

Vorticity Homework

Interior Sverdrup Transport:

$$M_y = -\frac{1}{\beta} \frac{\partial \tau_x}{\partial y}$$

$$M_y = -\frac{\partial \tau_x}{\partial f}, \text{ where } \beta = \frac{\partial f}{\partial y}$$

$$M_y = -\frac{(-0.05 - 0.05)}{2\Omega(\sin(30^\circ) - \sin(20^\circ))}$$

$$M_y = 4353.5 \text{ kg m}^{-1} \text{ s}^{-1}$$

Distance, x , from the non-slip Western boundary to the axis of the Florida Straits Current:

Using dimensional analysis to get a mass transport balance,

$$M_y \times \text{width of Atlantic at } 26^\circ \text{ N} = \rho z \int_0^{x_{max}} v(x) dx$$

Assume an approximate width of the North Atlantic, at 26° N , to be 6300 km, and substitute $v(x) = \frac{\partial v}{\partial x} x$, and $\frac{\partial \tau_x}{\partial y} = \rho k \frac{\partial v}{\partial x}$ to solve for x :

$$M_y \times 6300,000 = \rho z \frac{\partial \tau_x}{\partial y} \frac{1}{\rho k} \frac{x^2}{2}$$

$$4353.5 \times 6300,000 = z \left(\frac{0.1}{111,000 \times 10} \right) \frac{1}{k} \frac{x^2}{2}$$

Assume that vorticity and transport balance is achieved within the upper 800 m (depth, z) of the Florida Straits, and $k = 2 \times 10^{-6}$.

Therefore, $\boxed{x = 39 \text{ km (2 s.f.)}}$.

Maximum current, v , at the axis of the Florida Straits Current:

$$v = \frac{\partial v}{\partial x} x$$

$$= \frac{\partial \tau_x}{\partial y} \frac{1}{\rho k} x$$

$$= \frac{0.1}{111,000 \times 10} \frac{1}{1025 \times 2 \times 10^{-6}} \times 39,000$$

Therefore, $\boxed{v = 1.7 \text{ ms}^{-1} \text{ (2 s.f.)}}$