Homework 2

Student: Joy Jin   **Score:** _____

Description
The goal of this assignment is to improve your understanding of **lexical analysis** and **derivations**.

Due Date
Thursday, 10/16/2025 11:59 PM

[35 Points] Question 1: Regular Expression-to-Automaton
For each of the regular expressions, construct a deterministic finite automaton (DFA). You will provide both the diagram and table representations for the DFA. You must show all the steps from generating a nondeterministic finite automaton (NFA) to a DFA. To receive full credit, you must show *all* the workouts of DFA construction (i.e., Move(A, a), ε-closures, etc).

Regular Expressions:

1. [5 Points] $(a|b)^*$
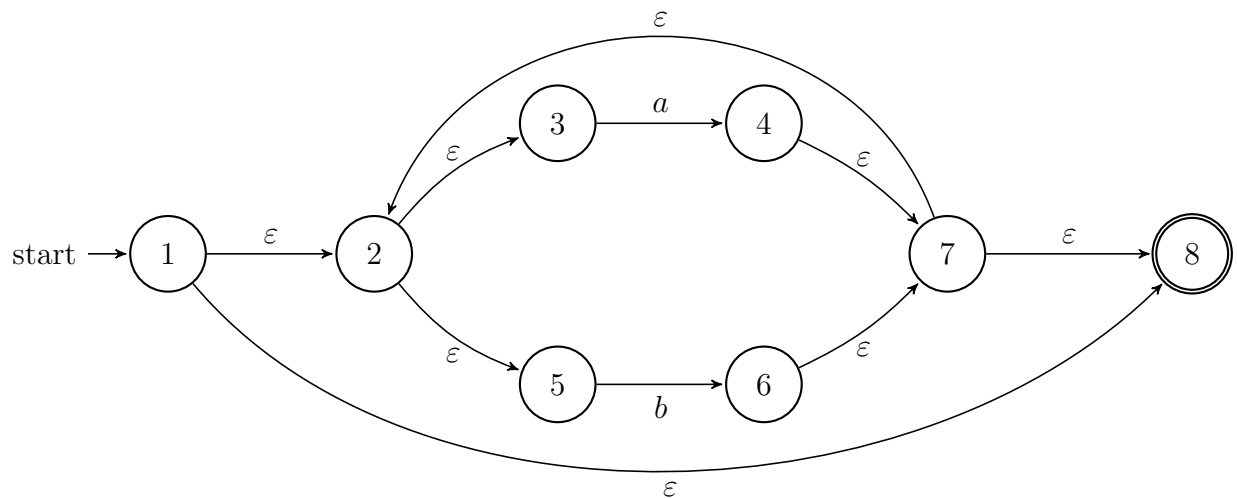
$$\epsilon\text{-closure}(1) = \{1, 2, 3, 5, 8\} : A \tag{1}$$
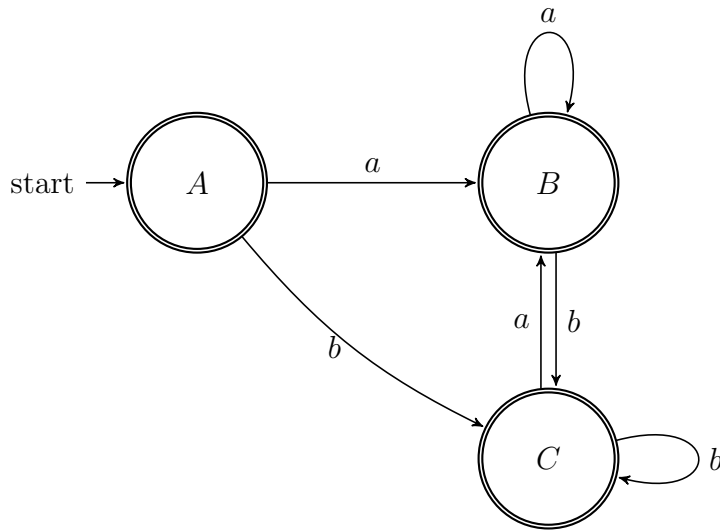$$\epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{4\}) = \{2, 3, 4, 5, 7, 8\} : B \tag{2}$$
$$\epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{6\}) = \{2, 3, 5, 6, 7, 8\} : C \tag{3}$$
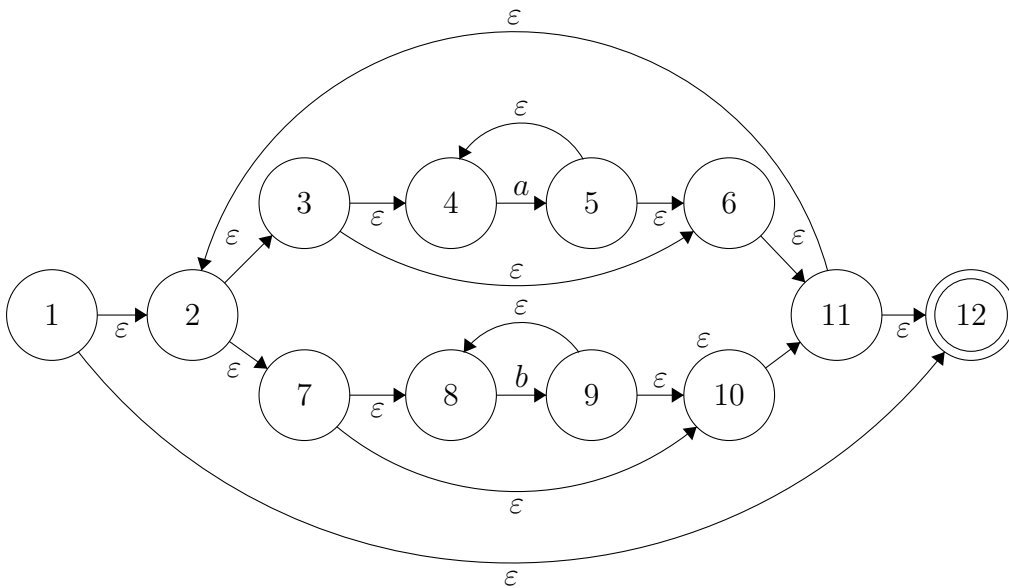$$\epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{4\}) = B \tag{4}$$
$$\epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{6\}) = C \tag{5}$$
$$\epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{4\}) = B \tag{6}$$
$$\epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{6\}) = C \tag{7}$$

2. [5 Points] $(a^*|b^*)^*$

$$\epsilon\text{-closure}(1) = \{1, 2, 3, 4, 6, 7, 8, 10, 11, 12\} : A \qquad (8)$$

$$\epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{5\}) = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12\} : B \qquad (9)$$

$$\epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{9\}) = \{2, 3, 4, 6, 7, 8, 9, 10, 11, 12\} : C \qquad (10)$$
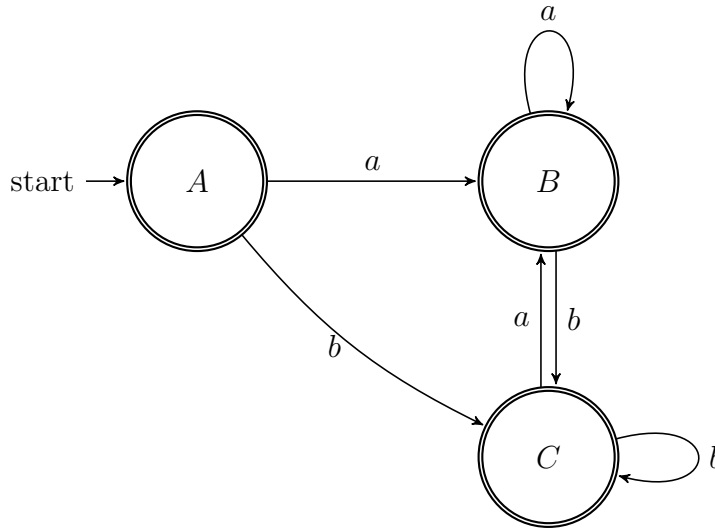
$$\epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{5\}) = B \qquad (11)$$

$$\epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{9\}) = C \qquad (12)$$
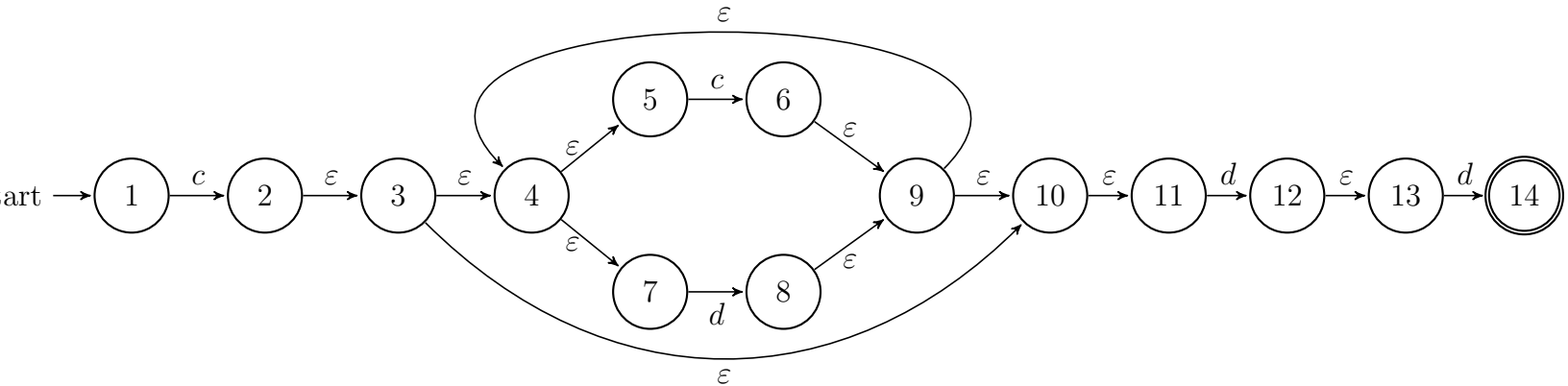
$$\epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{5\}) = B \qquad (13)$$

$$\epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{9\}) = C \qquad (14)$$

We get the same DFA, so we can use the graph earlier:

3. [10 Points] $c(c|d)^*dd$

3

$$\epsilon\text{-closure}(1) = \{1\} : A$$
$$\epsilon\text{-closure}(\text{move}(A, c)) = \epsilon\text{-closure}(\{2\}) = \{2, 3, 4, 5, 7, 10, 11\} : B$$
$$\epsilon\text{-closure}(\text{move}(A, d)) = \epsilon\text{-closure}(\{\}) = \{\} : C$$
$$\epsilon\text{-closure}(\text{move}(B, c)) = \epsilon\text{-closure}(\{6\}) = \{4, 5, 6, 7, 9, 10, 11\} : D$$
$$\epsilon\text{-closure}(\text{move}(B, d)) = \epsilon\text{-closure}(\{8, 12\}) = \{4, 5, 7, 8, 9, 10, 11, 12, 13\} : E$$
$$\epsilon\text{-closure}(\text{move}(C, c)) = \epsilon\text{-closure}(\{\}) = C$$
$$\epsilon\text{-closure}(\text{move}(C, d)) = \epsilon\text{-closure}(\{\}) = C$$
$$\epsilon\text{-closure}(\text{move}(D, c)) = \epsilon\text{-closure}(\{6\}) = D$$
$$\epsilon\text{-closure}(\text{move}(D, d)) = \epsilon\text{-closure}(\{8, 12\}) = E$$
$$\epsilon\text{-closure}(\text{move}(E, c)) = \epsilon\text{-closure}(\{6\}) = D$$
$$\epsilon\text{-closure}(\text{move}(E, d)) = \epsilon\text{-closure}(\{8, 12, 14\}) = \{4, 5, 7, 8, 9, 10, 11, 12, 13, 14\} : F$$
$$\epsilon\text{-closure}(\text{move}(F, c)) = \epsilon\text{-closure}(\{6\}) = D$$
$$\epsilon\text{-closure}(\text{move}(F, d)) = \epsilon\text{-closure}(\{8, 12, 14\}) = F$$
$$(15)$$
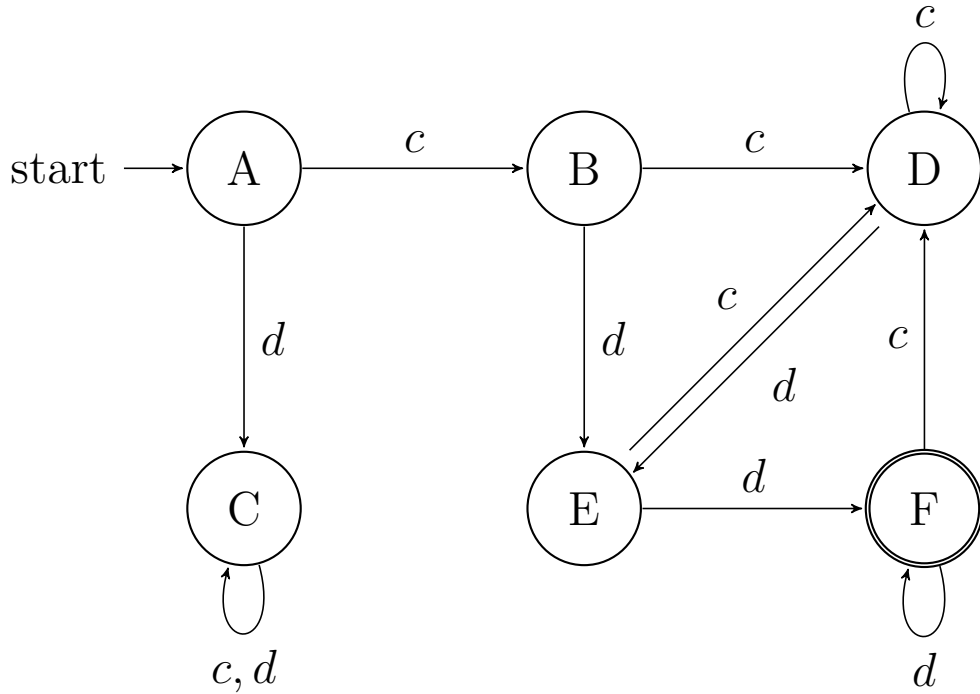
4. [15 Points] $(a|b)a(a|b)b$
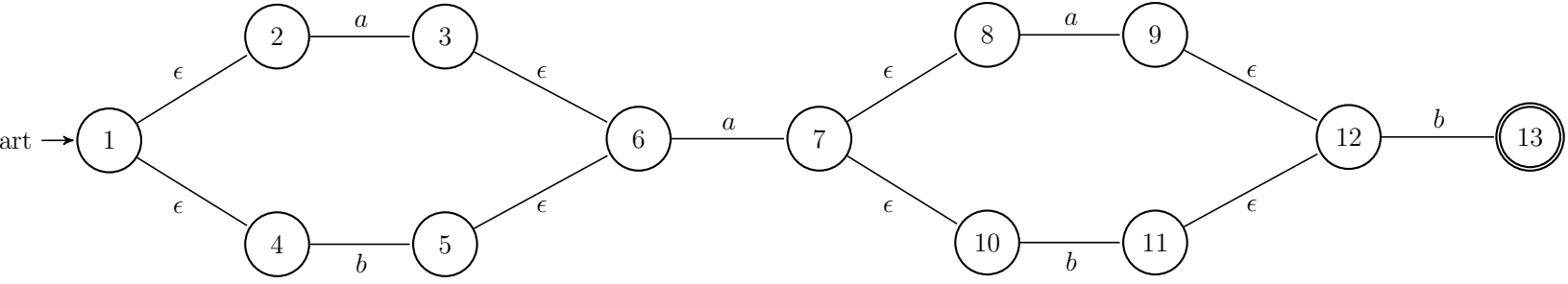
Here we removed the ε in concatenation to simplify.

$$\epsilon\text{-closure}(1) = \{1, 2, 4\} : A \tag{16}$$

$$\epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{3\}) = \{3, 6\} : B \tag{17}$$

$$\epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6\} : C \tag{18}$$

$$\epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{7\}) = \{7, 8, 10\} : D \tag{19}$$

$$\epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{\}) = \{\} : E \tag{20}$$

$$\epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{7\}) = D \tag{21}$$

$$\epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{\}) = E \tag{22}$$

$$\epsilon\text{-closure}(\text{move}(D, a)) = \epsilon\text{-closure}(\{9\}) = \{9, 12\} : F \tag{23}$$

$$\epsilon\text{-closure}(\text{move}(D, b)) = \epsilon\text{-closure}(\{11\}) = \{11, 12\} = G \tag{24}$$

$$\epsilon\text{-closure}(\text{move}(E, a)) = \epsilon\text{-closure}(\{\}) = E \tag{25}$$

$$\epsilon\text{-closure}(\text{move}(E, b)) = \epsilon\text{-closure}(\{\}) = E \tag{26}$$

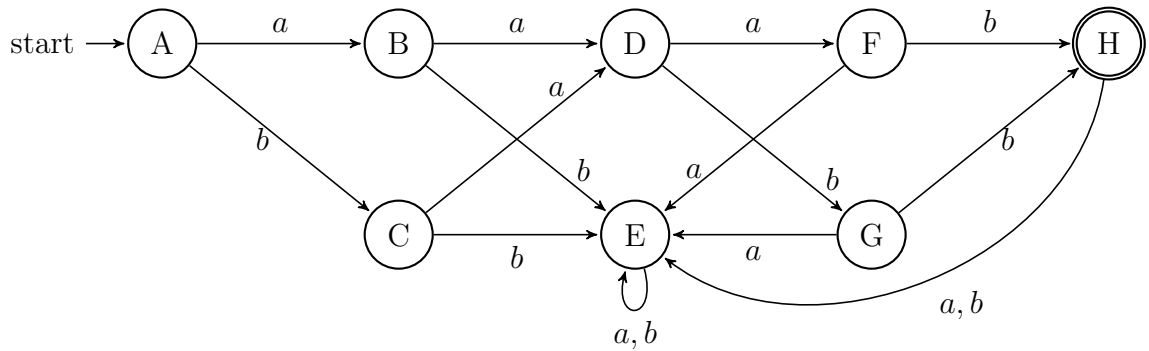$$\epsilon\text{-closure}(\text{move}(F, a)) = \epsilon\text{-closure}(\{\}) = E \tag{27}$$

$$\epsilon\text{-closure}(\text{move}(F, b)) = \epsilon\text{-closure}(\{13\}) = \{13\} : H \tag{28}$$

$$\epsilon\text{-closure}(\text{move}(G, a)) = \epsilon\text{-closure}(\{\}) = E \tag{29}$$

$$\epsilon\text{-closure}(\text{move}(G, b)) = \epsilon\text{-closure}(\{13\}) = H \tag{30}$$

$$\epsilon\text{-closure}(\text{move}(H, a)) = \epsilon\text{-closure}(\{\}) = E \tag{31}$$

$$\epsilon\text{-closure}(\text{move}(H, b)) = \epsilon\text{-closure}(\{\}) = E \tag{32}$$

Tables:

Q1	a	b
A	B	C
B	B	C
C	B	C

Q2	a	b
A	B	C
B	B	C
C	B	C

Q3	c	d
A	B	C
B	D	E
C	C	C
D	D	E
E	D	F
F	D	F

Q4:	a	b
A	B	C
B	D	E
C	D	E
D	F	G
E	E	E
F	E	H
G	E	H
H	E	E

[15 Points] Question 2: Derivations

For each of the grammar and target string, show all the steps of the leftmost and rightmost derivations. Clearly show each step.

1. [5 Points] Grammar:

$$S \to 0S1 \mid \epsilon$$

   String: 0011

$$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 00\epsilon11\checkmark \tag{33}$$

   Rightmost: identical.

2. [5 Points] Grammar:

$$S \to a \mid (L)$$

$$L \to L; S \mid S$$

   String: $w = ((a; a); a)$

$$
\begin{aligned}
S &\Rightarrow (L) \\
&\Rightarrow (L; S) \\
&\Rightarrow (S; S) \\
&\Rightarrow ((L); S) \\
&\Rightarrow ((L; S); S) \\
&\Rightarrow ((S; S); S) \\
&\Rightarrow ((a; S); S) \\
&\Rightarrow ((a; a); S) \\
&\Rightarrow ((a; a); a)
\end{aligned}
\tag{34}
$$

Right:

$$
\begin{aligned}
S &\Rightarrow (L) \\
&\Rightarrow (L; S) \\
&\Rightarrow (L; a) \\
&\Rightarrow (S; a) \\
&\Rightarrow ((L); a) \\
&\Rightarrow ((L; S); a) \\
&\Rightarrow ((L; a); a) \\
&\Rightarrow ((S; a); a) \\
&\Rightarrow ((a; a); a) \tag{35}
\end{aligned}
$$

3. [5 Points] Grammar:

$$
L \to L, E \mid E
$$

$$
E \to E + E \mid id
$$

String: $w = id, id + id$

$$
\begin{aligned}
L &\Rightarrow L, E \\
&\Rightarrow E, E \\
&\Rightarrow id, E \\
&\Rightarrow id, E + E \\
&\Rightarrow id, id + E \\
&\Rightarrow id, id + id \tag{36}
\end{aligned}
$$

Rightmost:

$$
\begin{aligned}
L &\Rightarrow L, E \\
&\Rightarrow L, E + E \\
&\Rightarrow L, E + id \\
&\Rightarrow L, id + id \\
&\Rightarrow E, id + id \\
&\Rightarrow id, id + id \tag{37}
\end{aligned}
$$