QEA 2 Teach In: Method of Moments for Planar Patch Antenna Radiation Modeling

Aaditya Rao and Charlie Sands

1 Learning Goal

Students should be able to understand the principles behind the method of moments technique for finite element analysis and apply it to build an intuitive understanding of electromagnetic modeling techniques. A grasp of this technique will help students develop an intuition of antenna design and the resultant wave propagation.

2 Method of Moments

To explore the method-of-moments (MoM), we will approach it from the perspective of designing a patch antenna on a dielectric substrate and ground plane. Given a current source on the antenna, we want to:

1. Solve for the current distribution $\mathbf{J}(\mathbf{r})$ in the antenna using MoM.

2. Calculate the radiated electric field strength $\mathbf{E}(\mathbf{r})$ in space.

3. Use the electric field equation to determine the radiation pattern of the antenna.

We can solve for the electric and magnetic fields using the Maxwell equations. Conventionally, Faraday's and Ampère's law are written in the Time Domain. These describe the relationship between time-varying electric and magnetic fields. When analyzing a structure with MoM to determine antenna gain, the system progression is unimportant; instead the steady-state behavior of the structure is required to calculate these parameters. While this could be done with the time-domain equations, we can abstract away time dependence of Maxwell's equations by using the frequency-domain equivalents, simplifying our work. Assume:

$$\mathbf{E}(t) = \mathrm{Re}\, \mathbf{E}(\mathbf{r})e^{j\omega t}\ , \quad \mathbf{H}(t) = \mathrm{Re}\, \mathbf{H}(\mathbf{r})e^{j\omega t} \tag{1}$$

Then the frequency-domain Maxwell equations become:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \quad \nabla \times \mathbf{H} = \mathbf{J} + j\omega\varepsilon\mathbf{E} \tag{2}$$

With time dependence removed, we can further simplify the equations by eliminating the magnetic field components according to Helmholtz's proof. Proving this relationship is beyond the scope of the project, but in essence, based on the vector calculus identities, taking the curl of the frequency domain Maxwell equations allows for the elimination of the magnetic effects and simplified the problem to one of only spatial propagation.

Moving forward, let us assume that we wish to analyze a rectangular planar antenna made of a good conductor. Our first goal is to discretize the antenna, separating it into a series of small rectangular cells that can be individually analyzed. Once the structure is discretized, we form an impedance matrix $\mathbf{Z}$ that characterizes how each discretized element interacts with the others. This matrix is calculated based on the Helmholtz equation and incorporates the mutual coupling between elements.

The relationship between the current on the antenna and the resulting electric field is governed by an integral form of Maxwell's equations, which incorporates the Green's function. The Green's function represents the impulse response between a source and an observation point. For planar structures in a homogeneous medium, we use the scalar Green's function:

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

This function describes the field at point $\mathbf{r}$ due to a unit source at $\mathbf{r}'$. The total electric field scattered by the antenna can then be expressed as a convolution of the Green function with the surface current density $\mathbf{J}(\mathbf{r}')$:

$$\mathbf{E}^{\text{scat}}(\mathbf{r}) = j\omega\mu \int_S G(\mathbf{r}, \mathbf{r}')\mathbf{J}(\mathbf{r}')\, dS'$$

The current distribution itself is not known at this point, so it is approximated using a finite set of basis functions:

$$\mathbf{J}(\mathbf{r}) \approx \sum_{n=1}^{N} I_n f_n(\mathbf{r})$$

Applying the method of weighted residuals with appropriate testing functions leads to a matrix equation of the form:

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$$

Here, $\mathbf{Z}$ contains mutual impedances between all pairs of basis functions, $\mathbf{I}$ is the vector of unknown current amplitudes, and $\mathbf{V}$ is the excitation vector due to the incident field.

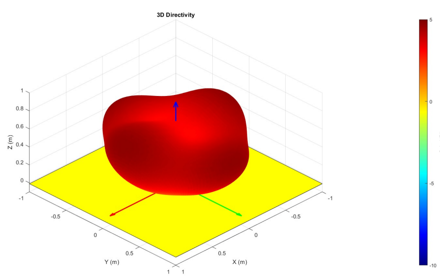
Using the discretized elements and a resonance-based solution (such as those used in the method of moments or standing wave models), we can calculate the surface current distribution on the antenna when a frequency is applied at a given feed point. Once this current distribution $\mathbf{J}(\mathbf{r})$ is known, the radiated electromagnetic field can be computed.

3 Implementation

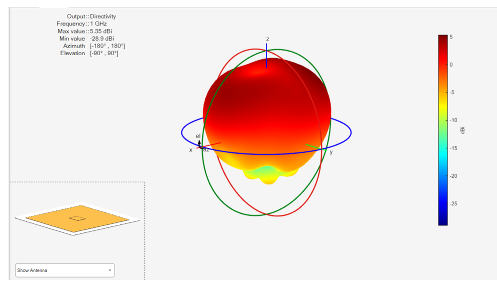
We implemented a method-of-moments solver in MATLAB. We used our modeling solver to verify the results of some standard "rule-of-thumb" design equations for rectangular patch antennas, as outlined in the attached worksheet. We constructed and profiled the antenna to validate both the simulation and the ”rule-of-thumb” calculations. Our example modeled a 1 GHz rectangular patch antenna positioned on a 1.6 mm FR4 fiberglass dielectric substrate above a large ground plane. Unfortunately, because of our lack of an anechoic chamber, we could not profile and validate the antenna radiation pattern.

3.1 Simulation

The directivity is visualized at a frequency of 1.0 GHz as part of our MATLAB simulation as a 3D function of θ and ϕ, with normalized results and plotted in dBi. The shape of the radiation pattern was compared with that of the same antenna simulated in the dedicated MATLAB toolbox. Differences are presumed to be caused by the more idealized model of the antenna used by the custom simulation.



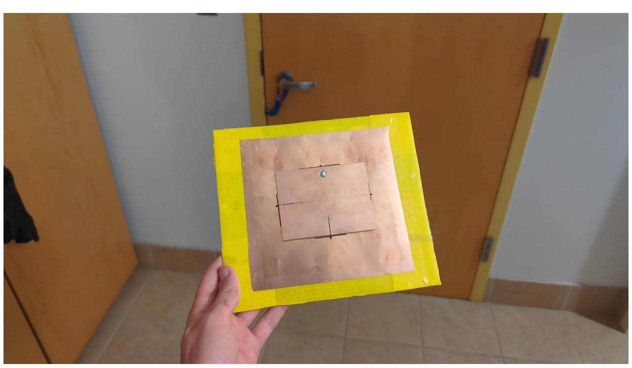
The MoM-simulated radiation pattern with the custom MATLAB simulation



The MoM-simulated radiation pattern with the MATLAB toolbox simulation

3.2 Fabrication

The antenna was fabricated from scrap copper and copper-clad fiberglass board.

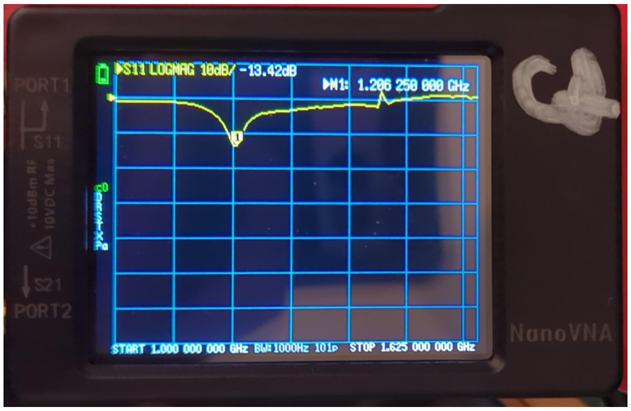


The fabricated patch antenna

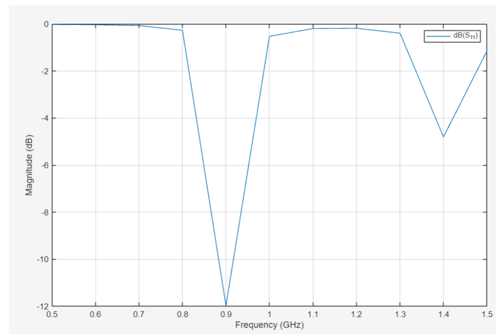
3.3 Validation

The antenna was profiled using a vector network analyzer (VNA) to understand its performance in the desired frequency range. The antenna has an acceptable return loss metric of

14 dB, which means that it radiated 96% of the energy it receives at the resonant frequency. The resonant frequency of the assembled antenna has an error of 20% (1.2 GHz vs. 1.0 GHz resonance). This error is likely due to the crude construction, poor mechanical tolerance, and lack of a correct ground-plane structure.



VNA measurement of the reflection coefficent of the fabricated antenna



The simulated reflection coefficent of the antenna with MATLAB

4 References

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3. Werner, D. H. "Lecture 9c – Method of Moments for Thin Wire Antennas." *EMPossible*.
   `https://empossible.net/wp-content/uploads/2019/08/Lecture-9c-Method-of-Moments-for-`
   `pdf`

4. Emagtech Wiki. "Basic Principles of The Method of Moments."
   `https://www.emagtech.com/wiki/index.php/Basic_Principles_of_The_Method_of_Moments`

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   `https://www.propagation.gatech.edu/2018/11/28/patch-antenna-notes/`

Design a Patch Antenna!

QEA II Teach-In Final Project Activity

Charlie Sands and Aaditya Rao

Designing a patch antenna is not complicated. When the rigorous forms of Maxwell's equations are analyzed and combined, the different electromagnetic effects of the antenna can be quantified and used to develop helpful "rule of thumb" modeling equations that allow engineers to easily design an antenna which is good enough for most use cases. The derivations of these formulas are beyond the scope of this activity, but those who are interested in the topic can read *Microwave Engineering* by David Pozar, which deeply covers this topic and similar ones.

The greatly reduced equations to design a patch antenna rely on three parameters:

The operational frequency of the system	f_0
The relative permittivity of the substrate	ε_R
The speed of light in the surrounding medium	c

The length (L) of the patch antenna can be calculated as:

$$L = \frac{c}{2f_0\sqrt{\varepsilon_R}}$$

The width (W) of the patch antenna can be calculated as:

$$W = \frac{c}{2f_0\sqrt{\frac{\varepsilon_R + 1}{2}}}$$

Exercise:

Try calculating the size of a patch antenna for a $1\ \mathrm{GHz}$ patch antenna in air constructed on an FR4 substrate with a relative dielectric permittivity of$4.4\ \frac{F}{m}$. Assume the speed of light in air is $3 \times 10^8\ \frac{m}{s}$.

Your name: Vanessa A. Flores

Name of students running the activity:

charlie sands aaditya rao

1. What was the learning goal for this activity?

Our presenters wanted us to gain a basic understanding of antennas and their electro magnetic fields

2. What went well in this activity?

- they were very good at explaining how the antenna works, why/how they created their own
- the worksheet was fun!

3. How could the team improve the activity?

- more time explaining the relationship between maxwells and Alhassans equations would've been a nice addition :p

4. To what degree did the activity help you achieve the learning goal?

- i found the activity very informative and interesting and feel confident that I have a basic understanding of antennas 9/10*

QEA2, Spring 2025
Teach-In Activity Feedback

Your name: Manuel de Tezanos Pinto

Name of students running the activity: Charlie Aadiya

1. What was the learning goal for this activity?

Antennas: what are they, what shapes can they come in
Method of ~~Moment~~ Moments: how do they correspond to antennas (frequency + radiation)
Simulation: found operational frequency + simulated mag field

2. What went well in this activity?

Fantastic explaining of topics with a proper equations showcase and connection to overall topics.
Very immersive simulations.
Great math activity and practical application.

3. How could the team improve the activity?

I'm not sure. ~~[illegible]~~

4. To what degree did the activity help you achieve the learning goal?

I gained a deep conceptual understanding of the topics in mind and was able to apply it through wellacrafted activity. Was also able to visualize the ideas through rich visualizations.

Your name: Kilan Rougeot

Name of students running the activity:
Charlie Sands
Aaditya Rao

1. What was the learning goal for this activity?

Using numerical methods like maxwells equation trying to solve for the ~~most effective antenna~~.
visualization of how an antenna works. Calculating the antenna area.

2. What went well in this activity?

- I liked the interactive worksheet.
- Good explanations
- Good preperation.

3. How could the team improve the activity?

Be more clear about the learning goal but overall a very good project. was very fun :)

4. To what degree did the activity help you achieve the learning goal?

Very much so, we got the equations and values and had to calculate the size of the antenna.

Your name: Shanna

Name of students running the activity: Charlie, Aaditya

1. What was the learning goal for this activity?

learn about designing antenna and the shape of their radiation and how to make one

2. What went well in this activity?

interactive activity was super fun and cool, and I liked their model / mini verson of the antenna

3. How could the team improve the activity?

maybe be a little more clear about what the learning goal is and the process of doing the actual project was like

4. To what degree did the activity help you achieve the learning goal?

very well :) interactivity was fun + engaging, they explained most topics in a simple enough way to understand, visual models were helpful + cool

Your name: Charlie Sands

Name of students running the activity: Manu and Vanessa

1. What was the learning goal for this activity?

The learning goal of this activity is to understand how ~~the~~ the gradient of a function can be formed ~~by~~ and how tangent plane can be visualized.

2. What went well in this activity?

I thought the visualization in this activity was good. I enjoyed all of the toggelable options in the simulation to help us understand the gradient.

3. How could the team improve the activity?

I would have liked it if you went a bit deeper into a new topic because I thought I already had a pretty good grasp of the gradient.

4. To what degree did the activity help you achieve the learning goal?

I thought the visualization activity was helpful in building my understanding of the topic. I think if it was faster it would have been a bit nicer, but that is probably more down to R than you...

Your name: Charlie Sands

Name of students running the activity: Kilom and Shamma

1. What was the learning goal for this activity?

The goal was to perform inverse kinematics to determine the ideal position of ~~an arm~~ a two linked arm and understand how gradient descent can be used to find the nearest position to a point the desired point.

2. What went well in this activity?

I liked the usage of the visual aid and I thought the MATLAB script looked really good. The explaination was pretty clear and concise.

3. How could the team improve the activity?

You could have rewritten the ChatGPT instead of screenshotting it. You also could have covered the gradient descent a bit more.

4. To what degree did the activity help you achieve the learning goal?

I think the activity was fun and did a good job helping us achieve the learning goal.

Your name: aaditya Rao

Name of students running the activity: manu, Vanessa

1. What was the learning goal for this activity?

Visualizing gradients with Rlang.

2. What went well in this activity?

I liked the visual implementation. It was super helpful in visualizing points and tangent planes.

3. How could the team improve the activity?

I would make the activity more interactive.

4. To what degree did the activity help you achieve the learning goal?

I understood how to find the gradient at a point from a visual perspective

Your name: Aaditya Rao

Name of students running the activity: Kilam Sharma

1. What was the learning goal for this activity?

In verse Kinimatics in the context of gradient decent

2. What went well in this activity?

I really like the demo and the physical prop

3. How could the team improve the activity?

I think the visual was Good, but I would like to understand the Gradient ~~you~~ are actully using.

4. To what degree did the activity help you achieve the learning goal?

I better understood what slope the gradient makes and why the minimum is the objective function.

Our project went well overall. Our activity generally seemed to communicate the point of our learning goal, and make effective abstractions of the concepts in order to do the activity without doing complex derivations. Two pieces of feedback we got, were:

1. Understanding the relationship between the different equations we were presenting. We abstracted a lot of the math, but It would have been helpful to have color or visual indication on how equations are used.
2. Our learning goal was unclear since we were talking both about the algorithm we made and the fundamental problem of antenna design. It would have been clear to have a goal at the start linking both points.

In the future, we would add more context and clear indicators between the different concepts.