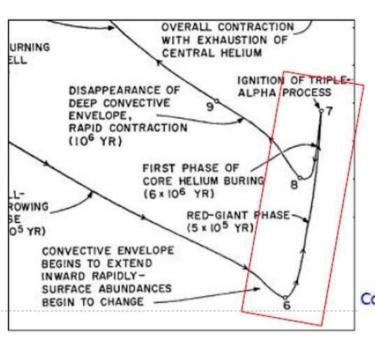


5 M_{\odot} star evolution after ignition of the triple- α process



- At point 7 :
- $T_c = 1.3 \times 10^8 \text{ K}$
- ρ_c=7700 g/cm³
- High central T and density → quantum-mechanical tunneling through the Coulomb barrier (acting between ⁴₂He nuclei) becomes effective → triple-α process begins

$$\epsilon_{3\alpha} \simeq \epsilon'_{\circ,3\alpha} \rho^2 Y^3 f_{3\alpha} T_8^{41.0}$$

(Strong T-dependence)

```
Core – He-b (3α p)

Contribution to Lum: 

Shell – H-b
```

```
In the core, new source of E \rightarrow core expands and cools \rightarrow Shell E-output \downarrow

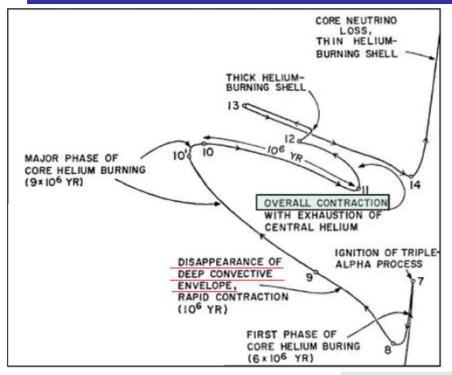
• envelope contracts \rightarrow T<sub>eff</sub> \uparrow
\rightarrow H-b shell compresses
```

→ Overall stellar E-output ↑ (8 → 10)

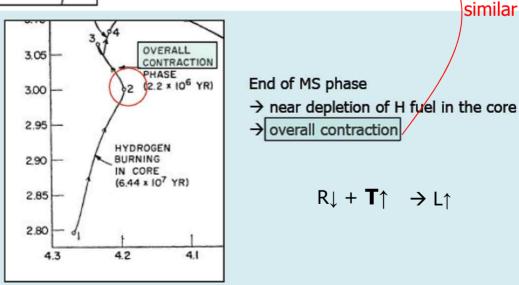
→ shell E-output ↑

Core He-b continues →

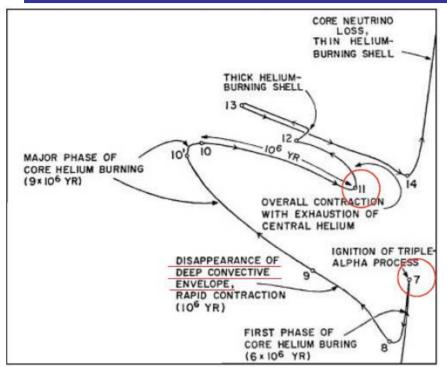
Horizontal Branch (HB) stage: core He-b



- At point 10'
 : core mean-molecular weight increases enough
 → core contracts + envelope expands and cools
- \rightarrow L \downarrow
- At point 11 : core He exhausted
 → entire star contracts

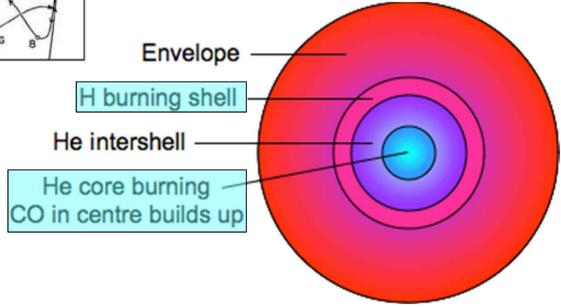


Post-MS Stellar Evolution



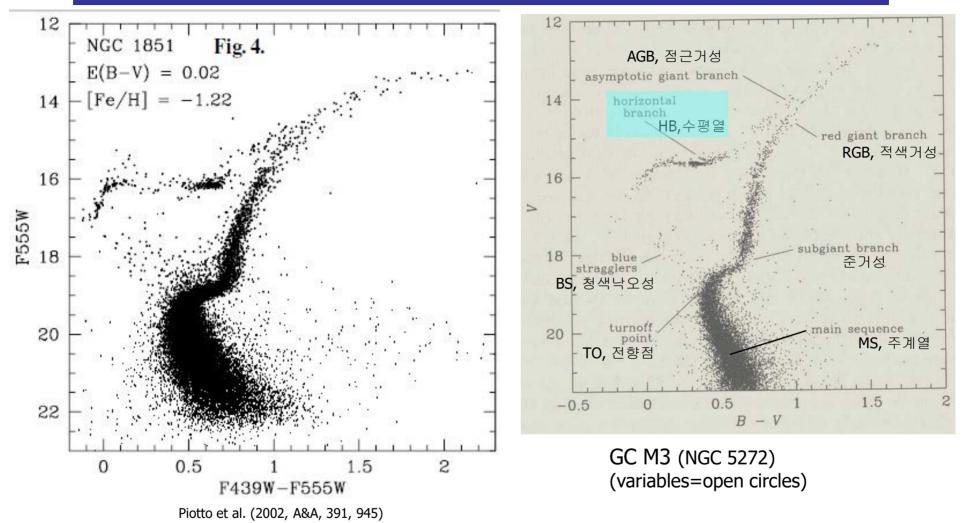
7-11 : generally horizontal evolution "Horizontal branch (HB)"

He-b core + H-b shell



https://astronomy.swin.edu.au/cosmos/H/Horizontal+Branch+stars

Color-Magnitude Diagrams for globular clusters (GCs)



Pulsating Variable Stars

HB stars in the instability strip (IS) = RR Lyrae stars

- →Instabilities in outer envelope
- → Periodic pulsations
- → Variations in L, T, R, surface radial vel.

Instability strip

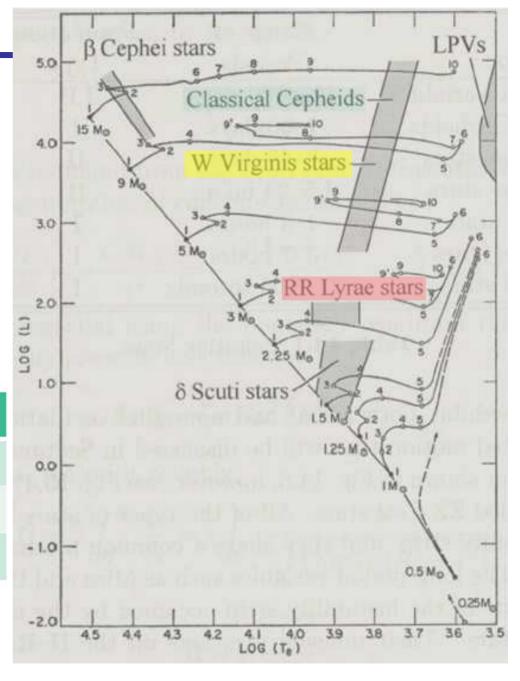
: narrow (~600 - 1000K wide)

Stars evolve horizontally along the IS

→Enters : starts pulsating →Leaves : cease pulsating

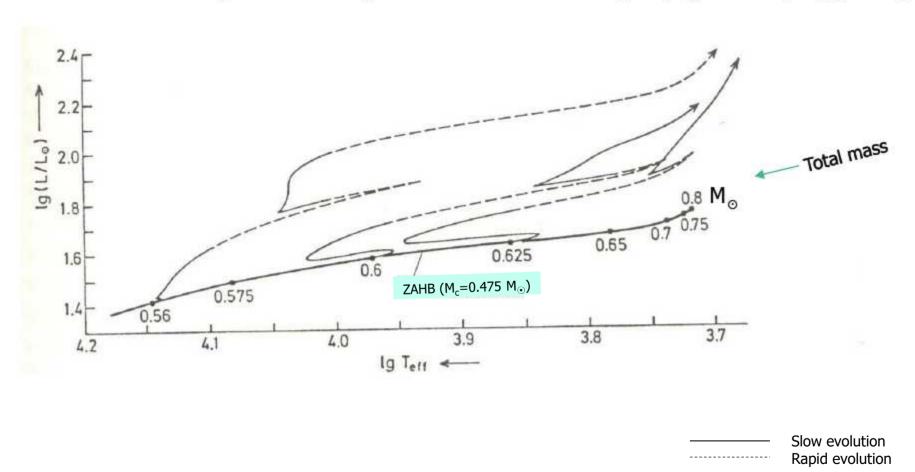
Туре	Periods	Population	Comments
Classical Cepheids	1-50 days	Ι	Radial Pulsation
W Virginis stars (Pop II Cep)	2-45 days	II	Radial Pulsation
RR Lyrae stars	1.5 – 24 hours	II	Radial Pulsation

These stars are distance indicators!

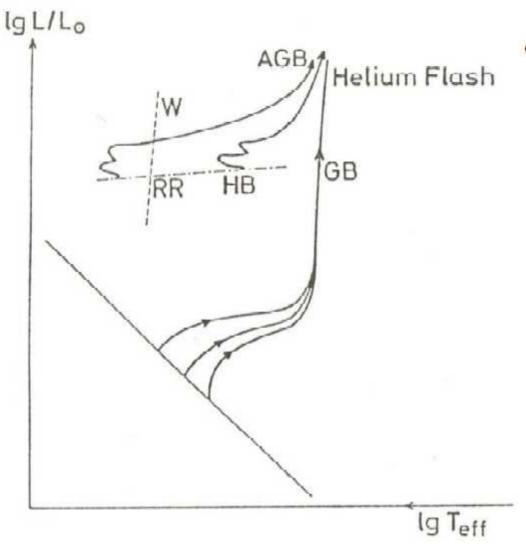


Post-HB Evolution – in HR diagram

Zero-age HB (ZAHB) and evolution afterwards For a He-core of $M_c=0.475~M_\odot$ and a H-rich envelope (XH=0.699, XHe=0.3)



Post-HB Evolution – in HR diagram



 Evolution of low-mass stars, with three different masses

(RR = RR Lyrae stars)

(W = W Virginis stars)

RR Lyrae stars

- Periodic variable stars
- P = 1.5 to 24 h (\sim 12 h), Peak M_V \sim 0-1 mag, spectra A2 to F6
- Current mass ~ 0.8 M_☉ (original MS mass ~ 1 M_☉), old, relatively metal-poor
- In GCs and in low-metallicity systems (population II) But, some have high metallicity

$$M_V(RR) = +0.71 \pm 0.12$$
 at $\langle \text{[Fe/H]} \rangle = -1.61$ for the halo (162 stars)
 $M_V(RR) = +0.79 \pm 0.30$ at $\langle \text{[Fe/H]} \rangle = -0.76$ for the thick disk (51 stars)
Layden et al. (1996, AJ, 112, 2110)

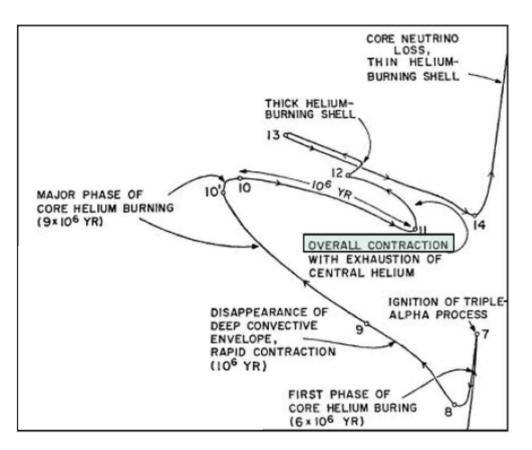
$$\langle M_V \rangle = (0.16 \pm 0.03)[\text{Fe/H}] + 1.02 \pm 0.03.$$
 (5.24)

$$\langle M_K \rangle = -(2.3 \pm 0.2) \log(P/1 \,\mathrm{d}) - 0.88 \pm 0.06$$
 (5.25)

$$\langle M_K \rangle = -(2.0 \pm 0.3) \log(P/1 \,\mathrm{d}) + (0.06 \pm 0.04) [\mathrm{Fe/H}] - 0.7 \pm 0.1$$
 (5.26)

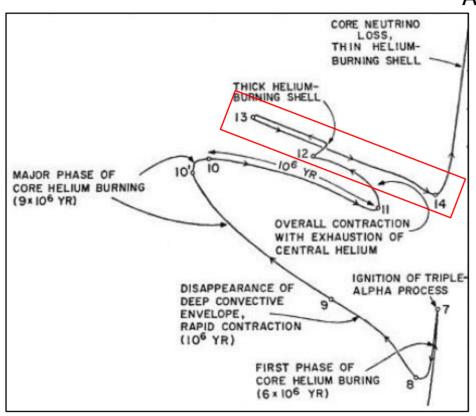
Galactic Astronomy (James Binney and Michael Merrifield, 1998) p. 296

Post-HB, early-AGB (E-AGB)



- At point 11 : core He exhausted
 - → entire star contracts
 - \rightarrow core T \uparrow
- During $11 \rightarrow 12$
 - : thick He-b shell develops

Early-AGB (E-AGB)

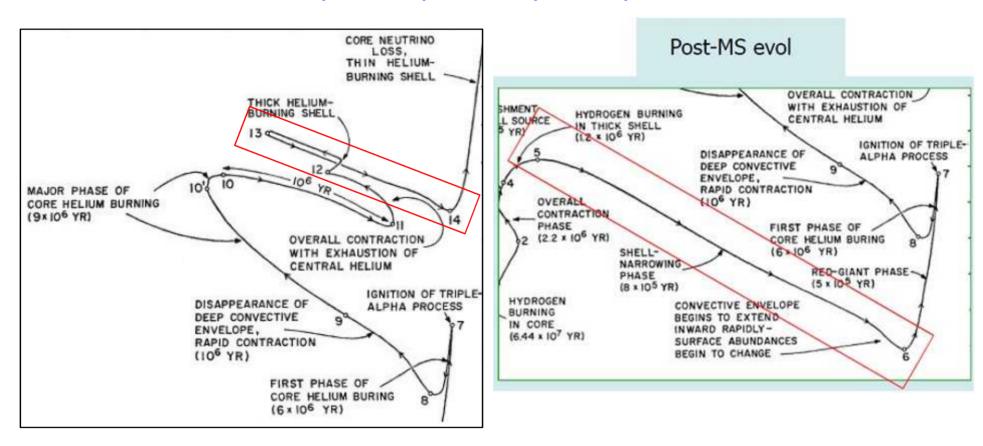


After point 12

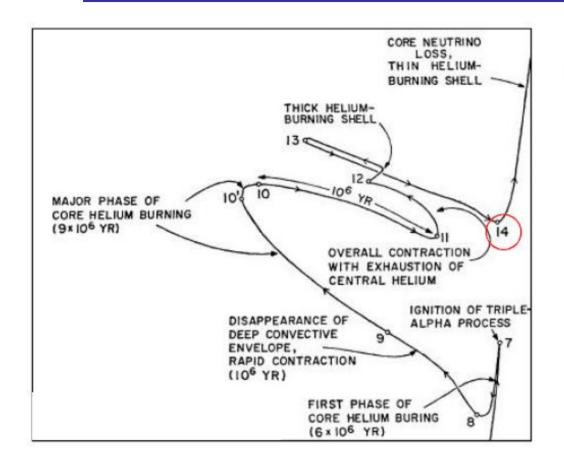
- Core continues to contract,
 He-b shell narrows + produce more E
- Envelope expands + cools
- T → convective envelope deepens again (extending downward to the chemical discontinuity between the H-rich outer layers and the He-rich region above He-b shell)
- → Mixing = second dredge-up
- → Increases He-, N-content of the envelope

Early-AGB (E-AGB)

 $(13\rightarrow14\rightarrow)$ similar to $(5\rightarrow6\rightarrow)$



Early-AGB (E-AGB)



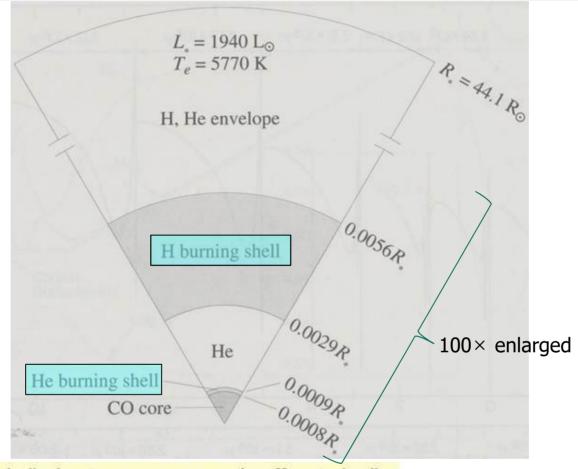
asymptotic giant branch (AGB)

At near point 14 : $T_c \sim 2 \times 10^8 \text{ K}$, $\rho_c \sim 10^6 \text{ g/cm}^3$

Interior structure at point 14 \rightarrow

5 M_☉ star - AGB (asymptotic giant branch)

Inert CO core + Two shell sources (not to scale)



Narrowing He-b shell: begins to turn-on and -off periodically

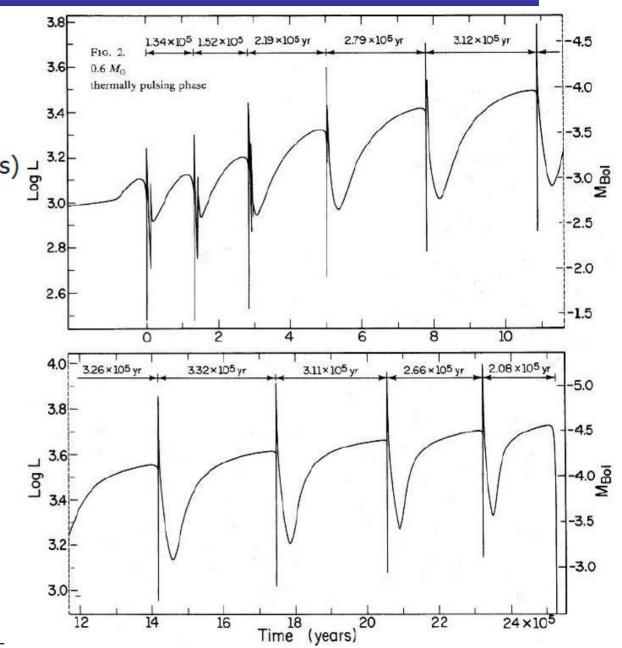
H-b shell dumps He onto the He-layer → the He-layer mass ↑, becomes slightly degenerate → as the He-shell T ↑ → He-shell flash occurs (~core He flash of low-mass stars, but much less energetic)

- → Drives H-b shell outward → cool, turn-off for a time
- thermally

 → Convection between the two shells → H-shell T↑ and burning recovers → repeats → pulsing AGB

Thermally pulsing AGB (TPAGB)

- 0.6 M_☉
- Pulse period = f(stellar mass) = from 10³ years (~5 M_o)
 to 10⁵ years (~0.6 M_o)
- Pulse amplitude grows w/successive event



Iben 1982 (ApJ, 260, 821 – Low mass AGB evolution. I.) – Fig. 2

AGB stars

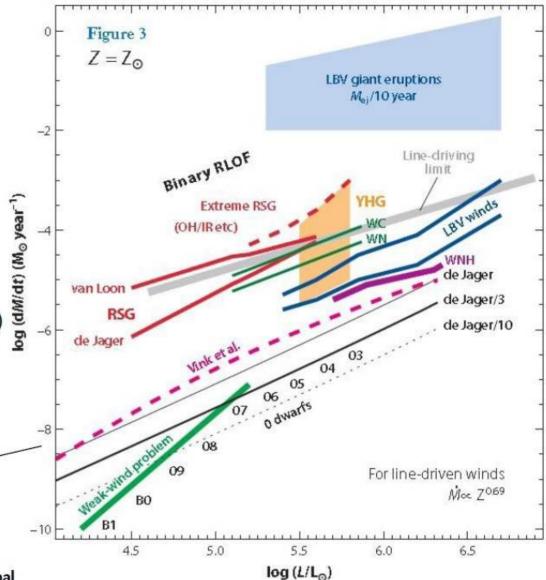
- AGB stars strong wind + rapid mass-loss $\dot{M}\sim 10^{-4}M_{\odot}~yr^{-1}$ Cool (effective T \sim 3,000 K)
 - → dust grains exist in the matter expelled
 - O-rich environment → Silicate grains form
 - C-rich environment → Graphite grains form
- Stellar initial masses evolving to AGB : $0.89 \le \text{M/M}_{\odot} \le 5.0$ (Vassiliadis & Wood 1993, ApJ, 413, 641 : Evolution of low- and intermediate-mass stars to the end of the AGB with mass loss)
- Evolution afterwards
- depends on initial mass and mass-loss
- massive stars burnings up to form Fe → neutron stars (NSs), black holes (BHs)
- low-mass stars white dwarfs (WDs) + planetary nebulae (PNe)
- dividing mass ~ 8 M_☉

AGB – mass loss

L↑, R↑ → Mass-loss rate↑
 Stellar mass↓ → surface gravity↓
 → surface material is less tightly bound

→ mass-loss becomes progressively more important as AGB evolution continues

• At the end of the mass-loss phase \rightarrow superwind ($\dot{M}\sim 10^{-4} M_{\odot}~yr^{-1}$



N. Smith 2014, ARA&A, 52, 487 (Mass Loss: Its Effect on the Evolution and Fate of High-Mass Stars)

- RSG: van Loon+05 (A&A 438 273)
- RSG: de Jager+ 88 (A&AS 72 259)
- · WNH: H-rich Wolf-Rayet stars
- O-type stars: Vink + 01 (A&A 531 A132)
- de Jager+88 (A&AS 72 259) = "standard" observational rates for O-type stars
- Weak-wind problem: lower mass-loss rates for late Otype and early B-type MS stars

Thermonuclear Energy Generation Stages

Process	Fuel	Major products	Temperat ure (K)	Minimum mass (M _☉)
H-burning	Н	He	1-3×10 ⁷	0.1
He-burning	He	¹² ₆ C, ¹⁶ ₈ O	2×108	1
C-burning	С	¹⁶ O, ²⁰ Ne, ²³ Na, ²³ Mg, ²⁴ Mg	8×10 ⁸	1.4
Ne-burning	Ne	¹⁶ ₈ O, Mg	1.5×10^{9}	5
O-burning	0	²⁴ ₁₂ Mg, ²⁷ ₁₃ AI, ²⁸ ₁₄ Si, ³¹ ₁₅ P, ³² ₁₆ S	2×10 ⁹	10
Si-burning	Mg to S	near Fe	3×10^9	20

- C-burning: near 6×10⁸ K (Carroll & Ostlie, p. 348)
- Ne-burning: 1.2-1.9×10⁹ K (El Eid+04 ApJ 611 452 Evolution of massive stars up to the end of central Oxygen burning) (https://en.wikipedia.org/wiki/Neon-burning_process)
- O-burning: 1.5-2.6×10⁹ K (El Eid+04 ApJ 611 452 Evolution of massive stars up to the end of central Oxygen burning) (Carroll & Ostlie, p. 348)

Thermonuclear Energy Generation Stages

Table 1 Evolution of a 15-solar-mass star.

Stage	Timescale	Fuel or product	Ash or product	Temperature (10 ⁹ K)	Density (gm cm ⁻³)	Luminosity (solar units)	Neutrino losses (solar units)
Hydrogen	11 Myr	Н	Не	0.035	5.8	28,000	1,800
Helium	2.0 Myr	He	C, 0	0.18	1,390	44,000	1,900
Carbon	2000 yr	С	Ne, Mg	0.81	2.8×10^{5}	72,000	3.7×10^{5}
Neon	0.7 yr	Ne	O, Mg	1.6	1.2×10^{7}	75,000	1.4×10^{8}
Oxygen	2.6 yr	O, Mg	Si, S, Ar, Ca	1.9	8.8×10^{6}	75,000	9.1×10^{8}
Silicon	18 d	Si, S, Ar, Ca	Fe, Ni, Cr, Ti,	3.3	4.8×10^{7}	75,000	1.3×10^{11}
Iron core collapse*	~1 s	Fe, Ni, Cr, Ti,	Neutron star	>7.1	$>\!7.3\!\times\!10^9$	75,000	$>\!3.6\times10^{15}$

^{*} The pre-supernova star is defined by the time at which the contraction speed anywhere in the iron core reaches 1,000 km s⁻¹.

Evolving up the AGB (asymptotic giant branch)

- Early-AGB (E-AGB): He shell burning Radius increases (up to 1 AU ~215 R_☉)
- Thermally pulsing AGB (TP-AGB): when He shell runs out of fuel
 H shell burning
 When He builds up → He-shell ignites explosively (He-shell flash)
- He-b shell converts more and more of the He into C and then into O
 - \rightarrow CO core mass \uparrow / core contracts slowly, $\rho_c \uparrow$
 - → electron degeneracy pressure begins to dominate
- Similar to the development of an electron-degenerate He-core in a low-mass star during its rise up the RGB

 $M_i < 4 M_{\odot} \rightarrow no$ CO-burning $4 < M/M_{\odot} < 8$, if no mass-loss \rightarrow CO-core mass increase \rightarrow catastrophic core-collapse

Massive ($M \ge 8 M_{\odot}$) star evolution

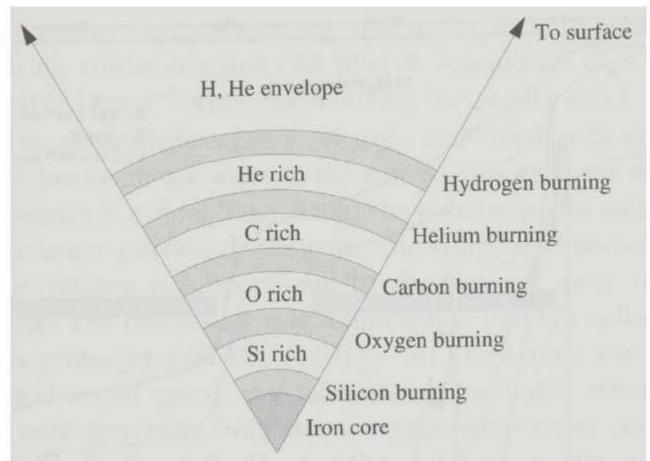
- He-b shell → add ash to the CO core
- CO core continues to contract
- ¹²₆C-burning starts
- By-products: ¹⁶₈O, ²⁰₁₀Ne, ²³₁₁Na, ²³₁₂Mg, ²⁴₁₂Mg
- NeO core
- ¹⁶₈O-burning starts
- → Making ²⁸₁₄Si-dominated (and ³²₁₆S) core

At T \sim 3×10⁹ K \rightarrow Si-burning starts

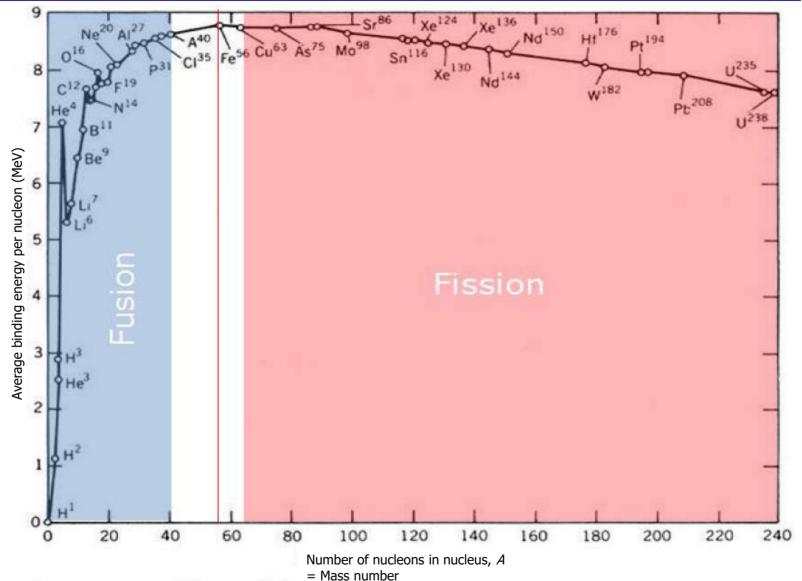
Making iron(
$${}^{56}_{26}$$
Fe)-peak elements, like ${}^{54}_{26}$ Fe, ${}^{56}_{26}$ Fe, ${}^{56}_{28}$ Fe, and finally iron-core
$$50 \le A \le 62$$
(Atomic mass)

Massive (M ≥ 8 M_{\odot}) star evolution

Onion-like interior → (not to scale)

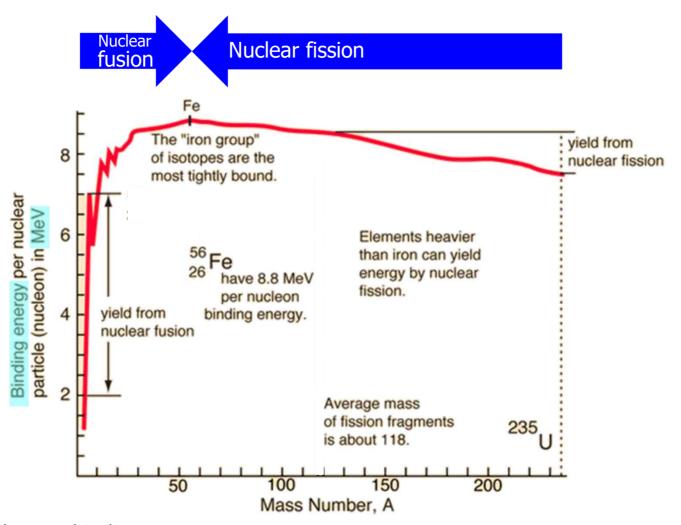


Binding energy per nuclear particle



⁵⁶₂₆Fe: the most stable nuclei

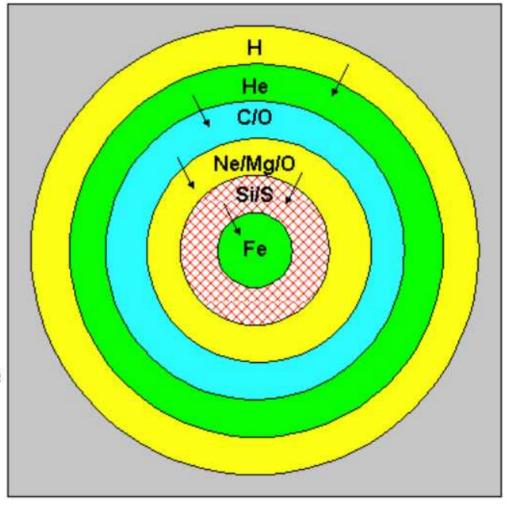
Binding energy per nucleon (E_b/A)



Fe: The largest binding energy
The most stable element
The final product of both nuclear fusion and nuclear fission

End of Massive stars → Core-Collapse Supernovae (CCSNe)

- Iron core collapse
- → shock wave propagates outward
- → outer layers (envelope) follows collapse
- → explosion!
- (expansion velocity ~a few 10⁴ km/s)



 $https://www.astro.umd.edu/{\sim} richard/ASTR680/A680_SNR_2019_lec1.pdf$

- Final stage of the evolution of massive stars
 - Outer part → SN explosion → interstellar media
 - Inner part → neutron stars (pulsars) or black holes

Supernova SN 1987A (in Large Magellanic Cloud, LMC)

SN 1987A (II peculiar, LMC)

Tarantula Nebula

d~49.97 kpc (Pietrzynski+ 13 Nature 495 76)

1987 Feb 23.316 (UT)

B3 I (supergiant)

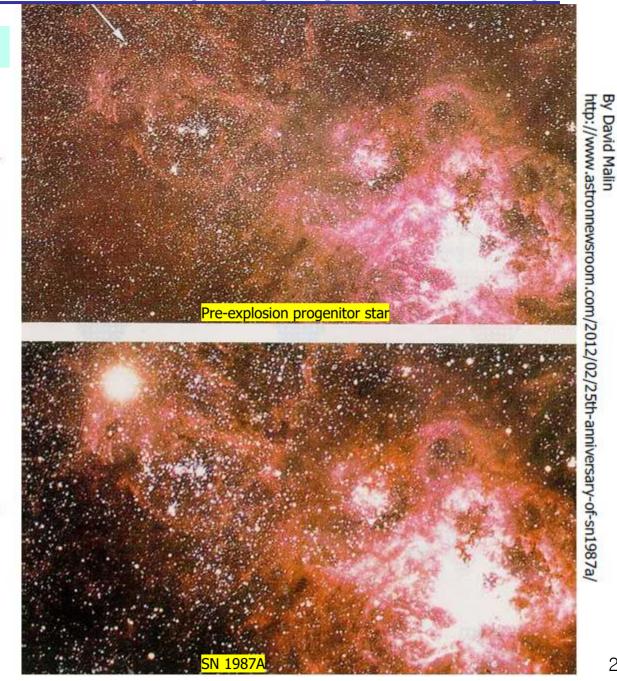
Peak: +2.9 mag

(B-V) = +0.085

 $T_{\rm eff} = 16,000 \text{ K}$

L ≥ 105 L_☉

 \rightarrow $M_{initial}\sim 20~M_{\odot}$ (N. Smith 2007 AJ 133 1034)



3. Star Deaths (별의 죽음) 3-2 Supernova Explosion (초신성 폭발)



Supernova (SN) types

- Brightest objects in galaxies $(M_V = -14 \sim -22)$
- Typical types

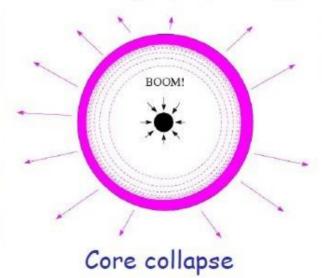
No H lines (pop II) → Type Ia Ib Ic

H lines (pop I) \rightarrow Type II



WD + Giant/MS/He *
(Single Degenerate, SD)

WD + WD (Double Degenerate, DD)



CC SNe

SNe Ia (thermonuclear stellar explosion)
(WD originated SNe)
백색왜성 기원 초신성

핵붕괴 초신성

http://dujs.dartmouth.edu/2008/05/type-ia-supernovae-properties-models-and-theories-of-their-progenitor-systems http://wwwmpa.mpa-garching.mpg.de/mpa/research/current_research/hl2013-8/hl2013-8-en.html http://spiff.rit.edu/richmond/sdss/sn_survey/sn_survey.html

Supernova (SN) types

- Brightest objects in galaxies $(M_V = -14 \sim -22)$
- Typical types

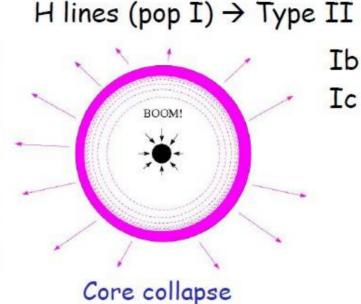
No H lines (pop II) → Type Ia



WD + Giant/MS/He *
(Single Degenerate, SD)

WD + WD (Double Degenerate, DD)

SNe Ia (thermonuclear stellar explosion)
(WD originated SNe)
백색왜성 기원 초신성



CC SNe

핵붕괴 초신성

http://dujs.dartmouth.edu/2008/05/type-ia-supernovae-properties-models-and-theories-of-their-progenitor-systems http://wwwmpa.mpa-garching.mpg.de/mpa/research/current_research/hl2013-8/hl2013-8-en.html http://spiff.rit.edu/richmond/sdss/sn_survey/sn_survey.html

Supernovae taxonomy

Туре	Sub-types				
I No H	Ia Si II (6150Å) absorption near peak light				
	Ib/c Weak/no Si absorption	Ib He I (5876Å) emi	ssion		
		Ic Weak/no He			
II	II-P/L/n	II-P/L No narrow lines	II-P Plateau in light curve(LC: mag vs time)	SNII-P	
			II-L Linear decrease in LC	SNII-L	
	Some narrow lines	IIn: Hines			
			Ibn: He lines		
She	IIb: Spectrum chan	ges to become like t	ype Ib		

Ic-BL: sometimes associated with GRBs and/or hypernova (broad lines: (2-3)×10⁴ km/s)

ultra-bright type II: ~10⁵¹ erg radiation energy

.Ia: changing rapidly

Superluminous SNe, pair-instability SNe, Superluminous Ia

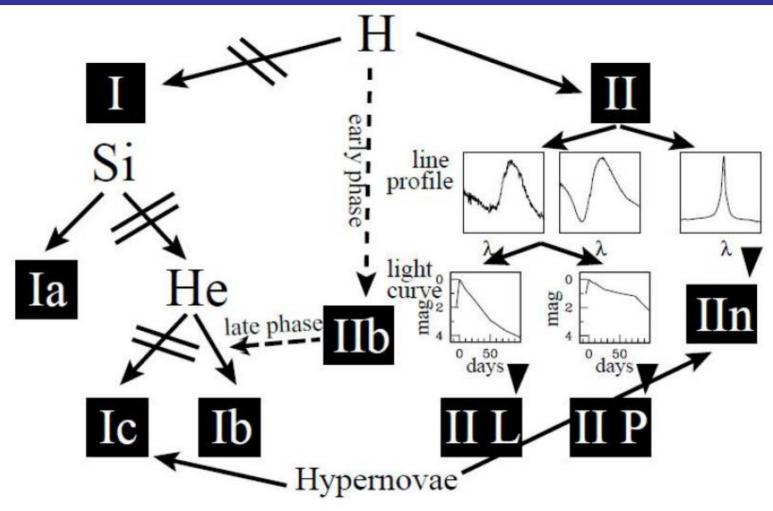
Subluminous SNe, Subluminous Ia

Super-Chandrasekhar Ia: mass > Chandrasekhar limit

Ia-IIn: CSM

Iax : Ia w/lower L, less E, less ejecta mass Kilonova/macronova, SN imposters, magnetar

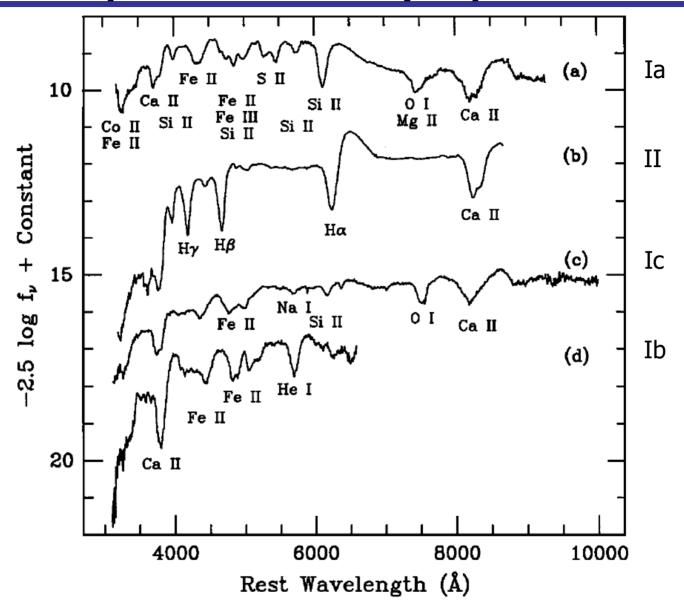
Supernova taxonomy



Cappellaro & Turatto (2000) Figure 2

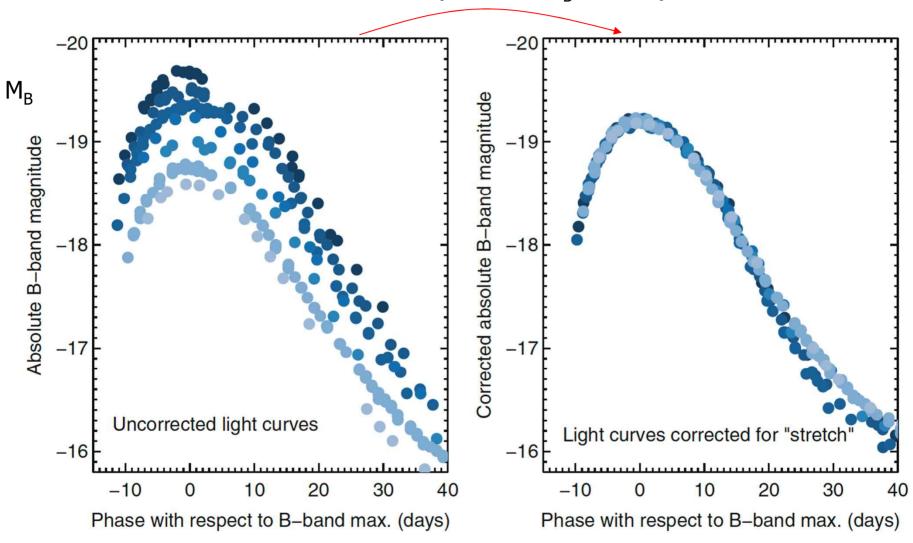
https://arxiv.org/abs/astro-ph/0012455

Supernova taxonomy - spectra



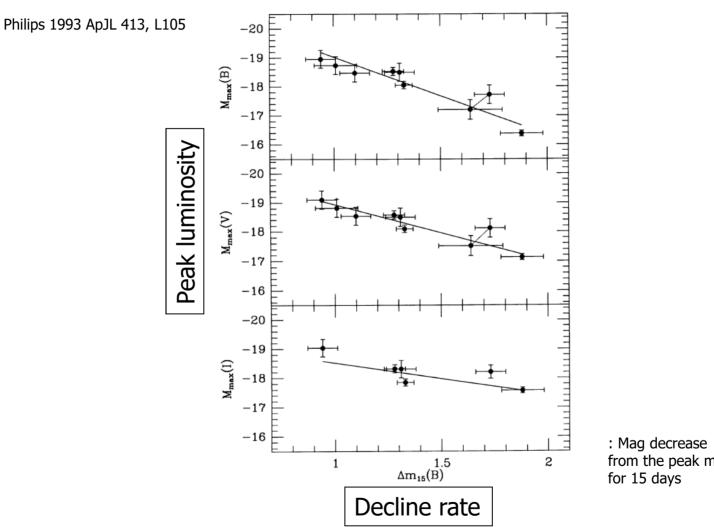
Supernova(SN) Ia light curve (LC)





Supernova(SN) Ia light curve (LC)

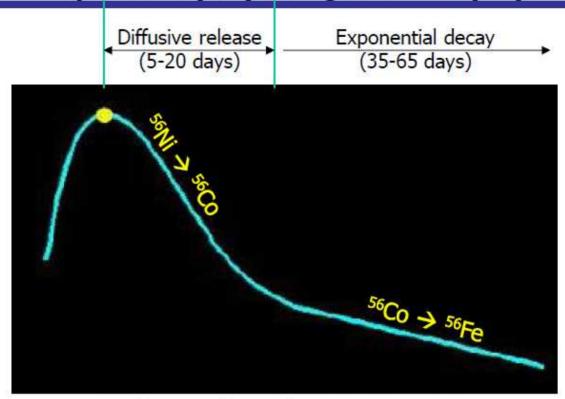
Correction (LC width-M_B relation)



from the peak mag

Fig. 1.—Decline rate-peak luminosity relation for the nine best-observed SN Ia's. Absolute magnitudes in B, V, and I are plotted vs. $\Delta m_{15}(B)$, which measures the amount in magnitudes that the B light curve drops during the first 15 days following maximum.

Supernova(SN) Ia light curve (LC)

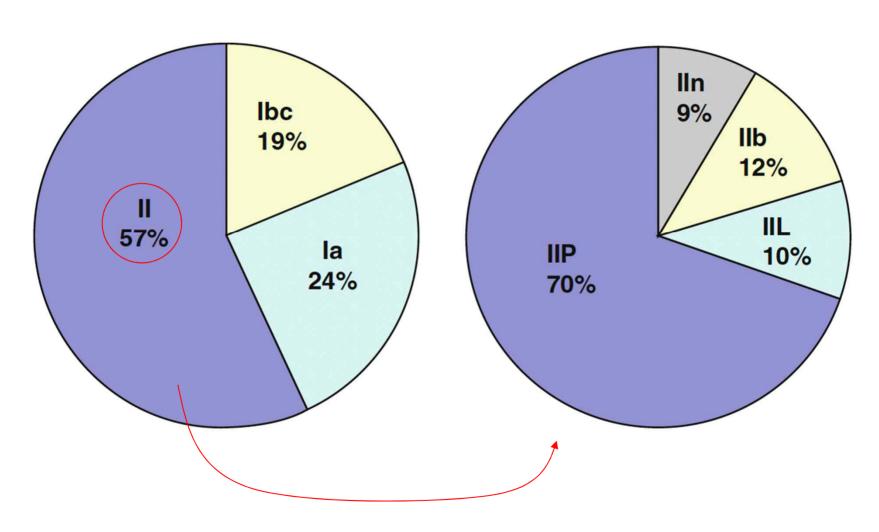


Radioactive decay of $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ (Colgate & McKee 69 ApJ 157 623; Arnett 82 ApJ 253 785)

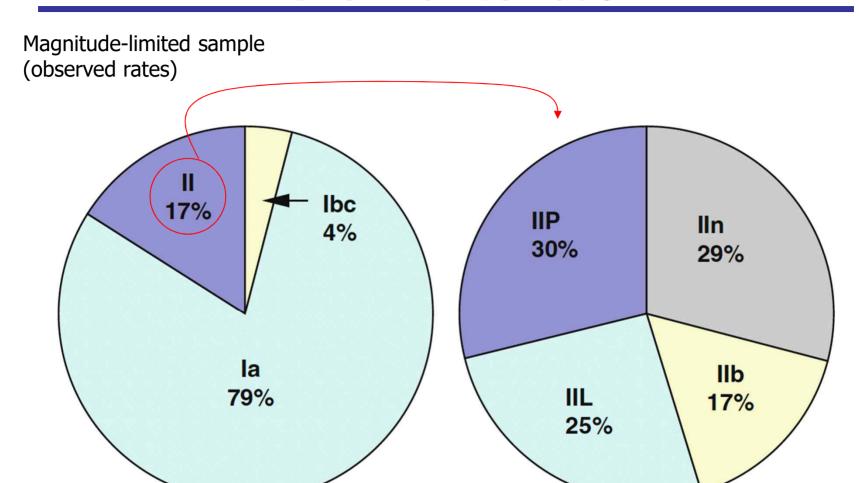
$$\begin{array}{c} {}^{56}{\rm Ni} \ \to \ ^{56}{}{\rm Co} + e^+ + \nu_e + \gamma & {}^{6.10}{} \, {\rm d, \, E=1.72 \, MeV} \\ {}^{56}{\rm Co} \ \to \ ^{56}{}{\rm Fe} + e^+ + \nu_e + \gamma & {}^{78.8}{} \, {\rm d, \, E=3.59 \, MeV} \end{array}$$

SNe – number ratio

Volume-limited sample (Intrinsic rates)



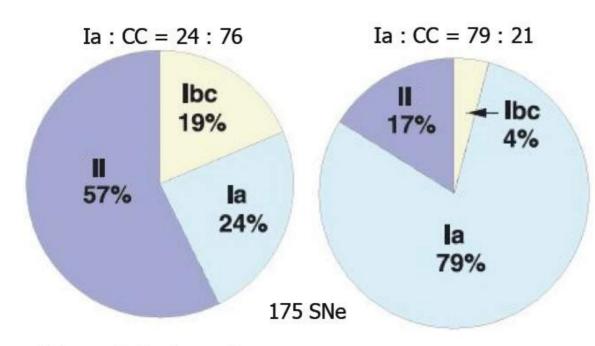
SNe – number ratio



SNe Ia – distance indicator

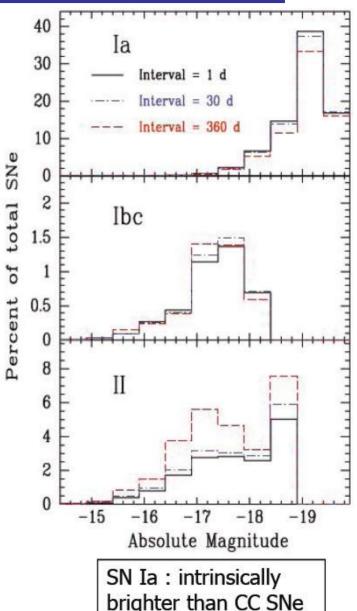
- SN Ia maximum brightness (peak luminosity) $M_V \approx -19.30 \pm 0.03 + 5 \log (H_0 /60)$
- Rising time ~ 20 days

Number ratio



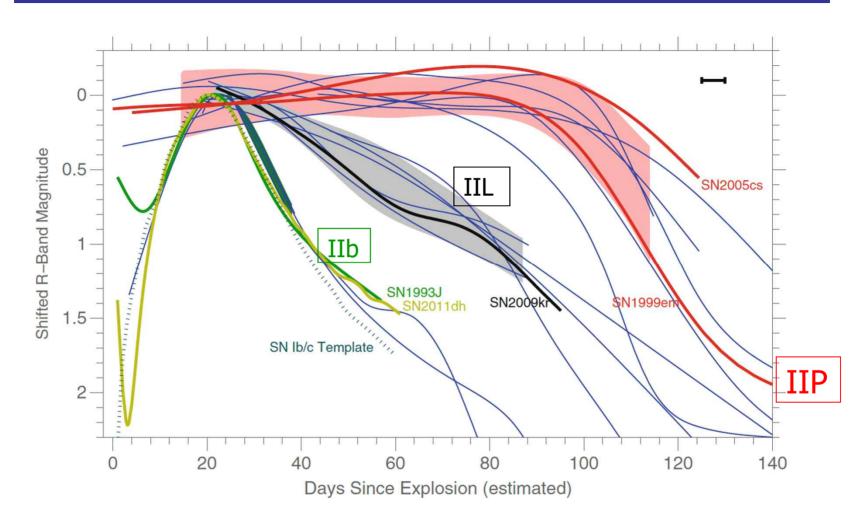
Volume-limited sample $D \le 60 \text{ Mpc} (CC \text{ SNe})$ $D \le 80 \text{ Mpc} (SNe Ia)$

Ideal magnitude-limited sample



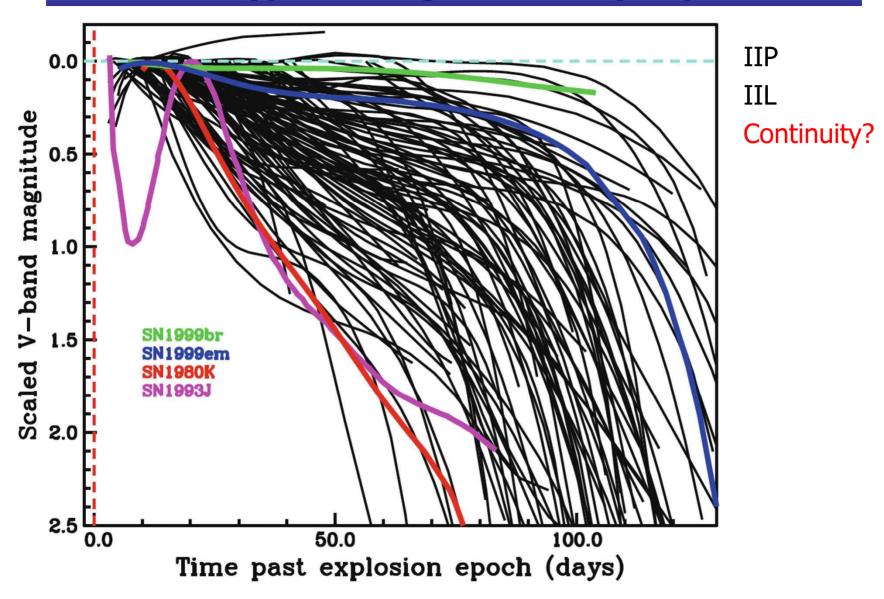
brighter than CC SNe

Type II – Light Curves (LCs)



Arcavi+12 (ApJL 756 L30) (see also Faran+14 MN 445 554)

Type II – Light Curves (LCs)



CC SNe vs. SNe Ia

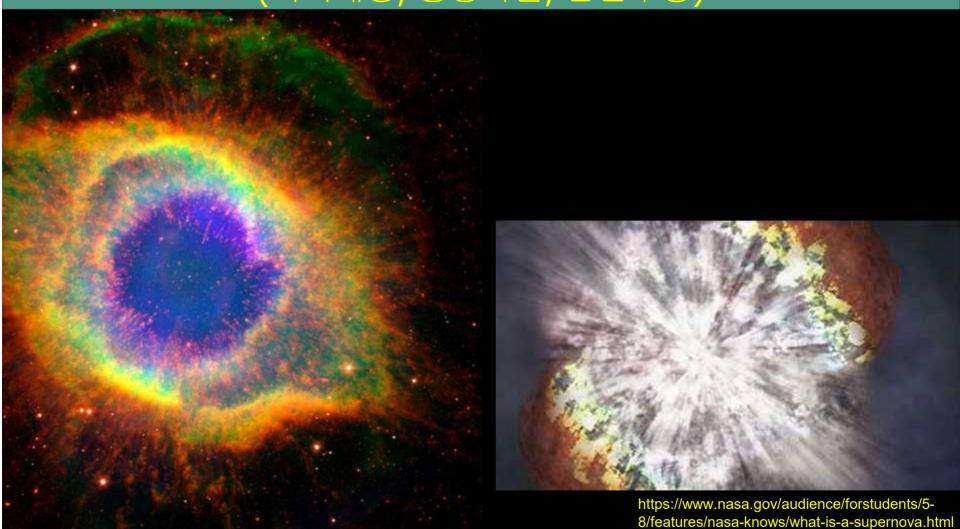
- CC SNe: not in early-type galaxies, Spiral galaxies spiral arms
- SN Ia: all types of galaxies (Spiral galaxies no spatial preference)
- spiral arms = short-lived massive stars
- → CC SNe = massive stars
 SN Ia = not that massive stars
- Total energy emitted in neutrinos 3×10⁵³ erg
- Total kinetic energy ~10⁵¹ erg = f.o.e. = foe
- ~1% of total kinetic energy comes in photons ~ 10^{49} erg (peak luminosity ~ 10^{43} erg/s, ~ 10^9 L $_{\odot}$, ~brightness of an entire galaxy)

CC SNe vs. SNe Ia

- Total energy emitted in neutrinos 3×10⁵³ erg
- Total kinetic energy ~10⁵¹ erg = f.o.e. = foe
- ~1% of total kinetic energy comes in photons ~10⁴⁹ erg (peak luminosity ~10⁴³ erg/s, ~10⁹ L_o, ~brightness of an entire galaxy)



3. Star Deaths (별의 죽음) 3-3 White Dwarfs, Neutron Stars, and Black Holes (백색왜성, 중성자별, 검은구멍)

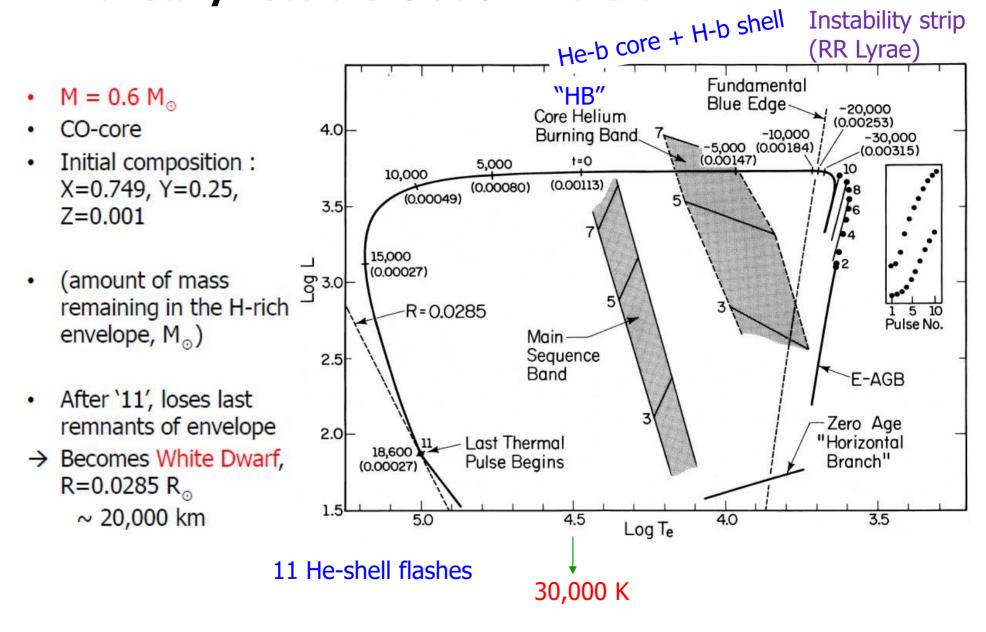


Helix Nebula in the Aquarius

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https://www.dailymail.co.uk/sciencetech/article-3517714/The-dying-star-rewrite-know-stellar-evolution-White-dwarf-bizarre-atmosphere-oxygen.html

1. Planetary nebula evolution in the CMD



WDs - general

RG

```
Extremely low-mass (M_i \le 0.2 M_{\odot}) \rightarrow \text{no He-b} \rightarrow \text{He WD}

M_i \le 8 M_{\odot}, T_c < 10^9 \text{ K, No C-burning} \rightarrow \text{CO-core} + \text{PN}

\rightarrow \text{CO WD}

8 < M_i < 10.5 M_{\odot} \rightarrow \text{C-b, but no Ne-b}

\rightarrow \text{ONeMg WD}
```

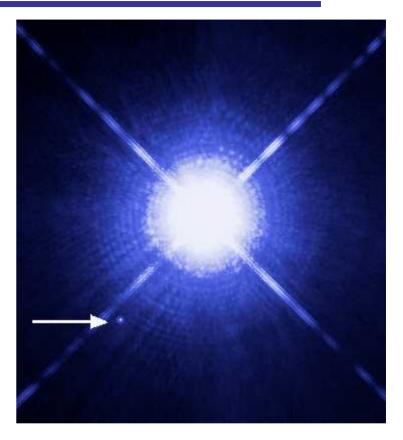
No fusion reactions, supported by electron degenerate pressure

```
(Current) Mass: 0.55-0.6~{\rm M}_{\odot}
- min mass \sim 0.4~{\rm M}_{\odot}
- max mass for a non-rotating WD = Chandrasekhar mass \sim 1.4~{\rm M}_{\odot}
Typical density \sim 10^6~{\rm g/cm}^3
Typical B \sim 10^6~{\rm Gauss}~(2\times 10^3 - 10^9~{\rm Gauss})
```

Binary – mass transfer from a companion → CO WD mass approaches the Chandrasekhar mass → C-detonation → SN Ia

Evolution: hot WD → E decreases, color reddens → black dwarf (timescale > 13.8 Gyr)

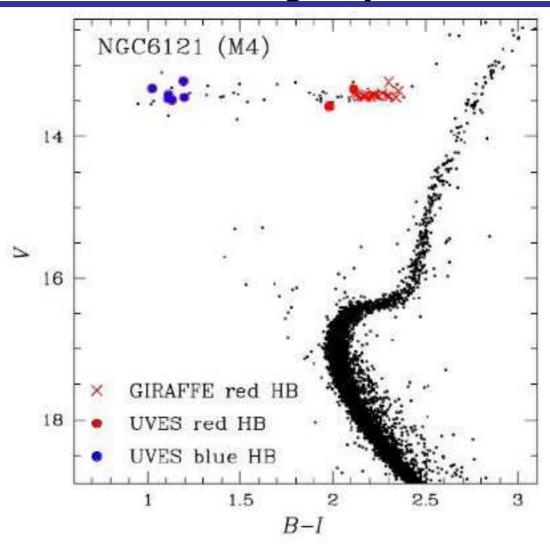
→ no black dwarf yet formed



Sirius A, Sirius B (WD) - HST

https://en.wikipedia.org/wiki/White_dwarf

WD – cooling sequence



Marino+11 (ApJ 730 L16 - Sodium-Oxygen Anticorrelation Among HB Stars in the GC M4)

WD – cooling sequence

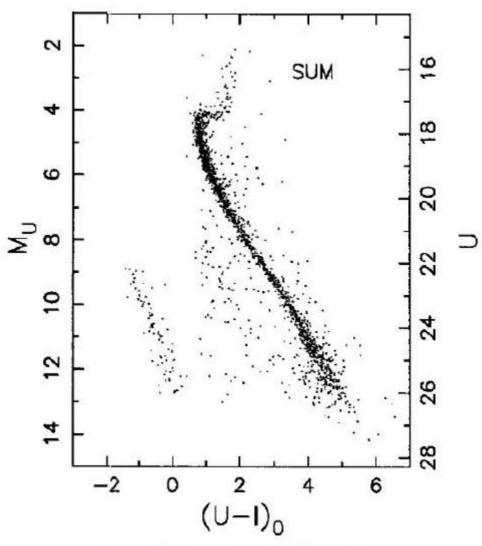
WD cooling line of the GC M4

: bluest stars

: U = 22 - 26 mag

$$L \propto T_{eff}^4$$
 or
$$M_{bol} = -10 \log T_{eff} + const.$$

- Constant radius line
- Roughly parallel to the MS
- Stretched not by mass, but by age (young → old)



U-(U-I) CMD of GC M4 (NGC 6121)

HST

Richer et al. 1995 (ApJ 451 L17 - HST Observations of WDs in the GC M4)

WD – cooling sequence

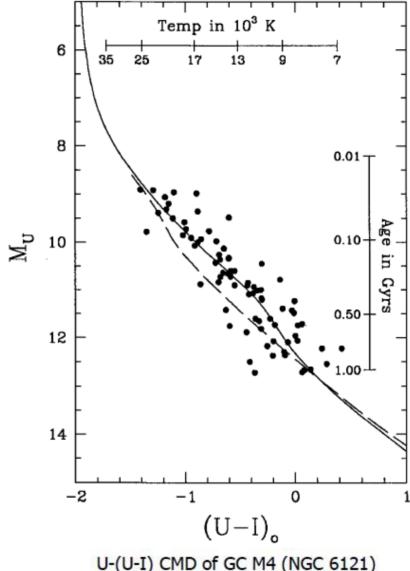
WD cooling line of the GC M4

: bluest stars

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U-(U-I) CMD of GC M4 (NGC 6121) HST

Richer et al. 1995 (ApJ 451 L17 - HST Observations of WDs in the GC M4)

2. Neutron Stars (NSs, 중성자별)

- M_i ≥8 M_☉
- $M_f > 1.4 M_{\odot} \rightarrow$ degenerate electron gas pressure cannot hold off gravity \rightarrow matter is crushed to very high densities \rightarrow inverse- β decay occurs :

$$p^+ + e^- \rightarrow n + \nu$$

(protons and electrons are squeezed into neutrons)

- Degenerate neutron gas → provides internal pressure
- Mass: 1.1 2 M_☉ (upper limit ~ 3 M_☉)

```
Typical mass \sim 1.4~{\rm M}_{\odot}

Typical radius \sim 10~{\rm km}

Typical density \sim 4 \times 10^{14}~{\rm g/cm^3}

(400,000~{\rm ton/mm^3})

B \sim 10^{12}~{\rm Gauss}

Surface temp \sim 6 \times 10^5~{\rm K}
```

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※ 1 Tesla = 10<sup>4</sup> Gauss

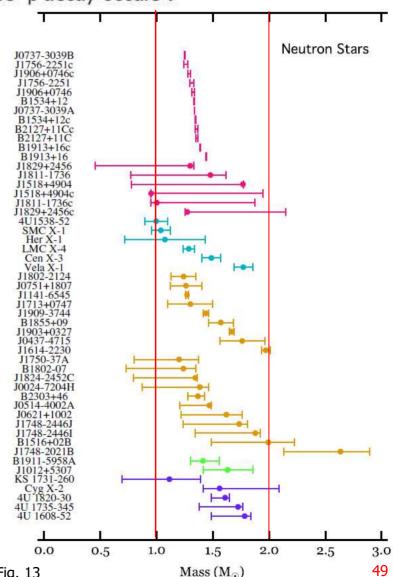
※ Sun: mean density ~ 1.408 g/cm<sup>3</sup>

B<sub>o</sub>~1-2 Gauss

※ Earth: B<sub>⊕</sub>~0.1 Gauss

※ Magnetic Resonance Imager (MRI, 자기공명영상):

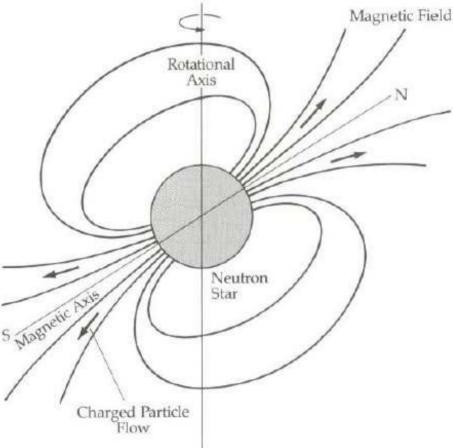
B~10<sup>5</sup> B<sub>⊕</sub>~1 Tesla
```



2-1. Pulsars (펄사)

- pulsating star = pulsar = rotating neutron stars
- 1967 discovered by Jocelyn Bell Burnell and Anthony Hewish (1974 Nobel prize)
 Radio pulses coming every 1.33730113s (at 81.5 MHz)+ 3 more objects
- Now, N \sim 2300, P \sim 1.6 \times 10⁻³ to 4.0 s (average 0.65 s)
- · Strong magnetic field
- → Synchrotron radiation along the magnetic dipolar axis
- Lighthouse model: transforms the rotational energy into electromagnetic energy → emits through the magnetic axes → pulse period = rotation period
- If the Sun becomes a NS → R = 7 km

TABLE 17–1 Properties of Selected Pulsars				
Name (PSR)	Period (s)	dP/dt (10 ⁻¹⁵ s/s)	DM(pc/cm ³)	
1937 + 21	0.001557	1.07×10^{-4}	71.2	
1855 + 09	0.005362	46421	13.3	
0531 + 21 (Crab)	0.033326	421	56.8	
0833 - 45 (Vela)	0.089234	124	69.1	



magnetic axis is inclined with respect to the rotation axis

2-1. Pulsars (펄사)

- Dispersion: a given pulse arrives at the Earth later as we look at lower frequencies
 - Due to a slowing down of the photon velocity by electrons in the line-of-sight (los)
 - Longer λ are slowed down more
 - Observations can tell us the mean electron density in the los
- If pulses of ν₁ and ν₂ (ν₁ > ν₂) are emitted at time t₀ → they arrive at t₁ and t₂,

- respectively $\text{Then,} \quad t_1-t_0=\frac{d}{v_1} \quad \text{and} \quad t_2-t_0=\frac{d}{v_2}$ $\text{We can measure} \quad t_2-t_1 \ \Rightarrow \text{equal to} \quad (\frac{1}{v_2}-\frac{1}{v_1})d$
- Velocities depend on the electron density → if we know the electron density, we can get the distance
- Dispersion Measure (DM) = integrated electron density : $DM = \int_{e}^{a} n_e dl$

Introductory Astronomy and Astrophysics (4th edition) Michael Zeilik & Stephen A. Gregory (1998), p. 340 Properties of Selected Pulsars

TABLE 1/-1 Troperdes of Selection Familia				
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2-1. Pulsars (펄사) - Discoverer



https://thewire.in/science/women-astrophysics-editors-stem

Jocelyn Bell Burnell (Source : YouTube)

Binary pulsars (쌍성 펄사)

- First binary pulsar, PSR B1913+16: 1974 at Arecibo by Joseph H. Taylor, Jr. (조세프 테일러, b. 1941), Russell Hulse (러셀 헐스, b. 1950)
- "Hulse-Taylor binary pulsar"
- Einstein's theory of general relativity → two NSs would emit gravitational waves as they orbit a common center of mass
- Gravitational waves carry away orbital energy → cause the two stars get closer → orbital P ↓

The Nobel Prize in Physics 1993

Russell A. Hulse and Joseph H. Taylor Jr. "for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"

https://www.nobelprize.org/prizes/lists/all-nobel-prizes-in-physics/

3. Black Holes (BHs, 검은구멍)

- A region of spacetime in which gravity is so strong that nothing (incl. light) can escape it
- Minimum mass ~ 3 M_o
- Theoretically zero volume and infinite density → "singularity" (breakdown of the laws of physics)
- An object with escape speed $v_{\it esc}$ at the surface of the BH :

Total Energy = KE + PE =
$$\frac{mv_{\it esc}^2}{2} - \frac{\it GmM}{\it R} = 0$$

Assuming max escape velocity = c

$$R = \frac{2GM}{c^2} = 3M \text{ km}$$

- If the Sun becomes a BH → density ~10¹⁶ g/cm³ (~nucleus of an atom)
- If an object cross the Schwarzshild radius, it crashes into a singularity (zero volume)

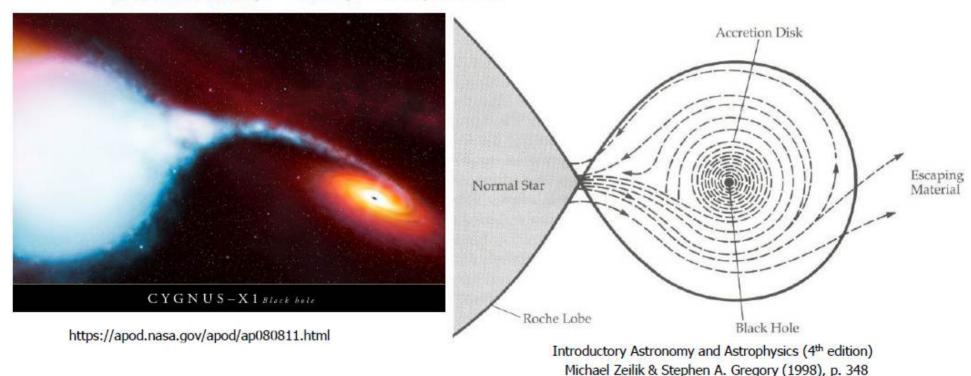
Schwarzshild radius



black_hole_baird.jpg http://sciencequestionswithsurprisinganswers.org/20 13/06/18/can-you-go-fast-enough-to-get-enoughmass-to-become-a-black-hole/

3. Black Holes (BHs, 검은구멍)

- How to observe/find a BH?
- not for an isolated BH → but for a BH in a binary system (interactions with other material)
- matter falling toward a BH gains kinetic energy → heats up, becomes ionized → emits electromagnetic radiation
- If T reaches a few × 10⁶ K → emits X-rays
- accreted material + initial angular momentum → form a disk around the BH : accretion disk (강착원반) = X-ray source



X-ray sources: good candidates for BHs