Homework 2

Student: Joy Jin **Score:** _____

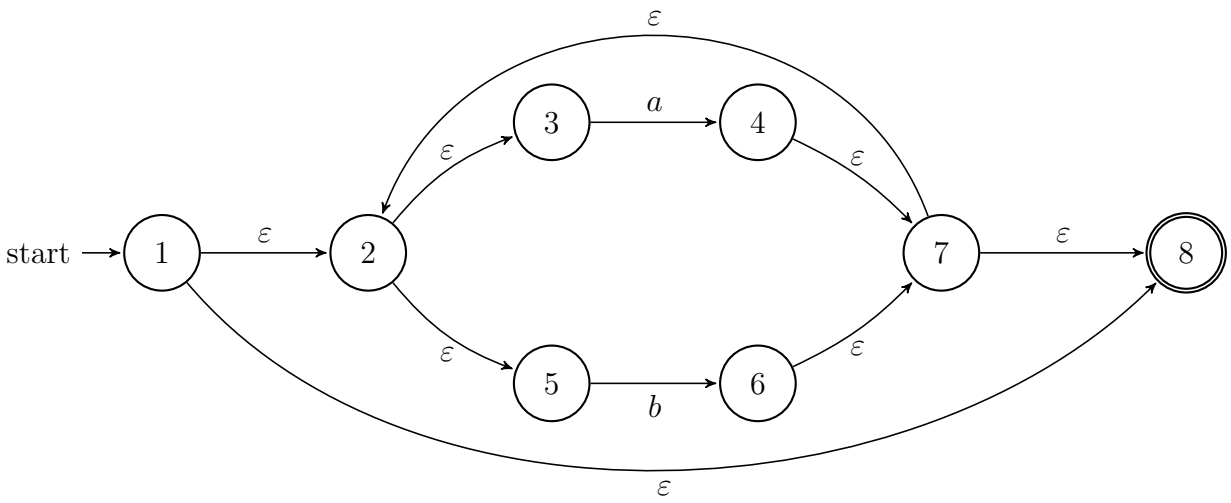
Description
The goal of this assignment is to improve your understanding of **lexical analysis** and **derivations**.

Due Date
Thursday, 10/16/2025 11:59 PM

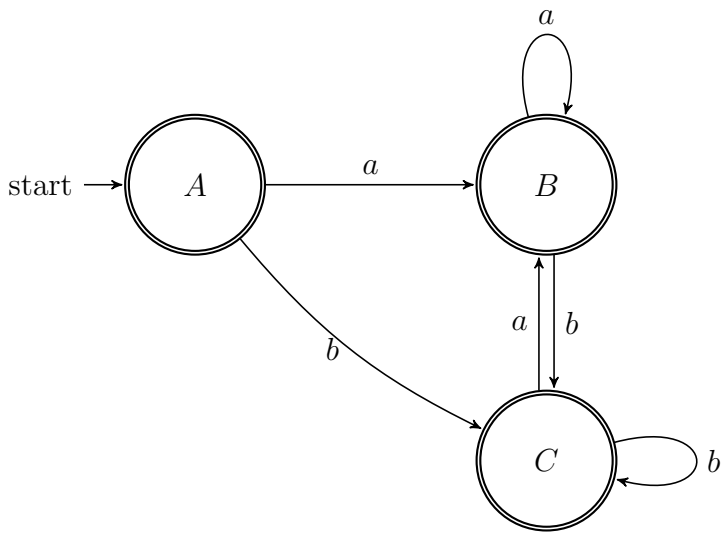
[35 Points] Question 1: Regular Expression-to-Automaton
For each of the regular expressions, construct a deterministic finite automaton (DFA). You will provide both the diagram and table representations for the DFA. You must show all the steps from generating a nondeterministic finite automaton (NFA) to a DFA. To receive full credit, you must show *all* the workouts of DFA construction (i.e., Move(A, a), ε-closures, etc).

Regular Expressions:

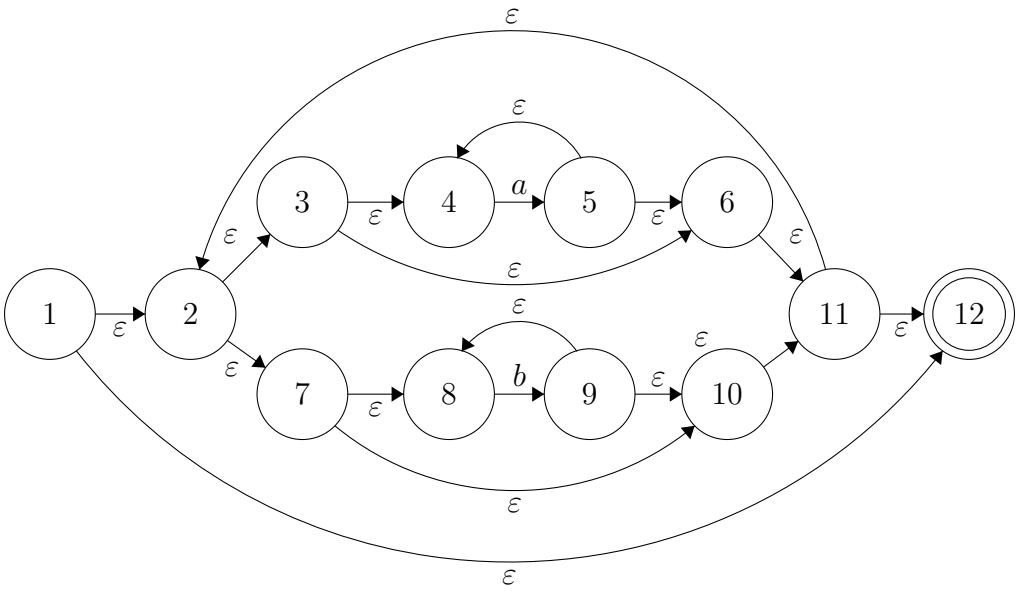
1. [5 Points] $(a|b)^*$



$$\epsilon\text{-closure}(1) = \{1, 2, 3, 5, 8\} : A \tag{1}$$
$$\epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{4\}) = \{2, 3, 4, 5, 7, 8\} : B \tag{2}$$
$$\epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{6\}) = \{2, 3, 5, 6, 7, 8\} : C \tag{3}$$
$$\epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{4\}) = B \tag{4}$$
$$\epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{6\}) = C \tag{5}$$
$$\epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{4\}) = B \tag{6}$$
$$\epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{6\}) = C \tag{7}$$

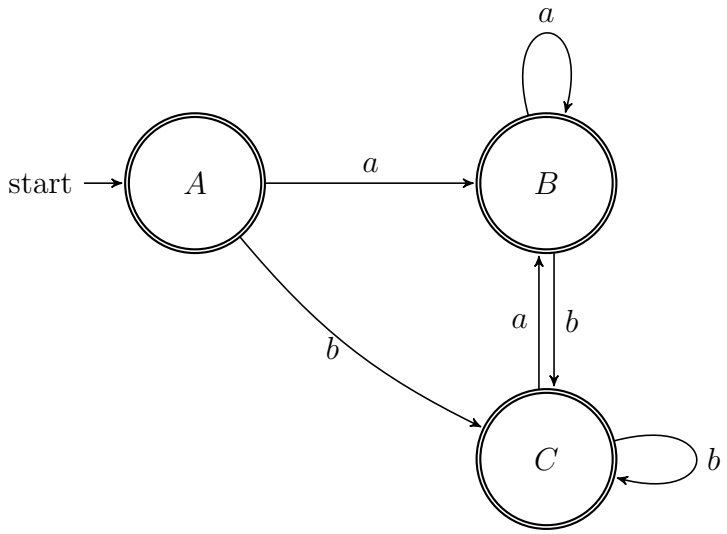


2. [5 Points] $(a^*|b^*)^*$

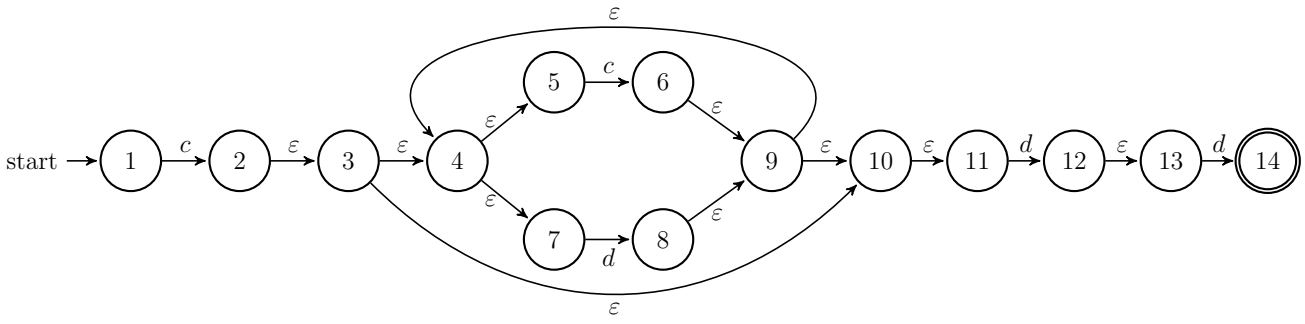


$$\epsilon\text{-closure}(1) = \{1, 2, 3, 4, 6, 7, 8, 10, 11, 12\} : A \tag{8}$$
$$\epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{5\}) = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12\} : B \tag{9}$$
$$\epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{9\}) = \{2, 3, 4, 6, 7, 8, 9, 10, 11, 12\} : C \tag{10}$$
$$\epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{5\}) = B \tag{11}$$
$$\epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{9\}) = C \tag{12}$$
$$\epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{5\}) = B \tag{13}$$
$$\epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{9\}) = C \tag{14}$$

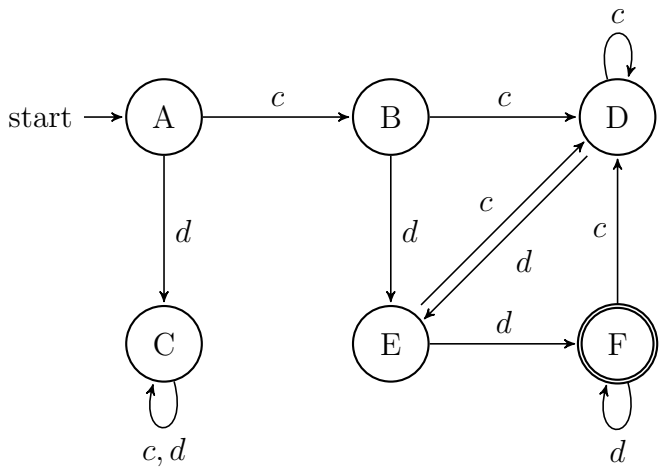
We get the same DFA, so we can use the graph earlier:



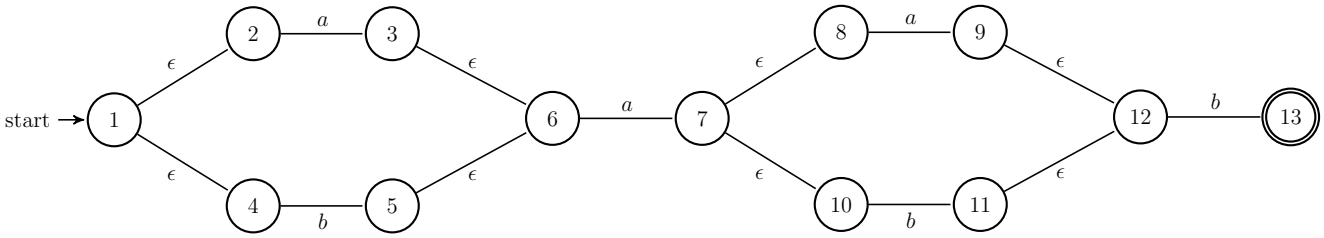
3. [10 Points] $c(c|d)^*dd$



$$
\begin{aligned}
\epsilon\text{-closure}(1) &= \{1\} : A \\
\epsilon\text{-closure}(\text{move}(A, c)) = \epsilon\text{-closure}(\{2\}) &= \{2, 3, 4, 5, 7, 10, 11\} : B \\
\epsilon\text{-closure}(\text{move}(A, d)) = \epsilon\text{-closure}(\{\}) &= \{\} : C \\
\epsilon\text{-closure}(\text{move}(B, c)) = \epsilon\text{-closure}(\{6\}) &= \{4, 5, 6, 7, 9, 10, 11\} : D \\
\epsilon\text{-closure}(\text{move}(B, d)) = \epsilon\text{-closure}(\{8, 12\}) &= \{4, 5, 7, 8, 9, 10, 11, 12, 13\} : E \\
\epsilon\text{-closure}(\text{move}(C, c)) = \epsilon\text{-closure}(\{\}) &= C \\
\epsilon\text{-closure}(\text{move}(C, d)) = \epsilon\text{-closure}(\{\}) &= C \\
\epsilon\text{-closure}(\text{move}(D, c)) = \epsilon\text{-closure}(\{6\}) &= D \\
\epsilon\text{-closure}(\text{move}(D, d)) = \epsilon\text{-closure}(\{8, 12\}) &= E \\
\epsilon\text{-closure}(\text{move}(E, c)) = \epsilon\text{-closure}(\{6\}) &= D \\
\epsilon\text{-closure}(\text{move}(E, d)) = \epsilon\text{-closure}(\{8, 12, 14\}) &= \{4, 5, 7, 8, 9, 10, 11, 12, 13, 14\} : F \\
\epsilon\text{-closure}(\text{move}(F, c)) = \epsilon\text{-closure}(\{6\}) &= D \\
\epsilon\text{-closure}(\text{move}(F, d)) = \epsilon\text{-closure}(\{8, 12, 14\}) &= F
\end{aligned}
\tag{15}
$$

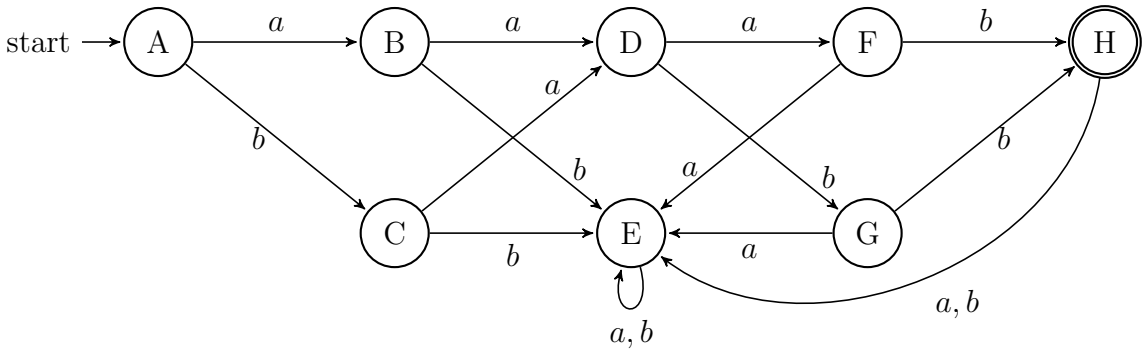


4. [15 Points] $(a|b)a(a|b)b$



Here we removed the ε in concatenation to simplify.

$$\epsilon\text{-closure}(1) = \{1, 2, 4\} : A \tag{16}$$
$$\epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{3\}) = \{3, 6\} : B \tag{17}$$
$$\epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6\} : C \tag{18}$$
$$\epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{7\}) = \{7, 8, 10\} : D \tag{19}$$
$$\epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{\}) = \{\} : E \tag{20}$$
$$\epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{7\}) = D \tag{21}$$
$$\epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{\}) = E \tag{22}$$
$$\epsilon\text{-closure}(\text{move}(D, a)) = \epsilon\text{-closure}(\{9\}) = \{9, 12\} : F \tag{23}$$
$$\epsilon\text{-closure}(\text{move}(D, b)) = \epsilon\text{-closure}(\{11\}) = \{11, 12\} = G \tag{24}$$
$$\epsilon\text{-closure}(\text{move}(E, a)) = \epsilon\text{-closure}(\{\}) = E \tag{25}$$
$$\epsilon\text{-closure}(\text{move}(E, b)) = \epsilon\text{-closure}(\{\}) = E \tag{26}$$
$$\epsilon\text{-closure}(\text{move}(F, a)) = \epsilon\text{-closure}(\{\}) = E \tag{27}$$
$$\epsilon\text{-closure}(\text{move}(F, b)) = \epsilon\text{-closure}(\{13\}) = \{13\} : H \tag{28}$$
$$\epsilon\text{-closure}(\text{move}(G, a)) = \epsilon\text{-closure}(\{\}) = E \tag{29}$$
$$\epsilon\text{-closure}(\text{move}(G, b)) = \epsilon\text{-closure}(\{13\}) = H \tag{30}$$
$$\epsilon\text{-closure}(\text{move}(H, a)) = \epsilon\text{-closure}(\{\}) = E \tag{31}$$
$$\epsilon\text{-closure}(\text{move}(H, b)) = \epsilon\text{-closure}(\{\}) = E \tag{32}$$



[15 Points] Question 2: Derivations

For each of the grammar and target string, show all the steps of the leftmost and rightmost derivations. Clearly show each step.

1. [5 Points] Grammar:

$$S \to 0S1 \mid \epsilon$$

String: 0011

$$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 00\epsilon11\checkmark \tag{33}$$

Rightmost: identical.

2. [5 Points] Grammar:

$$S \to a \mid (L)$$

$$L \to L; S \mid S$$

String: $w = ((a; a); a)$

$$\begin{aligned} S &\Rightarrow (L) \\ &\Rightarrow (L; S) \\ &\Rightarrow (S; S) \\ &\Rightarrow ((L); S) \\ &\Rightarrow ((L; S); S) \\ &\Rightarrow ((S; S); S) \\ &\Rightarrow ((a; S); S) \\ &\Rightarrow ((a; a); S) \\ &\Rightarrow ((a; a); a) \end{aligned} \tag{34}$$

Right:

$$\begin{aligned} S &\Rightarrow (L) \\ &\Rightarrow (L; S) \\ &\Rightarrow (L; a) \\ &\Rightarrow (S; a) \\ &\Rightarrow ((L); a) \\ &\Rightarrow ((L; S); a) \\ &\Rightarrow ((L; a); a) \\ &\Rightarrow ((S; a); a) \\ &\Rightarrow ((a; a); a) \end{aligned} \tag{35}$$

3. [5 Points] Grammar:

$$L \rightarrow L, E \mid E$$

$$E \rightarrow E + E \mid id$$

String: $w = id, id + id$

$$\begin{aligned} L &\Rightarrow L, E \\ &\Rightarrow E, E \\ &\Rightarrow id, E \\ &\Rightarrow id, E + E \\ &\Rightarrow id, id + E \\ &\Rightarrow id, id + id \end{aligned} \tag{36}$$

Rightmost:

$$\begin{aligned} L &\Rightarrow L, E \\ &\Rightarrow L, E + E \\ &\Rightarrow L, E + id \\ &\Rightarrow L, id + id \\ &\Rightarrow E, id + id \\ &\Rightarrow id, id + id \end{aligned} \tag{37}$$