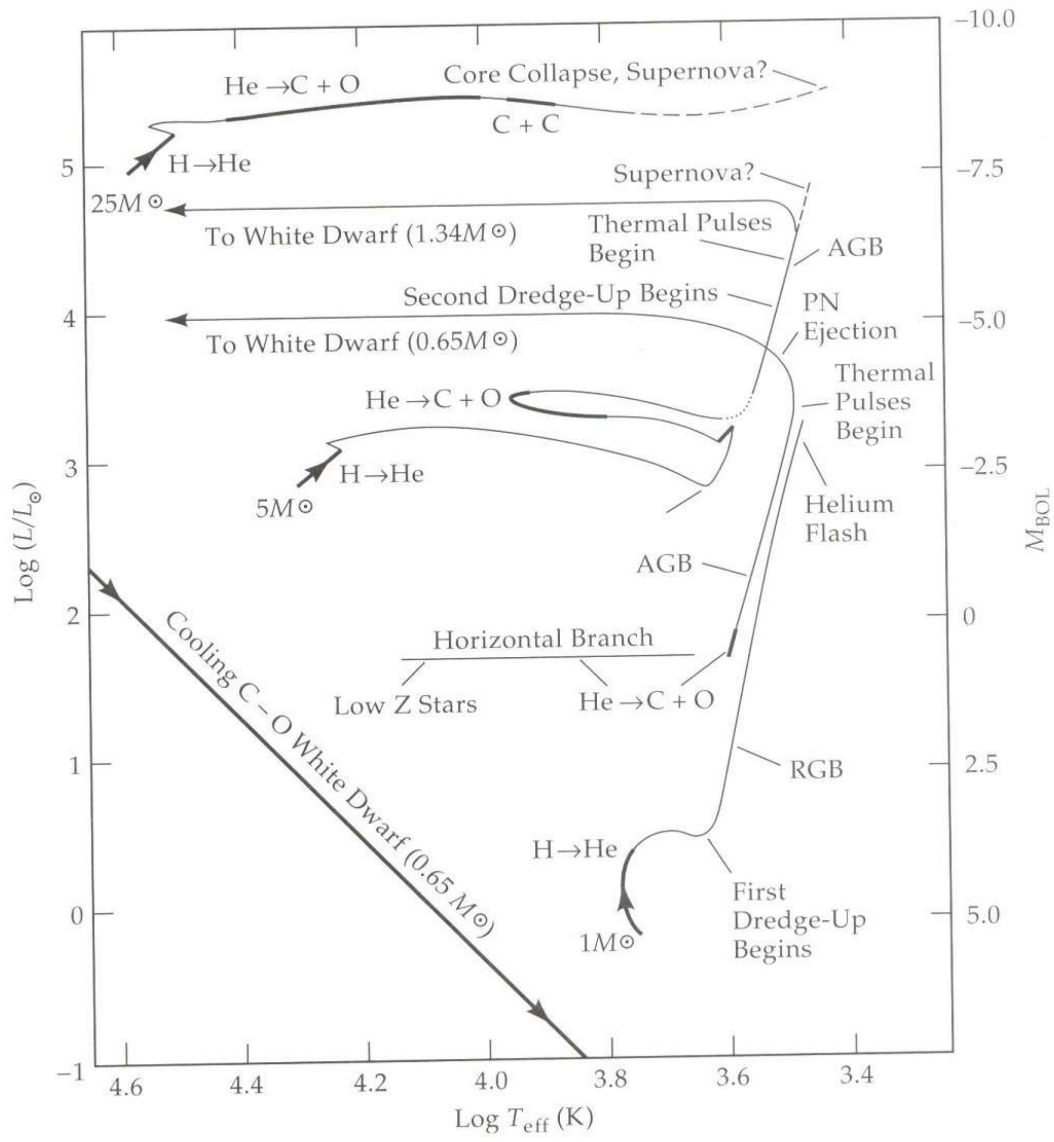
$$-2.2 < [\mathrm{Fe/H}] < -0.7 \text{ dex}$$

Lee et al. (1993, ApJ, 417, 553)

TRGB (tip of the red giant branch)

- Y-axis (L scale) : squeezed
- Time scale : different
- Low-mass stars : less structure variation, less contraction → more simple and smooth tracks

- Thicker parts : major phases of fusion burning
- Dashed lines : uncertain phases



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