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Theoretical studies of diurnal wind-structure variations in the planetary boundary layer

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SUMMARY

Averages of the wind vector at four times of the day were evaluated for 29 suitable days during the summer of 1951 at Wichita, Kansas, and Oklahoma City. The periodic portion of the wind variation vector is an ellipse at all levels with the major axis approximately in the direction of the wind vector. The amplitude reaches a maximum at 2,500 ft above the ground.

The cause of the variation is sought by theoretical methods, first by finding what periodic variations would occur when the eddy viscosity is periodic in time and constant with height, and secondly by finding solutions with the aid of an electronic analogue computer for cases when the eddy viscosity is distributed arbitrarily with height. It is concluded that variations of the type which were observed can occur only when both the average value of the eddy viscosity and the amplitude of its variations decrease rapidly with height above the lowest third of the friction layer.

1. INTRODUCTION

Fig. 1 is a composite diagram showing the average of the diurnal wind variations at different levels above the ground at Wichita, Kansas and Oklahoma City.

It was obtained from an analysis of routine 6-hourly pilot-balloon observations made on 29 days during the summer of 1951 which were selected so as to avoid marked changes of surface-pressure gradient. Owing to the fact that the direction of the pressure gradient differs from one day to another, the orientation of each wind was measured relative to the direction at 7,000 ft above sea level, at which level previous studies had indicated that

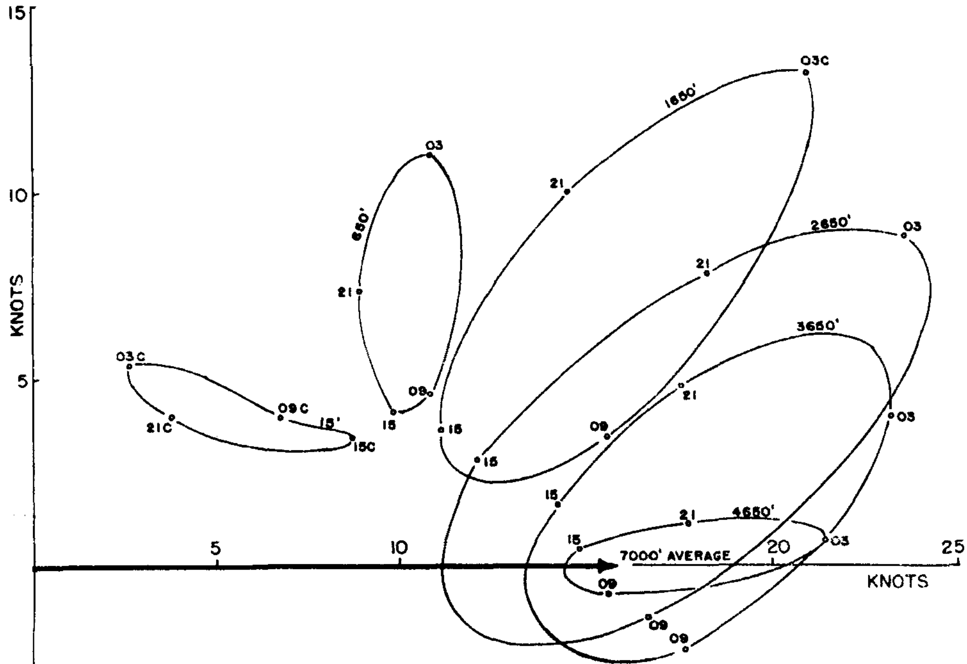

Figure 1. Composite diagram of diurnal wind variations at Wichita and Oklahoma City.

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the diurnal variation nearly disappears. The re-oriented wind vectors were averaged level-by-level and time-by-time so as to show the characteristic direction and speed of the wind at each level and time.

The odd values of height above the ground correspond to whole thousands of feet above sea level, both stations being at nearly the same elevation. Inasmuch as the days were not chosen consecutively, there was a fairly large difference between the mean wind at 1500 CST when the beginning of each day was reckoned and the value 24 hr later. This difference is ascribed to gradual changes of the pressure gradient during the period and was removed by subtracting from each wind an appropriate linear variation. Thus Fig. 1 represents only the periodic portion of the mean variation. The shape, orientation, and phase of the elliptical variations are similar to variations observed by Wagner (1939) in a study of wind variations in the mid-western United States. The variation of the wind at each level is much more pronounced in the direction of mean motion than in the direction at right angles thereto. Hence variations of speed are more noticeable than are variations of wind direction.

It is the purpose of this study to find what distribution of eddy viscosity in time and height best accounts for these features of the wind structure.

Before proceeding further it is well to consider whether the wind structure can actually be explained on the basis of the distribution of eddy viscosity with time and height. Wagner (1939) attributed the wind variations in the mid-western United States to a combination of (1) a circulation between the dry region of the south-west United States and its surroundings, (2) a large-scale land-sea breeze circulation, and (3) a large-scale mountain-plain circulation. Bleeker and Andre (1951) attributed the diurnal variation of convergence, which is probably associated with these same wind variations, to a large-scale mountain-valley wind system between the Rocky Mountains on the west side, the Appalachian Mountains on the east, and the broad Mississippi basin in between. That such circulations with a period of 24 hr and an amplitude of the order of 10 kt can arise through gravity drainage or thermal gradients seems quite implausible because of the tremendous mass of air involved in the circulations.

The most decisive argument against drainage as a cause of the observed variations concerns the phase of the variations. Heating and cooling of the lowest layers above a sloping surface result in a downhill force at night (in this case toward the east) and an uphill (westward) force during the day. Consider a coordinate system which is oriented so as to place the x-axis toward the east. The motions which result from a periodic force of the kind just described are given by the following equations, provided effects of advection and friction are ignored.

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= lv - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - F\cos\omega t \\ \frac{\partial v}{\partial t} &= -\,lu - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} \end{aligned}\right\} \qquad (1)$$

In these equations u and v are the velocity components in the x and y directions, l the Coriolis parameter, F the amplitude of the force, ρ the density, $\bar{p}$ the mean pressure over a 24 hr period, ω the angular velocity of the earth, and t the time after maximum surface temperature. It is sufficient for the purpose at hand to consider the pressure gradient as a constant. The solution may then be expressed by the following equations :

$$\left.\begin{aligned} u &= u_g + \omega F\,(l^2 - \omega^2)^{-1}\sin\omega t \\ v &= v_g + l\,F\,(l^2 - \omega^2)^{-1}\cos\omega t \end{aligned}\right\} \qquad (2)$$

in which u_g and v_g are the components of the mean geostrophic wind. According to these equations the wind vector moves around an ellipse with north-south major axis when the latitude exceeds 30°, and it reaches its maximum component towards the north in mid-afternoon. The mean wind direction at Wichita and Oklahoma City at 7,000 ft in summer is from the south-west. When the 7,000-ft wind direction in Fig. 1 is so oriented, it is seen that at lower levels the wind at mid-afternoon (1500 CST) actually has a minimum northward component. Therefore drainage and buoyancy effects do not offer a satisfactory explanation of the observed wind variations.

A theory of the diurnal variation of the wind has been given by Iswekow (1929) for eddy viscosity which is independent of height and varies sinusoidally with time. A solution was found by postulating that the ageostrophic wind is capable of representation as the product of a function of height and a function of time. The special solution thus found does not satisfy reasonable boundary conditions and is capable of satisfactory representation of the wind only in a restricted layer. In the next section this same problem is re-examined and a general solution is obtained which better explains several aspects of the observed variations.

Staley (1956) has criticized the use of sinusoidally varying exchange coefficient by several investigators and has found that simpler solutions based on a coefficient which is independent of time give equally good predictions of the height-time distributions of temperature changes. This simplification is unsatisfactory for a discussion of wind variations because, contrary to what is observed, it would result in a deviation from equilibrium which decreases with height. It is realized that the diurnal variation of exchange coefficient is in reality not exactly sinusoidal, but evidence to be presented herein indicates that the complications probably have little effect upon the wind variations above a few hundred metres. Thus no precise description of the form of the time variation should be expected from comparisons between theory and observations.

The procedure employed in this study is to find general solutions of the equations of motion for various distributions of eddy viscosity in time and height. Comparison with the observed variations then indicates the nature of the distributions of eddy viscosity which actually occurred. The geostrophic departure method, which was used by Lettau (1950) and by Sheppard and Omar (1952), was not attempted because a determination of the vertical distribution of the geostrophic wind was not feasible for so many days.

2. Analytical solution

The purpose of this section is to determine what type of wind variation occurs if the eddy viscosity is a periodic function of time and is independent of height. This analysis is not sufficiently powerful to be capable of explaining the observed variation at Wichita and Oklahoma City, but it serves a useful purpose in pointing the way for the possible solution of more difficult problems, and also in demonstrating the accuracy obtainable by an approximate method using a differential analyzer.

The wind variations which are associated with the diurnal and semi-diurnal variations of the pressure are much smaller than the variations observed in the boundary layer, and they are not limited to the surface layers (Bemmelen 1921, Johnson 1955). It is reasonable therefore to neglect time variations of the geostrophic wind. It will also be assumed that the geostrophic wind and the density are independent of height, that the mean wind vector is a function of height and time only, and that the vertical velocity is zero. The equations of motion can then be written

$$\left.\begin{aligned}\frac{\partial u^*}{\partial t} &= lv^* + \frac{\partial}{\partial z}\left[K(z,t)\frac{\partial u^*}{\partial z}\right]\\ \frac{\partial v^*}{\partial t} &= -lu^* + \frac{\partial}{\partial z}\left[K(z,t)\frac{\partial v^*}{\partial z}\right]\end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$

where u^* and v^* are the components of the deviation of the mean wind from the mean geostrophic wind, l is the Coriolis parameter, and $K(z, t)$ is the kinematic eddy viscosity coefficient. In order to express these equations in a form that is convenient for solution, let

$$W = u^* + iv^* \qquad (4)$$

Let the complex number θ be defined by

$$\theta(z, t) = e^{ilt}\, W(z, t) \qquad (5)$$

Then the differential Eq. (3) may be replaced by

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(z, t)\frac{\partial \theta}{\partial z}\right] \qquad (6)$$

The simplest case of diurnal variations occurs when $K(z, t)$, as given by

$$K(z, t) = k(1 + \eta \cos \omega t) \qquad (7)$$

in which η is a number between 0 and 1, ω is the angular velocity of the earth, k is a constant and t is measured from the time of maximum eddy viscosity. To find the solution for this case it is convenient to introduce the variables

$$\tau = t + \frac{\eta}{\omega}\sin \omega t \qquad (8)$$

and

$$\zeta = (l/2\pi^2 k)^{\frac{1}{2}}\, z \qquad (9)$$

Heights expressed by ζ are fractions of the gradient-wind height appropriate to k. For this case the differential equation reduces to

$$\frac{\partial \theta}{\partial \tau} = \frac{l}{2\pi^2}\frac{\partial^2 \theta}{\partial \zeta^2} \qquad (10)$$

It has been shown by Wagner (1936) that the daytime maximum of wind speed that occurs below a height of a few tens of metres is caused by the increase of eddy viscosity with height. The distribution of eddy viscosity assumed herein and the wind distribution which results from it can hardly be expected to apply to these layers. Therefore the lower boundary ($\zeta = 0$) is put near the top of this layer. It will be assumed that the variation of wind vanishes at this level and that it either vanishes or remains finite at infinity. After the resulting solution is found, the error resulting from actual variations at the lower boundary will be calculated and discussed. Accordingly, the following boundary conditions are adopted :

$$\left.\begin{aligned} &W(\tau, 0) = \theta(\tau, 0)\, e^{-ilt} = W_0 \\ &\lim_{\zeta \to \infty} \theta(\zeta, \tau) = \text{finite number} \end{aligned}\right\} \qquad (11)$$

in which W_0 is a constant.

Initial conditions need not be specified because only periodic solutions of the form

$$\theta(\tau, \zeta) = W_0\, e^{il\tau} \sum_{-\infty}^{+\infty} b_n(\zeta)\, e^{in\omega\tau} \qquad (12)$$

are desired. To find what functions must be used for $b_n(\zeta)$, one must substitute the periodic solution into the differential equation. After equating the coefficients of the various independent time functions, one obtains a set of differential equations

$$\partial^2 b_n/\partial \zeta^2 = 2i\pi^2(1 + n\omega/l)\, b_n \qquad (13)$$

each of which has an exponential solution. For n equal to 0, 1 and -1 respectively, the solutions which satisfy the boundary condition at infinity are

$$\left.\begin{aligned} b_0(\zeta) &= b_0(0)\exp\left[-(1+i)\,\pi\zeta\right] \\ b_1(\zeta) &= b_1(0)\exp\left[-(1+i)(1+\omega/l)^{\frac{1}{2}}\,\pi\zeta\right] \\ b_{-1}(\zeta) &= b_{-1}(0)\exp\left[-(1+i)\left|1-\omega/l\right|^{\frac{1}{2}}\,\pi\zeta\right] \end{aligned}\right\} \quad . \quad . \quad (14)$$

In order to get a first approximation to the complete solution, it will be assumed that the coefficients of the higher-order terms vanish, and that the powers of η higher than the first can be neglected. In order to satisfy the boundary condition at $\zeta = 0$, the following equation must be valid.

$$e^{il(\tau-t)}\left[b_0(0) + b_1(0)\,e^{i\omega\tau} + b_{-1}(0)\,e^{-i\omega\tau}\right] = 1 \quad . \quad . \quad (15)$$

The inverse of Eq. (8) may be expressed as a power series in η as follows :

$$\left.\begin{aligned} \omega t &= \omega\tau - \eta\sin\omega\tau + \eta^2\cos\omega\tau\sin\omega\tau \\ &\quad - \eta^3(\cos^2\omega\tau\sin\omega\tau - \tfrac{1}{2}\sin^3\omega\tau) + \ldots \end{aligned}\right\} \quad . \quad . \quad (16)$$

Since powers of η higher than the first are neglected, one obtains the expression

$$e^{il(\tau-t)} \simeq 1 + \frac{il\eta}{\omega}\sin\omega\tau = 1 + \frac{\eta l}{2\omega}\left(e^{i\omega\tau} - e^{-i\omega\tau}\right) \quad . \quad . \quad (17)$$

Substitution of this equation into (15) and requiring that the coefficient of each independent function of time be zero leads to the result :

$$b_0(0) = 1; \quad b_1(0) = -l\eta/2\omega; \quad b_{-1}(0) = +l\eta/2\omega \quad . \quad . \quad (18)$$

The final solution for the case $\phi \geqslant 30°$, is

$$\left.\begin{aligned} W(\zeta,\tau) &\simeq W_0\exp\left[-(1+i)\,\pi\zeta\right] \\ &+ (W_0\,\eta l/2\omega)\left\{\exp\left[-(1+i)\,\pi\zeta\right] - \exp\left[-(1+i)(1+\omega/l)^{\frac{1}{2}}\,\pi\zeta\right]\right\}e^{i\omega\tau} \\ &+ (W_0\,\eta l/2\omega)\left\{\exp\left[-(1+i)(1-\omega/l)^{\frac{1}{2}}\,\pi\zeta\right] - \exp\left[-(1+i)\,\pi\zeta\right]\right\}e^{-i\omega\tau} \end{aligned}\right\} \quad . \quad (19)$$

An example of this solution is plotted in Fig. 2 for latitude 37°N (the approximate latitude of Oklahoma City and Wichita). The wind at $\zeta = 0$, as computed from the theory described by Haurwitz (1941 p. 201), has been added in order to show the relation of the wind variations to the actual wind.

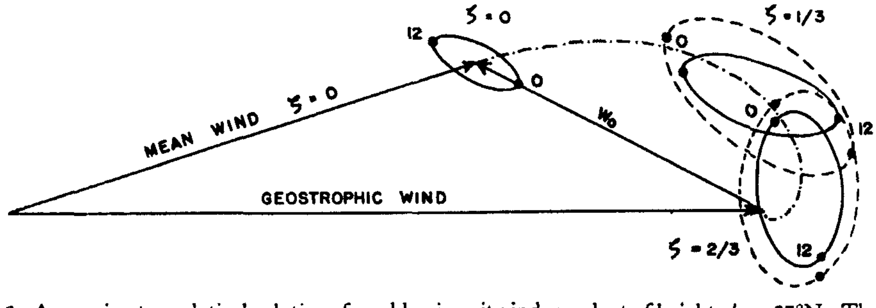

Figure 2. Approximate analytical solutions for eddy viscosity independent of height, $\phi = 37°$N. The velocity scale is arbitrary. Time is measured in hours after maximum eddy viscosity. Variations shown are for unit value of η and should be reduced in proportion to the value of η assumed. Dashed curve results from neglecting wind variations at $\zeta = 0$; solid curve takes such variations into account.

When η is zero there is no diurnal variation of eddy viscosity. According to Eq. (19) the wind vector is then distributed with height according to the Ekman spiral. When η is different from zero the wind vector executes an elliptical time variation, centred in the first approximation upon the equilibrium wind velocity. The amplitude of the time variation increases from zero at $\zeta = 0$ to a maximum and then decreases to zero again at great elevations; at any level, this amplitude is proportional, in the first approximation, to η.

Fig. 1 shows, as might have been expected from the work of Köppen (1921), that there is no level above the surface layer where the wind-vector variation actually vanishes. The error resulting from the erroneous boundary condition can be calculated in this case and will serve a useful purpose for estimating the error of more difficult subsequent calculations. Let the following boundary condition replace that in Eq. (11):

$$W(\tau, 0) = (1 + A\eta\, e^{i\omega\tau} + B\eta\, e^{-i\omega\tau})\, W_0 \qquad (11a)$$

in which A and B are complex constants to be chosen so as to represent any desired elliptical variation at the lower boundary. Then the following correction should be added to the solution already found above.

$$\left.\begin{aligned} \Delta W = A\eta\, W_0 \exp\left[-(1+i)(1+\omega/l)^{\frac{1}{2}}\,\pi\zeta\right] e^{i\omega\tau} \\ + B\eta\, W_0 \exp\left[-(1+i)\left|1-\omega/l\right|^{\frac{1}{2}}\,\pi\zeta\right] e^{-i\omega\tau} \end{aligned}\right\} \qquad (20)$$

This correction is a maximum at the lower boundary and decreases exponentially to zero at infinity, except at 30° latitude where its magnitude approaches a constant. An example showing the nature of this correction for a surface variation like that at the anemometer level in Fig. 1 is shown in Fig. 2. The correction has the effect of decreasing the amplitude and of rotating each of the ellipses slightly in a cyclonic sense.

Comparison of Figs. 1 and 2 indicates that η is probably not very small. To obtain better approximations when η is large, one must retain higher-order terms in Eq. (12) and correspondingly higher powers of η in Eq. (16). In this study the approximation was carried out to the third order, but because of its complexity the result is not repeated here. A plot of the third approximation of a representative case is shown in Fig. 3. These higher-order approximations result in considerable displacements of the points for large η, but the orientation and phase of the variations remain like those of the first approximation.

A fundamental discrepancy exists between the theoretical and observed distributions. The major axes of the elliptical variation in the theoretical case are inclined at a large angle to the wind direction, especially in the middle and upper portion of the boundary layer, whereas the actual ellipses have axes which are in all cases more nearly parallel to the direction of the wind at the same level. Thus the assumed distribution of eddy viscosity appears to be incapable of explaining the observed variations of the wind. Such an explanation requires a technique which allows other distributions of eddy viscosity with height. Also, in order to explain the magnitude of the variations which are observed, it is important to utilize a method which can deal with larger values of η.

3. Analogue method of solution

Even if some method can be found to remove the severe limitations of the analytical method of solution, there remains the difficulty that the computation and plotting of results are extremely laborious. A method is needed for finding quick solutions for any assumed distribution of eddy viscosity so that many different situations can be tried in a reasonable time. Such a method need not be exact. If the correct distribution of eddy viscosity can be indicated by an approximate result, it may later be feasible to verify the result by a more laborious method.

The Analogue Computer Laboratory of the Electrical Engineering Department of New York University has been of great assistance during these studies. The principal equipment of this laboratory is an electronic differential analyser, Model 16-31R, and two cathode-ray oscilloscopes which are connected directly to the analyser, one for visual observation, and the other for simultaneous photographic recording. The computer has a capacity of 20 summation units, 8 of which may also be used as integrators. It also has 4 multipliers, 32 coefficient potentiometers (which allow multiplication by a constant), and 6 output recorders.

In order to apply this type of analogue equipment to the problem, it is necessary to transform the two simultaneous partial differential Eq. (3) in two independent variables into a larger set of simultaneous ordinary differential equations in a single independent variable. The general method by which this is done is to replace the continuously distributed dependent variables u^* and v^* by a set of variable values at discrete levels. The differential equations for these discrete values are written by replacing the partial derivatives with respect to height by finite-difference approximations.

Let the index j refer to one particular level, $(j+1)$ and $(j-1)$ to the next higher and next lower level, respectively, and let a be the distance between levels. Then the simplest transformations of Eq. (3) are the following set of equations :

$$\left.\begin{aligned} \frac{du_j^*}{dt} &= lv_j^* + \left(\frac{K_j}{a^2}+\frac{K_j'}{2a}\right)u_{j+1}^* - \frac{2K_j}{a^2}u_j^* + \left(\frac{K_j}{a^2}-\frac{K_j'}{2a}\right)u_{j-1}^* \\ \frac{dv_j^*}{dt} &= -lu_j^* + \left(\frac{K_j}{a^2}+\frac{K_j'}{2a}\right)v_{j+1}^* - \frac{2K_j}{a^2}v_j^* + \left(\frac{K_j}{a^2}-\frac{K_j'}{2a}\right)v_{j-1}^* \end{aligned}\right\} . \tag{21}$$

In these equations, K_j is the kinematic-exchange coefficient at the jth level, and K_j' is its vertical derivative at the same level. For purposes of obtaining a solution, these quantities are specified functions of time and are either generated by the computer, or, in some cases, fed into it from an external voltage source. The experimental procedure is to vary the K_j s and K_j' s to fit different trial patterns and to observe the solutions which result.

The number of levels which may be included in such a scheme is limited by the number of integrators in the computer. In this case, two integrators are required to generate a cosine function, leaving six to solve for dependent variables. Since the equations are of first order in time, six dependent variables can be accommodated. The model which is adopted consists of five levels numbered from 0 at the bottom to 4 at the top. At the bottom level the deviation from geostrophic wind is assumed to be constant, and the values

$$u_0^* = -100 \text{ and } v_0^* = 0$$

expressed in arbitrary units are adopted. At the top level, both components of the deviation are assumed to vanish. These conditions are similar to those which were imposed in the analytical problem and are acceptable if the height scaling is chosen so as to place the top level high enough to neglect the diurnal variation and if an allowance is made for non-negligible surface variations. When applied to the three intervening levels, the set of Eq. (21) gives six simultaneous ordinary differential equations to be solved for the three pairs of values of the wind components.

The choice of a suitable value for the height unit a is a compromise. The smaller a is allowed to be, the better is the finite-difference approximation of the vertical derivatives; however, a must be large enough so that the error involved in the upper-boundary condition is not objectionable. Trial comparisons between finite-difference calculations and analytical calculations indicate that a is best chosen so as to place the gradient-wind level appropriate to k at the third level. This scaling is accomplished by the relation :

$$a^2 = \bar{k}/0{\cdot}456\, l$$

$\bar{k}$ being the average value of the kinematic eddy viscosity at the three levels.

TABLE 1. COMPARISON OF COMPUTER AND ALGEBRAIC SOLUTIONS OF THE DISCRETE-LEVEL MODEL AND THE EXACT SOLUTION (EKMAN SPIRAL) WHEN THE GRADIENT-WIND LEVEL IS LOCATED AT $z = 3a$

	a		$2a$		$3a$		$4a$	
	u^*	v^*	u^*	v^*	u^*	v^*	u^*	v^*
(1) Algebraic	− 19·4	+ 26·4	+ 3·3	+ 10·4	+ 3·3	+ 1·5	0·0	0·0
(2) Computer	− 19·0	+ 27·5	+ 3·6	+ 11·0	+ 3·5	+ 1·5	–	–
(3) Theoretical	− 17·5	+ 30·3	+ 6·2	+ 10·7	+ 4·3	0·0	+ 0·8	− 1·3

The simplest estimate of the errors of this method is furnished by a comparison of three solutions of the wind distribution for constant eddy viscosity : (1) an exact algebraic solution of the finite-difference equations, (2) a computer solution of the same, and (3) an exact calculation of the Ekman-spiral solution for the same conditions. Comparison of (1) and (3) in Table 1 indicates that the use of the discrete-level model involves an error of about 2 per cent of the wind deviation at the surface. The error which the computer makes in solving the finite-difference equations, which is shown by a comparison of solutions (1) and (2), is less than 1 per cent of the surface deviation. A more realistic comparison can be made between the results of the analytical solution for identical parameters (see Fig. 3). The two solutions are displaced from each other by a nearly constant amount, and this displacement is about equal to that between the algebraic and theoretical solutions in Table 1. Its cause is the crudeness of the discrete-level model. The remaining errors are small and do not affect the amplitude, shape, or phase significantly, as may be judged by referring to Fig. 3, in which the computer solution has been shifted a constant amount

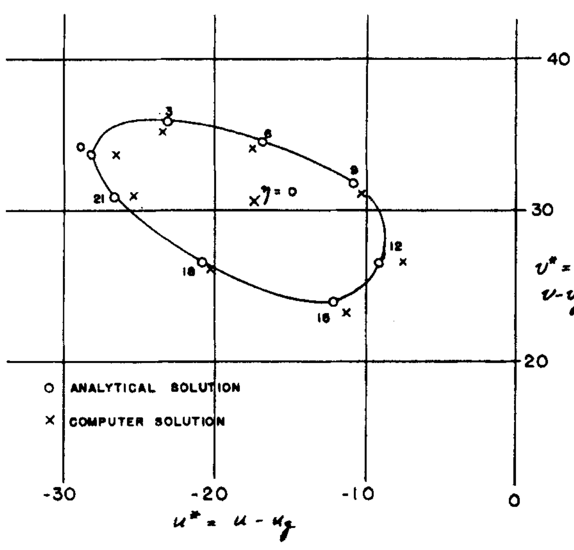

Figure 3. Comparison of the analytical solution (third approximation) and the modified computer solution for the case $K = k\,(1 + \eta \cos \omega t)$, $\zeta = \frac{1}{3}$, $\phi = 45°$, and $\eta = \frac{1}{3}$. The computer solution has been given a constant displacement so as to make the solutions agree when $\eta = 0$.

in order to bring the equilibrium solutions into agreement. The analogue method thus appears to be a very adequate tool for exploring the effect of various distributions of eddy viscosity in time and height upon the nature of the resulting variations of the wind.

Three varieties of functions were used for $K(z, t)$ during the course of the experiments, each of which was associated with a different programme for the computer. These functions are

$$\left.\begin{aligned} K_1(z, t) &= k(z)(1 + \eta \cos \omega t) \quad (\eta = \text{const}) \\ K_2(z, t) &= k(z) + L(z) \cos \omega t \quad L(z) \leqslant k(z) \\ K_3(z, t) &= k(z) + L(z) f(t) \end{aligned}\right\} \quad . \quad . \quad (22)$$

in which $k(z)$, $L(z)$ and $f(t)$ are arbitrary functions. The function $K_1(z, t)$ is a special case of the functions $K_2(z, t)$ and $K_3(z, t)$ but is treated separately because it uses a simpler programme. The functions $K_2(z, t)$ could not be completely programmed because of limitation in the number of summation units; the programme which was adopted allows L to be given different values at each level, but requires $\partial L/\partial z$ at each level to be neglected. That the programmes for $K_2(z, t)$ and $K_3(z, t)$ nevertheless produce solutions of value is indicated by the fact that a repetition of some solutions of the K_1 programme using the K_2 programme gave similar though not identical results despite the fact that $\partial L/\partial z$ was actually different from zero.

4. Analogue studies of wind variations

Once the analogue computer has been programmed for the form of $K(z, t)$, it is only necessary to adjust the potentiometers for appropriate values of k, $\partial k/\partial z$, and either η or L at each of the levels. The machine is then allowed to run from arbitrary initial conditions through several cycles until inspection of the recorder indicates that the solution is completely periodic, usually within one minute. In a sense the computer functions as a model of the atmosphere which is amenable to controlled experiments. More than 100 experiments representing different sets of distributions of exchange coefficient in time and height were conducted. Only a few of the more interesting ones will be discussed here.

The graphs which are given in Figs. 4-10 represent in each case the deviations from the geostrophic wind in arbitrary units. The value of this deviation vector which has been assumed at the lowest level as boundary condition is also shown. In attempting to compare the computed wind variations with the observed values which are shown in Fig. 1, one should bear in mind the relation of the vector W_0 to the surface wind and the geostrophic wind. The conditions which have been assumed about the distribution of eddy viscosity during the conduct of the experiments do not specify this relation between W_0 and the geostrophic wind; it is determined instead by the distribution of eddy viscosity in the surface layers. Comparison of the ensuing figures with Fig. 1 therefore requires some inference about the wind at $z = 0$. Perhaps the vectors shown in Fig. 2 will serve this purpose.

Whenever an attempt is made to compare computer solutions with the variations of the wind shown in Fig. 1, it should be kept in mind that the times shown in Figs. 4 to 10 are hours after the time of maximum eddy viscosity, which may be presumed to occur at about the time of maximum surface temperature. In most cases the time 0 hr in the computer solution may be compared with the time 1500 CST in Fig. 1.

The distributions of eddy viscosity which were used in the experiments which are about to be described are summarized in Table 2.

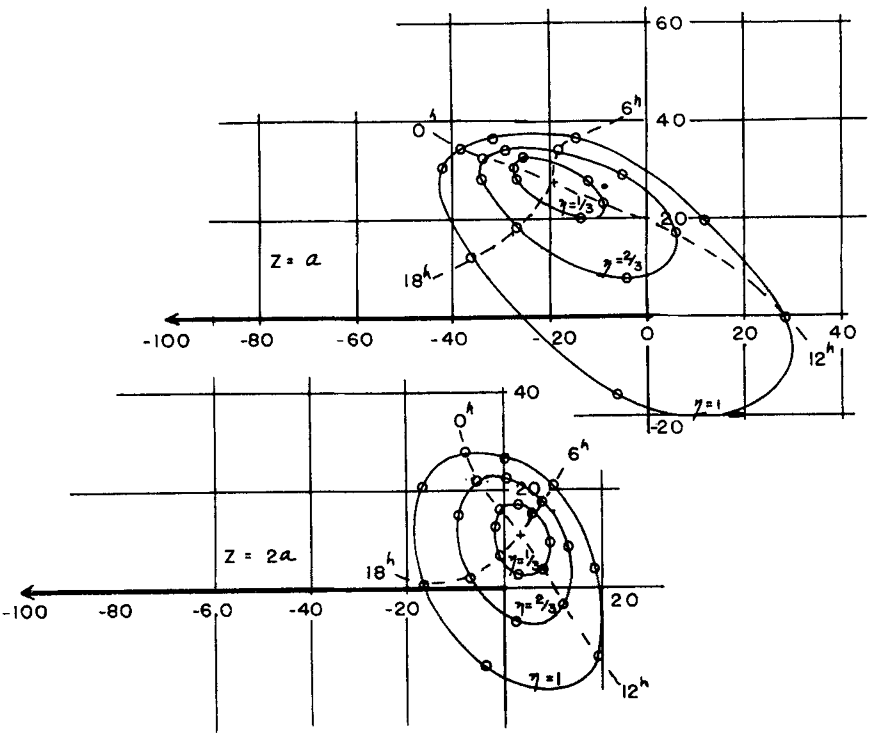

Figure 4. Wind variations at $z = a$ and $z = 2a$ for various values of η when $k\,(z) = 0{\cdot}456\ a^2\,l$ at latitude 45°.

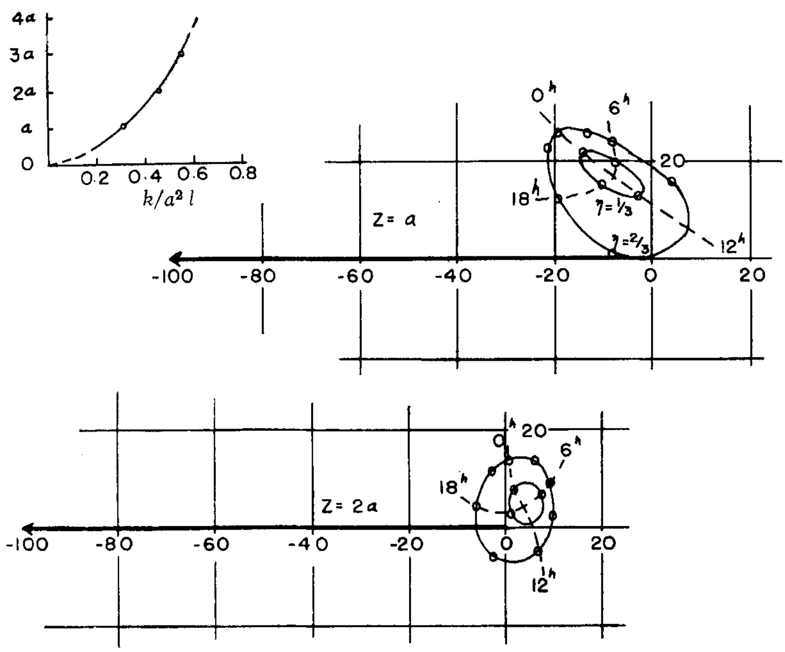

Figure 5. Wind variations at 45°N for values of $\eta = \frac{1}{3}$ (inner) and $\frac{2}{3}$ (outer) with mean eddy viscosity and amplitude of eddy viscosity both increasing with height as shown in the inset.

TABLE 2. Parameters used for experiments described in this report. (In Fig. 8 the values given for L/k were not used).

Figure	$k_1/a^2 l$	$k_2/a^2 l$	$k_3/a^2 l$	$k_1'/a l$	$k_2'/a l$	$k_3'/a l$	L_1/k_1	L_2/k_2	L_3/k_3
4	0·456	0·456	0·456	0·000	0·000	0·000	–	–	–
5	0·322	0·456	0·558	0·162	0·114	0·094	–	–	–
6	0·370	0·400	0·450	0·060	0·030	0·090	0·230	0·160	0·050
7, 8, 10	1·000	0·400	0·100	– 0·500	– 0·225	– 0·075	0·800	0·800	0·800

A comparative study of Figs. 4 to 7 reveals a number of significant facts :

(1) The relation between the amplitude of the eddy viscosity and the amplitude of the wind variations is non-linear; large values of η bring disproportionately larger amplitudes than smaller values.

(2) With similar amplitudes of eddy viscosity, the largest amplitude oscillations of wind occur when both the mean value and the amplitude of the eddy viscosity decrease rapidly with height at and above the first level (such a distribution implies a lower-level maximum, since the eddy viscosity increases upward at the surface).

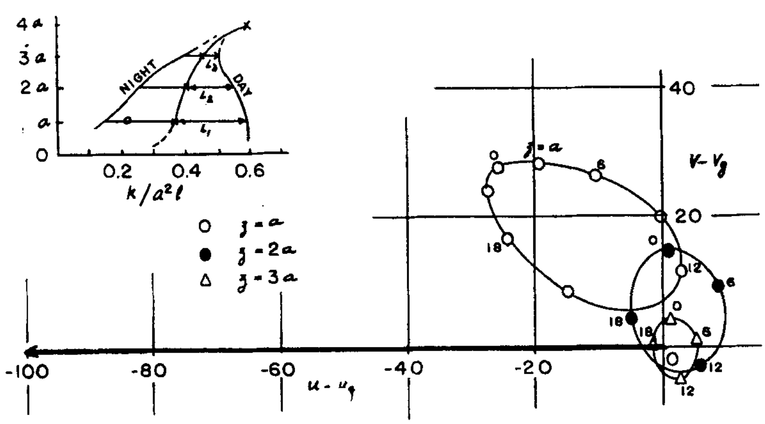

Figure 6. Wind variations at 45°N when the mean eddy viscosity increases with height and the amplitude of the variation of eddy viscosity decreases with height as shown in the inset.

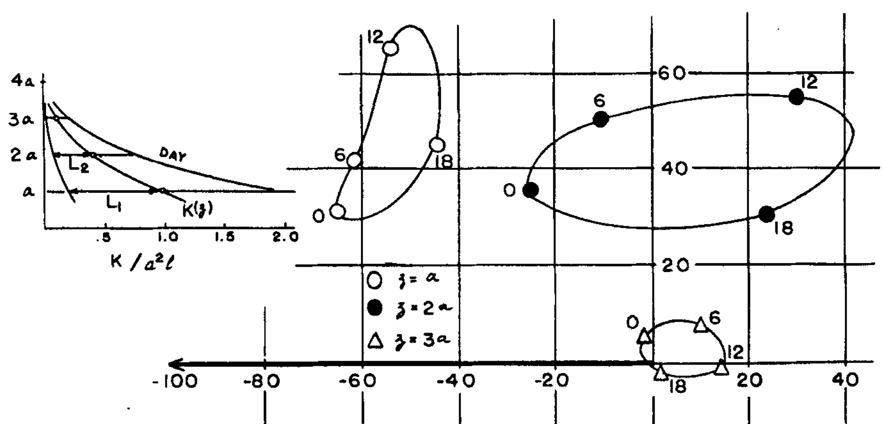

Figure 7. Wind variations at 45°N when both the mean eddy viscosity and the amplitude of the variation of eddy viscosity decrease with height at and above the first level, as shown in the inset diagram, using the computer programme for $K(z, t) = k(z) + L(z) \cos \omega t$.

(3) The orientation and phase of the elliptical variations are most nearly like those which are observed when both the mean eddy viscosity and *the amplitude of its variation* decrease rapidly with elevation.

(4) The largest of the computed amplitudes is not more than half as large as that of the observed oscillations in Fig. 1. It is recalled that if allowance is to be made for more realistic surface boundary conditions the computed amplitude of the oscillations must be even smaller. It thus appears that an even larger diurnal range of eddy viscosity than that used in Fig. 7 must actually occur.

(5) There is an inverse relation between the magnitude of the wind and the eddy viscosity. For example, the wind difference between the surface and the first level is smaller in Fig. 7, where the eddy viscosity is large, than in Fig. 5 where it is small. Between the second and third level in the same two figures the situation is reversed, with, however, the same relation between wind shear and eddy viscosity.

The amplitude, orientation and phase of the observed variations are best accounted for in Fig. 7 in which both the mean value and amplitude of the eddy viscosity decrease with height. Since the programme used in obtaining this result necessitated a neglect of the vertical derivative of the amplitude of eddy viscosity, another calculation, using a similar distribution which is described by giving η a constant value, is instructive. Such a case is shown in Fig. 8.

The mean position of the wind vector agrees very closely with that of Fig. 7, but the orientation of the ellipses is different, agreeing more nearly with that of the other experiments. The rather different orientation in Fig. 7 may therefore be due to neglect of the terms involving the vertical derivative of the amplitude, since the earlier examples did not indicate much change of orientation with variations of η. Although the results shown in Fig. 8 do not agree as well with the observed wind distribution as the results in Fig. 7, they are nevertheless much better in agreement than any of the others. For example, Fig. 8 shows also the variations at corresponding levels when $k(z)$ is constant with height and η is also 0·4. The result for k decreasing gives more nearly the observed orientation and a slightly better phase at both levels. The larger variation of amplitude of the wind at the second level shows better promise of being able, with larger values of η (which unfortunately were not attainable without overloading components of the computer), to explain the enormous amplitude of the observed wind variations. It may be remarked that because of its interest the experiment shown in Fig. 8 has since been recomputed using an analytical method which is being developed by K. Ooyama. The orientation, phase and amplitude agree very closely, but there is some discrepancy of the mean location of the ellipses, such as has been noted in the earlier comparisons between computer and analytical solutions.

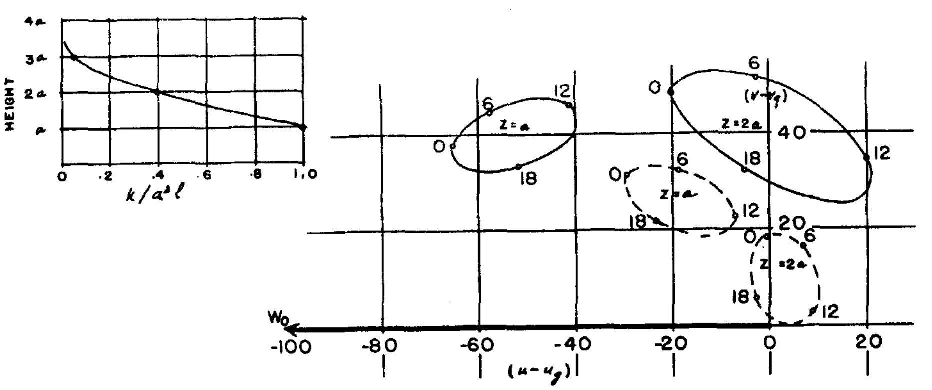

Figure 8. Solutions for a case when $k(z)$ decreases rapidly with height (as shown in the inset); $\eta = 0{\cdot}4$; $\phi = 45°$ (solid lines). Dashed curves are solutions for $k(z)$ constant, $\eta = 0{\cdot}4$, and $\phi = 45°$.

The behaviour of the wind variation with change of latitude is illustrated by an experiment shown in Fig. 9. The eddy-viscosity distribution is the same as that shown in Fig. 7 except that the range of the variation has been reduced slightly to prevent overloading of the computer at low latitudes. The experiment shows a typical decrease of of amplitude with increasing latitude between 30° and 55°N. The larger amplitude of 30° could be expected from the fact that the free period of inertia oscillations becomes equal at that latitude to the period of the forcing function. The tendency for resonance is actually not excessive because a large amount of viscous damping is characteristic of these layers.

It is of interest to investigate the sensitivity of the solutions to a change of the time function used to describe the diurnal variation of eddy viscosity. The inset of Fig. 10 shows one of several selections of function $f(t)$ for use in the third equation of (22); it is characterized by a relatively brief maximum and long minimum and in this respect approximates observed low-level variations of exchange coefficient more closely than a simple cosine curve. It was obtained by a single integration of a manually-generated

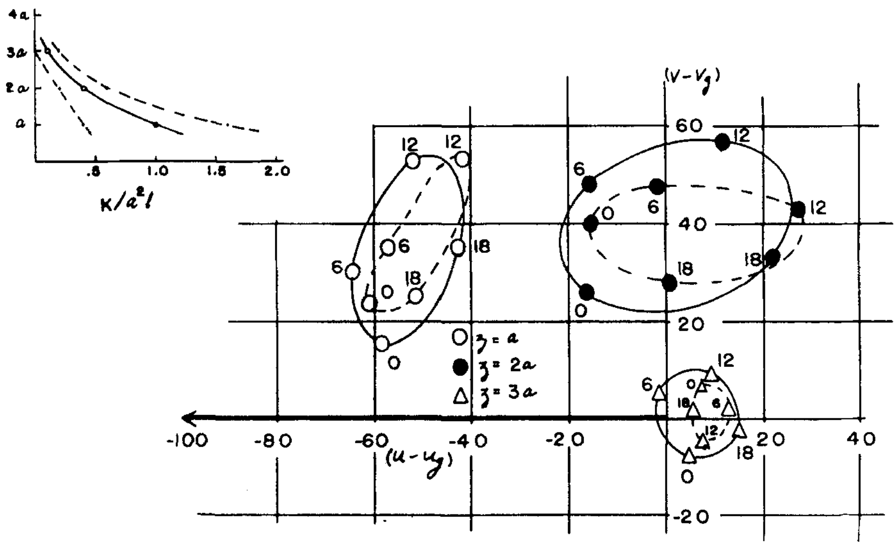

Figure 9. Wind variations at 30°N (solid curves) and at 55°N (dashed) when both the mean eddy viscosity and amplitude of the eddy viscosity decrease with elevation as shown in the inset.

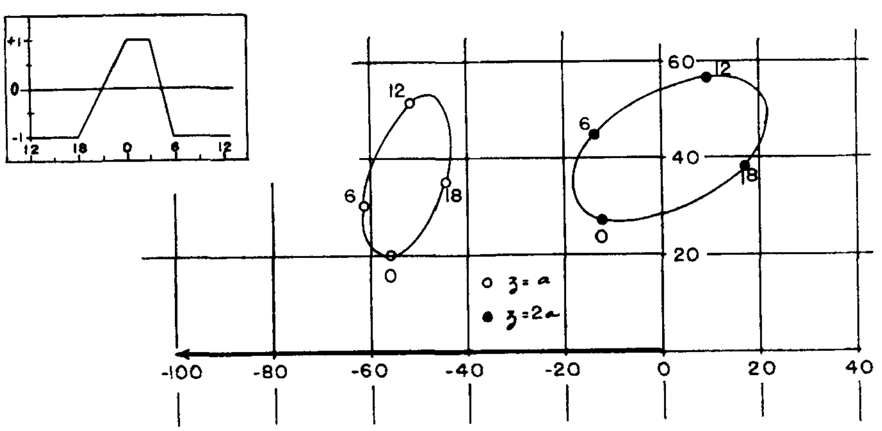

Figure 10. Wind variations at 45°N with a non-sinusoidal variation of eddy viscosity. Inset shows the function $f(t)$ in Eq. (22). The vertical distributions of the mean eddy viscosity and the amplitude are the same as in Fig. 7.

square wave. Except for this substitution of input the programme and constants used were the same as for the case shown in Fig. 7. The resulting wind variations are shown in Fig. 10. Some differences ascribed to the alteration are discernible, but the general conclusion reached from this and several other experiments is that the substitution of any periodic function bearing a vague similarity to the cosine function does not alter the wind variation in any important respect.

Natural variations are subject to disturbances due to non-periodic variations of stability, latitude, cloudiness and physical properties of underlying surface. A rapid change of physical conditions is followed by a period of transition from one kind of wind variation to the periodic variation appropriate to the altered conditions. If the transition period is very long, the form of the observed quasi-periodic variation may not be related so much to the existing physical condition as it is to some combination of conditions during the past history of the air mass. It is a simple matter to observe this period on the computer by starting the solution at an initial value which is removed from the final solution and observing how many oscillations occur before the deviation from the completely periodic solution becomes imperceptible. In most cases not involving extreme initial conditions the transient period lasts for only $\frac{1}{2}$ to $\frac{3}{4}$ period. Therefore, the variations which occur on a given day are not likely to display any perceptible effect of conditions existing during the previous day. An air mass which moves inland from the sea where there was no diurnal wind variation at all may acquire a fully developed frictional-type diurnal wind variation in as little as 12 hours.

5. Summary and conclusions

None of the more than 100 experiments which were carried out on the computer gives a satisfactory explanation of the orientation and amplitude of the observed diurnal wind-structure variation at Oklahoma City and Wichita. The best agreement occurs when both the mean value of the eddy viscosity and the amplitude of the variation decrease rapidly with height above the surface layers. The distribution of eddy viscosity with height probably depends upon the temperature structure of the air mass; therefore the distribution of eddy viscosity which appears to have occurred in connection with the wind variations shown in Fig. 1 is not necessarily typical of that which would be found in other air masses. The prevailing wind direction in this case was southerly. Brown (1955) found that diurnal variations of the wind are much smaller in amplitude when the prevailing wind direction is from the north, a difference which may be the result of a different temperature structure. The distribution which is indicated in this study is similar to that found by Mildner (1932) from an examination of the average of a large number of pilot-balloon observations made in Leipzig. He found that the dynamic-exchange coefficient reaches a maximum value of 500 g cm^{-1} sec^{-1} at the height 240 m and decreases rapidly to a value of 70 g cm^{-1} sec^{-1} at 500 m. A re-examination of the same profile by Lettau (1950) has yielded a qualitatively similar distribution of eddy viscosity with, however, a smaller value of the maximum and a smaller rate of decrease with height.

Even when η is 1, so that amplitude of eddy viscosity is the maximum, the amplitude of the computed wind variation is too small to account for the observed amplitude so long as the eddy viscosity increases with height or remains constant. When the eddy viscosity decreases rapidly with height it may be possible to account for the large observed variation if η exceeds the value 0·8. Thus it appears that the eddy viscosity during the day reaches a value which is considerably larger than ten times the night-time value. The large range thus indicated is in agreement with the range of the exchange coefficient for heat transfer which has been observed near the ground (Staley 1956). The smallness of the night-time values of eddy viscosity helps to account for the large super-geostrophic winds observed in connection with the boundary layer wind maximum which occurs regularly in the mid-western United States (Blackadar 1957).

The mean wind direction at 2,500 ft above the surface in the mid-western United States in summer is from the south-west. During the daytime the wind is more retarded in the south than it is in the north (see Fig. 9). Therefore, with a tendency for decreasing wind downstream, a field of divergence tends to become established. At night (between 12 and 18 hr after the time of maximum eddy viscosity in Fig. 9) the situation is reversed and convergence tends to be set up. Thus diurnal changes in the field of convergence occur which are qualitatively similar to that observed by Bleeker and Andre (1951). The magnitude of the latitude effect by itself is at least an order of magnitude too small. It is probable, however, that differences in the nature of the underlying surface (as for instance must be present between the area around the Great Lakes and the semi-arid region of the southwest) may be responsible for large enough differences of amplitude to account for the observed fields of convergence.

Inasmuch as theoretical methods have not given a completely satisfactory account of the observed wind variations, further observations and analysis of the wind structure and its relation to the geostrophic wind in this area will be of great interest.

6. Acknowledgments

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