# Programming Languages: Functional Programming Practicals 7. Types and Logic

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1. Prove the following propositions:

   (a) P → Q → P.

   (b) (P → Q → R) → Q → P → R.

   (c) (P → Q) → (Q → R) → P → R.

   (d) P → (P → Q) → (P → Q → R) → R.

   (e) (P → Q → R) → (P ∧ Q) → R.

   (f) (P ∧ Q) → ((P ∨ Q) → R) → R).

   (g) (P → Q → R) → (P → Q) → P → R.

2. Reduce the following expressions to normal form, if possible.

   (a) $(\lambda x . x + x)\ 3$.

   (b) $(\lambda f\ x . f\ x\ x)\ (\lambda y\ z . y + z)\ 3$.

   (c) $(\lambda x . x\ x)\ (\lambda x . x)$.

   (d) $(\lambda x . x\ x)\ (\lambda x . x\ x)$.

   (e) $(\lambda x . f\ (x\ x))\ (\lambda x . f\ (x\ x))$.

3. Write down the type derivation trees of the following expressions, if possible.

   (a) $(\lambda x\ y . x)$.

   (b) $(\lambda p . (snd\ p, fst\ p))$.

   (c) $(\lambda f\ g\ x . f\ x\ (g\ x))$.

   (d) $(\lambda x . x\ x)\ (\lambda x . x\ x)$.

4. Given the following types, construct (simply typed) lambda expressions having the types.

   (a) $(P \rightarrow Q \rightarrow R) \rightarrow Q \rightarrow P \rightarrow R$.

   (b) $(P \wedge Q) \rightarrow ((P \vee Q) \rightarrow R) \rightarrow R$.