HWK1 _ BAROTROPIC VORTICITY

1. derive the barotropic vorticity budget for a non-flat ocean bottom

PG mom^m eqn.: $\quad \underset{\sim}{f} \times \underset{\sim}{u} = -\nabla_z \phi + \dfrac{\partial \underset{\sim}{\tau}}{\partial z}$ $\qquad$ where $\dfrac{\partial \phi}{\partial z} = b$

and $\dfrac{Db}{Dt} = \dot{b}$

We ignore acceleration terms due to small Rossby no.

and $\nabla . \vec{v} = 0$

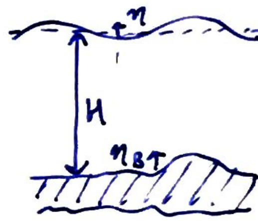
- Derive vorticity eqn. (take the curl of momentum)

$$\hat{k} . \nabla \times \left[\underset{\sim}{f} \times \underset{\sim}{u} = -\nabla_z \phi + \frac{\partial \underset{\sim}{\tau}}{\partial z} \right]$$

$$= -\frac{\partial}{\partial y}\left[-fv = -\frac{\partial \phi}{\partial x} + \frac{\partial \tau^x}{\partial z} \right] + \frac{\partial}{\partial x}\left[fu = -\frac{\partial \phi}{\partial y} + \frac{\partial \tau^y}{\partial z} \right]$$

$$= \frac{\partial}{\partial y}(fv) + \frac{\partial}{\partial x}(fu) + \cancel{\frac{\partial^2 \phi}{\partial x \partial y}} - \cancel{\frac{\partial^2 \phi}{\partial x \partial y}} = -\frac{\partial}{\partial y}\left(\frac{\partial \tau^x}{\partial z}\right) + \frac{\partial}{\partial x}\left(\frac{\partial \tau^y}{\partial z}\right)$$

$$\Rightarrow \beta v - f \frac{\partial w}{\partial z} = \text{curl}\left(\frac{\partial \underset{\sim}{\tau}}{\partial z}\right)$$



- Integrate over vertical - from η to $-(H) + \eta_B$

$$\int_{-H+\eta_B}^{\eta} \beta v \, dz - f \int_{-H+\eta_B}^{\eta} \frac{\partial w}{\cancel{\partial z}} \cancel{dz} = \text{curl} \int_{-H+\eta_B}^{\eta} \frac{\partial \underset{\sim}{\tau}}{\cancel{\partial z}} \cancel{dz}$$

$$\beta V - f\Big[\overset{0 \text{ rigid lid approx.}}{\cancel{W\big|_{z=\eta}}} - \underbrace{W\big|_{z=-H+\eta_B}}_{} \Big] = \text{curl}_z(\underset{\sim}{\tau}_s - \underset{\sim}{\tau}_b)$$

- $$W\big|_{z=-H+\eta_B} = \frac{D}{Dt}(H + \eta_B) = \frac{D\eta_B}{Dt} = \overset{\text{not changing}}{\cancel{\frac{\partial \eta_B}{\partial t}}} + \underset{\sim}{u} . \nabla \eta_B$$

$$\therefore \boxed{\beta V + \underset{\sim}{u} . \nabla \eta_B = \text{curl}_z(\underset{\sim}{\tau}_s - \underset{\sim}{\tau}_b)}$$