Stackelberg Pricing Game for Ride-Hailing Platforms With Combined Travel Modes

Weina Xu, Gui-Hua Lin, Tingsong Wang, and Xide Zhu

***Abstract*— This paper proposes a pricing strategy based on the variable-ratio charging-compensation scheme for a ride-hailing platform with combined travel modes (Express or Carpool) of riders in a general network composed of multiple corridors. We establish a Stackelberg game model with the platform as a leader and the riders as followers to capture the decision-making process of stakeholders, in which the pricing decision is determined by the platform in the upper level taking each rider's optimal response into account and travel modes are selected by riders independently in the lower level. The built model is a mixed-integer bilevel programming problem, which is difficult to solve due to its inherent hierarchical structure and discrete variables. By means of some mathematical techniques, we transform the bilevel model equivalently into a single-level mixed-integer nonlinear programming problem. Based on the numerical examples and results analysis, we bring some interesting managerial insights into the pricing of ride-hailing platforms, one of which is that booking fees of Express, unit time and distance fees of Express, compensations for Express riders and Express drivers are the core pricing factors to affect travel mode choices of riders.**

***Index Terms*— Ride-hailing platform, variable-ratio charging-compensation scheme, travel mode, Stackelberg game, mixed-integer nonlinear programming.**

I. Introduction

ACCORDING to the research report [1], the global shared mobility market is surprisingly estimated to be 114 billion in 2026. Top players are DiDi Chuxing, Uber Technologies Inc., Cabify, ANI Technologies Pvt. Ltd. (OLA), Gett, Lyft Inc., Europcar, Grab, Careem, Taxify OÜ, BlaBlaCar, Wingz Inc., Curb Mobility, etc. Over the past few years, the market value in China, China's shared travel transaction amount has been able to reach over 32 billion USD, accounting for about 11.3% of travel consumption expenditure [2]. With the popularity of cutting-edge technologies such as 5G, big data, and cloud computing, the scale of ride-hailing industry will continue to expand and the number of shared mobility participants and consumption levels will continue to increase year by year.In China, a ride-hailing platform like DiDi Chuxing provides diversified services for nearly 550 million person-time in more than 400 cities [3]. In the ride-hailing market, Express and Carpool are the top two representative businesses. Express is the dominant business of ride-hailing platform and modes such as premium cars and luxury cars can be regarded as its derivatives. Express offers more effective, more affordable and more comfortable transportation services with flexibility and quick response. Carpool allows riders to share seats with others who take the same route with a discount fare, which is another hot trafficked travel mode in the ride-hailing market.

The ride-hailing platform utilizes idle vehicles, which can bring additional income, and accordingly, it attracts people who want to earn additional income into the platform. Charging commissions from each transaction is the business base for the ride-hailing platform. In the field of ride-hailing platform, in addition to discussing typical mode choices, the pricing strategy has long been a core element of operations on ride-hailing platforms. Compared with other management factors, price is a more manageable factor. The balance of supply and demand in the market can generally be adjusted by a reasonable price. Specifically, the pricing strategy (also known as the charging-compensation scheme) is about charging fare by riders, paying remuneration to drivers, determining the difference between the fare and wages withheld by the platform as a commission, and rewarding both drivers and riders from a portion of its revenue. Fare, wage, compensation and commission are the core of the operation of ride-hailing platform. Besides, as the shared mobility capital market returns to rationality in recent years, the era of everlasting capital investment in the ride-hailing industry has come to an end. Instead of generous subsidies, these ride-hailing platforms are gradually shifting from the full-input market share to efficient management and benign development. In addition, the ride-hailing has profound effects on transportation and environmental sustainability. Therefore, it is of great practical significance to study and improve the operating mode and pricing strategy of shared travel.

The ride-hailing platform consists of a typical two-sided market [4], [5]. There are two common pricing schemes: fixed-ratio charging-compensation scheme (FCS) and variable-ratio charging-compensation scheme (VCS). However, FCS does not offer much flexibility for the ride-hailing platform to achieve its desirable objectives [6]. In this paper, through optimizing the interactive and integrated pricing strategy involving price, wage, commission and compensation, we introduce a pricing strategy based on the VCS for the ride-hailing platform to maximize its generalized profit.

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In general, travel assignment models are conducted either on a single corridor or in a general network. Note that single corridor studies dynamic travel problems while general network focuses on static traffic assignments, which means that the complexity of the network structure and traffic problems cannot be unified simultaneously. In this paper, a Stackelberg game pricing model is conducted in a general network composed of multiple corridors. This network includes some time and geographic information of orders, which reflects some structural properties of the general network. The built model captures the decision-making process of stakeholders, in which the pricing strategy is determined by the ride-hailing platform (leader) and then the travel modes of riders (followers) are chosen sequentially, and mainly aims to answer the following two issues, that is, how the ride-hailing platform is impacted by the pricing strategy and how the riders' mode decisions result in repositioning of limited ride-hailing resource.

The main contributions of this paper are summarized as follows:

(i) This paper studies the travel mode choices of riders with respect to Express and Carpool. We introduce VCS into the ride-hailing platform and investigate the optimal VCS to achieve generalized profit of the ride-hailing platform. The proposed pricing strategy is a pre-strategy (not a real-time dynamic adjustment) by estimating historical data to obtain a general strategy and give the platform some price references.

(ii) The proposed model is a mixed-integer bilevel programming problem, where the upper level is used to choose pricing strategy and the lower level is used to determine each rider's decision. It is known that solving a bilevel programming problem is difficult due to its inherent hierarchical structure. We provide a solvable single-level reformulation by equivalently transforming the model into a mixed-integer nonlinear programming problem.

(iii) With the help of DiDi Chuxing GAIA Open Dataset Initiative and BONMIN (Basic Open-source Nonlinear Mixed Integer) solver, a large number of numerical examples are constructed. The results indicate that booking fees of Express, unit time and distance fees of Express, compensations for Express riders and Express drivers are the core factors to affect travel mode choices of riders. These interesting findings gain some management insights into the pricing of ride-hailing platforms.

The remainder of this paper is organized as follows. Section II provides a review of the existing ride-hailing literature including the concepts of ride-hailing and ride-sharing, network design, mode choice, pricing strategies and Stackelberg game. Section III introduces the Stackelberg game model to choose the optimal pricing scheme of the ride-hailing platform and optimal travel modes of riders. Taking into account the characterizations of this model, a solvable single-level reformulation is proposed and their equivalence is also proved. In Section IV, we give numerical instances based on DiDi Chuxing GAIA Open Dataset Initiative and present some interesting results. The effects of pricing strategy and travel mode choice are also extensively studied in this section. Finally, Section V concludes the work of this paper and states the future work.

II. Literature Review

A. Ride-Hailing Concepts and Its Network Design

Shared mobility is mainly divided into sharing a vehicle (e.g., car-sharing, bike-sharing, motorcycle-sharing), sharing a ride with ride-sharing (carpooling, vanpooling), on-demand ride services (ride-hailing, ride-splitting, e-hailing) [7], [8]. Ride-hailing (or ride-sourcing) refers to transportation services that connect private-car drivers with riders via mobile phones and applications [7], [9], [10]. It includes traditional taxis and car services, with companies describing themselves as transportation network companies or mobility service providers [11]. Successful TNCs include DiDi Chuxing, Uber, Lyft, Gett, Mytaxi, Ola Cabs, BlaBla Car, Careem, and Kako Taxi.

Network design problems in traffic markets are typically addressed on corridor networks or general networks. A single bottleneck corridor network is a linear geographic band [12]. Most models focus on corridor networks, which cater to travel demand between specific areas with fixed routes, such as morning commutes or school bus routines [13], [14], [15], [16], [17], [18], [19]. In a scenario with substantial traffic flow, a complex traffic network can be seen as a collection of bottleneck corridor problems occurring simultaneously in various locations. Corridor network problems help unravel intricate traffic issues. While a single corridor tackles dynamic travel challenges, a general network deals with static traffic assignment and varying ridership for each ride-sharing vehicle [20], [21], [22]. The network structure's complexity and problem complexity remain distinct. Network design decisions involve selecting which links to activate, connecting users and facilities, and potentially linking facilities together [23]. For example, the fixed-charge network design problem is a well-known issue in network design, explored in [24], [25]. Operational costs, encompassing allocation and routing costs, vary based on user demand and total flow per arc, with a single allocation functioning as a network.

In ride-hailing systems, it may be necessary to assess user functionality and performance requirements to determine the scale of the system. Researching the demand response issues of ride-hailing platforms, including the impact of matching mechanisms on the efficiency of the transportation system and comparisons with traditional street-hailing systems, may be involved. This could involve an assessment of the system scale to determine the effectiveness of the matching mechanism [26], [27].

B. Travel Mode Choice

The travel mode choice of riders has become a more significant consideration for the ride-hailing platform. If the service cost and duration do not perform as expected, riders can choose any travel mode during their commutes, which leads to competition for riders across modes. The travel mode choice problem is essentially the traffic flow assignment problem.

Ride-hailing platforms typically offer a range of service options differentiated by price and quality [9]. The choice may depend on various factors such as the characteristics of the service providers, riders themselves, and other physical conditions, including wage, waiting time, travel time, vehicle type, number of people requesting a trip, budget, value of time (VOT), weather, and congestion. An important extension is to consider individuals with different VOTs to distinguish between populations and individuals [17]. The travel mode choice problem commonly involves the following four issues: (i) Influential travel mode choice models include general models [17], while extensive work has also been conducted in diverse specific contexts, such as high-occupancy vehicle lanes [28] and rail-based long-distance public transport trips [29]. (ii) The choice between a ride-hailing platform and conventional modes, such as taxis and public transit [15], [21], [22]. (iii) The choice between diverse service options on ride-hailing platforms, especially the choice between a solo ride and a shared ride [17], [30], [31], [32], which is also known as Express and Carpool in practice. (iv) The behavior and impact of rider choice between diverse service options on a platform [33].

Users have a self-understanding of the product value, generally referred to as user experience. Designers need to understand how to design for user experience and how the product's design achieves specific user experience goals. Forlizzi and Ford propose an initial framework for understanding experience as it relates to user-product interactions [34]. Empathy is used to help position some emerging design and user experience methodologies in terms of dynamically shifting relationships between designers, users, and artefacts [35]. The UX Curve aims to assist users in retrospectively reporting how and why their experience with a product has changed over time, determining the quality of long-term user experience, and identifying influences that either improve user experience over time or cause it to deteriorate [37].

C. Pricing Strategy

Ride-hailing platforms consist of a typical two-sided market. In the most influential work on two-sided markets [4], [5], price affects both demand (riders) and supply (drivers). There has been considerable interest in various modes associated with travel costs consisting of time cost (e.g., travel time and VOT), and non-time cost (e.g. fare, wages, compensation, and commission). Previous studies have primarily priced static ride-hailing modes by applying a fixed discount rate to share the cost of ride-hailing, which addresses the cost issue but overlooks the inconvenience to passengers caused by detours during ride-hailing. Some other factors such as safety, convenience, provider behavior, and traffic congestion mentioned in [6] have been individually considered to comprehensively improve the current static ride-hailing pricing model.

There are two pricing strategies: the fixed-ratio charging-compensation scheme (FCS) and the variable-ratio charging-compensation scheme (VCS). The FCS is the most common scheme studied so far to solve the traffic assignment problem and to achieve the social optimum [13], [16], [17], [20], [22]. However, cross-group externalities with VCS in the platform are often weaker than that with FCS, which could increase platform profits [4], [5]. On the other hand, different with selective charging in [36], FCS does not offer much flexibility for the ride-hailing platform to performance its disutility, that is, it simplifies some realities.

VCS in the ride-sharing platform has a strong temporal to employ origin-based pricing and origin-destination-based pricing in order to decrease geospatial supply and demand imbalances [9], [38], [39]. In the meanwhile, Zha et al. find that the platform and drivers could benefit from higher prices while riders may be made worse off. Another important feature is spatial price segmentation, i.e., setting different prices in different regions to balance the demand and the supply in the spatial dimension [40]. Guda and Subramanian explicitly account for the spatial pricing strategic interaction in their decisions to move between adjacent zones, which is useful even in supply-exceed-demand regions to increase the total platform profit [41]. Luo and Saigal deal with the complicated spatiotemporal pricing problem, which allows the agency to determine the optimal price-compensations without suffering from combinatorial complexities. Of particular note is the demand elasticity with respect to prices, which is a critical input for pricing problems and is examined in several industry reports [42]. Guo and Xu discuss a new method for optimizing both operation cost and passenger quality of service, taking into account the rebalancing of idle vehicles within the system [43]. Before Mobility-on-Demand (MoD) mentioned by [44], integrates vehicle rebalancing with charging scheduling to address issues with electric vehicles has been discussed in MoD systems, focusing on performance indicators such as service rate, average waiting time, and travel distance [45]. The demand for a single ride-hailing platform may be more elastic than that for the entire market. Consumers are more responsive to price changes because they can easily switch to another company [46].

D. Stackelberg Game in Ride-Hailing Markets

Given the network topology, origin-destination (OD) travel demands and link performance, the classical traffic assignment model aims to determine travelers' path choices having the minimum travel cost [47]. Additionally, the use of model-free deep reinforcement learning has been shown to dynamically learn and optimize the relocation of vehicles in ride-hailing systems [48]. Mixed equilibrium has been proposed as a consequence to study the equilibration behavior of multiple players in the ride-sharing market [6], [21], [22], [31], [49]. Many studies on pricing strategies are based on Stackelberg games to access the effects of transportation policies on traffic flows and system performance. Typically, the decisions are made by policy-makers or operators as leaders in the upper level, and travelers as followers respond through an equilibrate adjustment process in the lower level [50]. Stackelberg game provides a good perspective to integrate the ride-hailing platform and users (both drivers and riders) in the transportation system.

TABLE I
STACKELBERG GAME MODELS IN THE EXISTING RIDE-HAILING RESEARCHES

	Upper and lower level objects	Factors	Variables
Zhao et al. [51]	U: greenhouse gas emissions L: total travel cost	system time, travel cost, emission toll travel time	continuous integer
Di et al. [20]	U: total vehicle travel time L: ride-sharing UE with HOT lans	toll-pricing imposed to solo drivers building high occupancy toll lanes	continuous 0-1
Zha et al. [40]	U: profit L: equilibrium of drivers' work hour	fares and commission average hourly revenue	continuous continuous
Mofidi and Pazour [52]	U: platform's expected benefit L: all suppliers' utilities	request and supplier matching suppliers' alternative selection	0-1 0-1

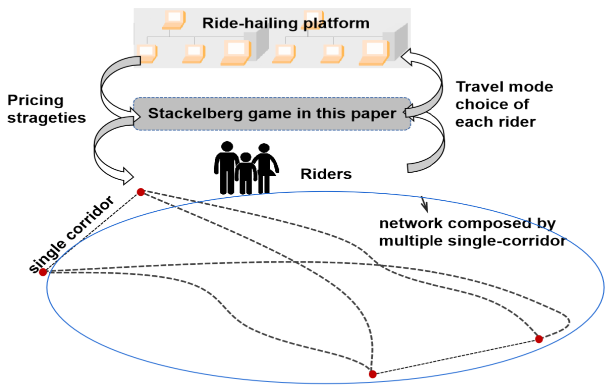

Fig. 1. The network structure.

There are many candidate objects in the upper level, and different objective choices will lead to different optimal decisions. One reason for choosing to minimize the total generalized travel cost, instead of maximizing the overall occupancy ratio or minimizing the total number of solo drivers, is that the last two problems often generate multiple solutions that are not conducive to decision making. Table I provides a systematic comparison of the key modeling components in the existing ride-hailing researches.

III. STACKELBERG GAME BETWEEN THE RIDE-HAILING PLATFORM AND RIDERS

In this section, we first discuss the costs and utilities of Express and Carpool. Then, we give a Stackelberg game in a general network composed of multiple corridors, see Figure 1 in which the pricing strategy is determined by the ride-hailing platform in the upper level and the travel mode is selected by each rider in the lower level. Before the presence of ride-hailing programs, we consider a multiple corridors network consists of two dynamic travel choice modes, i.e., Express and Carpool. The network design problem in the ride-hailing platform is mainly performed on single corridor networks or a general network. A single corridor network is largely defined as a linear geographic band. Most models are conducted on single corridor networks, which are special networks that serve between two specific areas with well-defined routes and travel demand. Considering realistic high traffic demand scenarios, complex transportation networks can be viewed as a combination of multiple single corridor problems occurring simultaneously at multiple locations due to the complexity and variability of the situation in the network. In each bottleneck corridor problem, the origin-destination pair and the departure time are easy to attach.

Before investigating the travel mode choice of riders, we make the following assumptions:

A1: Drivers decide whether to purchase cars based on their primary travel needs before the emergence of the dynamic ride-sharing platform. Riders and drivers do not change their car ownership in the short run since they only can have one identity.

A2: Each Carpool driver can be matched with at most three riders through the ride-hailing platform, and each Express rider is traveling independently.

A3: Each Express driver can obtain compensation from the ride-hailing platform. There is no compensation for Carpool drivers as they have travel purpose whether or not they receive an order.

A4: Each rider can obtain compensation from the ride-hailing platform. Riders are only heterogeneous in terms of their values of time.

A. Travel Mode Choice of Riders

When a rider $i \in \{1, \cdots, N\}$ places an order, there are some identified informations including the departure time, the latitude and longitude of the origin O_i and destination D_i. The travel process from O_i to D_i involves an expected travel time t_i^s and congestion time t_i^c. Specifically, t_i^s is a fixed time of free flow, and t_i^c is an elastic value involving wait time and other uncertain time depending on road conditions.

We now present the charging structure of Express, which consists of booking fees (base or initial fees), time fees over the base time t_0, distance fees over the base distance S_0, long distance fees between S_l and S_h, and ultra-long distance fees exceeding S_h ($S_h > S_l > S_0$). Some notations are given as follows.

- c_0 Booking fee
- c_t Time fee per unit over t_0
- c_d Distance fee per unit over S_0
- c_l Long distance fee per unit within $[S_l, S_h]$
- c_u Ultra-long distance fee per unit exceeding S_h

Let S_i denote the travel distance of a rider i from O_i to D_i. In order to measure how far S_i exceeds the base distance S_0, it is usually denoted as $\max\{S_i - S_0, 0\}$. Similarly, $\max\{S_i - S_l, 0\}$ and $\max\{S_i - S_h, 0\}$ are used to determine the distances over S_l and S_h, respectively. The time exceeding the base time t_0 is given by $\max\{t_i^s + t_i^c - t_0, 0\}$. Consequently, for a rider i, the travel cost of choosing Express, denoted by C_e^i, can be computed as follows:

$$\begin{aligned} C_e^i := &\underbrace{c_0}_{\text{booking fee}} + \underbrace{c_d \max\{S_i - S_0, 0\}}_{\text{distance fee}} + \underbrace{c_t \max\{t_i^s + t_i^c - t_0, 0\}}_{\text{time fee}} \\ &+ \underbrace{c_l\big(\max\{S_i - S_l, 0\} - \max\{S_i - S_h, 0\}\big)}_{\text{long distance fee}} \\ &+ \underbrace{c_u \max\{S_i - S_h, 0\}}_{\text{ultra-long distance fee}} \end{aligned}$$

where those items are concluded the booking fee, regular distance fee and time fee, long distance fee, and ultra-long distance fee, as well as the notation ':=' means 'equal by definition'.

Let δ_e indicate the compensation for a rider choosing Express from the ride-hailing platform. Then, the actual fare paid by the rider i is $C_e^i - \delta_e$. User experience of Express, such as emotions, perceptions, responses (both physical and psychological) and behaviors, is a significant consideration for the platform, which is quantified as θ_e.

A positive θ_e means a good user experience while a negative value indicates a bad experience. Let ω_i be the value of time (VOT) per unit of rider i considering individual differences. Then, the utility of rider i choosing Express is given by

$$U_e^i := \underbrace{\theta_e}_{\text{user experience}} - \underbrace{\omega_i(t_i^s + t_i^c)}_{\text{time cost}} - \underbrace{\left(C_e^i - \delta_e\right)}_{\text{actual fare}} \quad (1)$$

where those items represent the user experience of Express, the time cost, and the actual fare paid by the rider respectively.

Next, let us discuss another travel mode, i.e., Carpool. Since Carpool is diverse in detouring and waiting in real life, it is difficult to estimate the exact travel time or distance. Hence, it is reasonable to assume that the fare of each Carpool rider i is determined by the expected travel time t_i^s (equivalently by the distance S_i). Let α be the unit time fee of Carpool, which is given by the ride-hailing platform.

Thus, for a rider i, the travel cost of choosing Carpool, denoted by C_c^i, is given as

$$C_c^i := \alpha t_i^s.$$

Compared with Express, Carpool usually has an extra detouring time. It should be noted that obtaining the exact time for Carpool may be difficult. In this paper, we propose employing the average coefficient β as an approximate representation of each rider's detouring time. This average coefficient is assumed to be a fixed parameter, which can be estimated through the analysis of extensive historical data. Thus, for each rider i, the travel time of Carpool is $t_i^s + \beta t_i^s$.

Let δ_c indicate the compensation for a rider choosing Carpool from the ride-hailing platform. Then, the actual fare paid by the rider i is $C_c^i - \delta_c$. Similar to θ_e, we denote by θ_c the user experience of Carpool. Then, the utility of rider i choosing Carpool is given by

$$U_c^i := \underbrace{\theta_c}_{\text{user experience}} - \underbrace{\omega_i(t_i^s + \beta t_i^s + t_i^c)}_{\text{time cost}} - \underbrace{(C_c^i - \delta_c)}_{\text{actual fare}} \quad (2)$$

where those items are the user experience of Carpool, the time cost, and the actual fare paid by the rider in turn.

B. Stackelberg Game Between the Platform and Riders

Denote by $\boldsymbol{x} := (c_0, c_t, c_d, c_l, c_u, \alpha, \lambda, \delta_e, \delta_c, \xi) \in \mathcal{X}$ the charging-compensation strategy of the ride-hailing platform, where λ is the commission per order of the platform, ξ is the fixed fee per order to compensate Express drivers and $\mathcal{X}$ is the feasible region. For any $\boldsymbol{x} \in \mathcal{X}$ given by the platform, by introducing 0-1 variable $y_i \in \{0, 1\}$, we can simply rewrite the utility of each rider i as

$$U_i(\boldsymbol{x}, y_i) := y_i U_e^i(\boldsymbol{x}) + (1 - y_i) U_c^i(\boldsymbol{x}) \quad (3)$$

where $U_i(\boldsymbol{x}, y_i)$ is equal to $U_e^i(\boldsymbol{x})$ when $y_i = 1$ and, on the contrary, $U_i(\boldsymbol{x}, y_i)$ becomes $U_c^i(\boldsymbol{x})$ when $y_i = 0$. The objective of each rider i is to maximize its utility, that is,

$$\mathcal{S}_i(\boldsymbol{x}) := \arg\max_{y_i \in \{0,1\}} U_i(\boldsymbol{x}, y_i). \quad (4)$$

For each i, the optimal solution $z_i \in \mathcal{S}_i(\boldsymbol{x})$ represents

$$z_i = \begin{cases} 1, & \text{if the rider } i \text{ selects Express,} \\ 0, & \text{if the rider } i \text{ selects Carpool.} \end{cases}$$

Particularly, $\mathcal{S}_i(\boldsymbol{x}) = \{0, 1\}$ means that there is no difference in choosing Express or Carpool for rider i.

Given $\boldsymbol{x} \in \mathcal{X}$ and $z_i \in \mathcal{S}_i(\boldsymbol{x})$, the platform's income from rider i is $\lambda\big(z_i C_e^i + (1 - z_i) C_c^i\big)$ and the compensation for rider i is $z_i \delta_e + (1 - z_i)\delta_c$. As per Assumption A3, the platform's compensation for all Express drivers is $\sum_{i=1}^N \xi z_i$. Clearly, for each Express driver, the more orders received the higher the compensation. Taking into account each response $z_i \in \mathcal{S}_i(\boldsymbol{x})$, the payoff (utility) of the platform is

$$\begin{aligned} P(\boldsymbol{x}, \boldsymbol{z}) := &\underbrace{\sum_{i=1}^N \lambda\Big(z_i C_e^i + (1 - z_i) C_c^i\Big)}_{\text{income}} - \underbrace{\sum_{i=1}^N (z_i \delta_e + (1 - z_i)\delta_c)}_{\text{compensation for riders}} \\ &- \underbrace{\sum_{i=1}^N \xi z_i}_{\text{compensation for Express drivers}} \end{aligned}$$

where the first item represents the overall income from order fares, the second item denotes the overall compensation for riders, and the third item denotes the overall compensation for Express drivers. Suppose that, for the platform, the amount of compensations will not exceed the total subsidy budget b, that is,

$$\sum_{i=1}^N \big(z_i \delta_e + (1 - z_i)\delta_c\big) + \sum_{i=1}^N \xi z_i \le b.$$

The platform is intended to maximize the payoff with considering each rider's mode choice. Suppose that each rider is cooperative and the platform can decide the most suitable mode when there is no difference in choosing Express or Carpool for each rider. Based on the above consideration, a Stackelberg game model for the platform is built as

$$\begin{aligned}\textbf{SGM}\max_{\boldsymbol{x},\boldsymbol{z}}\ & P(\boldsymbol{x},\boldsymbol{z})\\ \text{s.t.}\ & \sum_{i=1}^{N}\left(z_i\delta_e+(1-z_i)\delta_c\right)+\sum_{i=1}^{N}\xi z_i\le b,\ \boldsymbol{x}\in\mathcal{X},\\ & z_i\in\arg\max_{y_i\in\{0,1\}}U_i(\boldsymbol{x},y_i),\ i=1,\cdots,N.\end{aligned}$$

Comments: The model **SGM** is a mixed-integer bilevel programming problem (BLPP) where $\boldsymbol{x}$ is the upper level decision vector representing the pricing strategy of the ride-hailing platform, and z_i related to $\boldsymbol{x}$ denotes the optimal solution of each lower level problem (4). So far, some numerical algorithms for solving discrete or mixed-integer BLPPs have been studied by researchers, such as branch-and-bound [53], [54], Benders decomposition [55], decomposition with nonlinear optimization techniques [56], trust-region mechanism [57], and some heuristic algorithms [58]. Although these numerical algorithms are widely used in practice, their solutions are generally not optimal and theoretically they are not guaranteed to be stationary. From the perspective of constrained optimization, the commonly used method for a BLPP is to replace the lower level problem by its first-order optimality condition and solve a mathematical program with equilibrium constraint (MPEC) [59], [60]. However, the MPEC reformulation requires that, for any given upper level variable, the lower level is a smooth convex optimization problem while satisfying some appropriate regularity conditions. It is clear that the MPEC reformulation method can not be applied to **SGM** due to the existence of 0-1 variables in the lower level problem. In the following subsection, we will propose a solvable single-level reformulation for this problem.

C. Model Reformulation

Due to the existence of the generalized equation (implicitly determined) constraint

$$z_i\in\arg\max_{y_i\in\{0,1\}}U_i(\boldsymbol{x},y_i),\tag{5}$$

solving **SGM** is challenging. We first give a theorem to indicate that (5) can be equivalently transformed into a certain system.

Theorem 1: Equation (5) is equivalent to the following system with inequalities and 0-1 variables:

$$\left(U_c^i(\boldsymbol{x})-U_e^i(\boldsymbol{x})\right)(1-2z_i)\ge 0,\quad z_i\in\{0,1\}.\tag{6}$$

Proof: The solution of (5) has two cases:

(i) $z_i=1$ if and only if z_i is an optimal solution of

$$\begin{aligned}\max_{y_i}\ & y_iU_e^i(\boldsymbol{x})+(1-y_i)U_c^i(\boldsymbol{x})\\ \text{s.t.}\ & U_e^i(\boldsymbol{x})\ge U_c^i(\boldsymbol{x}),\quad y_i\in\{0,1\}.\end{aligned}$$

(ii) $z_i=0$ if and only if z_i is an optimal solution of

$$\begin{aligned}\max_{y_i}\ & y_iU_e^i(\boldsymbol{x})+(1-y_i)U_c^i(\boldsymbol{x})\\ \text{s.t.}\ & U_e^i(\boldsymbol{x})\le U_c^i(\boldsymbol{x}),\quad y_i\in\{0,1\}.\end{aligned}$$

Especially for the situation that $U_e^i(\boldsymbol{x})=U_c^i(\boldsymbol{x})$, z_i can be 0 or 1. Thus, for any given $\boldsymbol{x}\in\mathcal{X}$, (5) can be equivalently written as

$$z_i=\begin{cases}1, & U_e^i(\boldsymbol{x})>U_c^i(\boldsymbol{x}),\\ 0, & U_e^i(\boldsymbol{x})<U_c^i(\boldsymbol{x}),\\ 0\text{ or }1, & U_e^i(\boldsymbol{x})=U_c^i(\boldsymbol{x}),\end{cases}\qquad i\in\mathcal{I}.\tag{7}$$

Next, we show the equivalence between (7) and (6) for each $i\in\mathcal{I}$.

(i) Suppose that z_i is a solution of (7). In the case of $U_e^i(\boldsymbol{x})>U_c^i(\boldsymbol{x})$, we have $z_i=1$. It is easy to verify that in this case, $z_i=1$ satisfies (6). In the case of $U_e^i(\boldsymbol{x})<U_c^i(\boldsymbol{x})$, we have $z_i=0$. Clearly, in this case, $z_i=0$ also satisfies (6). In the case of $U_e^i(\boldsymbol{x})=U_c^i(\boldsymbol{x})$, we have $z_i=0$ or $z_i=1$. Clearly, in this case, z_i satisfies (6). This shows that (7) implies (6) for each $i\in\mathcal{I}$.

(ii) Suppose that z_i is a solution of (6). In the case of $U_e^i(\boldsymbol{x})>U_c^i(\boldsymbol{x})$, we have $1-2z_i\le 0$ as per (6) and hence, as per $z_i\in\{0,1\}$, we know $z_i=1$. It is clear that in this case, $z_i=1$ satisfies (7). In the case of $U_e^i(\boldsymbol{x})<U_c^i(\boldsymbol{x})$, we have $1-2z_i\ge 0$ as per (6) and hence, as per $z_i\in\{0,1\}$, we know $z_i=0$. Clearly, in this case, $z_i=0$ satisfies (7). In the case of $U_e^i(\boldsymbol{x})=U_c^i(\boldsymbol{x})$, we have $z_i=0$ or $z_i=1$. Clearly, in this case, z_i satisfies (7). This shows that (6) implies (7) for each $i\in\mathcal{I}$.

The above analysis shows the equivalence among (5), (6), and (7). □

Based on Theorem 1, we can equivalently transform **SGM** into the following mixed-integer nonlinear programming problem:

$$\begin{aligned}\textbf{MINP}\ \max_{\boldsymbol{x},\boldsymbol{z}}\ & P(\boldsymbol{x},\boldsymbol{z})\\ \text{s.t.}\ & \sum_{i=1}^{N}\left(z_i\delta_e+(1-z_i)\delta_c\right)+\sum_{i=1}^{N}\xi z_i\le b,\ \boldsymbol{x}\in\mathcal{X},\\ & \left(U_c^i(\boldsymbol{x})-U_e^i(\boldsymbol{x})\right)(1-2z_i)\ge 0,\ z_i\in\{0,1\},\\ & i=1,\cdots,N.\end{aligned}$$

In the rest of this subsection, we investigate the platform's preference when there is no difference in choosing Express or Carpool for a rider. Suppose that $(\bar{\boldsymbol{x}},\bar{\boldsymbol{z}})$ is an optimal solution of **MINP**. It is not difficult to understand that $\bar{z}_i=0$ if $U_c^i(\bar{\boldsymbol{x}})>U_e^i(\bar{\boldsymbol{x}})$ and $\bar{z}_i=1$ if $U_c^i(\bar{\boldsymbol{x}})<U_e^i(\bar{\boldsymbol{x}})$. The following theorem shows the solution property for the situation of $U_e^i(\bar{\boldsymbol{x}})=U_c^i(\bar{\boldsymbol{x}})$.

Theorem 2: Let $(\bar{\boldsymbol{x}},\bar{\boldsymbol{z}})$ be an optimal solution of **MINP**. In the situation of $U_e^i(\bar{\boldsymbol{x}})=U_c^i(\bar{\boldsymbol{x}})$ for i,

(i) if $\bar{\delta}_c-\bar{\delta}_e=\frac{\bar{\lambda}}{1-\bar{\lambda}}(\bar{\theta}_e-\bar{\theta}_c+\beta\omega_it_i^s)$, then $\bar{z}_i=1$ or $\bar{z}_i=0$;

(ii) if $\bar{\delta}_c-\bar{\delta}_e<\frac{\bar{\lambda}}{1-\bar{\lambda}}(\bar{\theta}_e-\bar{\theta}_c+\beta\omega_it_i^s)$, then $\bar{z}_i=1$;

(iii) if $\bar{\delta}_c-\bar{\delta}_e>\frac{\bar{\lambda}}{1-\bar{\lambda}}(\bar{\theta}_e-\bar{\theta}_c+\beta\omega_it_i^s)$, then $\bar{z}_i=0$.

Proof: It is obvious that $U_e^i(\bar{\boldsymbol{x}})=U_c^i(\bar{\boldsymbol{x}})$ is equivalent to

$$C_e^i(\bar{\boldsymbol{x}})-\bar{\delta}_e-(C_c^i(\bar{\boldsymbol{x}})-\bar{\delta}_c)+\beta\omega_it_i^s+(\bar{\theta}_e-\bar{\theta}_c)=0.\tag{8}$$

(i) If $\bar{\delta}_c - \bar{\delta}_e = \dfrac{\bar{\lambda}}{1-\bar{\lambda}}(\bar{\theta}_e - \bar{\theta}_c + \beta\omega_i t_i^s)$, it follows from (8) that $\bar{\lambda}C_c^i(\bar{\boldsymbol{x}}) - \bar{\delta}_c = \lambda C_e^i(\bar{\boldsymbol{x}}) - \bar{\delta}_e$ and hence

$$\begin{aligned}&\bar{\lambda}\Big(0C_e^i(\bar{\boldsymbol{x}}) + (1-0)C_c^i(\bar{\boldsymbol{x}})\Big) - 0\bar{\delta}_e - (1-0)\bar{\delta}_c\\ &= \bar{\lambda}\Big(1C_e^i(\bar{\boldsymbol{x}}) + (1-1)C_c^i(\bar{\boldsymbol{x}})\Big) - 1\bar{\delta}_e - (1-1)\bar{\delta}_c.\end{aligned}$$

The above equation means that $P(\bar{\boldsymbol{x}}, z)$ takes the same value at $z_i = 0$ and $z_i = 1$ when other elements are fixed. Hence, we have $\mathcal{S}_i(\bar{\boldsymbol{x}}) = \{0, 1\}$, i.e., $\bar{z}_i = 0$ or $\bar{z}_i = 1$.

(ii) If $\bar{\delta}_c - \bar{\delta}_e < \dfrac{\bar{\lambda}}{1-\bar{\lambda}}(\bar{\theta}_e - \bar{\theta}_c + \beta\omega_i t_i^s)$, it follows from (8) that $\bar{\lambda}C_c^i(\bar{\boldsymbol{x}}) - \bar{\delta}_c < \lambda C_e^i(\bar{\boldsymbol{x}}) - \bar{\delta}_e$ and hence

$$\begin{aligned}&\bar{\lambda}\Big(0C_e^i(\bar{\boldsymbol{x}}) + (1-0)C_c^i(\bar{\boldsymbol{x}})\Big) - 0\bar{\delta}_e - (1-0)\bar{\delta}_c\\ &< \bar{\lambda}\Big(1C_e^i(\bar{\boldsymbol{x}}) + (1-1)C_c^i(\bar{\boldsymbol{x}})\Big) - 1\bar{\delta}_e - (1-1)\bar{\delta}_c.\end{aligned}$$

The above inequation means the value of $P(\bar{\boldsymbol{x}}, z)$ at $z_i = 1$ is more than the one at $z_i = 0$ when other elements are fixed. Then, we have $\bar{z}_i = 1$.

(iii) In a similar way to the proof of (ii), we can show $\bar{z}_i = 0$ if $\bar{\delta}_c - \bar{\delta}_e > \dfrac{\bar{\lambda}}{1-\bar{\lambda}}(\bar{\theta}_e - \bar{\theta}_c + \beta\omega_i t_i^s)$. □

Theorem 2 indicates that when there is no difference in choosing Express or Carpool for a rider, the ride-hailing platform has no preference on Express or Carpool in case (i); on the contrary, the platform prefers to choose Express for case (ii) and Carpool for case (iii).

IV. Numerical Examples

In this section, we report our numerical experiments by adopting the BONMIN (Basic Open-source Nonlinear Mixed Integer) solver to solve the reformulated model **MINP** [53]. The basic idea of BONMIN is a branch-and-bound algorithm, which divides the problem into a series of subproblems according to the pre-selected branching criteria and then selects one subproblem to test with a set delimiting function to judge whether it goes forward or backward on the current.

We tested some numerical examples based on the shared data contributing to DiDi Chuxing GAIA Open Dataset Initiative. These data were obtained from the trajectory data of the drivers with orders from the Drip Express platform in the Second Ring Road area of Chengdu City in China over November 2016, including order set, ride start time, ride stop time, pick-up longitude, pick-up latitude, drop-off longitude, and drop-off latitude. The measurement interval of the track points was set to be approximately 2-4 seconds. The trajectory points were processed by tying the roads to ensure that the data could all correspond to the actual road information. Driver and order information was encrypted and desensitized for anonymity.

A. Data Processing

The longitude and latitude (GCJ-02 Coordinate System) of the data is a three-dimensional coordinate, which needs to be converted in calculating the plane distance. The area studied in this paper has a small plotting scale and low latitude, which is suitable for Mercator Projection to plot plane distances. The meridians projected by Mercator Projection are evenly distributed with almost no distortion. Let L_n $(0 \le L_n \le \pi)$ be the longitude of the geographic location. The projection formula for the horizontal coordinate is

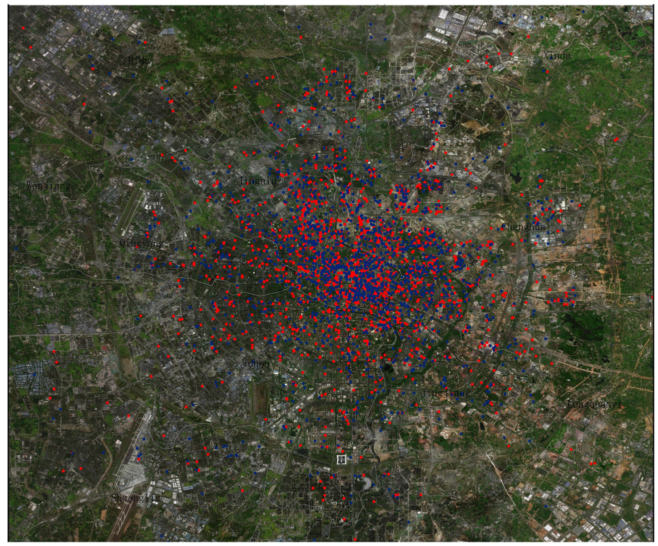

Fig. 2. Location samples of riders in Chengdu City.

$$H(L_n) = R_e L_n,$$

where the earth radius R_e is 6378.137 kilometers.

Next, we focus on the transformation method for the latitude. Let L_a $(-\pi/2 \le L_a \le \pi/2)$ be the latitude. Then, the latitude L_a can be projected into the vertical coordinate as

$$V(L_a) = 0.5R_e \ln \frac{1+\sin L_a}{1-\sin L_a}.$$

Therefore, the Euclidean distance between O_i and D_i can be determined as

$$S_i = \sqrt{\big(H(L_n^{i\,\prime}) - H(L_n^{i\,\prime\prime})\big)^2 + \big(V(L_a^{i\,\prime}) - V(L_a^{i\,\prime\prime})\big)^2},$$

where $L_n^{i\,\prime}$ and $L_a^{i\,\prime}$ are the longitude and latitude of the origin respectively, $L_n^{i\,\prime\prime}$ and $L_a^{i\,\prime\prime}$ are the longitude and latitude of the destination respectively, while $O_i(H(L_n^{i\,\prime}), V(L_a^{i\,\prime}))$ and $D_i(H(L_n^{i\,\prime\prime}), V(L_a^{i\,\prime\prime}))$ are the origin and the destination of i in plane coordinates respectively. Figure 2 shows the geographic locations of riders, where the red points represent the origins and the blue points represent the destinations.

Let ω stand for the VOT (CNY per hour). Suppose that ω follows a (truncated) log-normal distribution with mean $\mathbb{E}(\omega) = e^{\mu+\sigma^2/2}$ and variance $\mathbb{D}(\omega) = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$, that is, $\ln\omega \sim \mathcal{N}(\mu, \sigma^2)$ (Zhao et al. [51]; Wang et al. [17]). Then, the log-density function of ω is

$$f(\omega, \sigma, \mu) = \frac{1}{\omega\sigma\sqrt{2\pi}} \exp\bigg(-\frac{(\ln\omega - \mu)^2}{2\sigma^2}\bigg).$$

In our tests, we set $\mathbb{E}(\omega) = 40$ and $\mathbb{D}(\omega) = 900$. We chose the support set of ω as [0, 200]. Figure 3 and Figure 4 show the probability histograms (displayed in blue) and the probability

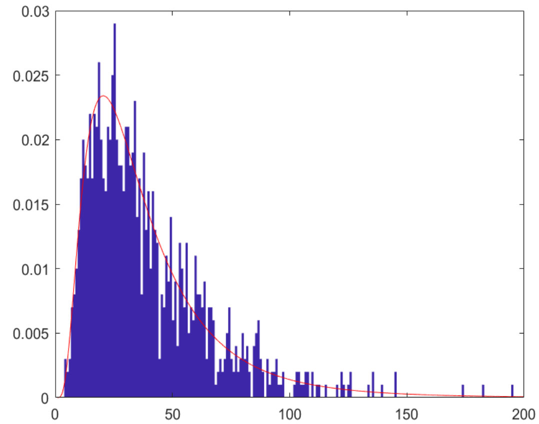

Fig. 3. Probability histograms and densities of VOT with 1000 samples.

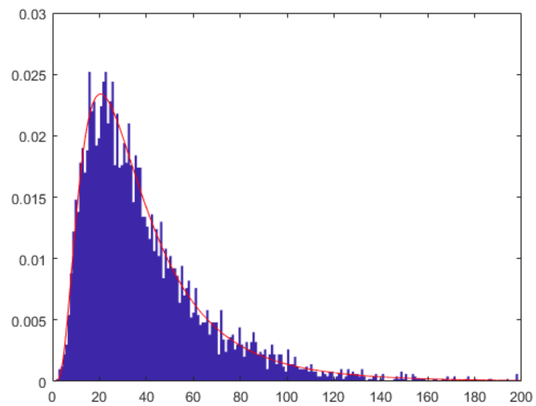

Fig. 4. Probability histograms and densities of VOT with 5000 samples.

densities (displayed in red) of VOT when the sample sizes were 1000 and 5000 in turn.

The parameters in our tests were set as follows. The base distance S_0 was 3 kilometers and the base time t_0 was 10 minutes. S_l and S_h were 15 and 40 kilometers, respectively. The average coefficient of detouring time of Carpool β was 0.3. The total subsidy budget b was one million CNY. The user experiences of Express θ_e and Carpool θ_c were 3 and 0 CNY, respectively. The lower and upper bounds of $\boldsymbol{x}$ were $(5, 1, 0.4, 0.5, 0.8, 1, 0.1, 0.5, 0.5, 10)$ and $(15, 3, 1, 1, 1.5, 2, 0.5, 5, 2, 100)$, respectively.

In Carpool services, the quality of user experience is crucial as it directly affects customer satisfaction and the likelihood of repeat use. Here are some key aspects of user experience in Express and Carpool: comfort and cleanliness u_1, safety u_2, as well as convenience and flexibility u_3. Comfort and cleanliness mainly refer to clean and well-maintained vehicles provided a more comfortable experience for riders. Safety means that ride-hailing service ensures all users are verified enhances the sense of security. Convenience and flexibility mean that detour routes and times should be as short as possible. These factors are linked by a linear correlation. For Express rider, user experience can be quantified as $\theta_e = 1 \cdot u_1 + 1 \cdot u_2 + 1 \cdot u_3 = 2 + 1 + 0 = 3$ in numerical examples in this paper. In the meantime, user experience of Carpool can be quantified as $\theta_c = 1 \cdot u_1 + 1 \cdot u_2 + 1 \cdot u_3 = 1 + 0 - 1 = 0$.

B. Pricing Strategy and Results Analysis

In the ride-hailing field, there is no uniform formula for calculating fixed fees, as different ride-hailing services, regions, and drivers may adopt different pricing standards. However, we can discuss some common calculation methods for Express:

(1) Base fee plus per mile charge like follows:

$$F_e^1 = \underbrace{c_0}_{\text{booking fee}} + \underbrace{c_d S_i}_{\text{distance fee per unit} \cdot \text{travel distance}}$$

where the first item is the booking fee and the second item is distance fee. If the booking fee is 10 CNY and the cost per unit is 2 CNY, then the cost for a 10-kilometer trip would be 10 CNY + (2 CNY/unit × 10 units) = 30 CNY.

(2) Base fee plus time and distance charges like

$$F_e^2 = \underbrace{c_0}_{\text{booking fee}} + \underbrace{c_d S_i}_{\text{distance fee per unit} \cdot \text{travel distance}} + \underbrace{c_t S_i}_{\text{time fee per unit} \cdot \text{travel time}}$$

where the first item is the booking fee, the second item is distance fee and the third item is time fee. If the booking fee is 10 CNY, the cost per kilometer is 2 CNY, and the cost per minute is 0.5 CNY, and the trip is 10 kilometers taking 20 minutes, then the total cost would be 10 CNY + (2 CNY/kilometer × 10 kilometers) + (0.5 CNY/minute × 20 minutes) = 40 CNY.

(3) Dynamic pricing (surge pricing during peak hours) is

$$F_e^3 = \kappa \left[\underbrace{c_0}_{\text{booking fee}} + \underbrace{c_d S_i}_{\text{distance fee}} + \underbrace{c_t S_i}_{\text{time fee}} \right]$$

where κ is dynamic pricing multiplier. During peak hours, if the dynamic pricing multiplier is 1.5, then the total cost for the second example would be 40 CNY × 1.5 = 60 CNY.

(4) Minimum charge limit can be described as

$$F_e^4 = \max(\min \text{charge}, \text{actual cost}).$$

If the minimum charge is 40 CNY, and the actual cost like gas cost is only 30 CNY, the rider will need to pay 40 CNY.

(5) Combined pricing for distance and time has general form like

F_e^5 = booking fee + (distance fee per unit · travel distance) + (distance fee per unit · travel distance within long distance) + (time fee per unit · travel time)

Combined pricing integrated the previous four charge methods, taking into account time, distance, and long distance. Our work C_e^i is based on this combination method, which gives a more reasonable setting.

These fees may vary based on the region, type of service, vehicle type, demand and supply conditions (such as weather, traffic conditions, special events). Many ride-hailing platforms might also include other fees, like cancellation fees, booking fees, or special service charges. In summary, the calculation formulas for fixed fees vary depending on the service and region, and are typically based on a set of standard charges,

TABLE II
OPTIMAL PRICING STRATEGIES WITH DIFFERENT RIDER NUMBERS

Components of pricing strategy $\boldsymbol{x}$	N=1000	2000	3000	4000	5000
Booking fee of Express c_0	5.00	5.00	5.00	5.00	5.00
Unit time fee of Express over 10 c_t	0.40	0.40	0.40	0.40	0.40
Unit distance fee of Express over 3 c_d	1.00	1.03	1.23	1.00	1.00
Unit long distance fee of Express within [15,40] c_l	0.68	1.00	1.00	0.51	1.00
Unit ultra-long distance fee of Express over 40 c_u	1.00	1.50	1.50	0.81	1.50
Unit time fee of Carpool α	2.00	2.00	2.00	2.00	2.00
Platform commission per order λ	0.30	0.30	0.30	0.30	0.30
Compensation for Express rider δ_e	5.00	3.50	2.88	5.00	3.15
Compensation for Carpool rider δ_c	0.50	0.50	0.50	0.50	0.50
Compensation for Express driver ξ	10.03	10.00	10.00	10.30	10.00

including base fees, per kilometer charges, and per minute charges.

The pricing formulas for carpool services need to take into account various factors to ensure fairness and reasonableness for both drivers and riders. Here are some common carpool pricing strategies.

(1) Cost split among carpoolers:

$$F_c^1 = \text{total cost / number of riders}$$

In this method, the total cost of the trip is shared equally among all riders, making it more economical for each rider.

(2) Dynamic pricing during peak times:

$$F_c^2 = \alpha' \times \text{original cost.}$$

where α' is the peak time multiplier. During high-demand periods, the cost may dynamically increase based on market demand. Besides, to encourage the use of carpool services, specific discounts or promotions might be offered. Carpool cost with discounts and promotions also could be the difference between original cost and discounts.

(3) Fixed rates for specific routes or areas:

For certain specific popular routes or areas, a fixed rate might be set regardless of the actual distance or time, than the total cost $F_c^3 =$ fixed rate.

(4) Minimum charge limit can be described as

$$F_c^4 = \max(\text{min charge, actual cost}).$$

This ensures that drivers receive a reasonable minimum income, even if the shared cost is low.

The pricing strategy for carpool services should consider factors like market competitiveness, operational costs, and encouraging shared rides to reduce environmental impact. Pricing formulas may vary based on region, policy, and type of service. These pricing formulas are designed to make carpool services an attractive option for both drivers and passengers, balancing cost, convenience, and environmental considerations.

We utilized the BONMIN solver mentioned in the first paragraph of this section to obtain the optimal pricing strategies. Table II shows the optimal pricing strategies of the platform on November 3, 2016 when the numbers of riders were 1000, 2000, 3000, 4000, and 5000, respectively.

We further analyze the impact of the optimal pricing strategies with 2000 orders in the same set of VOT, see Figure 5 and Figure 6. 2000 riders were randomly selected each day

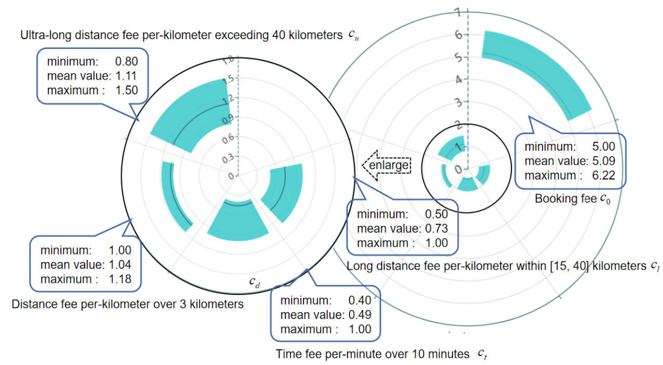

Fig. 5. Optimal pricing strategies of the ride-hailing platform (c_0, c_d, c_t, c_l, c_u).

between November 1 and November 30 in 2016 as samples. We got some analysis of each component of $\boldsymbol{x}$. Figure 5 shows that the booking fees of Express c_0 on the third, seventh, tenth and twelfth days of data sets were 6.22, 5.19, 5.78 and 5.71 (CNY) respectively, while the values at the rest days were all 5.00. On these four days with higher booking fees of Express, there were more short-haul orders of less than 20 minutes. The unit time fee of Express c_t was fluctuating a lot but mainly concentrated in the lower segments. The unit distance fee of Express c_d was concentrated at 1.04. The unit long and ultra-long distance fees of Express c_l and c_u stably lied in [0.5, 1.00] and [0.8, 1.50], respectively. Figure 6 shows that the unit time fee of Carpool α, platform commission λ, and compensation for a Carpool rider δ_c remained at 2.00, 0.30 and 0.50, respectively. The compensation for an Express driver ξ remained steady at 10.00 for a long period, while the compensation for an Express rider δ_e experienced dramatic fluctuations over the whole interval.

To be more specific, only one example is shown in this part to additional clarification of Figure 5 and Figure 6, i.e., unit time fee of Express over 10c_t. According to the unit time fee data c_t, the basic statistics are as follows: the minimum value is 1.00 (appearing multiple times), the maximum value is 2.84, the average value is 1.14, and the median is 1.05. In terms of data distribution, the data is not completely symmetrical and is slightly right-skewed. Most data points are concentrated between 1.0 and 1.2, indicating that the unit time fee is generally close. There are some larger values (such as 2.84), which may represent high rates in special circumstances. In terms of data characteristics, there are multiple data points

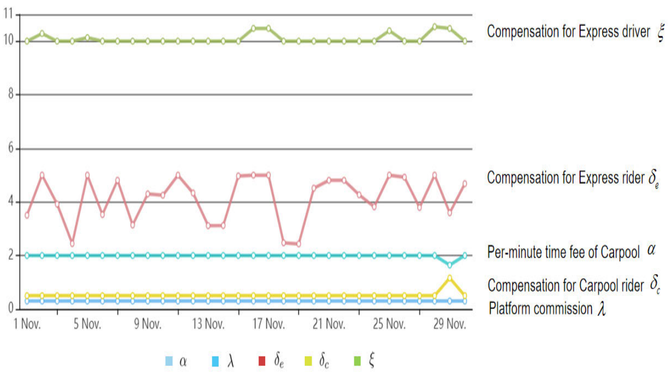

Fig. 6. Optimal pricing strategies of the ride-hailing platform(α, λ, δ_e, δ_c, ξ).

very close to 1 (numerical representation is 0.99999999), possibly indicating some base rate. The differences between data points reflect rate fluctuations under different times, locations, or supply and demand conditions to some extent. Possible influencing factors for this situation include peak periods or nighttime that may affect rates. Rate standards may vary across different cities or areas. Rates may increase during tight vehicle supply. Adverse weather conditions or major events may lead to temporary rate adjustments. There are some disparities in the unit time fee of taxi rides, but most rates are relatively concentrated, indicating a possible industry benchmark rate. A few higher values may reflect dynamic pricing in special circumstances.

The minimum value in the result data, booking fee of Express c_0, is 5.00. The maximum value is 7.40, the average value is 5.05, the median is 5.00, the standard deviation is 0.60, and the variance is 0.36. These statistics summarize the variation in values of A over these 30 days. Most of the data values are clustered around 5, but there are also a few higher values, which cause the mean to be slightly higher than 5 and significantly impact the standard deviation.

The minimum value in the cost per unit distance for Express exceeds 3 c_d is 0.40. Its maximum is 1.00, the mean is 0.44, the median is 0.41, and the standard deviation is 0.10. From these statistics, it can be seen that the difference between the minimum and maximum values is relatively large, indicating that the range of the data is quite wide. The median is 0.41, which is relatively close to the mean, indicating that the data distribution may be relatively symmetric, without too many extreme values. The standard deviation is 0.099, reflecting the degree of dispersion of the data. It can be observed that the range of values for the cost per unit distance is quite wide, but most of the data is concentrated between 0.39 and 0.42, which may be the common charging standard. The average cost per unit distance is approximately 0.44, but there are some higher values that have raised the average level. Overall, the data distribution is relatively symmetric, without extremely extreme values.

The results for the unit long distance fee of Express within [15,40] c_l data are shown below. The unit distance fee of taxi services fluctuates considerably in the long distance case, ranging from about 0.5 to about 1. The average charge is close to 0.7126, indicating that the average charge is about 71.26% of the base rate without considering the extremes. The median charge is about 0.616, which indicates that half of the services charge less than this value and the other half more than this value. Based on the calculations, the dataset indicates that the per unit distance charge for long distance taxi rides fluctuates between 0.5 and 1, with a mean of approximately 0.627, indicating that the majority of the charges are slightly below 0.63. The standard deviation is 0.205, indicating that the distribution of the data varies within a range, but is not terribly dispersed. The median is 0.6217, indicating that half of the data are below this value.

Next, analyze unit ultra-long distance fee of Express over 40 c_u. Most of the values in this data set are concentrated between 0.8 and 1.5, with a mean value of about 1.03. The data show some dispersion, but are mainly concentrated around 1.0. There are a few extreme values, such as those around 1.5 and 0.8. There is no clear regularity or pattern in the distribution of the data. The cost per unit traveled over long distances is concentrated around 1.0, with an average of about 1.03. There is some fluctuation in the cost, which may be adjusted for different circumstances. There are a few extreme highs and lows, which may be special cases or outliers. The overall distribution of the data is relatively decentralized and lacks a clear regular pattern.

Descriptive statistics of data commission λ. The commission data is basically concentrated around 0.3, showing a single peaked distribution. Most of the data values are between 0.299-0.301, which is very close to the expected commission of 0.3. There are two obvious outliers of 0.30, which may require further verification of their causes. Overall, the commission data is concentrated and in line with the expected level. In summary, although there are individual outliers, overall the commission data extracted by the platform is relatively centralized at around 0.3, which is in line with the expected level.

As for compensation for Express rider δ_e. The distribution of subsidy data is relatively decentralized, and there are two peak ranges, around 3–4 and 5 respectively. The average subsidy amount is 4.34, and the median is 4.52, indicating that most of the subsidy amount is concentrated in the range of 4–5. There are some extreme values, such as the minimum value of 2.43 and the maximum value of 5.0, which may be due to special circumstances or policy adjustments. Overall, the amount of subsidy fluctuates within a wide range, which may be related to different passengers' travel situations, distances and other factors.

Compensation for Carpool rider's subsidy δ_c is mainly concentrated around 0.5, showing a single-peaked distribution. The vast majority of the values is very close to the expected subsidy of 0.5. There is an obvious outlier of 1.16, which may need to be further verified. Overall, the subsidy data for Carpool riders is relatively concentrated and in line with the expected level. In summary, with the exception of one extreme outlier, the subsidy data for Carpool riders are mostly concentrated around the expected level of 0.5, which is more concentrated overall. If the outliers have a large impact on the analysis results, we can consider removing them and reanalyzing the data.

In conclusion, there were only minor changes in the values of booking fees of Express, unit time and distance fees of Express, and compensations for Carpool riders and Express drivers. It is generally assumed that the ride-hailing platform aims for setting the highest fare as far as possible to obtain the maximum profit. However, from the above results analysis, we observed that the platform preferred to reduce the cost to encourage more long distance Express orders. We also found that, if there were too many short-haul Express orders, the booking fee of Express increased. Furthermore, it is worth to mention that the platform was inclined to think that subsidizing Express riders and Express drivers might be better than subsidizing Carpool riders, which could help enterprises improve utilities efficiently. Since the unit time fee of Carpool has always reached the maximum, the platform is inclined to attract Express riders due to the increasing detouring distance and inconvenience of Carpool.

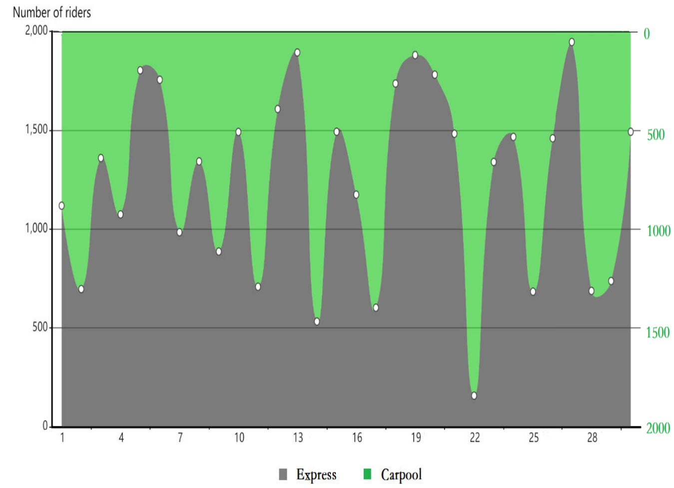

Fig. 7. Rider numbers of Express and Carpool.

C. Mode Choice Results

In this subsection, we report the mode choice results of riders after presenting optimal pricing strategies. Figure 7 gives the numbers of Express and Carpool riders per day during 30 days of November in 2016 and the results indicates that the riders were more likely to take Express on weekends while to be carpoolers on workdays. There were more than 1500 Express riders on the days of 5, 6, 12, 13, 18, 19, 20 and 27. These days were all weekends except for the 18th (Friday). This reveals that the ride-hailing platforms should focus more on pricing strategies for Express on weekends and on carpooling schemes on weekdays. The information provided reveals several key insights about the behavior and preferences of ride-hailing platform users. Here's a detailed analysis of what these points indicate. Riders are more inclined to use the Express service during weekends. This could be due to the convenience and speed of the Express option when individuals have leisure plans, social activities, or other engagements that require quick and direct transportation. During weekdays, riders show a higher tendency to opt for carpooling. This might be due to cost-saving benefits, as carpooling is typically cheaper than solo rides, which suits the daily commuting needs of workers. Environmental consciousness and an effort to reduce traffic congestion during peak hours might also contribute to this trend. Ride-hailing platforms should consider dynamic pricing strategies that capitalize on the higher demand for Express services during weekends. This behavioral data underlines distinct patterns in user preferences based on the day of the week, highlighting the need for ride-hailing platforms to tailor their services and promotional efforts accordingly. By focusing on targeted pricing strategies for Express on weekends and optimizing carpooling schemes on weekdays, these platforms can better meet user demands, enhance customer satisfaction, and potentially increase their overall revenue.

Figure 8 exhibits two boxplots of VOT for Express and Carpool riders per day during 30 days of November in 2016. Among these days, the average VOTs of Express and Carpool were 39.43 and 36.78, respectively. Moreover, we found that there were only three days on which the mean VOT of Express was less than that of Carpool, while the median VOT of Express was larger than that of Carpool every day. These results reveal that the VOT of Express riders was generally higher than that of Carpool riders. Therefore, for the ride-hailing platform, it is important to design pricing strategies based on the VOT characteristics of riders in different regions.

These results reveal significant insights about the value of time for different types of ride-hailing riders, which can have important implications for designing targeted pricing strategies. The fact that riders are more likely to choose Express services on weekends suggests that these riders value their time highly. They prefer faster, more direct services even if it costs more, indicating a preference for convenience and time savings over cost efficiency. As carpool riders' lower VOT, carpooling is more popular on workdays implying that these riders are more cost-conscious and willing to spend additional time in transit to save money. Given that Express riders value time more, ride-hailing platforms could optimize pricing strategies to reflect this higher value of time. Higher pricing, premium features (such as guaranteed pickup times, luxury vehicles), or surge pricing during peak demand periods (e.g., weekend evenings) can be implemented to maximize revenue. Tailoring pricing strategies to VOT characteristics of riders in different regions can enhance customer satisfaction and optimize revenues, by ensuring services are aligned with the preferences and willingness to pay of various rider segments.

D. Relationship Between the Optimal Pricing Strategy and Travel Modes

In this part, we discuss robustness analysis, the relationship between the optimal pricing strategy and optimal travel modes, and find the main pricing factors that influence the travel mode choice of riders.

In the first, robustness analysis studies the performance of a model in the face of uncertainty, that is, the robustness to changes in parameters, data disturbances, or changes in assumptions. Robustness analysis can be performed through parameter disturbance, data uncertainty, or changes in model structure. This section provides a robustness analysis in terms

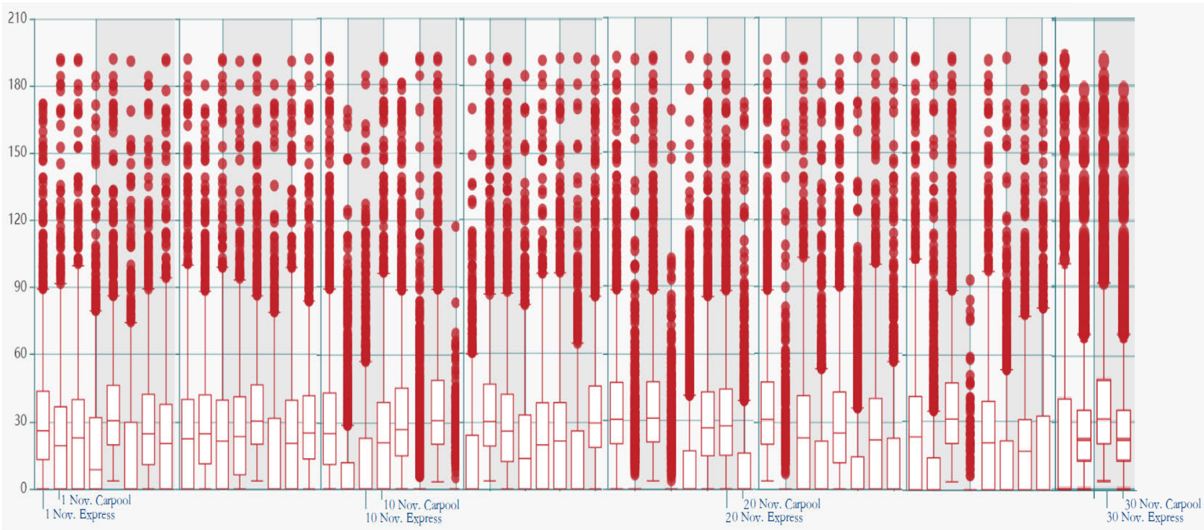

Fig. 8. Boxplots of VOT for Express and Carpool riders.

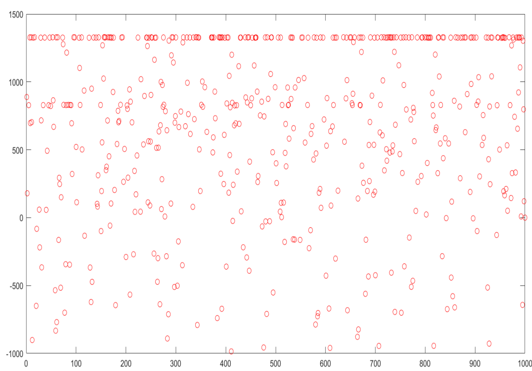

Fig. 9. Robustness experiments.

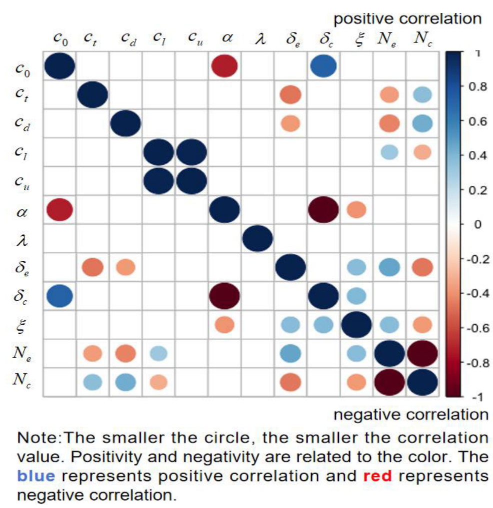

Fig. 10. Correlations among pricing strategies, numbers of Express and Carpool riders.

of numerical verification rather than theoretically. A common method is to use Monte Carlo simulation to evaluate the model's robustness under parameter uncertainty. The objective function was subjected to multiple random disturbances, each disturbance being a random value sampled from a standard normal distribution, then the disturbed optimization problem was re-solved, and the value of the objective function after each disturbance was recorded. Finally, the robustness of the model was evaluated by calculating the variance of these objective function values. This part conducted 1,000 experiments, and the robustness data ranged from −197,698.77 to 1,329.60, with an average value of −3,176.72, showing a negative skew distribution, as shown in Figure 9. The data is highly dispersed and distributed more scattered.

Figure 10 shows correlations among each component of $\boldsymbol{x}$, the number of Express riders $N_e = \sum_{i=1}^{N} z_i$, and the number of Carpool riders $N_c = N - N_e$ below 10% confidence level. Here, blue represents positive correlation and red represents negative correlation. The darker the color, the higher the correlation. This shows that there were correlations between c_0 and α, c_0 and δ_c, c_t and δ_e, c_t and N_e, c_d and δ_e, c_d and N_e, c_l and N_e, c_l and c_u, α and δ_c, α and ξ, δ_e and ξ, δ_e and N_e, δ_c and ξ, as well as ξ and N_e. Excluding some virtually unchanged variables $\alpha, c_0, \delta_c, \xi$, correlations existed between c_t and δ_e, c_t and N_e, c_d and δ_e, c_d and N_e, c_l and N_e, c_l and c_u, δ_e and ξ, δ_e and N_e, as well as ξ and N_e.

In order to understand which was the most influential pricing strategy for mode choice of riders, we carried out a Granger causality test. Recall that the Granger causality test applies to stationary time series or unit root procedures with co-integration relationships. To verify the stationary of time series, an augmented Dickey-Fuller (ADF) test was conducted. The results were given in Table III.

Since $\boldsymbol{x}$ and N_e had stationary time series of data, causalities between the variables could be proved with the Granger causality test. Table IV reports the p-values of Granger causality tests through 'vargranger' in Stata 16. It shows an interesting empirical result that c_0, c_t, c_d, δ_e and ξ were said to Granger cause N_e respectively after testing N_e and each component of $\boldsymbol{x}$. Granger causality test is a statistical hypothesis test for determining whether time series $\boldsymbol{x}$ is useful

TABLE III
ADF TEST RESULTS

Description	ADF test	1% Critical value	5% Critical value	10% Critical value	Test result
c_0	-7.877	-3.723	-2.989	-2.625	stationary
c_d	-7.877	-3.723	-2.989	-2.625	stationary
c_t	-7.876	-3.723	-2.989	-2.625	stationary
c_l	-5.902	-3.723	-2.989	-2.625	stationary
c_u	-4.391	-3.723	-2.989	-2.625	stationary
α	-5.112	-3.723	-2.989	-2.625	stationary
λ	-5.035	-3.723	-2.989	-2.625	stationary
δ_e	-5.042	-3.723	-2.989	-2.625	stationary
δ_c	-5.385	-3.723	-2.989	-2.625	stationary
ξ	-4.168	-3.723	-2.989	-2.625	stationary
N_e	-5.54	-3.723	-2.989	-2.625	stationary
N_c	-4.118	-3.723	-2.989	-2.625	stationary

TABLE IV
GRANGER CAUSALITY TEST RESULTS OF OPTIMAL PRICING STRATEGY $\boldsymbol{x}$ AND NUMBER OF EXPRESS RIDERS N_e

Equation	Excluded	Null hypothesis	p-values	Result
N_e	c_0	c_0 does not Granger cause N_e	0.021	Refuse
N_e	c_t	c_t does not Granger cause N_e	0.360	Refuse
N_e	c_d	c_d does not Granger cause N_e	0.020	Refuse
N_e	c_l	c_l does not Granger cause N_e	0.000	Accept
N_e	c_u	c_u does not Granger cause N_e	0.000	Accept
N_e	α	α does not Granger cause N_e	0.000	Accept
N_e	λ	λ does not Granger cause N_e	0.000	Accept
N_e	δ_e	δ_e does not Granger cause N_e	0.281	Refuse
N_e	δ_c	δ_c does not Granger cause N_e	0.001	Accept
N_e	ξ	ξ does not Granger cause N_e	0.340	Refuse

5% is the critical value, i.e., the probability greater than 5% is considered to reject the null hypothesis.

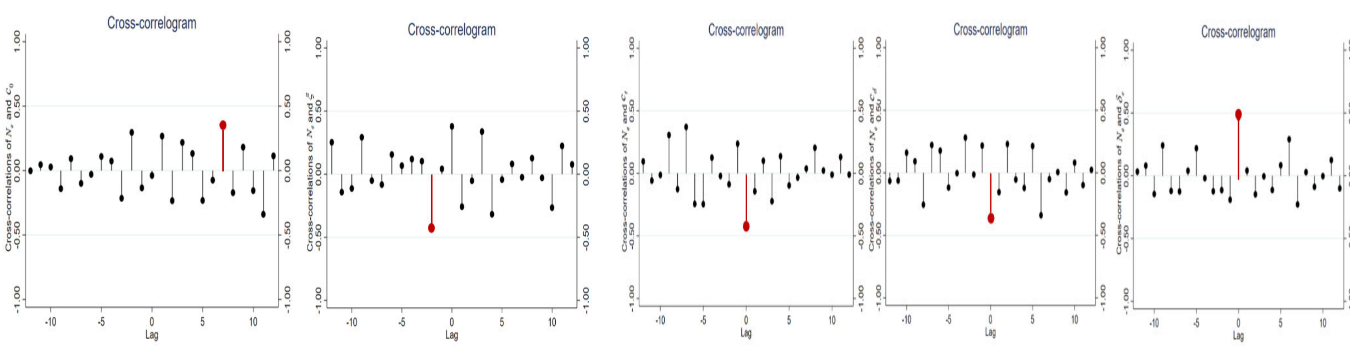

Fig. 11. Cross-correlogram of N_e and core pricing factors c_0, c_t, c_d, δ_e and ξ.

for forecasting N_e. Instead of testing the cause of N_e, the Granger causality test found the key component of $\boldsymbol{x}$ being able to forecast N_e. For instance, if c_0 was said to Granger cause N_e, the cause was partially or totally responsible for N_e. In other words, N_e was partially or totally dependent on c_0.

We further examined the cross-correlations between N_e and c_0, N_e and c_t, N_e and c_d, N_e and δ_e, N_e and ξ. The results are depicted in Figure 11. Figure 11(a) shows that, seven days after, c_0 was the most relevant to N_e and, two days ahead, ξ was the most relevant to N_e. Figure 11(b) shows that c_t, c_d and δ_e were the most relevant to N_e on the day.

V. CONCLUSION

We have provided a new perspective for the variable-ratio pricing strategies of Express and Carpool riders and their choices of travel modes on the ride-hailing platform in a general network composed of multiple corridors. By maximizing the utility of each rider in the lower level to obtain travel mode choice and maximizing the profit of the ride-hailing platform in the upper level to optimize the pricing strategy, we have presented the Stackelberg game pricing model. This model has an inherent hierarchical structure and has been transformed into the single-level optimization problem equivalently, which can be solved as a mixed-integer nonlinear programming problem.

Applying the value of time with a log-normal distribution and data from DiDi Chuxing GAIA Open Dataset Initiative, we have examined the effects on ride-hailing platform utilities of pricing strategies including booking fees of Express, unit time and distance fees of Express, unit long and ultra-long distance fees of Express, unit time fees of Carpool, platform commission, compensations for each rider and Express driver. It has been observed from our numerical experiments that riders always choose to be carpoolers on workdays because Carpool offers a travel cost saving; on the other hand, increasing compensations for Express riders or Express drivers, or decreasing booking fees, unit time or distance fees, will increase the number of Express riders, while increasing unit time fees of Carpool, platform commissions or compensations for Carpool riders, will not necessarily reduce the total travel cost of riders. We have also investigated that booking fees of Express, unit time and distance fees of Express, compensations for Express riders and Express drivers are the main factors for increasing Express riders.

In the future, we will study the ride-hailing platforms by considering elastic travel demand, the rise of computing capacity, and the influence of mode choice switching between traditional modes and ride-hailing modes, introducing more travel modes such as taxi, bus and metro. The research on ride-hailing platforms is challenging not only for inherent theoretical difficulties but also for dealing with the big data. For example, how to calculate accurately the exact time for Carpool instead of using the average coefficient of detouring time. Therefore, in our opinion, this is a very promising area in terms of devising efficient solution methods for these hierarchical programming problems in the field of ride-hailing platform pricing strategy design.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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