A Game Theoretic Solution for the Territory Sharing Problem in Social Taxi Networks

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***Abstract*— Recent years have witnessed a surge of new methods by which commuters are accessing transportation modals. The use of social taxi networks is one of the most promising door-to-door transportation methods. In social taxi networks, commuters use their smart devices to contact social taxi service providers based on their geographical proximity and the listed fare prices. These taxis can be traditional taxis or privately owned vehicles. However, with the success of this new car hailing mechanism, there is an emergence of a number of problems. One of these problems is the territory allocation problem. i.e., the majority of social taxi drivers choose areas in which the most likely number of customers will be present. Therefore, they negatively impact each other's profit by offering a high supply of services. This paper develops a cooperative territory allocation approach such that, through negotiation between service providers, can reduce the conflict between taxi drivers. Game theory is used to formulate the territory sharing problem, which can be solved using bargaining-based solution model. The solution model is designed to correspond to a no regret game for which the outcome corresponds to a coarse correlated equilibrium. Simulation work results are provided to support the findings in this paper.**

***Index Terms*— Cooperative trip planning, intelligent cooperation systems, bargaining models, intelligent transportation system, social taxi networks, ride-hailing systems.**

Nomenclature

X	Group of drivers where $X = \{x_1, x_2, , , , , x_N\}$.
A^{x_i}	Territories shared by the drivers.
$C^{A^{x_i}}$	Pre-assigned cost values for the territories.
η	Regulatory body that moderates the territory sharing game.
$a_j^{x_i}$	Route chosen by a driver x_i as a strategy.
$a_{j*}^{x_i}$	The optimum strategy of player x_i.
T	Time window and $t \in T$.
λ, μ	Smoothness parameters such that $\lambda > 0$ and $\mu < 1$.
$\Delta_{x_i,t}$	The gain/cost incurred by driver x_i at instance t.
ϑ	Retaliation/threat factor in form of strategy.
x_{init}	Driver who initiates a bargaining deal.
x_{resp}	Driver who responds to a bargaining deal.
core	The strategic assignment of all players in a game.
S	Coalition of drivers.
$C(S)$	Total cost per coaltion.
S^{Gnd}	The grand coalition encompassing all drivers.
$A^{S^{Gnd}}$	The final strategy set as agreed upon by all drivers.
d	point of disagreement between any two players in a bargaining game.

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I. Introduction

A NEW trend has emerged in recent years in the field of transportation systems: the use of social taxi networks [1]–[4]. We *coin* the term social taxi networks to describe ride-sourcing applications where the service is delivered to riders by utilizing a network of private vehicles. Taxi drivers in these networks communicate with their customers through mobile smart apps. Lyft and Uber are prime examples of such networks [5]. In social taxi-networks, an increasing number of commuters rely on smartphone apps that allow them to find a transportation service based on their locations. Likewise, service providers (i.e., taxi drivers) are dependent on smartphone apps to connect them with potential customers based on their geographical proximity. Since these apps operate on a location basis, and the drivers choose their own territory, a disturbance in the supply and demand chain is inevitable. The rational thinking of each taxi driver is to choose an area with an elevated chance of having potential customers. For example, downtown areas and shopping districts are areas that have high popularity among customers. Many of these customers are in need for door-to-door transportation services. However, since this is common knowledge, most service providers will target these areas. This will lead to a situation in which we have some areas oversaturated with service providers, and other areas that have low to no existence of service providers. Furthermore, the majority of the smartphone apps governing the social taxi networks have a dynamic fare-rate that changes based on the availability of the service. For peak time periods, the fare-prices are increased and vice versa.

The existing solutions for social taxi networks (for example, Uber and lyft) employ demand-based policies such as price surging or upfront pricing [6], [7]. These policies include a dynamic multiplier in the pricing scheme. This model is susceptible to the phenomenon known as the tragedy of the commons [8], which is pertinent to situations in which the resources are common and the decision makers are selfish. The price surging approach is based on self-scheduling, and it is mostly effective and consistent for both the drivers and the service customers [6]. Therefore, the aim of this research is not to propose an alternative, but to introduce an approach that

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improves the existing self-scheduling approach. The proposed approach relies on solving the territory sharing problem. We call the problem of effectively distributing the routes among drivers based on their preferences and according to their personal schedules the territory sharing game.

There are various issues that can render the territory sharing game a challenging problem. First, there is the issue of formulating the game such that a solution model can be designed. The game consists of three parties: the players, the smartphone app, and the resources. The smart application can be a participant or a game moderator. Depending on its intended role, the game set-up will be different. Furthermore, if the players use the resources as strategies to play among themselves, then the game can be either symmetric or asymmetric. In symmetric games, the players possess the same sets of strategies. For example, if there are only three possible actions and all players have these actions as possible strategies, then the game is symmetric. However, asymmetric games are more general in the sense that the players may have non-identical sets of strategies. Hence, it is important to determine whether the game is symmetric or asymmetric.

Furthermore, once a game's nature is defined, an additional problem arises in determining a solution algorithm that can yield a stable outcome. A stable outcome is a self enforcing outcome that meets certain criteria. This is the second challenge in defining the territory sharing game. In regards to game theory, a stable solution constitutes a stable equilibrium. In general, we can define three main types of equilibria: dominant strategy equilibrium, Nash equilibrium, correlated equilibrium [9]. Identifying the equilibrium sought will assist in designing the solution model. It will also guide the assessment process of the obtained solution, which leads to the third challenge: the evaluation of the effect of the solution model on the overall performance (i.e., the welfare of the system). As discussed thus far, the territory sharing problem suffers from the tragedy of the commons; hence, a stable outcome does not guarantee the efficiency of resource utilization. To assess the (in)efficiency of the proposed solution, the Price of Anarchy (PoA) is used.

In this paper we develop a game theoretic formulation for the territory sharing problem. The model describes the problem as a cooperative game in which players with asymmetric sets of strategy cooperate to reach an agreement over their strategic choices despite the competition over the resources. The bargaining-based solution model is chosen to allow for the competition procedure to take place while it facilitates for the final agreement to be reached. The agreement can be enforced using the social taxi networks mediator, i.e., the smart app. To ensure that the final solution corresponds to a stable equilibrium, the bargaining framework utilizes the no-regret approach [10] which results in a coarse correlated equilibrium.

The remainder of the paper is organized as follows: Section II provides a literature review on the work related to the territory sharing problem. Section III provides a description of the territory sharing problem, problem formulation, and solution formulation. Section IV provides a description of the developed solution model. Section V discusses several scenarios as a proof of concept regarding the functionality of the developed system. Discussion and conclusion are presented in Section VI.

II. Related Work

The territory sharing game in social taxi networks is a new class of games. However, there are several related areas that can be found in literature: 1) social taxi networks, 2) cooperative game theory, 3) the stability of game theoretic solutions.

A. Social Taxi Networks

Triggered by the ubiquitous use of the information technology, a new economical business model has emerged: the sharing economy. The sharing economy is a platform in which people cooperate using technology to share what can be otherwise an un-utilized inventory on fee/non-fee basis [11]. One application of the sharing economy of interest is the social taxi networks; known as the ride-sourcing platform and free-floating car sharing systems. Social taxi networks are platforms in which users communicate through social media to obtain/provide a fee-based ride-sharing experience. In this particular case, the social taxi is similar to a traditional taxi since the provider of the service does not share the destination as the service user [12].

The treatment of the territory sharing problem can be observed in literature when examining some of the relocation algorithms for traditional taxis [13] as well as the ride-sharing applications [14]. The main goal of these algorithms is to (re)distribute service providers, such as taxis, in a manner that guarantees that the maximum number of customers is served. With regards to traditional taxis, the operator has complete control over a fleet of vehicles. Thus, the relocation of vehicles is subject to cost/benefit criteria. This model is similar to the models proposed for some of the social ride-sharing platforms. For example, in [15], Weikl and Bogenberger investigated the possible relocation strategies of the so-called free-floating car sharing systems. They describe two possible strategies: user-based and operator-based. According to the user-based strategy, the users are incentivized by lower costs to use the system despite its long waiting times and intermittent service availability. Nevertheless, not all customers can be influenced by such incentives. According to the operator-based strategy, the number of service-providers is increased to cover all areas. This case assumes that there is complete control over the service providers. The problem with using these two models in social taxi networks is that they do not accommodate the autonomy of the service-providers. For instance, a vehicle owner may prefer to operate in a low density residential area due to its close proximity to their living/work place. On the other hand, most traditional taxi companies might regard that area as a low priority area due to its low population density. If social networks had the same relocation model, the low density areas suffer from the scarcity of taxi services.

In the current social taxi networks such as Uber and Lyft, service-providers and service-users are independent from the operator. Therefore, similar to the user-based strategy in free-floating car sharing applications, the only way the operator can influence the process is by controlling the fare prices

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according to the state of demand and supply. This scheme is problematic since it creates situations in which either the service-users have to pay an expensive fare due to the lack of services, or the service-provider receives cheap service prices due to high service supply. The main concern with the traditional relocation systems is that they operate on the basis of optimizing the overall system performance neglecting the welfare of the system users: service-providers and service-users. Cooperative systems are proposed in the literature to deal with this shortcoming of the traditional control systems.

B. Cooperative Game Theory

Game theory is the theory of rational decision making. Players in various games are expected to exercise forms of strategic interaction. Games in game theory are categorized into two branches: Cooperative games and non-cooperative games. These two branches are different in terms of the relationship between the players. In non-cooperative games players do not communicate as they make their decisions. On the other hand, players engaging in cooperative games communicate and agree on their chosen strategies [16]. Although, non-cooperative games are concerned with the procedure by which players can compete and therefore achieve certain selfish gains, the non-cooperative games do not lack cooperation. In fact, players do cooperate as they share their sets of strategies and expected pay-offs prior to the decision making phase which assume a non-cooperative state. On the other hand, in cooperative games, despite the mutual endeavour of the players to create a beneficial binding agreement, the players do compete within their coalitions [17].

The territory sharing problem is essentially a resource sharing problem which was extensively studied in the literature of game theory. Although the targeted application is different, the fundamentals are similar for all applications including the ride-sourcing application. The approaches of solving the resource sharing allocation are mostly viewed in terms of various marketing mechanisms. For example, in [18] MacKie-Mason and Varian describe a smart marketing for resource auctioning through which system users start bidding for their traffic. Users are prioritized based on their biddings; the more they are willing to bid, the higher their priority gets. Kelly proposed in [19] that the market, the resource manager, accepts bids from users and assigns the prices based on the received bids and the actual market price. This approach has been adopted and modified for resource sharing in various applications such as time-sharing systems [20], internet competition systems [21] and cooperative relay networks [22].

Auctioning algorithms in general guarantee that bidders are "honest" regarding their bids, but they do not prevent monopolies. Consider the case in which one bidder can always outbid all other players, a monopoly is bound to happen. Although auctioning algorithms allow for the resources to be used efficiently, fairness among players is not assured. For the territory sharing problem, we might end up with various types of players. The first type is a driver who is willing to pay more or get less profit. Consequently, all "good" spots are awarded to this driver. In contrast, there will be another driver who is not able to make the same offer. The latter will lose interest in the game and eventually abandon the game leading to a monopoly that causes the prices to increase due to the unbalance in the supply-and-demand process. On the other hand, there is the counter concept of network neutrality [23] which is the basis of the tragedy of the commons. Balancing the two concepts requires a great deal of micro-management.

For the territory sharing problem, if we consider that the targeted areas for the social taxi drivers are sources of revenue, the territory sharing problem can be treated as a classical atomic resource sharing problem [24]. Resource sharing problems revolve around the aim to perform system wide optimization while maintaining the fairness in allocating resources among a group of decision makers [25]. However, as cited in [24], centralized systemwide optimization can be difficult to achieve for most systems. Therefore, the best alternative is to adopt a distributed approach for resource allocation. Cooperative game theory have proven to be useful in a variety of applications concerning resource management and revenue/cost allocation [26]–[28], which posits it as a plausible solution model for the territory sharing problem. In the case of social taxis, it can be considered to be favourable to use cooperative game theory, for territory sharing among drivers, since drivers and customers use smart apps as a platform by which their transactions are governed.

C. Existence and (In)Efficiency of Game Theoretic Solutions

In literature, there is a wide range of problems with a game theoretic presentation. A game theoretic set-up does not guarantee the existence of a solution. A solution exists in a game when an equilibrium outcome converges at the end of the game. Generally, there are three types of equilibria: dominant equilibria, Nash equilibria, and correlated equilibria [9], [29], with each type having its own variants. The dominant equilibria exist if players within a game have strategies that can provide them with the best outcome regardless of the possible strategies of other players. However, for the majority of resource sharing games such a scenario is rare. Nash equilibrium has two main variants: pure equilibria and mixed equilibria. Pure equilibria can be found and analyzed, but they don't exist at all times. On the other hand, the mixed equilibria always exist, but it is difficult to be found. The correlated equilibrium can be seen as a general case of Nash's mixed equilibrium. The correlated equilibrium has two attractive features: first, it always exists, and it can be found; second, the correlated equilibrium analysis is suitable for games with subjective strategies. The correlated equilibrium is primarily described as an outcome of a game in which a random device sends a signal to the players describing/assessing a situation of interest. Therefore, through the signal, the device can affect the players and correlate their choices [29]. In [30], Cigler and Faltings have argued that the existence of a smart device is not needed to produce a correlated equilibrium. Alternatively, for the equilibrium to be produced, it is sufficient to play the game repeatedly and as such, the players will learn from previous rounds such that they incorporate their knowledge in subsequent rounds [31]. In this paper, we analyze our solution in the form of the correlated equilibrium.

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The most important aspect of analyzing the various solutions of the resource sharing problem is the efficiency of the outcome. Price of anarchy (PoA) is the most used criterion in assessing the (in)efficiency of a solution. PoA is an index that measures the impact of the selfish behaviour of system users on the overall system performance by comparing the worst equilibrium with the best possible solution that is not necessarily an equilibrium. For the resource sharing problems there are several assessments of the PoA values under various assumptions. For example, Johari and Tsitsiklis argue in [32] that for users who are anticipating the effect of their actions on the prices, and who share the same resources, the lower bound for the aggregated utility is 75% of the best case scenario. However, the assumption driving the lower bound to be high is that the players are conscious of the consequences of their actions which affect their choices. For the other scenarios that have less desirable working conditions, the PoA lower bounds decrease. In [33] Bachrach *et al.* propose that for a wide range of coalitional games, in which players expect their share to be at least their individual contribution/cost without participating in the cooperative games, the strong PoA is 50% of the optimum scenario. The variation of the upper and lower bounds is attributed to the fine details of each game as well as the proposed solution method. Furthermore, these bounds provide an inside look to the gains and losses of adopting the cooperational approaches to certain game as opposed to the non-cooperational approaches which varies from one game to another [33].

III. Territory Sharing Game

The nature of the social taxi networks puts the smart app in a good position to monitor and moderate, to a certain degree, the territory sharing game. The smart app should not force drivers to make certain decisions. However, it can force the drivers to participate in the game on the promise that the outcome of this game would outweigh any alternatives. Furthermore, the smart app can limit the number of drivers who have access to the system according to the locational demands. Therefore, the problem of finding the right number of players can be solved.

Once the game begins, drivers will agree collectively on certain strategies and communicate that to the smart app. The smart app will combine the localization functionality with the decisions committed by the drivers and will only grant access to the drivers who commit to their announced decisions. Through this arrangement, a binding agreement is established. The role of the smart app ends here. The smart app will not run the game nor control its flow. Alternatively, drivers will make their agreement based on their best interest to minimize their joint cost while maximizing their individual profits. Hence, the aim of our research of this problem is to find a game theoretic model for the territory sharing problem that balances the process of supply and demand such that the services are available at most times for all customers with profitable gain for each service provider. Next, we present the problem formulation of the territory sharing problem for social taxi networks.

A. Problem Formulation

We consider a group of drivers $X = \{x_1, x_2, , , , , x_N\}$, each of which is requesting to have ownership of specific territories. The territories are denoted as A^{x_i}. Each driver picks areas of interest $A^{x_i} = \left\{A_1^{x_i}, A_2^{x_i}, , A_n^{x_i}\right\}$ with the pre-assigned prices $C^{A^{x_i}} = \left\{C^{A_1^{x_i}}, C^{A_2^{x_i}}, , C^{A_N^{x_i}}\right\}$. These territories are in the form of routes in the same area such that $A^{x_i} = \left\{a_1^{x_i}, a_2^{x_i}, , a_n^{x_i}\right\}$. For each route, $a_j^{x_i}$, a regulatory body η assigns the tag price $c^{a_j^{x_i}}$. The price is equal for all drivers. Let A^{x_i} be the set of strategies, represented by their actions, and $C^{a_j^{x_i}}$ be the utility function for driver x_i over $a_j^{x_i}$. A^{x_i} is a compact, differentiable, convex set for which the cost set $C^{A^{x_i}}$ is computed via a positive non decreasing function. That is, A^{x_i} is a bounded closed set which contains all of the desired strategies such that each strategy will yield a positive utility value.

The game is a 3-tuple $\Sigma(X, A^{x_i}, C^{A^{x_i}})$ cooperative territory sharing game. The regulatory body, η receives requests of reserving areas from the drivers. The regulatory body computes $C^{A^{x_i}}$ based on the drivers' concentration in the areas of interest and send this information to the drivers as well as the information about where the drivers are situated. $C^{A^{x_i}}$ represent the drivers' expected fare price deduction for each territory, i.e., the "loss" for every driver due to the declining fare rate. Once the drivers receive $C^{A^{x_i}}$, they compute their $C^{A^{x_i}}$ and then they start communicating to agree on their chosen strategies, i.e., territories.

Since the drivers are impacted by their individual decisions, a cooperative scheme is needed to achieve the following:

$$C(S) = \min_{a_j^{x_i} \in A^{x_i}} \sum_{1}^{N} C^{a_j^{x_i}} \tag{1}$$

such that

$$C(S) \leq \sum_{i=1}^{N} C(a_{j*}^{x_i}, a_j^{x-i}) \tag{2}$$

Where S is the coalition of drivers in the game and $C(S)$ is the cost value of the coalition. $C(a_{j*}^{x_i}, a_j^{x-i})$ is the cost value of player x_i using his/her best strategy, route, given that all other players x_{-i} use their different strategies.

(1) describes the game as a game in which the goal is to have a minimum overall cost in the coalition S. The overall cost is the sum of the individual cost values incurred by the drivers according to their strategic choices. Therefore, each drive is expected to find the strategy $a_j^{x_i}$ that minimize the overall cost value. Furthermore, in (2) the game is expected to arrive at an equilibrium as a competitive game such that if any player unilaterally changes his/her strategy to another strategy, even to the optimum strategy a_{j*}, the overall outcome would not improve.

B. Solution Formulation

The game described so far has two aspects. The first aspect is the individual cost for each driver. Drivers would like to choose strategies that guarantee them the lowest possible

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conflict cost, conversely the highest possible profit. The second aspect is the overall cost value. An overall cost minimization might require the drivers to cooperate. Therefore, the game should be approached from both sides of the spectrum. Drivers who are most interested in bringing up their gain or reducing their cost should form coalitions, S. The purpose of these coalitions is to provide their members with a platform in which they can make strategic agreements with regard to the mutual resources. The resource sharing game is faced usually with the problem of forming cooperating groups that adhere to their agreements. For our targeted application, this is made simple. Customers contact the smart app expressing their interest in having a door-to-door transportation service. The drivers contact the app expressing their availability to provide this service. Both parties have no direct interaction, and their communications go through the smart app. Therefore, the smart app can force the drivers to abide by whichever agreement they made among themselves.

Nevertheless, drivers are not expected to have an interest in joining an agreement that will not benefit them. Therefore, the solution should guarantee two outcomes. The first outcome is related to the drivers' geographical distribution that needs to be aligned with the customer concentration areas which can be reflected on the cost values defined by the smart app. The second outcome is achieved when the drivers increase their gain by joining in the cooperative process. These outcomes are best described by the following formulation:

$$\text{Core(c)} = \left\{ \chi \in \mathbb{R}^N | \sum_{i \in N} \chi_i = c(N) \text{ and } \sum_{i \in S} \chi_i \leq c(s) \forall s \in 2^N \setminus \varnothing \right\} \quad (3)$$

where χ is an imputation of the core, a core allocation, $c(N)$ is the total cost of all players, and $c(s)$ is the cost incurred by any individual coalition of players. The core is the set all feasible outcomes of the game such that no player will benefit by not cooperating [34]. Expression $\sum_{i \in N} \chi_i = c(N)$ guarantees the efficiency of the outcome. That is, the payoff for the grand coalition is equal to the sum of all posable core imputations. Expression $\sum_{i \in s} \chi_i \leq c(s)\ \forall s \in 2^N \setminus \varnothing$ guarantees the stability of the game. That is, each cost value paid by an coalition is less than the cost value of the players outside the coalition.

As seen in (3), drivers are interested in joining a cooperative endeavour if they are sure that they will be individually benefited while improving through their cooperation. The formulation in (3) is described as the core and it responds to the requirements in (1) and (2). The core is one of the solution concepts for cooperative games.

IV. The Bargaining Model

We use the bargaining model in this paper to deal with the cooperative and competitive aspects of the territory sharing game. Although the game is constructed in a cooperative manner, the tools of finding the solution as well as analyzing the solution are heavily used in the non-cooperative branch of game theory. This is hardly a dichotomous employment of these tools and solution methods. To better highlight the relationship between cooperative and non-cooperative games, we need to quote R. J. Aumann's comment on the nature of the relationship between the cooperative and non-cooperative games: "Formally, cooperative games may be considered a special case of non-cooperative games, in the sense that one may build the negotiation and enforcement procedure explicitly into the extensive form of the game. Historically, however, this has not been the mainstream approach. Rather, cooperative theory starts out with a formalization of games (the coalitional form) that abstracts away altogether from procedures and form the question of how each player can best manipulate them for his own benefits; it concentrates, instead, on the possibilities for agreement." [35].

The solution model in this paper follows the formal definition of cooperative games solutions. In contrast to the work in many of the recent publications, we develop a solution model that has a negotiation procedure between the players while the final agreement are enforced by the game moderator, the smart app. We utilize the properties of Nash's bargaining model, described in [36], to create a framework for the game's negotiation procedure. The bargaining model deals with a scenario in which competitive players attempt to minimize their disagreements such as for two players we will have

$$A^S = \min_{a_i, a_j} (u_1(a_i) - u_1(d)) * (u_2(a_j) - u_2(d)) \quad (4)$$

where u is the utility value that determines the cost/benefit, d is the point of disagreement between the two players, and S is the coalition of two drivers.

Our bargaining solution extends the objective function in Equation 4 to deal with multi-player scenarios by utilizing the coalitional aspect of cooperative game theory such that

$$A^{S^{Gnd}} = \arg \min_{a_i, a_{-i}} \sum_j \prod_k (u_j(a_i) - u_j(d_{j,k})) * (u_k(a_{-i}) - u_k(d_{j,k})) \quad (5)$$

where

S^{Gnd} is the grand coalition encompassing all drivers,

$A^{S^{Gnd}}$ is the final strategy set as agreed upon by all drivers

The bargaining solution for the territory sharing game can be represented by the core as described in Equation 3.

Definition 1: For a player, x_i, there exist a set of strategies A^{x_i} in which each point can be less profitable that the solution $a^{x_{i*}}$ such that $a^{x_{i*}} \in A^{x_i}$. If a preferred strategy exists, player x_i is allowed to choose any other strategy in A^{x_i} which might yield a less payoff, or a higher cost, than a^{x_i}.

The previous definition permits, within the game, any volunteer choice made by any player to give in some of his/her resources to other players to the benefit of the group rather than the individual. However, this proposition might contradict with (2) since drivers have better alternatives, i.e., no equilibrium is reached. Therefore, we start by reformulating (2) such that for a series of sequential negotiation rounds and agreed upon sets of actions $A^1, A^2, A^3, , , A^T$ over T time,

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(2) becomes

$$\frac{1}{T}C(s^t) \leq \frac{1}{T}\sum_{i=1}^{N} C(a_{j*}^{x_i{}^t}, a_j^{x_{-i}^t}) \tag{6}$$

Proof

$$\sum_{1}^{T} C(s^t) = \sum_{1}^{T}\sum_{1}^{N} C(s^{x_i{}^t}) \tag{7}$$

$$\sum_{1}^{T} C(s^t) = \sum_{1}^{T}\sum_{1}^{N} [C(a_{j*}^{x_i}, a_j^{x_{-i}}) + \Delta_{x_i,t}] \tag{8}$$

$$\Delta_{x_i,t} = C(s^t) - C(a_{j*}^{x_i}, a_j^{x_{-i}^t}) \tag{9}$$

using the smoothness assumption detailed in [9], [37], and [38], we can have the following relationship

$$\sum_{i=1}^{N} C(a_{j*}^{x_i}, a_j^{x_{-i}}) \leq \lambda \cdot C(a_{j*}^{x_i}) + \mu \cdot C(a_j^{x_i}) \tag{10}$$

Therefore,

$$\sum_{1}^{T} C(s^t) \leq \sum_{1}^{T} \lambda \cdot C(a_{j*}^{x_i}) + \sum_{1}^{T} \mu \cdot C(a_j^{x_i{}^t}) + \sum_{1}^{T}\sum_{1}^{N} \Delta_{X_i,t} \tag{11}$$

For each x_i we use the no-regret model to present the following assumption

$$\sum_{1}^{T} \Delta_{x_i,t} \leq 0 \tag{12}$$

Hence

$$\frac{1}{T}C(s^t) \leq \frac{1}{T}\lambda \cdot C(a_{j*}^{x_i}) + \frac{1}{T}\mu \cdot C(a_j^{x_i{}^t}) \tag{13}$$

■

For cost saving games, i.e., our game, (10) and (11) can be expressed as follows

$$\sum_{1}^{T} C(s^t) \leq \sum_{1}^{T} \lambda \cdot C(a_{j*}^{x_i}) + \sum_{1}^{T} \mu \cdot C(a_j^{x_i{}^t}) - \sum_{1}^{T}\sum_{1}^{N} \Delta_{x_i,t} \tag{14}$$

and

$$\sum_{1}^{T} \Delta_{x_i,t} \geq 0 \tag{15}$$

where

$a_{j*}^{x_i}$ = The optimum strategy of player x_i,
T = Time window and $t \in T$,
λ, μ = Smoothness parameters such that $\lambda > 0$ and $\mu < 1$,
$\Delta_{x_i,t}$ = The gain/cost incurred by driver x_i at instance t.

The game according to (8)-(13) corresponds to a "no regret" game that results in a coarse correlated equilibrium (CCE) which is a general case of Nash equilibrium [39].

Groups of players engaging in these bargaining games are called coalitions, S, as seen in Figure 1. The coalitions in our game are created based on power play (i.e., the ability of the players to use their strategies to increase their influence in the game to gain higher rewards) rather than fairness in resource allocation. Players will list their strategies and associated payoffs. For the conflicting set of strategies, $\hat{A} : \hat{A} \subset A$, players will list the actual payoffs caused by the conflict of interest. The player who is having the less risk, when changing his/her strategy, will swerve. In this game, every player makes his/her strategies known to other players. If there are contradicting strategies, players with such contradictions start to contact each other to resolve the *conflict of interest*.

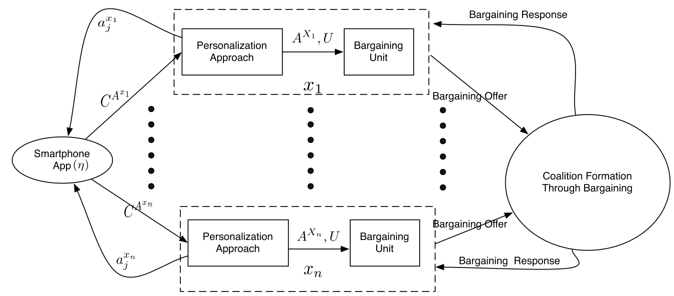

Fig. 1. Territory sharing game using the bargaining model.

As the name suggests, one party will initiate the bargaining procedure by proposing a *deal*. The initiator, x_{init} will suggest that the other player, x_{resp}, should swerve. Furthermore, after making the strategies and the associated payoffs/costs known, the value of the shown payoff/cost will be recalculated. The deal offered by the fist party has a retaliation/threat factor $\vartheta^{x_{init}}$. $\vartheta^{x_{init}}$ represents the risk which x_{resp} has to face when swerving. $\vartheta^{x_{init}}$ can be any mixed strategy which x_{init} might use in case the negotiation was not successful. $\vartheta^{x_{init}}$ is computed as follows:

$$\vartheta^{x_{init}} = \min_{\hat{A}}(|(u(a^{x_{init}}) - u(\hat{a}^{x_{init}})|, |u(a^{x_{resp}}) - u(\hat{a}^{x_{resp}})|) \tag{16}$$

where $a^{x_{init}}$ is x_{init}'s original strategy and $\hat{a}^{x_{init}}$ is the alternative strategy. Similarly, $a^{x_{resp}}$ and $\hat{a}^{x_{resp}}$ are x_{resp}'s original and alternative strategies respectively.

The other party will check the offer and the available options. If there was another deal in which he/she has a $\vartheta^{x_{resp}} < \vartheta^{x_{init}}$, then x_{init} will be the one swerving. Each party will revisit their set of available strategies as the negotiation progresses until an agreement is made. If $\vartheta^{x_{resp}} = \vartheta^{x_{init}}$, the deal offered by the initiator will be the conclusion of the negotiation. Both parties will have a binding agreement regarding their chosen strategies and they will change their set of available strategies according to the deal. Communications between both parties are maintained at all times. Algorithm 1 explains our developed bargaining model.

V. Simulation Work

In this simulation work, we simulate N drivers who have been given the option with two business areas: downtown Waterloo and downtown Kitchener. Each area has two avenues. Each driver is required to choose at least two non-identical avenues. These avenues will represent the elements of their sets of strategies. The number of drivers, N, can vary from

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Algorithm 1 Territory Sharing Game

Interested drivers communicate with η
η sends $C^{A^{x_i}}$ to each player
while Decision not reached **do**
  procedure COALITION–CREATION
Broadcast A^{x_i} to other players
    for each player x_i **do**
Compute $(|(u(x_{init}(a)) - u(x_{init}(\hat{a})))|)|x_{-i}$
Idenify a potential coalition partner
Compute the conflict factor $(|(u(x_{init}(a)) - (u(x_{init}(\hat{a})))|x_{resp}(a^{resp} = a_i, i \neq 1))|)$
Compute the threat factor ϑ^{init}
      if X_{resp} responded with an offer **then**
Compute $\vartheta^{respond}$
        if $\vartheta^{init} < \vartheta^{resp}$ **then**
x_{init} will choose $a^{init} = a_i, i \neq 1$
        else
x_{resp}will choose $a^{resp} = a_i, i \neq 1$
        end if
      else
x_{init} will not join any coalition
      end if
Broadcast A^{s^i}
    end for
  end procedure
  procedure NEGOTIATION–ROUNDS
    for each player x_i with new strategy a^{new} **do**
Compute the cost $((u(x_i(a))|x_{-i}(a))$
      if $u(x_i(a^{new}) > u(x_i(a^{old})$ **then**
        if $u(x_i(a^j : a^j \neq a^{new}) < u(x_i(a^{new})$ **then**
          if $\Sigma_k^N A^{x_k}|(x_i(a^j)) \leq \Sigma_k^N A^{X_k}|(X_i(a^{new}))$ **then**
Change strategy
          end if
        end if
      else
Do nothing
      end if
    end for
  end procedure
end while
Each player x_i will communicate his/her A^{x_i} to η.
η broadcasts the final strategy set A^S

TABLE I
COALITION FORMATION IN THE TERRITORY SHARING

Coalition No.	Coalition Members	Strategy Agreement
1	x_1, x_2	$\{a_1^{x_1}, a_2^{x_2}\}$
2	x_3, x_4	$\{a_1^{x_3}, a_2^{x_4}\}$
3	x_5, x_7	$\{a_1^{x_5}, a_2^{x_7}\}$
4	x_6, x_9	$\{a_1^{x_6}, a_2^{x_9}\}$
5	x_8, x_{10}	$\{a_1^{x_8}, a_2^{x_{10}}\}$

TABLE II
NEGOTIATIONS FOR COOPERATIVE TERRITORY SHARING

Players	Selfish Game	Cooperative Game Outcome			Δ_{X_i}
		1st Round	2nd Round	3rd Round	
x_1	15	12	9	6	0
x_2	15	6	3	3	6
x_3	9	9	9	9	9
x_4	9	18	9	9	9
x_5	15	12	9	6	0
x_6	9	3	6	9	11
x_7	15	10	7	5	4
x_8	15	12	9	6	4
x_9	9	17	20	9	11
x_{10}	15	5	5	2	7
$\sum_1^{N=10} C^{a_j^{x_i}}$	126	104	86	64	NA

one experimental set up to another. However, once the game begins all drivers who express their desire to participate and were approved by η will be considered to be players.

Through this simulation, we first demonstrate how our model operates for a specific number of players and show the expected gains/losses. We will also examine the validity of (13) for our model through rigorous simulation work. The simulation work will examine the value of $\sum_1^N \Delta_{x_i}$ for all players if the game was to be played for a period of 30 days with the same players. Furthermore, the efficiency of the game model is examined using the Price of Anarchy (PoA). PoA is a measure used to examine the effect of the cooperation or the lack thereof. PoA is computed as the following

$$PoA = \frac{\max_{a_j} \sum_1^N C^{a^{x_i}}}{\min_{a_j} \sum_1^N C^{a^{x_i}}} \tag{17}$$

For the purposes of this simulation, each driver was represented by a thread. There a set of personalization approaches are available for all drivers, and each driver will randomly choose one personalization approach based on which two alternatives are found. Each thread can send independently a query to the smart app simulated on an independent processor. Python was used for the simulation work.

Next we demonstrate the behaviour of our model for 10 drivers.

A. Territory Sharing for 10 Drivers

In this scenario, 10 drivers choose randomly their sets of strategies. Their total profit is penalized by the smart app according to their chosen territory. If two drivers chose the same avenue, the avenue price will be penalized by 2 price units for each driver and so on. Therefore, their collective goal from playing this game is to minimize their overall sharing incidents as well as their individual penalties.

The players start the game by broadcasting their sets of strategies each of which has only two alternatives $\{a_1^{x_i}, a_2^{x_i}\}$. All drivers announce that their preferred strategy is a_1, and each driver will compute the overall sharing incidents as shown in Table II. According to (14) a few coalitions are formed as shown in Table I. After the coalitions are established the first round of negotiation begins. The coalition formation rounds are governed by the two conditions of rationality and efficiency as indicated in (3). As seen in Table II, For the first round the second condition is met. However, for X_4 and X_9 their individual prices have increased since they joined their respective coalitions. Therefore, over two rounds we allow each one of them to examine independently their

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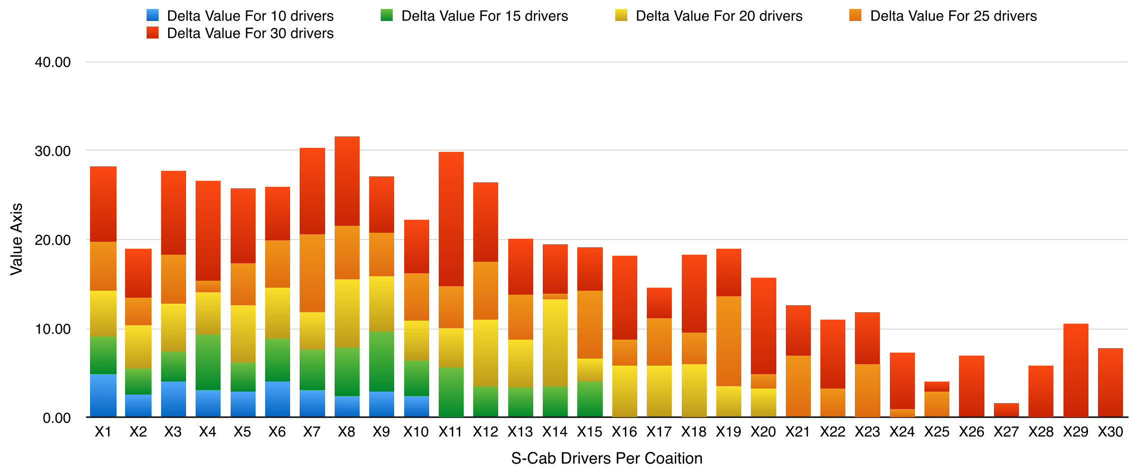

Fig. 2. Values of Δ computed over 30 days for $N = (10, 15, 20, 25, 30)$.

costs outside of their coalitions. After the third round they confirm that indeed their cost outside of their sub coalition is better and therefore they secede from their coalitions. For this first scenario the final grand coalition is found as follows:

$$A^{S^{Gnd}} = \{a_1^{x_1}, a_2^{x_2}, a_1^{x_3}, a_2^{x_4}, a_1^{x_5}, a_1^{x_6}, a_2^{x_7}, a_1^{x_8}, a_2^{x_9}, a_2^{x_{10}}\} \tag{18}$$

B. Smoothness and the Convergence of Nash Equilibrium

The previous scenario included 10 drivers for which a cooperative solution is found. By examining Δ_{X_i} in the previous scenario we can indeed confirm that the cooperative solution corresponds to a Nash Equilibrium for 10 drivers. However, the possibility of this outcome being consistent for all possible cases can be challenged if Definition (1) is applied. If Definition (1) prevented the Nash equilibrium from converging, the no-regret model should come into play and in any case we should have $\sum_1^T \Delta_{x_i,t} \leq 0$, and the final solution will converge to the CCE. To verify this hypothesis, we have run more than 1000 scenarios for various values of N simulating situations in which games are played repeatedly over a period of time up to 30 days, $T \leq 30$.

As we can see from Figure 2, for up to 30 drivers the $\sum_1^T \Delta_{x_i,t}$ has a positive value reflecting the gain achieved by cooperation. This result proves the ability of our model to provide drivers with an acceptable state of equilibrium. However, we need to keep in mind that this case is true for when $N \leq 30$, $|A^{x_i}| = 2$, and $|A^S| = 4$. If we are to arbitrary increase the number of players without offering more options to the drivers, the final result may not converge to an equilibrium even if a momentary gain is found.

As can be seen in Figure 3, as we allow for more drivers to join the game, the overall results will not conform to (13). Therefore, to deal with this issue we have two options: either to increase the observation period T with the assumption that $\sum_1^T \Delta_{x_i,t}$ will be positive, or to increase the number of offered strategies. With the former option we may or may not get an equilibrium. Whereas, with the latter option, by diversifying and broadening the search space, we are assured to get a CCE. With regards to our application, one of our goals is to distribute the drivers among all possible areas which the customers frequent the most. Therefore, as more drivers express their interest in playing the game, it is logical to increase the coverage areas. The burden of dealing with this issue will fall on η as highlighted in Algorithm 1. For our scenario, we have added two more avenues, i.e., $|A^S| = 6$. For each driver, the value of $|A^{x_i}|$ will remain to be 2. However, now each driver has a bigger "pool" of strategies to choose from. As shown in Figure 3, we can see that for the same coalition formulation and over the same period of time an overall equilibrium, CCE, has converged.

Although the assumption in this game is that the areas are fixed, and thereby their cost as per the system is fixed, the drivers who compete for the ownership of these areas may change. In other words, drivers may change their strategies, areas of interest, and engage in the competition with different drivers under the supervision of η. However, for the drivers to guarantee a benefiting equilibrium, they need to play the game repeatedly. Over an extended period of time, while choosing different strategies, the Coarse Correlated Equilibrium (CCE) will still converge.

C. Overall System Performance

The simulation work presented thus far showed the effectiveness of our model in terms of individual cost distribution and the converged equilibrium. Nevertheless, since our bargaining model is a heuristic one, a stronger form of validation might be required. For this purpose, we will use the PoA as a tool of assessment. For N $<=$ 20 we can compare the converged equilibrium with the best case scenario. However, for $N > 20$ this might be difficult since the search space for $|A^{x_i}| = 2$ is 2×2^N. Therefore, our assessment will include a comparison with the deterministic best equilibrium as well as a comparison with an estimated "best" equilibrium.

1) Deterministic Equilibrium vs. Achieved Equilibrium: We will use the case of $N = 20$ drivers to compare the overall

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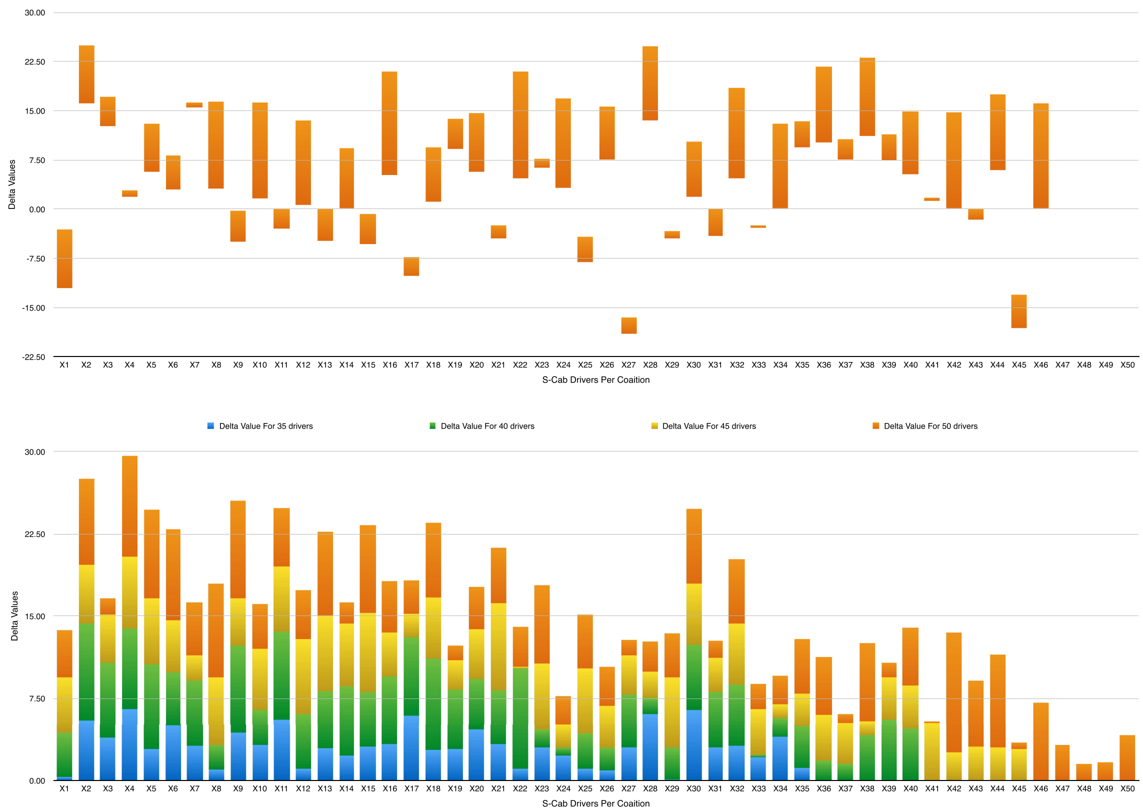

Fig. 3. Values of Δ computed over 30 Days for $N = (35, 40, 45, 50)$.

TABLE III

CONVERGED EQUILIBRIUM FOR 20 DRIVER DURING 10 DAYS

Days	Best Equilibrium	Worst Equilibrium	Achieved Equilibrium	PoA	Achieved PoA
1	370	984	390	2.66	1.05
2	382	1044	392	2.73	1.03
3	564	1120	592	1.99	1.05
4	418	1260	432	3.01	1.03
5	306	816	320	2.67	1.05
6	336	1212	402	3.61	1.20
7	382	1120	392	2.93	1.03
8	364	1022	396	2.81	1.09
9	348	1038	352	2.98	1.01
10	348	1038	388	2.98	1.11

TABLE IV

CONVERGED EQUILIBRIUM FOR 30 DRIVERS DURING 10 DAYS

Days	Best Equilibrium	Worst Equilibrium	Achieved Equilibrium	PoA $=\sqrt{N}$	Achieved PoA
1	502	2712	788	5.40	1.57
2	447	2414	628	5.40	1.40
3	453	2448	795	5.40	1.75
4	468	2526	576	5.40	1.23
5	502	2712	589	5.40	1.17
6	441	2380	556	5.40	1.26
7	488	2634	602	5.40	1.23
8	478	2580	685	5.40	1.43
9	468	2526	816	5.40	1.74
10	459	2480	582	5.40	1.27

cost of our achieved equilibrium with the deterministic best and worst equilibria which were determined through brute search. As can be seen in Table III, the PoA for 10 days is consistently high. In comparison, the PoA from our model is close to optimum, i.e., close to 1.

2) Estimated Equilibrium vs. Achieved Equilibrium: As mentioned before, as the number of drivers increases, the number of possible outcomes increases exponentially. For example, for 30 drivers with 2 strategies per driver, there are more than 1 billion possible outcomes. Therefore, it becomes computationally infeasible to deterministically find the best and worst possible outcome (equilibrium) through brute force search. Alternatively, we will make few assumptions. First, we will use the case in which all drivers will choose their least favoured strategy. The worst equilibrium is either equal to this one or even worse. Therefore, this can be considered as a strong upper bound. For the lower bound, i.e., the best equilibrium, it's rather difficult to make any form of direct estimation to the best equilibrium. Therefore, we assume that the system's PoA is $\sqrt{N} = \sqrt{30} = 5.4$, which incidentally is the same for a 3^{rd} degree polynomial PoA cost function, and from this value it is possible to compute the best equilibrium. Obviously these are indeed strong assumptions. However, as seen in Table IV, even though the PoA is higher than the deterministic case, our model's efficiency with regard to the overall performance is proven.

VI. CONCLUSIONS

In this paper we described a territory sharing game for social taxi network. In this game, a regulatory body in form of smart app will allow a group of drivers to engage in a cooperative endeavour by which they earn the right to operate in certain attraction areas. We have developed a bargaining based solution for this game by which an efficient and effective solution is found. A no regret model was developed to ensure

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that the final outcome of our game will converge to a coerced correlated equilibrium.

To validate the developed model we conducted an extensive experimental work. Through this work we showcase the implementation of our model, the effectiveness of the no-regret model and an analysis of the overall system efficiency by examining the price of anarchy (PoA).

The main concern with the existing solutions (price-surging based approaches) is the welfare the system users: drivers and customers. The PoA, as a measure of system efficiency, is employed to examine the the social welfare of the system. The developed solution of the territory sharing game has been shown to have an improved PoA over the existing self-scheduling approaches, which subsequently leads to an improved social welfare of the system.

For future work, it is possible to add to the set of strategies some form of time stamps. That is, the model will not only operate on location basis, but it will be a locationally and temporally aware model. Furthermore, the time window (T) in the no regret model can be dynamic to serve both short and long term goals. The main limitation of the developed system is the issue of finding the maximum number of drivers to participate in the game prior to the commencement of the game. The Price of Anarchy can be used as a method of upper bounding the number of drivers based on general assessment of various equilibria. The problem with this approach is that it is localized to each game. Alternatively, dynamic bounding of the different set-ups should be investigated.

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