# Tricky Limit Problems

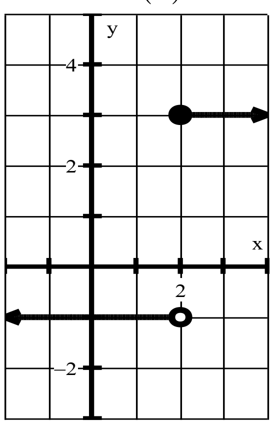
The limit problems on this worksheet are inspired by these AP test problems:

AP Calculus BC Practice Exam, 2016: #86

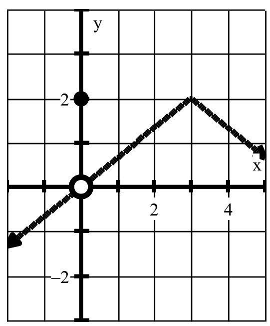
AP Calculus BC International Exam, 2016, #88

AP Calculus AB International Exam, 2017, #15

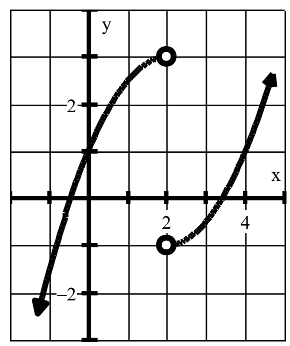
$y = f(x)$



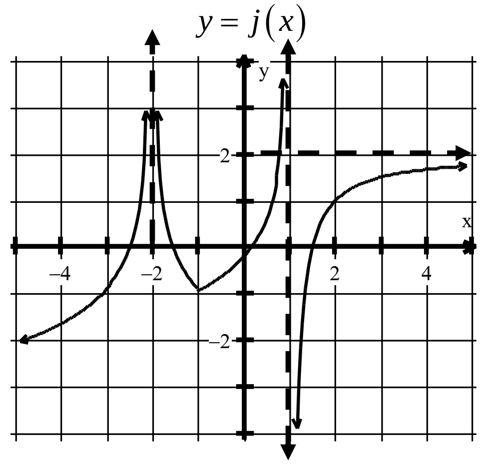

$y = g(x)$




$y = h(x)$




$y = j(x)$




Find the following limits or state that the limit does not exist.

1. $\lim\limits_{x \to -2} j(x) =$

2. $\lim\limits_{x \to 1} j(x) =$

3. $\lim\limits_{x \to 3} f(g(x)) =$

4. $\lim\limits_{x \to 0} f(|x| + 2) =$

5. $\lim\limits_{x \to 0} \left(g(x) \bullet f(x+2)\right) =$

6. $\lim\limits_{x \to 2} \left((f(x) - 1)^2 - 6\right) =$

7. $\lim\limits_{x \to 2} \left(h(x) + f(x)\right) =$

8. $\lim\limits_{x \to -2} j(j(x)) =$

9. $\lim\limits_{x \to 1.5} \dfrac{g(x) - 1}{2x - 3} =$

10. $\lim\limits_{x \to 0} g(f(x) + 1)$

# Tricky Limit Problems Solutions

1. $\lim_{x\to -2} j(x) = \infty$ or DNE

2. $\lim_{x\to 1} j(x)$ DNE

3. $\lim_{x\to 3} f(g(x)) =$

$\lim_{x\to 2^-} f(x) =$

$-1$

4. $\lim_{x\to 0} f(|x|+2) =$

$\lim_{x\to 2^+} f(x) =$

$3$

5. $\lim_{x\to 0}(g(x) \bullet f(x+2)) =$

$\lim_{x\to 0^-}(g(x) \bullet f(x+2)) = 0 \bullet (-1)$

$= 0$

$\lim_{x\to 0^+}(g(x) \bullet f(x+2)) = 0 \bullet 3$

$= 0$

therefore $\lim_{x\to 0}(g(x) \bullet f(x+2)) = 0$

6. $\lim_{x\to 2}\left((f(x)-1)^2 - 6\right) =$

$\lim_{x\to 2^-}\left((f(x)-1)^2 - 6\right) =$

$\left((-1-1)^2 - 6\right) =$

$-2$

$\lim_{x\to 2^+}\left((f(x)-1)^2 - 6\right) =$

$\left((3-1)^2 - 6\right) =$

$-2$

Therefore $\lim_{x\to 2}\left((f(x)-1)^2 - 6\right) = -2$

7. $\lim_{x\to 2}(h(x) + f(x)) =$

$\lim_{x\to 2^-}(h(x) + f(x)) =$

$(3 + (-1)) =$

$2$

$\lim_{x\to 2^+}(h(x) + f(x)) =$

$((-1) + 3) =$

$2$

Therefore $\lim_{x\to 2}(h(x) + f(x)) = 2$

8. $\lim_{x \to -2} j\left(j\left(x\right)\right) =$

$\lim_{x \to \infty} j\left(x\right) =$

$2$

9. $\lim_{x \to 1.5} \frac{g\left(x\right)-1}{2x-3} =$

Since $\lim_{x \to 1.5} g\left(x\right)-1=0$ and $\lim_{x \to 1.5} \left(2x-3\right)=0$ use L'Hospital's Rule.

$$\lim_{x \to 1.5} \frac{g\left(x\right)-1}{2x-3} = \lim_{x \to 1.5} \frac{g'\left(x\right)}{2}$$

$$= \frac{2/3}{2}$$

$$= \frac{1}{3}$$

10. $\lim_{x \to 0} g\left(f\left(x\right)+1\right) = g\left(-1+1\right)$

$= g\left(0\right)$

$= 2$

This one is unique because *f* (*x*) is a constant -1 near *x* = 0. It is not approaching -1 from above or below but is exactly -1.