Solutions Estimate the amount of heat, Q, that must pass across the stratified layer to maintain a steady state.

Set up a heat budget calculation:

heat in = heat out

$$Q_{geo} + Q_{mixing} + Q_{GINS} + Q_{Lab} + Q_{Med} = Q_{upwelling} + Q_{so}$$

$$\begin{aligned} Q &= Q_{mixing} - Q_{upwelling} \\ &= Q_{s0} - (Q_{geo} + Q_{GINS} + Q_{Lab} + Q_{Med}) \end{aligned}$$

We want to calculate all terms in units of Watts. The geothermal heating, Q_{geo} is given to be 2×10^{13} W, and the cooling of the abyssal box in the Southern Ocean, Q_{so}, is given to be 2.8×10^{14} W. For the other terms, use:

$$\rho C_p \Delta T \times \text{production rate}$$

where the density of water is $\rho = 1025$ kgm^{-3}, the specific heat capacity of water is $C_p = 3990$ Jkg^{-1}K^{-1}, ΔT is the change in temperature of the water, and the production rate is given in *Sverdrups*.

$Q_{GINS} = 1025 \times 3990 \times (2 - 1.7) \times 13 \times 10^6 = 1.60 \times 10^{13}$ W (3 s.f.)
$Q_{Lab} = 1025 \times 3990 \times (3 - 1.7) \times 3 \times 10^6 = 1.60 \times 10^{13}$ W (3 s.f.)
$Q_{Med} = 1025 \times 3990 \times (12 - 1.7) \times 1 \times 10^6 = 4.21 \times 10^{13}$ W (3 s.f.)

$$\begin{aligned} \Rightarrow Q &= 2.8 \times 10^{14} - (2 \times 10^{13} + 1.60 \times 10^{13} + 1.60 \times 10^{13} + 4.21 \times 10^{13}) \\ &= 1.86 \times 10^{14} \text{ W} \quad (3 \text{ s.f.}) \end{aligned}$$

Therefore, the diffusive heat flux, Q_{flux}, is:

$$\begin{aligned} &= \frac{Q}{\text{area of stratified layer}} \\ &= \frac{1.86 \times 10^{14}}{0.65 \times 3.62 \times 10^{14}} \\ \therefore Q_{flux} &= 0.79 \text{ Wm}^{-2} \end{aligned}$$

To find K_z, use the 1-D Advection/Diffusion balance equation, and integrate by dz:

$$\begin{aligned} w\ C_p \frac{d\theta}{dz} &= C_p\ \frac{K_z}{\rho} \frac{d^2\theta}{dz^2} \\ \int \left[w\ C_p \frac{d\theta}{dz} \right. &= \left. C_p\ \frac{K_z}{\rho} \frac{d^2\theta}{dz^2} \right] dz \\ w\ C_p \Delta\theta &= C_p\ \frac{K_z}{\rho} \frac{d\theta}{dz} + constant \end{aligned}$$

$$\Rightarrow \frac{w\ C_p\theta - constant}{C_p\dfrac{d\theta}{dz}} = \frac{K_z}{\rho}$$

When advection and diffusion are balanced, the constant term would be equal to zero, due to no throughflow of heat. In this case, the abyssal box has a throughflow of heat out, $-Q_{flux}$:

$$\Rightarrow \frac{w\ C_p\theta + constant\ Q_{flux}}{C_p\dfrac{d\theta}{dz}} = \frac{K_z}{\rho}$$

With dimensional analysis, the exact form of the constant can be determined by equating the units. The units of terms $[wC_p\theta]$ must match the units for $[constant\ Q_{flux}]$:

$$ms^{-1}\ Jkg^{-1}K^{-1}\ K = [constant]\ Js^{-1}m^{-2}$$
$$\Rightarrow [constant] = kg^{-1}m^3 = \left[\frac{1}{\rho}\right]$$

$$\begin{aligned}\Rightarrow \frac{K_z}{\rho} &= \frac{w\ C_p\theta + \dfrac{Q_{flux}}{\rho}}{C_p\dfrac{d\theta}{dz}}\\ &= \frac{(3990 \times 7.2 \times 10^{-8}(13 - 1.7)) + \dfrac{0.79}{1025}}{3990 \times \dfrac{(13-1.7)}{500}}\\ &= 4.45 \times 10^{-5} mks\end{aligned}$$

Where $w = \dfrac{\text{production rate}}{\text{Area of Ocean}} = \dfrac{17 \times 10^6}{0.65 \times 3.62 \times 10^{14}} = 7.2 \times 10^{-8}\ \text{m}s^{-1}$. This value of K_z is close to typical values for mixing depths of 500 m, $\sim 5 \times 10^{-5}$ mks.